Mixing shear and dilation in marginal solids

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with **René Pecnik**

and **Ana Martin Calvo**

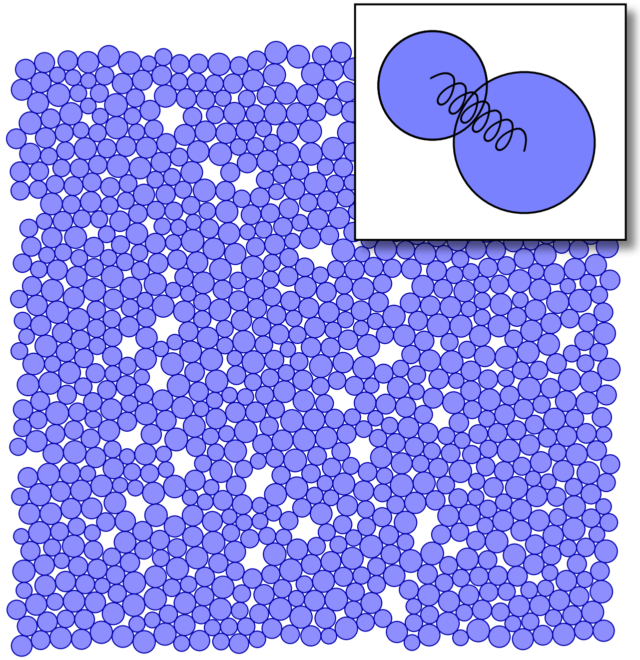
TU Delft

Mixing shear and dilation...

I. Dilation induced by shear

II. Tuning shear compliance
with pre-tension

Mixing shear and dilation...



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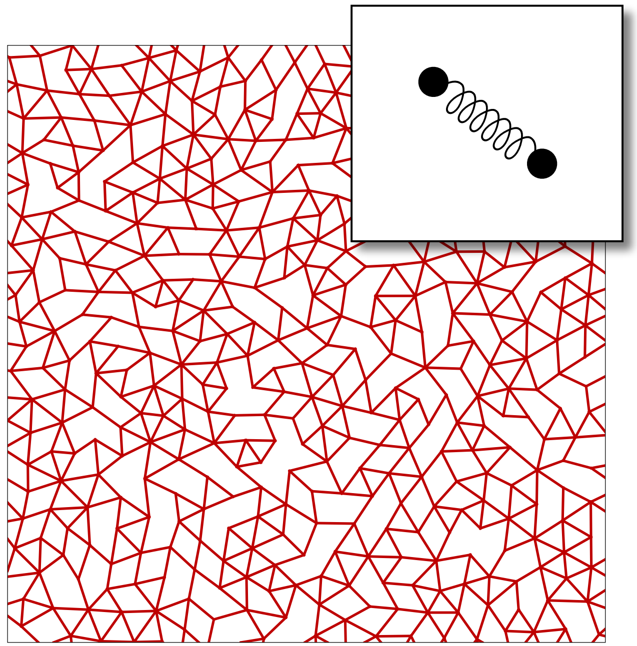


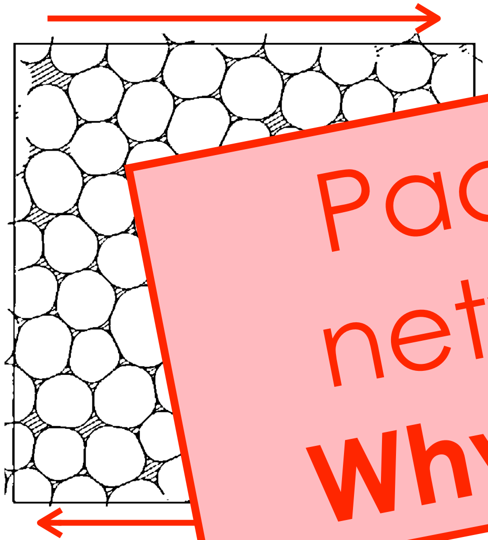
Counterintuitive dilatancy

O. Reynolds 1885

Weaire & Hutzler, Phil. Mag. 2003

Janmey et al. Nature Materials 2006

Conti & MacKintosh PRL 2008



Packings expand,
networks contract:
Why the difference?

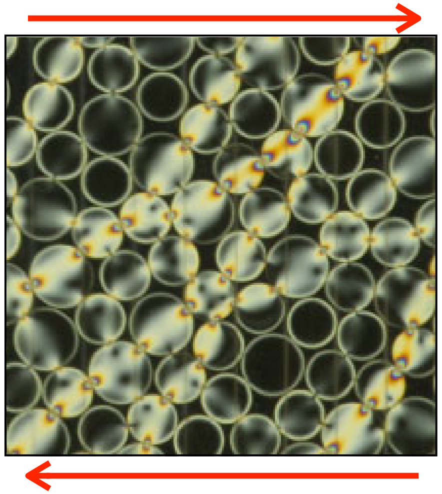
packings:

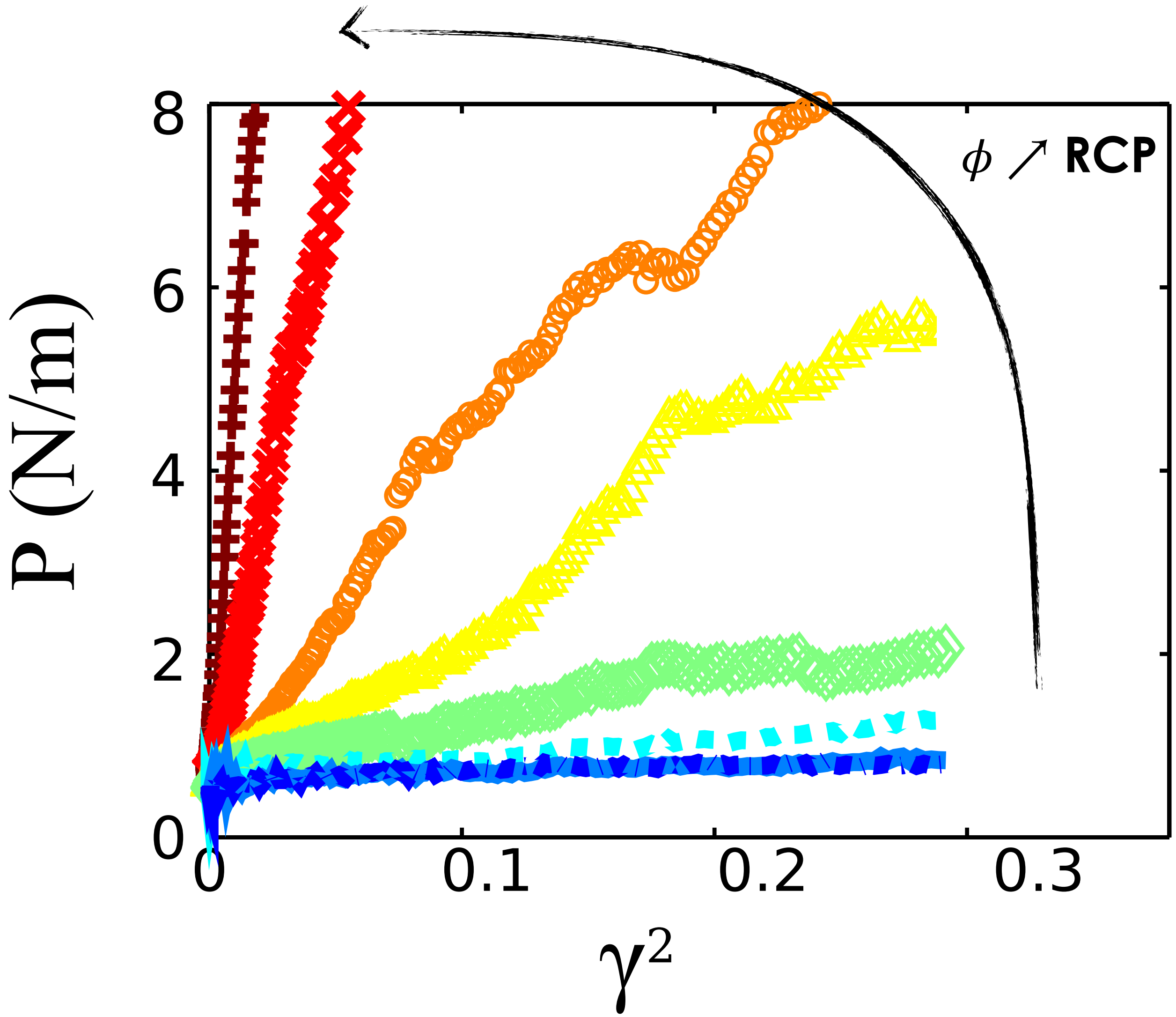
system expands
or pressure increases

networks:

system contracts
or pressure decreases

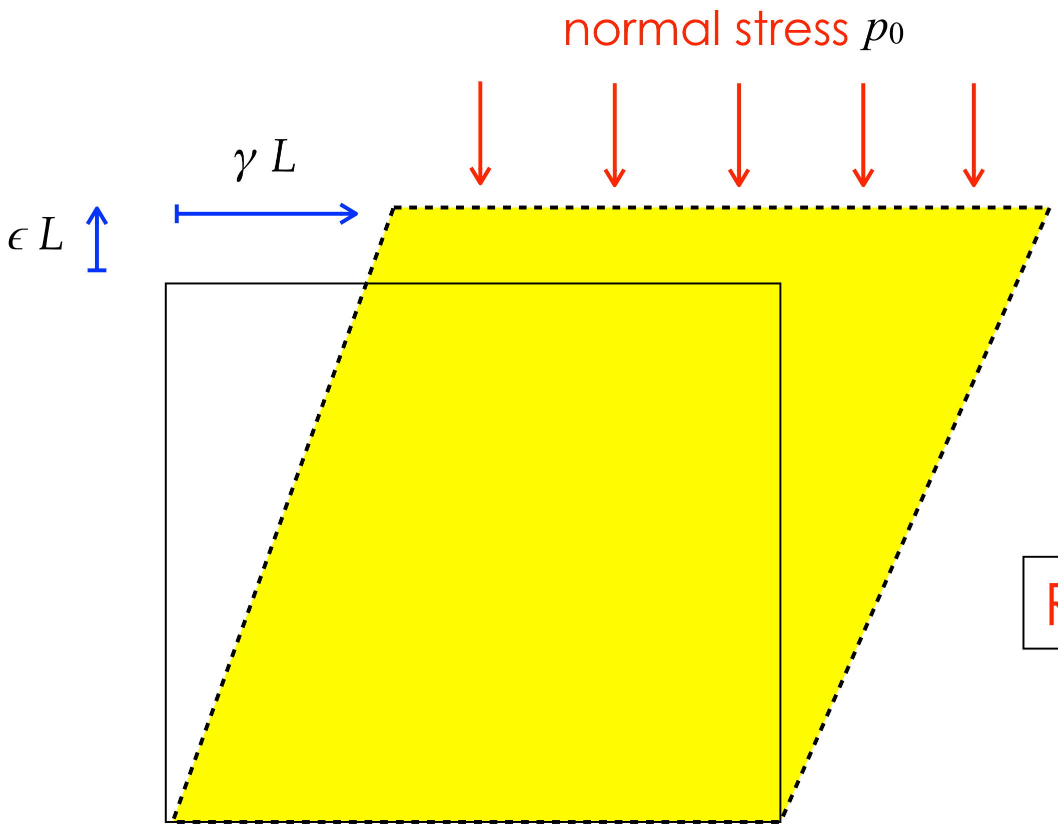
Dilatancy enhanced near jamming





Ren, Dijksman & Behringer
PRL 2013

A nonlinear effect



symmetry:

$$\epsilon = \frac{1}{2} R_p\, \gamma^2 + \ldots$$

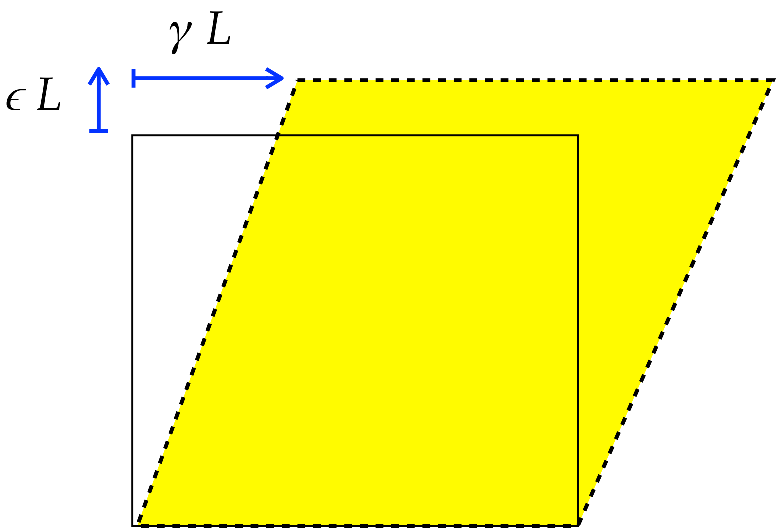
Reynolds dilatancy coefficient

$$R_p = \left(\frac{\partial^2 \epsilon}{\partial \gamma^2}\right)_\gamma$$

Ren, Dijksman & Behringer, PRL 2013

Weaire & Hutzler, Phil. Mag. 2003

Reynolds coefficient



assume a hyperelastic solid:

energy $\mathrm{d}U = -p\,\mathrm{d}V - \sigma V\,\mathrm{d}\gamma$

"enthalpy" $\mathrm{d}H = V\,\mathrm{d}p - \sigma V\,\mathrm{d}\gamma$

Maxwell $$\left(\frac{\partial V}{\partial \gamma}\right)_p = -\left(\frac{\partial\, \sigma V}{\partial p}\right)_\gamma$$

$\longrightarrow$ expression for R_p

Weaire & Hutzler, Phil. Mag. 2003

BPT, Gran. Matt. 2013

Reynolds coefficient

$$R_p = \left(\frac{\partial G}{\partial p}\right)_\gamma - \boxed{\frac{G}{E}}$$

shear modulus G > 0
Young's modulus E > 0

typically E > G

Weaire & Hutzler, Phil. Mag. 2003

BPT, Gran. Matt. 2013

Reynolds coefficient

$$R_p = \boxed{\left(\frac{\partial G}{\partial p}\right)_\gamma} - \frac{G}{E}$$

shear modulus G > 0
Young's modulus E > 0

typically E > G

magnitude >> 1 in marginal solids

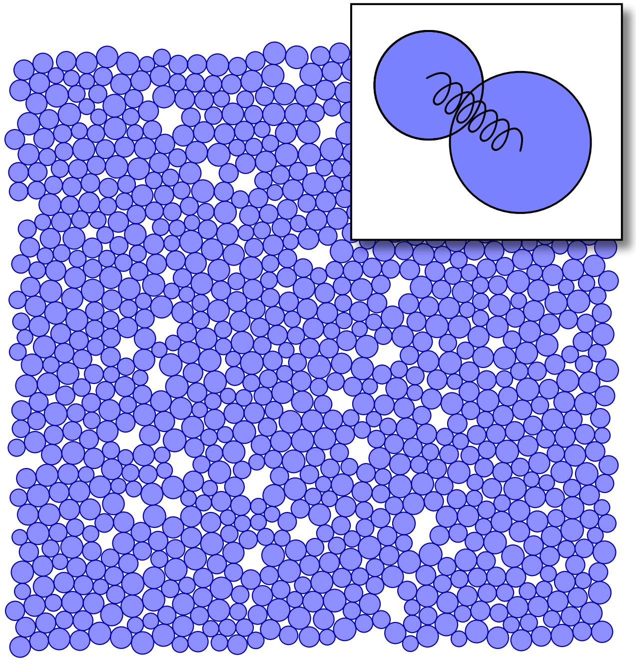
$$\simeq \left(\frac{\partial G}{\partial p}\right)_\gamma$$

does compression stiffen or soften the shear modulus?

Weaire & Hutzler, Phil. Mag. 2003

BPT, Gran. Matt. 2013

Soft spheres



$$R_p \simeq \left(\frac{\partial G}{\partial p}\right)_\gamma$$

$$G \sim p^{1/2}$$ (Hookean)

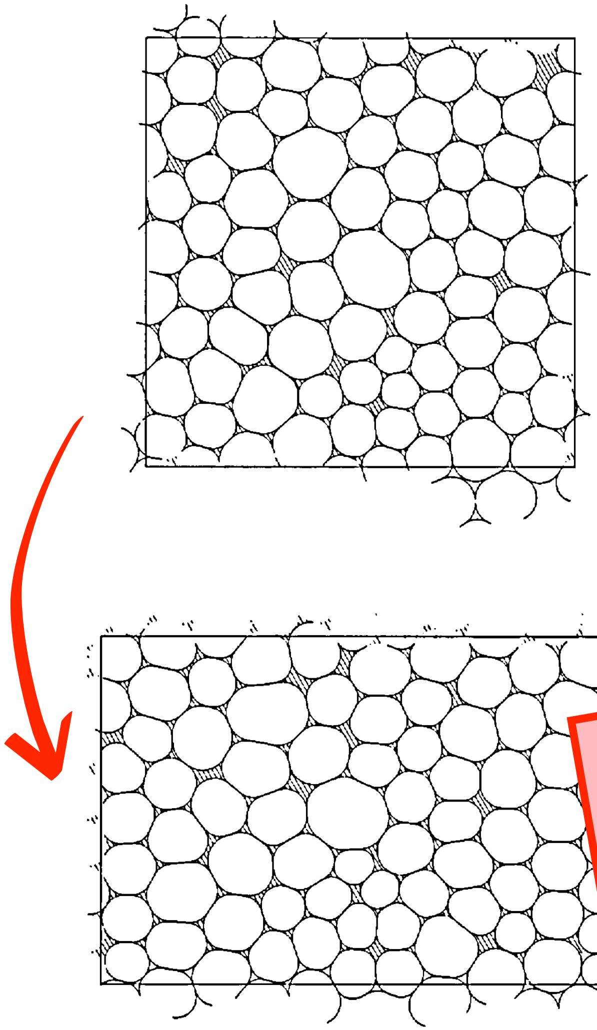
O'Hern, Silbert, Liu & Nagel, PRE 2003

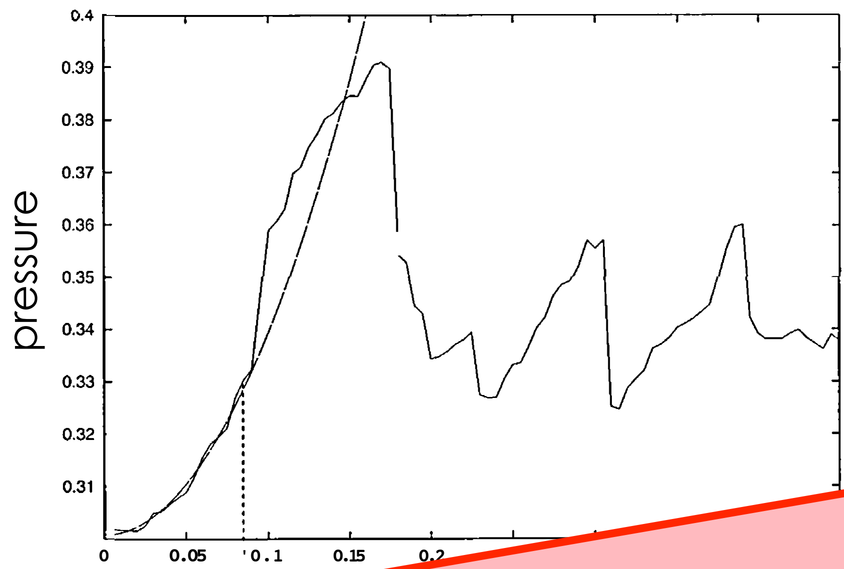
$$R_p \sim \frac{1}{p^{1/2}} > 0$$

packings **expand**

BPT, Gran. Matt. 2013

Packings expand:
verified in model foams

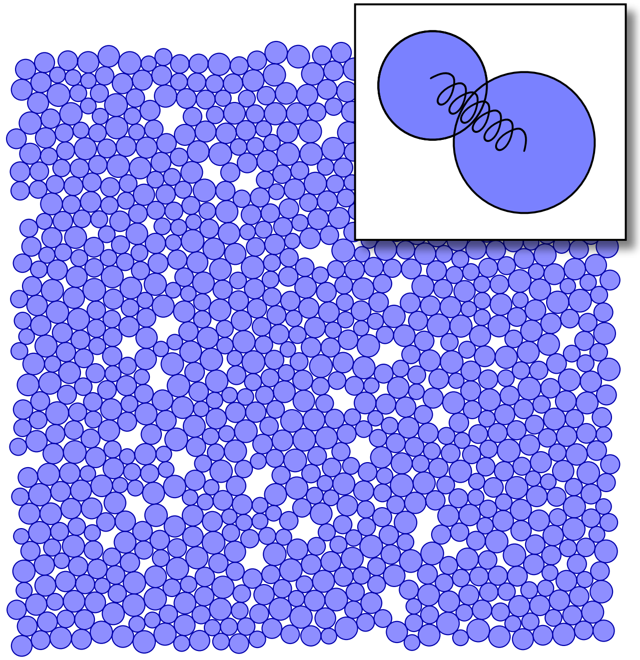




1. Physical intuition?
2. What about networks?

Weaire & Hutzler, Phil. Mag. 2003

Mixing shear and dilation...



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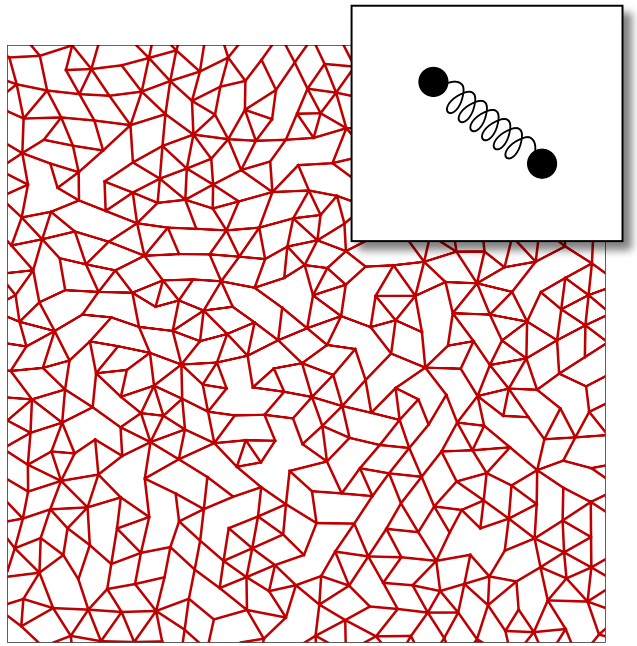
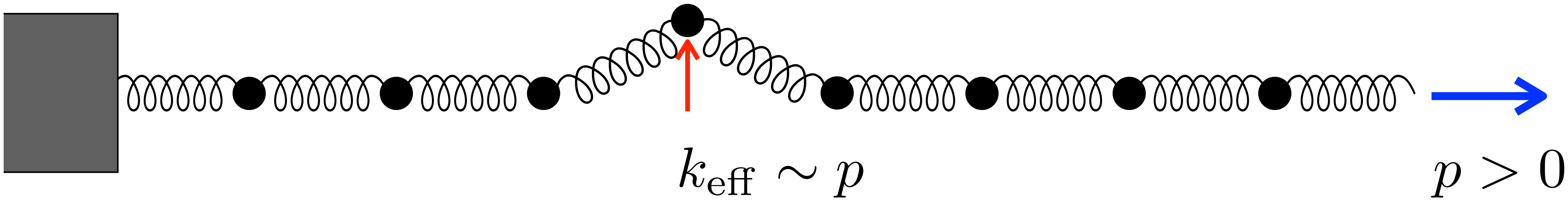
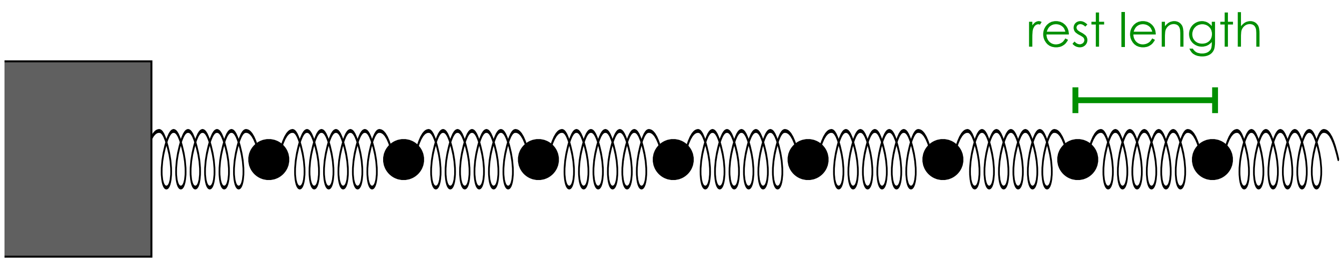
II. Tuning shear compliance with pre-tension

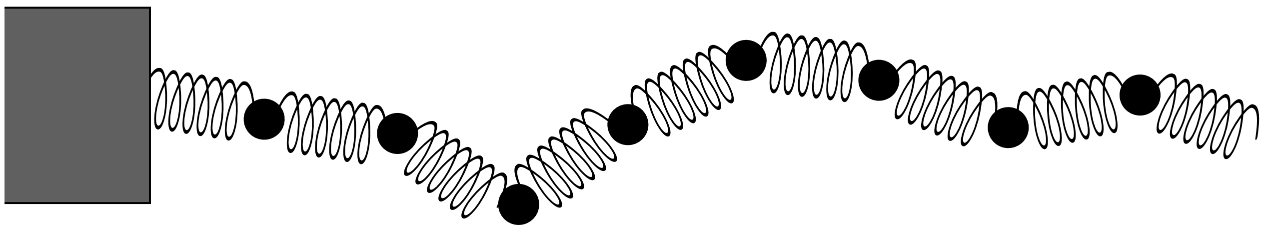


Tuning with tension

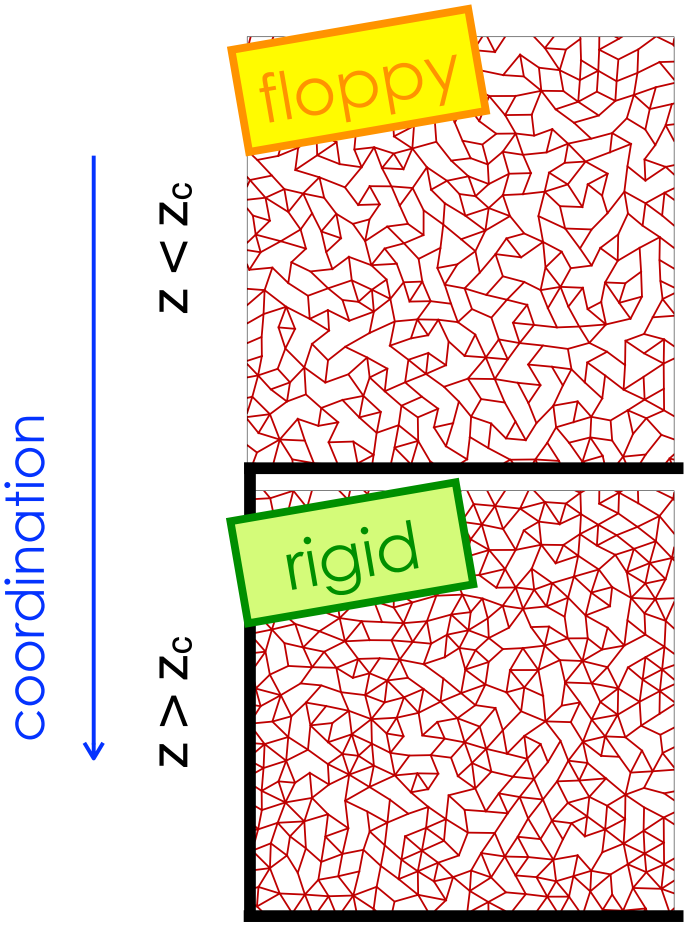




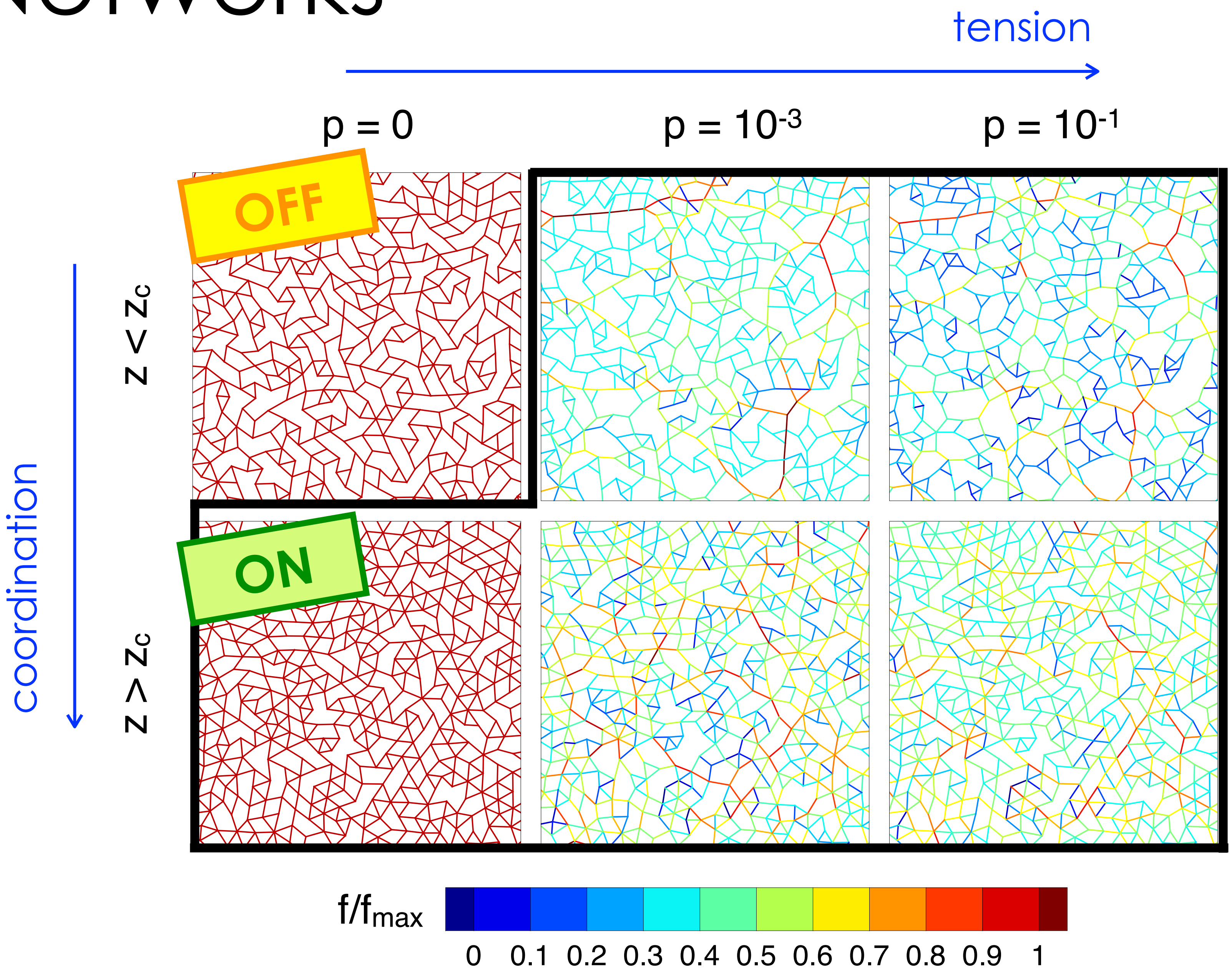
unloaded state = floppy = tunable!



Networks



Networks


tension
p = 0
p = 10-3
p = 10-1
OFF
ON
z < zc
z > zc
coordination
f/fmax
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

Manipulating marginal matter

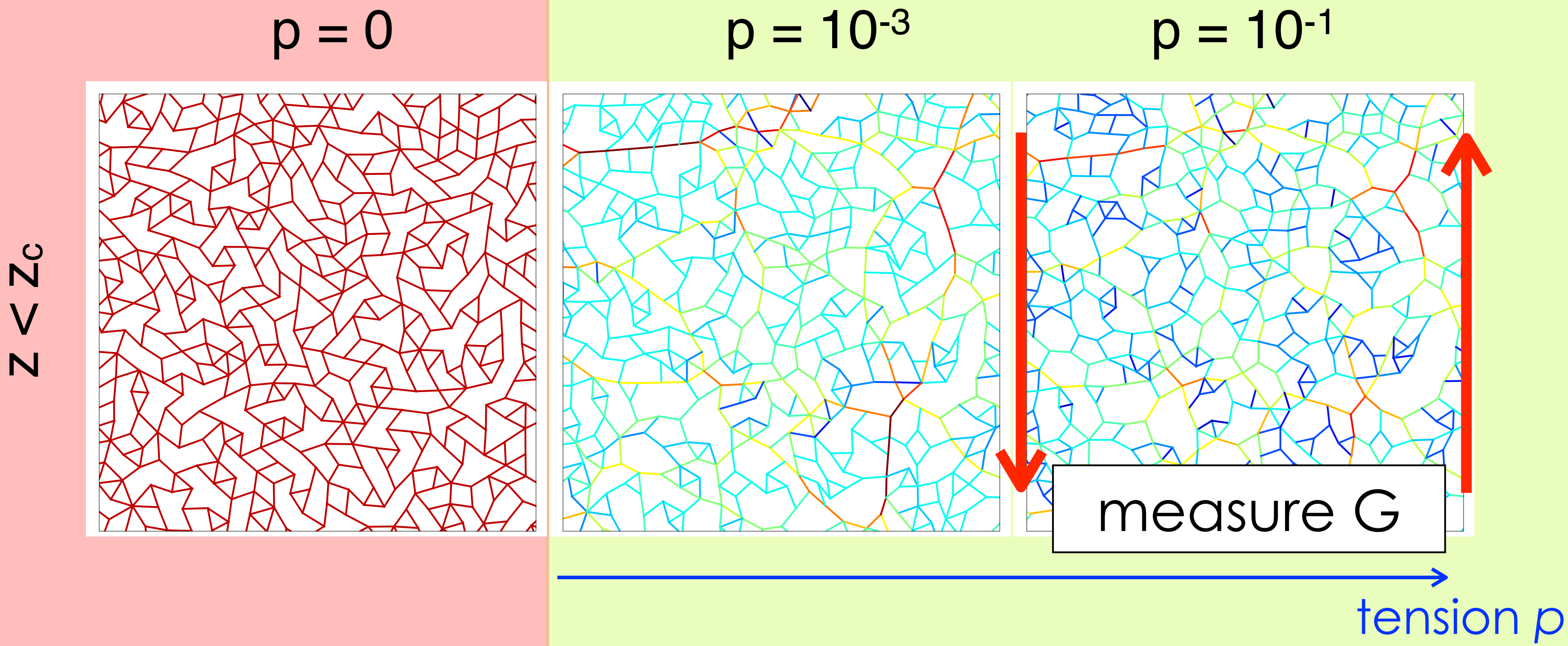
Brown et al, PNAS 2010

unjammed = **OFF**

jammed = **ON**

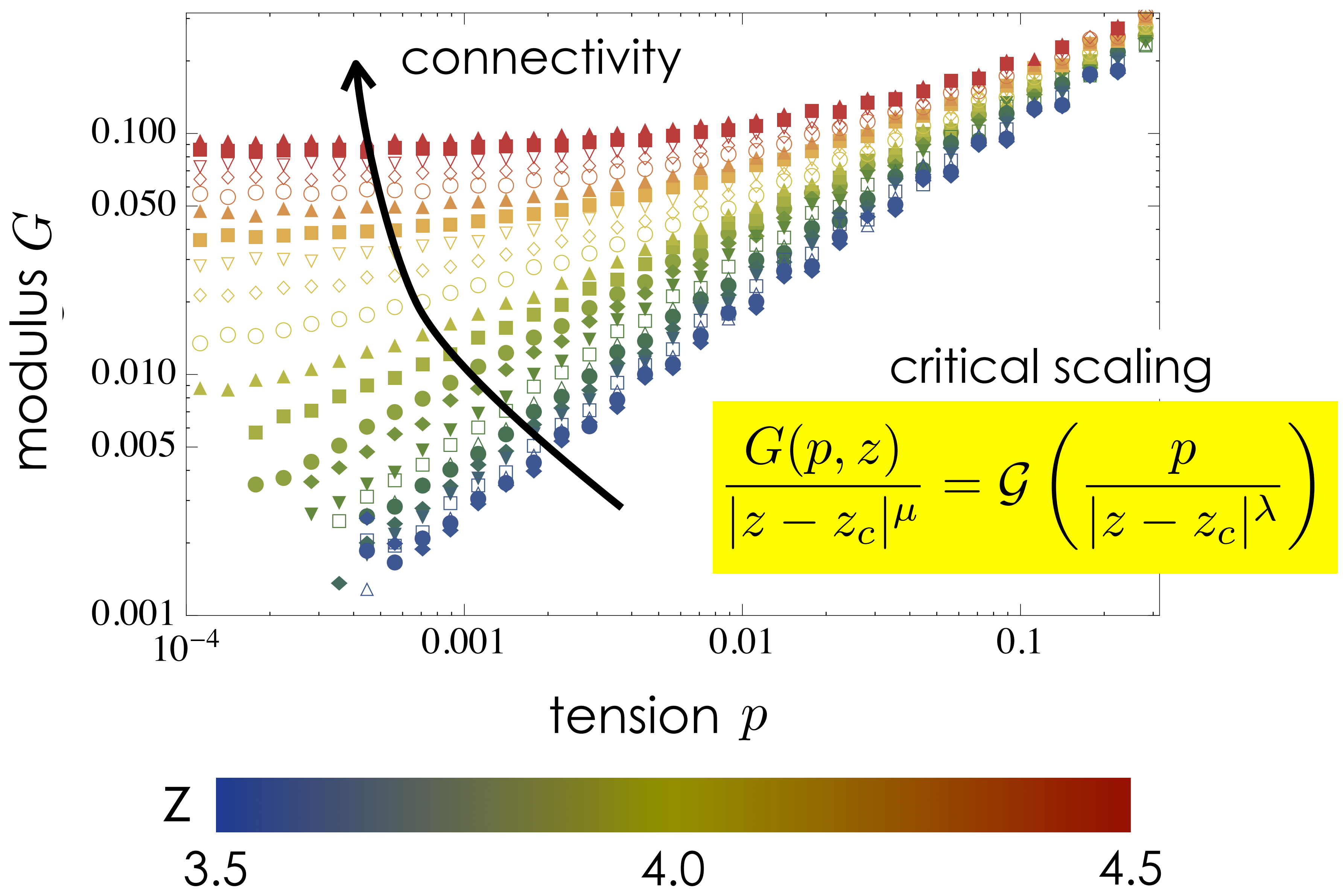
jamming transition
as a **switch**

Rigidity induced by tension

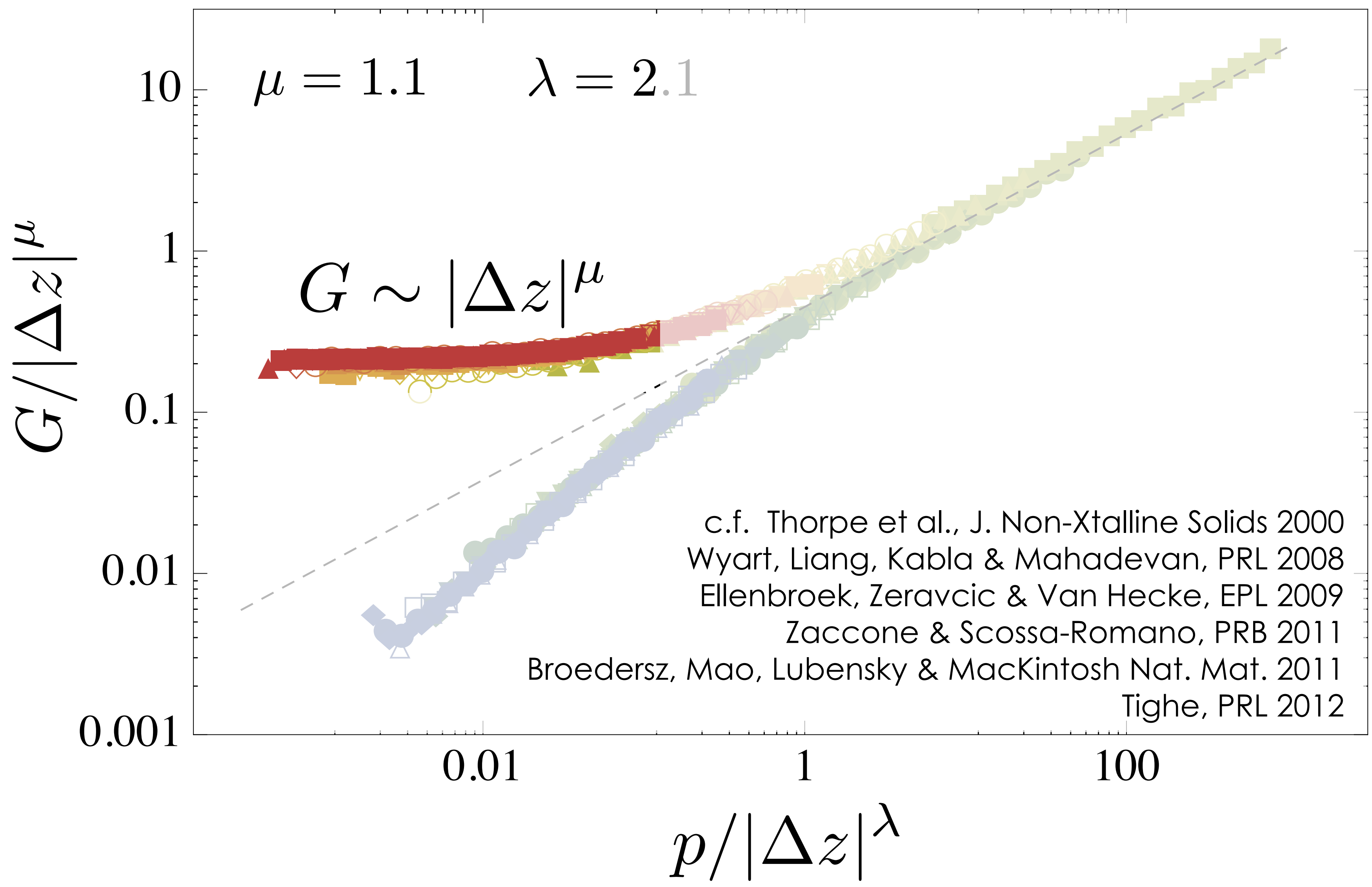


jamming transition
as a **switch**
...or a **knob**

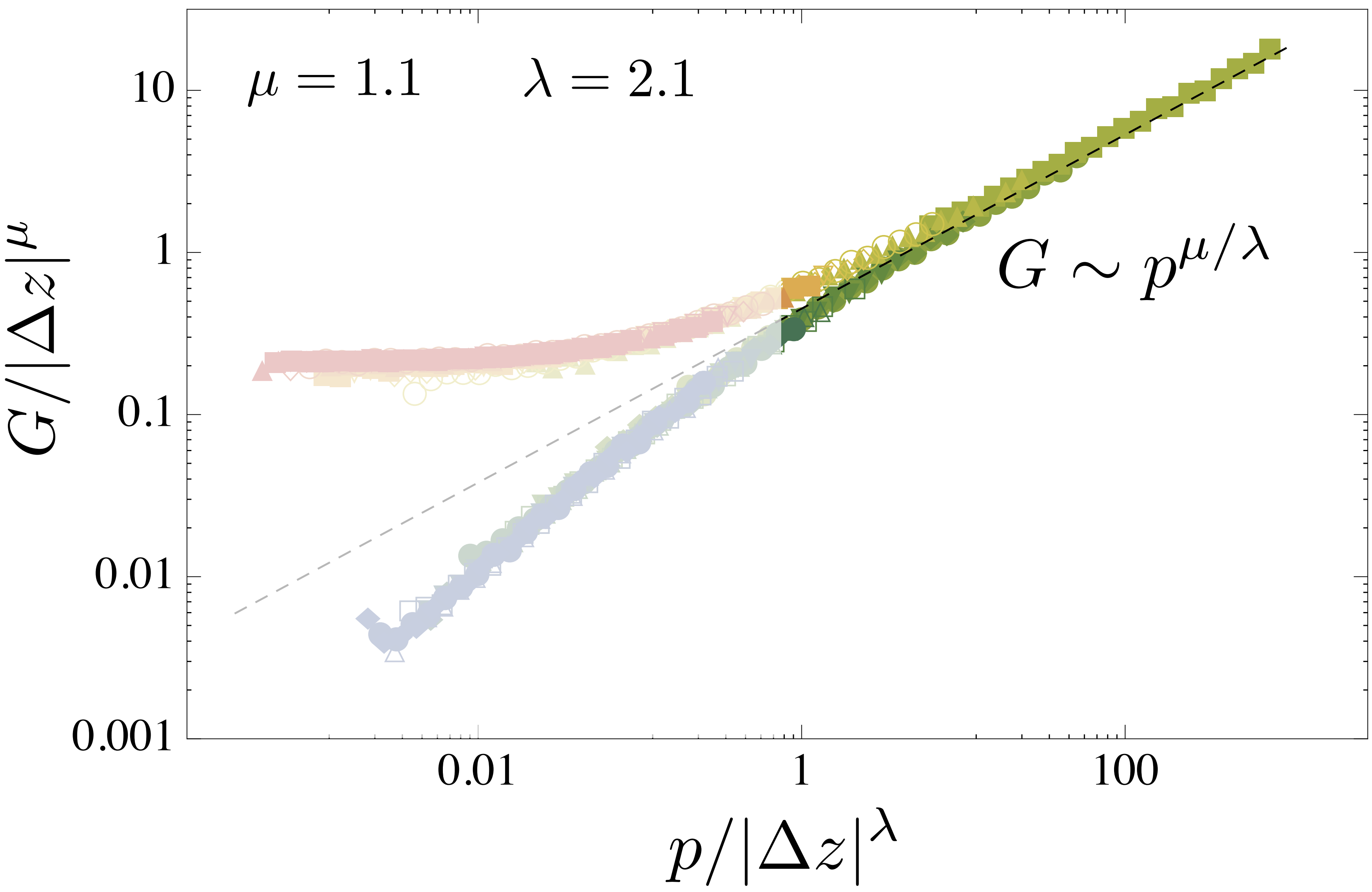
Shear modulus



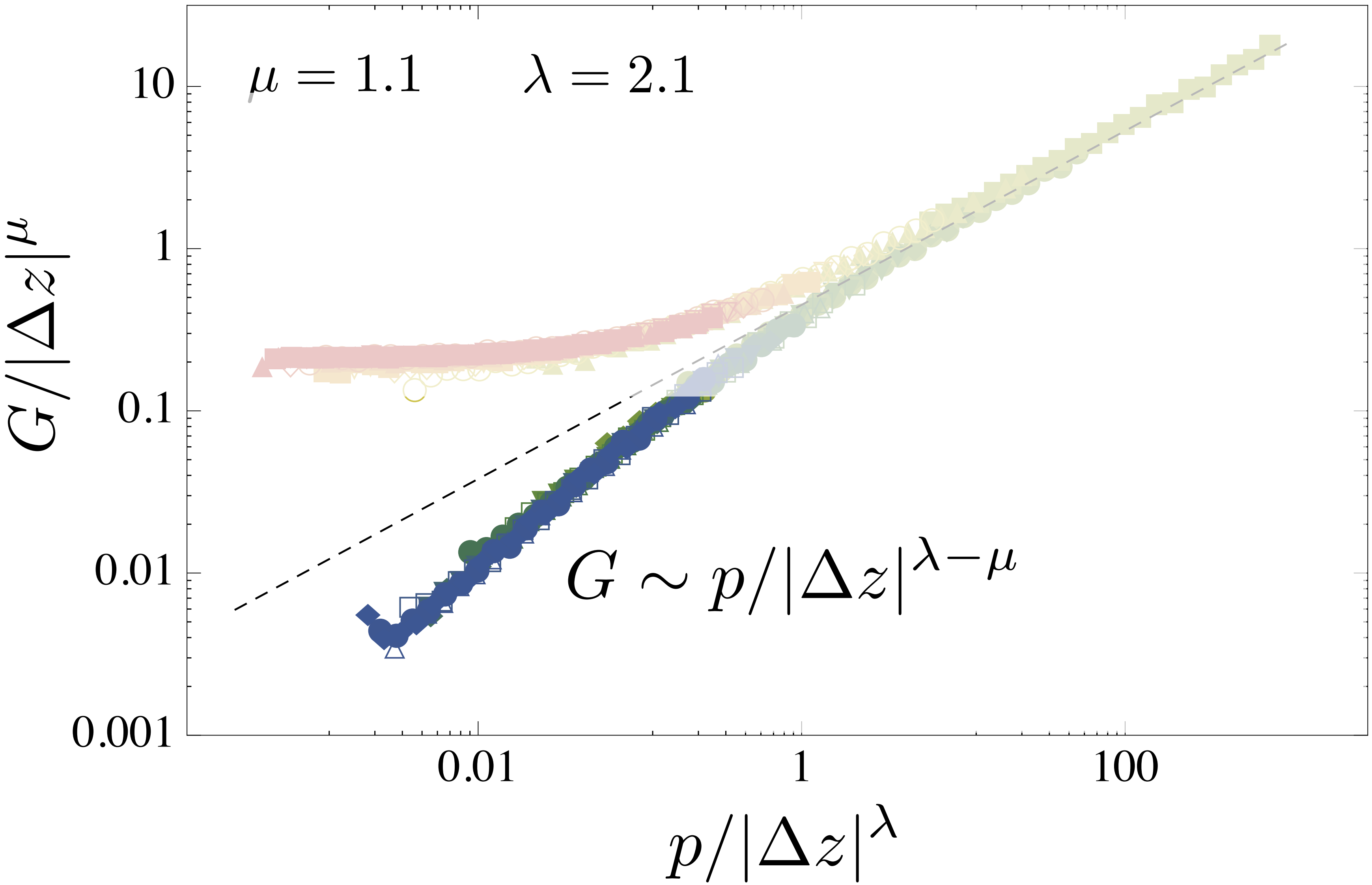
Critical scaling



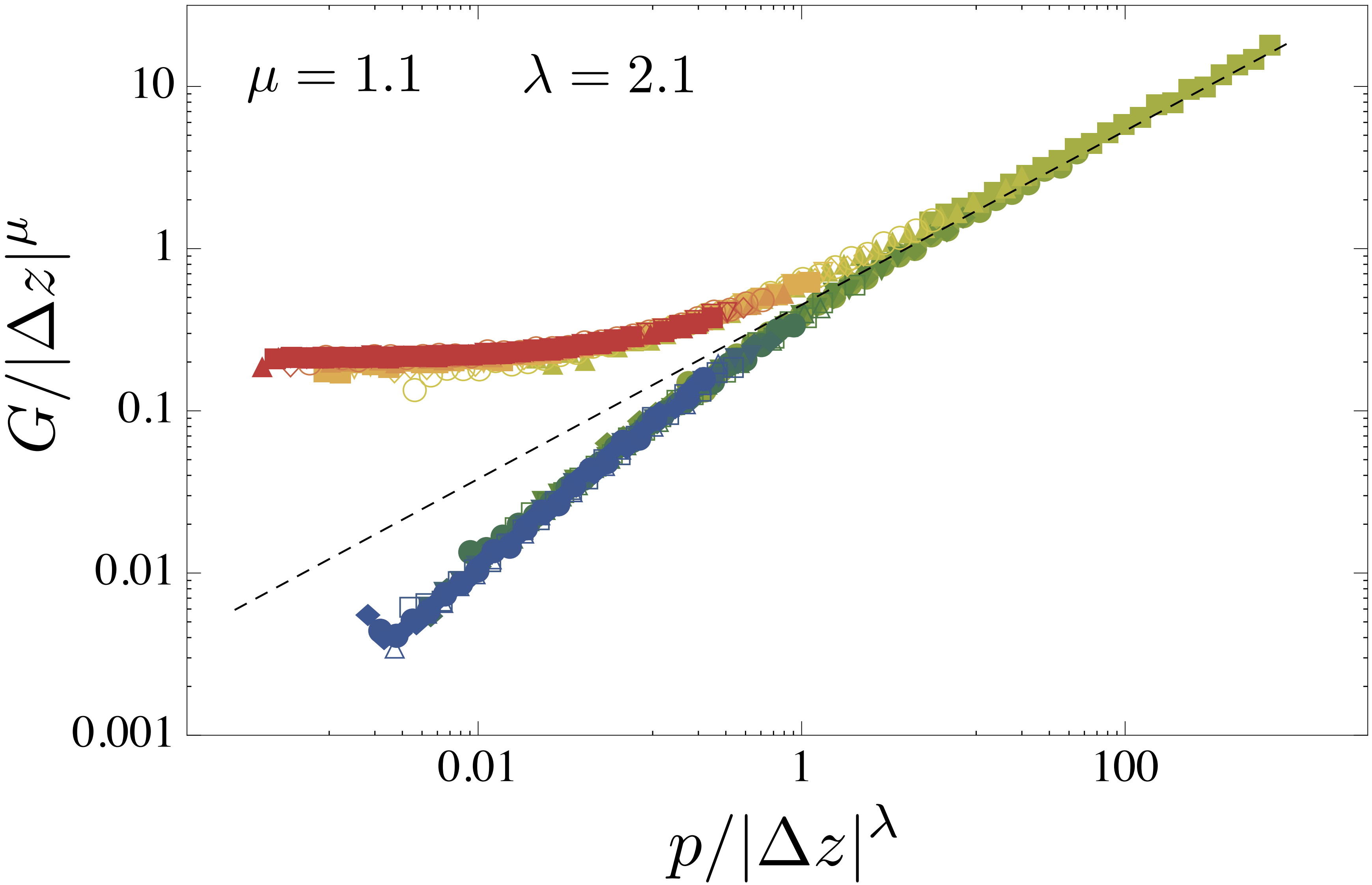
Critical scaling



Critical scaling

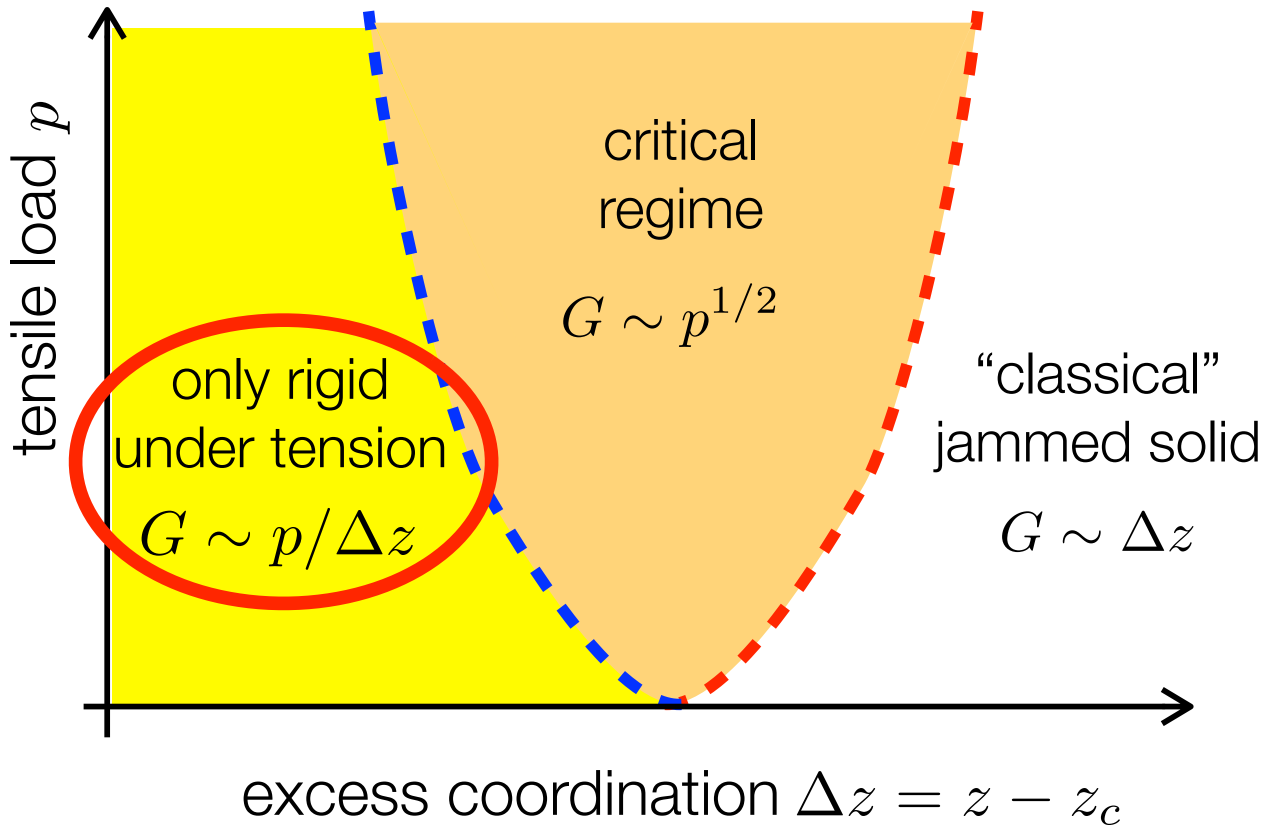


Critical scaling

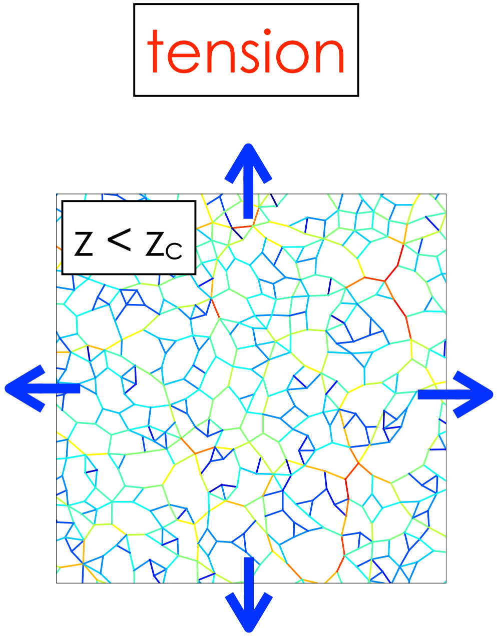


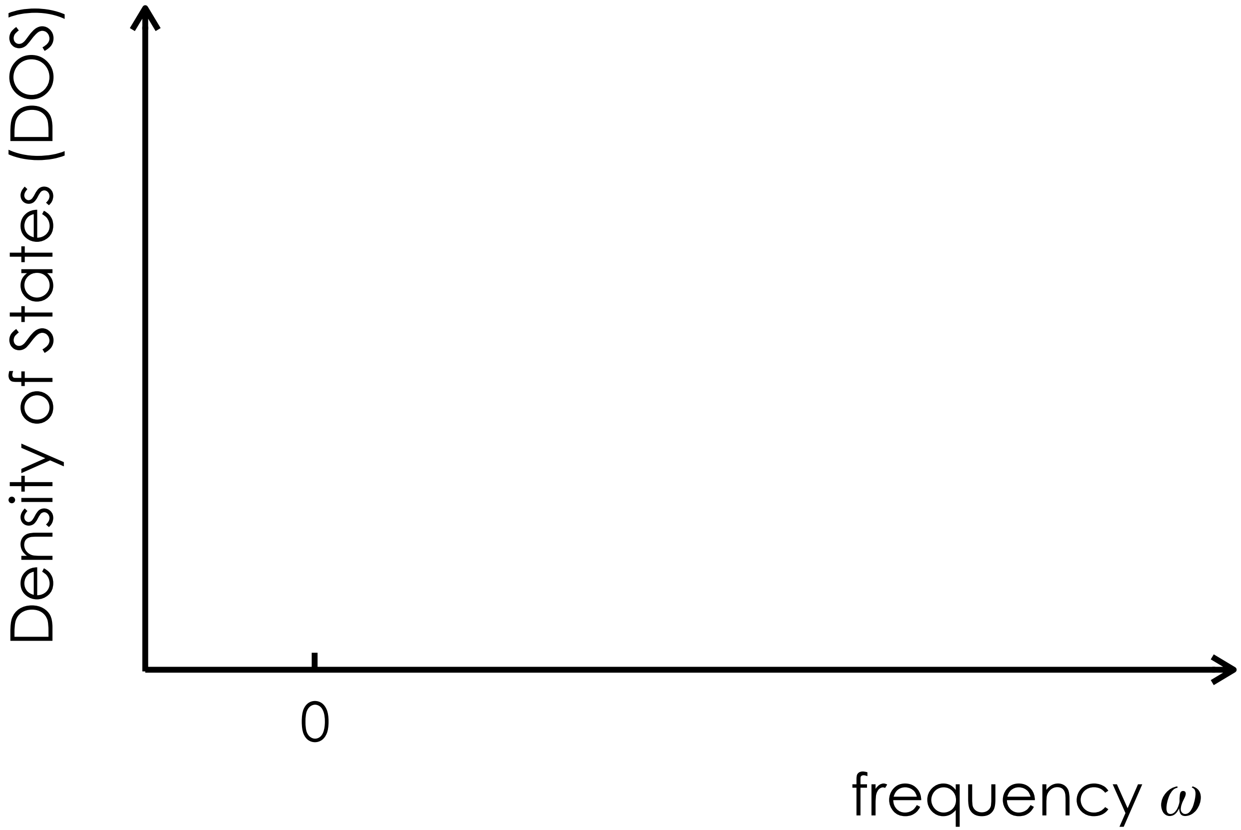
reversible tuning with tension

diverging susceptibility



Spectra

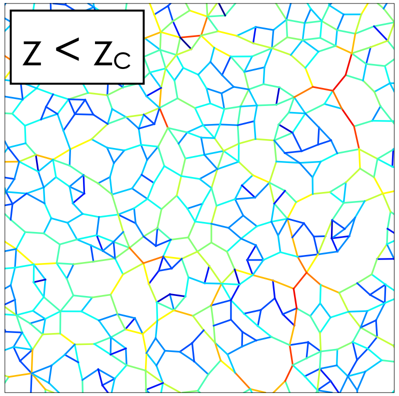


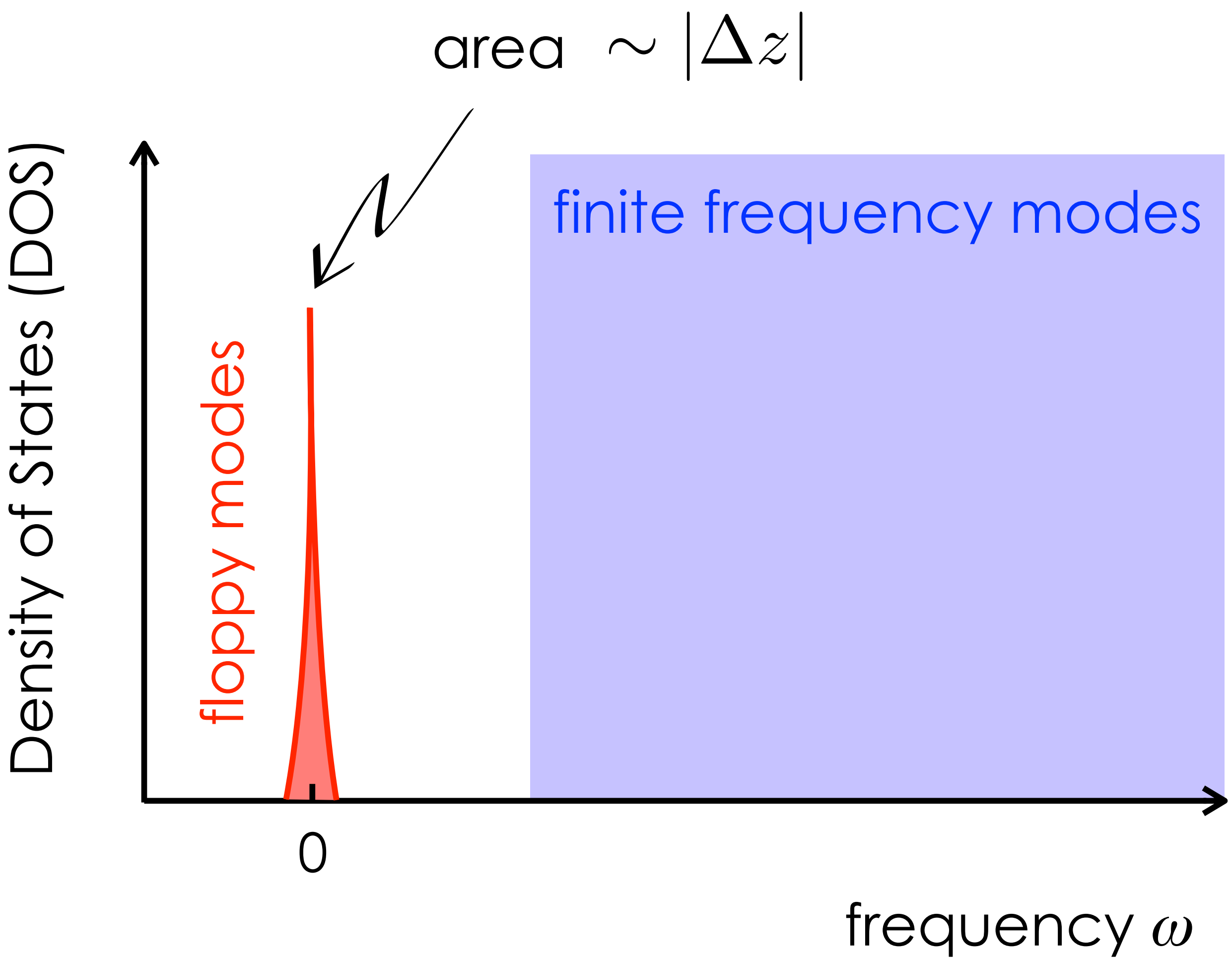


gap: Düring et al, Soft Matter 2012

Spectra

zero tension





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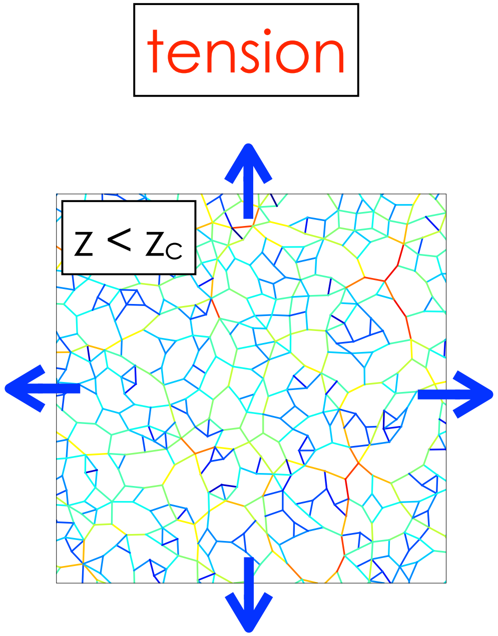

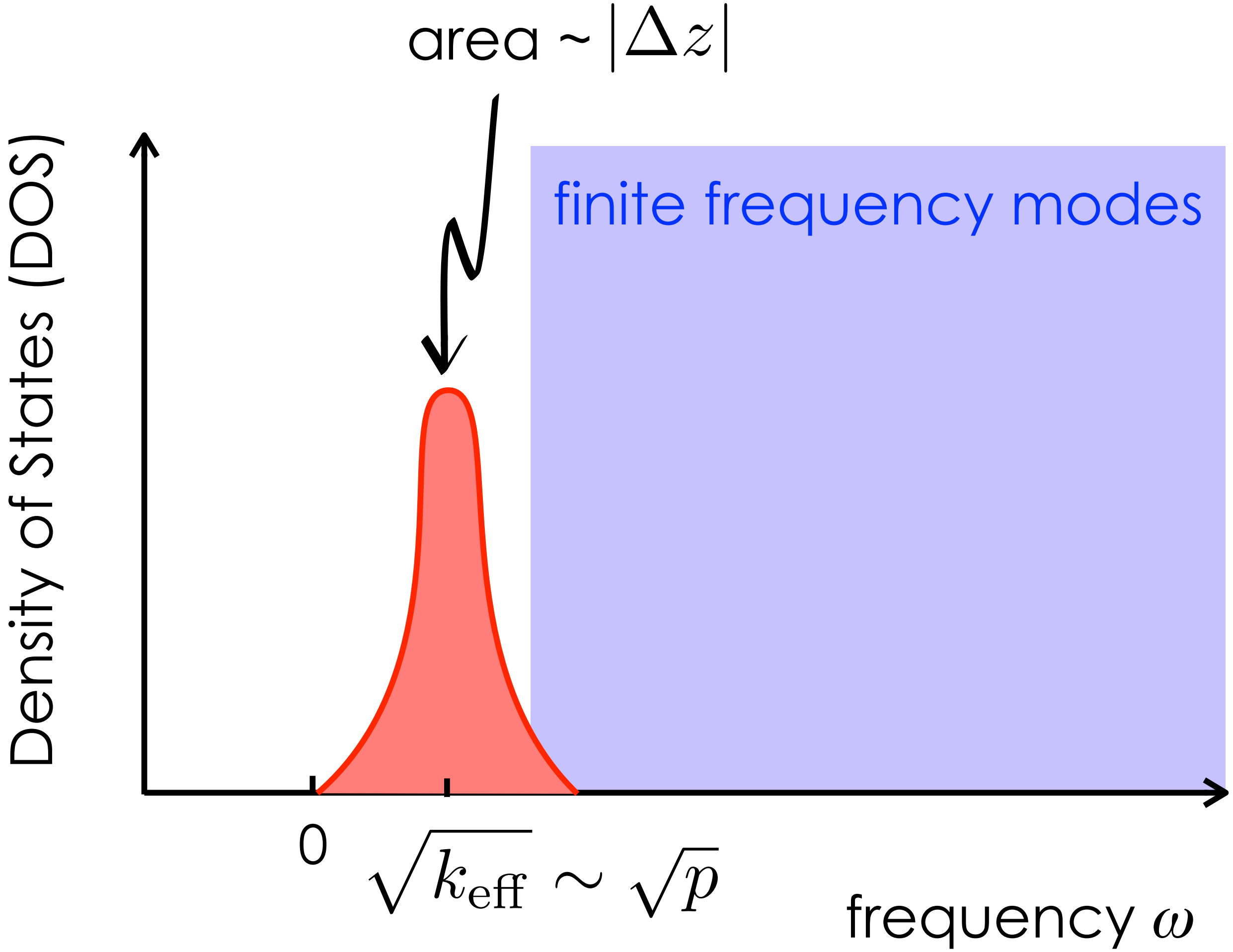

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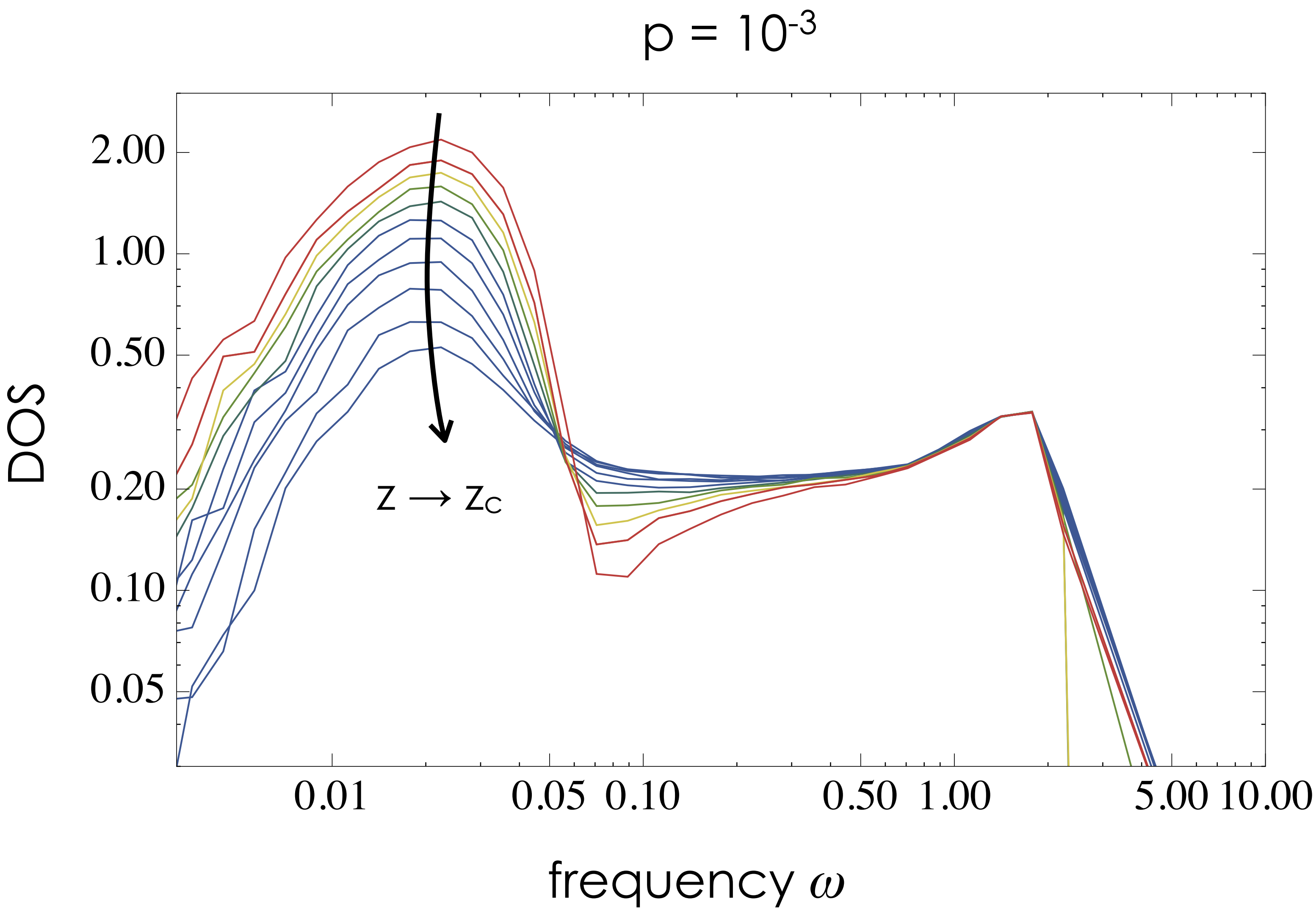
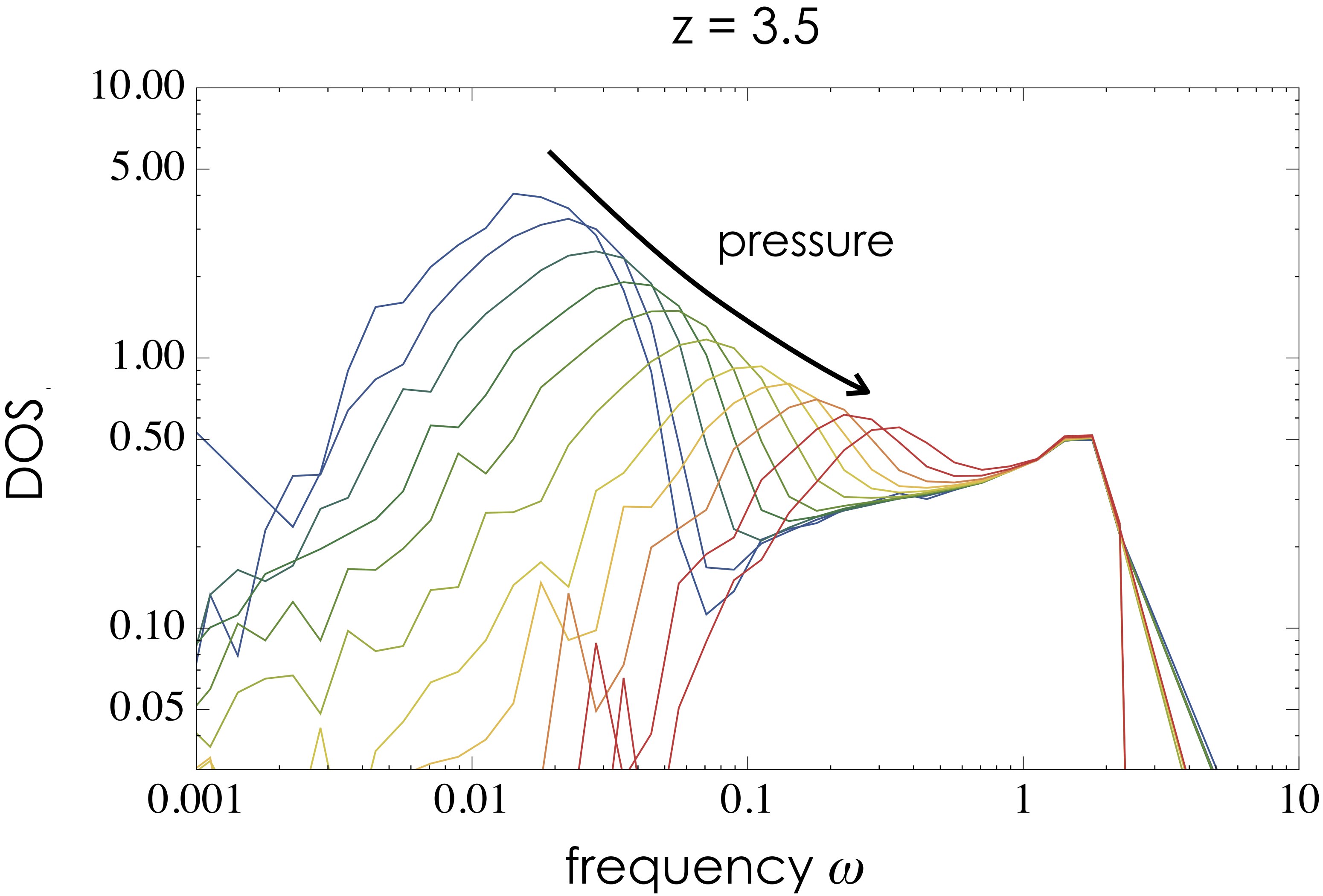
Spectra



p = 10-3
2.00
1.00
0.50
0.20
0.10
0.05
DOS
z → zc
0.01
0.05
0.10
0.50
1.00
5.00
10.00
frequency ω

Spectra

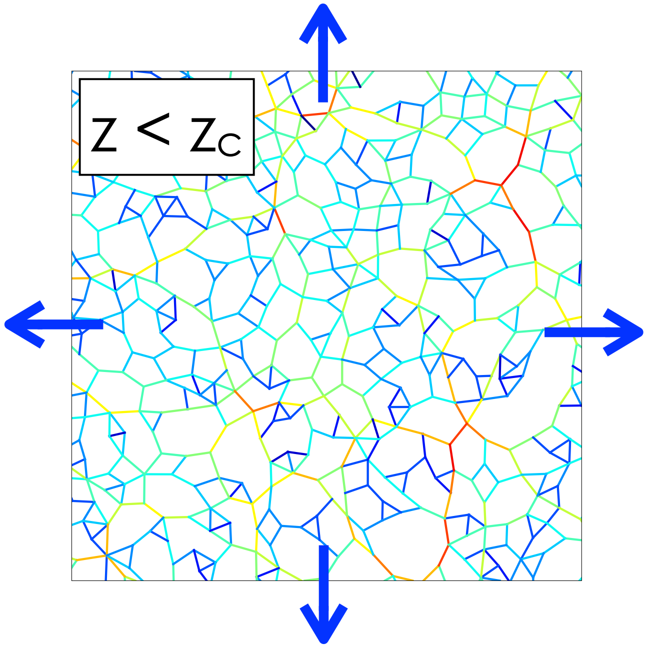


z = 3.5
pressure
10.00
5.00
1.00
0.50
0.10
0.05
DOS
0.001
0.01
0.1
1
10
frequency ω

Simple scaling argument

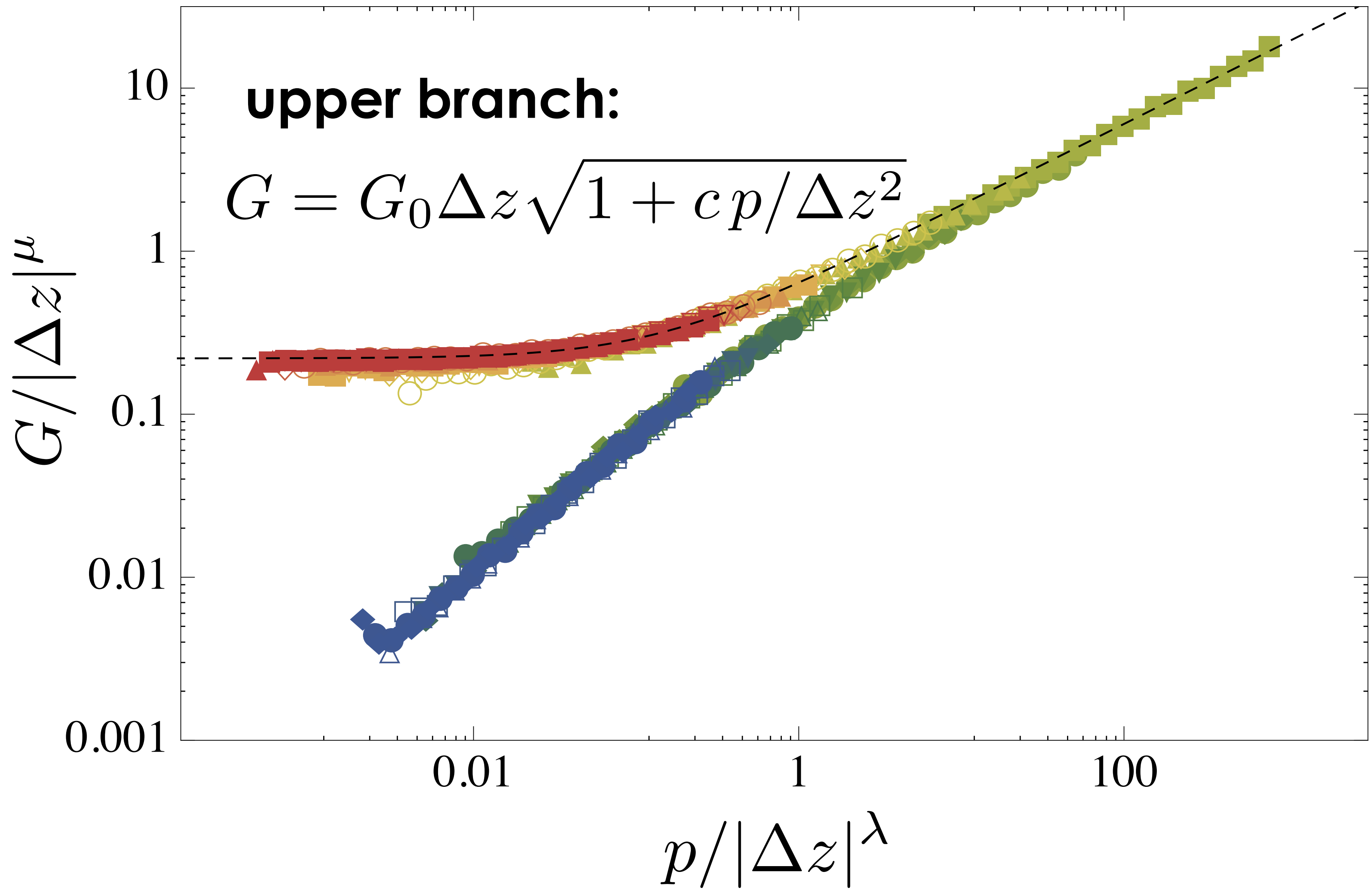
modulus ↔ modes

$$\frac{1}{G} \sim \frac{1}{N}\sum_n \frac{1}{\omega_n^2}$$

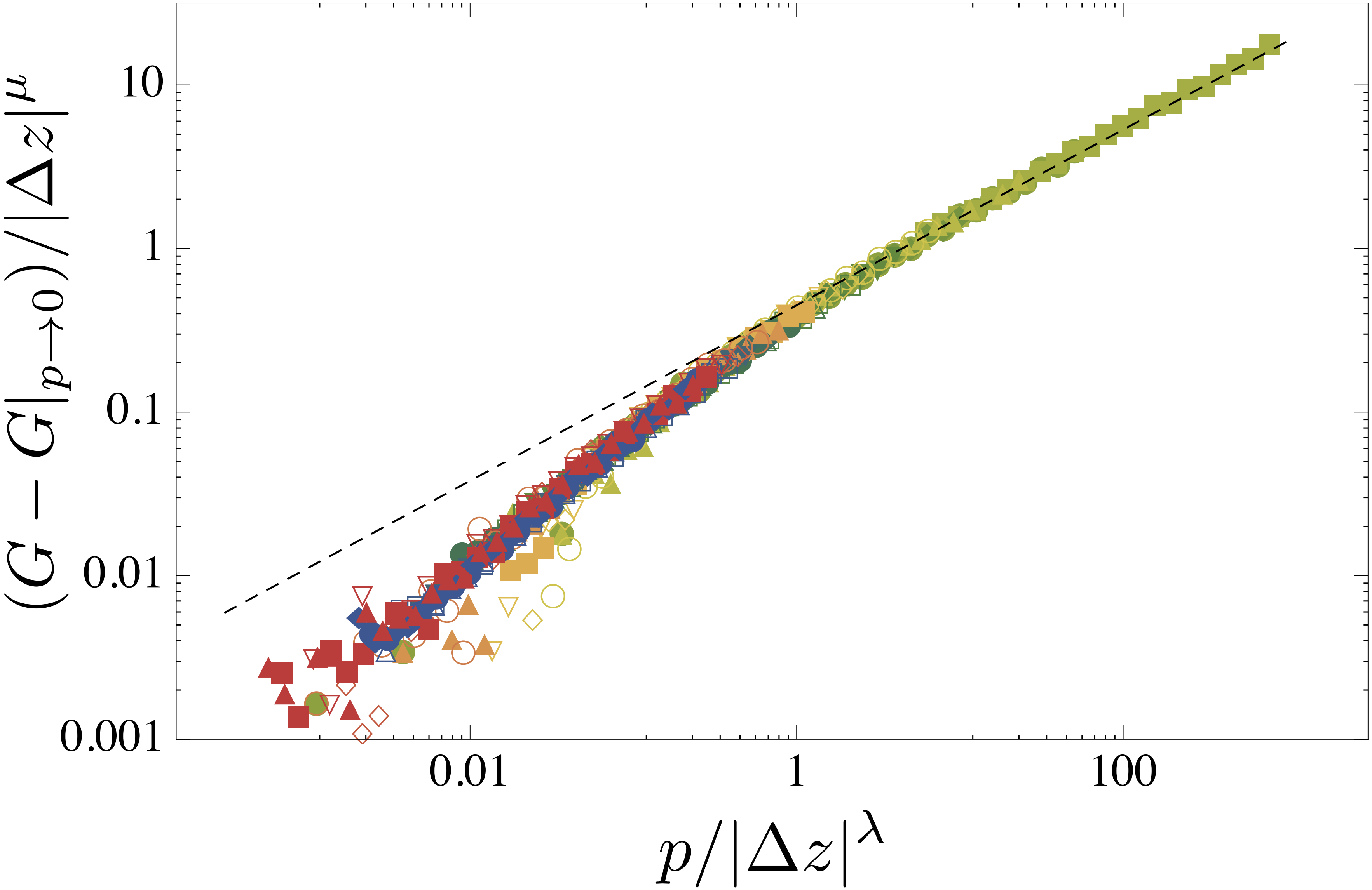


$$G \sim \frac{p}{|\Delta z|}$$ w/ high susceptibility $$\frac{\partial G}{\partial p} \sim \frac{1}{|\Delta z|}$$

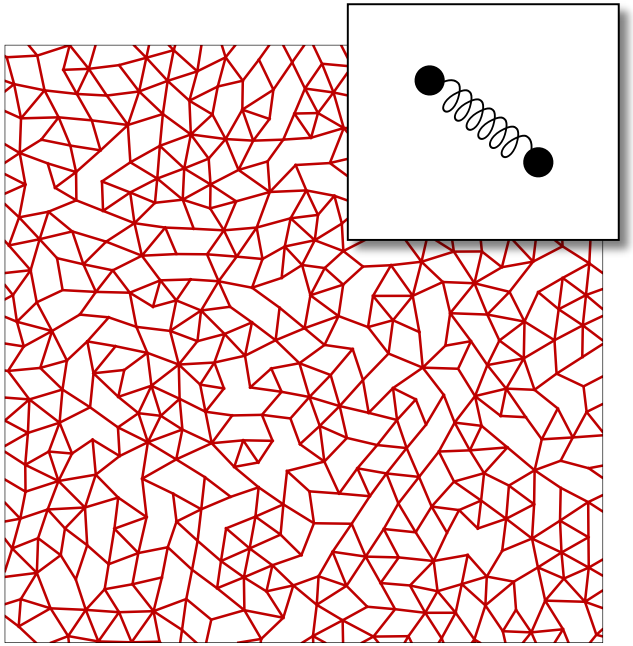
Critical scaling



Critical scaling



Network dilatancy



$$R_p \simeq \left(\frac{\partial G}{\partial p}\right)_\gamma$$

note sign
convention

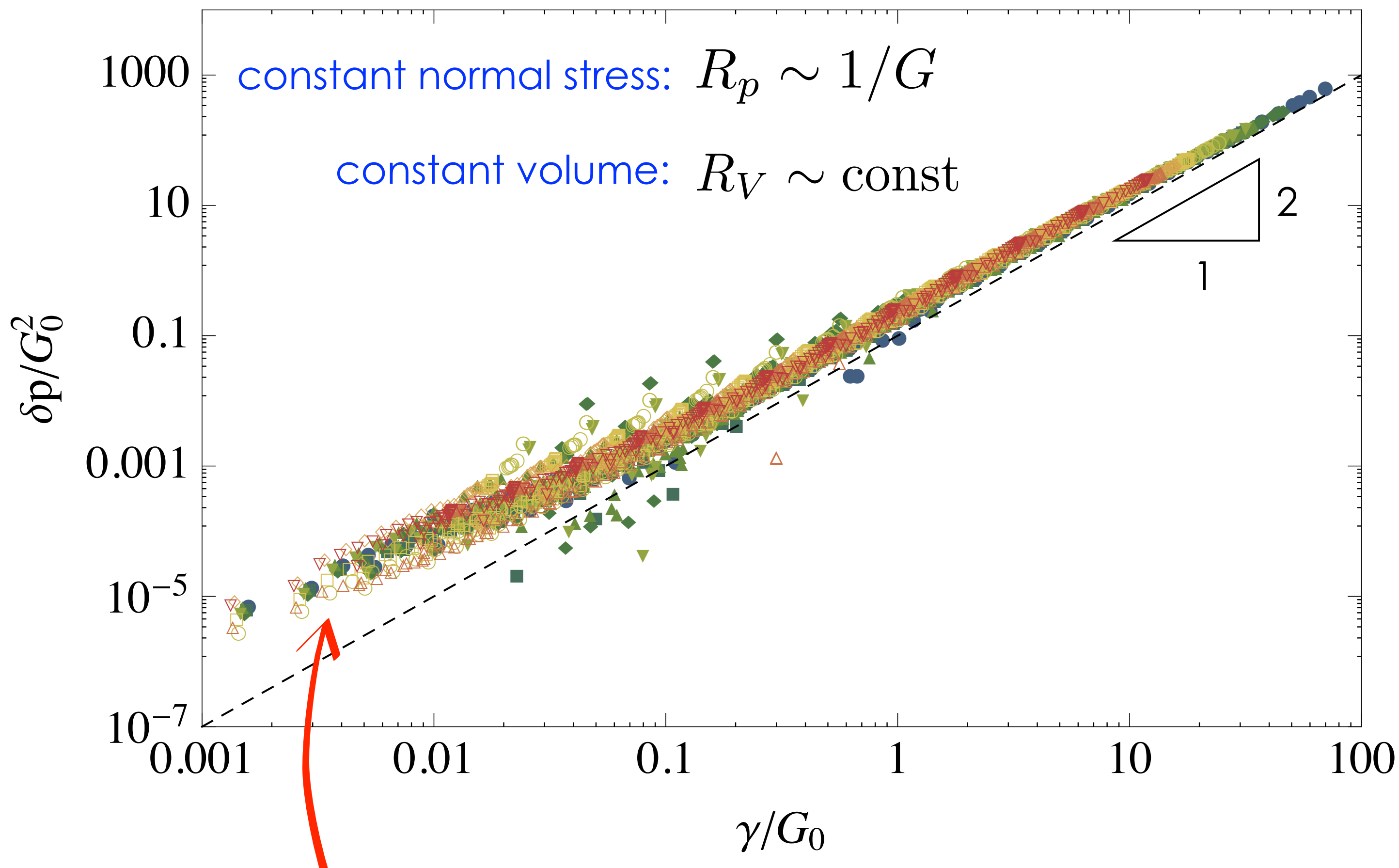
$$G \propto \Delta z \sqrt{1 - \mathrm{const} \cdot \frac{p}{\Delta z^2}}$$

$$R_p \sim -\frac{1}{G} < 0$$

networks **contract**

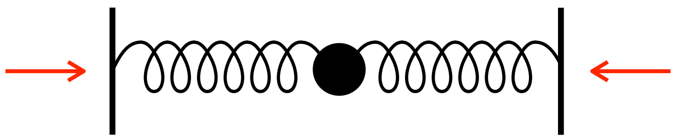
BPT, Gran. Matt. 2013

Network dilatancy



finite size effect:
Goodrich, Dagois-Bohy
et al. (in prep)

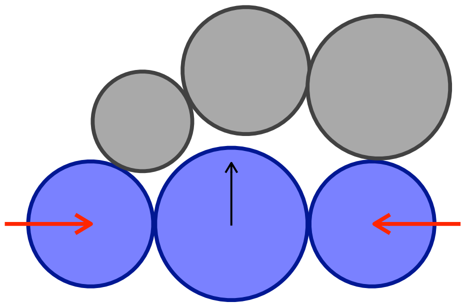
Intuiting dilatancy near jamming



compression
destabilizes

$$R_p \sim \frac{\partial G}{\partial p} < 0$$

networks
contract

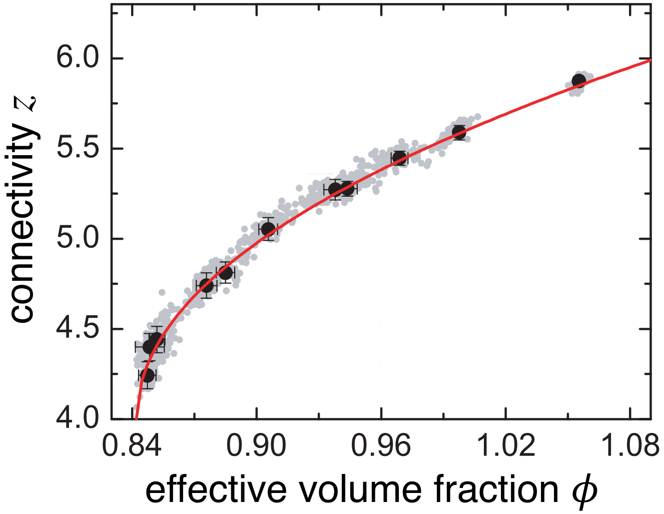


compression
stabilizes

$$R_p \simeq \frac{\partial G}{\partial p} > 0$$

packings
expand

Relating networks and packings



Katgert & Van Hecke, EPL 2010

$$G \propto \Delta z \sqrt{1 - \mathrm{const} \cdot \frac{p}{\Delta z^2}}$$

stability: $G > 0$

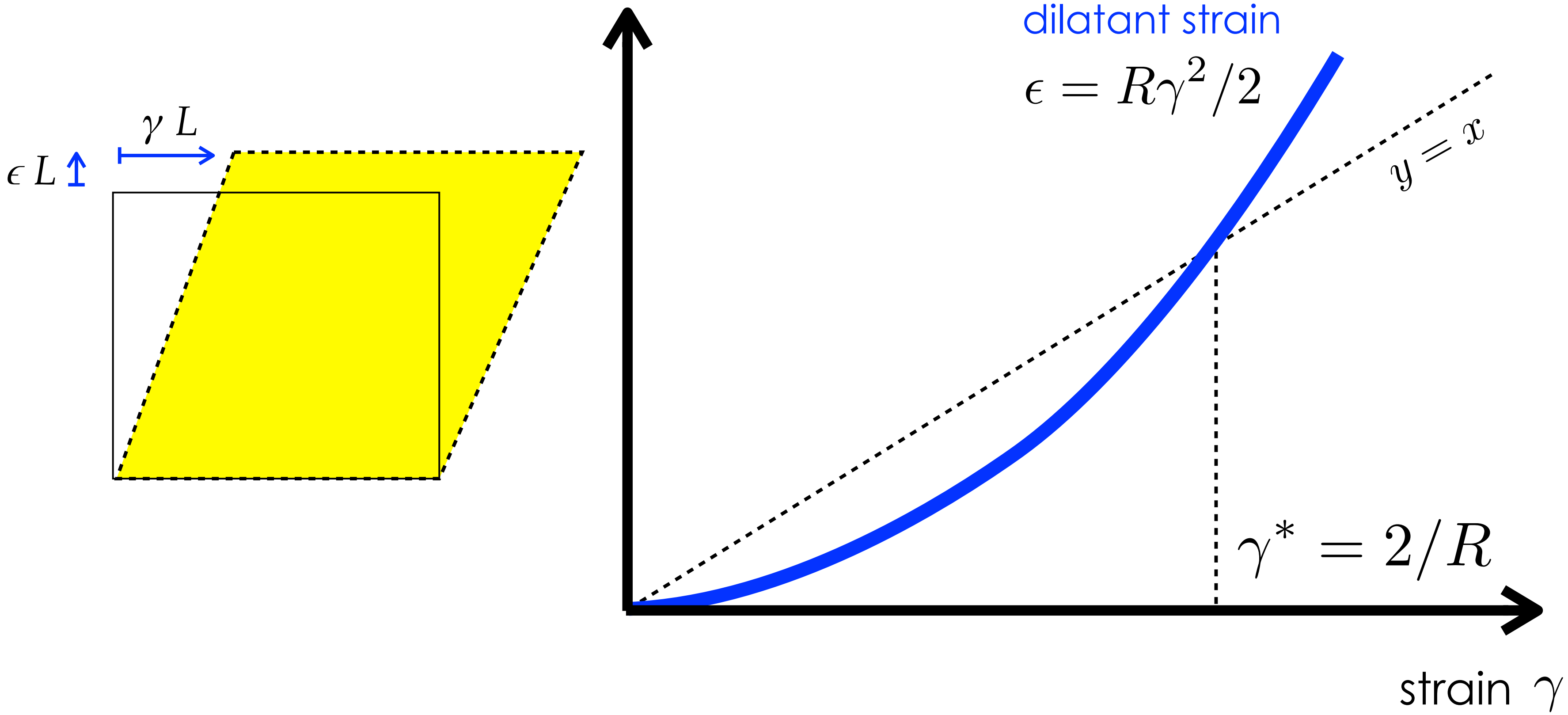
$$\Delta z \geq \mathrm{const} \cdot p^{1/2}$$

cf. Wyart et al, PRE 2005

higher pressure = more contacts

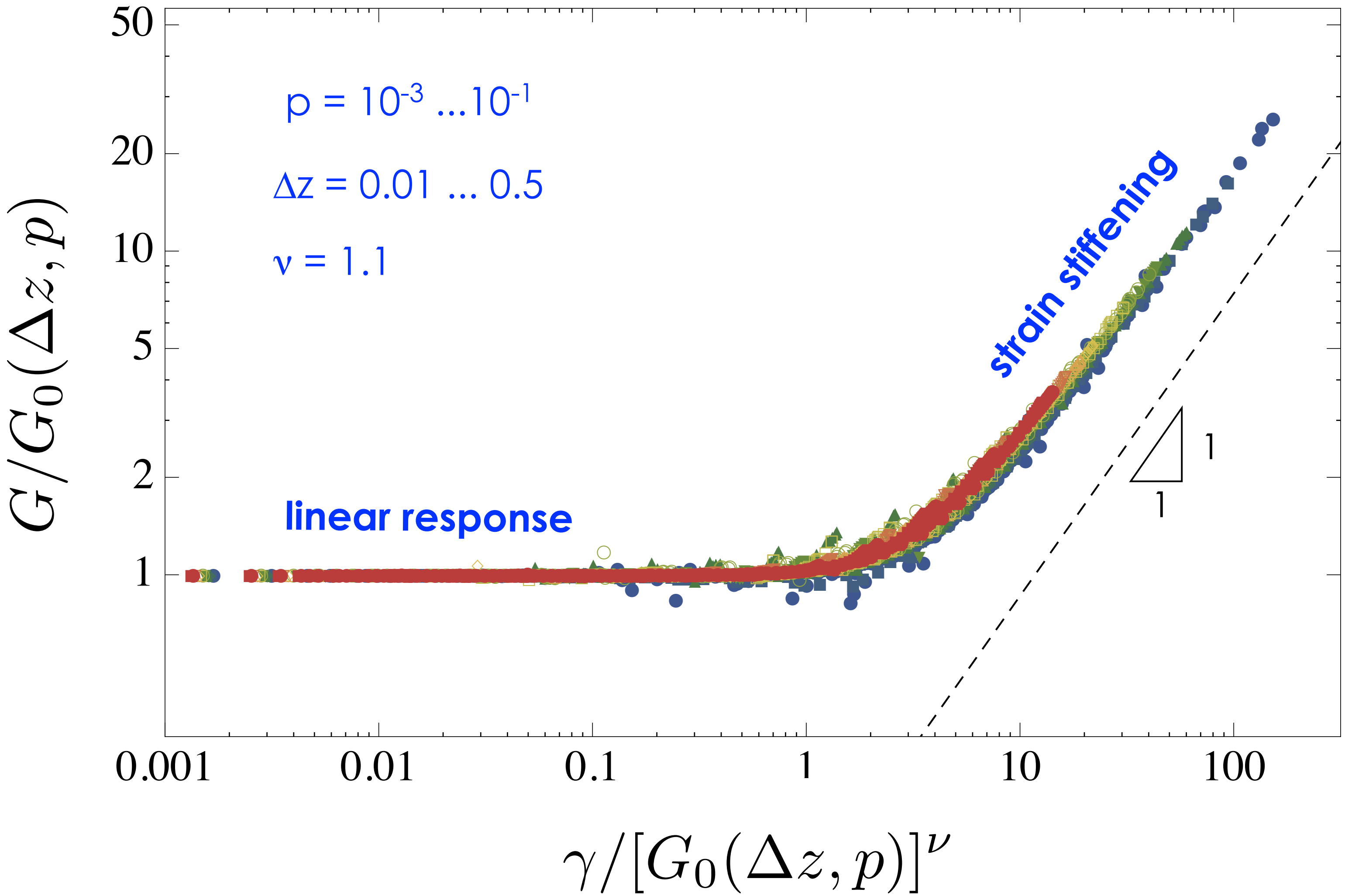
$$\longrightarrow G \sim p^{1/2}$$

Dilatancy and strain stiffening



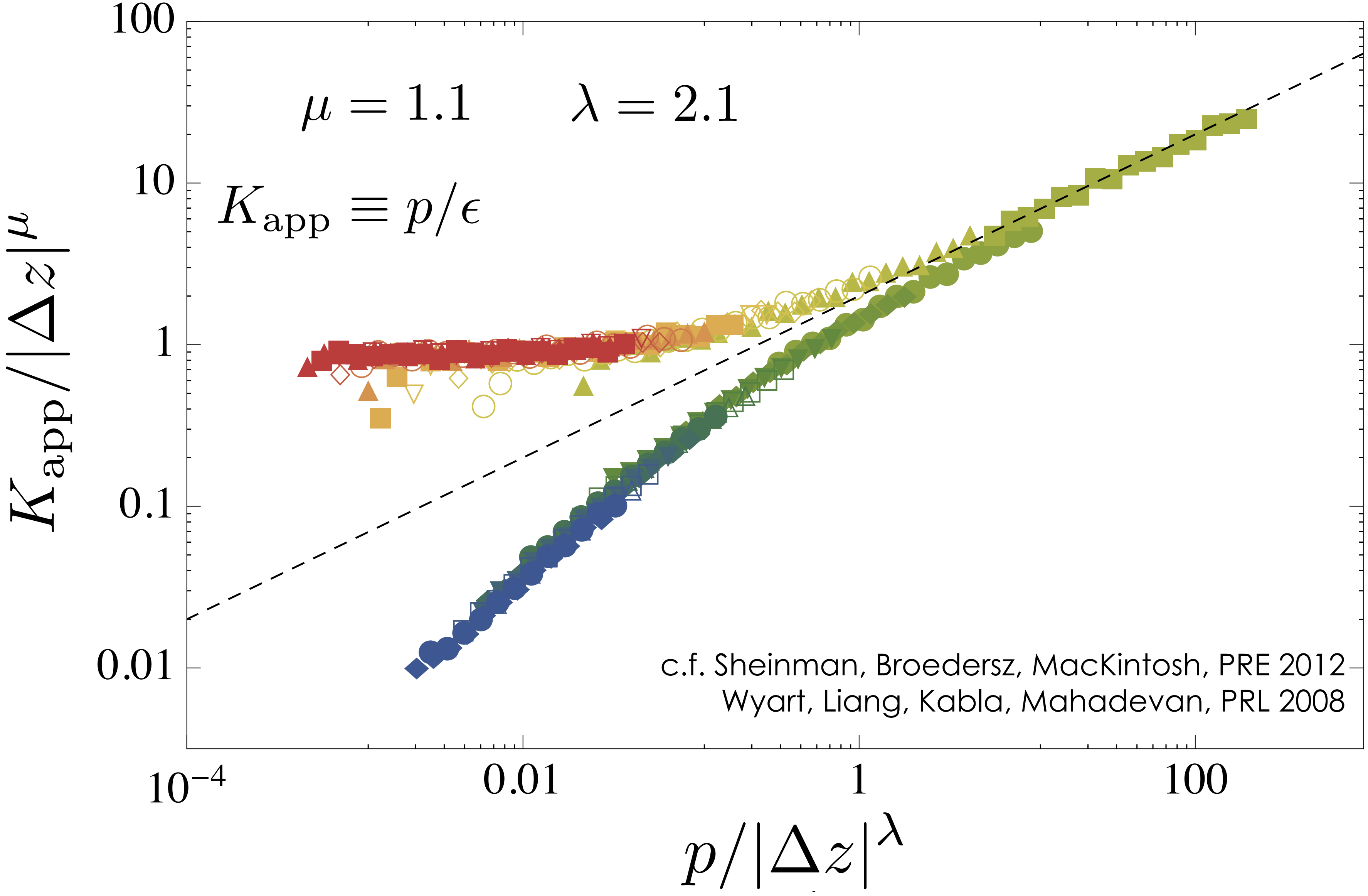
Ansatz: $G(\Delta z, p, \gamma) = G_0(\Delta z, p)\, f(\gamma/\gamma^*)$

Dilatancy and strain stiffening



cf Wyart et al., PRL 2008

Nonlinear bulk modulus



packings expand / networks contract

tunable shear modulus

enhanced near jamming

PhD and Postdoc positions available