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**ELLIPTIC FLOW FROM THERMAL AND KLN INITIAL CONDITIONS**

Based on collaboration with:  
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In this talk:

- Very short introduction to heavy ion collisions
- Transport theory and heavy ion collisions
- Thermalization
- Elliptic flow computation
- Conclusions and Outlook
**QGP in Heavy Ion Collisions**

**A, B:** Cu, Au (RHIC@BNL) 
Pb (LHC@CERN).

\[ \sqrt{s} \text{ up to } 200 \times A \text{ GeV}, \quad \text{RHIC} \]
\[ \sqrt{s} \text{ up to } 2.76 \times A \text{ TeV}, \quad \text{LHC} \]

**FIREBALL:**
Hot and dense expanding parton mixture:
QUARK-GLUON-PLASMA (QGP)
T about \(10^{12}\) K,
t about \(10^{-23}\) seconds
Initial temperature much larger than QCD critical temperature: 
*Description in terms of partons is appropriate.*
Elliptic flow

Immediately after the collision, pressure gradient along \( X \) is larger than that along \( Y \). As a consequence, the medium expands preferentially along the short axis of the ellipse, creating a flow.

Elliptic flow: leading contribution to anisotropy in momentum space

Particle multiplicity in momentum space

\[
\frac{d^3 N}{d\eta dy_{pT} dp_{T} d\phi} = \frac{1}{2\pi} \frac{d^2 N}{dy_{pT} dp_{T}} \left[ 1 + 2v_2(y, p_{T}) \cos 2\phi \right]
\]

\[v_2 = \langle \frac{p_{x}^2 - p_{y}^2}{p_{x}^2 + p_{y}^2} \rangle = \langle \frac{p_{x}^2}{p_{T}^2} - \frac{p_{y}^2}{p_{T}^2} \rangle\]
Elliptic flow

Transfer of anisotropy

J. Y. Ollitraut, PRD46 (1992)

M. R. et al., in preparation
Boltzmann equation and QGP

In order to simulate the temporal evolution of the fireball we solve the *Boltzmann equation* for the parton distribution function $f$:

$$\left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) f(x, p, t) = C[f]$$

**Drift term**: change of $f$ due to particles flowing into and out of the phase space volume centered at $(x,p)$.

**Collision integral**: change of $f$ due to collision processes in the phase space volume centered at $(x,p)$.

For the case of $12 \rightarrow 1'2'$ processes:

$$C[f] = \frac{1}{2} \int dp_2 \int dp'_1 \int dp'_2 \ w(12 \rightarrow 1'2') \times [f(x, p'_1, t)f(x, p'_2, t) - f(x, p_1, t)f(x, p_2, t)]$$

L. Boltzmann, 1872
eta/s: hydro “by” transport

We use **Boltzmann equation** to simulate a fluid at **fixed eta/s**. **Total Cross section** is computed in each **configuration space cell** according to **Chapman-Enskog equation** to give the wished value of eta/s.

\[
\frac{\eta}{s} = \frac{\langle p \rangle}{g(m_D)\rho\sigma} \frac{1}{1}
\]

We use Boltzmann equation to simulate a fluid at fixed $\eta/s$. Total Cross section is computed in each configuration space cell according to Chapman-Enskog equation to give the wished value of $\eta/s$.

Collision integral is gauged in each cell to assure that the fluid dissipates according to the desired value of $\eta/s$. Microscopic details are not important: the specific microscopic process producing $\eta/s$ is not relevant, only macroscopic quantities are, in analogy with hydrodynamics.

We use *Boltzmann equation* to simulate a fluid at *fixed* *\( \eta/s \).* *Total Cross section* is *computed* in *each configuration space cell* according to *Chapman-Enskog equation* to give the *wished value of* *\( \eta/s \).*

(Collision integral is gauged in each cell to assure that the fluid dissipates according to the desired value of *\( \eta/s \).*

Microscopic details are not important: the specific microscopic process producing *\( \eta/s \)* is not relevant, only macroscopic quantities are, in analogy with hydrodynamics.

\[
\frac{\eta}{s} = \frac{\langle p \rangle}{g(m_D) \rho \sigma} \frac{1}{\langle \sigma \rangle}\]

**Transport**

Description in terms of parton distribution function

**Hydro**

Dynamical evolution governed by macroscopic quantities.
We use the Boltzmann equation to simulate a fluid at fixed \( \eta/s \). Total Cross section is computed in each configuration space cell according to the Chapman-Enskog equation to give the wished value of \( \eta/s \).

Collision integral is gauged in each cell to assure that the fluid dissipates according to the desired value of \( \eta/s \).

Microscopic details are not important: the specific microscopic process producing \( \eta/s \) is not relevant, only macroscopic quantities are, in analogy with hydrodynamics.

\[
\eta = \frac{\langle p \rangle}{s g(m_D) \rho \sigma} 1
\]

**Transport** ← “bridge” → **Hydro**

**Non perturbative description:** we never assume coupling is small.
eta/s: hydro “by” transport

We use *Boltzmann equation* to simulate a fluid at *fixed* eta/s. *Total Cross section* is computed in *each configuration space cell* according to *Chapman-Enskog equation* to give the *wished value of eta/s*.

There is agreement of hydro with transport also in the non dilute limit

Huovinen and Molnar, PRC79 (2009)
eta/s: hydro “by” transport

We use *Boltzmann equation* to simulate a fluid at *fixed eta/s*. *Total Cross section* is *computed* in *each configuration space cell* according to *Chapman-Enskog equation* to give the *wished value of eta/s*.

There is agreement of hydro with transport also in the non dilute limit.

Bhalerao et al., PLB627 (2005)
We use Boltzmann equation to simulate a fluid at fixed $\eta/s$. Total Cross section is computed in each configuration space cell according to Chapman-Enskog equation to give the wished value of $\eta/s$.

A smooth kinetic freezeout is implemented in order to gradually reduce the strength of the interactions as the temperature decreases below the critical temperature.
eta/s: hydro “by” transport

Temperature dependence of eta/s already appeared in the literature recently.

H. Niemi et al., PRC86 (2012), PRL106 (2011)
Shen and Heinz, PRC83 (2011)
Initial condition: Glasma

Reviews/Lectures
McLerran, 2011
Iancu, 2009
McLerran, 2009
Lappi, 2010
Gelis, 2010
Fukushima, 2011

Decay of flux tubes to parton liquid should occur on a timescale $1/Q_s$. 

\[ \mathcal{L} = -\frac{1}{2} \text{tr} F_{\mu\nu} F^{\mu\nu} + (J_1^\mu + J_2^\mu) A_\mu \]

Gluon dynamics: fast partons


McLerran and Venugopalan, PRD 49, 2233 (1994)
McLerran and Venugopalan, PRD 49, 3352 (1994)
Initial condition: fKLN

(f)KLN spectrum

\[
\frac{dN_g}{d^2x_T dy} \propto \int \frac{d^2p_T}{p_T^2} \int_0^{p_T} d^2k_T \alpha_s(Q^2) \times \phi_A \left( x_A, \frac{(p_T + k_T)^2}{4} ; x_T \right) \\
\times \phi_B \left( x_B, \frac{(p_T - k_T)^2}{4} ; x_T \right)
\]

Saturation effects built in the uGDFs.

For Pb-Pb collision average $Q_0$ can be larger [Lappi, EPJC71 (2011)]

Drescher and Nara, PRC75, 034905 (2007)
Hirano and Nara, PRC79, 064904 (2009)
Albacete and Dumitru, arXiv:1011.5161[hep-ph]
Few remarks on KLN

- fKLN is not glasma [Blaizot et al., NPA846 (2010)]
- It is not our purpose to insist on exact reproduction of experimental data [Gale et al., PRL110 (2013)]

Rather we want to check the role of the initial distribution in momentum space

- Hydro widely uses KLN, and we are interested to compare the two approaches

**Viscometer**: Schen et al., arXiv1308:2111
**Thermometer**: Schen et al., arXiv1308:2440
**Flow computations**: Ollitrault et al., arXiv1311:5339
Drescher and Nara, PRC 75 (2007)
Hirano and Nara, PRC 79 (2009)
Hirano and Nara, NPA 743 (2004)
Initial condition: Th-Glauber

(Almost) Geometrical description of the fireball:

Assuming a nucleon distribution in the parents nuclei (typically a Woods-Saxon), one counts how many particles from each nucleus are present in the overlap region; among them, the participants are the nucleons that effectively can have an interaction (in fact, the particles that are in the overlap region but do not interact, are not considered).

Our novelty:
For fKLN we consider the *initial spectrum given by the theory at small transverse momenta*.
Initial spectra

Assume different initial times

For fKLN we consider the initial spectrum given by the theory at small transverse momenta.
Final spectra of fKLN and Th-Glauber coincide
Thermalization

In less than 1 fm/c, in agreement with: Greiner et al., Nucl. Phys. A806, 287 (2008).

Not so surprising: Because eta/s is small, large cross sections naturally lead to fast thermalization. However, interesting: We have dynamics in the early stages of the simulation, which prepares the momentum distribution to build up the elliptic flow.
Thermalization in less than 1 fm/c, in agreement with: Greiner et al., Nucl. Phys. A806, 287 (2008).

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However, interesting:
We have dynamics in the early stages of the simulation, which prepares the momentum distribution to build up the elliptic flow.

\[
\sigma_{tot} = \frac{\langle p \rangle}{\rho g(a)} \frac{1}{\eta/s}
\]
Elliptic flow from Transport

Au-Au collision
RHIC energy

**Hydro-like**  **KLN initial.**

Larger eccentricity of KLN implies larger $v_2$

Results in fair agreement with hydro:
Song et al., PRC83 (2011)
Elliptic flow from Transport

Au-Au collision
RHIC energy

Hydro-like  KLN initial.

Thermalized spectra  Nonthermal spectra

M. R. et al., PLB727 (2013)
M. R. et al., in preparation
Elliptic flow shows that this quantity is very sensitive to the initial conditions:

1. Initial anisotropy (eccentricity)
2. Initial momentum distribution

Measurements of elliptic flow in experiments might permit to identify the best theoretical initial conditions.
Elliptic flow from Transport

**Au-Au collision**

**RHIC energy**

*Summary of the effect on differential $v_2$*

For more central collisions the effect on $v_2$ becomes milder.
Are micro-details important?

\[ \frac{d\sigma_{gg\rightarrow gg}}{dt} = \frac{9\pi^2\alpha_s^2}{2} \frac{1}{(t - m_D^2)^2} \left(1 + \frac{m_D^2}{s}\right) \]

**Same cross section used in:**
Zhang et al., PLB 455 (1999)
Molnar and Gyulassy, NPA 697 (2002)
Greco et al., PLB 670 (2009)

Increasing \( m_D \) makes the cross section isotropic. However:
**Strong change of the cross section does not result in a strong change of the elliptic flow.**

\[ Q_{s}^2 = 5 \text{ GeV}^2 \]

\[ Q_{s, A}^2(x, x_{\perp}) \propto Q_{s}^2 T_A(x_{\perp}) x^{-\lambda} \]

M. R. et al., in preparation
Invariant distributions

M. R. et al., in preparation
M. R. et al., work in progress

Longitudinal and transverse expansions help to dilute the system, lowering the invariant distribution functions.

$1+f$ factors in the collision integral, arising from the bosonic nature of gluons, should not modify in a substantial way our results on elliptic flow.

Preliminary result: no change due to $1+f$ (at RHIC energy).
Conclusions

- QGP produced in heavy ion collisions behaves as a liquid rather than a gas, developing collective flows.
- *Kinetic Theory* permits to compute *elliptic flow* of plasma, as well as its *thermalization times* and *isotropization efficiency*.
- *Initial distribution in momentum space affects the flow and the building up of momentum anisotropy.*
Outlook

(.) *Bose-Einstein condensate*
   BE condensation, in particular at LHC energy
   [Blaizot et al., NPA920 (2013), NPA873 (2012)]

(.) *Initial conditions from classical field dynamics*
   Implementation of the *proper* initialization from glasma spectrum & eccentricity

(.) *Fluctuations in the initial condition*
   Systematic study of higher order harmonics

(.) *Inelastic processes*
   Implementation of 2 to 3 and 3 to 2 processes in the collision integral
Freedom creates doubts.
(James Douglas Morrison)
Spectra and data

Au-Au@200 A GeV

\[ \frac{dN}{2\pi p_T \, dp_T} [\text{GeV}^2] \]

- STAR 20-30%, charged
- fKLN, \( \tau = 8.2 \text{ fm/c} \)
- Th-Glauber, \( \tau = 8.2 \text{ fm/c} \)
Pressures: weak coupling

- Pb-Pb@2.76 TeV
- Au-Au@200 GeV

\[ \frac{P_L}{P_T} \]

- \(4\pi\eta/s=1\)
- \(4\pi\eta/s=4\)
- \(4\pi\eta/s=10\)

\(\tau\) [fm/c]
Eccentricities

M. R. et al., in preparation
QGP in Heavy Ion Collisions

Initial temperature much larger than QCD critical temperature:

*Description in terms of partons is appropriate.*
Thermalization

$\text{AuAu@200A GeV Spectra}$

$\text{AuAu@200A GeV Multiplicity}$

M. R. et al., PLB727 (2013)
M. R. et al., in preparation
Fireball Isotropization

\[ T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E} f(x, p) \]

\[ P_T = \frac{1}{V} \int_{\Omega} d^2 x_\perp d\eta \frac{T_{xx} + T_{yy}}{2} \]

\[ P_L = \frac{1}{V} \int_{\Omega} d^2 x_\perp d\eta T_{zz} \]

Complete isotropization in strong coupling
(perfect gas would not be efficient to isotropize pressure)
Fireball Isotropization

\[ T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E} f(x,p) \]

\[ P_T = \frac{1}{V} \int_\Omega d^2x_\perp d\eta \frac{T_{xx} + T_{yy}}{2} , \]

\[ P_L = \frac{1}{V} \int_\Omega d^2x_\perp d\eta T_{zz} , \]

Complete isotropization in strong coupling
(perfect gas would not be efficient to isotropize pressure)