Giant Nernst Effect due to Berry Phase Fluctuation in Chiral Superconductors

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Collaborators and Acknowledgment

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Takuya Yamashita

Outline

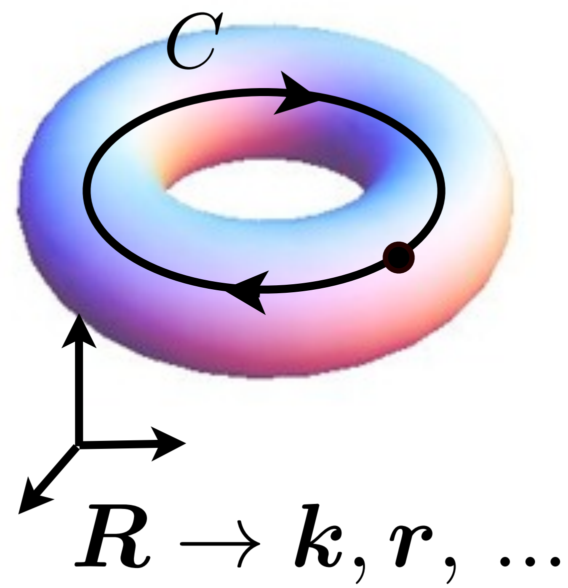
- Introduction : Berry phase in chiral superconductors and motivation of this study

- Colossal Nernst effect in chiral superconductor URu_2Si_2

- Novel mechanism of giant Nernst effect due to Berry phase fluctuation (chiral SC fluc.)

Berry Phase in Condensed Matter Physics

Berry phase $|\psi'\rangle = e^{i\gamma(C)}|\psi\rangle$



Topological quantum phenomena associated with Berry phase

- Topological transport such as anomalous Hall effect, Spin Hall effect, etc
- Skyrmion textures in chiral magnets, MnSi, FeCoSi
- Topological insulators, Topological superconductors
- Weyl semi-metal, Weyl superconductors

Berry Phase in Condensed Matter Physics

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$$\Phi \equiv \gamma(C) = \int dS \cdot \mathbf{\Omega}$$

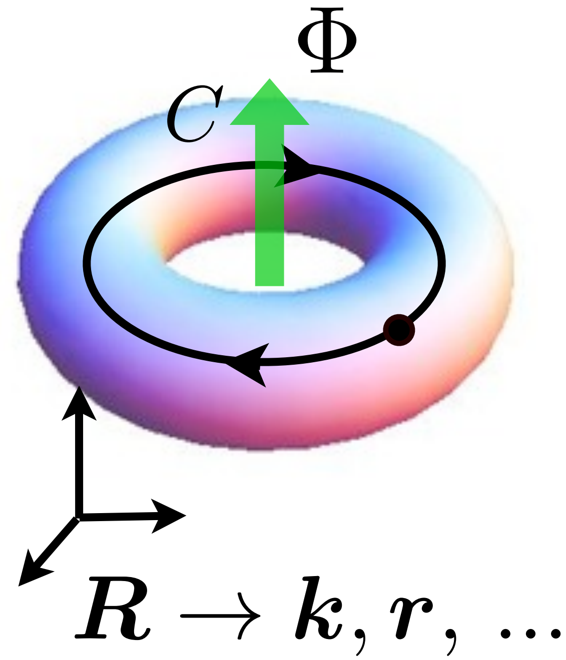
Berry curvature

(fictitious magnetic field)

$$\mathbf{\Omega} = \nabla_{\mathbf{R}} \times \mathbf{A}(\mathbf{R})$$

Berry connection

$$\mathbf{A}(\mathbf{R}) = i\langle\psi|\frac{\partial}{\partial \mathbf{R}}|\psi\rangle$$



Fictitious "magnetic" flux in *k*-space, or *r*-space

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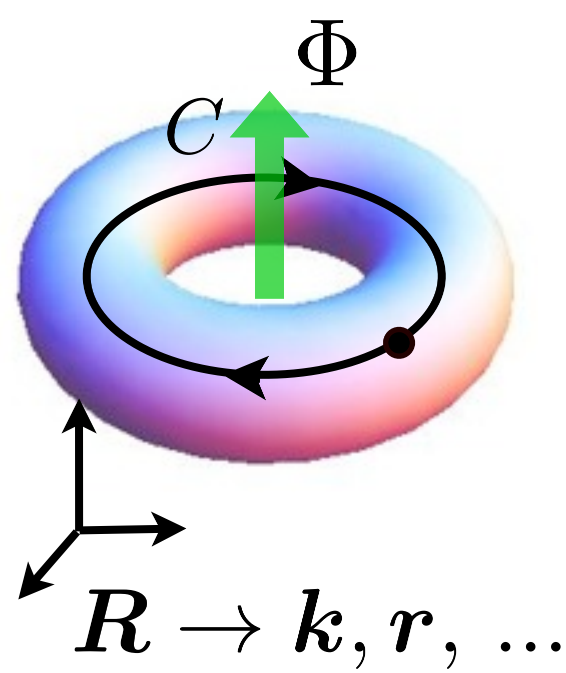

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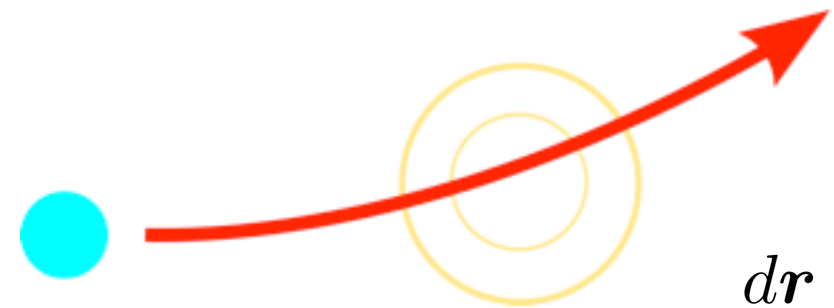
Berry Phase in Condensed Matter Physics

Berry curvature

(Effective Magnetic Field
in ***r***-space and ***k***-space)

Exotic Transport Phenomena



$$\frac{d\boldsymbol{r}}{dt} = \frac{1}{\hbar}\frac{\partial\varepsilon(\boldsymbol{k})}{\partial\boldsymbol{k}} + \frac{d\boldsymbol{k}}{dt} \times \boldsymbol{\Omega}_{kk}$$

$$\hbar\frac{d\boldsymbol{k}}{dt} = e\boldsymbol{E} + e(\boldsymbol{v} \times \boldsymbol{B}) + \boldsymbol{v} \times \boldsymbol{\Omega}_{rr}$$

Insulator/ Metal ...with SOI	Quantum (Spin) Hall Effect	Thouless, et al. /Kane, Mele
	Anomalous (Spin) Hall Effect	Karplus, Luttinger / Nagaosa et al. /Niu et al.
	Topological Hall Effect (Skyrmion)	Bruno et al.
Superconductor ...Chiral SC	Anomalous Quantum Thermal Hall Effect in Topological Superconductor (Majorana fermion)	Read, Green /Nomura et al./ Sumiyoshi, S.F.

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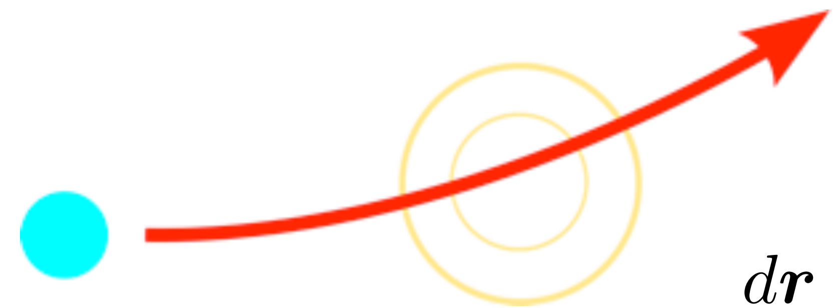

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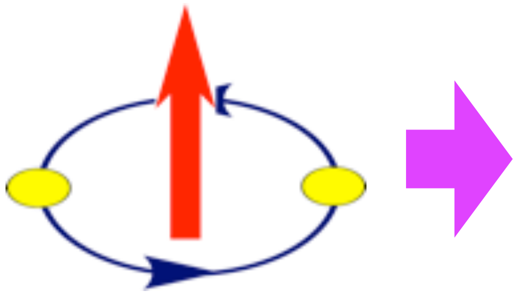
Chiral superconductor (SC) with broken time-reversal symmetry

Chiral Cooper pair :

$$p_x + ip_y \quad d_{x^2-y^2} + id_{xy} \cdots$$



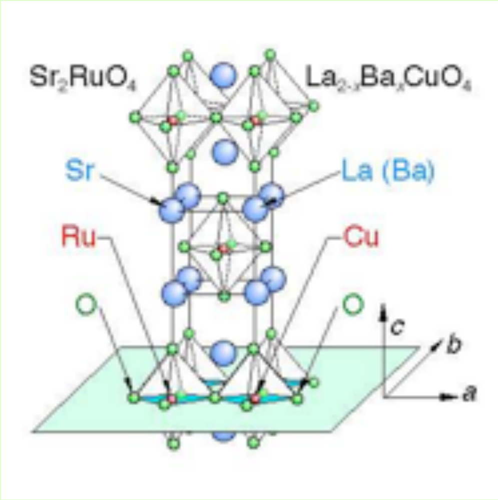
Non-zero Berry curvature

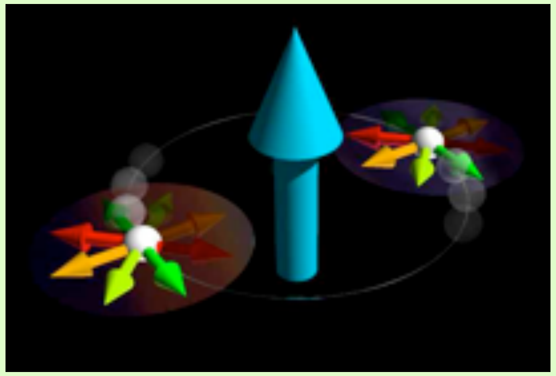
$$B(\boldsymbol{k}) = \nabla_{\boldsymbol{k}} \times \boldsymbol{A}(\boldsymbol{k})$$

Sr_2RuO_4

Chiral p_x+ip_y SC

Topological Superconductor





(Y. Maeno et al.)

non-zero Chern number

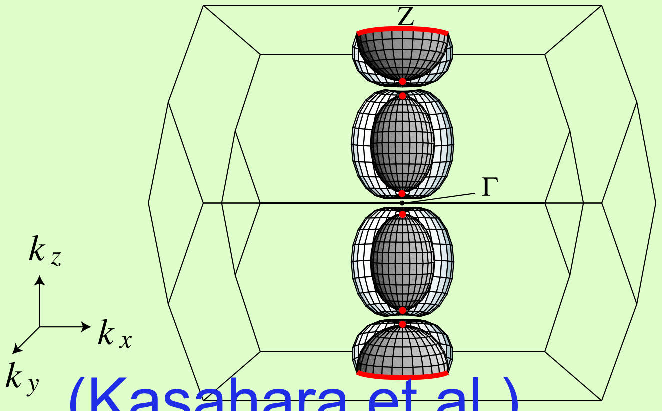
Superconductivity-analogue of QHE

Quantum thermal Hall effect

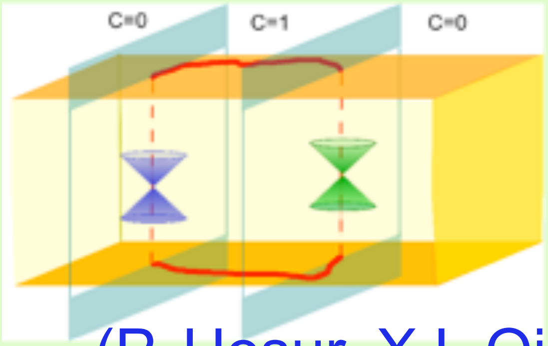
URu_2Si_2

Chiral d_{zx}+id_{yz} SC

Weyl Superconductor



(Kasahara et al.)



(P. Hosur, X.L.Qi)

Majorana arc on surface

Superconductivity-analogue of Weyl semi-metal

Anomalous thermal Hall effect

Thermal anomalous Hall conductivity for superconductors

Thermal Hall effect due to Berry curvature (not due to magnetic field)

$$\kappa_{xy}^{tr} = \kappa_{xy}^{\mathrm{Kubo}} + \frac{2M_E}{TV}$$

(here, we put $\mu = 0$.
Then, heat current is equal
to energy current. justified for SCs)

$$\kappa_{xy}^{tr} = -\frac{1}{TV}\int \mathrm{d}E\, E^2 \sum_{\substack{kn \\ E_{kn}\le E}} \underbrace{\mathrm{Im}\left\langle \frac{\partial u_{kn}}{\partial k_x} \middle| \frac{\partial u_{kn}}{\partial k_y} \right\rangle}_{\text{Berry curvature}} f'(E)$$

$|u_{kn}\rangle$: BdG eigen functions

In the low-*T* limit,

$$\kappa_{xy}^{tr} = \frac{C_1(0)}{2}\frac{\pi^2 k_B^2 T}{6\pi\hbar}$$

$C_1(0)$ is 1st Chern number, *when there is no gap-node*.

$$C_1(E) \equiv \sum_n \int \frac{\mathrm{d}^2 k}{\pi}\, \mathrm{Im}\left\langle \frac{\partial u_{kn}}{\partial k_x} \middle| \frac{\partial u_{kn}}{\partial k_y} \right\rangle \Theta(E - E_{kn})$$

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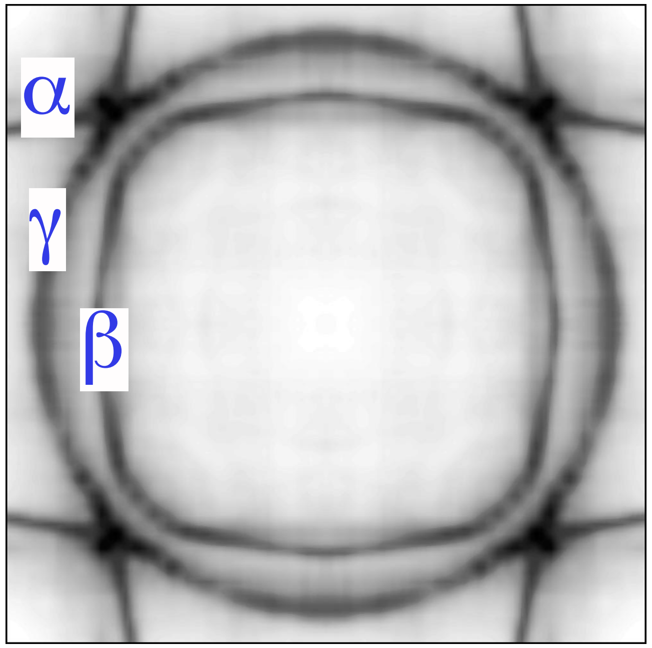
note that 1/2 of the value of Chern insulator (IQHE state)

Application to Sr_2RuO_4

-Chiral p+ip Superconductor-

Fermi surfaces (3-bands)



(MacKenzie and Maeno, R.M.P)

assuming full-gapped p+ip state

$$C_1^{\alpha}(0) = -2$$

$$C_1^{\beta}(0) = 2$$

$$C_1^{\gamma}(0) = 2$$

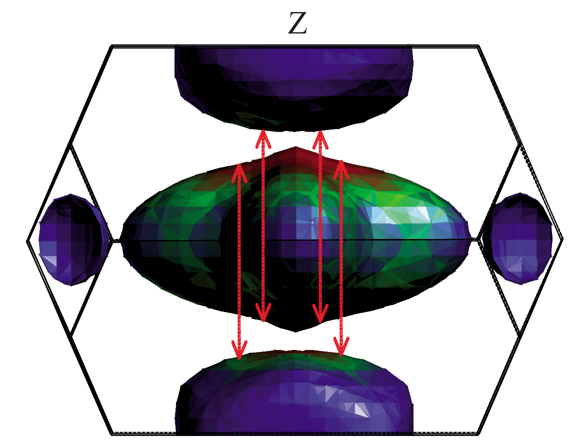
$$\kappa_{xy}^{tr} = \frac{1}{2}\frac{\pi^2 k_B^2 T}{6\pi\hbar}(2+2-2) = \frac{\pi^2 k_B^2 T}{6\pi\hbar}$$

$$\sim 10^{-4}\ T\ (\mathrm{W/Km})$$

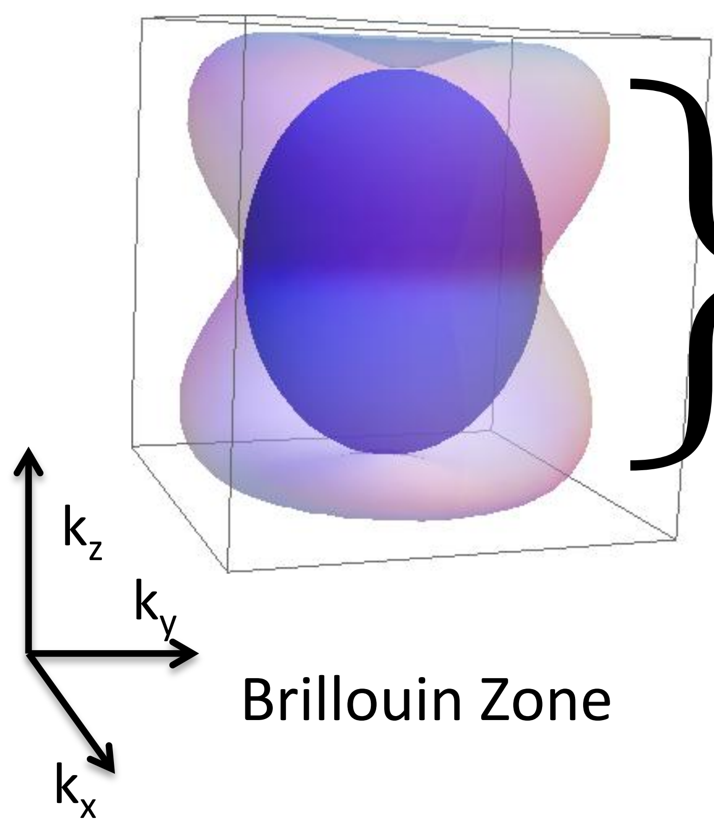
Application to URu_2Si_2

-Chiral $d_{xz}+id_{yz}$ Superconductor-

$$\Delta_k = \Delta \sin k_z (\sin k_x + i \sin k_y)$$
$$\xi_k = -2t(\cos k_x + \cos k_y) - 2t' \cos k_z + u$$



(Oppeneer et al.)



$\Big\} -4$

Brillouin Zone

The TKNN number of $k_x k_y$-plane

$Ch_1(k_z) = -2, \quad \text{for } -k_{zF} < k_z < k_{zF}$

topological phase tr.
at $k_z = \pm k_{zF}$

$Ch_1(k_z) = 0, \quad \text{for otherwise}$

Majorana arc on surface BZ

$$\frac{\kappa_{xy}}{T} = -\frac{\pi k_B^2}{12\hbar}\frac{4k_{zF}}{2\pi} \sim 10^{-4}\ T\ (\mathrm{W/Km})$$

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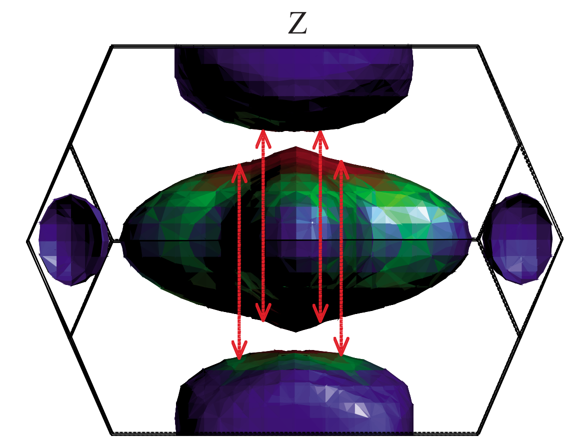

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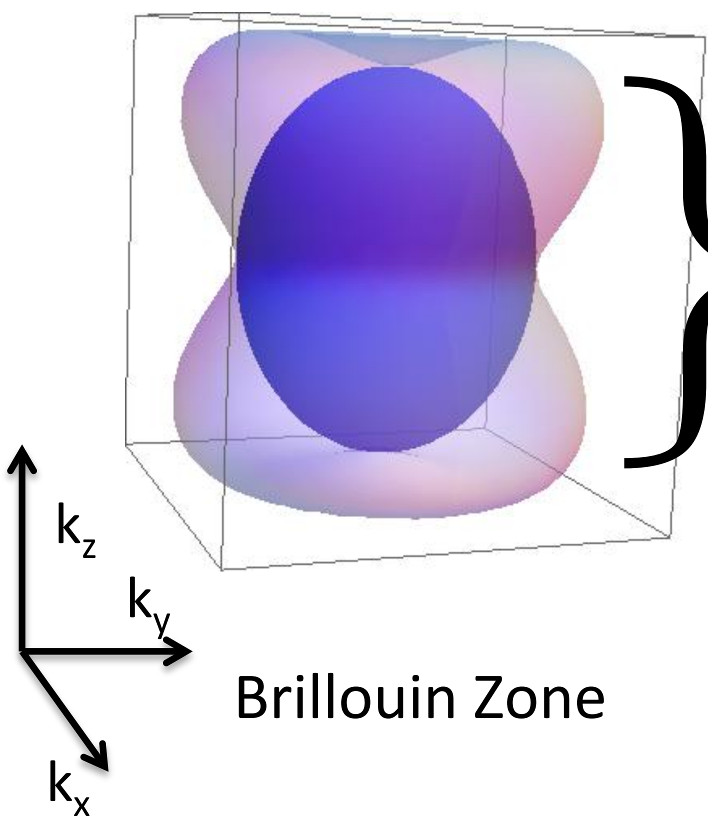

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➡ **experimental detection is very difficult**

Dynamical Berry Phase Effect ?

Does Fluctuation of Berry curvature raise novel effects ?

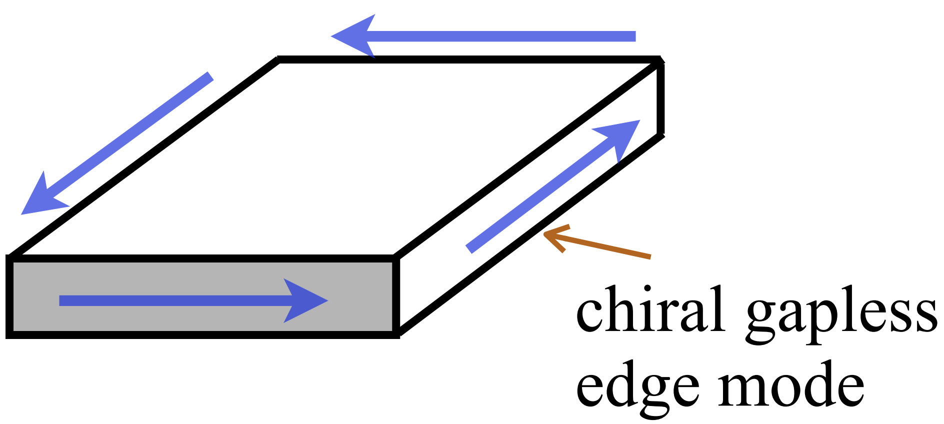
Realistic example :

Chiral Superconductor

p+ip wave pairing Sr_2RuO_4

d+id wave pairing URu_2Si_2

……



Non-zero Berry curvature associated with chiral SC order parameter below T_c

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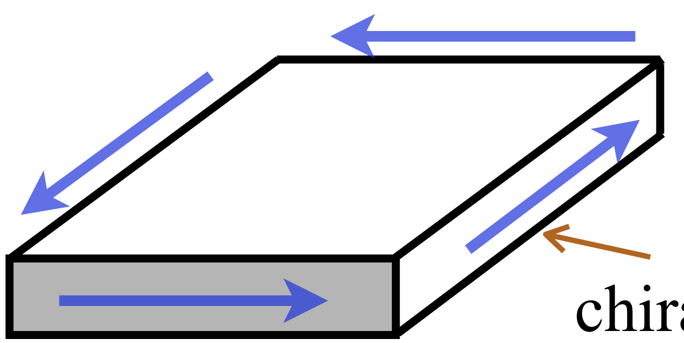


Non-zero Berry curvature associated with chiral SC order parameter below T_c

Fluctuations of SC order above and near T_c

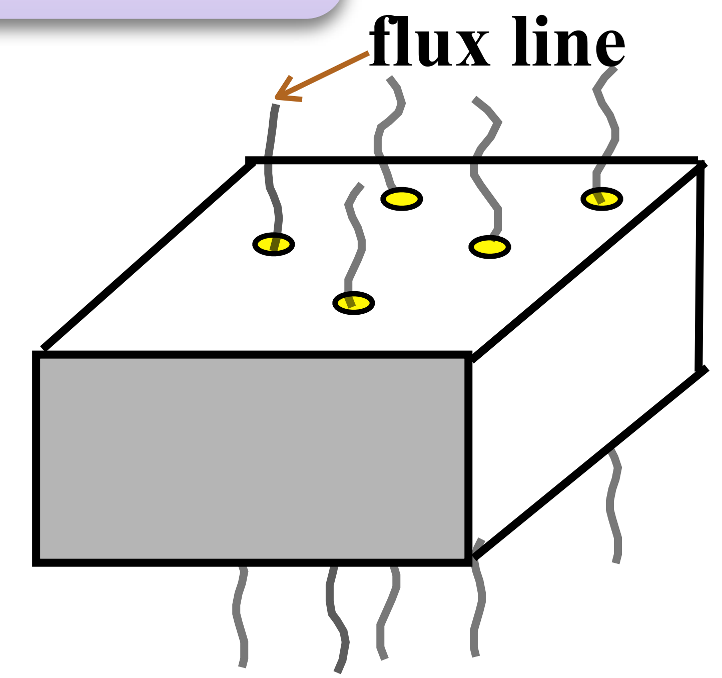
→

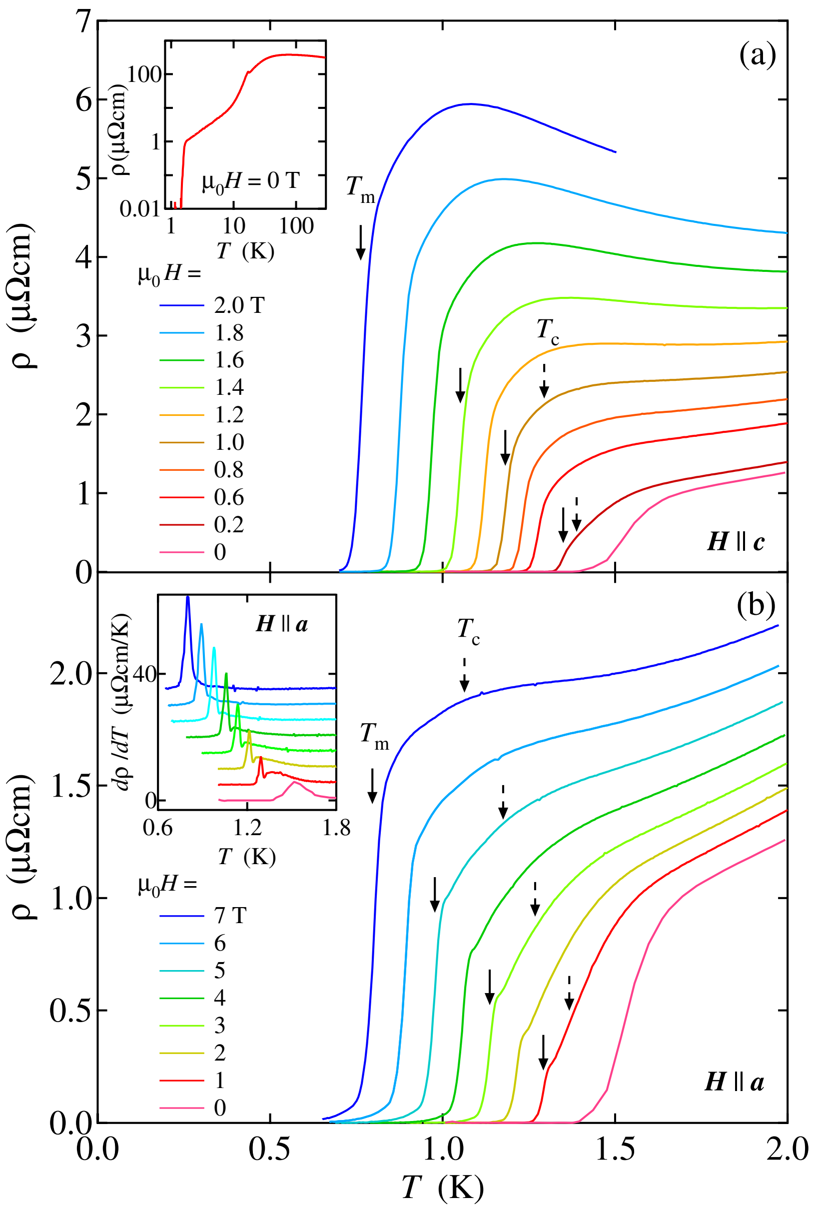
Berry Phase fluctuation !

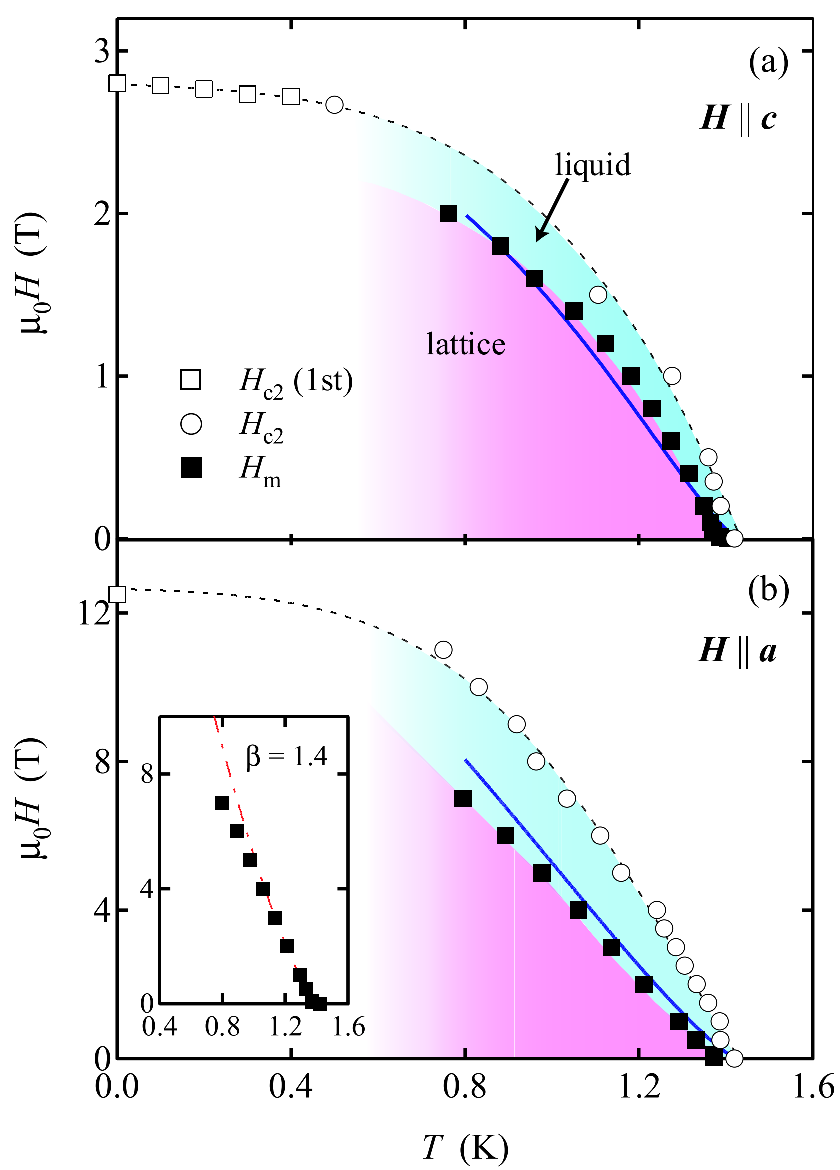
novel phenomena due to SC fluctuations accompanying topological Berry phase !

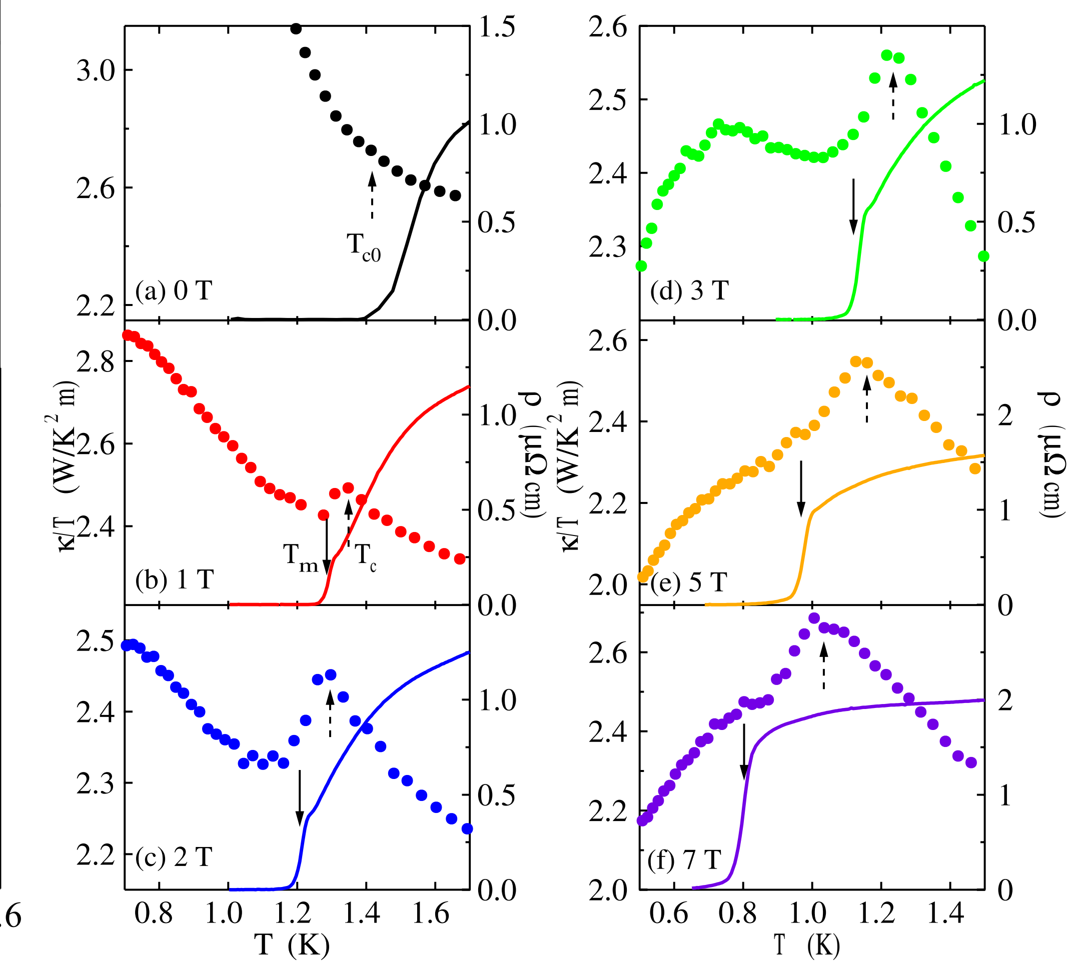
Strong superconducting fluctuations in URu_2Si_2

Flux line lattice melting ⟷ strong SC fluc.









(Okazaki et al., PRL 2008)

Strong superconducting fluctuations in URu_2Si_2

Ginzburg parameter : $G_i = [\epsilon k_B T_c / H_c(0)^2 \xi_a^3]^2/2$

ordinary SC : $G_i \sim 10^{-11} - 10^{-7}$

high Tc SC (YBCO) : $G_i \sim 10^{-2}$

URu_2Si_2 : $G_i \sim 10^{-4}$

because of small fermi energy due to heavy fermion state and reconstruction raised by hidden order

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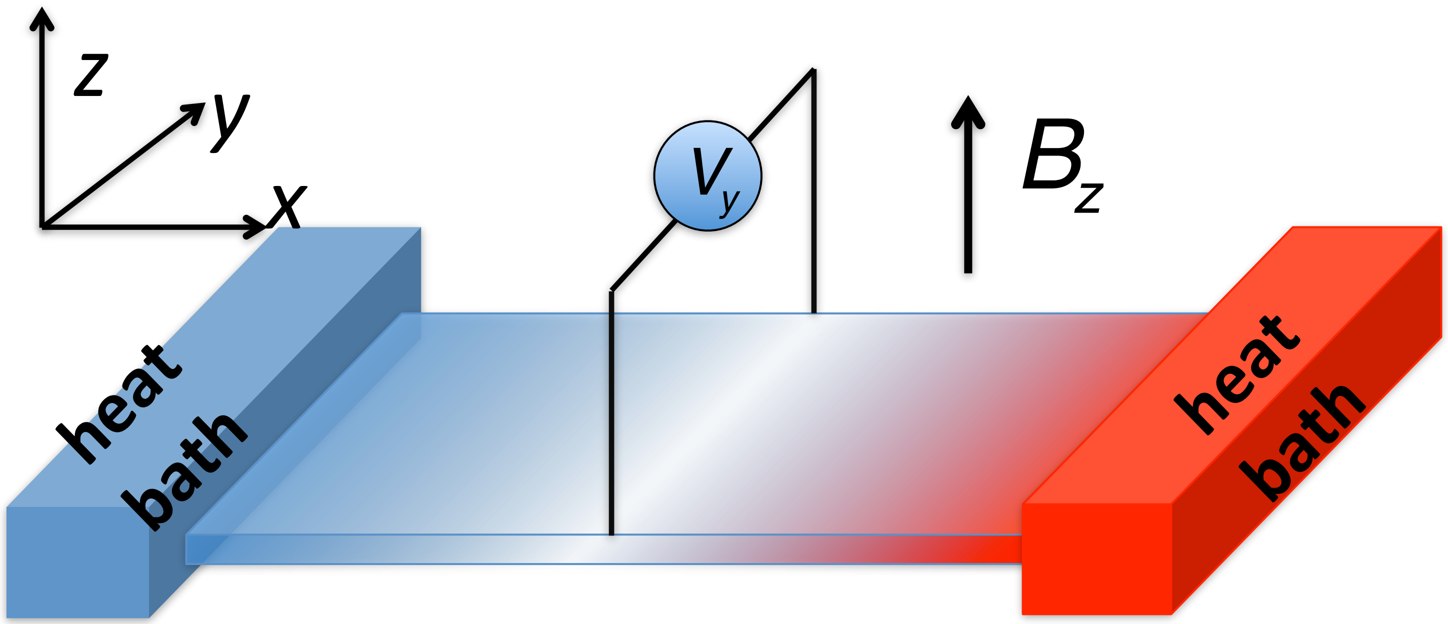
Heavy fermion superconductor with chiral SC order, URu_2Si_2 is suitable for investigation of Berry phase fluctuation phenomena

Nernst effect : a good probe for superconducting fluctuations



$$\nabla_x T \neq 0 \qquad \nabla_y T = 0$$

$$\vec{J}_e = 0 \Rightarrow E_y \neq 0$$

$$\vec{J}_e = \sigma \vec{E} + \underline{\alpha}(-\vec{\nabla} T)$$

σ, α : conductivity tensors

Nernst coefficient :

S : Seebeck coefficient

$$\nu := \frac{E_y}{B_z(-\nabla_x T)} = \frac{1}{B_z} \frac{\alpha_{xy}\sigma_{xx} - \alpha_{xx}\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2} = \frac{1}{B_z}\left(\frac{\alpha_{xy}}{\sigma_{xx}} - S \tan \Theta_H\right)$$

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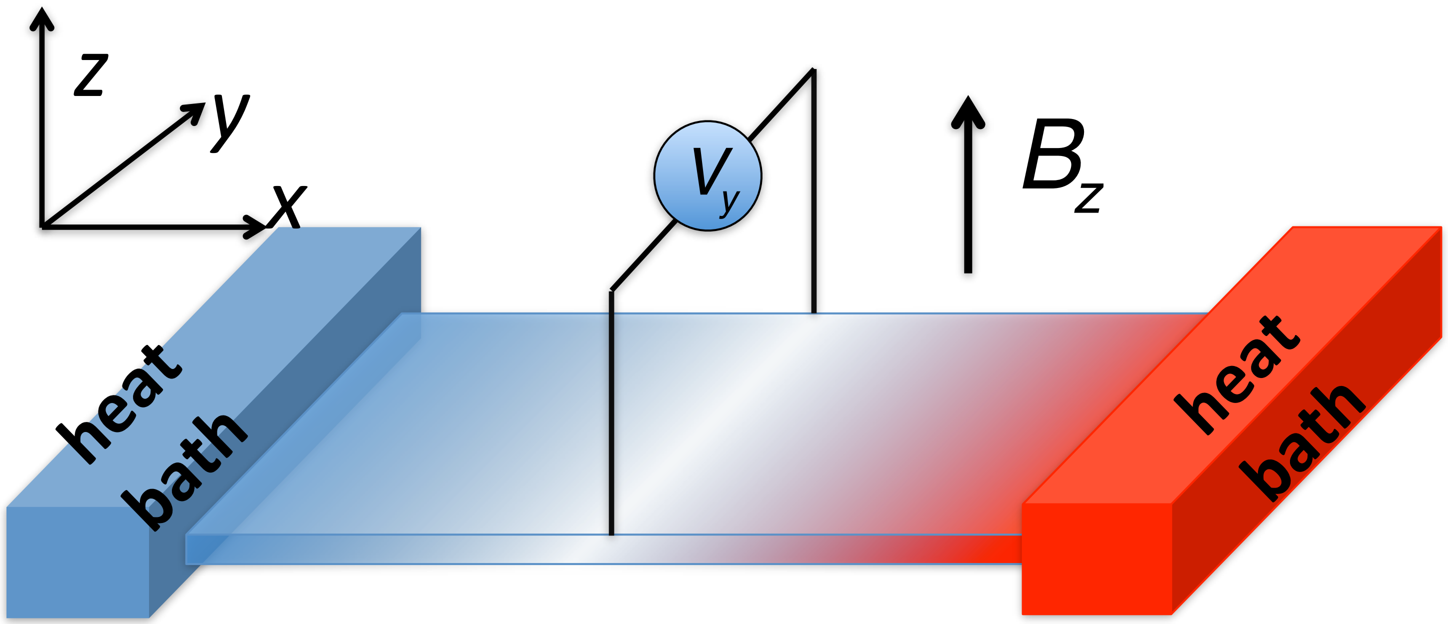

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α_{xy} : Nernst conductivity

σ_{xy} : Hall conductivity

in ordinary metal, cancel with each other

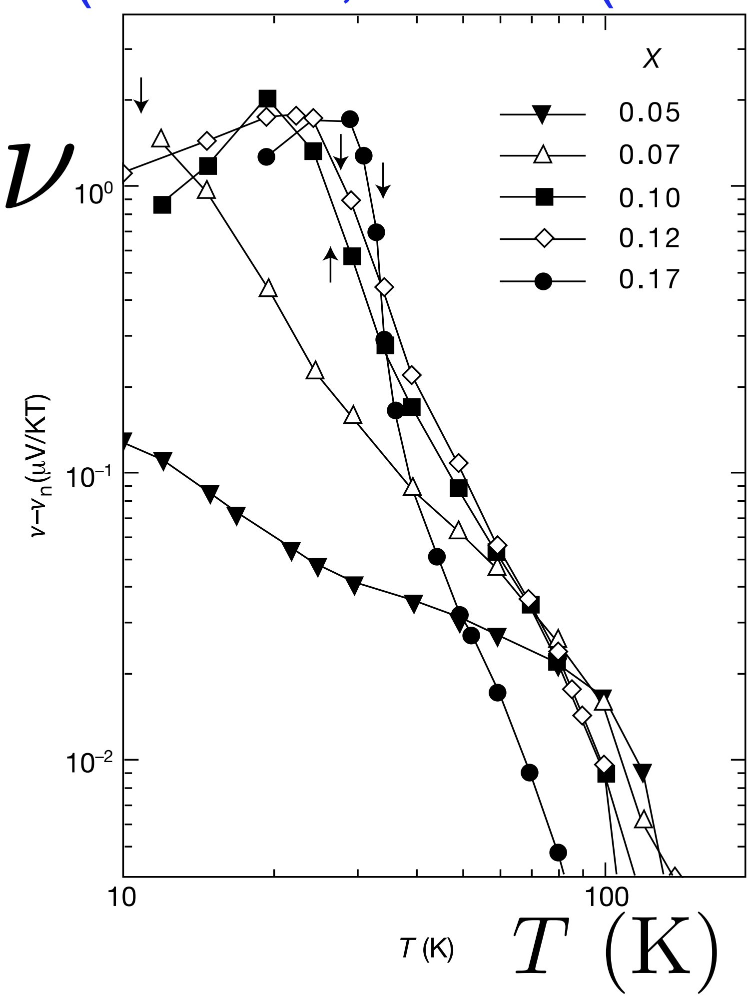
Thus, ν is, usually, very small

Sondheimer cancellation !

Giant Nernst effect due to superconducting fluctuations

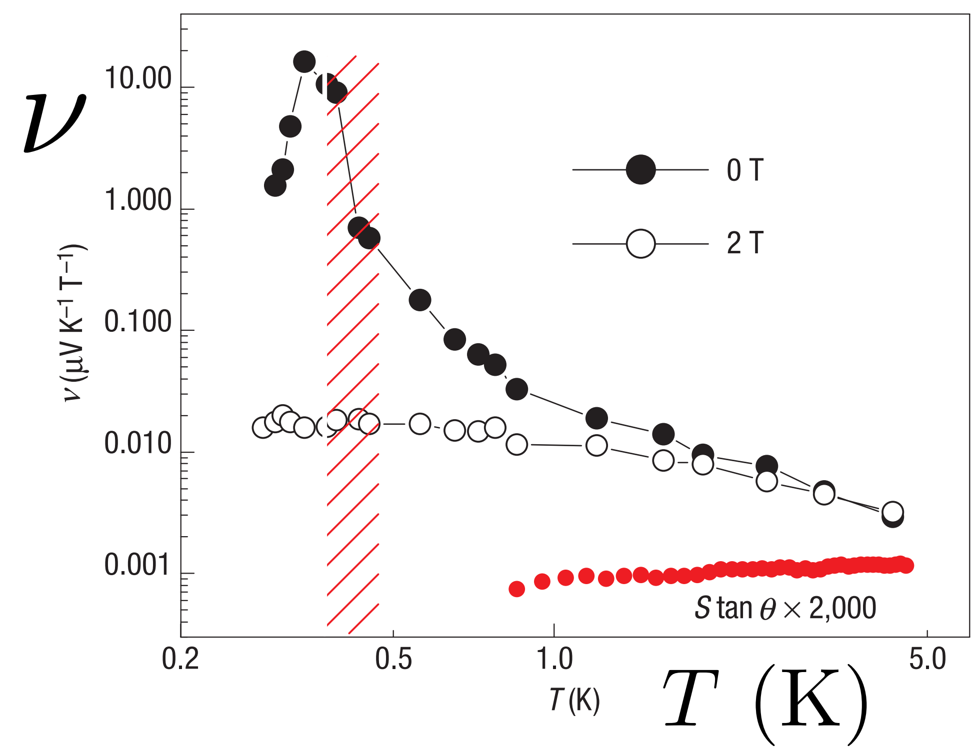
Pseudo-gap phase
of High-Tc cuprate

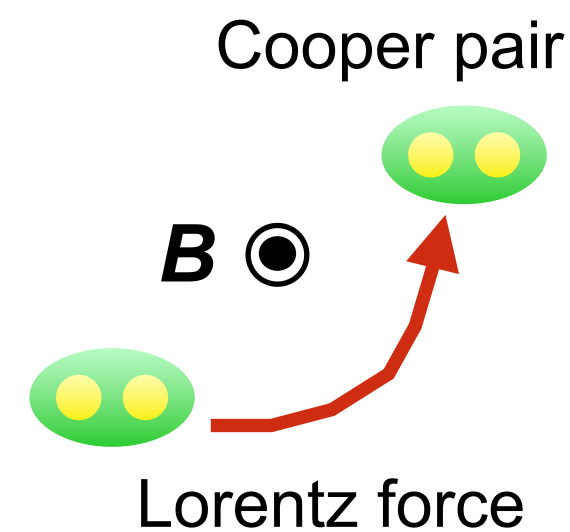
(Xu et al., Nature (2000))



NbSi thin film

(Pourret et al., Nature Physics (2006))





Nernst effect due to SC fluctuations

- short-lived Cooper pair
  (Aslamazov-Larkin type SC fluc.)
- vortex motion

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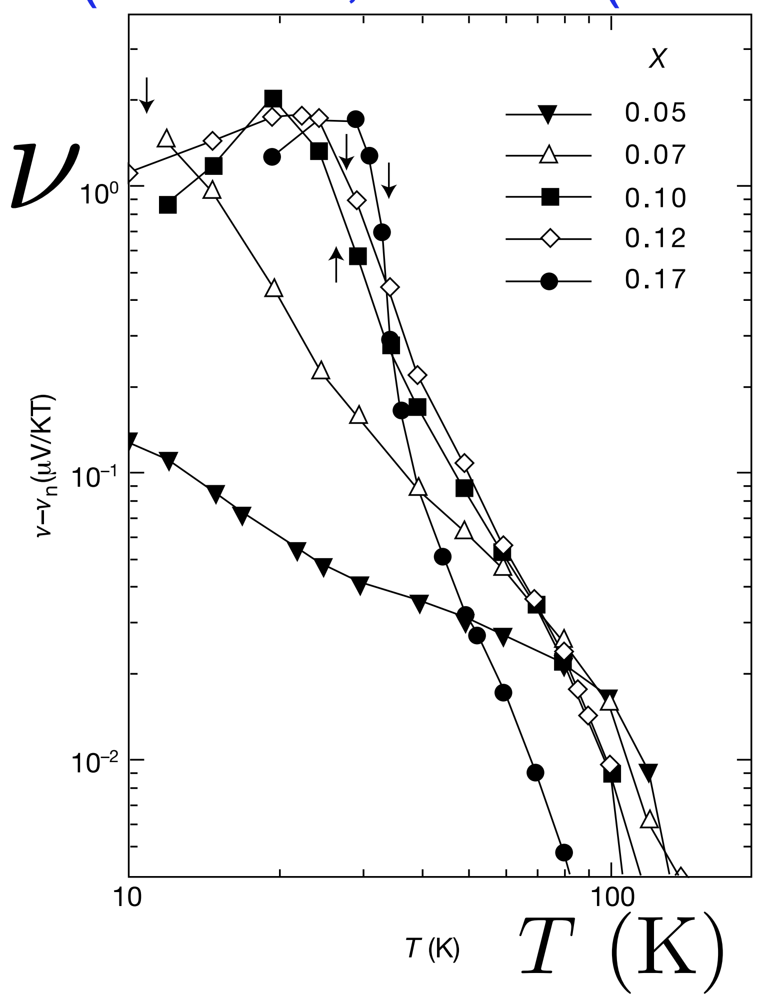

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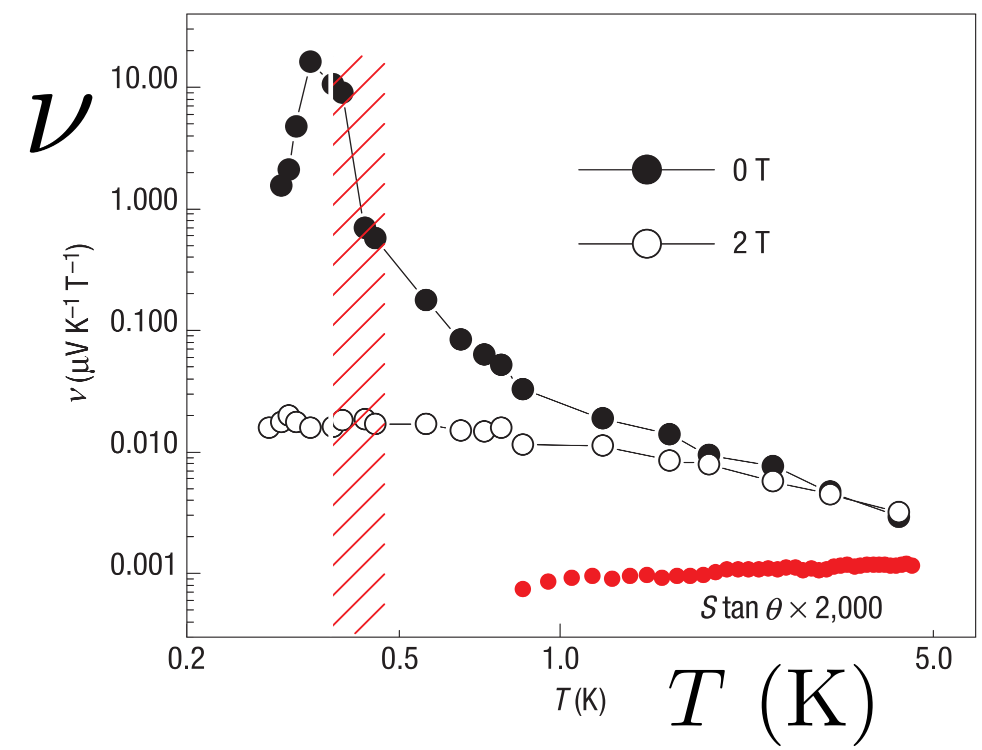

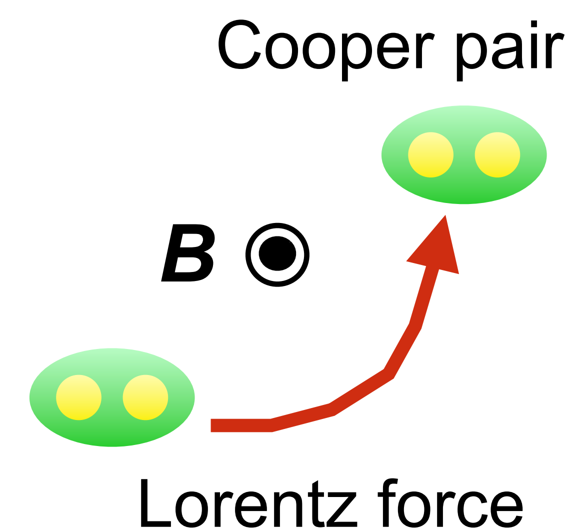

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$$\nu \sim \frac{\alpha_{xy}}{B_z \sigma_{xx}} \propto \frac{1}{\tau}$$

(τ : scattering rate)

smaller for cleaner samples

Novel mechanism : Berry phase fluctuation in chiral superconductors

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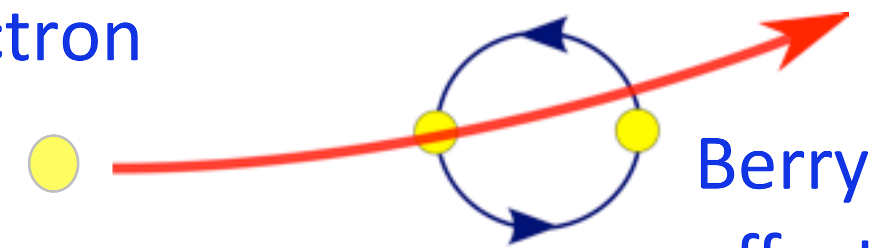
below T_c

Berry curvature due to chiral Cooper pairs

electron



Berry curvature : Ω_{kk}
effective magnetic field
in ***k***-space

e.g. anomalous
thermal Hall effect

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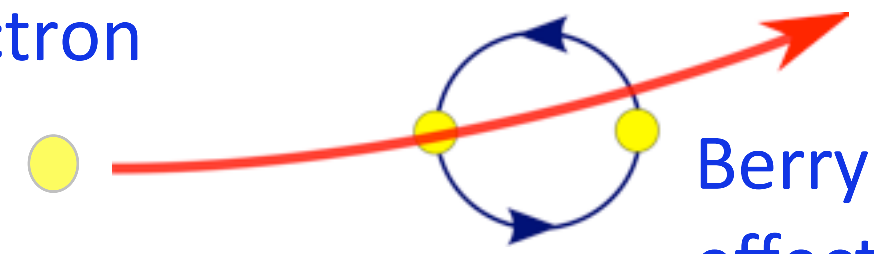

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above but near T_c

Berry curvature is zero, however ....

Novel mechanism : Berry phase fluctuation in chiral superconductors

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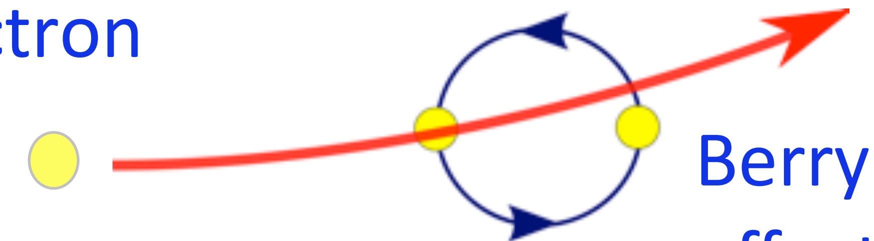

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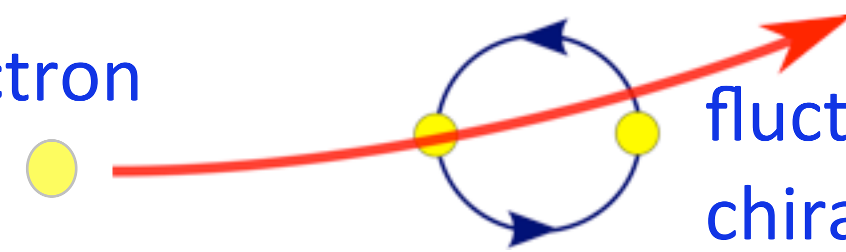
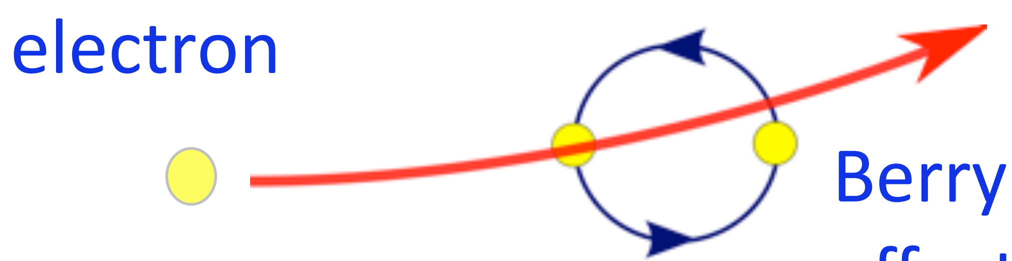


fluctuation of
chiral Cooper pair
(Berry phase fluctuation)

Novel mechanism : Berry phase fluctuation in chiral superconductors

below T_c

Berry curvature due to chiral Cooper pairs



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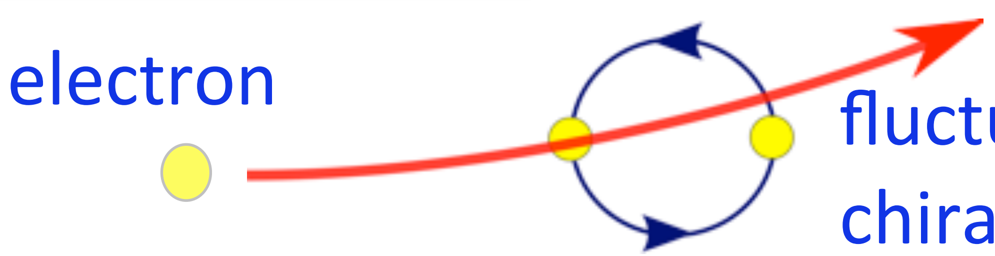


fluctuation of chiral Cooper pair
(Berry phase fluctuation)

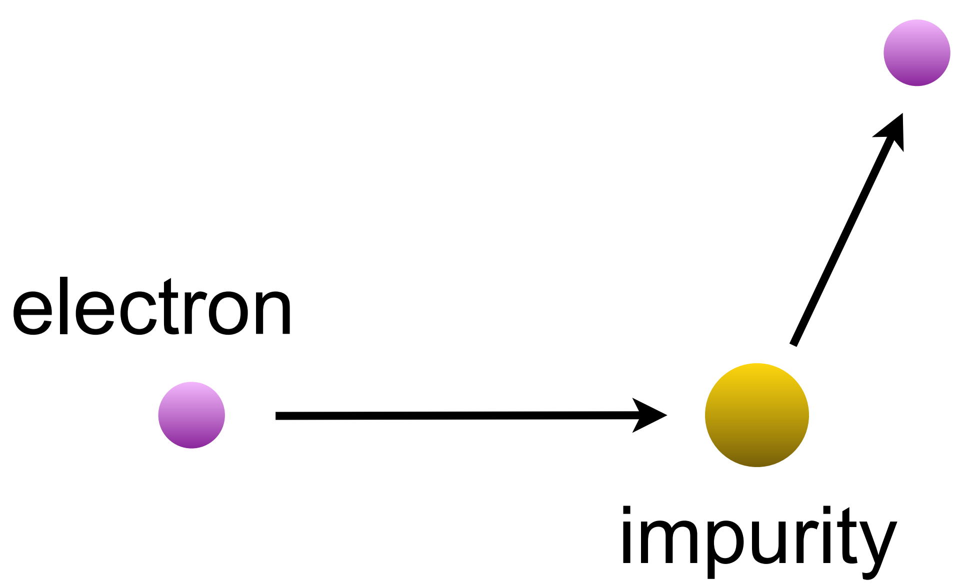
➡ ***asymmetric scattering due to Berry phase fluc.***

akin to side-jump, skew scatt. mechanism of AHE

Anomalous Nernst effect raised by dynamical skew-scattering due to chiral superconducting fluctuation (Berry-phase fluctuation)

Not due to Lorentz force !!

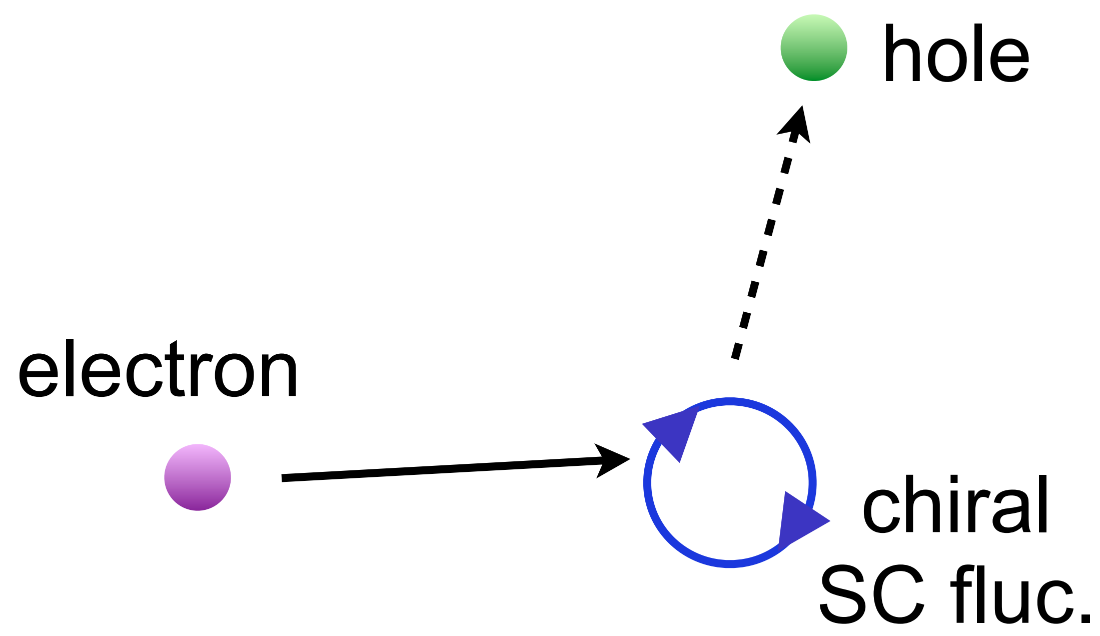
c.f. Skew-scattering mechanism of anomalous Hall effect due to SO int.



Anomalous Nernst effect raised by dynamical skew-scattering due to chiral superconducting fluctuation (Berry-phase fluctuation)

Anomalous Nernst effect raised by dynamical skew-scattering due to chiral superconducting fluctuation (Berry-phase fluctuation)

Andreev skew-scattering due to chiral superconducting fluctuation



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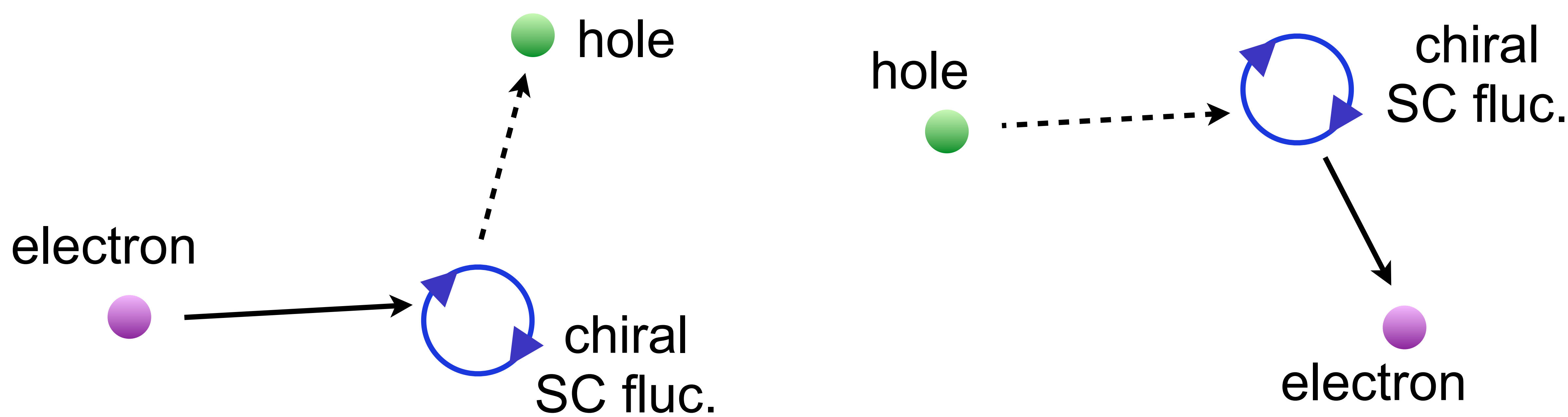
Andreev skew-scattering due to chiral superconducting fluctuation



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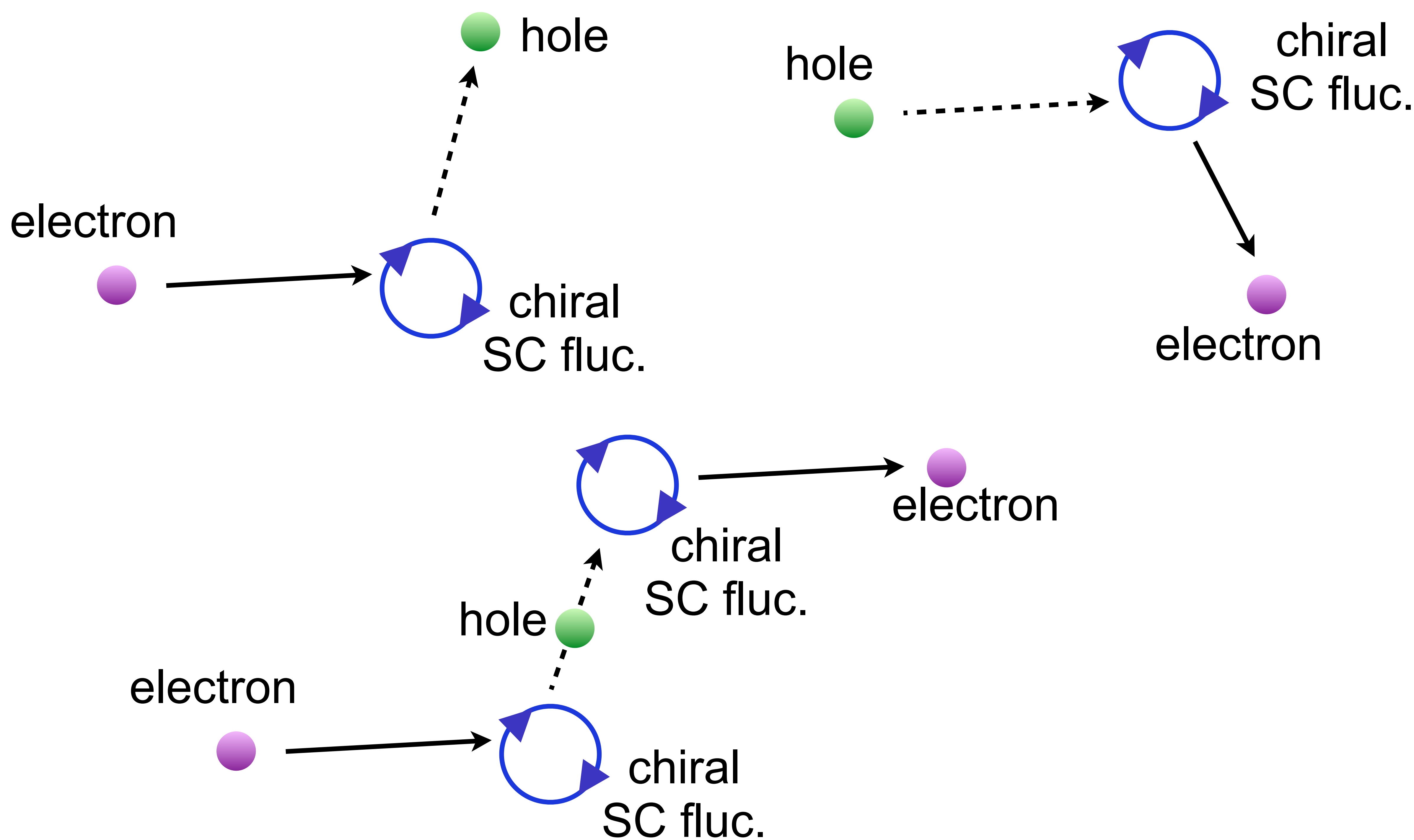

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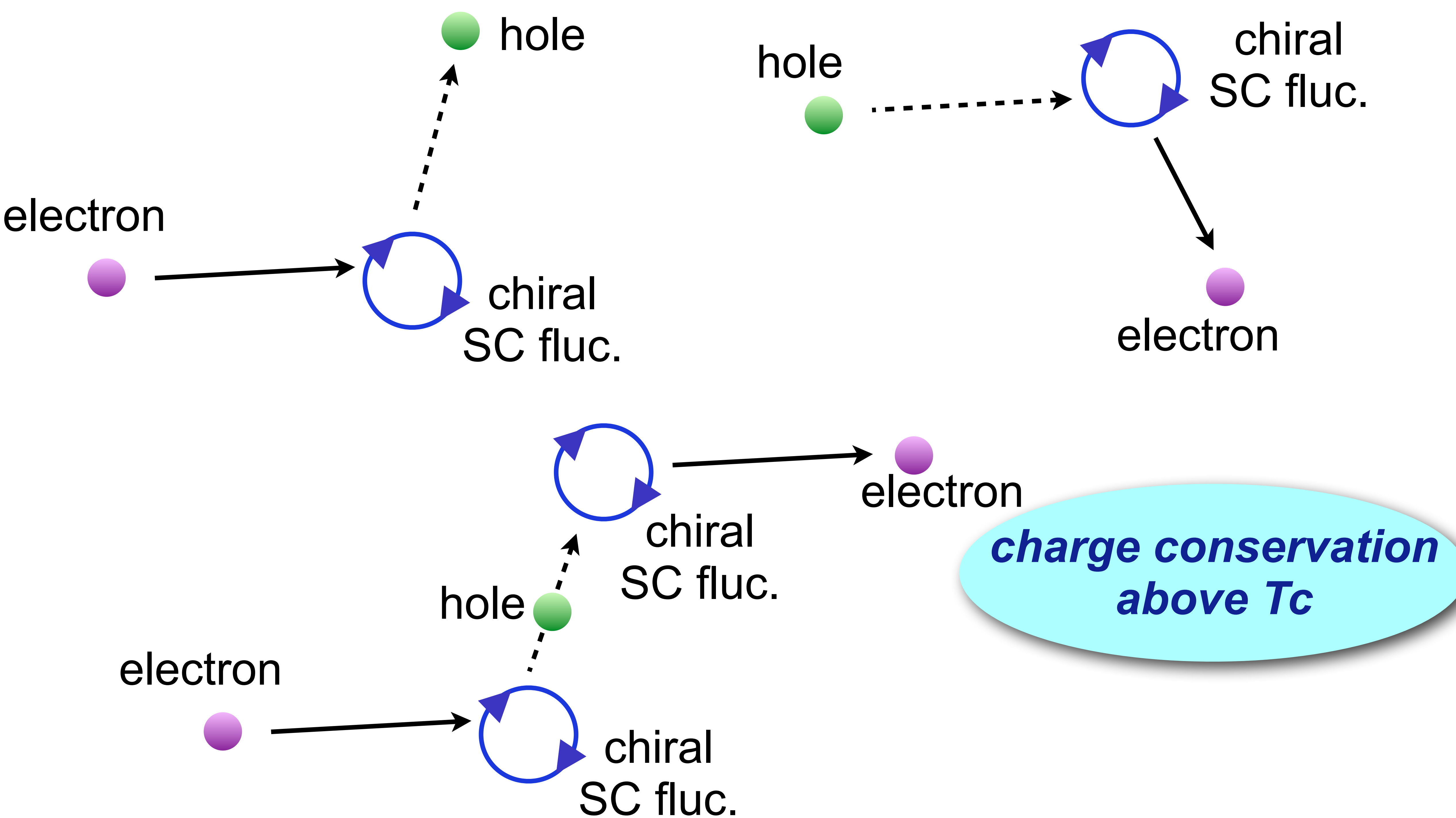

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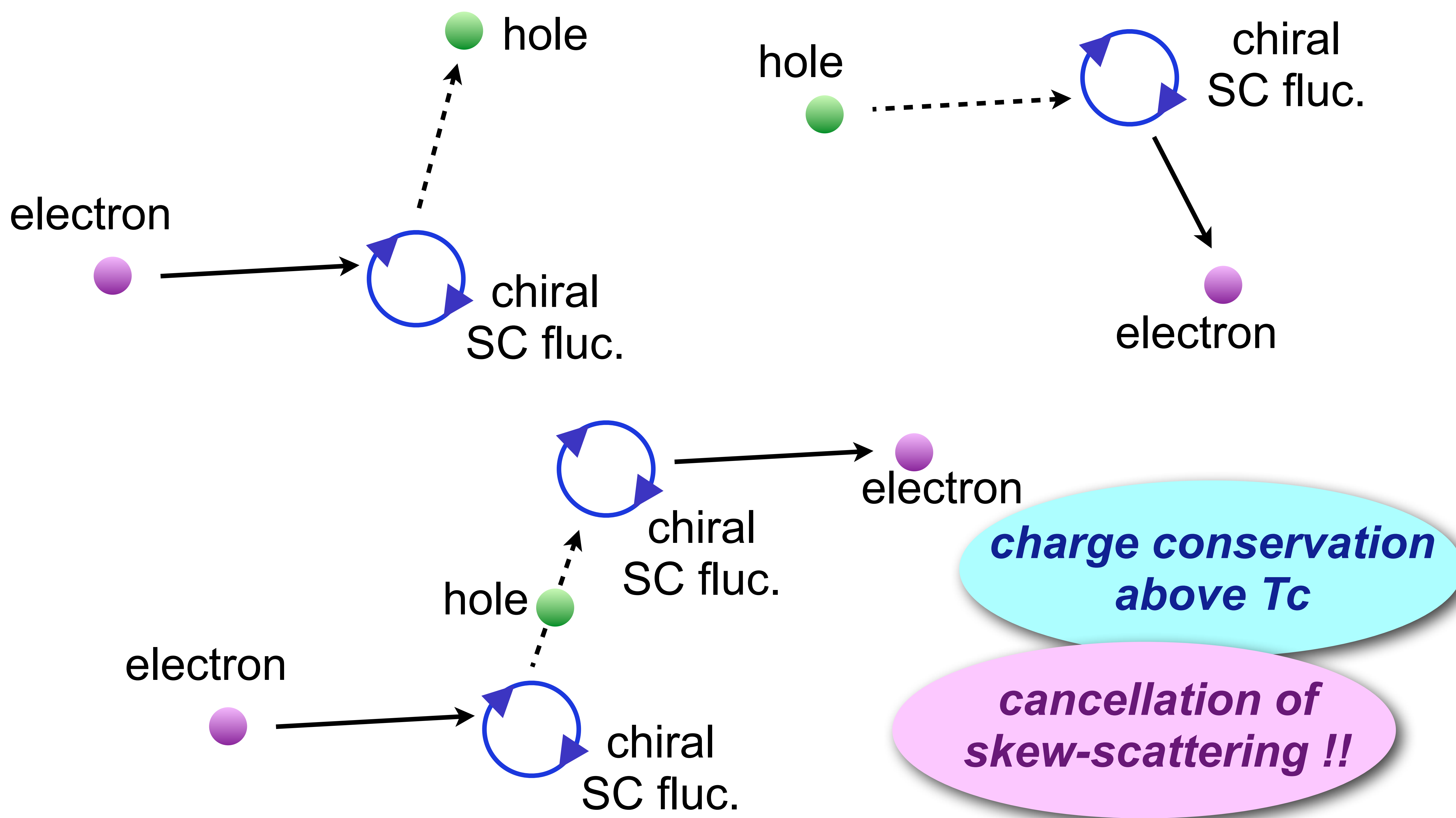


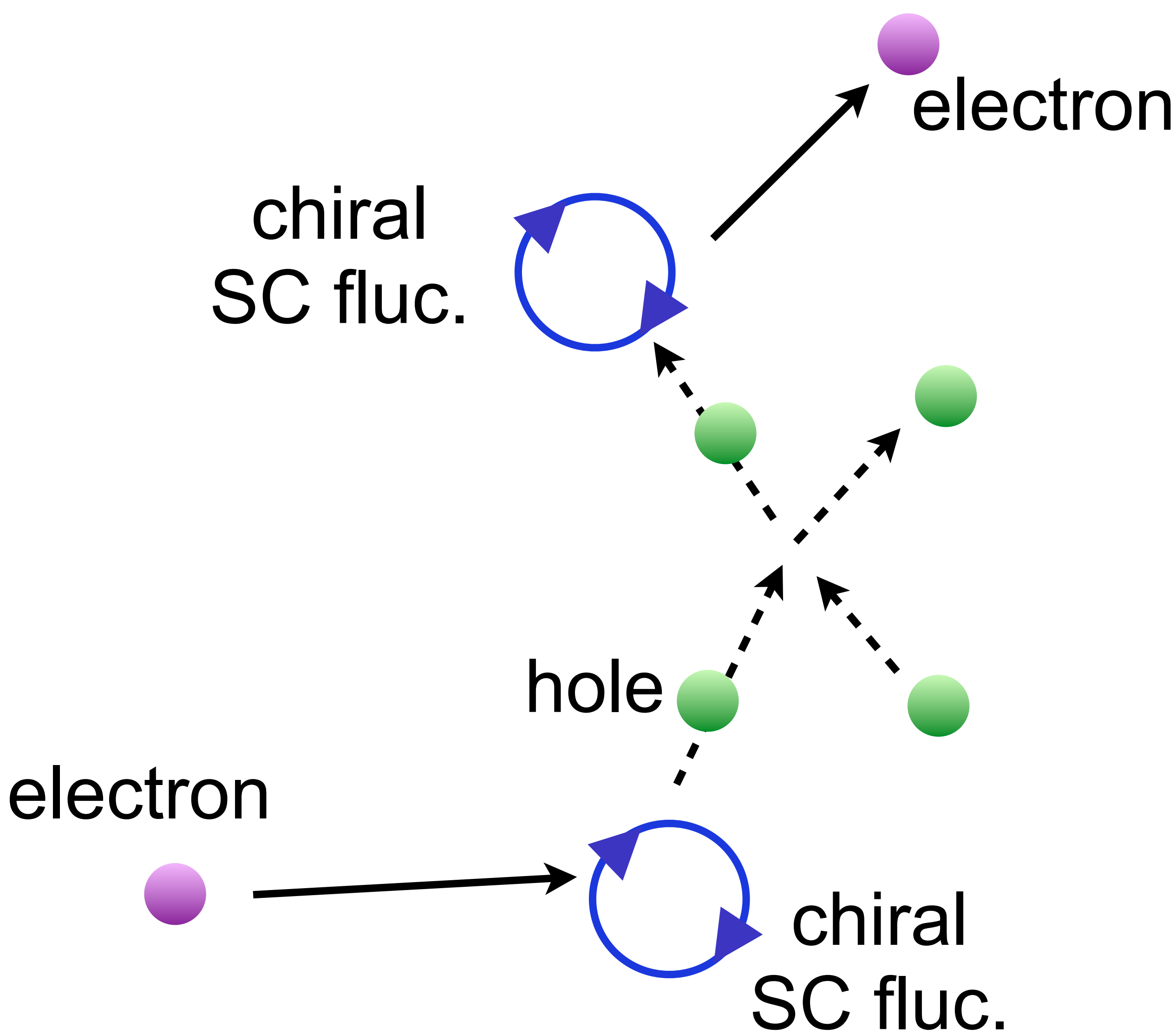
Anomalous Nernst effect raised by dynamical skew-scattering due to chiral superconducting fluctuation (Berry-phase fluctuation)

How to get around cancellation ?

Anomalous Nernst effect raised by dynamical skew-scattering due to chiral superconducting fluctuation (Berry-phase fluctuation)

How to get around cancellation ?

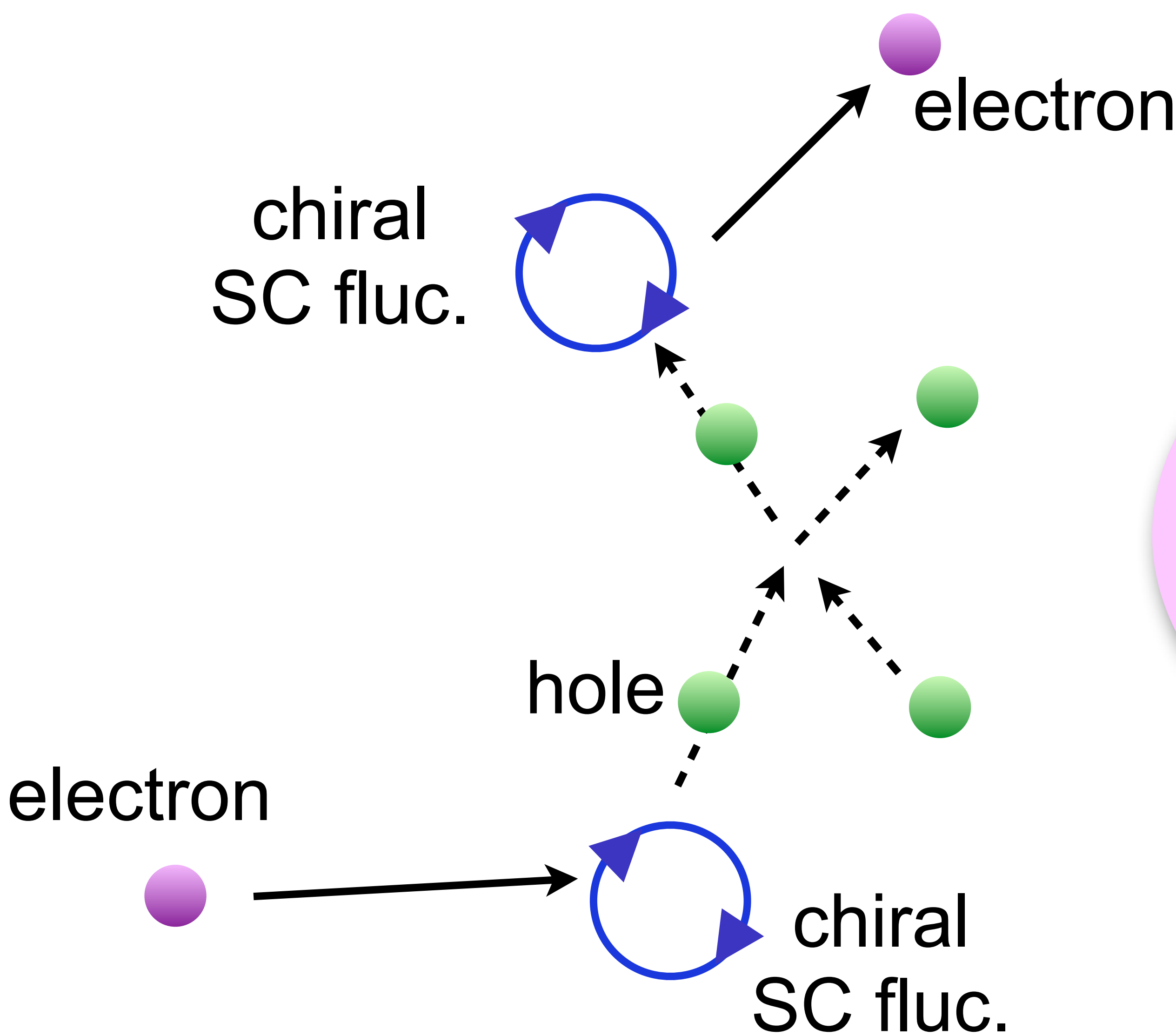
additional scatterings due e-e interaction, e-phonon int., impurity, etc. suppress cancellation



Anomalous Nernst effect raised by dynamical skew-scattering due to chiral superconducting fluctuation (Berry-phase fluctuation)

How to get around cancellation ?

additional scatterings due e-e interaction, e-phonon int., impurity, etc. suppress cancellation



Transverse transport due to dynamical skew-scattering (Berry phase fluctuation) !!

Model and Method

$$H = \sum_{p,\sigma} \xi(p) + \sum_{k,k',q} V_+(k,k') c^*_{k+q/2,\uparrow} c^*_{-k+q/2,\downarrow} c_{-k'+q/2,\downarrow} c_{k'+q/2,\uparrow}$$

$$+ [\, V_+ \to V_- \,]$$

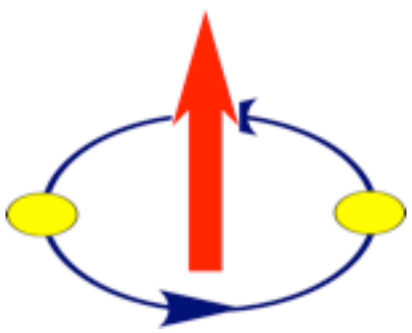
$$+ \sum_{k,k',q} W(q) c^*_{k-q\sigma} c^*_{k'+q\sigma'} c_{k'\sigma'} c_{k\sigma}$$

$$\xi(p) := p^2/2m - \mu$$

$$V_\pm(k,k') := -\lambda \phi_\pm(k) \phi^*_\pm(k')$$ effective attractive interaction

$$\phi_\pm(k) := \hat{k}_z(\hat{k}_x \pm \mathrm{i}\hat{k}_y)$$

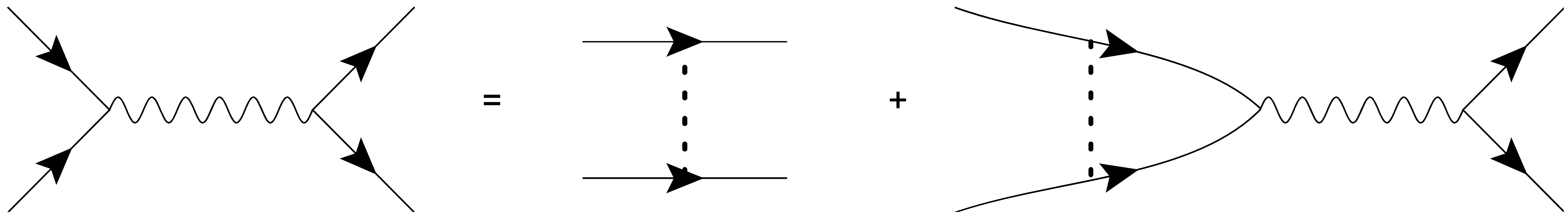
chiral $d_{zx} + id_{yz}$ ($d_{zx} - id_{yz}$) **SC pairing**

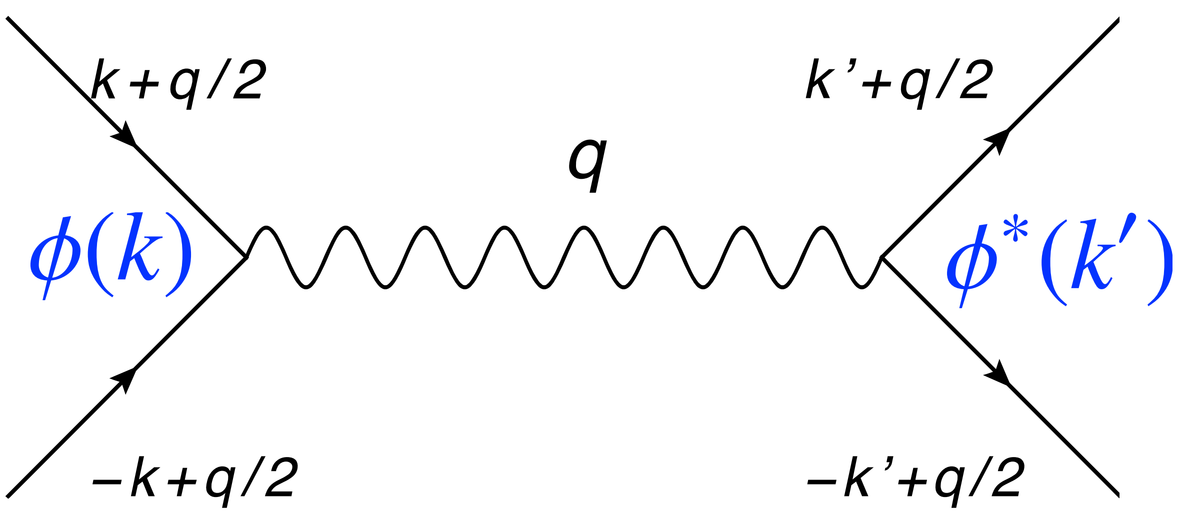


$W(q)$ effective repulsive interaction due to e-e int.

Superconducting Fluctuation Propagator

Gaussian Fluctuation propagator





Fluctuation Propagator :

$$L_{kk'}(q,\omega_l) = \phi(k)\phi(k')L(q,\omega_l)$$

$$L^{-1}(q,\omega_l) = -\tilde{a}\nu_{DOS}\left(\varepsilon + \frac{\pi|\Omega_l|}{8T} + \sum_{i=x,y,z}\xi_i q_i^2\right)$$

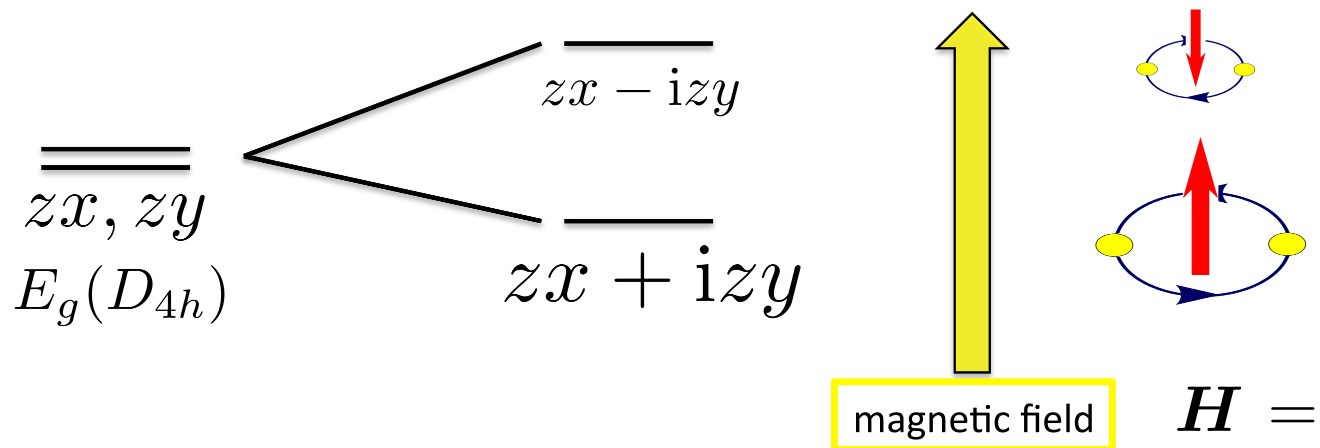
$$\varepsilon := \log(T/T_C)$$

$\phi(k) := \hat{k}_z(\hat{k}_x + \mathrm{i}\hat{k}_y)$: chiral d+id SC paring ***k***-dependence

Polarization of chirality fluc. due to magnetic field

TRS is ***not*** broken spontaneously above T_c. Net chirality fluc. vanish.

However Applied magnetic field induces polarization of chiral SC fluc. !!



$\boldsymbol{H} = (0, 0, H)$

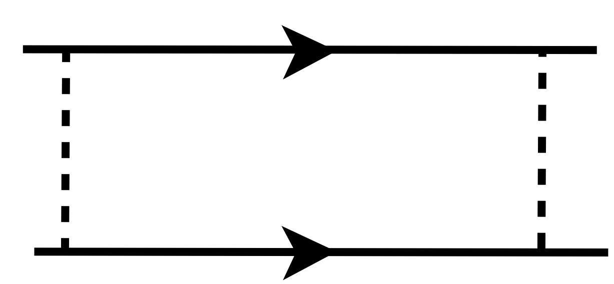
SC fluctuation propagator :

$$\tilde{L}_C^{-1}(\boldsymbol{x}, \boldsymbol{y}, \omega_q; H) = -\frac{\delta(\boldsymbol{x}-\boldsymbol{y})}{g} + \tilde{\Pi}_C(\boldsymbol{x}, \boldsymbol{y}, \omega_q; H)$$

$C = \pm 1$: chirality

$$\tilde{\Pi}_C(\boldsymbol{x}, \boldsymbol{y}, \omega_q; H) = e^{-\mathrm{i}2e\Phi(\boldsymbol{x},\boldsymbol{y})}\left[\Pi(\boldsymbol{x}-\boldsymbol{y}, \omega_q; H) - \underline{C\frac{5eH}{4k_F^2}\Pi'(\boldsymbol{x}-\boldsymbol{y}, \omega_q; H)}\right]$$

$$\Phi(\boldsymbol{x}, \boldsymbol{y}) = \int_{\boldsymbol{x}}^{\boldsymbol{y}} \boldsymbol{A}(\boldsymbol{r}) d\boldsymbol{r}$$

$\tilde{\Pi}_C(\boldsymbol{x}, \boldsymbol{y}, \omega_q; H) =$ 

Nernst Conductivity

Neglecting Seebeck coefficient which is small for URu_2Si_2

$$\nu = \frac{1}{B_z} \frac{\alpha_{xy}}{\sigma_{xx}}$$

$$\boxed{\alpha_{xy} = \alpha_{xy}^{\mathrm{Kubo}} - \frac{M_z}{TV}}$$

(Xiao et al. (2006), Qin, Niu, Shi (2011))

M_z : magnetization

$$\alpha_{xy}^{\mathrm{Kubo}} = \lim_{\omega \to 0} \frac{1}{T\omega} \int_0^\infty dt e^{i\omega t} \langle [\underbrace{J_{Qx}(t)}_{\text{heat current}}, \underbrace{J_y(0)}_{\text{electric current}}] \rangle$$

It turns out $\underbrace{\alpha_{xy}^{\mathrm{Kubo}} \sim \tau^2}_{\text{dominant for clean systems}}$ $\quad M_z/TV \sim O(\tau^0)$

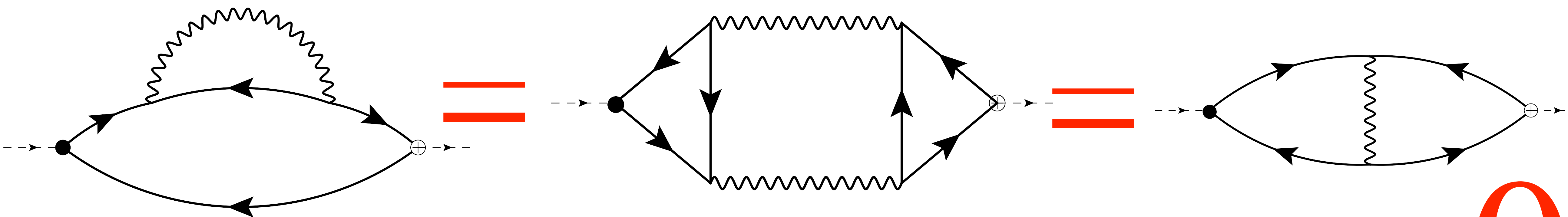
(τ relaxation time)

Cancellation of Andreev skew-scattering

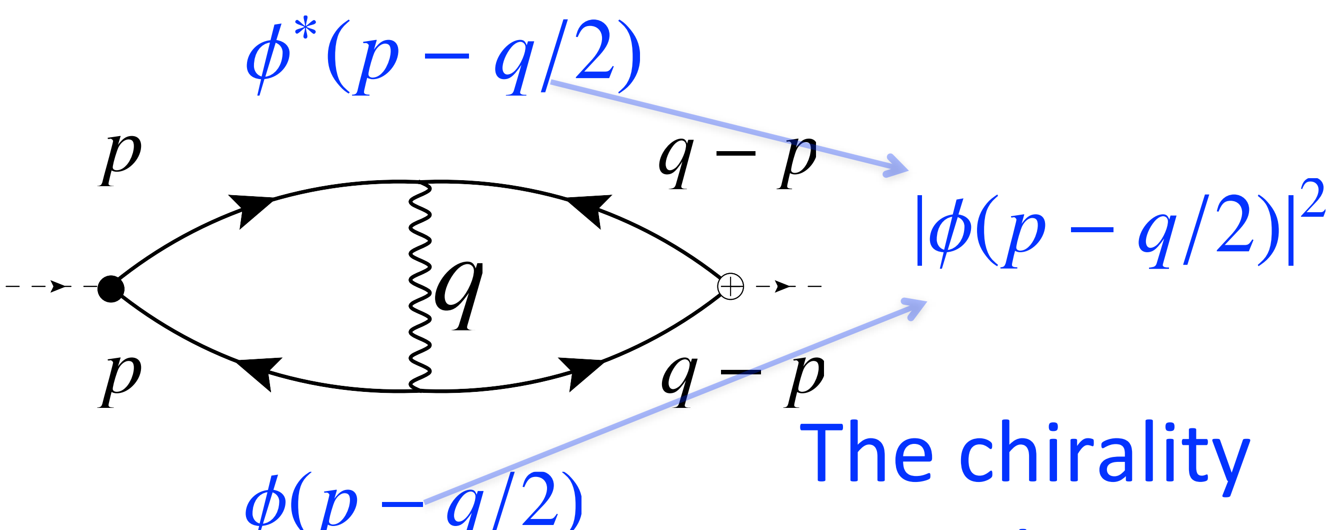
AL, MT, and DOS diagrams for $\alpha_{xy}^{\mathrm{Kubo}}$

AL

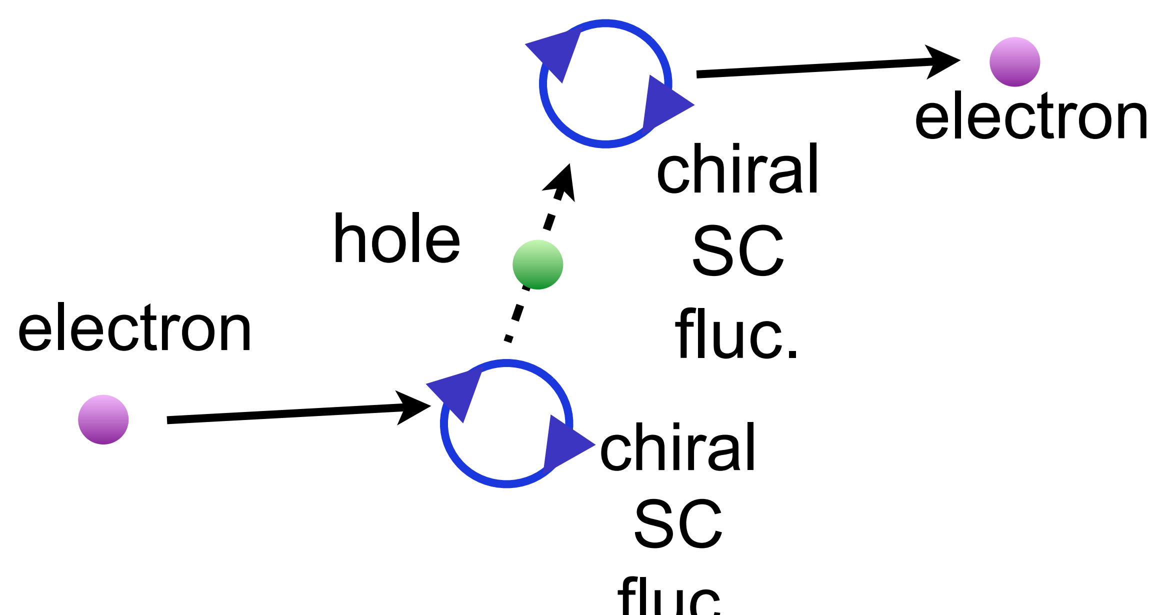
note: without Lorentz force



$= 0$

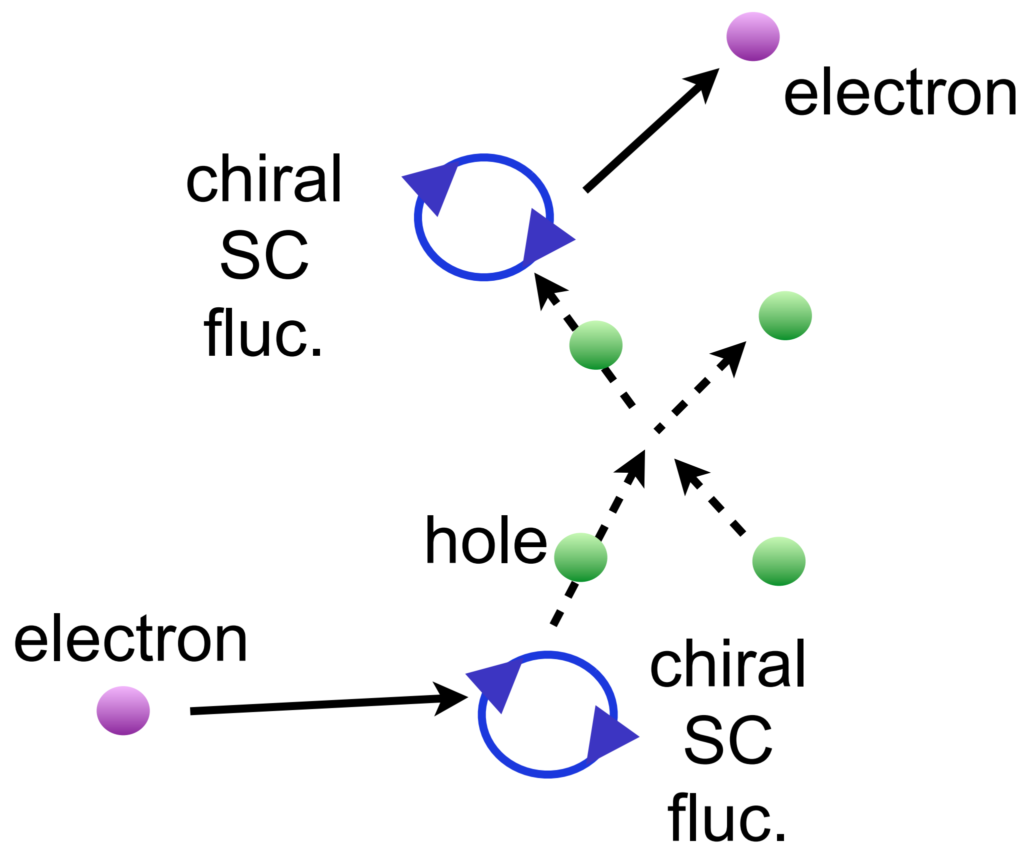


The chirality cancels out.



We need additional scattering processes suppressing the cancellation.

Disturbing the cancellation



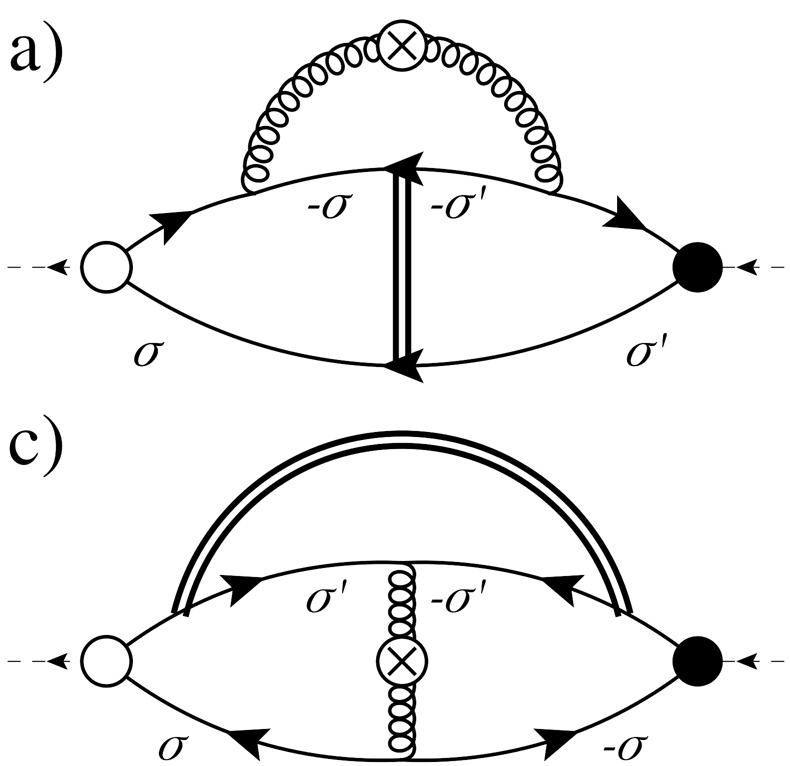
scattering due to short-range spin fluctuation

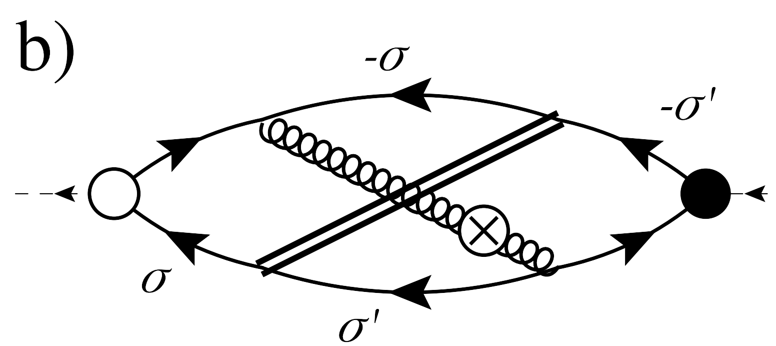
$$W(\boldsymbol{k},\omega_j) = W_0/(1+|\omega_j|/\Gamma)$$

$=$ 

c.f. Inelastic Neutron Scattering experiment for URu_2Si_2

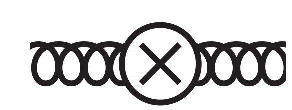
F. Bourdarot, et al., PRL(2003)





+ (Mirror Images)

chiral SC fluc. with chirality-polarization

$=$ 

Result

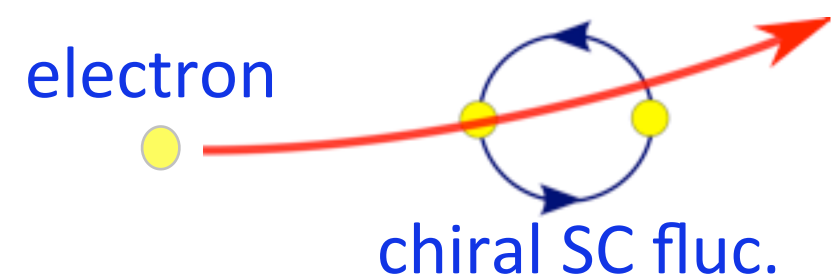
$$\boxed{\alpha_{xy} = \tau^2 \frac{e^2 \nu_{DOS} \Lambda^3 v_F^4 U^2 \chi(Q_0) f(\frac{\Omega_{AF}}{2\pi T}) B}{2304 k_F^2 \xi^2 T^3} \left(1 - \frac{3}{\xi^2 \Lambda^2} \log(T/T_C)\right)}$$

$\propto \tau^2$!

peak at $T=T_C$

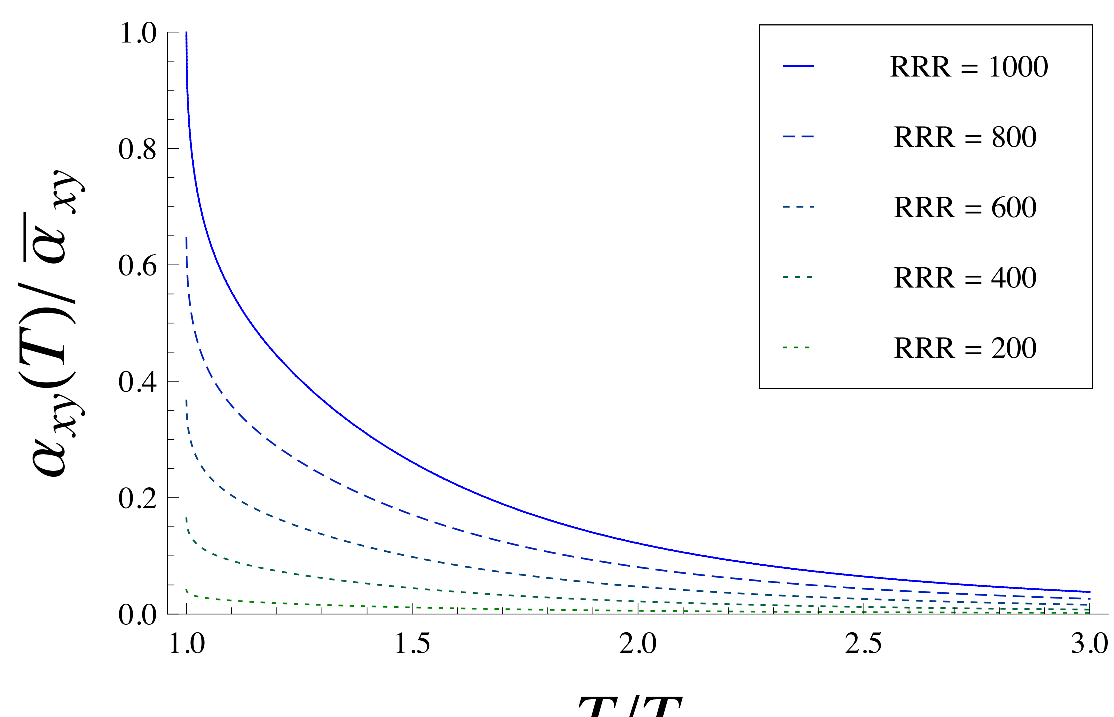
- not due to Lorentz force acting on electrons
- but raised by asymmetric scattering due to

chirality polarized SC fluc. (Berry phase fluc.) !!



✓ $\nu \propto \tau$

✓ Peak at $T=T_C$



Chirality fluctuation $\approx$ Berry-phase fluctuation

Berry curvature below and near T_c:

$$\Omega_k = \frac{\Delta_0^2 k_z^2 \varepsilon_k}{2[\Delta_0^2 k_z^2 (k_x^2 + k_y^2) + \varepsilon_k^2]} \propto \Delta_0^2$$

$d_{zx}+id_{yz}$ SC gap $\quad \Delta(\boldsymbol{k}) = \Delta_0 k_z (k_x + i k_y)$

normal, static $\quad \Delta_0 = 0 \rightarrow \Omega_k = 0$

normal, dynamical $\quad \Delta_0 = 0$, $\langle \Delta_0^2 \rangle \neq 0 \rightarrow \Omega_k \neq 0$

Chirality fluctuation ≈ Berry-phase fluctuation

Berry curvature below and near T_c:

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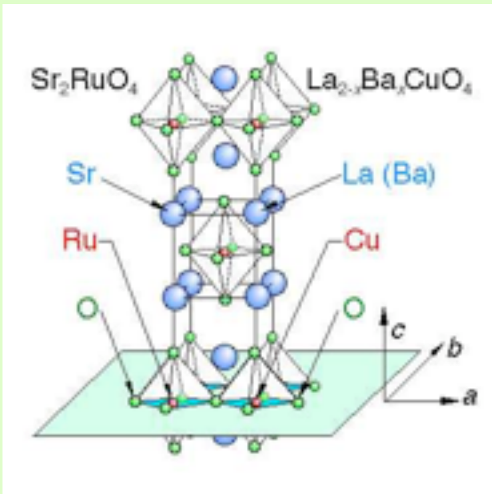
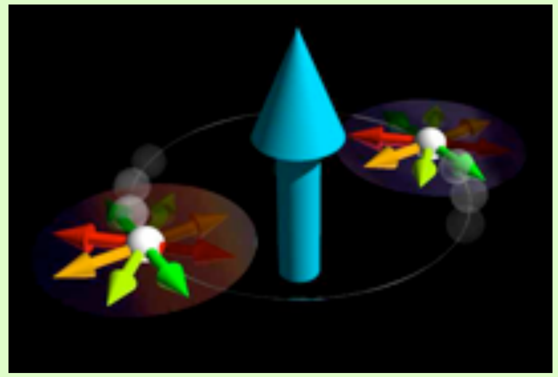
normal, dynamical $\quad \Delta_0 = 0$, $\langle \Delta_0^2 \rangle \neq 0 \Rightarrow \Omega_k \neq 0$

Berry-phase (curvature) fluctuation effect

Case of Sr_2RuO_4



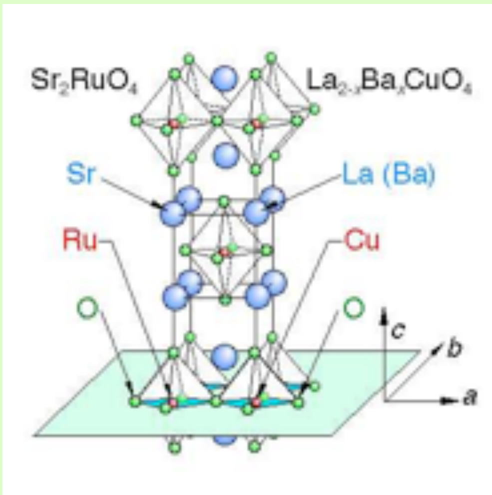


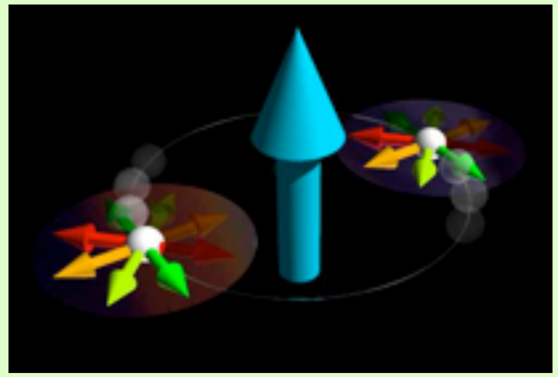
(Y. Maeno et al.)

- chiral p+ip wave SC
- quasi-2D system

$$\alpha_{xy} \propto \tau^2 \times \frac{1}{T - T_c}$$

Case of Sr_2RuO_4





(Y. Maeno et al.)

- chiral p+ip wave SC
- quasi-2D system

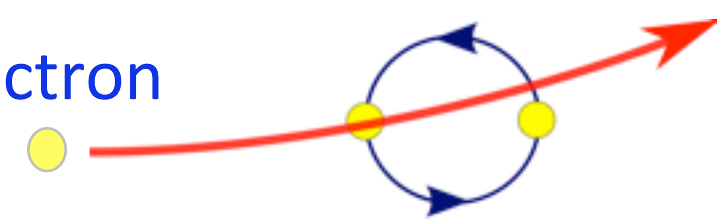
$$\alpha_{xy} \propto \tau^2 \times \frac{1}{T - T_c}$$

However, Ginzburg parameter ...

$$G_i^{\mathrm{Sr_2RuO_4}} = \frac{7\zeta(3)}{32\pi^3 k_F \xi} \sim 10^{-5} \ll G_i^{\mathrm{URu_2Si_2}} \sim 10^{-4}$$

SUMMARY

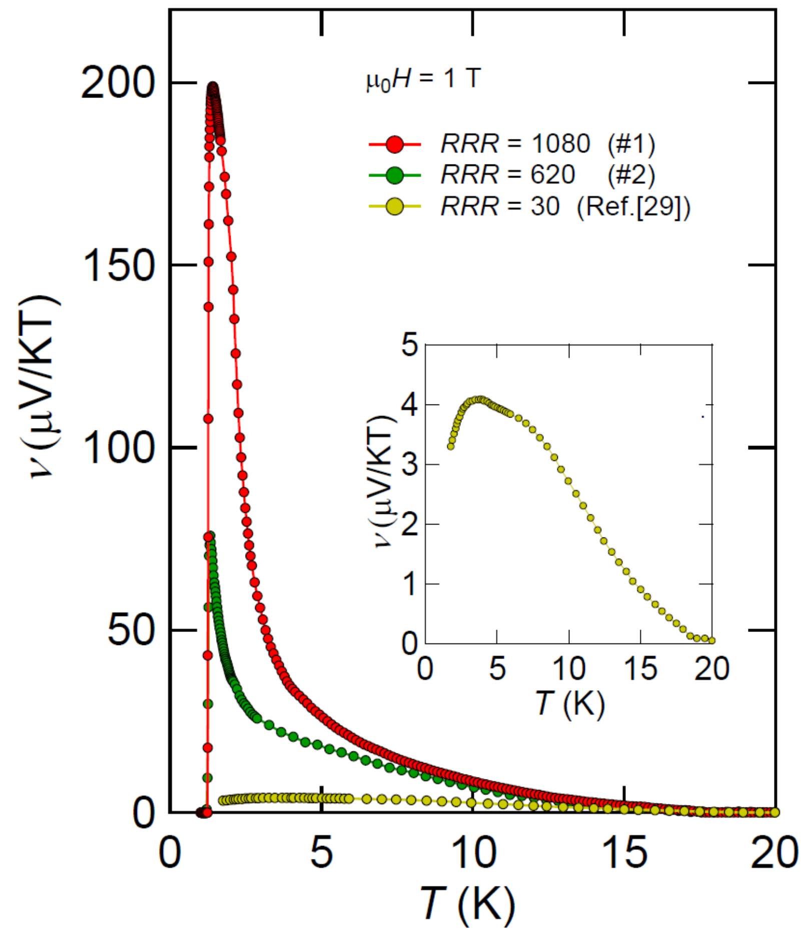
- We investigated the novel mechanism of anomalous Nernst effect due to Berry phase fluctuation in chiral superconductors.
- This novel effect is experimentally observed in URu_2Si_2 !



$$\nu \propto \tau$$

peak structure

near T_c



Similar to side-jump mechanism of anomalous Hall effect (but, this well-known effect is static mechanism)

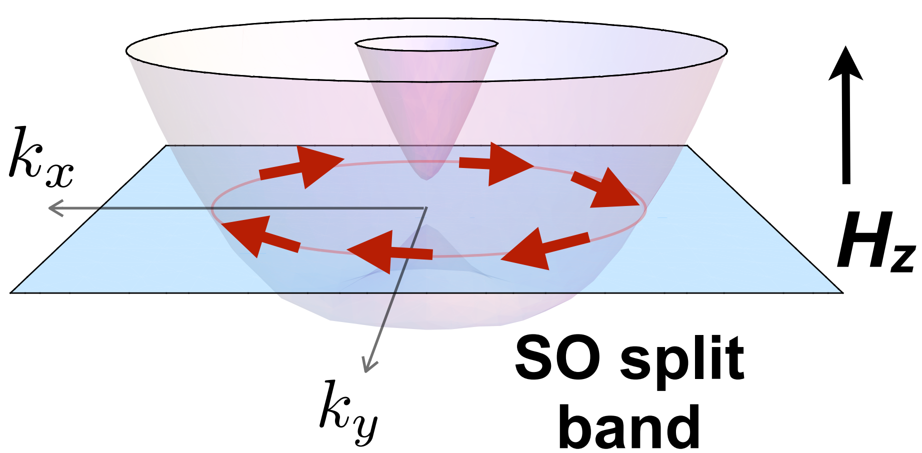
Asymmetric scattering due to Berry phase fluctuation !!

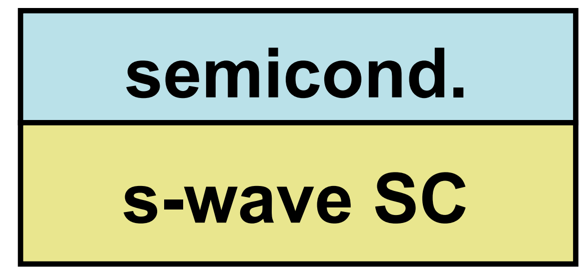
Outlook

- Berry-phase fluctuation mechanism is applicable any superconductors accompanying nontrivial Berry-curvature

c.f. Sr_2RuO_4

e.g. s-wave SC with Rashba SO int. and Zeeman field





Heterostructure system

Topological SC

Non-zero Berry curvature

(M. Sato, Y. Takahashi, S.F., PRL (2009))

c.f. quantum nanowire, Lutchyn et al./ Oreg et al. Mourik et al. (InSb)/ Das et al. (InAs)

For $H_z \gg \lambda_{\mathrm{SO}} k_F,\ g$

$$-g\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow \Rightarrow -g\left(\frac{\lambda_{\mathrm{SO}}k_F}{H_z}\right)^2(k_x+ik_y)(k'_x-ik'_y)a_k^\dagger a_{-k}^\dagger a_{-k'}a_{k'}$$

Non-chiral s-wave SC fluc. induces anomalous Nernst effect !!

Berry phase fluctuation is more essential than chiral SC fluc. !!

Thank you for your attention