Amplification of magnetic fields in non-rotating core collapse

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Magnetic fields in core collapse

- magnetic fields need to be strong to have an effect on SNe
- **But:** stellar evolution theory predicts rather weak fields in the pre-collapse core
  → efficient amplification required
    - compression
    - linear winding by differential rotation
    - **hydromagnetic instabilities:** convection, magnetorotational instability (MRI), SASI

Meier et al., 1976
## Magnetic fields in core collapse

### Summary

- **SASI convection MRI**
- **energy accretion flow**
- **mechanism advective-acoustic cycle**
- **role of \( \vec{b} \) passive; turbulent dynamo**
- **convection thermal buoyant transport of energy/species**
- **role of \( \vec{b} \) passive; turbulent dynamo**
- **MRI diff. rotation magnetic transport of angular momentum**
- **instability driver; turbulent dynamo**

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**Endeve et al., 2008**
The neutrino distribution

The $\nu$ field is equivalently described by its

- **distribution function**, $f(\vec{p}, \vec{x}, t)$, the probability to find a neutrino with a momentum $\vec{p}$ (i.e., energy $\epsilon = |\vec{p}|c$) at position $\vec{x}$, time $t$

- **radiative intensity**, $I(\vec{n}, \epsilon, \vec{x}, t)$, the energy carried by all $\nu$ of energy $\epsilon$ in direction $\vec{n} = \vec{p}/|\vec{p}|$ through a unit surface $dA$ at position $\vec{x}$, time $t$

**Properties**

- $f = \frac{(hc)^3}{c\epsilon^3}I$ is a relativistic invariant
- in local equilibrium with matter, $f$ is given by the Fermi-Dirac distribution, $f_{FD} = \frac{1}{\exp \frac{\epsilon - \mu_{\nu}}{k_BT} + 1}$
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Boltzmann transport equation (mixed frame)

\[
\partial_t I + \vec{n} \cdot \vec{\nabla} I = \eta_0(\epsilon) - \chi_0(\epsilon)I + \vec{n} \cdot \vec{v} (2\eta_0(\epsilon) - \epsilon \partial_\epsilon \eta_0) + [\chi_0(\epsilon) + \epsilon \partial_\epsilon \chi_0(\epsilon)] I
\]

with advection, emission, absorption, and Doppler shift, aberration.
Radiation moments

Expand the intensity in angular moments, $M_{i_1i_2...i_m} = \int d\vec{n}n_{i_1}n_{i_2}...n_{i_m}I$.

- **0\textsuperscript{th} moment:** $E = \int \frac{d\vec{n}}{4\pi} \left( \frac{\epsilon}{hc} \right)^3 f$, the energy density
- **1\textsuperscript{st} moment:** $F^i = \int \frac{d\vec{n}}{4\pi} \left( \frac{\epsilon}{hc} \right)^3 n^i f$, the momentum density
- **2\textsuperscript{nd} moment:** $P^{ij} = \int \frac{d\vec{n}}{4\pi} \left( \frac{\epsilon}{hc} \right)^3 n^in^jf$, the pressure tensor
- etc. ad inf. (with no straightforward physical interpretation)

Moment equations

equation for the $m\textsuperscript{th}$ moment contains the $(m + 1)\textsuperscript{th}$ moment as a flux

$$\partial_t M_{i_1...i_m} + \nabla_j M_{i_1...i_mj} + \text{velocity terms} = S_{i_1...i_m}$$

$\Rightarrow$ infinite series of equations $\equiv$ transport equation
**Truncated moment system**

evolve only the first 2 moments and obtain the higher (2\textsuperscript{nd} and 3\textsuperscript{rd}) moments by a local algebraic closure as a function of the lower moments
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0\textsuperscript{th} moment

assumption for the neutrino momentum

- **free streaming**: $\vec{F} = cE\vec{n}$; fails in optically thick regions
- **diffusion**: $\vec{F} = -\frac{c}{3\kappa} \vec{\nabla}E$; unphysical in vacuum
- **flux-limited diffusion**: ensure physical (not necessarily correct) vacuum limit $\Rightarrow$ parabolic equation
Truncated moment system

evolve only the first 2 moments and obtain the higher (2\textsuperscript{nd} and 3\textsuperscript{rd}) moments by a local algebraic closure as a function of the lower moments

\begin{align*}
\text{1\textsuperscript{st} moment} \\
\text{set } P_{ij} = \frac{1-\chi}{2} \delta_{ij} + \frac{3\chi-1}{2} n^i n^j \text{ with a variable Eddington factor } \chi(E, \frac{|\vec{F}|}{cE}). \\
\quad \text{local approximation } \Rightarrow \text{ simpler and less expensive than Boltzmann solvers (but less accurate)} \\
\quad \text{genuinely multidimensional} \\
\quad \text{physical consistency: } \chi \to \{1/3, 1\} \\
\quad \text{for } |\vec{F}|/(cE) \to \{0, 1\} \\
\quad \text{hyperbolic for suitable choice of } \chi
\end{align*}
Neutrino interactions

Reactions with matter

- $n + \nu_e \rightleftharpoons p^+ + e^-$
- $p^+ + \bar{\nu}_e \rightleftharpoons n + e^-$
- $(A, Z) + \nu_e \rightleftharpoons (A, Z + 1) + e^-$
- $n/p + \nu_X \rightleftharpoons n/p + \nu_X$
- $(A, Z) + \nu_X \rightleftharpoons (A, Z) + \nu_X$
- $e + \nu_X \rightleftharpoons e + \nu_X$
- $e^- + e^+ \rightleftharpoons \nu_X + \bar{\nu}_X$
- $N + N \rightleftharpoons N + N + \nu_X + \bar{\nu}_X$

- implementation following Rampp & Janka (2002)
- 2d simulations below neglect the reactions in grey
- limited scope of our models
Tests

- homogeneous radiating sphere
- differentially expanding atmosphere
- neutrino transport in a post-bounce core
- spherical core collapse

Figure 1. Comparison of the numerical results obtained using different codes with the analytic solution for the homogeneous sphere test problem. On the left (right) side the results for the case of moderate (high) opacity are plotted. The panels show from top to bottom the energy density, the luminosity, the flux factor and the Eddington factor against radius. In the second row the values of the (constant) luminosities outside of the sphere are given.
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Conclusion

in most regimes close to results of Boltzmann codes, but some limitations exist
Open issues

We want to study

▶ Where do the magnetic fields of young neutron stars come from?
▶ Under what conditions does the field amplification during a supernova explosion lead to dynamically important fields?
▶ How do the magnetic fields react back onto the flow?

But we are not aiming to answer

▶ Does a particular core produce a robust $\nu$-driven SN explosion and, if so, how large is the explosion energy?
▶ How does the dynamics depend on the details of the equation of state and the neutrino physics?
▶ What are the consequences for nucleosynthesis?
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Collapse of magnetised cores

- progenitor star: $15 \, M_\odot$, solar metallicity
- no rotation → restricts possible field amplification mechanisms
- poloidal (off-centre dipole) magnetic fields of field strength $\leq 10^{12} \, G$
- axisymmetric simulations
- spectral transport (16 energy bins) of $\nu_e$ and $\bar{\nu}_e$

Field structure and entropy at $t = 0$
Overview

without magnetic field:

→ PNS convection, SASI and convection in the hot-bubble region
amplification of a **weak field** by compression, stretching and folding of field lines, and unstable Alfvén waves
feedback of a **strong field**: field resists bending, slows down motion across field lines
→ modifies the growth of SASI, convection
→ development of very persistent large-scale patterns of upflows and downflows, stronger shock expansion
Field amplification

- most pronounced amplification occurs during collapse by compression
- for weak fields: significant increase in the hot-bubble on long time scales, caused by stretching and folding of field lines
- stronger initial fields experience less amplification, indicating dynamic feedback
- max. fields are magnetar-like

Time evolution of the (rescaled) r.m.s. field strength: entire volume and hot bubble
Dynamic feedback

- feedback on the flow limit the amplification of stronger seed fields
- strong fields lead to much larger unstable modes and persistent downflows
  → strong shock expansion

2d structure of a model with strong initial field
Dynamic feedback

- Feedback on the flow limit the amplification of stronger seed fields.
- Strong fields lead to much larger unstable modes and persistent downflows.
- Strong shock expansion.

Angularly averaged profiles of the strongest magnetised model as a function of time.
Summary

- the moment system with local closure is a good compromise between accuracy and effort
- we study the interplay of neutrino transport, hydrodynamics and magnetic fields, finding efficient amplification - compression - stretching by hydro instabilities and dynamic feedback
- next steps: switch on pair processes; rotating models