Excitonic mass generation in Honeycomb lattice

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Outline

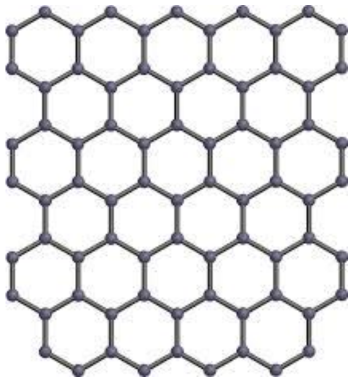
1. Introduction
   - Dirac electrons
   - Coulomb interaction
   - Excitonic order parameters(Previous works)

2. Honeycomb lattice model
   - Formulation
   - Self-consistent equation
   - Results

3. Summary

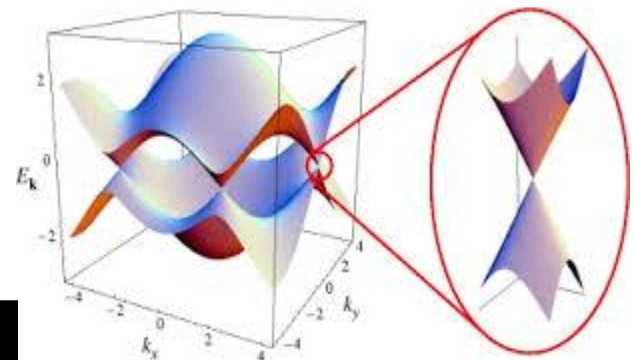
Massless Dirac electron system in solid

Graphene

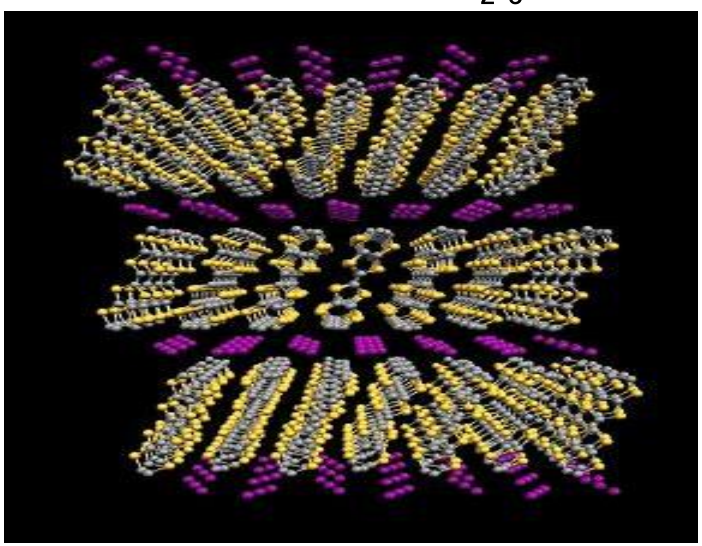


P. Wallace, Physical Review 71: 622-634. (1947)
(Graphite)

A. Geim, K. Novoselov Nature 438 (2005) 197.

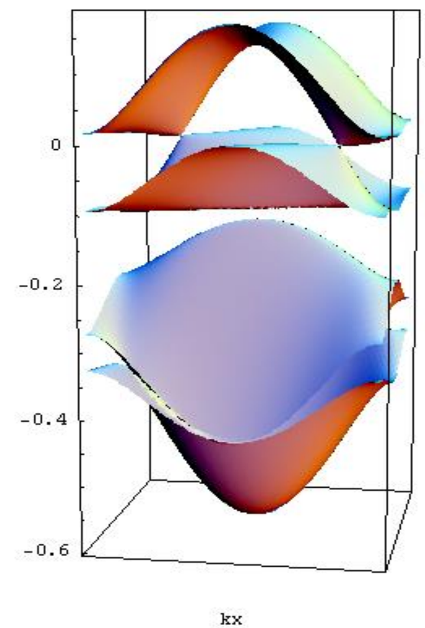


α−(BEDT−TTF)$_2$I$_3$



$$H_{eff} = \hbar \sum_{\rho=0}^{3} \mathbf{v}_\rho \cdot \tilde{\mathbf{k}} \sigma_\rho$$

Effective Weyl Hamiltonian



K. Kajita, et al., J. Phys. Soc. Jpn. 61, 23 (1992).

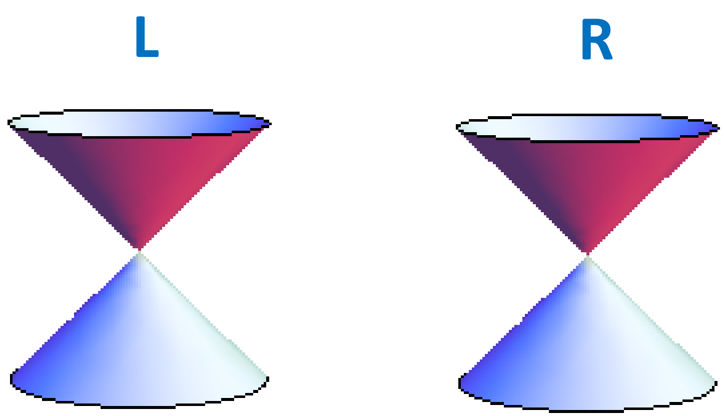
N. Tajima, et al., J. Phys.Soc. Jpn. 75, 051010 (2006).

S. Katayama, A. Kobayashi, Y. Suzumura, JPSJ. 75 (2006) 054705

Weyl Hamiltonian

sublattice-representation $\Psi_k^+ = (A_{k,R}^+ \quad B_{k,R}^+ \quad A_{k,L}^+ \quad B_{k,L}^+)$

$$H_0 = \sum_{ks} \Psi_k^+ \begin{array}{c|cccc} & \mathbf{R} & & \mathbf{L} & \\ \hline \begin{array}{c}\mathbf{R}\\ \\ \\ \mathbf{L}\end{array} & \multicolumn{4}{c}{\left(\begin{array}{cc|cc} 0 & \hbar v_F(k_x - ik_y) & 0 & 0 \\ \hbar v_F(k_x + ik_y) & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \hbar v_F(-k_x - ik_y) \\ 0 & 0 & \hbar v_F(-k_x + ik_y) & 0 \end{array}\right)} \end{array} \Psi_k$$



$k - K_L \to k$ $\qquad$ $k - K_R \to k$

$$H_{0,k}^{R,L} = \pm k_x \hat{\sigma}_x + k_y \hat{\sigma}_y$$

$$V_0(q) = \frac{2\pi e^2}{\varepsilon_0 |q|}$$

$$H' \equiv \int d\mathbf{r} \int d\mathbf{r}' V_0(\mathbf{r} - \mathbf{r}') n(\mathbf{r}) n(\mathbf{r}')$$

$$H = H_0 + H'$$

Electron correlation

Coulomb interaction(2dimensional)

$$V_0(q)=\frac{2\pi e^2}{\varepsilon_0|q|} \quad \Rightarrow \quad V^{\mathrm{screened}}(q,\omega)=\frac{V_0(q)}{1+V_0(q)\Pi(q,\omega)}=\frac{2\pi e^2}{\varepsilon q}$$

Polarization function

$$\Pi(q,\omega)=\frac{Nq^2}{16\sqrt{\hbar^2 v^2 q^2-\omega^2}} \quad \propto q$$

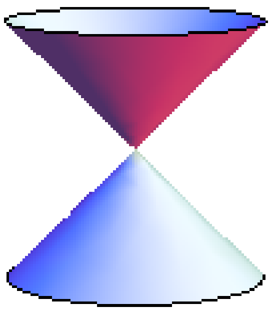
B. Wunsch et al. New J. Phys. 8 318.

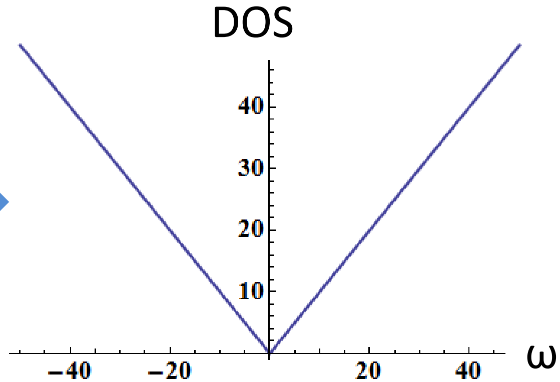
screening effects

Normal metal

$$V^{(RPA)}(q)=\frac{e^2}{q^2+q_{TF}^2}$$

2D Dirac electron



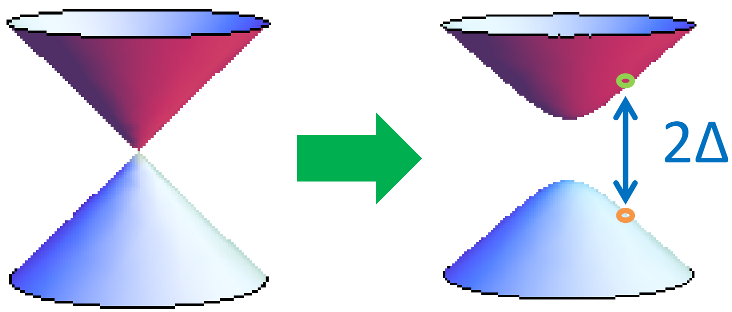


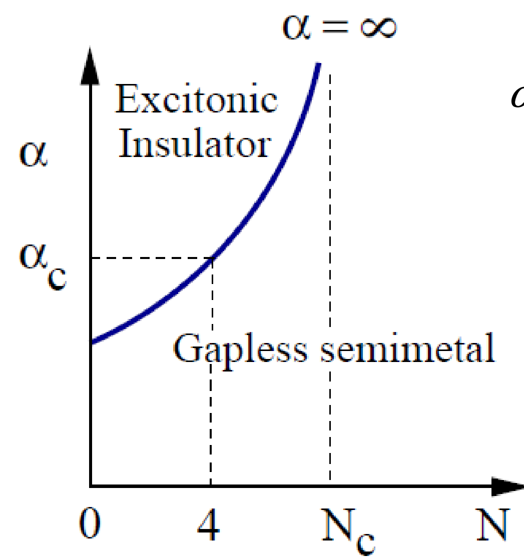
$$V^{(RPA)}(q)=\frac{e^2}{q(1+const.)}$$

still diverse at q=0.

Long-range Coulomb interaction is important in Dirac electron system.

Excitonic mass generation





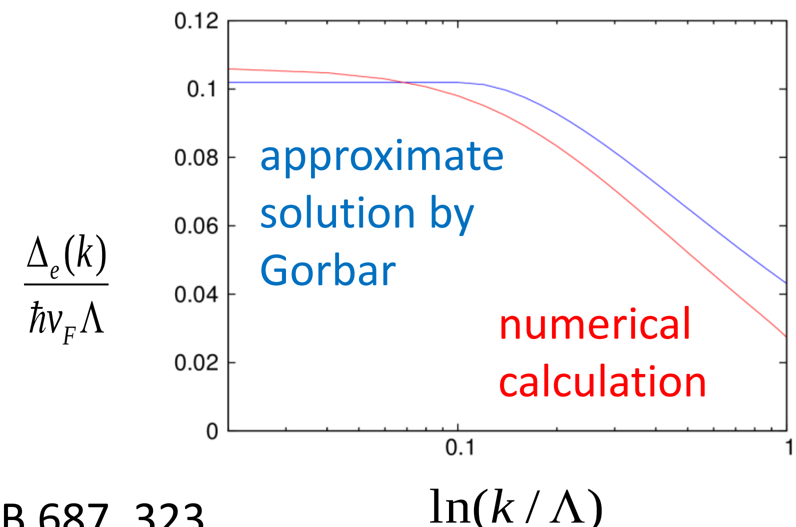
$$\alpha = \frac{e^2}{\varepsilon \hbar v} = 2.2/\varepsilon$$

$$\downarrow$$

$$\frac{\alpha}{2(1+\pi N\alpha/4)}$$

$$\Delta_k = \sum_p V(k-p)\frac{\Delta_p}{2E_p}\tanh\frac{E_p}{2k_\mathrm{B}T}$$

$$E_k = \sqrt{(\hbar v_F|k|)^2 + |\Delta_k|^2}$$



Gorbar et.al. PRB 66, 045108 (2002)

D. V. Khveshchenko, et al., (2004) Nucl. Phys. B 687, 323

Mean field order parameters

which contribute to mass generation

$\hat{\sigma}$: *sublattice*

$\hat{\tau}$: *valley*

4^2=16 degrees of freedom

$$\left\{\begin{aligned}
\Psi_k^+(\hat{\sigma}_3\otimes\hat{\tau}_0)\Psi_k &= \sum_{\tau=R,L}\left(A_k^{\tau+}A_k^{\tau}-B_k^{\tau+}B_k^{\tau}\right)\\
\Psi_k^+(\hat{\sigma}_3\otimes\hat{\tau}_3)\Psi_k &= \sum_{\tau=R,L} sign(\tau)\left(A_k^{\tau+}A_k^{\tau}-B_k^{\tau+}B_k^{\tau}\right)\\
\Psi_k^+(\hat{\sigma}_1\otimes\hat{\tau}_1)\Psi_k &= A_k^{R+}B_k^{L}-B_k^{R+}A_k^{L}+h.c.\\
\Psi_k^+(\hat{\sigma}_1\otimes\hat{\tau}_2)\Psi_k &= -iA_k^{R+}B_k^{L}-iB_k^{R+}A_k^{L}+h.c.
\end{aligned}\right\}\otimes\hat{s}:spin$$

singlet
triplet

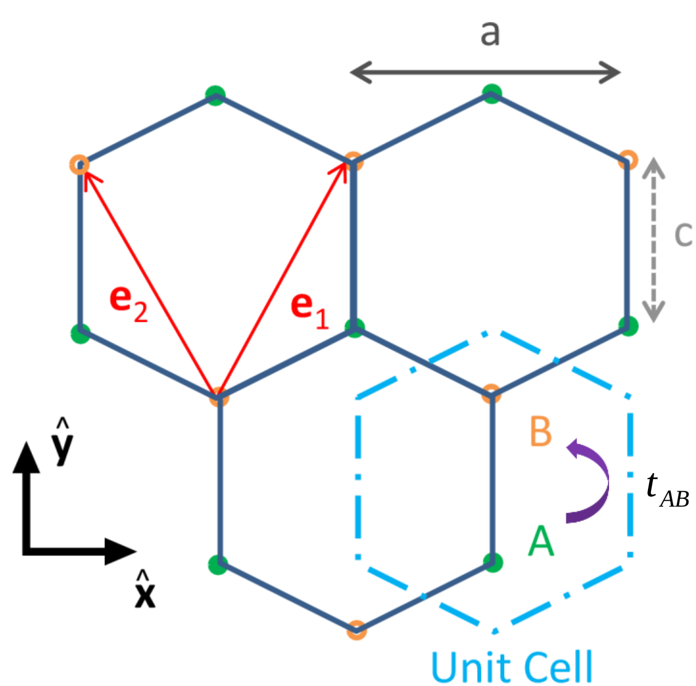
D.V.Khveshchenko, J. Phys.: Condens. Matter **21** (2009) 075303

S. Raghu et al., Phys. Rev. Lett. **100** (2008) 156401

	q	R ↔ L	physical content	$\langle C^\dagger C' \rangle$	matrix
$\Delta_{\mathbf{0}}^{\text{even}}$	$\mathbf{0}$	even	CDW/SDW	$\langle A^\dagger A - B^\dagger B\rangle$	$\sigma_3\otimes\tau_0$
$\Delta_{\mathbf{0}}^{\text{odd}}$	$\mathbf{0}$	odd	QAH/ QSH	$\langle A^\dagger A - B^\dagger B\rangle$	$\sigma_3\otimes\tau_3$
$\Delta_{\boldsymbol{Q}}^{\text{even}}$	$\boldsymbol{Q}$	even	BOW (charge/spin)	$\langle A^\dagger B + B^\dagger A\rangle$	$\sigma_1\otimes\tau_1$
$\Delta_{\boldsymbol{Q}}^{\text{odd}}$	$\boldsymbol{Q}$	odd	flux	$\langle A^\dagger B + B^\dagger A\rangle$	$\sigma_1\otimes\tau_2$

Honeycomb lattice

Real Space



$$\begin{cases} e_1 = \frac{a}{2}\left(1, \sqrt{3}\right) \\ e_2 = \frac{a}{2}\left(-1, \sqrt{3}\right) \end{cases}$$

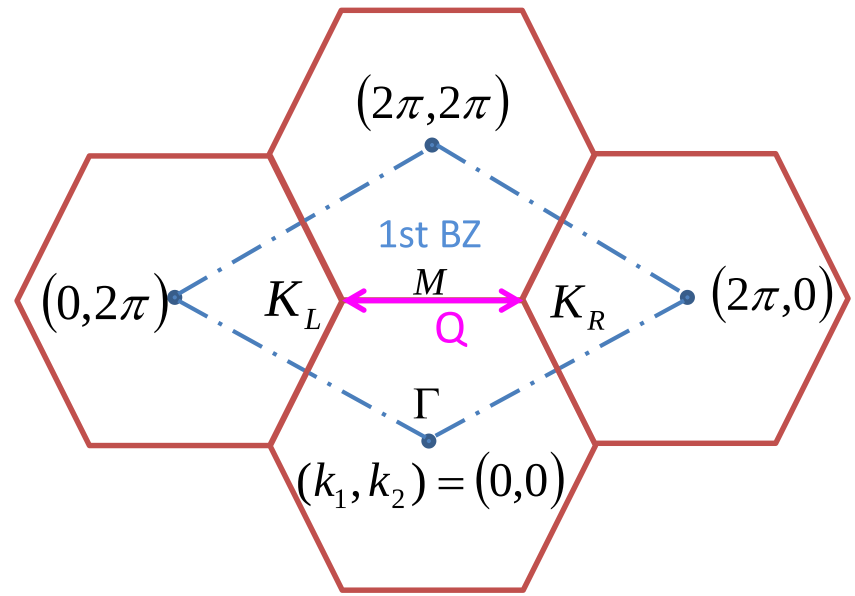
$$a = 2.5486\ \text{Å}$$

$$c = \frac{a}{\sqrt{3}}$$

$$\mathbf{R}_{nm} = n\mathbf{e}_1 + m\mathbf{e}_2$$

$$t_{AB}a = 1$$

Momentum Space



$$Q = \begin{pmatrix} \frac{2\pi}{3} & \frac{4\pi}{3} \end{pmatrix}, \begin{pmatrix} \frac{4\pi}{3} & \frac{2\pi}{3} \end{pmatrix}$$

$$\hat{H}_0 = \sum_{k,s} \begin{pmatrix} A^+_{k,s} & B^+_{k,s} \end{pmatrix} \begin{pmatrix} 0 & z_k^* \\ z_k & 0 \end{pmatrix} \begin{pmatrix} A_{k,s} \\ B_{k,s} \end{pmatrix}$$

$$z_k = 1 + \cos(k_x/2)\exp(-i\sqrt{3}k_y/2) \approx \frac{\sqrt{3}}{2}(s\tilde{k}_x + i\tilde{k}_y)$$

Formulation

$$H = t\sum_{\langle i,j\rangle}\left(c_i^{+}c_j + c_j^{+}c_i\right) + \frac{1}{2}\sum_{i,j}V_{ij}n_i n_j - \left(\frac{U}{2}+\mu\right)\sum_i n_i$$

$$V_{ij} = \begin{cases} \dfrac{e^2}{\varepsilon\left|R_i - R_j\right|} & i \neq j \\ U & i = j \end{cases}$$

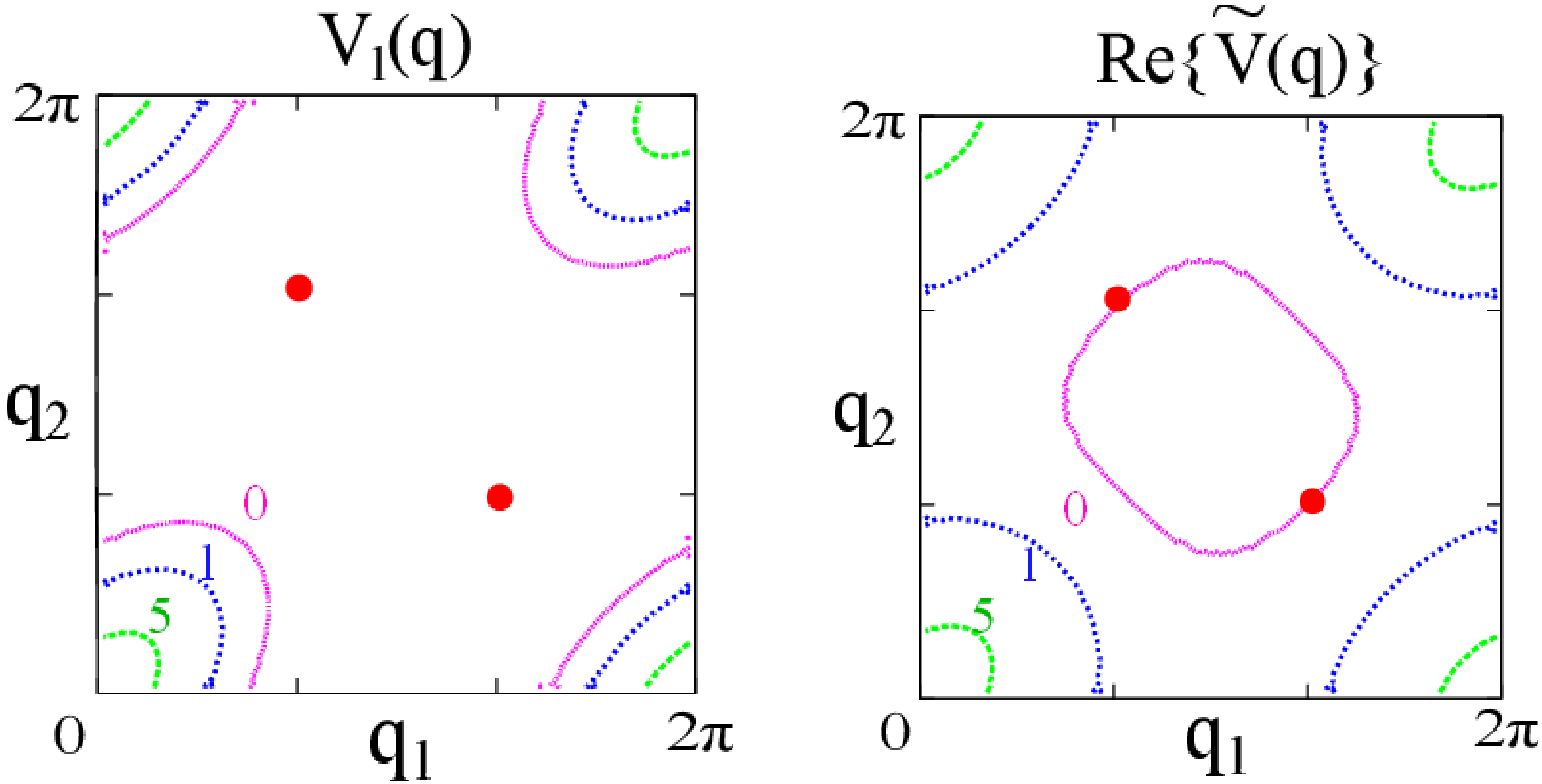
$$H = H_0 + H'$$

$$n_i = c_{i\uparrow}^{+}c_{i\uparrow} + c_{i\downarrow}^{+}c_{i\downarrow}$$

Mean field theory

$$\begin{aligned} H_{MF}^{'} = {} & U\sum_k\left\{\left[\sum_{k'}\left\langle A_{k'}^{+}A_{k'}\right\rangle\right]A_k^{+}A_k + \left[\sum_{k'}\left\langle B_{k'}^{+}B_{k'}\right\rangle\right]B_k^{+}B_k\right\} \\ & - \sum_{k,k',q}V(q)\left\{\left\langle A_{k+q}^{+}A_{k'+q}\right\rangle A_k^{+}A_k + \left\langle B_{k+q}^{+}B_{k'+q}\right\rangle B_k^{+}B_k\right\} \\ & - \sum_{k,k',q}\tilde{V}(q)\left\{\left\langle A_{k+q}^{+}B_{k'+q}\right\rangle B_k^{+}A_k + \left\langle B_{k+q}^{+}A_{k'+q}\right\rangle A_k^{+}B_k\right\} \end{aligned}$$

$$V_{s,s'}(q) = U\delta_{s,-s'} + \frac{e^2}{\varepsilon} \sum_{(n,m)\neq(0,0)} \frac{e^{-i(nq_1+mq_2)}}{\sqrt{n^2+m^2+mn}} = U\delta_{s,-s'} + V_l(q)$$



$$\tilde{V}(q) = \frac{e^2}{\tilde{\varepsilon}} \sum_{(n,m)} \frac{e^{-i(nq_1+mq_2)}}{\sqrt{n^2+m^2+mn+m+n+1/3}}$$

Self-consistent equation

excitonic order parameters

$$\Delta_{k,s,s'}^{q=0} = \Sigma_{k,s,s'}^{11} - \Sigma_{k,s,s'}^{22}$$

$$\Delta_{k,s,s'}^{q=Q} = \left(\Sigma_{k,s,s'}^{41} + \Sigma_{k,s,s'}^{32} + c.c.\right)/2$$

singlet・triplet

$$\Delta_k^{\text{singlet}} = \Delta_{k,s,s'}\delta_{s,s'}$$

$$\Delta_{i,k}^{\text{triplet}} = \sum_{s,s'} \left(\hat{s}_i\right)_{s,s'} \Delta_{k,s,s'}$$

$$\Sigma_{k,s,s'}^{ab} = \frac{1}{N}\sum_q W(q)\left\langle \Psi_{k+q,s'}^{b+} \Psi_{k+q,s}^{a} \right\rangle$$

linearized self-consistent field equation

$$\varepsilon_{k+q} = \hbar v_F |k+q|$$

$$\lambda\Delta_k = \sum_q W(q) \frac{\Delta_{k+q}}{2\varepsilon_{k+q}} \tanh\frac{\varepsilon_{k+q}}{2T}$$

q=0,singlet case

$$W(q) = V_l(q) - U + \left[V_l(q) - \tilde{V}(q)\right]_{q=0}$$

Comes from neutrality condition.

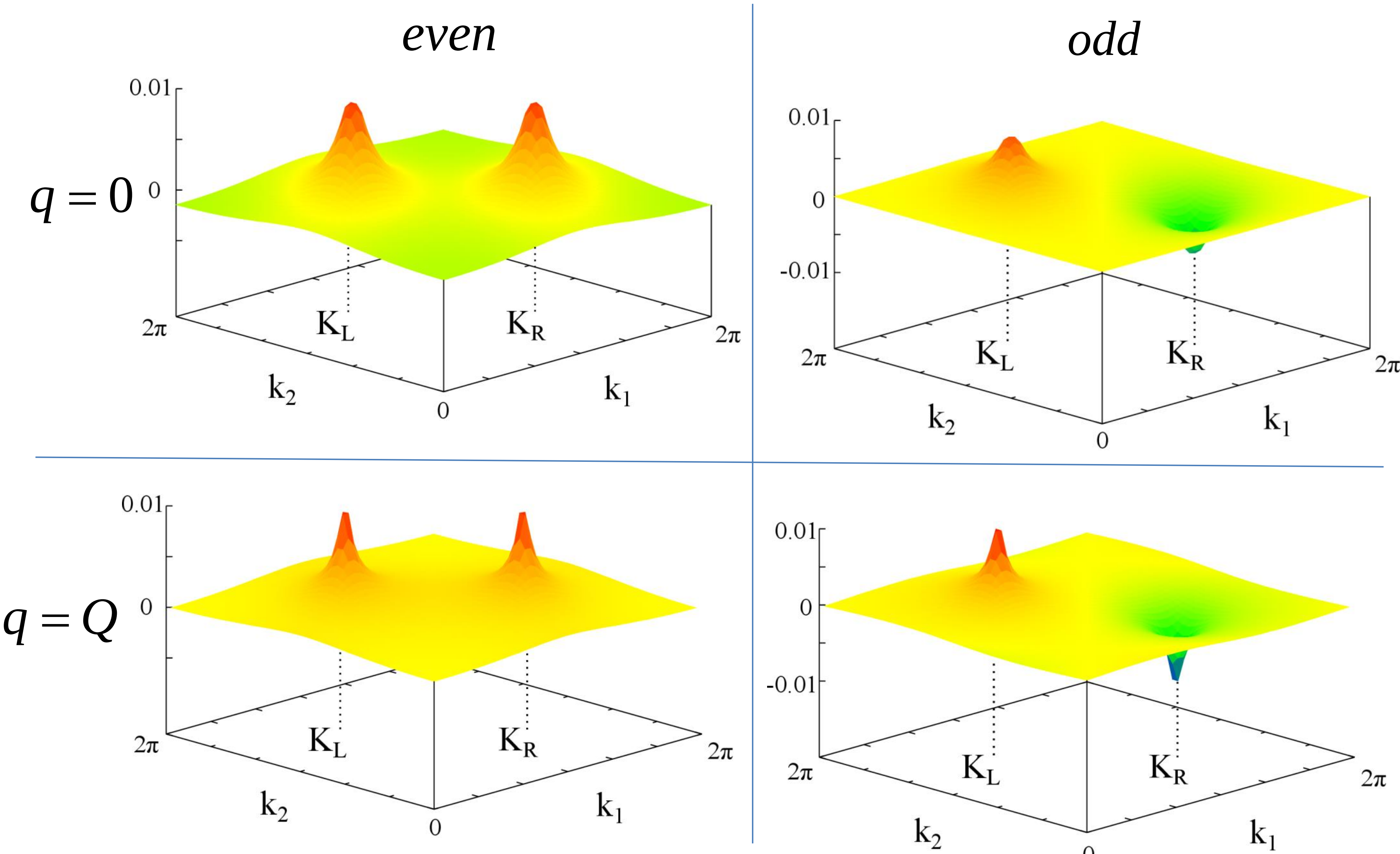
q=0,triplet case

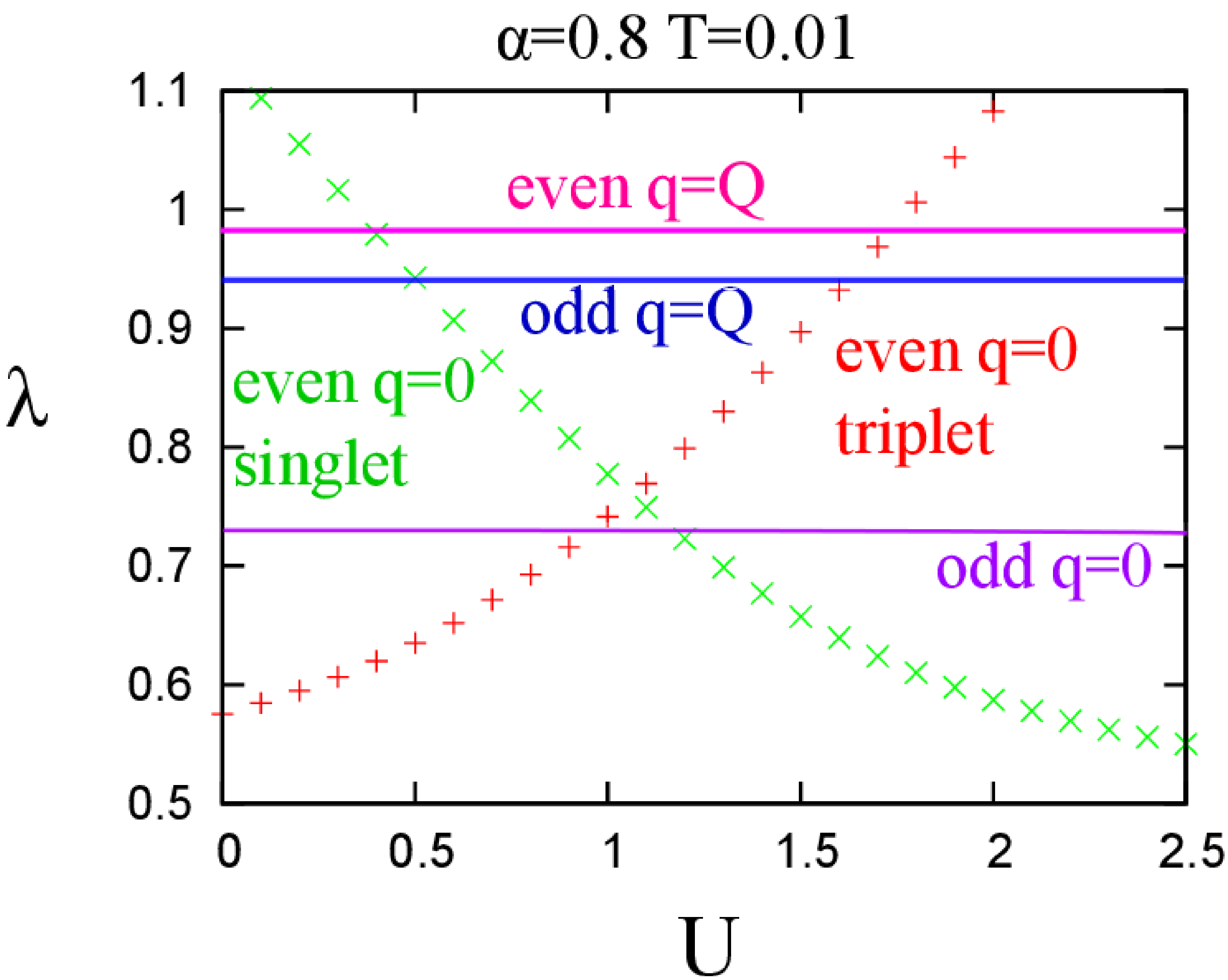
$$W(q) = V_l(q) + U$$

q=Q case

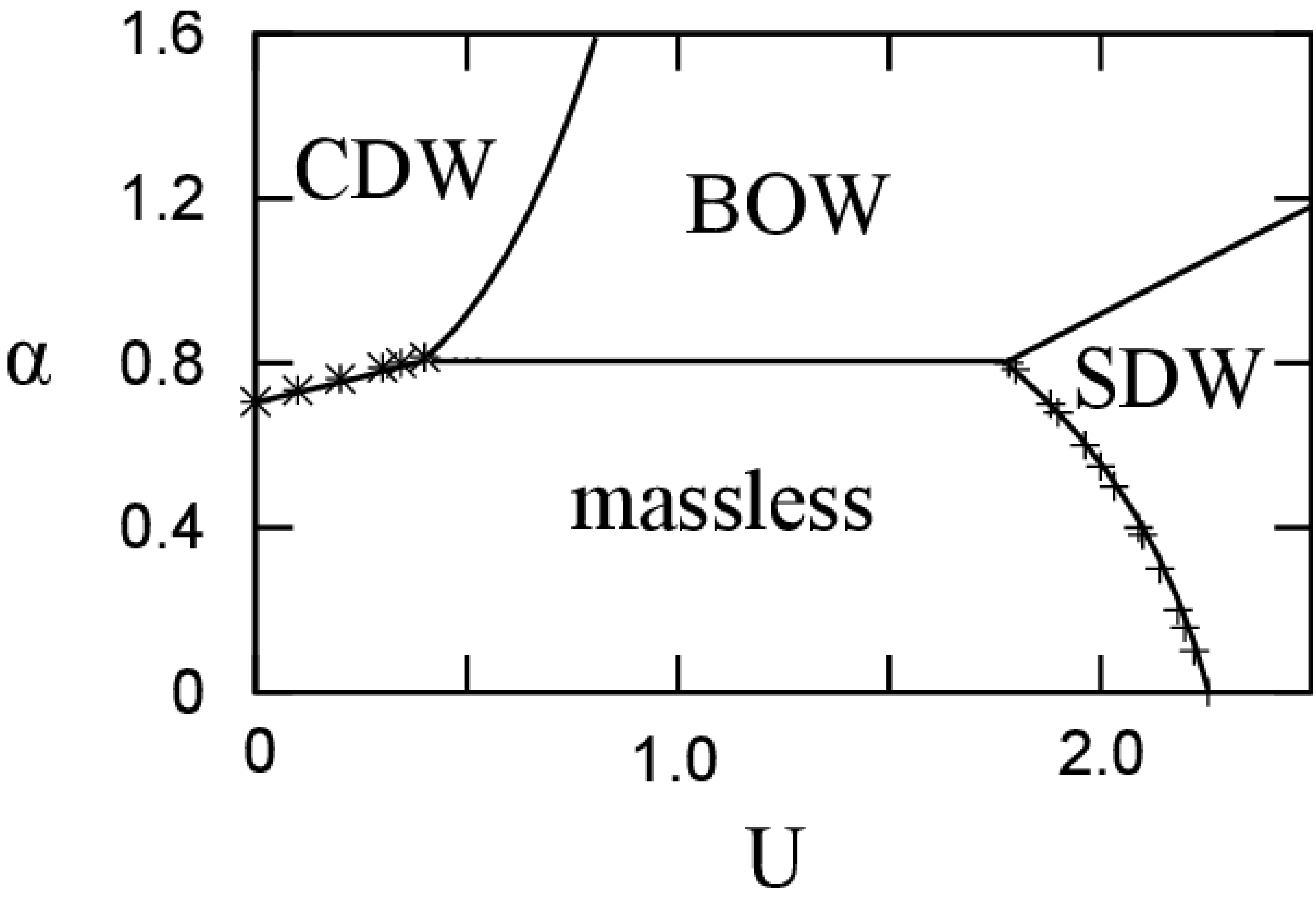
$$W(q) = \frac{\tilde{V}(q) + \tilde{V}(-q)}{2}$$

Order parameters in momentum space





α=0.8 T=0.01
λ
U
even q=Q
odd q=Q
even q=0
singlet
even q=0
triplet
odd q=0
1.1
1
0.9
0.8
0.7
0.6
0.5
0
0.5
1
1.5
2
2.5


1.6
1.2
0.8
0.4
0
α
CDW
BOW
SDW
massless
0
1.0
2.0
U

Summary

We study about excitonic mass generation in Honeycomb lattice model.
In continuum model, several excitonic orders are degenerated.
We find that in lattice model, the degeneracy of excitonic orders is broken.
For realistic parameter U=1.2t(graphene), BOW state is stable for $\alpha > 0.8$.

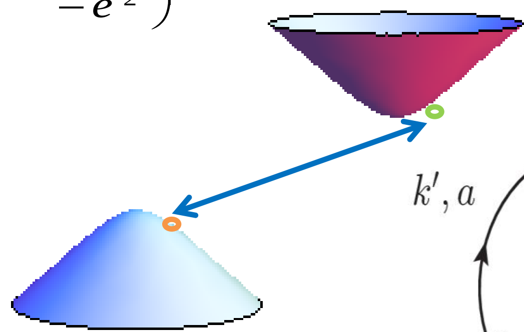
band-representation

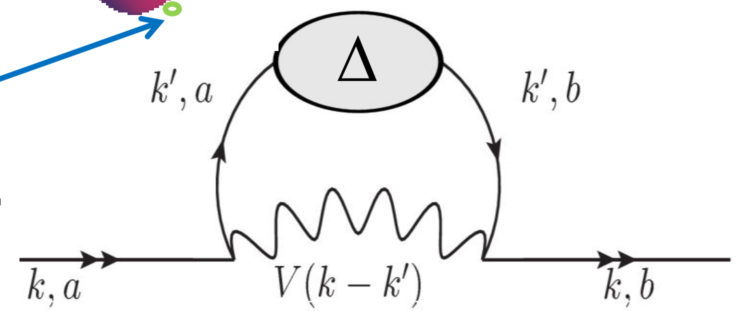
$$\Psi^{+}_{k,s}\hat{U}^{+}\hat{U}\hat{H}^{MF}_{k,s,s'}\hat{U}^{+}\hat{U}\Psi_{k,s'}=\Phi^{+}_{k,s}\hat{H}^{band}_{k,s,s'}\Phi_{k,s'} \qquad \Phi^{+}_{k,s}=\begin{pmatrix} a^{R+}_{\tilde{k},s} & b^{R+}_{\tilde{k},s} & b^{L+}_{\tilde{k},s} & a^{L+}_{\tilde{k},s}\end{pmatrix}$$

$$U_R=\frac{1}{\sqrt{2}}\begin{pmatrix} e^{\frac{i\theta}{2}} & e^{\frac{-i\theta}{2}} \\ e^{\frac{i\theta}{2}} & -e^{\frac{-i\theta}{2}}\end{pmatrix} \qquad U_L=\frac{1}{\sqrt{2}}\begin{pmatrix} e^{\frac{-i\theta}{2}} & e^{\frac{i\theta}{2}} \\ e^{\frac{-i\theta}{2}} & -e^{\frac{i\theta}{2}}\end{pmatrix}$$

$a^{+}_{\tilde{k},\sigma}$: *conduction*

$$H^{band(0)}_{k}=\frac{\sqrt{3}}{2}\begin{pmatrix} |\tilde{k}| & 0 & 0 & 0 \\ 0 & -|\tilde{k}| & 0 & 0 \\ 0 & 0 & -|\tilde{k}| & 0 \\ 0 & 0 & 0 & |\tilde{k}|\end{pmatrix}$$





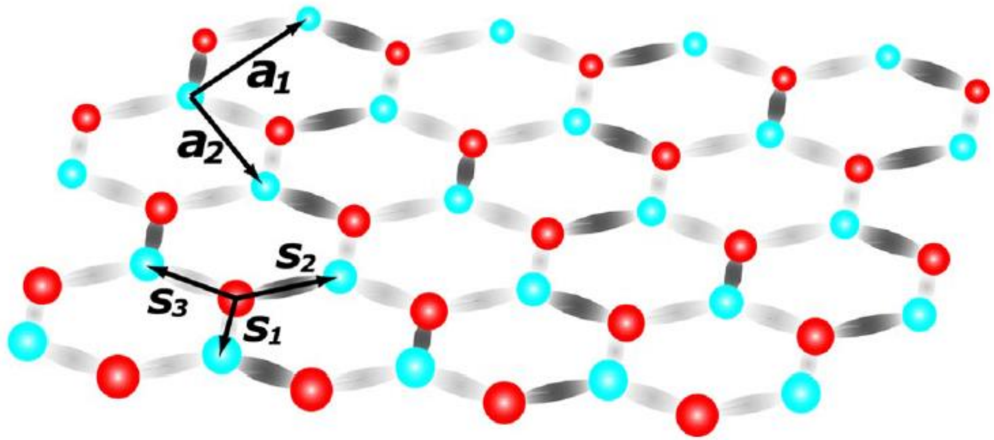
$b^{+}_{\tilde{k},\sigma}$: *valence*

sublattice -representation ⟷ band-representation

$$\Delta^{q=Q}_{\tilde{k},s,s'}=[\frac{1}{N}\sum_{q}\left\{\tilde{V}(q)\left\langle A^{R+}_{\tilde{k}+q,s'}B^{L}_{\tilde{k}+q,s}\right\rangle+\tilde{V}(-q)\left\langle B^{R+}_{\tilde{k}+q,s'}A^{L}_{\tilde{k}+q,s}\right\rangle+c.c.\right\}]\Big/2$$

$$=\frac{1}{N}\sum_{q}\frac{\tilde{V}(q)+\tilde{V}(-q)}{2}\left(\left\langle a^{R+}_{\tilde{k}+q,s'}b^{L}_{\tilde{k}+q,s}\right\rangle-\left\langle b^{R+}_{\tilde{k}+q,s'}a^{L}_{\tilde{k}+q,s}\right\rangle+c.c.\right)$$

BOW(Honeycomb)



$$H = -\sum_{r\in\Lambda_A}\sum_{i=1}^{3}(t+\delta t_{\mathbf{r},i})a_{\boldsymbol{r}}^{\dagger}b_{\boldsymbol{r}+\boldsymbol{s}_i} + \text{H.c.} \qquad (1)$$

$$\delta t_{\boldsymbol{r},i} = \Delta(\boldsymbol{r})e^{i\boldsymbol{K}_+\cdot\boldsymbol{s}_i}e^{i\boldsymbol{G}\cdot\boldsymbol{r}}/3 + \text{c.c.},$$

$$\boldsymbol{K}_{\pm} = \pm(\frac{4\pi}{3\sqrt{3}a}, 0).$$

$$\boldsymbol{G} := \boldsymbol{K}_+ - \boldsymbol{K}_-,$$

BOW in Honeycomb lattice correspond to the Kekulé distortion.

C. Y. Hou et al., Phys. Rev. Lett. 98 186809(2007)

Topological aspect of the quantum mechanics

Berry接続

$$\mathbf{A}_n = -\mathrm{i}\langle n|\partial_\kappa|n\rangle$$

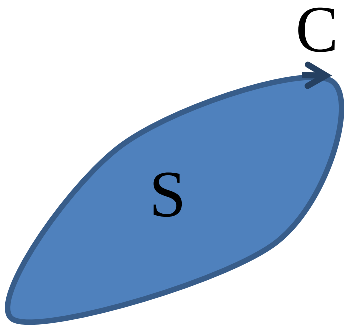
n: band index

Berry曲率

$$\mathbf{B}_n = \nabla\times\mathbf{A}_n$$

$$B_n^z = -\mathrm{i}\sum_{m(\neq n)}\frac{\langle n|\partial_{k_x}H|m\rangle\langle m|\partial_{k_y}H|n\rangle}{\left(E_n - E_m\right)^2} + \mathrm{c.c.}$$

Berry phase $\Phi = \int_S \mathbf{B}\cdot\mathrm{d}\mathbf{S} = \int_S \nabla\times\mathbf{A}\cdot\mathrm{d}\mathbf{S} = \oint_C \mathbf{A}\cdot\mathrm{d}\mathbf{X}$



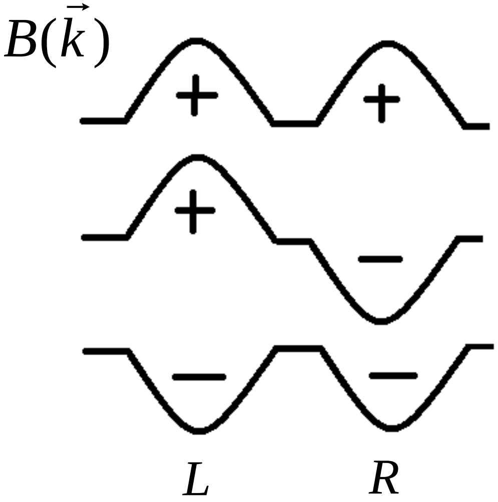
Chern number (2D) $\nu_{n\sigma} = \dfrac{1}{2\pi}\int_{\mathrm{BZ}}\mathrm{d}^2k B_{n\sigma}^z = 0,\pm1,\pm2,\ldots$

ホール伝導率

TKNN公式

D. J. Thouless et al. PRL 49 (1982) 405

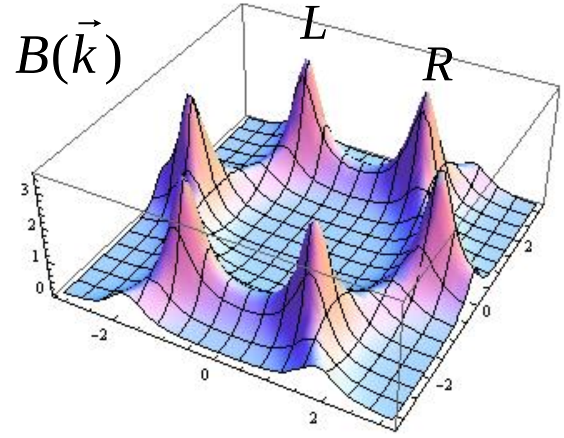
$$\sigma_{xy} = \frac{e^2}{\hbar}\int \frac{d\vec{k}}{(2\pi)^2}\sum_{\alpha} f_\alpha(\vec{k})[\nabla_{\vec{k}} \times \hat{A}_\alpha(\vec{k})]_z = \frac{e^2}{h}\nu$$



$\Delta_L > 0, \quad \Delta_R < 0 \qquad \upsilon = +1$

$\Delta_L > 0, \quad \Delta_R > 0 \qquad \upsilon = 0$

$\Delta_L < 0, \quad \Delta_R > 0 \qquad \upsilon = -1$



$$\sigma_{xy} = \frac{e^2}{h}\nu \neq 0$$
量子異常ホール状態（QAH）

$$\sigma_{xy}^{s} = \frac{e^2}{h}\left(\nu_\uparrow - \nu_\downarrow\right) \neq 0$$
量子スピンホール状態（QSH）

F. D. M. Haldane, PRL 61, 2015 (1988).

S. Raghu, X. L. Qi, C. Honerkamp, and S. C. Zhang, PRL 100, 156401 (2008).

The simplest Hamiltonian with non-zero Chern number

$$H = \alpha_x k_x \sigma_x + \alpha_y k_y \sigma_y + \alpha_z \Delta \sigma_z \qquad \left(\alpha_x, \alpha_y, \alpha_z = \pm 1\right)$$

Parameter space $\mathbf{X} = \left(k_x, k_y, \Delta\right)$

Eigen energy $E_\pm = \pm X$

Eigen function
$$\begin{cases} \Psi_+ = \dfrac{1}{\sqrt{2X\left(X - \alpha_z \Delta\right)}} \begin{pmatrix} \alpha_x k_x - \mathrm{i}\alpha_y k_y \\ -\alpha_z \Delta + X \end{pmatrix} \\ \\ \Psi_- = \dfrac{1}{\sqrt{2X\left(X + \alpha_z \Delta\right)}} \begin{pmatrix} \alpha_x k_x - \mathrm{i}\alpha_y k_y \\ -\alpha_z \Delta - X \end{pmatrix} \end{cases}$$

using $\partial_{k_x} H = \alpha_x \sigma_x \quad \partial_{k_y} H = \alpha_y \sigma_y$

$$B_-^\Delta = -\mathrm{i} \frac{\left(\psi_-^+ \cdot \partial_{k_x} H \cdot \psi_+\right)\left(\psi_+^+ \cdot \partial_{k_y} H \cdot \psi_-\right)}{\left(E_- - E_+\right)^2} + \mathrm{c.c.} = -\alpha_x \alpha_y \alpha_z \frac{\Delta}{2X^3} \Rightarrow Ch_- = -\frac{\alpha_x \alpha_y \alpha_z}{2}$$

Lower (filled) band

"Magnetic monopole"

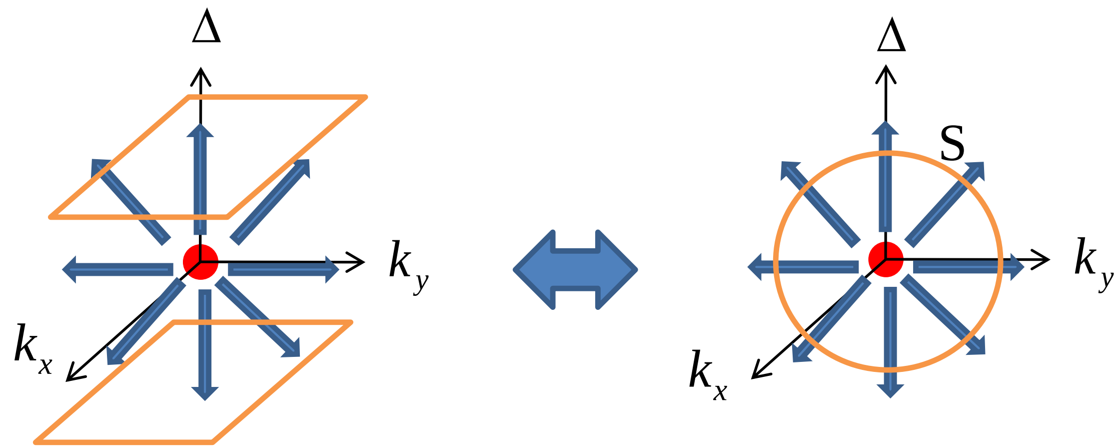
"magnetic monopole" and discontinuity of Chern number

$$\mathbf{B} = \frac{\mathbf{X}}{2X^3} \qquad \mathbf{X} = (k_x, k_y, \Delta)$$

$$Ch_n = \frac{1}{2\pi}\int_{\mathrm{BZ}} \mathrm{d}^2k \cdot B_n^{\Delta}$$

$$\left\{\begin{array}{l} Ch(\Delta_0) = \dfrac{1}{2} \\[2em] Ch(-\Delta_0) = -\dfrac{1}{2} \end{array}\right.$$

$$(\Delta_0 > 0)$$



$$Ch(\Delta_0) - Ch(-\Delta_0) = \frac{1}{2\pi}\int_{\mathrm{S}} \mathbf{B}\cdot\mathrm{d}\mathbf{S} = \frac{1}{2\pi}\frac{4\pi X^2}{2X^2} = 1$$

Topological insulator with TRS (time-reversal symmetry)

= a combined system of two subsystems with opposite sign Chern numbers

Kane-Mele model (graphene + spin-orbit interaction) PRL **95, 226801 (2005)**

Topological insulator with TRS = Quantum spin Hall system

$$H = v\left(k_x\sigma_x\tau_z + k_y\sigma_y\right) - \Delta\sigma_z\tau_z s_z$$

$$\left(\alpha_x, \alpha_y, \alpha_z\right) = (+,+,-)$$

$$\left(\alpha_x, \alpha_y, \alpha_z\right) = (-,+,+)$$

$$k_\pm = k_x \pm \mathrm{i}k_y$$

$$\psi = \begin{pmatrix} \varphi_{A\uparrow K} \\ \varphi_{B\uparrow K} \\ \varphi_{A\uparrow K'} \\ \varphi_{B\uparrow K'} \\ \varphi_{A\downarrow K} \\ \varphi_{B\downarrow K} \\ \varphi_{A\downarrow K'} \\ \varphi_{B\downarrow K'} \end{pmatrix} \qquad H = \begin{pmatrix} -\Delta & vk_- & 0 & 0 & 0 & 0 & 0 & 0 \\ vk_+ & \Delta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Delta & -vk_+ & 0 & 0 & 0 & 0 \\ 0 & 0 & -vk_- & -\Delta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Delta & vk_- & 0 & 0 \\ 0 & 0 & 0 & 0 & vk_+ & -\Delta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\Delta & -vk_+ \\ 0 & 0 & 0 & 0 & 0 & 0 & -vk_- & \Delta \end{pmatrix}$$

Ch= 1/2

Ch= 1/2

Ch= -1/2

Ch= -1/2

Spin Chern number: $SpCh = \frac{\mathrm{i}}{4\pi}\int d^2k\nabla_{\mathbf{k}} \times \left\{\left\langle u_{\mathbf{k}}^{\uparrow}\middle|\nabla_{\mathbf{k}}\middle|u_{\mathbf{k}}^{\uparrow}\right\rangle - \left\langle u_{\mathbf{k}}^{\downarrow}\middle|\nabla_{\mathbf{k}}\middle|u_{\mathbf{k}}^{\downarrow}\right\rangle\right\} = 1$

Direct calculation of Berry's phase gauge field using the Hamiltonian for alpha-(BEDT-TTF)$_2$I$_3$

Fictitious "magnetic field" given by Berry's phase gauge field

$$B_{n\sigma}^{\Delta} = -\mathrm{i} \sum_{m(\neq n)} \frac{\langle n\sigma | \partial_{k_x} H | m\sigma \rangle \langle m\sigma | \partial_{k_y} H | n\sigma \rangle}{\left(E_{n\sigma} - E_{m\sigma}\right)^2} + \mathrm{c.c.}$$

$$= -\mathrm{i} \sum_{m(\neq n), \alpha_1 \sim \alpha_4} \frac{\langle n | \alpha_1 \rangle \langle \alpha_1 | \partial_{k_x} H | \alpha_2 \rangle \langle \alpha_2 | m \rangle \langle m | \alpha_3 \rangle \langle \alpha_3 | \partial_{k_y} H | \alpha_4 \rangle \langle \alpha_4 | n \rangle}{\left(E_{n\sigma} - E_{m\sigma}\right)^2} + \mathrm{c.c.}$$

$$= -\mathrm{i} \sum_{m(\neq n), \alpha_1 \sim \alpha_4} \frac{d_{n\alpha_1\sigma}(\mathbf{k}) \partial_{k_x} H_{\alpha_1\alpha_2\sigma}(\mathbf{k}) d^{*}_{m\alpha_2\sigma}(\mathbf{k}) d_{m\alpha_3\sigma}(\mathbf{k}) \partial_{k_y} H_{\alpha_3\alpha_4\sigma}(\mathbf{k}) d^{*}_{n\alpha_4\sigma}(\mathbf{k})}{\left(E_{n\sigma} - E_{m\sigma}\right)^2} + \mathrm{c.c.}$$

$$\mathrm{n} = 2, \mathrm{m} = 1$$

$$\alpha_1, \alpha_2, \alpha_3, \alpha_4 = 1,2,3,4$$

$\mathbf{k}$: in BZ

excitonic mass generation in graphene

Static RPA： Khveshchenko, D. V., and H. Leal, 2004, Nucl. Phys. B 687, 323

Strong k-dependence of $\Delta(k)$ $\alpha_c = 1.13$

Effect of renormalized velocity: $\varepsilon_b(k) = \hbar v |k|^{1-\eta}$

Khveshchenko, D. V., 2009, J. Phys.: Condens. Matter 21, 075303.

$$T_c \approx \Delta \big/ \left|\ln\left(1-\tilde{\alpha}_c/\alpha_c\right)\right| \quad \left(\tilde{\alpha} = \frac{\alpha}{1+\pi N\alpha/4}\right) \qquad \frac{\Delta(0)}{\hbar v\Lambda} \approx 0.001 \qquad \Delta \approx k_B T_c \approx 4\text{mev}$$

Dynamical RPA on-shell: $\Delta(p, \varepsilon = \hbar v p)$

Gamayun, O. V., E. V. Gorbar, and V. P. Gusynin, 2010, Phys. Rev. B 81, 075429. $\alpha_c = 0.92$

面直磁場によりα_c減少、Δ増大

Monte Carlo calculations:

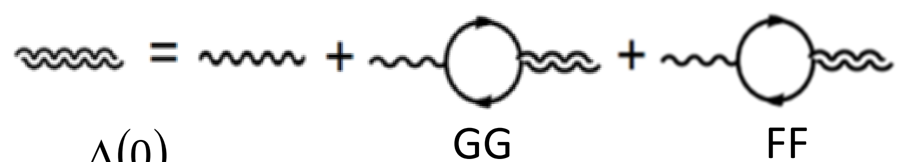
Hands, S., and C. Strouthos, 2008, PRB 78, 165423.

Drut et al.,2009, PRB 79, 241405(R); RPL 102, 026802; PRB 79, 165425. $\alpha_c = 1.1$

Polarization function with Δ(k) and anomalous Green function

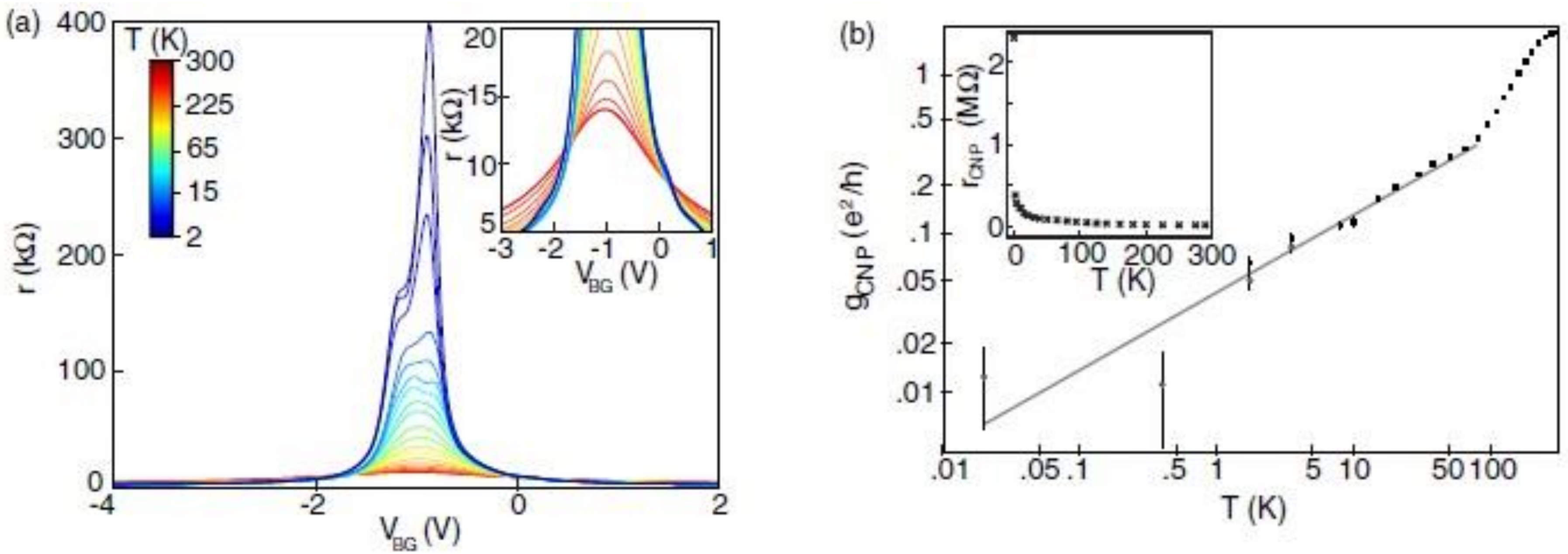
J-R. Wang et al., J. Phys.: Condens. Matter 23 (2011) 155602

$\Pi(\mathbf{q})$



Δ増大 α=32では $\dfrac{\Delta(0)}{\hbar v\Lambda} \approx 0.01$

グラフェンにおける電気抵抗



F. Amet, J. R. Williams, K. Watanabe, T. Taniguchi, and D. Goldhaber-Gordon
PRL 110, 216601 (2013)

シリコン基板上においたグラフェンの電気抵抗(右図インセット)

２次元ディラック電子系に普遍的な振る舞い？