Rayleigh-Taylor and Richtmyer-Meshkov Instabilities in Relativistic Hydrodynamic Jets

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What a relativistic jet?

collimated bipolar outflow from gravitationally bounded object

- active galactic nuclei (AGN) jet: $\gamma \sim 10$
- microquasar jet: $\nu \sim 0.9c$
- Gamma-ray burst: $\gamma > 100$

Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - (\nu/c)^2}}$$

A relativistic jet is considered to be launched form the central engine and propagates the progenitor star.
many numerical works in order to investigate the propagation dynamics of the relativistic jet (e.g., Marti+ 97, Aloy+ 00, Zhang+ 03,04, Mizuta+ 06, Morsony+ 07, Lazzati+ 09, Nagakura+ 11, Lopez-Camara+ 13)

- reconfinement shock (Norman et al. 1982; Sanders 1983)
- radial oscillating motion and repeated excitation of the reconfinement region (e.g., Gomez+ 97, JM+ 12)
To investigate the propagation dynamics and stability of the relativistic jet

- using relativistic hydrodynamic simulations
  focus on the transverse structure of the jet

- 2D simulations: evolution of the cross section of the relativistic jet
- 3D simulations: evolution of the cross section of the relativistic jet
- 3D simulation: propagation of the relativistic jet
2D simulations: evolution of the cross section of the relativistic jet

Numerical Setting: 2D Toy Model

- Cylindrical coordinate: \((r - \theta)\) plane
- Relativistically hot jet (z-direction)
- Ideal gas
- Numerical scheme: HLLC (Mignone & Bodo 05)
- Uniform grid: \(\Delta r = \Delta z = 10/320, \Delta \theta = 2\pi/200\)

- \(\rho_{\text{ext},0}c^2 = 1\)
- \(P_{\text{ext},0} = 0.1\)
- \(v_r = v_\theta = v_z = 0\)
- \(\rho_{\text{jet},0}c^2 = 0.1\)
- \(P_{\text{jet},0} = 1\)
- \(v_z = v_{\text{jet},0} = 0.99c\)
- \(\gamma \approx 7\)
Basic Equations

mass conservation
\[ \frac{\partial}{\partial t}(\gamma \rho) + \frac{1}{r} \frac{\partial}{\partial r}(r \gamma \rho v_r) + \frac{1}{r} \frac{\partial}{\partial \theta}(\gamma \rho v_\theta) = 0 \]

momentum conservation
\[ r \frac{\partial}{\partial t}(\gamma^2 \rho hv_r) + \frac{1}{r} \frac{\partial}{\partial r}(r(\gamma^2 \rho hv_r^2 + P)) + \frac{1}{r} \frac{\partial}{\partial \theta}(\gamma^2 \rho hv_r v_\theta) = \frac{P}{r} \]
\[ r \frac{\partial}{\partial t}(\gamma^2 \rho hv_\theta) + \frac{1}{r} \frac{\partial}{\partial r}(r(\gamma^2 \rho hv_\theta v_r)) + \frac{1}{r} \frac{\partial}{\partial \theta}(\gamma^2 \rho hv_\theta^2 + P) = -\frac{\gamma^2 \rho hv_r v_\theta}{r} \]
\[ \frac{\partial}{\partial t}(\gamma^2 \rho hv_z) + \frac{1}{r} \frac{\partial}{\partial r}(r(\gamma^2 \rho hv_z v_r)) + \frac{1}{r} \frac{\partial}{\partial \theta}(\gamma^2 \rho hv_z v_\theta) = 0 \]

energy conservation
\[ \frac{\partial}{\partial t}(\gamma^2 \rho h - P) + \frac{1}{r} \frac{\partial}{\partial r}(r(\gamma^2 \rho h v_r)) + \frac{1}{r} \frac{\partial}{\partial \theta}(\gamma^2 \rho h v_\theta) = 0 \]

specific enthalpy
\[ \frac{h}{c^2} = 1 + \frac{\Gamma}{\Gamma - 1} \frac{P}{\rho c^2} \]

ratio of specific heats
\[ \Gamma = \frac{4}{3} \]

Lorentz factor
\[ \gamma = \frac{1}{\sqrt{1 - (v_r^2 + v_\theta^2 + v_z^2)}} \]
The amplitude of the corrugated jet interface grows as time passes. A finger-like structure is a typical outcome of the Rayleigh-Taylor instability.
The Richtmyer-Meshkov instability is induced by impulsive acceleration due to shock passage.

The perturbation amplitude grows linearly in time (Richtmyer 1960)

$$\frac{\partial \delta}{\partial t} = k \delta^* A^* v^* , \quad A^* = \frac{\rho_1^* - \rho_2^*}{\rho_1^* + \rho_2^*}$$
Time Evolution of Jet Cross Section

Effective inertia: $\log \gamma^2 \rho h$

(a) $t=50$

(b) $t=70$

(c) $t=90$

Contact discontinuity

Shock

(d) $t=96$

(e) $t=100$

(f) $t=120$

Model A1

reconfinement shock
The transverse structure of the jet is dramatically deformed by a synergetic growth of the Rayleigh-Taylor and Richtmyer-Meshkov instabilities once the jet-external medium interface is corrugated in the case with the pressure-mismatched jet.
Stability Condition of the Jet

2D simulations of transverse structure of the jet is excluding the destabilization effects by the Kelvin-Helmholtz mode.

The stability criterion of the jet:

\[ \eta_0 = \frac{\gamma_{\text{jet},0}^2 \rho_{\text{jet},0} h_{\text{jet},0}}{\rho_{\text{ext},0} h_{\text{ext},0}} > 1 \]

\[ \begin{cases} P_{\text{jet},0}/P_{\text{ext},0} = 10 \\ \gamma_{\text{jet}} = 7 \end{cases} \]

\[ h_{\text{jet},0} = 1 + \frac{\Gamma}{\Gamma - 1} \frac{P_{\text{jet},0}}{\rho_{\text{jet},0} c^2} \]

Jet cross section

\[ \log \gamma \rho h: t=120 \]

Richtmyer-Meshkov
Rayleigh-Taylor

\[ |v_{\theta}|_{\text{ave}} = \frac{\int_{|v_z|>0} |v_{\theta}| r dr d\theta}{\int_{|v_z|>0} r dr d\theta} \]

\[ |v_{\theta}|_{\text{ave}}/c \]

\[ h_{\text{jet},0} - 1 \]

\[ \eta_0 \]

Model A1
Model A4

\[ \bigcirc \quad \bigcirc \quad \bigcirc \quad \bigcirc \]

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Stable
Unstable

Fixed
3D simulations: evolution of the cross section of the relativistic jet
Numerical Setting: 3D Toy Model 1

\[ \rho_{\text{ext},0} c^2 = 1 \]
\[ P_{\text{ext},0} = 0.1 \]
\[ v_r = v_\theta = v_z = 0 \]

\[ \rho_{\text{jet,0}} c^2 = 0.1 \]
\[ P_{\text{jet,0}} = 1 \]
\[ v_z = v_{\text{jet,0}} = 0.99c \quad \gamma \sim 7 \]
\[ v_r = v_\theta = 0 \]

- cylindrical coordinate
- relativistic jet (z-direction)
- ideal gas
- numerical scheme: HLLC (Mignone & Bodo 05)
- uniform grid: \( \Delta r = \Delta z = 10/320, \Delta \theta = 2\pi/200 \)
Basic Equations

**Mass Conservation**
\[ \frac{\partial}{\partial t}(\gamma \rho) + \nabla \cdot (\gamma \rho \mathbf{v}) = 0 \]

**Momentum Conservation**
\[ \frac{\partial}{\partial t}(\gamma^2 \rho \mathbf{v}) + \nabla \cdot (\gamma^2 \rho \mathbf{v} \mathbf{v} + Pc^2 \mathbf{I}) = 0 \]

**Energy Conservation**
\[ \frac{\partial}{\partial t}(\gamma^2 \rho h - P) + \nabla \cdot (\gamma^2 \rho h \mathbf{v}) = 0 \]

**Specific Enthalpy**
\[ \frac{h}{c^2} = 1 + \frac{\Gamma}{\Gamma - 1} \frac{P}{\rho c^2} \]

**Ratio of Specific Heats**
\[ \Gamma = \frac{4}{3} \]

**Lorentz Factor**
\[ \gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \]
finger-like structure emerges at the jet-external medium interface.

The interface deformation gradually grows.

radial oscillating motion of the jet.
We can not find the destabilization effect by the Kelvin-Helmholtz mode in the case of the radial oscillation motion of the jet although such effect is not excluded in the settings.

Synergetic Growth of Rayleigh-Taylor and Richtmyer-Meshkov Instabilities

development of the Rayleigh-Taylor instability at the jet interface

\[ |v_\theta|_{ave} \text{ increases exponentially.} \]

excitation of the Richtmyer-Meshkov instability at the jet interface

\[ |v_\theta|_{ave} \text{ grows linearly with time.} \]
Without Oscillation

The total energy of the jet is the same as the oscillation case.

\[(\gamma^2 \rho h V_{\text{jet}})_{\text{osci}} = (\gamma^2 \rho h V_{\text{jet}})_{\text{non-osci}} \sim 200\]
Without Oscillation

Kelvin-Helmholtz instability grows at the jet interface.

The interface deformation gradually grows.
The growth rate of the volume-averaged azimuthal velocity due to the Kelvin-Helmholtz instability is greater than the oscillation-induced Rayleigh-Taylor and Richtmyer-Meshkov instabilities.

The synergetic growth of the Rayleigh-Taylor and Richtmyer-Meshkov instabilities trigger the deformation of the radially oscillating jet.
3D simulation: propagation of the relativistic jet
Numerical Setting: 3D Toy Model 2

- cylindrical coordinate
- relativistic jet \((z\text{-direction})\)
- ideal gas
- numerical scheme: HLLC (Mignone & Bodo 05)
- uniform grid: \(\Delta r = 0.0666, \Delta \theta = 2\pi/160, \Delta z = 1\)

\[
\begin{align*}
\rho_{\text{jet}} c^2 &= 0.1 & v_z &= 0.99c \\
P_{\text{jet}} &= 1 & \gamma_{\text{jet}} &\sim 7 \\
\rho_{\text{amb}} c^2 &= 1 \\
P_{\text{amb}} &= 0.1 \\
v_r &= v_\theta = v_z = 0
\end{align*}
\]
The amplitude of the corrugated jet interface grows due to the oscillation-induced Rayleigh-Taylor and Richtmyer-Meshkov instabilities.
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3D vs Axisymmetric

Deceleration of the jet due to the mixing between the jet and surrounding medium in the 3D case.
Deceleration of the jet due to mixing between the jet and surrounding medium in the 3D case.
Deceleration of the jet due to mixing

- 3D case
- axisymmetric case

\( \gamma_h: t=2000 \)

\( \gamma_h \sim 300 \)

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\( \gamma_h \sim 300 \)

- relativistic Bernoulli equation: \( \gamma_h \sim \text{const.} \)

\( \gamma_h \) gives the maximum Lorentz factor of the jet after adiabatic expansion. However, \( \gamma_h \) drops to \( \sim 10 \) due to the mixing in this case.
Deceleration of the jet due to mixing

In the 3D case, the maximum Lorentz factor \( \gamma_h \) after adiabatic expansion is given by the relativistic Bernoulli equation: \( \gamma h \sim \text{const.} \)

\( \gamma h \) gives the maximum Lorentz factor of the jet after adiabatic expansion. However, \( \gamma h \) drops to \( \sim 10 \) due to the mixing in this case.
Summary

Propagation dynamics and stability of the relativistically hot jet is studied through 2D and 3D relativistic hydrodynamic simulations.

- A pressure mismatch between the jet and surrounding medium leads to the radial oscillating motion of the jet.
- The jet-ambient medium interface is unstable due to the oscillation-induced Rayleigh-Taylor instability and Richtmyer-Meshkov instability.
- Deceleration of the jet due to the mixing between the jet and surrounding medium

Next Study:
- More realistic situation for relativistic jets such as AGN jets and GRBs
- Effect of the magnetic field on the dynamics and stability of the jet
Comparison of Grid Points

In higher resolution case, you can find smaller structures due to the growth of the Rayleigh-Taylor and Richtmyer-Meshkov instabilities.
Comparison of Numerical Scheme

It is not easy to find Rayleigh-Taylor and Richtmyer-Meshkov fingers in the model with HLL scheme although the completely same initial settings and grid spacing (320 x 200 zones r- and \theta directions) are adopted in both models.
Propagation of Rarefaction Wave through the Origin
Propagation of Shock Wave through the Origin

Density: Time $t=0000$, $z=0.500$
Relaxation of Initial non-equilibrium State

Energy conversion from thermal energy into bulk motion energy

Transition stage (oscillation)
- Steady state
- Hydrostatic balance
- Energy conversion from thermal energy into bulk motion energy

relativistic Bernoulli: $\gamma h \sim \text{const.}$
oscillation timescale: propagation time of the sound wave over the jet width

\[ \tau = \sqrt{3} \gamma_{\text{jet}} W_{\text{jet}} / c \]

total energy conservation neglecting rest mass energy

\[ W_{\text{jet}}^2 \gamma_{\text{jet}}^2 P_{\text{amb,0}} = W_{\text{jet,0}}^2 \gamma_{\text{jet,0}}^2 P_{\text{jet,0}} \]

oscillation timescale

\[ \tau = \sqrt{3} \gamma_{\text{jet,0}} \left( \frac{W_{\text{jet,0}}}{c} \right) \left( \frac{P_{\text{jet,0}}}{P_{\text{amb,0}}} \right)^{1/2} \]

proportional to the square root of the initial pressure ratio between the jet and ambient medium.