UMR Gulliver

Effets Collectifs
&
Matière Molle

Vibrated granular experiments: Probing the vicinity of Jamming

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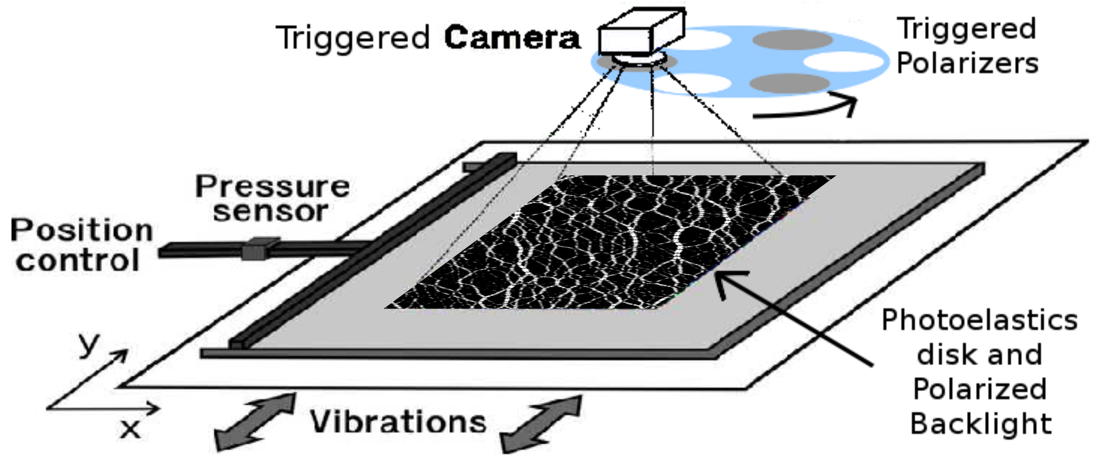
Overview

- Vibrated granular experiments : probing the jamming critical regime
  - Two distinct signatures of criticality at finite vibration
  - Approaching the zero vibration limit

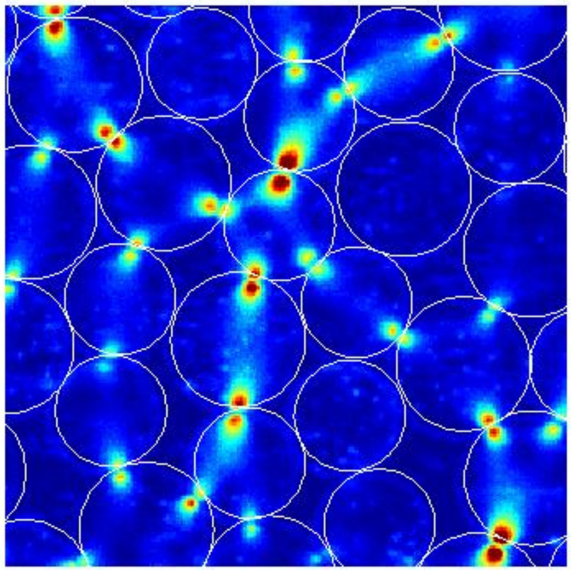
- Yielding close to jamming in hard discs
  - Yield stress of “vibration” origin below jamming

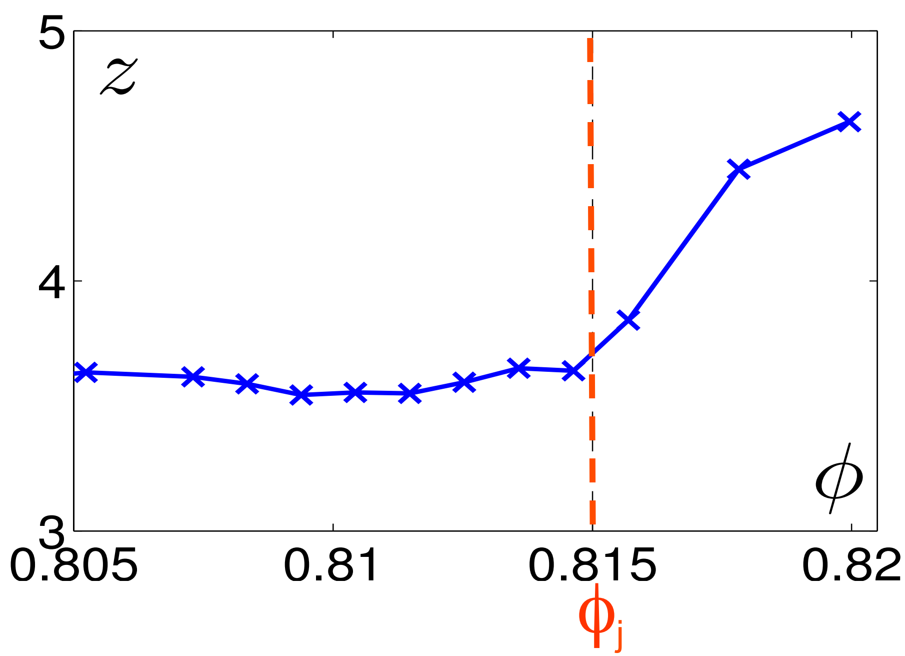
- Probing elasticity
  - Inflating an intruder : experimental set up and the linear elastic framework
  - Integrated vs Local measurements
  - Discussion : the interplay between shear and dilatancy

Vibrating soft photo-elastic discs



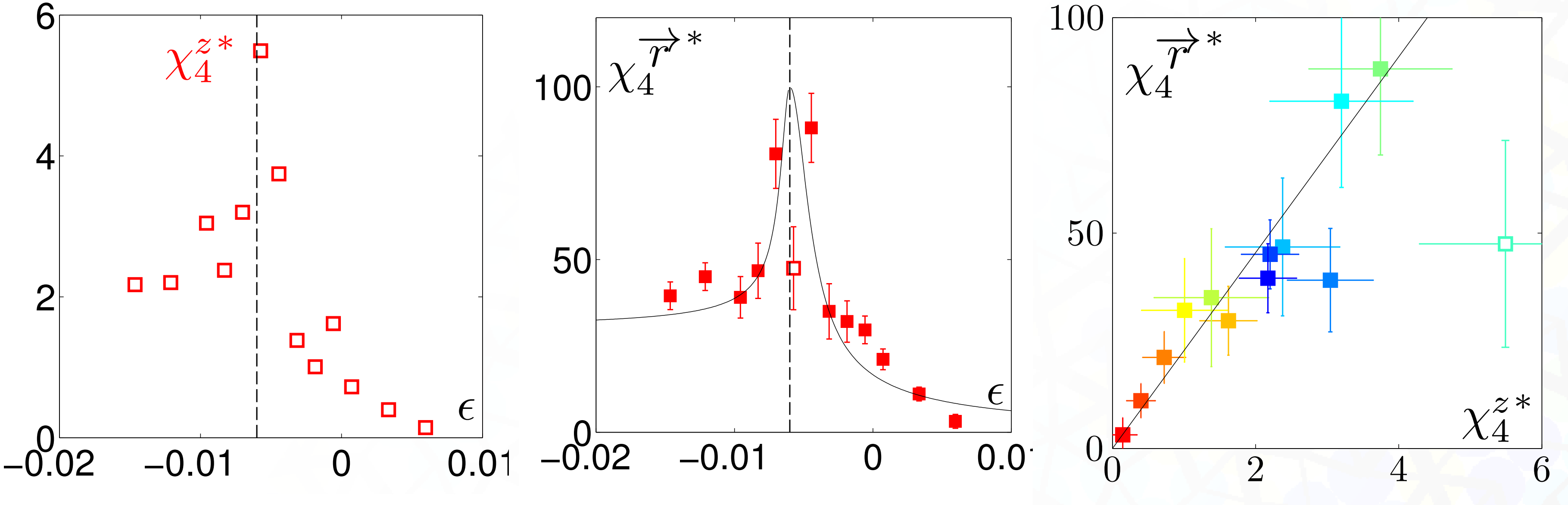
- 8000 soft discs
- Bi-disperse : d_s = 4 & 5mm
- Horizontal vibration (a=1cm, ω = 5 to 10 Hz)
- Acquisition:
  - Stroboscopic
  - Fast inside the cycles



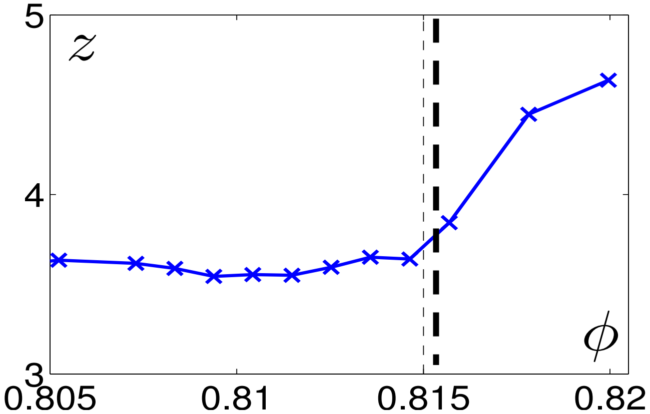


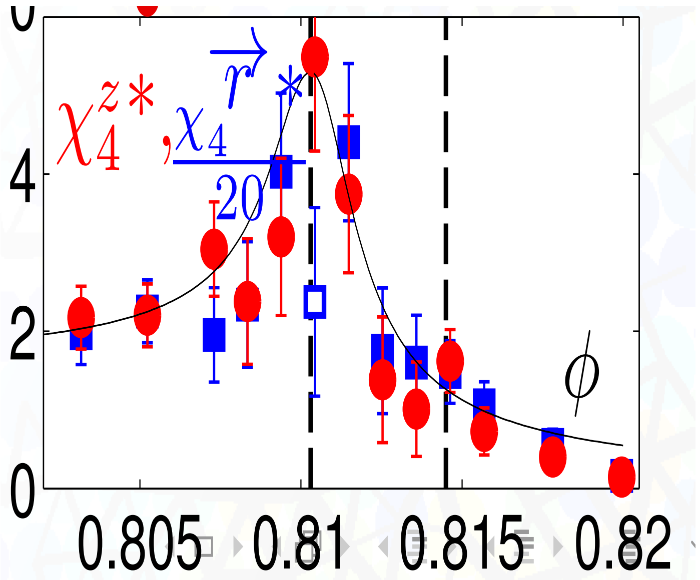
Heterogeneous Dynamics of the contacts

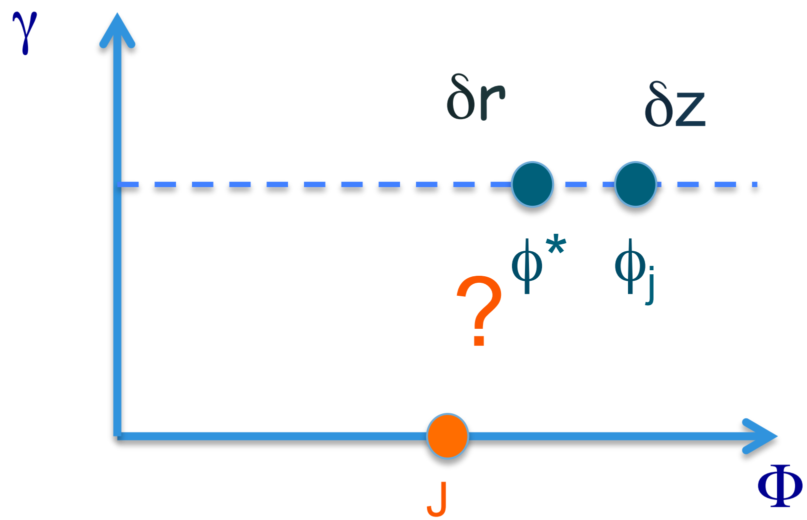
$$\chi_4^{z,r}(\tau) \equiv N Var\left(\left\langle Q_i^{z,r}\right\rangle_i\right)_t \quad \left\{ \begin{array}{l} Q_i^z(t,\tau) = \begin{cases} 1 \text{ if } |z_i(t+\tau) - z_i(t)| \leq 1 \\ 0 \text{ if } |z_i(t+\tau) - z_i(t)| > 1 \end{cases} \\ Q_i^r(t,\tau) \equiv \exp\left(- \dfrac{\Delta r_i^2}{2\left\langle \Delta r_i^2 \right\rangle} \right) \end{array} \right.$$



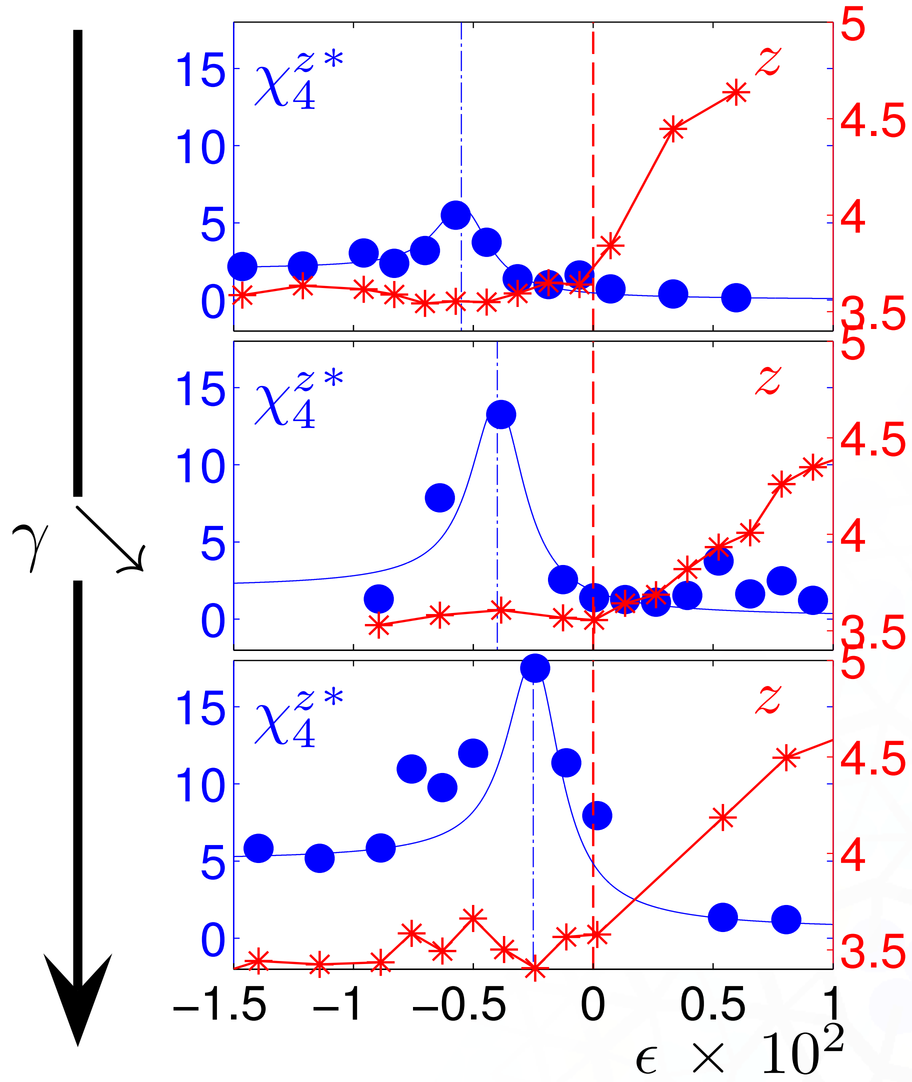
Two distinct signatures crossovers



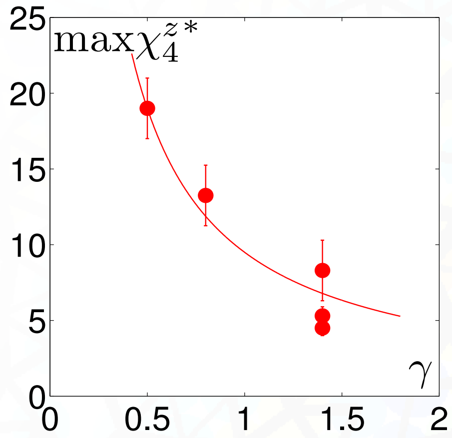




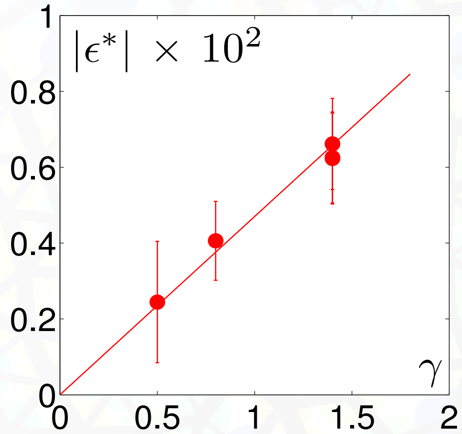
Decreasing the vibration



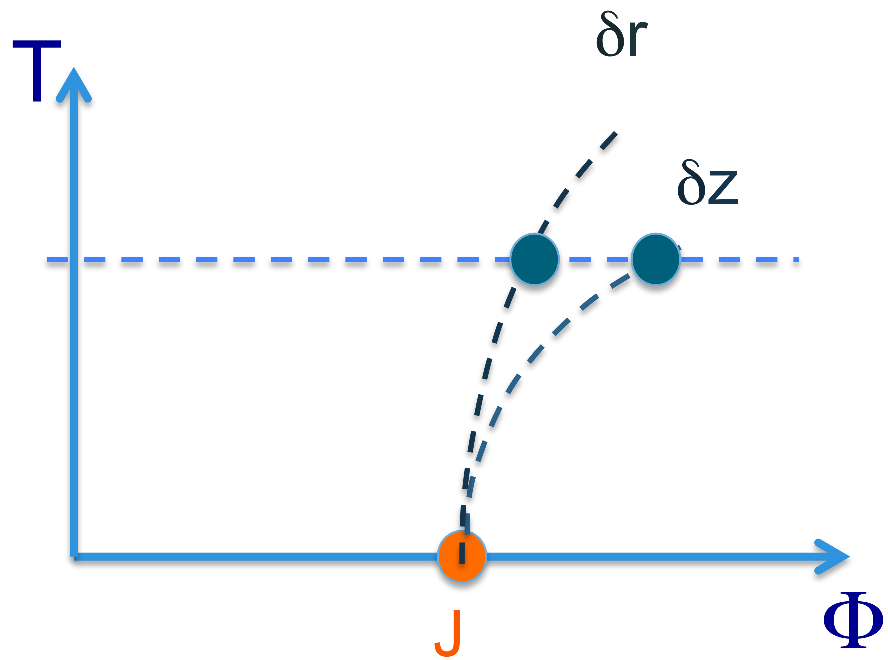
The fluctuations increase

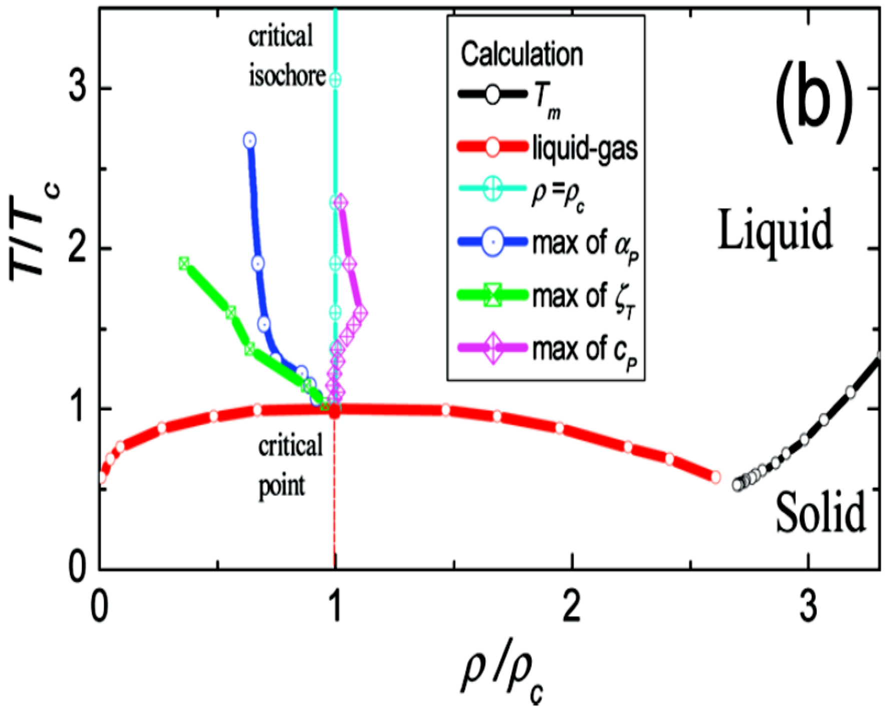


The crossovers merge



Hence two crossover lines : Widom lines



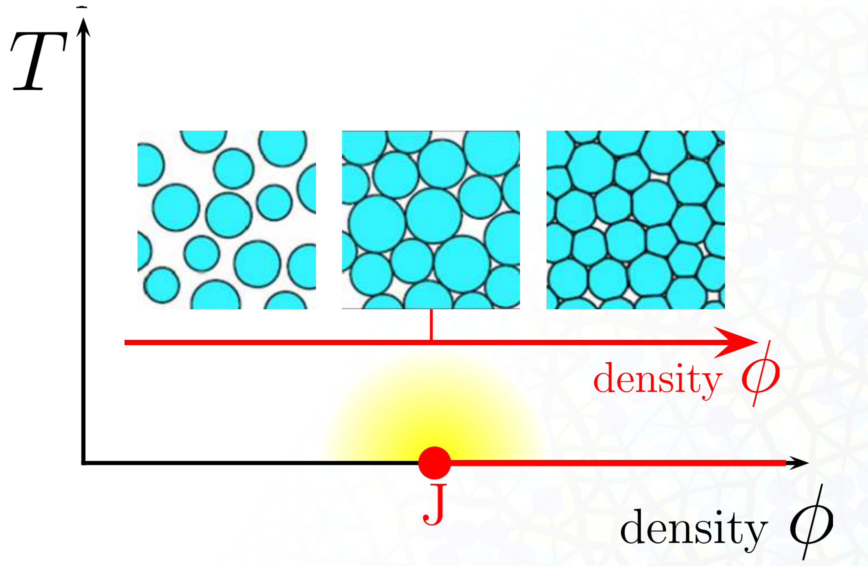


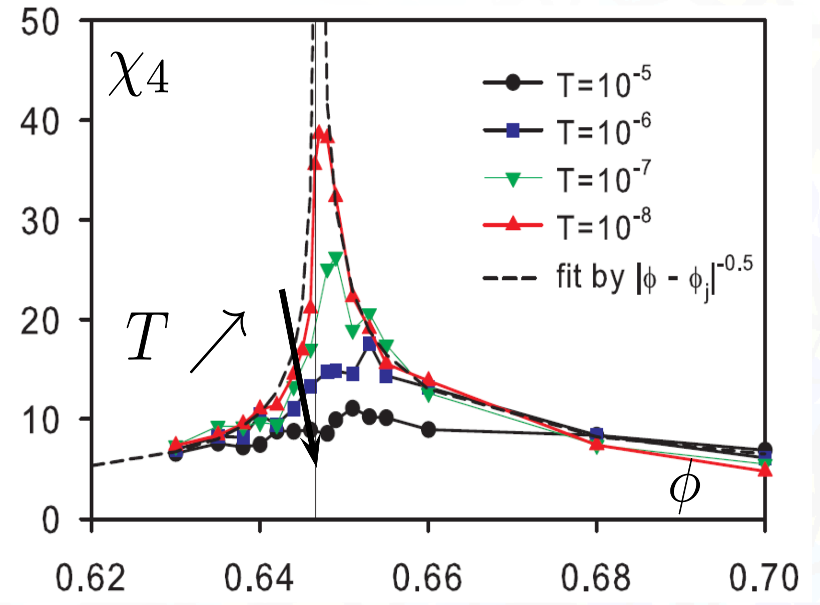
How far from the critical point ?

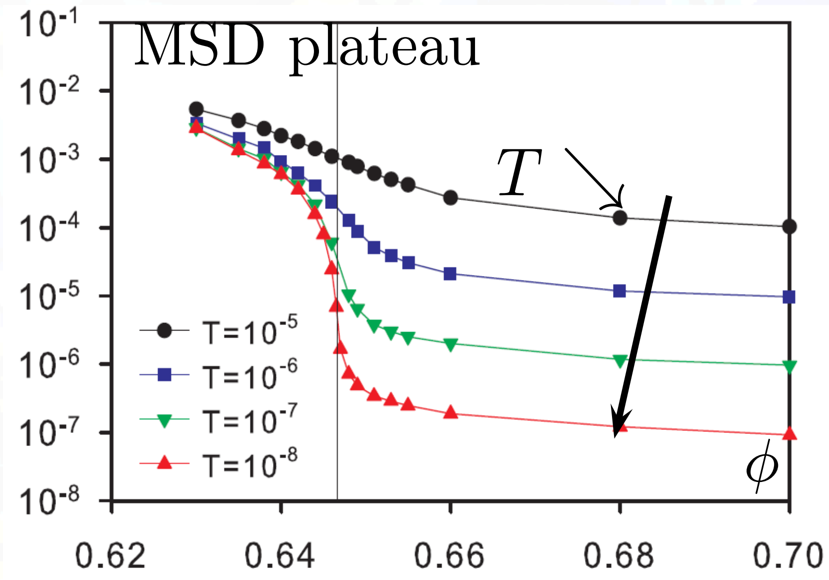
Comparison with thermal soft spheres…

Simulation of thermal soft-spheres

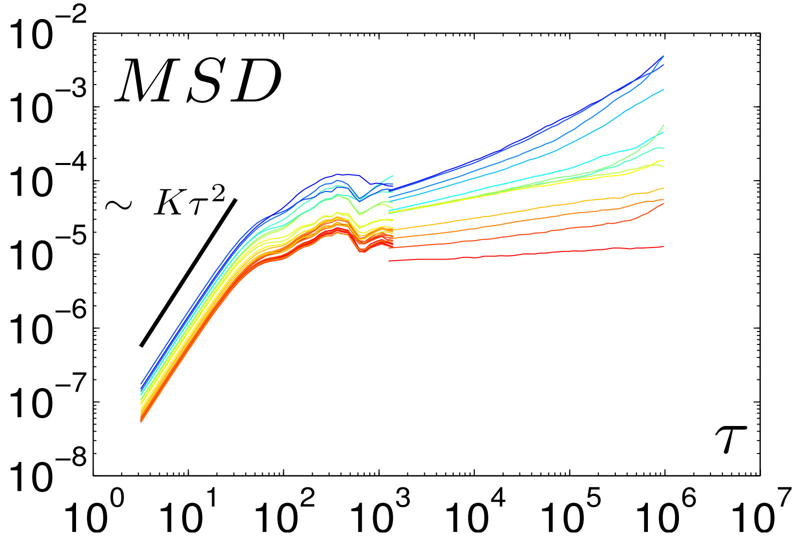
Ikeda et al, 2012

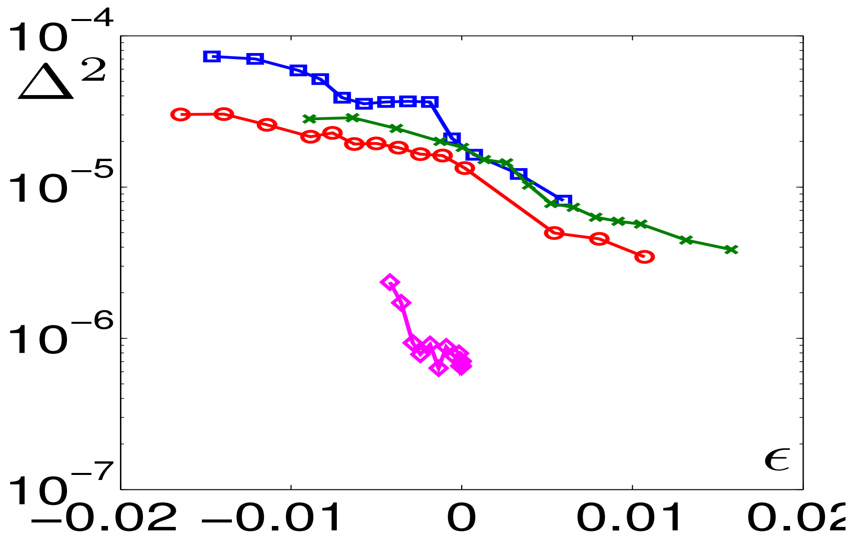


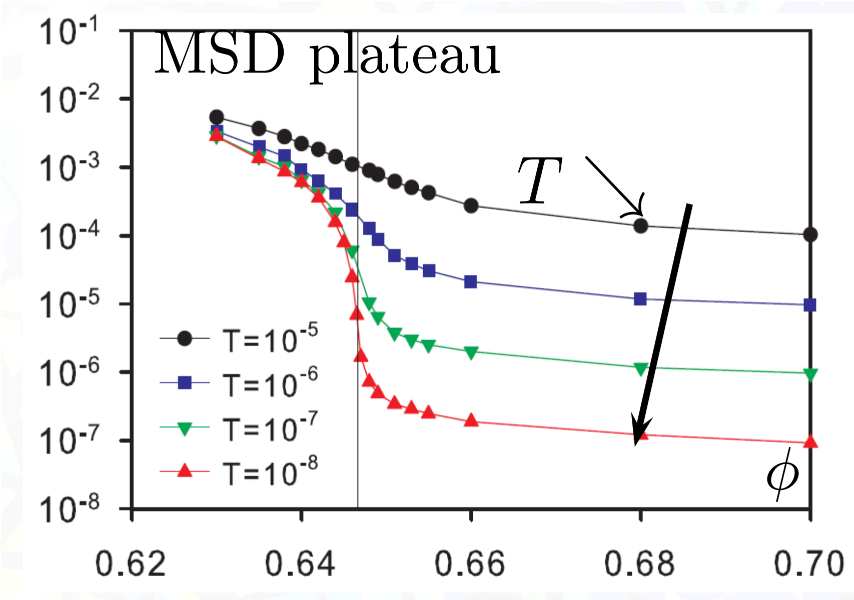


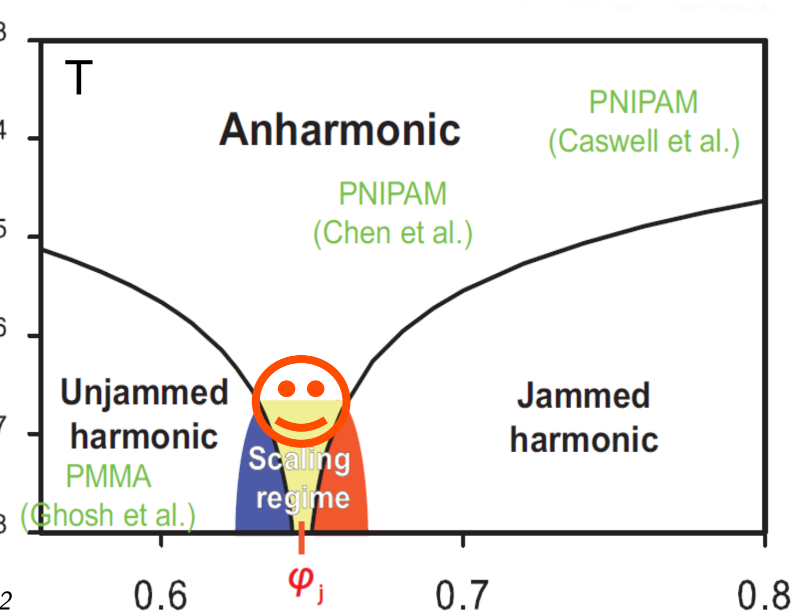


Discussion: in the light of the street-lamp



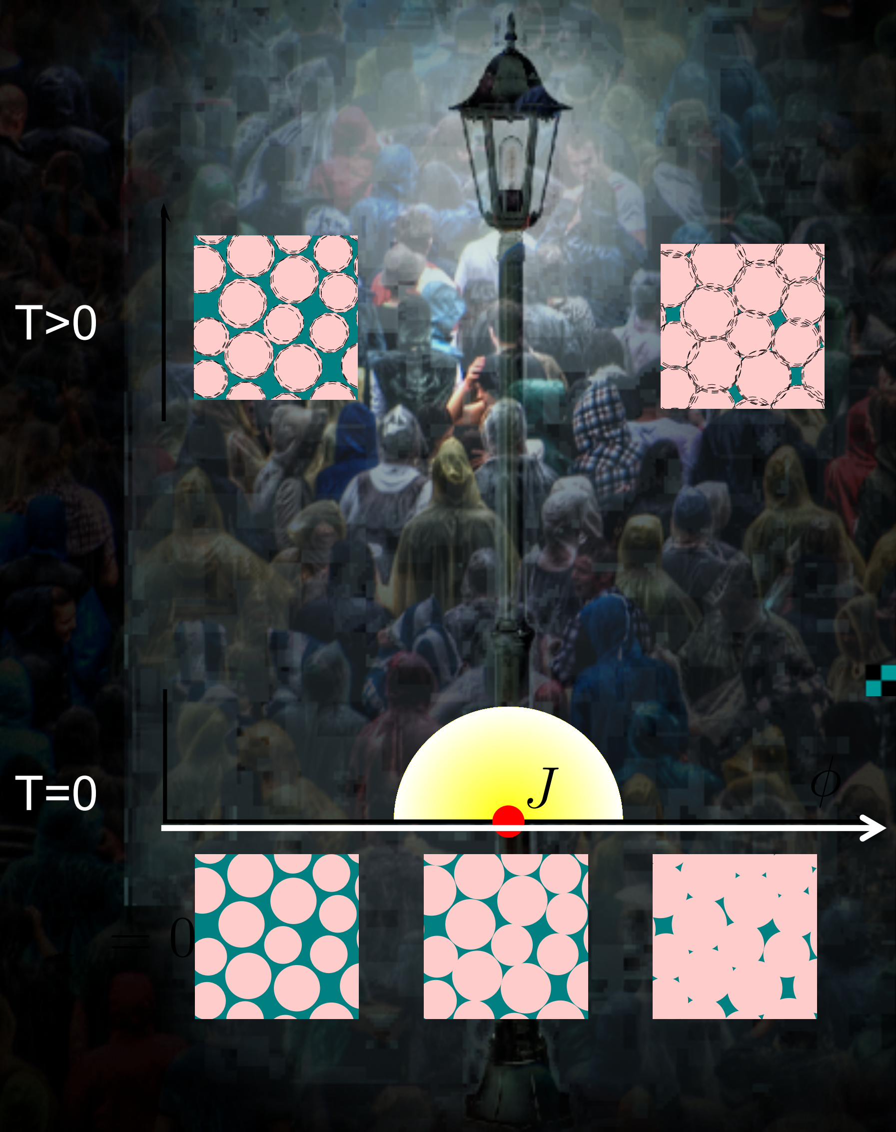






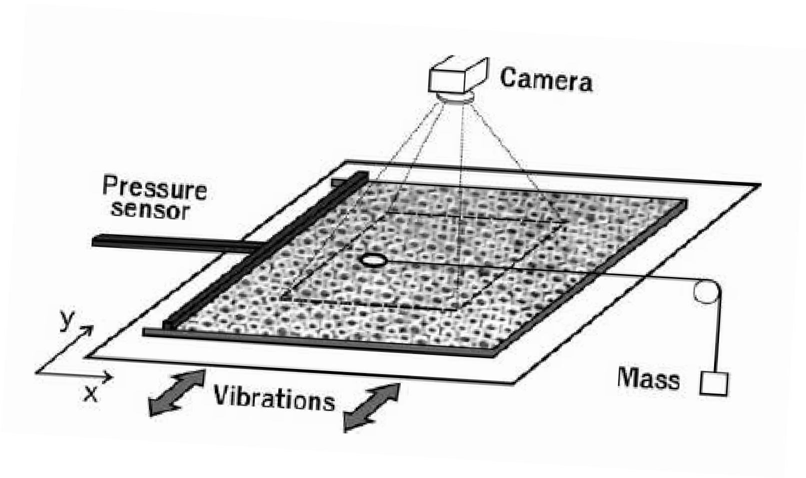
al, 2012

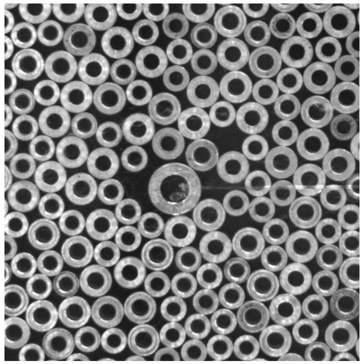
Conclusion of the first part

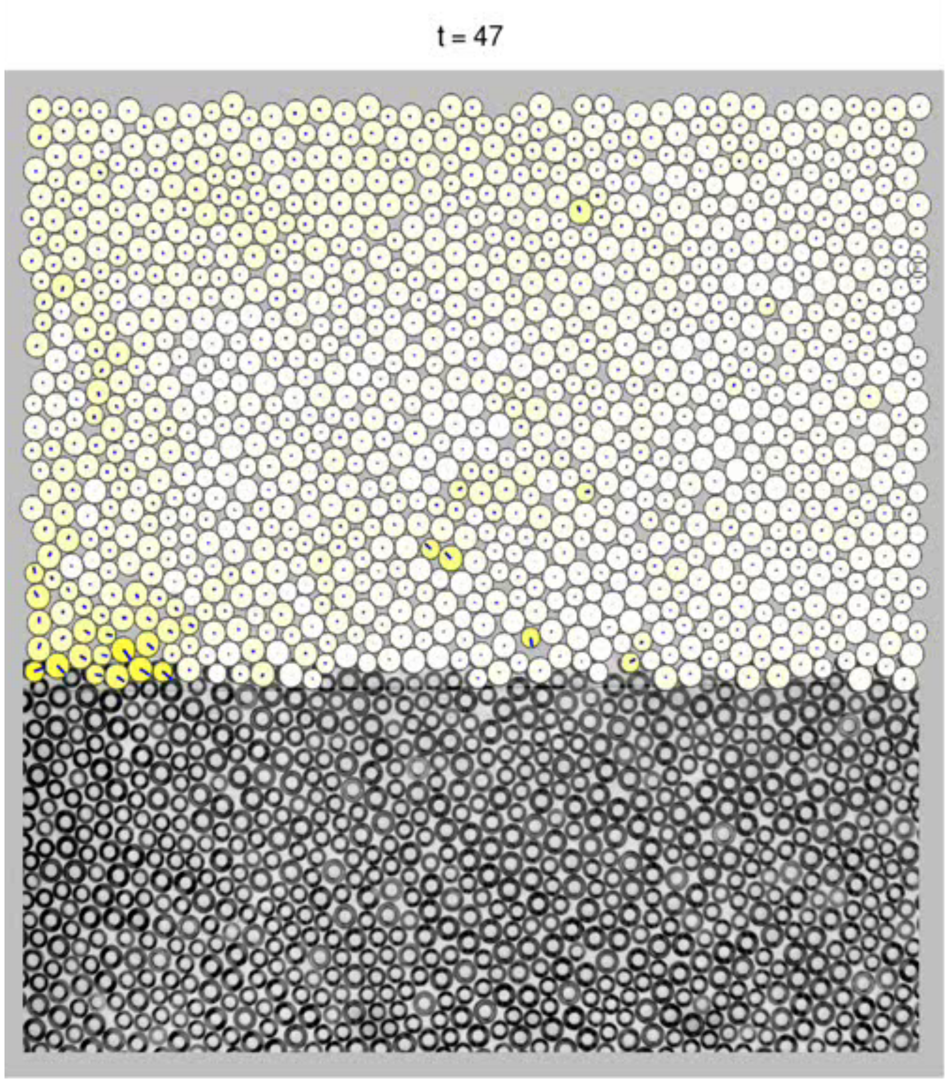


- Shaken Granular Experiments are in the street lamp halo of the J point:
  - They can constrain existing theories
  - Theories have something to say about the real world…
  - One cannot exclude effects of friction at the quantitative level
- => One step further (in the dark…)
  - Yielding close to jamming
  - A first attempt to probe elasticity close to jamming

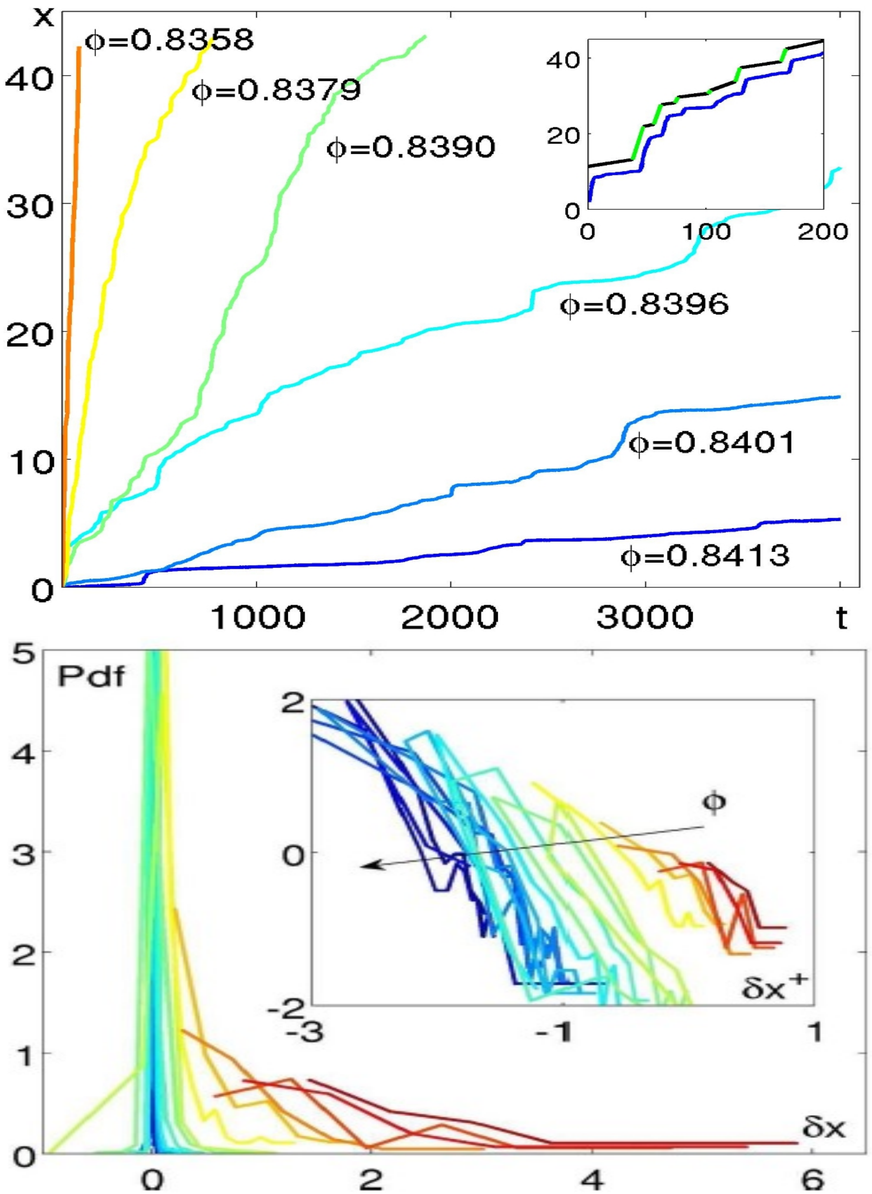
Yielding close to jamming : the motion of an intruder …



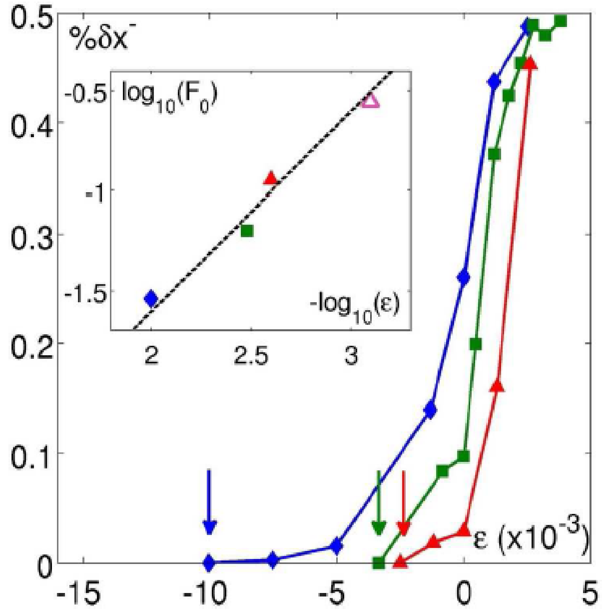




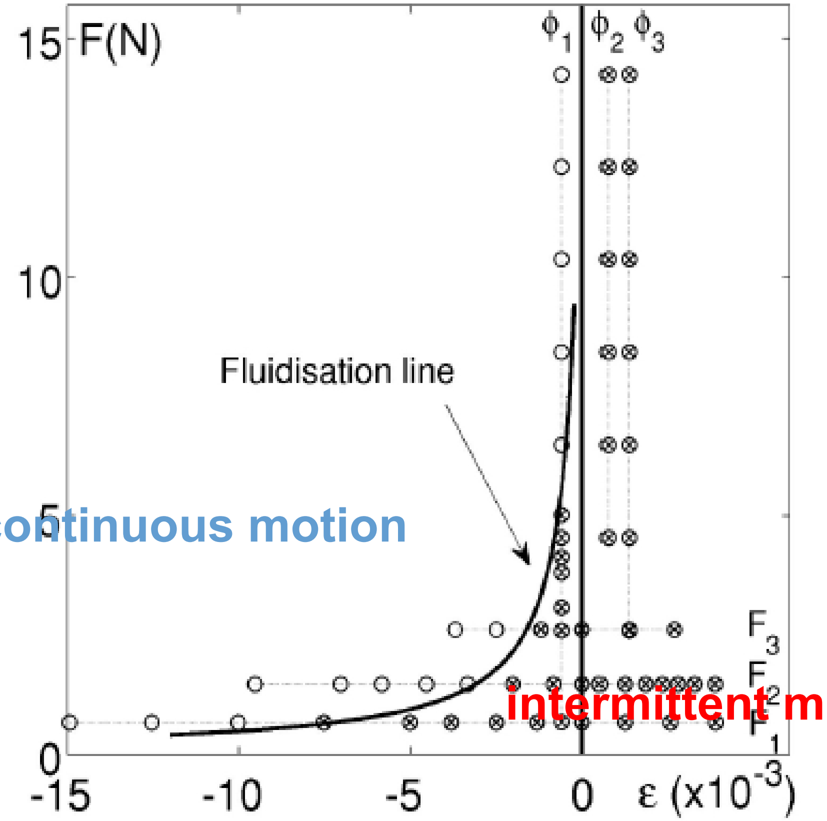
Evidence of a fluidization transition

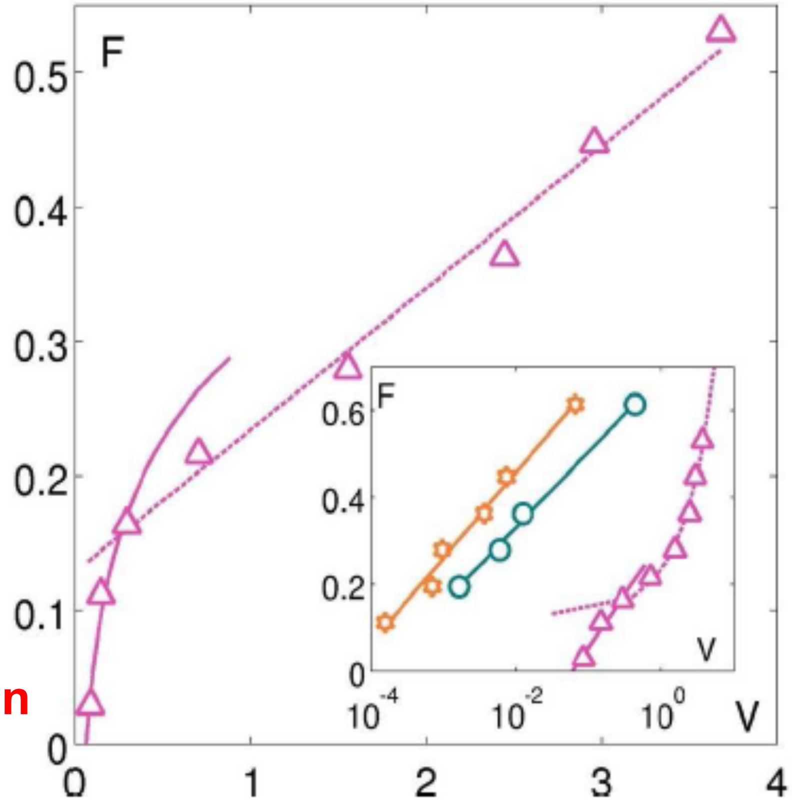


Transition : % δx < 0 --> 0



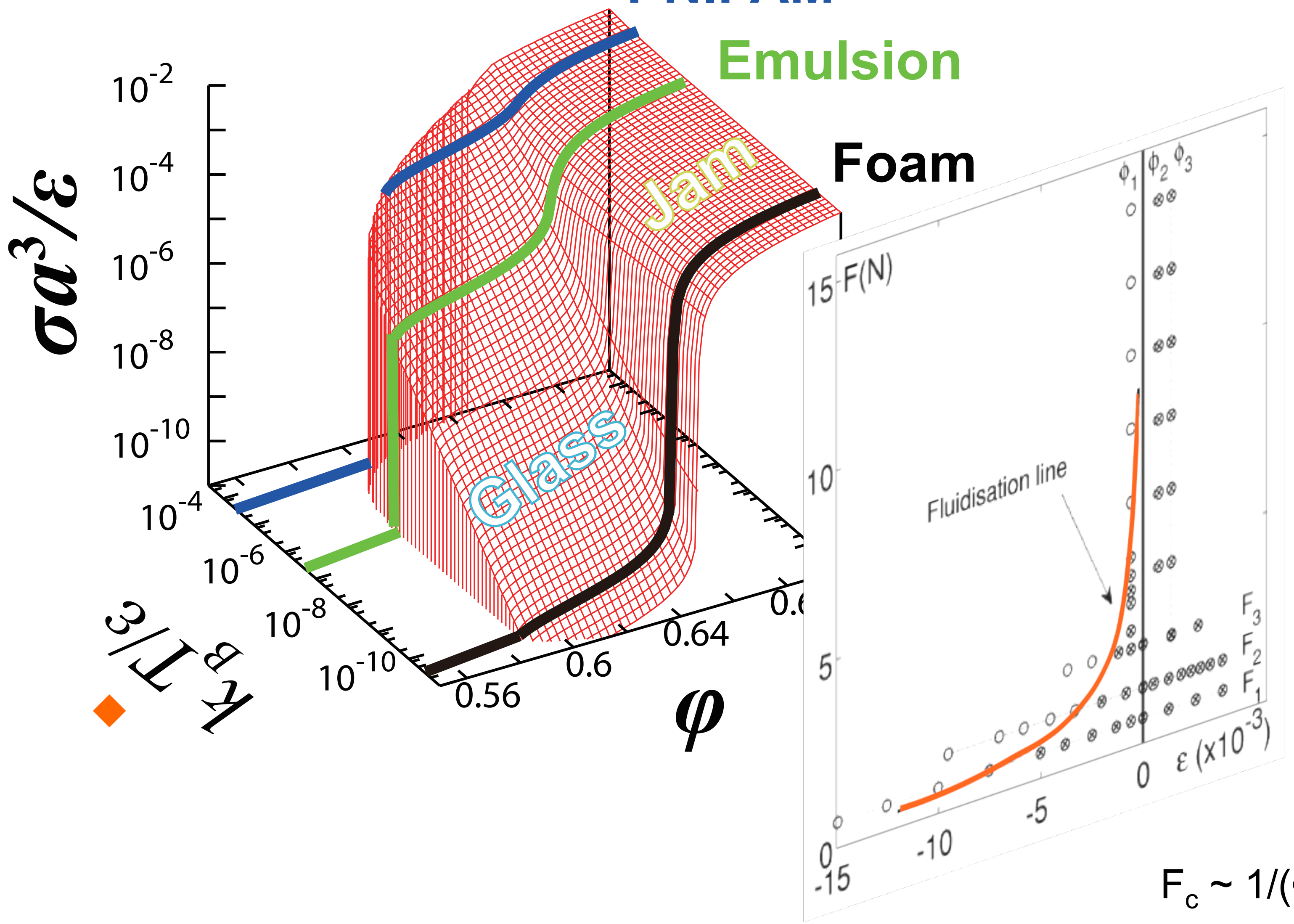
Indeed two very different rheological behaviors





- Fluidized regime : F ∝ <V> :
- Intermittent regime : F ∝ ln <V>

Critical force : “thermal” yield stress



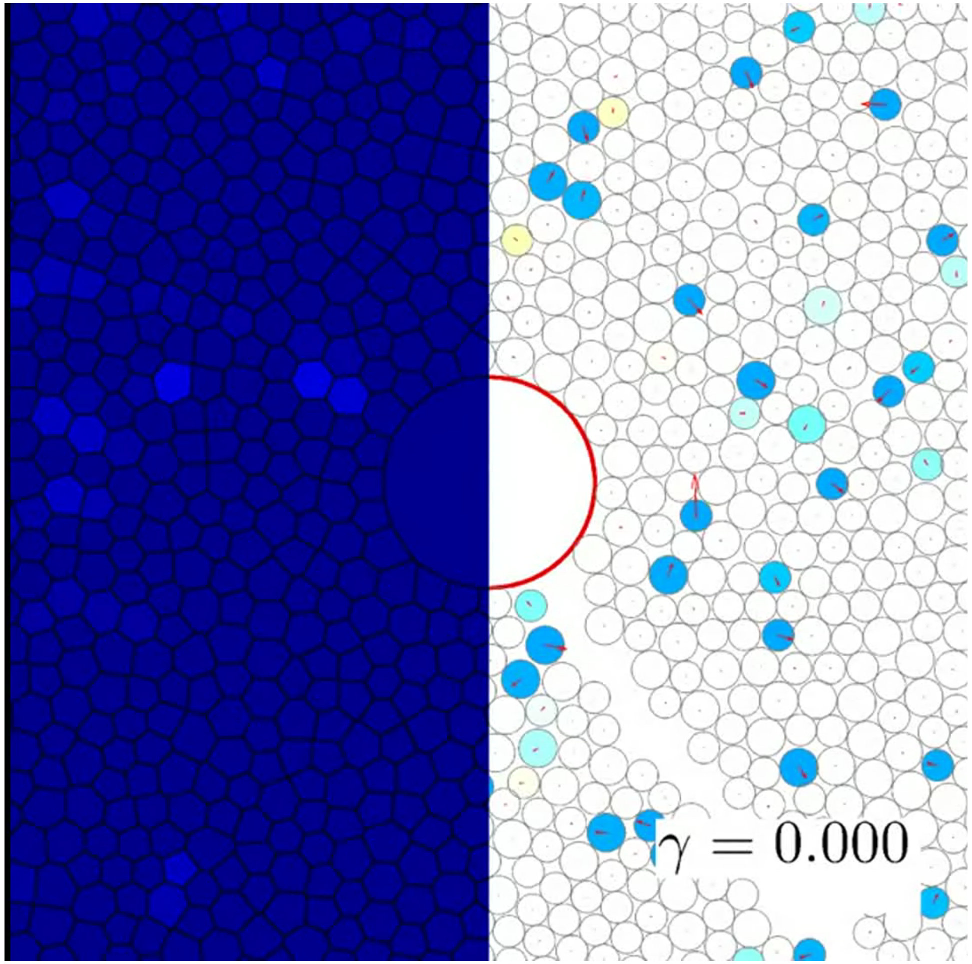
$F_c \sim 1/(\Phi_J-\Phi)$

Probing elasticity : set up

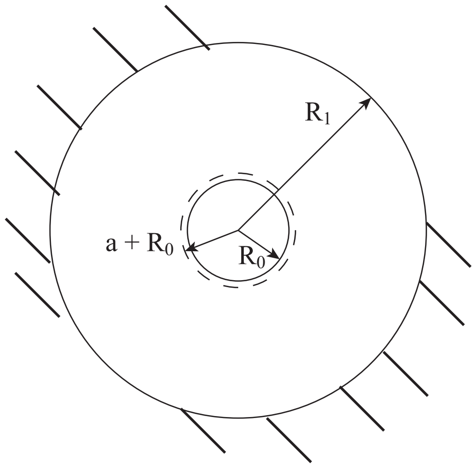
- Prepare the system at large packing fraction under vibration
- Inflate an intruder in the center (the vibration is stopped)
- Decrease the packing fraction while vibrating
- iterate

$R0 \to R0 + a$

$\gamma = a/R0$



Probing elasticity : the linear elastic framework



$$div\left(\underline{\underline{\sigma}}\right) = 0$$

$$\underline{\underline{\sigma}} = \frac{1}{2} Tr(\underline{\underline{\sigma}})\underline{\underline{1}} + \underline{\underline{\tau}}$$

$$\underline{\underline{\sigma}} = KTr(\underline{\underline{\varepsilon}})\underline{\underline{1}} + 2G\underline{\underline{\gamma}}$$

$$\underline{\underline{\varepsilon}} = \frac{1}{2}\left[\underline{\underline{\nabla U}} + \underline{\underline{{}^t\nabla U}}\right] = \frac{1}{2} Tr(\underline{\underline{\varepsilon}})\underline{\underline{1}} + \underline{\underline{\gamma}}$$

$$U(R_0) = a$$
$$U(R_1) = 0$$

$$\delta \equiv Tr\left(\underline{\underline{\varepsilon}}\right) = -2\frac{a}{R_0} A$$

$$\gamma \equiv J_2\left(\underline{\underline{\gamma}}\right) \equiv \sqrt{\frac{1}{2}\underline{\underline{\gamma}} \circ \underline{\underline{\gamma}}} = \frac{a}{R_0} B \left(\frac{R_0}{r}\right)^2$$

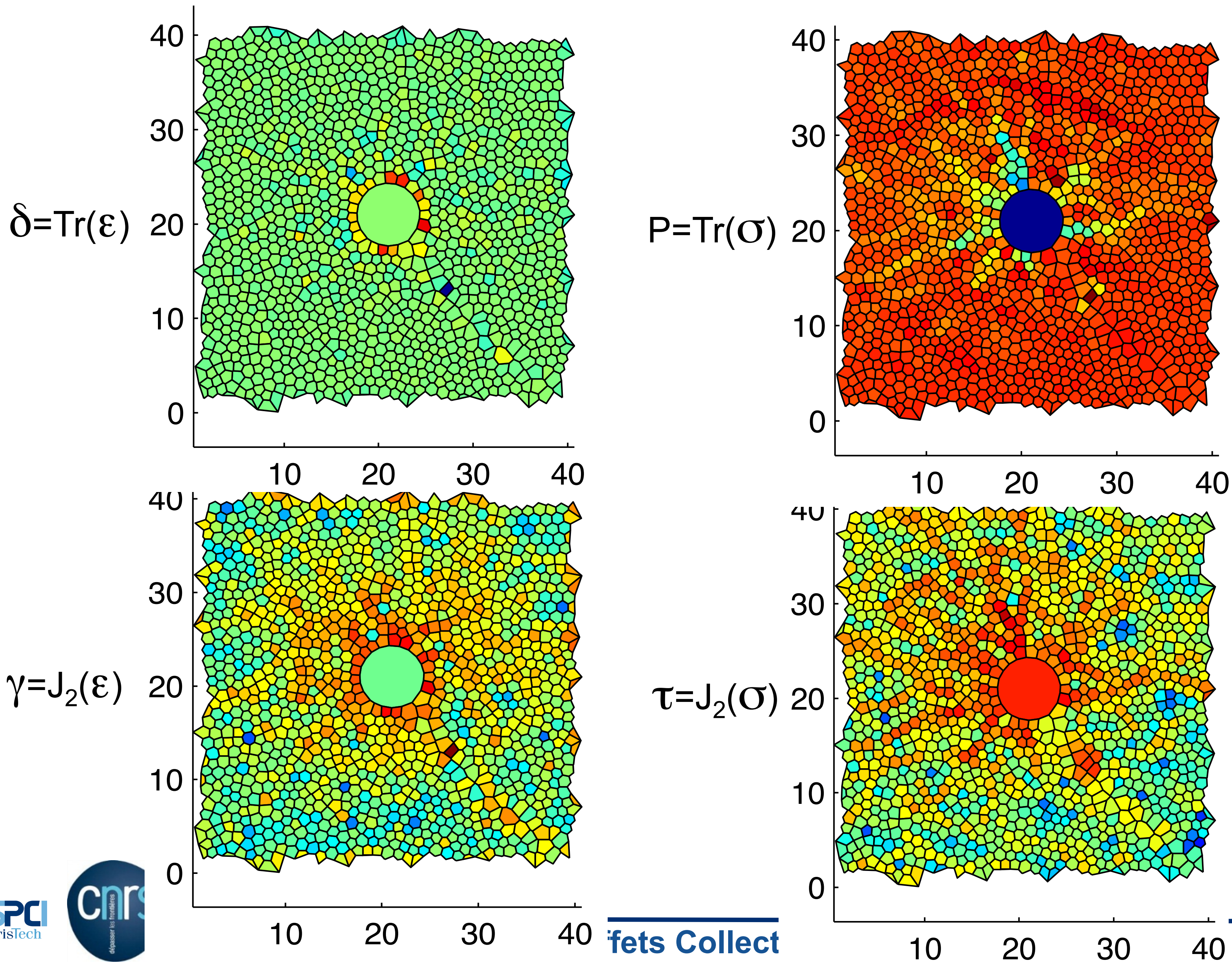
$$P \equiv Tr\left(\underline{\underline{\sigma}}\right) = KTr\left(\underline{\underline{\varepsilon}}\right)$$

$$\tau \equiv J_2\left(\underline{\underline{\tau}}\right) = \sqrt{\frac{1}{2}\underline{\underline{\tau}} \circ \underline{\underline{\tau}}} = 2GJ_2\left(\underline{\underline{\gamma}}\right)$$

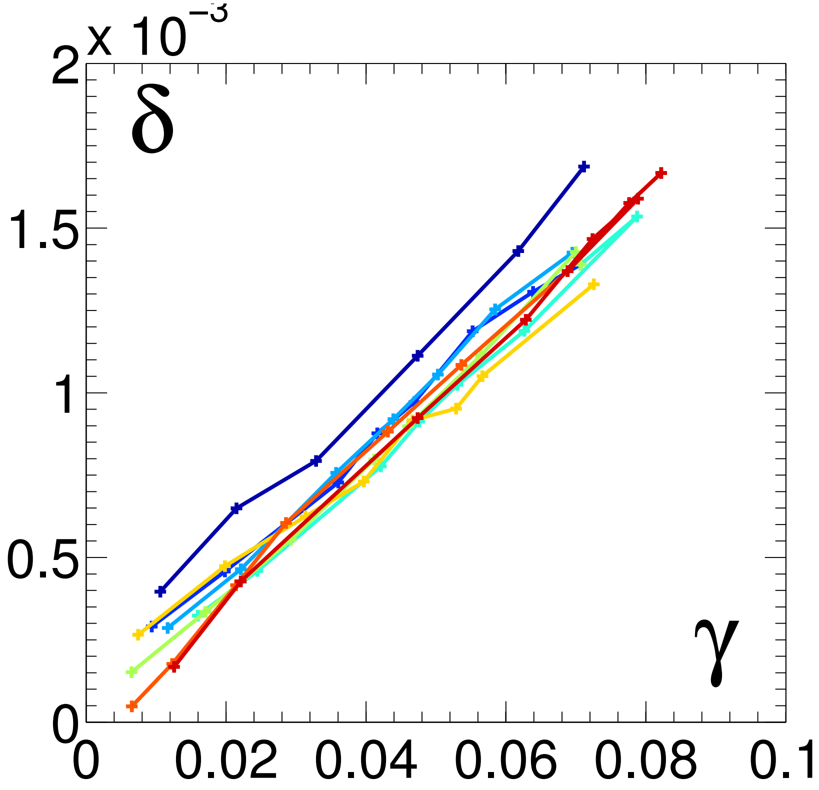
$$A = R_0^2 \Big/ \left(R_1^2 - R_0^2\right) ; B = R_1^2 \Big/ \left(R_1^2 - R_0^2\right)$$

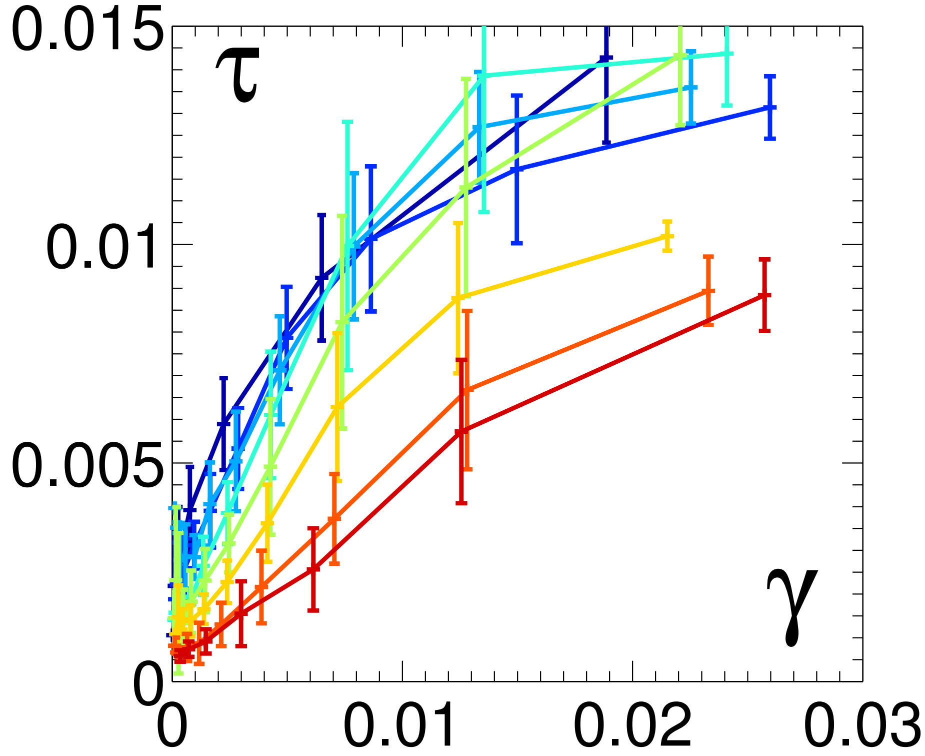
- Nota Bene
  - In the limit of large R_1, A->0, B->1 : this is a shear test!
  - G and K are simply obtained by the ratio of the stress and strain tensor invariants

For each packing fraction and each a/R_0



Salient features :





- Overall dilatant behaviour in the region close to the intruder
- Non linear constitutive law

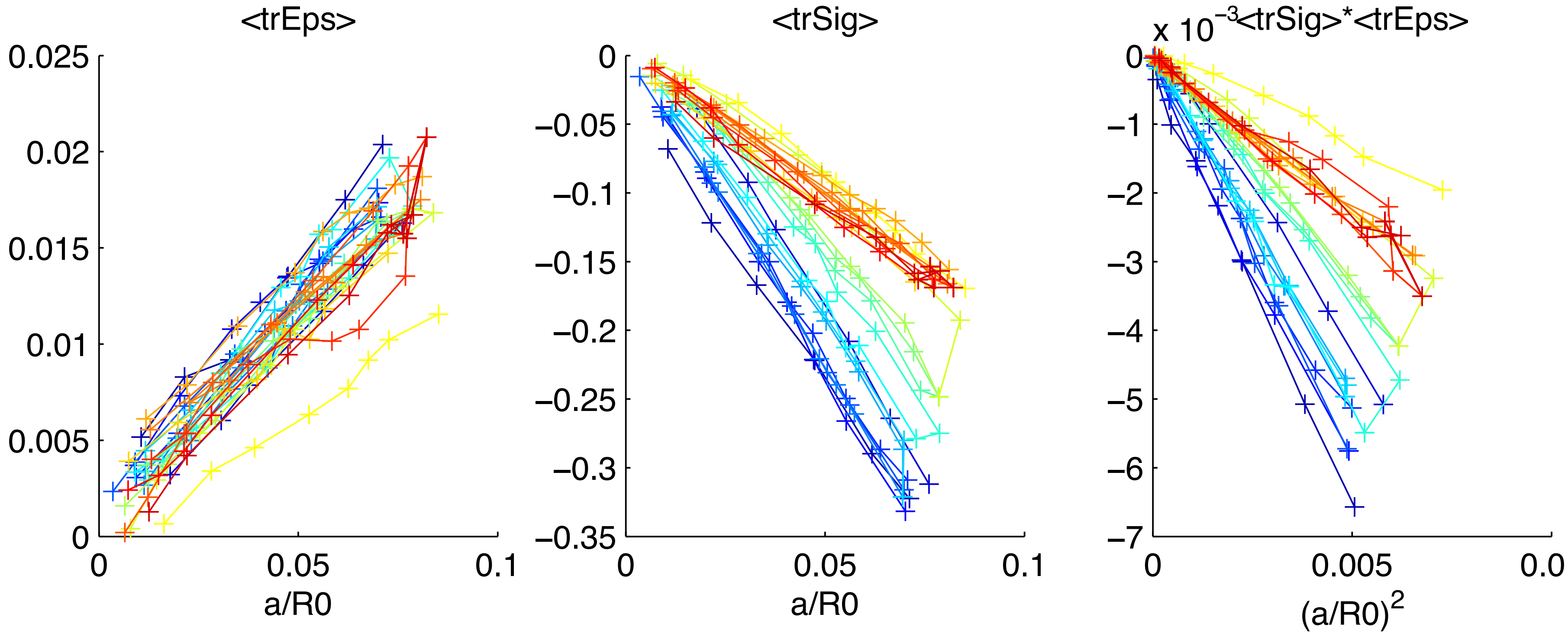
? Pressure stiffening = > Dilatancy => Shear weakening ?

Conclusion

- Vibrated granular media are suitable tools for probing the vicinity of jamming, (in particular low enough T_eff)
- Two distinct crossovers (one dynamical, one structural) converge toward J-point in the limit of low vibration
- Pulling an intruder in vibrated hard discs has allowed us to probe the yield stress of “thermal origin” => Suggest to try in the soft photo-elastic discs to capture the yield stress of “jamming origin”
- Inflating an intruder in soft photo-elastic discs => First indications of intricate interplay between dilatancy and non linear shear law.
- Further readings :
  - Europhysics Letters, 83, 46003, (2008).
  - Soft Matter, 6 (13), 3059–3064, (2010).
  - Phys Rev Lett 103 12800 (2009).
  - Europhysics Letters, 100, 44005 (2012).
  - Soft Matter (2013) to appear.

Integrated quantities vs. control parameter a/R_0

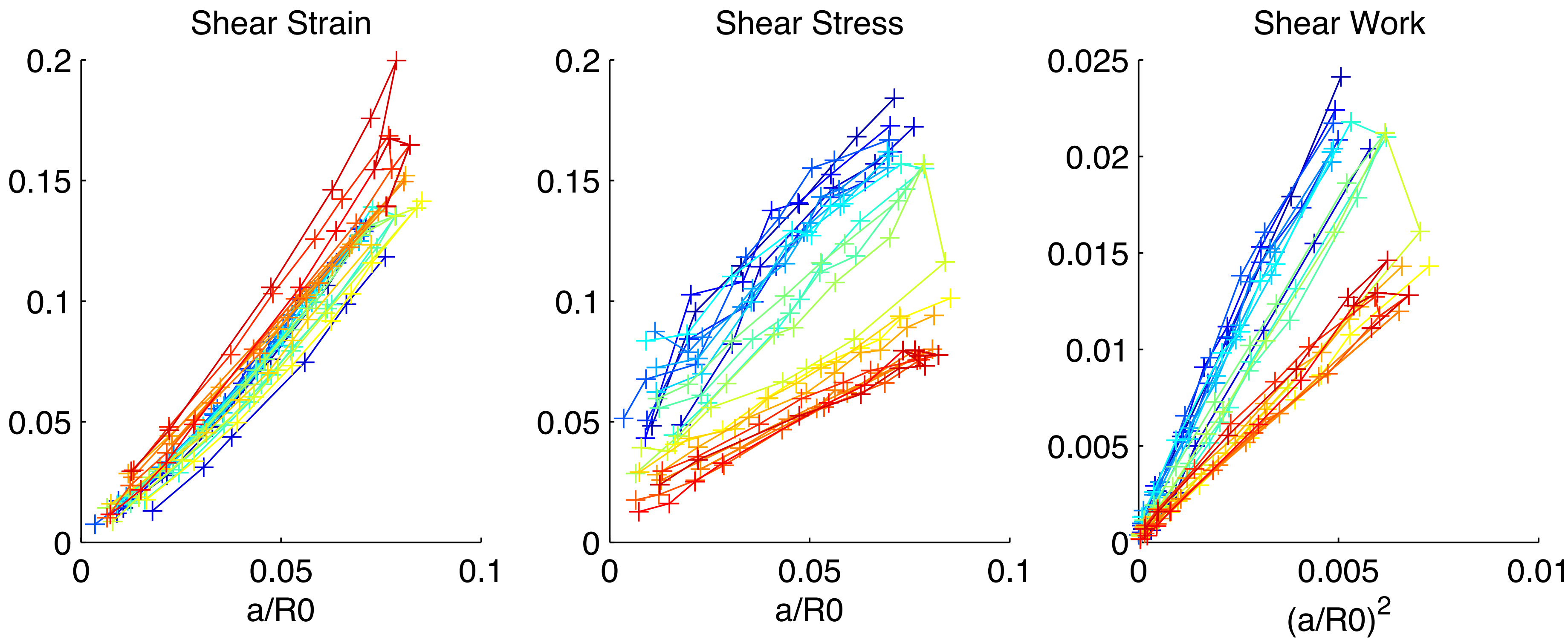
- Compressive part



- Linear with a/R_0
- Nota Bene : Tr(ε)>0 => **Overall dilatant behaviour.**

Integrated quantities vs. control parameter a/R_0

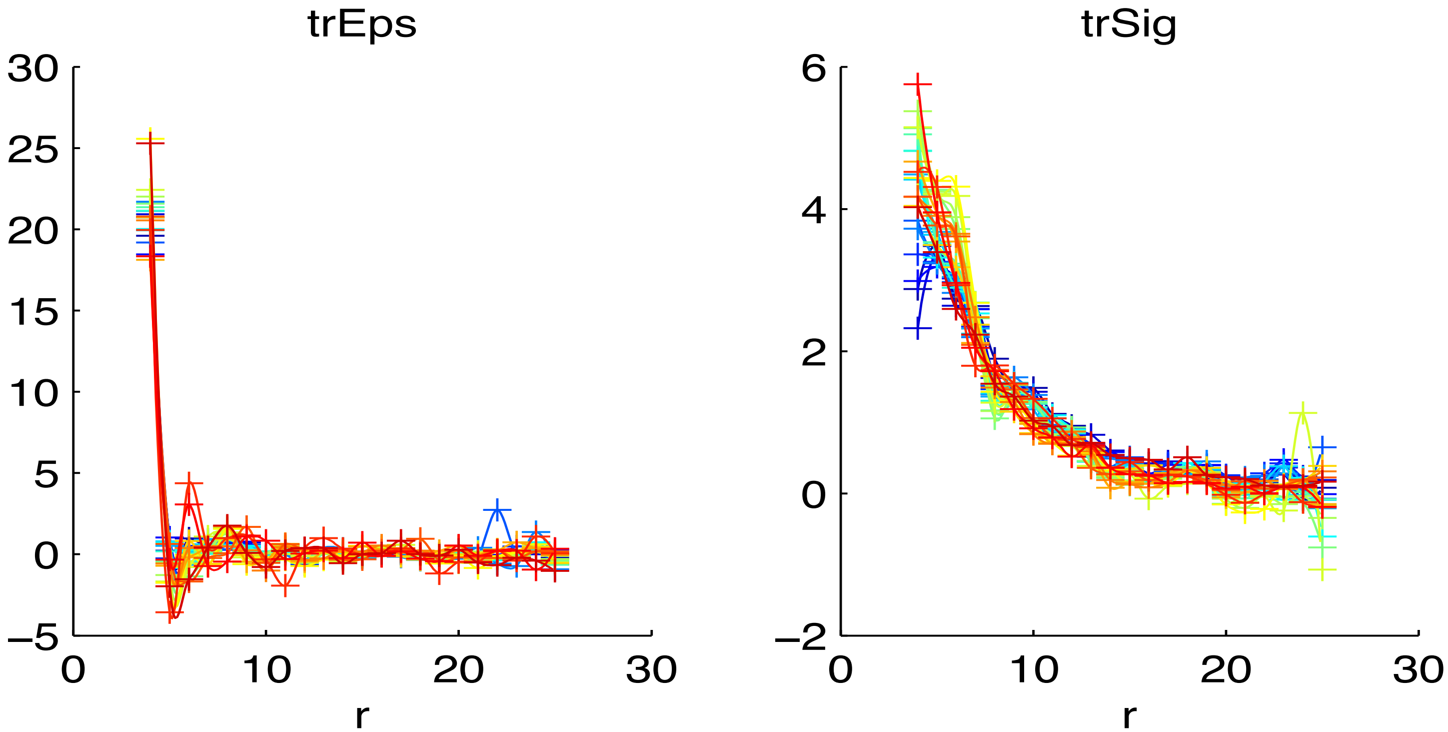
- Shear part



- Non linear behaviour of shear strain => a/R0 does not strictly control strain
- Both shear strain and shear stress are responses and non linear
- The shear work however is quadratic in a/R0 as prescribed by linear elasticity

Radial profiles (azimuthally averaged)

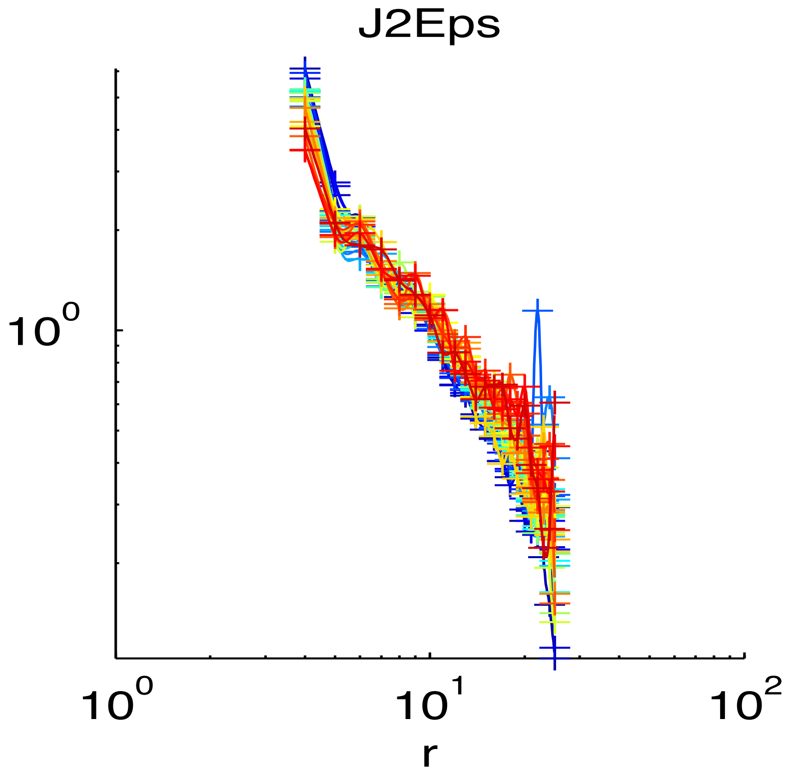
- Compressive part

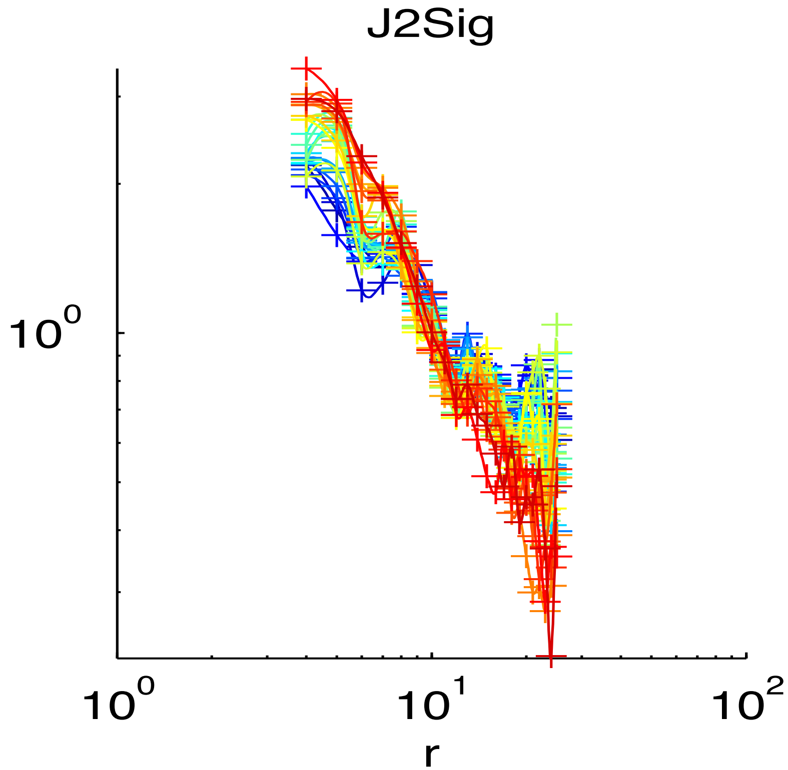


- Dilatancy strongly localized close to inflating intruder
- Pressure decreases exponentially

Radial profiles (azimuthally averaged)

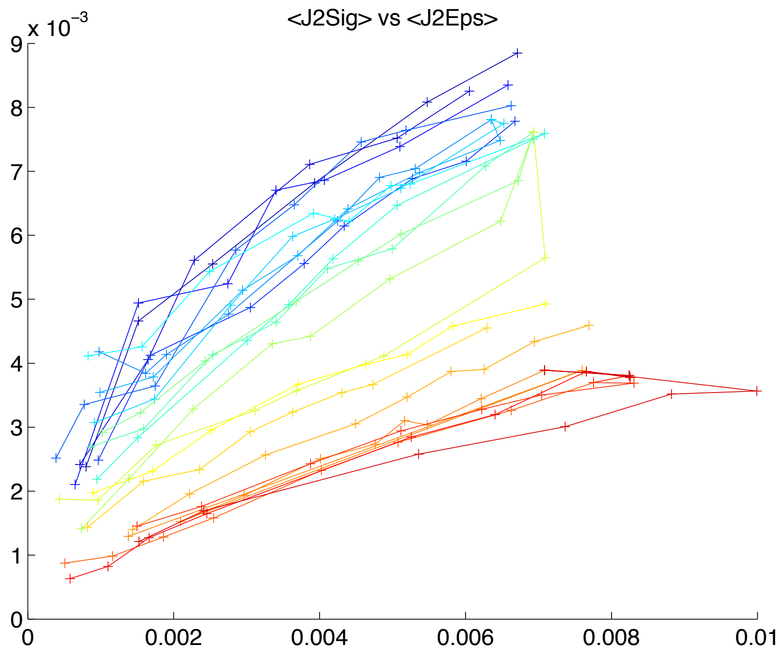
- Shear Part



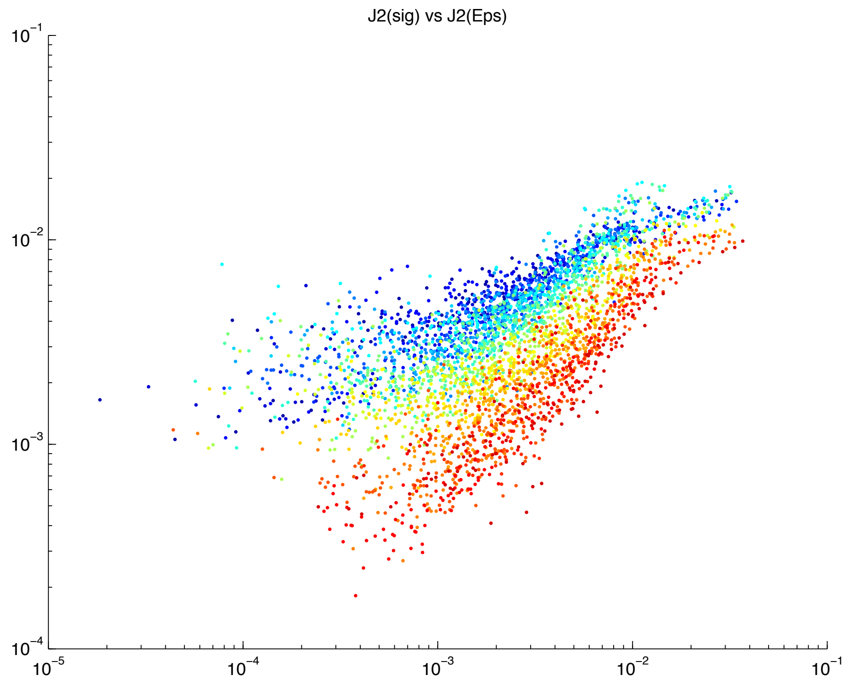


- Deviation from the $1/r^2$ law, expected from linear elasticity
- Some dependence with the packing fraction

Parametric plot shear stress vs. shear strain



Integrated quantities



Quantities local in r

- In both case, clear evidence for **non linear constitutive law.**
- G_eff increases with the packing fraction, however shear weakening

=> Suggest rather complex non linear interplay between shear and dilatancy