エンタングルメントと
ブラックホール防火壁パラドクス

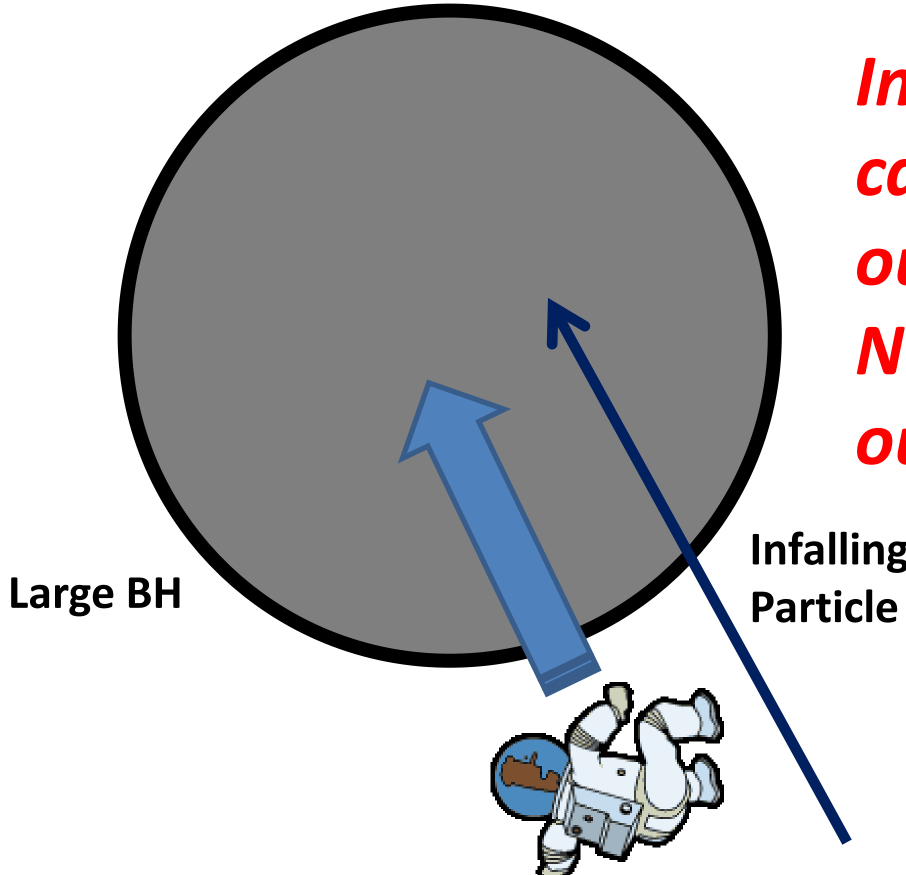
Entanglement and Black Hole Firewall Paradox

Based on arXiv:1306.5057

M. Hotta, J. Matsumoto and K. Funo

Introduction

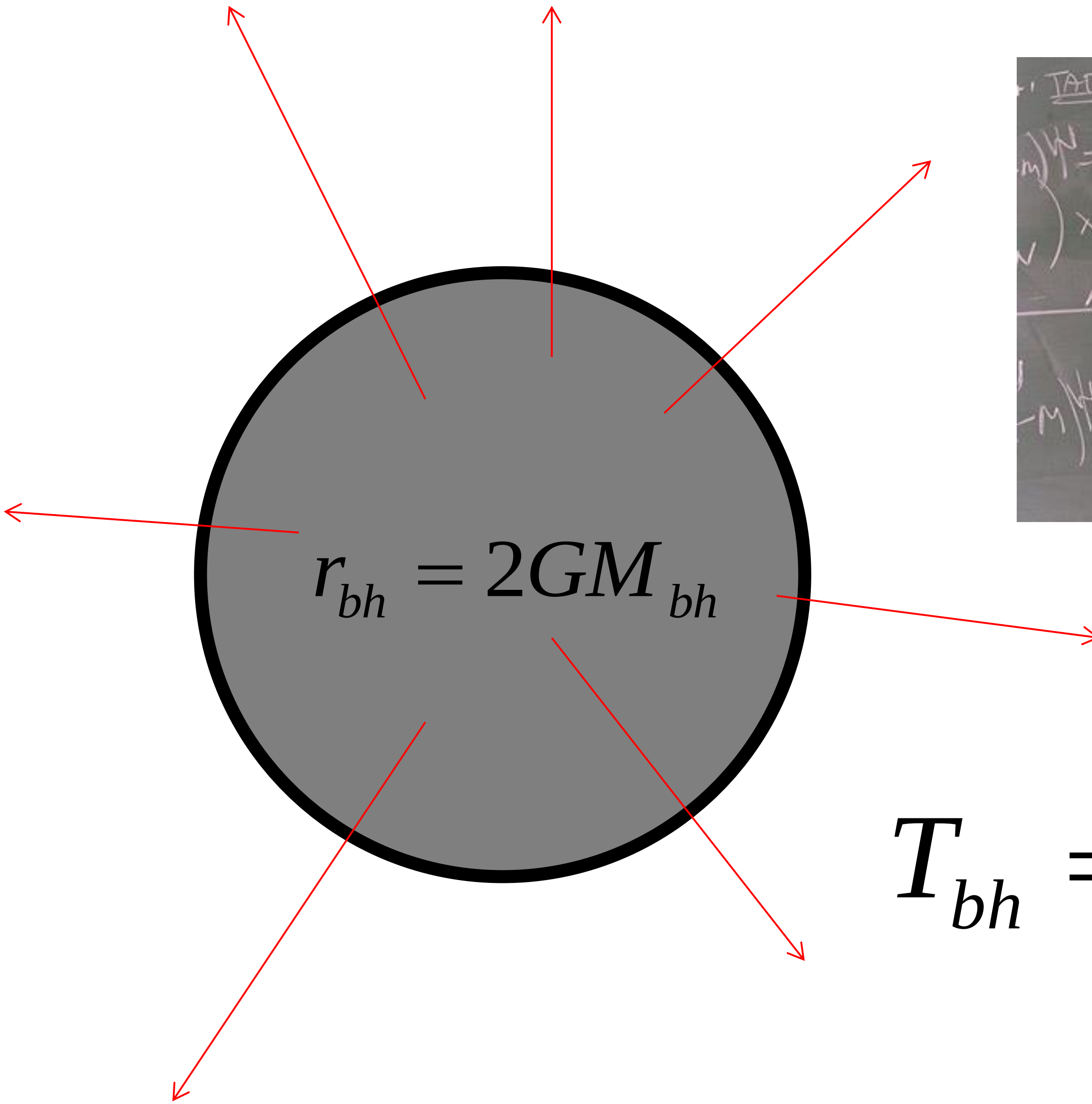
Large black-hole spacetimes are conventionally described by classical geometry.

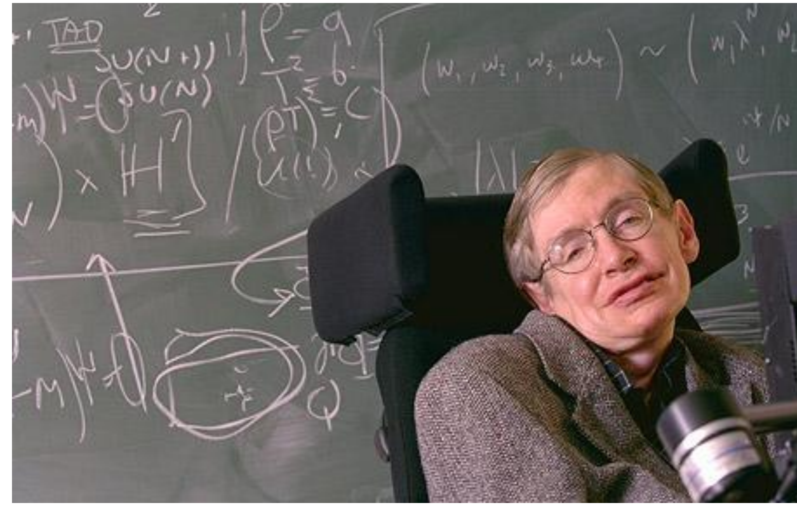


Inside cannot causally influence outside.
Nothing returns out of horizon.

Black holes emit thermal flux due to quantum effect.

(Hawking, 1974)





Special & General Relativity

Quantum Theory

$$T_{bh} = \frac{\hbar c^3}{8\pi k_B G M_{bh}}$$

Statistical Mechanics

$$c = \hbar = k_B = 1$$

Black Hole Temperature: $$T_{bh} = \frac{1}{8\pi G M_{bh}}$$

First Thermodynamics Law:

$$dS_{bh} = \frac{dM_{bh}}{T_{bh}} = 8\pi G M_{bh} dM_{bh}$$

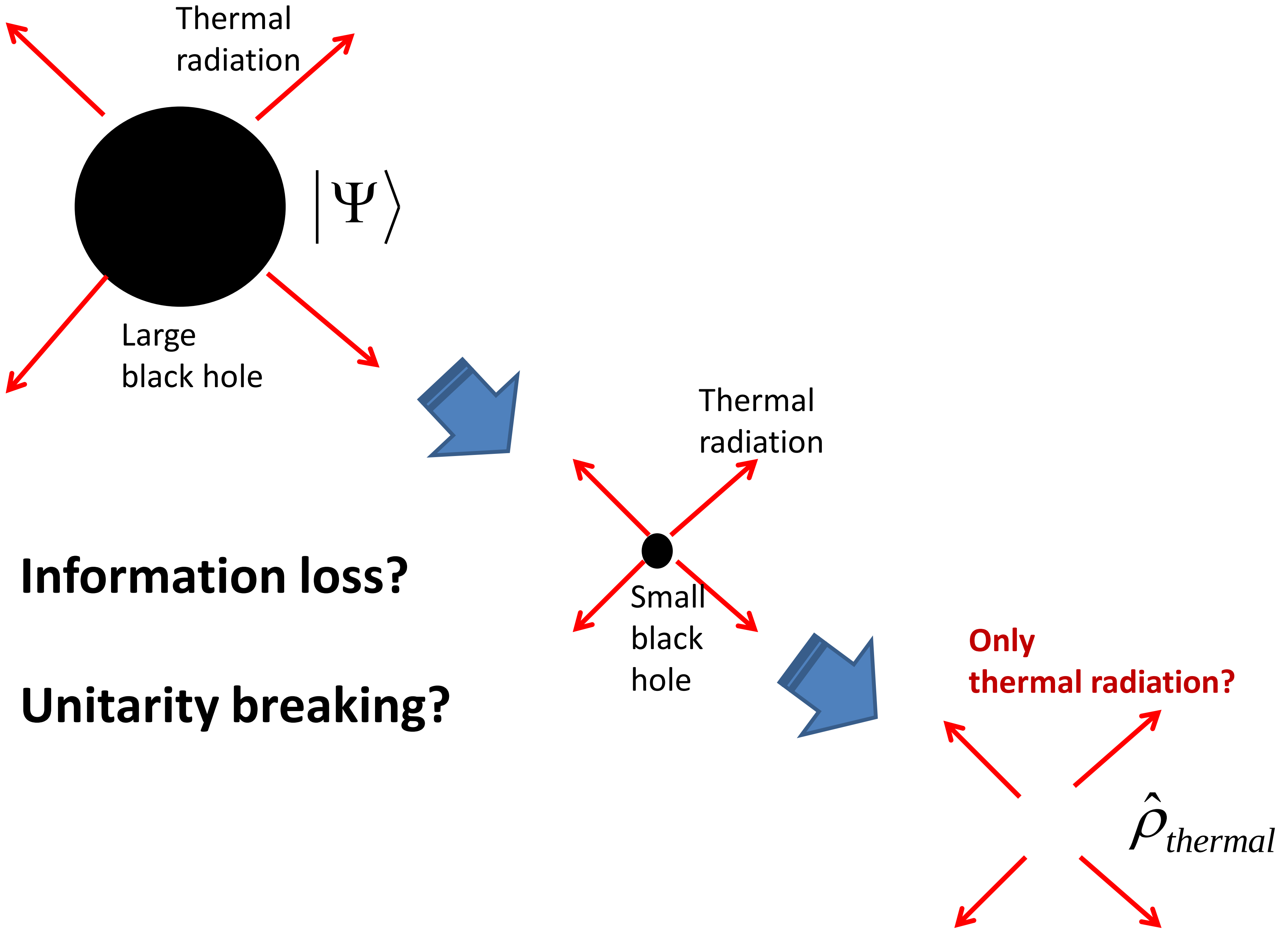
Black Hole Entropy: $$\boxed{S_{bh} = \frac{A_{bh}}{4G}}$$

$$r_{bh} = 2GM_{bh}$$

Area of Event Horizon: $$A_{bh} = 4\pi r_{bh}^{\ 2}$$

Since the advent of
Hawking radiation…

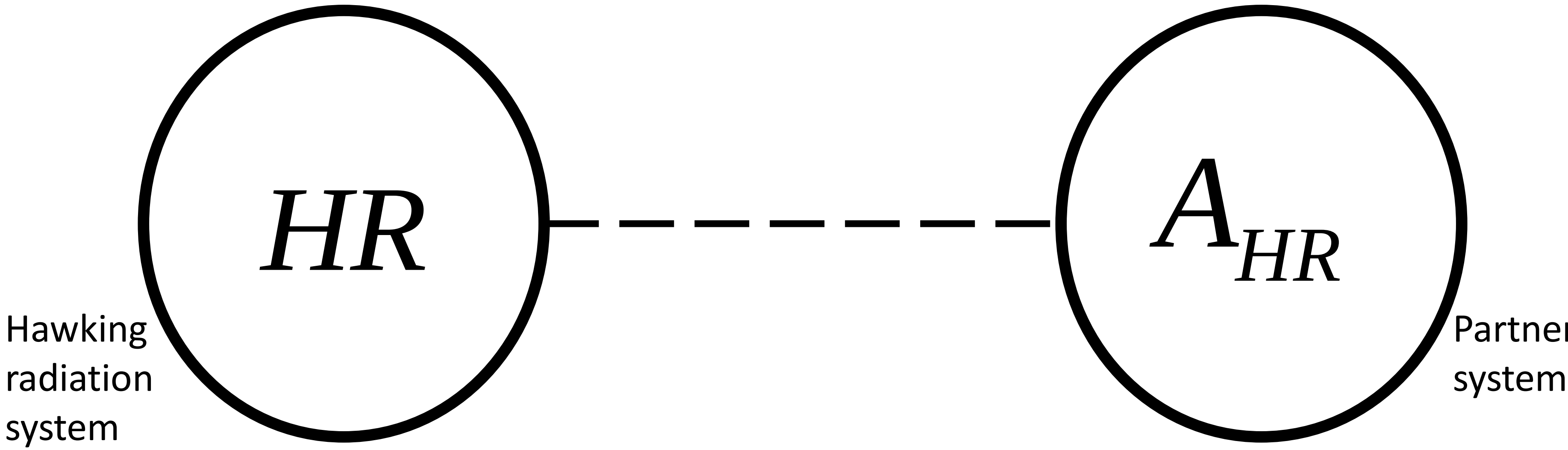
Information Loss Problem of Black Hole Evaporation


Thermal
radiation
$|\Psi\rangle$
Large
black hole
Thermal
radiation
Information loss?
Small
black
hole
Unitarity breaking?
Only
thermal radiation?
$\hat{\rho}_{thermal}$

Purification of Hawking Radiation?

$$\hat{\rho}_{HR} = \sum_n p_n |n\rangle_{HR} \langle n|_{HR}$$

Mixed state



$$|\Psi\rangle_{HRA_{HR}} = \sum_n \sqrt{p_n} |n\rangle_{HR} |u_n\rangle_{A_{HR}}$$

Composite system in a pure state

What is the final entangled partner of Hawking radiation?

(1) Nothing, Information Loss

(2) Exotic Remnant (Aharonov, Giddings,…)

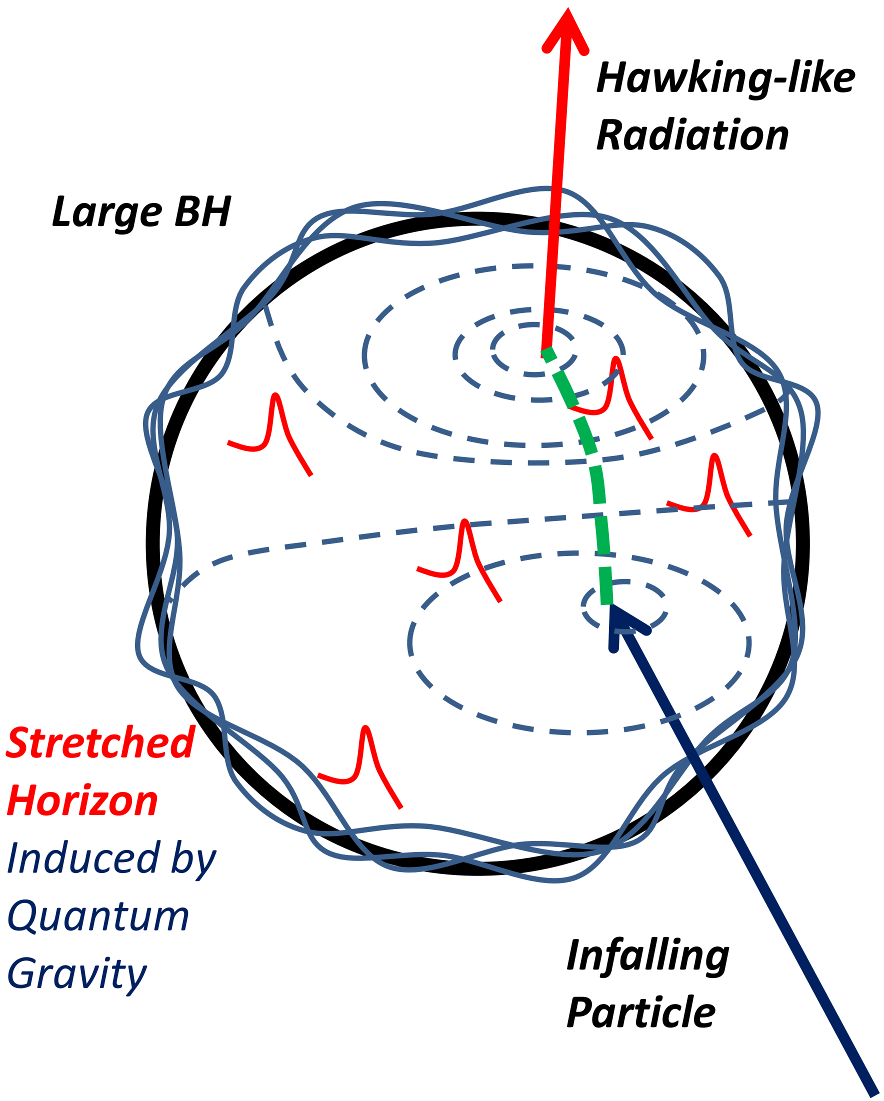
(3) Baby Universe (Dyson,..)

(4) Emitted Radiation Itself ←Today's Talk

- *Black Hole Complementarity (t' Hooft, Susskind, …)*
- *Fuzzi ball, Firewall (Mathur, AMPS, …)*

(5) Zero-Point Fluctuation in Local Vacuum regions
(Wilczek, Hotta-Matsumoto-Funo) *←Today's Talk*

○ Black Hole Complementarity

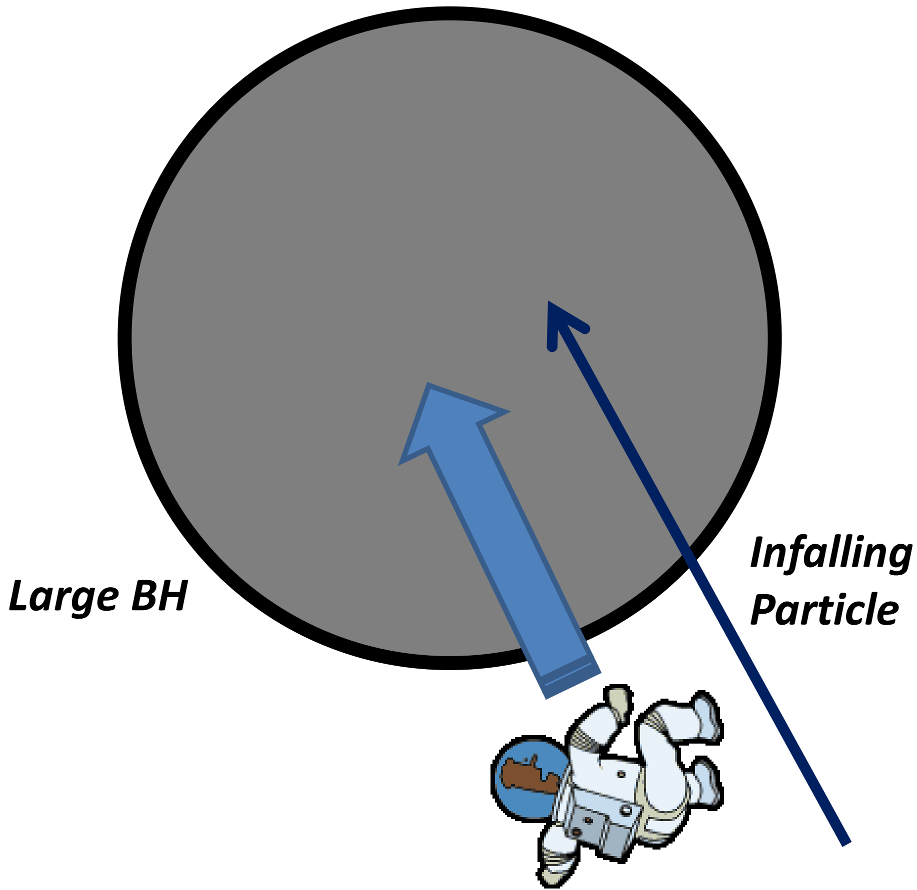


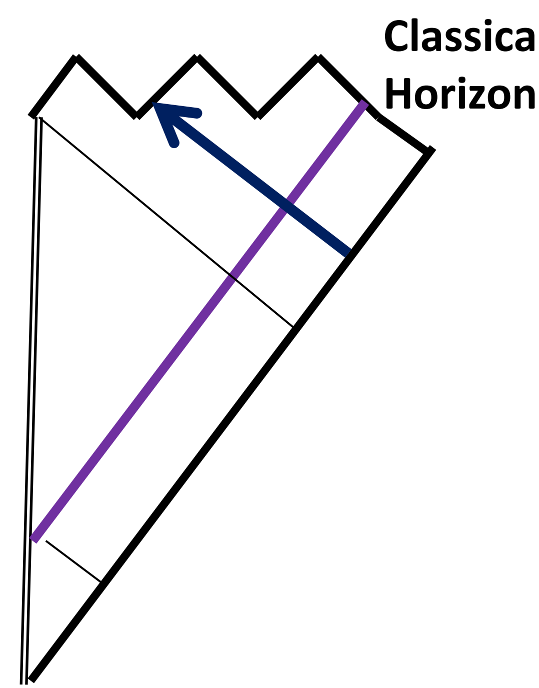
From the viewpoint of outside observer, the stretched horizon absorbs and emits quantum information so as to maintain the unitarity.

In the future, the whole radiation is not in a mixed state, but in a pure state.

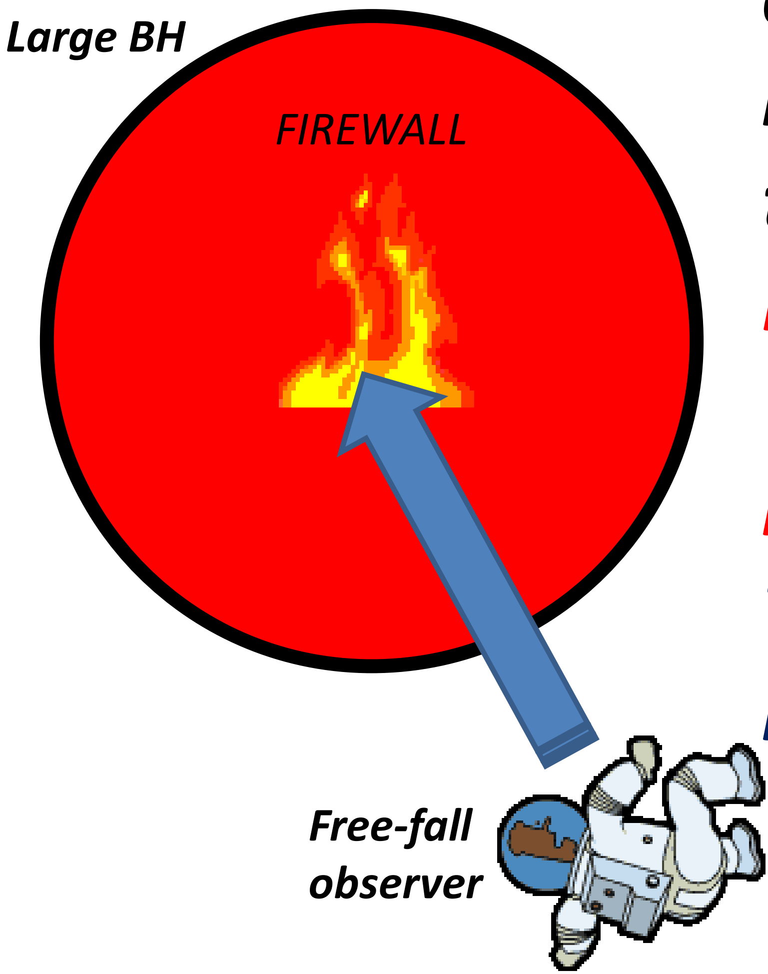
○ *Black Hole Complementarity*

From the viewpoint of free-fall observer, the stretched horizon disappears. No drama happens across the horizon.





○ *Firewall*



From the viewpoint of outside observer, the stretched horizon absorbs and emits quantum information so as to maintain the unitarity.

However, FIREWALL on the horizon burns out free-fall observers.

The inside region of BH does not exist!

Mathur and AMPS derive this conclusion from quantum information theoretical point of view.

In this talk, we argue that the information theoretical reason of Mathur and AMPS , which derives the existence of firewalls, are wrong. They misuse the results of quantum information theory.

However, another firewall paradox can be posed using quantum measurement theory.

The new paradox is resolved from the viewpoint of measurement energy cost.

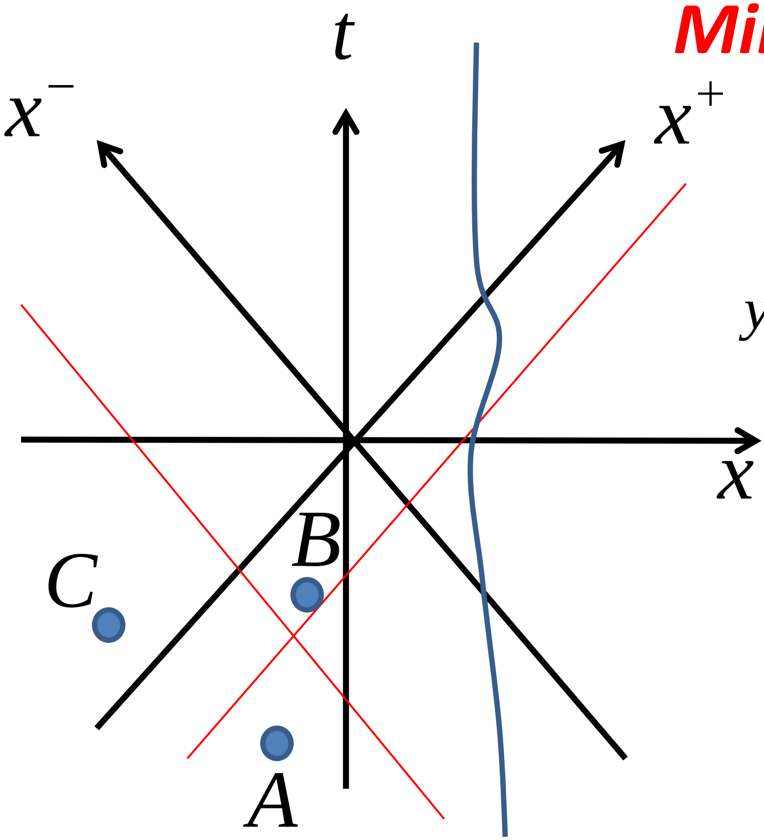
M.H., Jiro Matsumoto and Ken Funo, arXiv:1306.5057

Review:
Mathur-AMPS Strong Subaddivity Argument

Preparation for the Argument

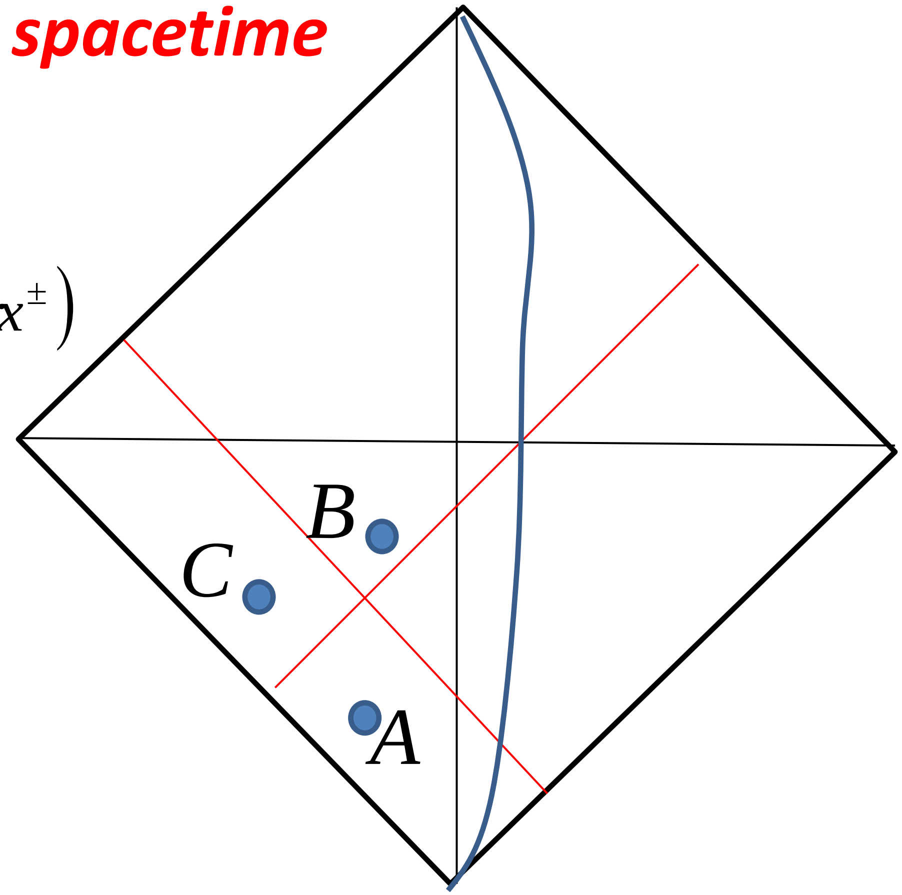
Penrose Diagram

Preserving causality relations among events, a spacetime is mapped into a finite region.



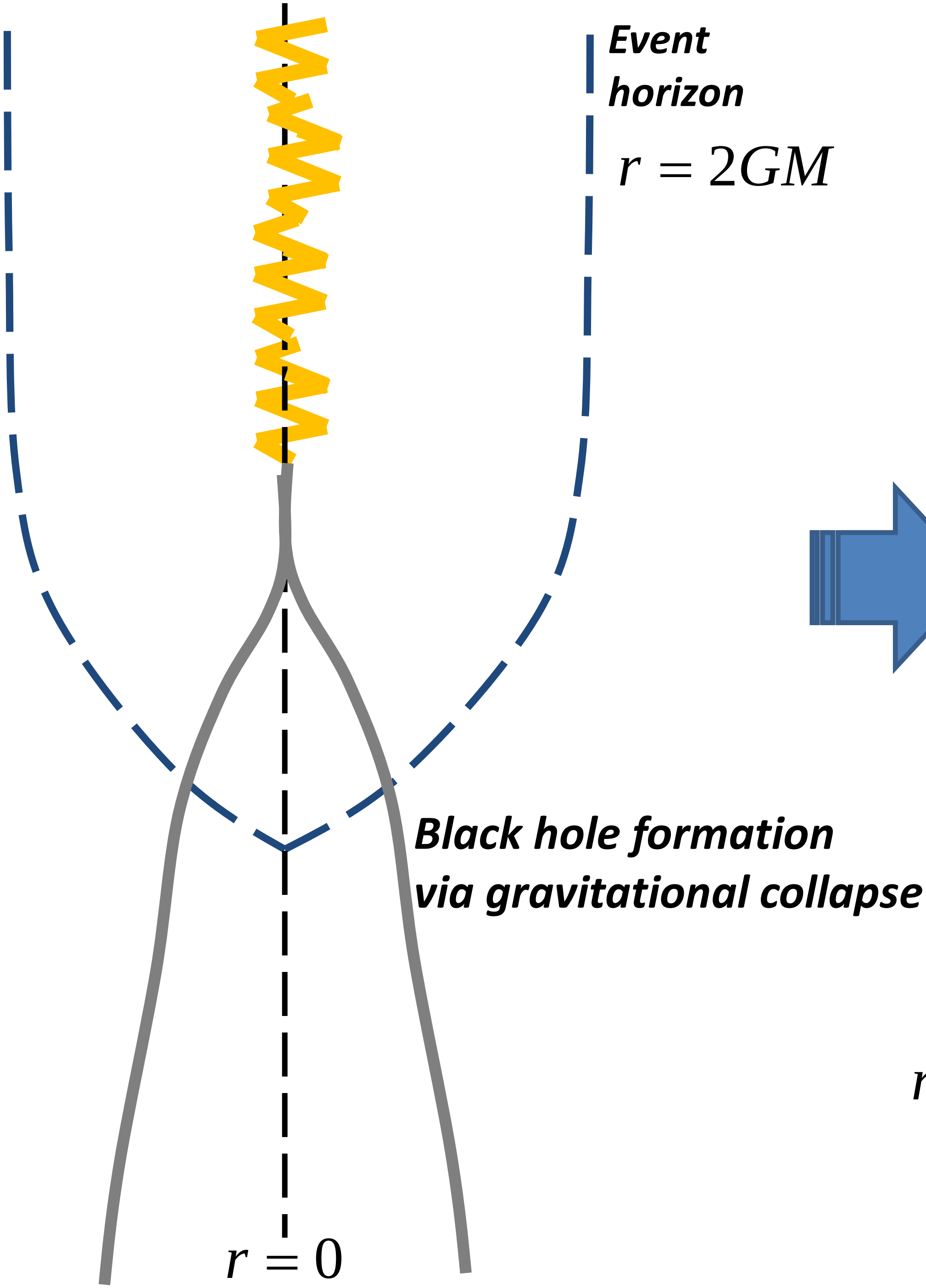
$$y^{\pm} = \tanh\left(\kappa x^{\pm}\right)$$

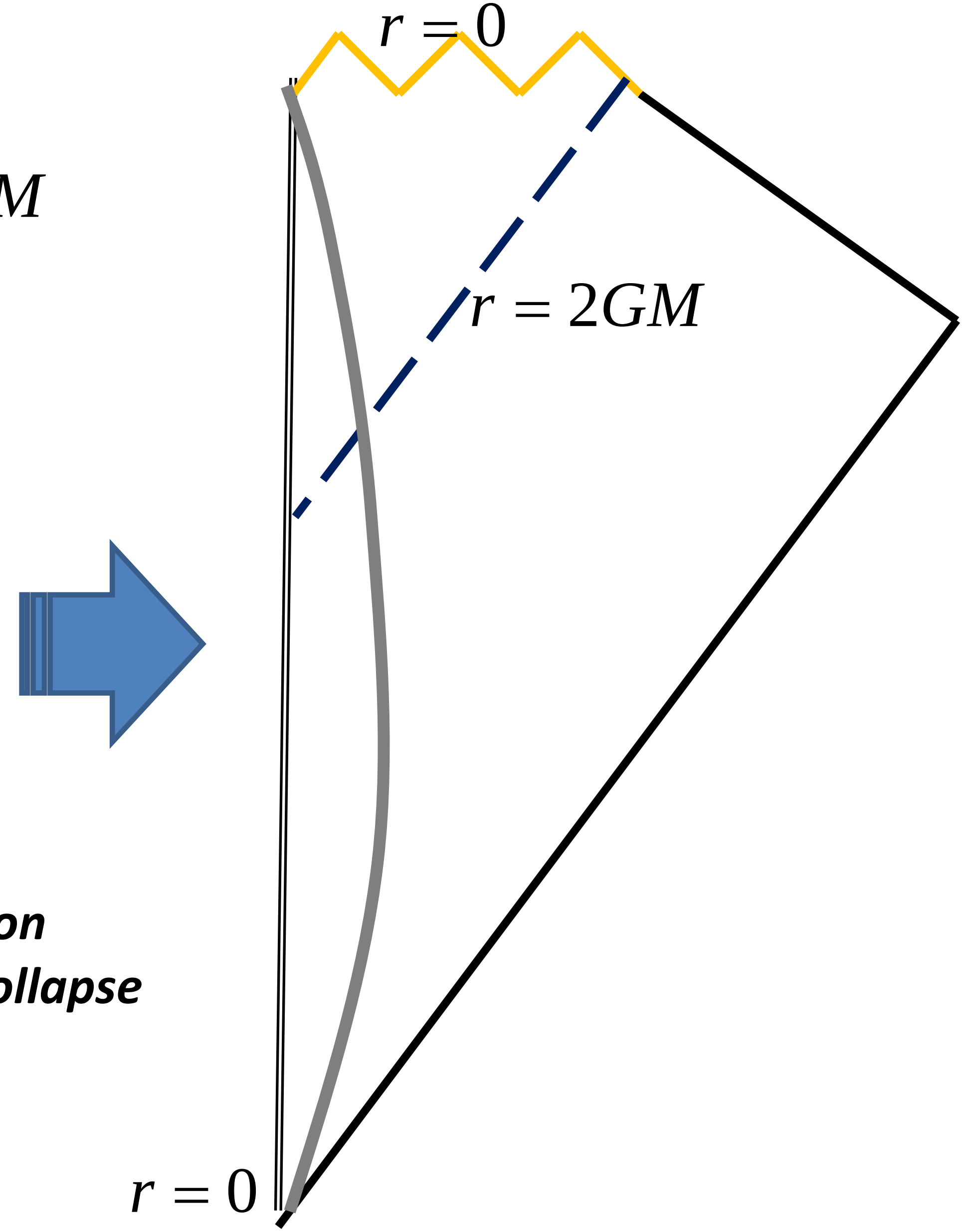
Penrose diagram of Minkowski spacetime



$$ds^2 = dt^2 - dx^2 = dx^+ dx^-$$
$$x^{\pm} = t \pm x$$

$$ds^2 = \frac{dy^+ dy^-}{\kappa^2\left(1 + y^{+2}\right)\left(1 + y^{-2}\right)}$$





(Radial) Penrose diagram of black hole formation

Maximal Entanglement

$$\left|Max\right\rangle_{AB} = \frac{1}{\sqrt{N}}\sum_{n=1}^{N}\left|u_n\right\rangle_A\left|v_n\right\rangle_B$$

$\left\{\left|u_n\right\rangle_A\right\}, \quad \left\{\left|v_n\right\rangle_B\right\}$ ***: complete orthonormal basis***

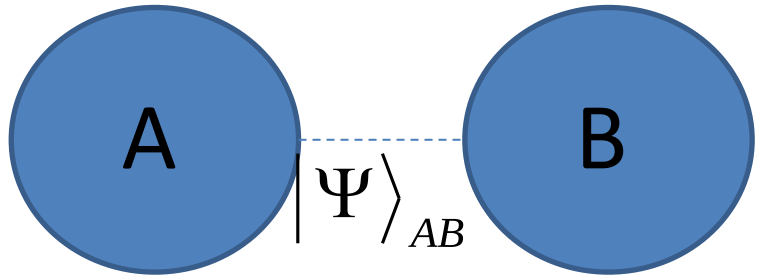
$$\hat{\rho}_A = Tr_B\left[\left|Max\right\rangle_{AB}\left\langle Max\right|_{AB}\right] = \frac{1}{N}\hat{I}_A,$$

$$\hat{\rho}_B = Tr_A\left[\left|Max\right\rangle_{AB}\left\langle Max\right|_{AB}\right] = \frac{1}{N}\hat{I}_B$$

$$\boxed{S_{AB} = -Tr\left[\hat{\rho}_A \ln \hat{\rho}_A\right] = -Tr\left[\hat{\rho}_B \ln \hat{\rho}_B\right] = \ln N}$$

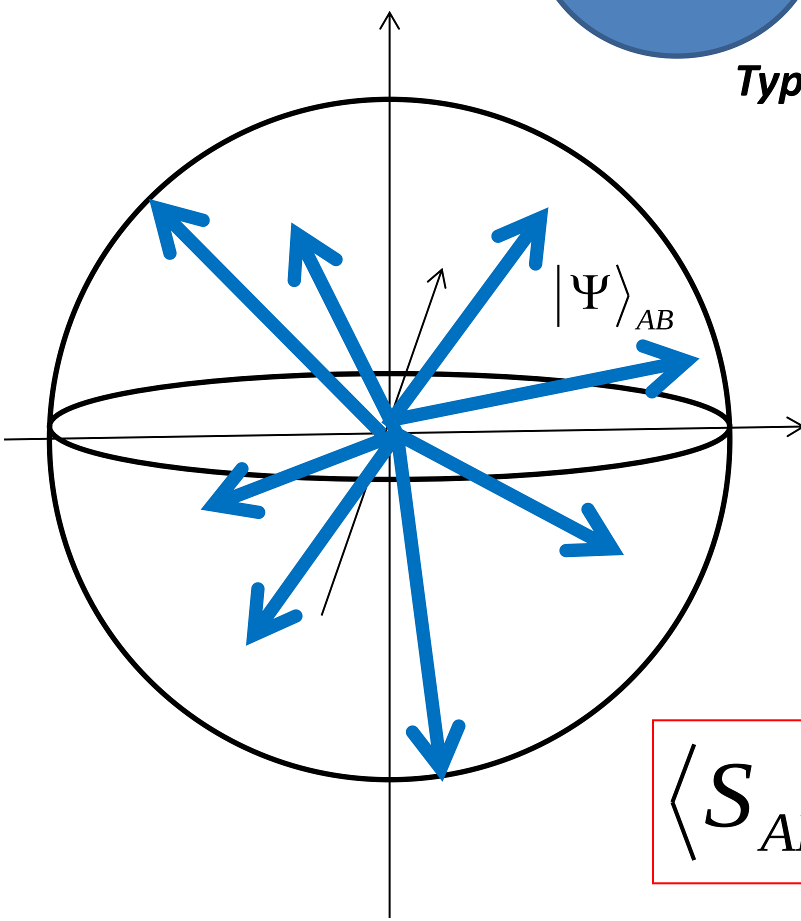
Typical states of A and B are almost maximally entangled when the systems are large.

$$\dim H_A = N_A$$



$$\dim H_B = N_B$$

Typical State of AB



$$\hat{\rho}_B = Tr_A\left[|\Psi\rangle_{AB}\langle\Psi_{AB}|\right]$$

$$\langle S_{AB}\rangle = -\langle Tr_B[\hat{\rho}_B \ln \hat{\rho}_B]\rangle$$

$$N_A \geq N_B >> 1$$

$$\boxed{\langle S_{AB}\rangle \approx \ln N_B} \Longrightarrow \boxed{\hat{\rho}_B \approx \frac{1}{N_B}\hat{I}_B}$$

Lubkin, Lloyd-Pagels, Page

Let us assume that Hilbert-space dimensions of black holes and Hawking radiation become finite due to quantum gravity effect.

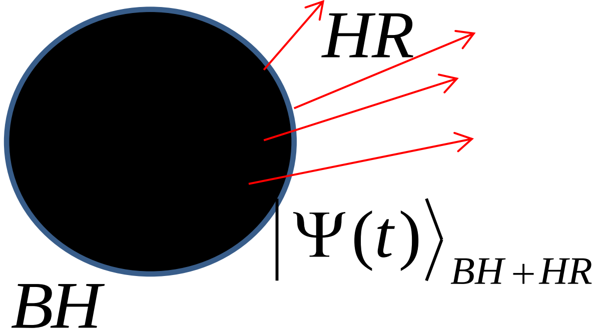
Page Strategy for Final State of BH Evaporation:

Nobody knows exact quantum gravity dynamics. So let's gamble that the final state scrambled by quantum gravity is one of TYPICAL pure states of the finite-dimensional composite system! That may not be so bad!

Then the discretized model suggests:

In a typical pure state of old black hole(BH) and Hawking radiation(HR) after Page time,

the internal system of BH is almost maximally entangled with a part of HR, and BH entropy is almost equal to entanglement entropy. (Page)

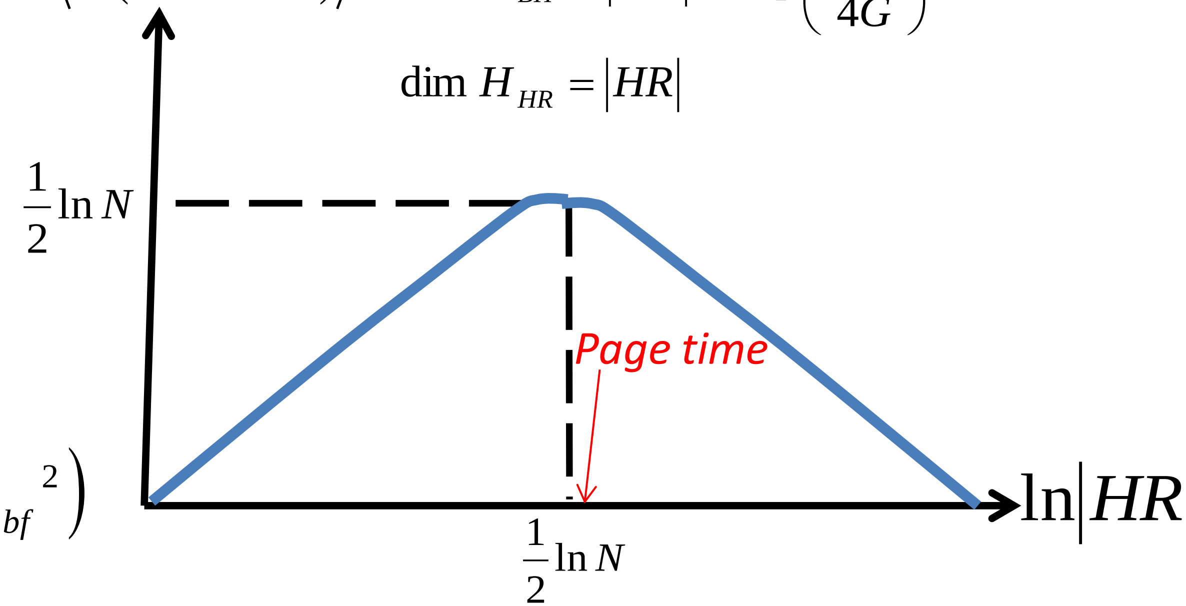


$$|BH||HR| = N, \ fixed$$

$$N = \exp\left(\frac{A_{bf}}{4G}\right) = \exp\left(4\pi G M_{bf}{}^{2}\right)$$

$$\dim H_{BH} = |BH| = \exp\left(\frac{A_{BH}}{4G}\right),$$

$$\dim H_{HR} = |HR|$$



$$1 << |BH| \le |HR| \Rightarrow \langle S(HR, BH)\rangle \approx \ln|BH| = \frac{A_{BH}}{4G}$$

<<Page Time>>

- $\ln|BH| = \ln|HR| = \frac{1}{2}\ln N$
- $M_{page} \approx 0.7 M_{bh}$

Mathur and AMPS apply this Page's argument to late-time Hawking radiation.

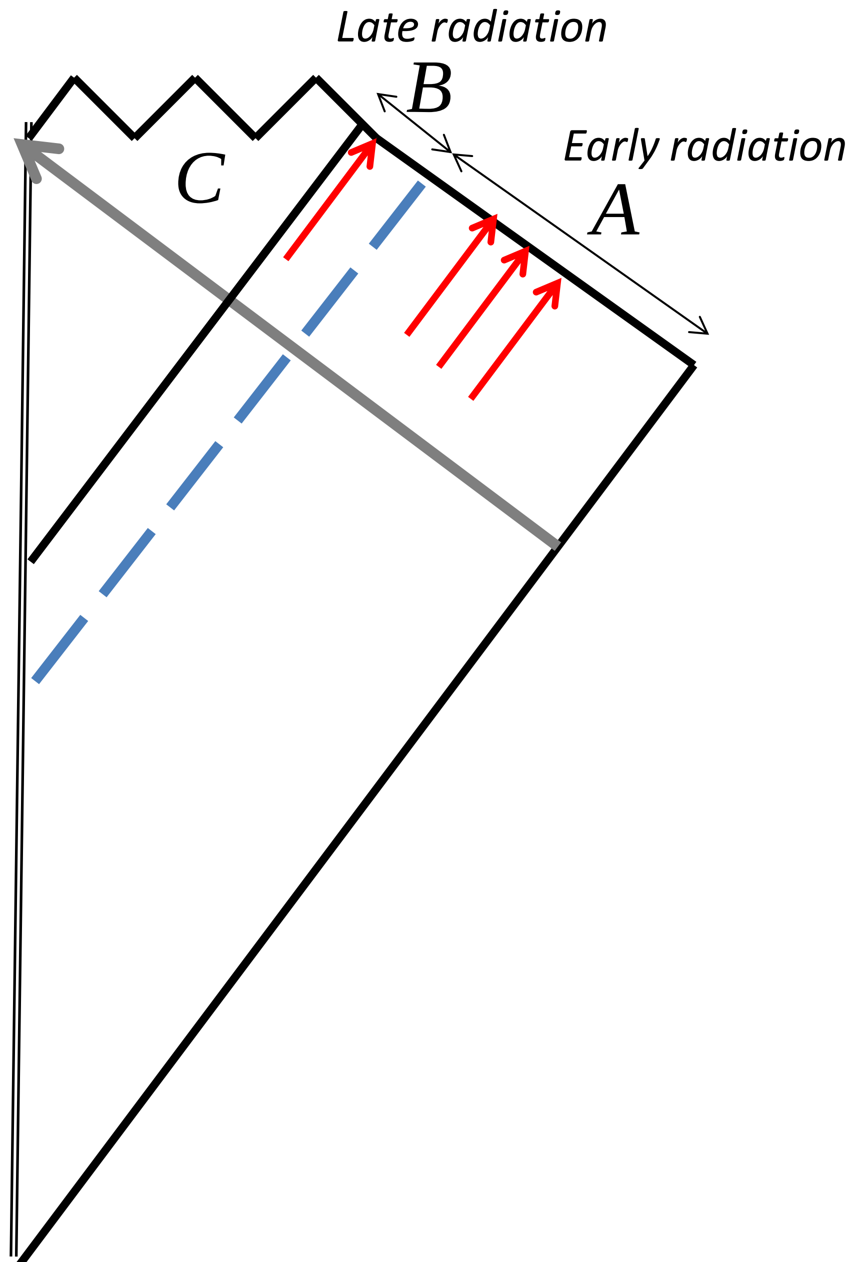
$$HR = A \cup B$$

$$BH = C$$

$$1 << |A|, |B|, |C|$$

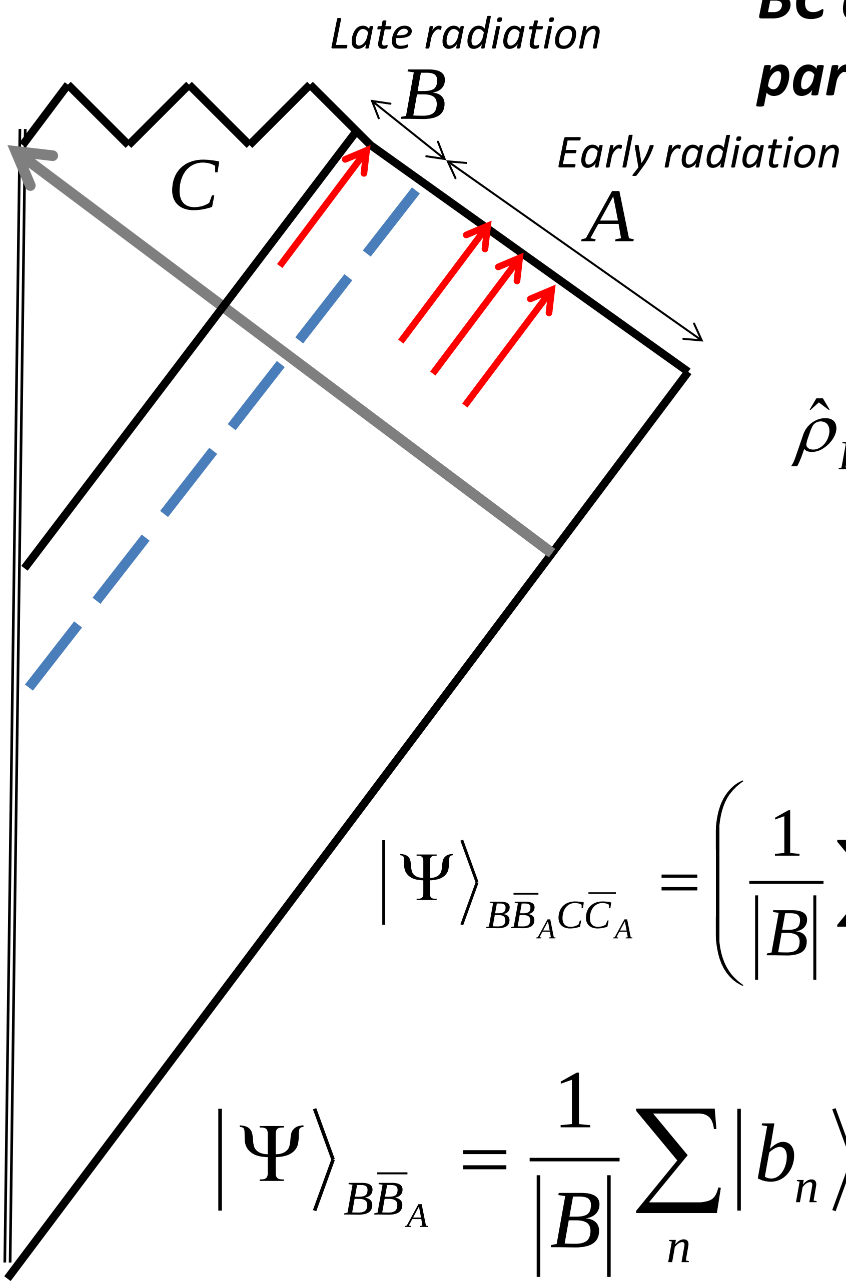
After Page time,

$$|B||C| < |A|$$



Mathur-AMPS strong subadditivity paradox:

BC are almost maximally entangled with a part of A in a typical state after Page time.



$$|B||C| < |A|$$

$$\hat{\rho}_{BC} = \frac{1}{|B||C|}\hat{I}_{BC} = \left(\frac{1}{|B|}\hat{I}_B\right)\otimes\left(\frac{1}{|C|}\hat{I}_C\right)$$

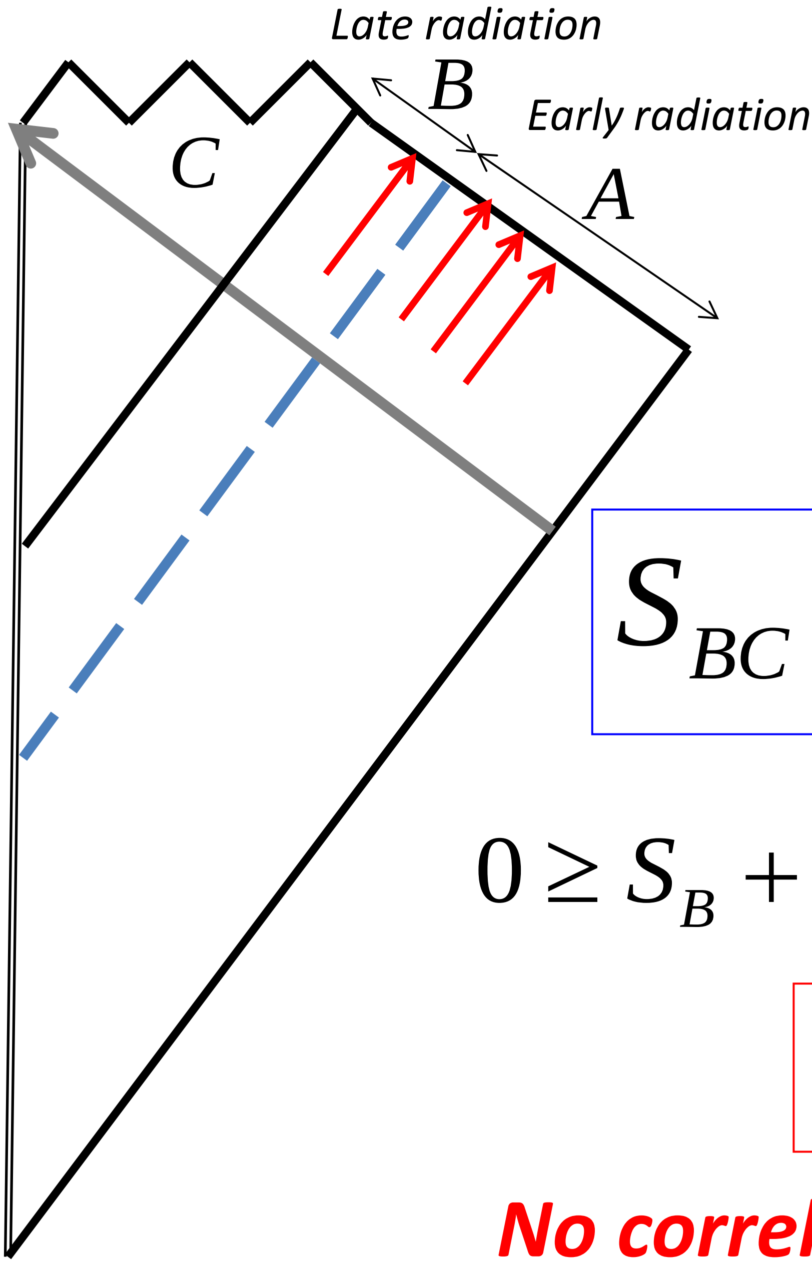
$$\bar{B}_A \subseteq A, \bar{C}_A \subseteq A$$

$$|\Psi\rangle_{B\bar{B}_A C\bar{C}_A} = \left(\frac{1}{|B|}\sum_n |b_n\rangle_B |\bar{b}_n\rangle_{\bar{B}_A}\right)\otimes\left(\frac{1}{|C|}\sum_m |c_m\rangle_B |\bar{c}_m\rangle_{\bar{C}_A}\right)$$

Harrow-Hayden

$$|\Psi\rangle_{B\bar{B}_A} = \frac{1}{|B|}\sum_n |b_n\rangle_B |\bar{b}_n\rangle_{\bar{B}_A} \Rightarrow S_{B\bar{B}_A} = 0,\ S_{B\bar{B}_A C} = S_C$$

Typical-State Condition:



$$S_{B\bar{B}_A} = 0,\; S_{B\bar{B}_A C} = S_C$$

Strong subadditivy:

$$\boxed{S_{BC} + S_{B\bar{B}_A} \geq S_B + S_{B\bar{B}_A C}}$$

$$0 \geq S_B + S_C - S_{BC} = I(B \| C) \geq 0$$

$$\boxed{I(B \| C) = 0}$$

No correlation between B and C!

Summary of Typical-State Condition:

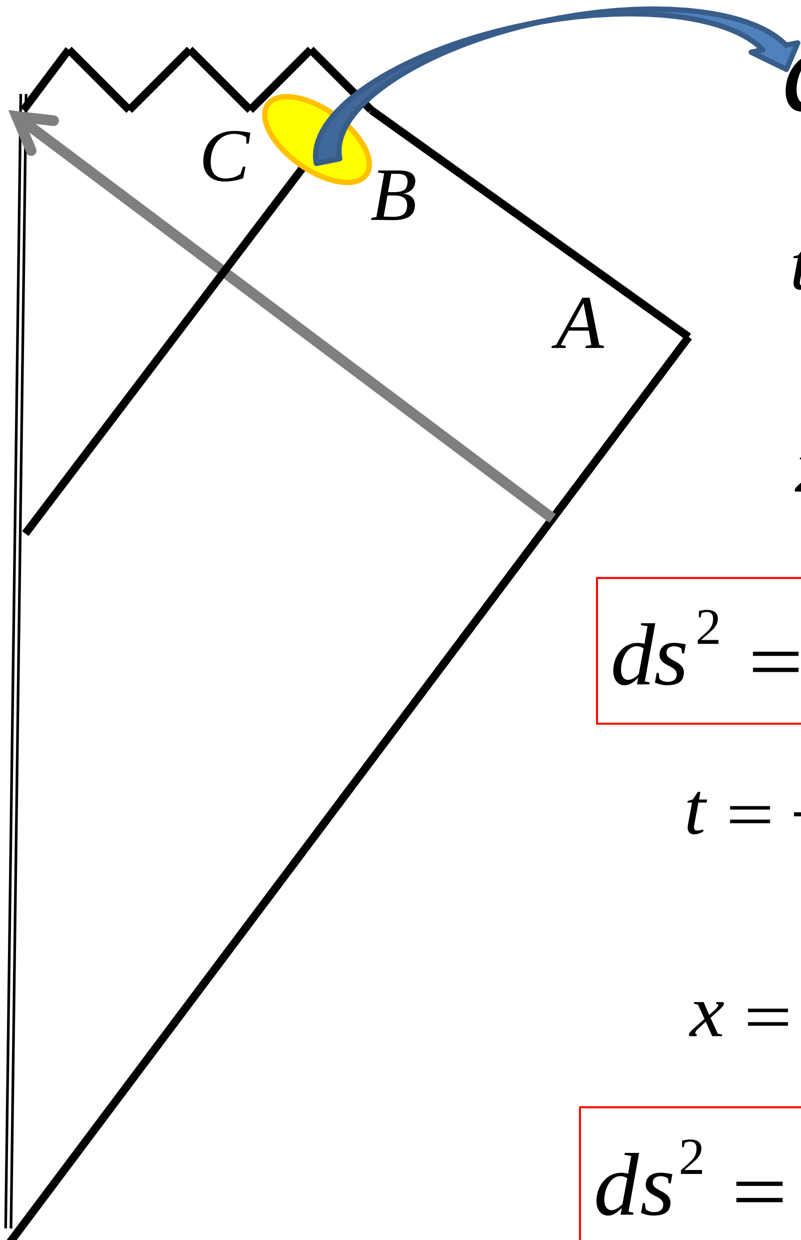
B is almost maximally entangled with a part of A!

$$\boxed{I(A \| B) >> 1}$$

No correlation between B and C!

$$\boxed{I(B \| C) = 0}$$

No-Drama Condition across Horizon:



$$ds^2 = -dt^2 + dx^2$$

$$t = \frac{1}{\kappa}\exp(\kappa\sigma_B)\sinh(\kappa\tau),$$

$$x = \frac{1}{\kappa}\exp(\kappa\sigma_B)\cosh(\kappa\tau)$$

$$\boxed{ds^2 = \exp(2\sigma_B)\left(-d\tau^2 + d\sigma_B{}^2\right)}$$

$$t = -\frac{1}{\kappa}\exp(-\kappa\sigma_C)\sinh(\kappa\tau),$$

$$x = -\frac{1}{\kappa}\exp(-\kappa\sigma_C)\cosh(\kappa\tau)$$

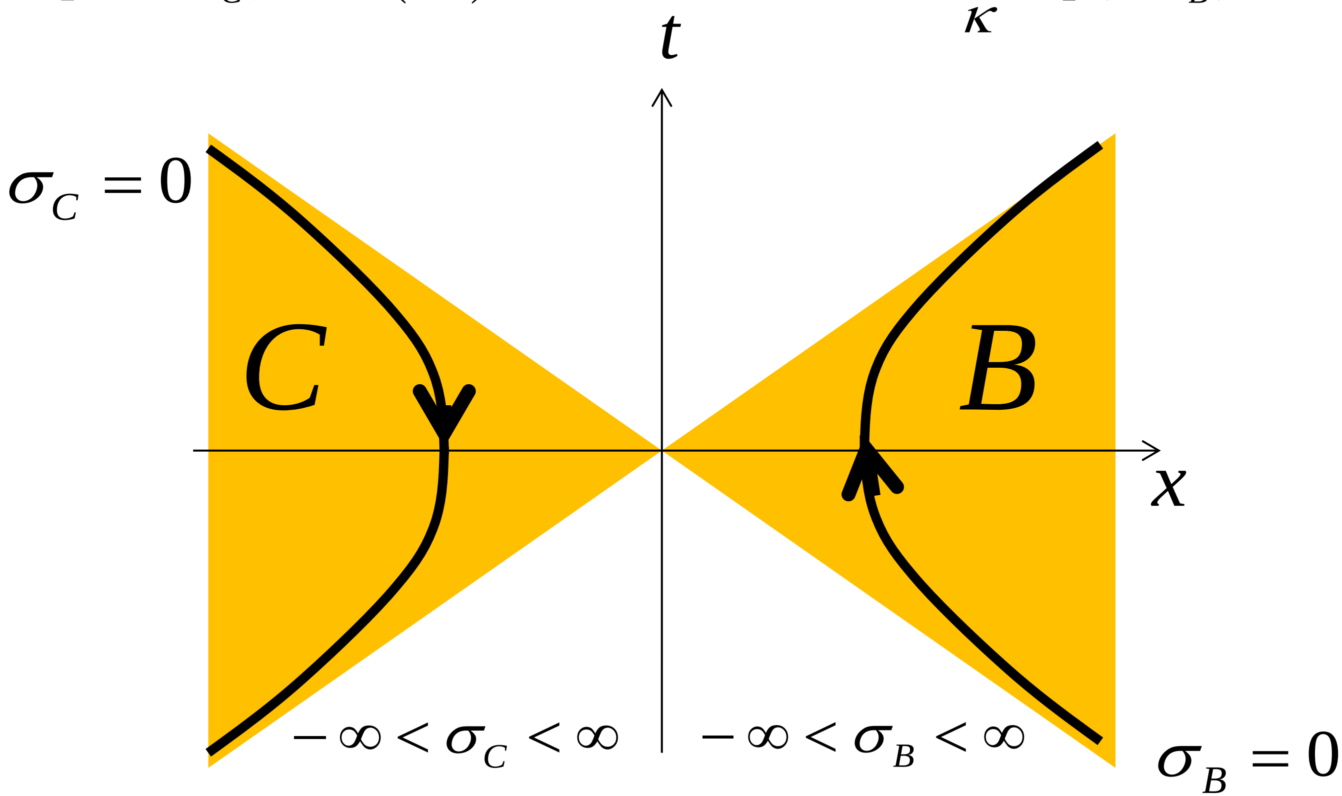
$$\boxed{ds^2 = \exp(-2\sigma_C)\left(-d\tau^2 + d\sigma_C{}^2\right)}$$

No-Drama Condition across Horizon:

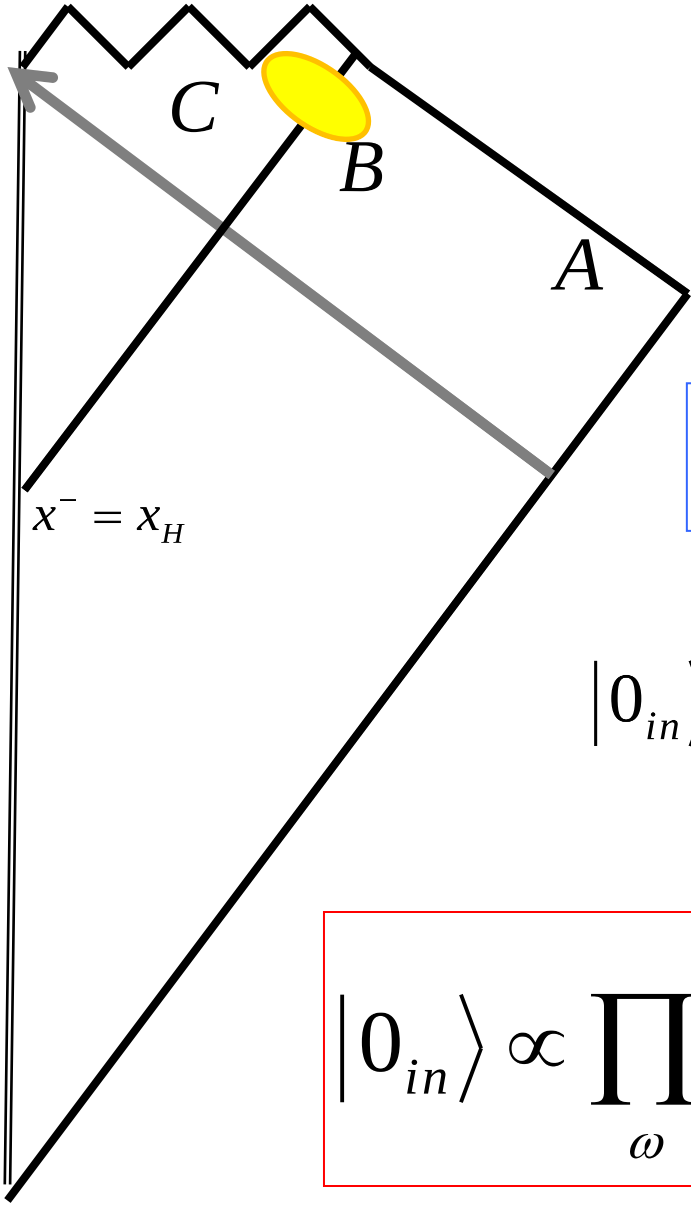
$$ds^2 = -dt^2 + dx^2$$

$$t = -\frac{1}{\kappa}\exp(-\kappa\sigma_C)\sinh(\kappa\tau), \qquad t = \frac{1}{\kappa}\exp(\kappa\sigma_B)\sinh(\kappa\tau),$$

$$x = -\frac{1}{\kappa}\exp(-\kappa\sigma_C)\cosh(\kappa\tau) \qquad x = \frac{1}{\kappa}\exp(\kappa\sigma_B)\cosh(\kappa\tau)$$



No-Drama Condition across Horizon:



Rindler mode function

$$\hat{\varphi}(\tau,\sigma)$$

$$= \Theta(x^- < x_H)\int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}}\left[\hat{a}_\omega^{(B)} \exp\left(i\omega(\sigma_B - \tau)\right) + h.c.\right]$$

$$+ \Theta(x^- > x_H)\int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}}\left[\hat{a}_\omega^{(C)} \exp\left(i\omega(\sigma_C + \tau)\right) + h.c.\right]$$

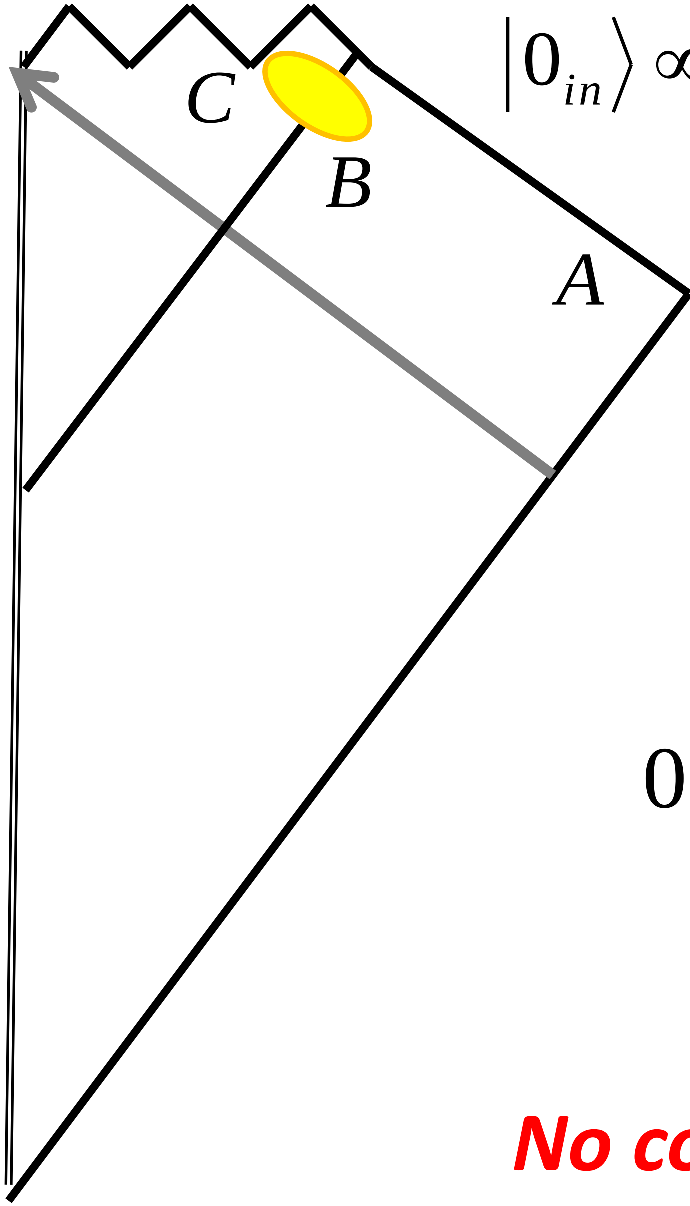
$$\boxed{\hat{a}_\omega^{(B)}\left|0_{Rindler}\right\rangle = \hat{a}_\omega^{(C)}\left|0_{Rindler}\right\rangle = 0}$$

$$\left|0_{in}\right\rangle \propto \prod_\omega \left[\exp\left(-\frac{\pi\omega}{\kappa}\hat{a}_\omega^{(B)\dagger}\hat{a}_\omega^{(C)\dagger}\right)\right]\left|0_{Rindler}\right\rangle$$

Unruh Relation:

$$\boxed{\left|0_{in}\right\rangle \propto \prod_\omega \left[\sum_n \exp\left(-\frac{n\pi\omega}{\kappa}\right)\left|n,\omega\right\rangle_B \left|n,\omega\right\rangle_C\right]}$$

No-Drama Condition across Horizon:



$$|0_{in}\rangle \propto \prod_{\omega}\left[\sum_{n}\exp\left(-\frac{n\pi\omega}{\kappa}\right)|n,\omega\rangle_B |n,\omega\rangle_C\right]$$

$$S_{BC}=0,\ S_{ABC}=S_A$$

$$\boxed{S_{AB}+S_{BC}\geq S_B+S_{ABC}}$$

$$0\geq S_A+S_B-S_{AB}=I(A\|B)\geq 0$$

$$\boxed{I(A\|B)=0}$$

No correlation between A and B!

Summary of No-drama Condition:

B is highly entangled with C!

$$\boxed{I(B \| C) >> 1}$$

No correlation between A and B!

$$\boxed{I(A \| B) = 0}$$

Strong Subadditivity Paradox:

Typical-State Condition of A and B:

$$\boxed{S_{\overline{B}_A B} = 0}$$

$$\boxed{I(A \| B) >> 1}$$

B is almost maximally entangled with A.

$$\boxed{I(B \| C) = 0}$$

No correlation between B and C.

No-drama Condition across Horizon:

$$\boxed{S_{BC} = 0}$$

$$\boxed{I(B \| C) >> 1}$$

B is highly entangled with C in a pure state.

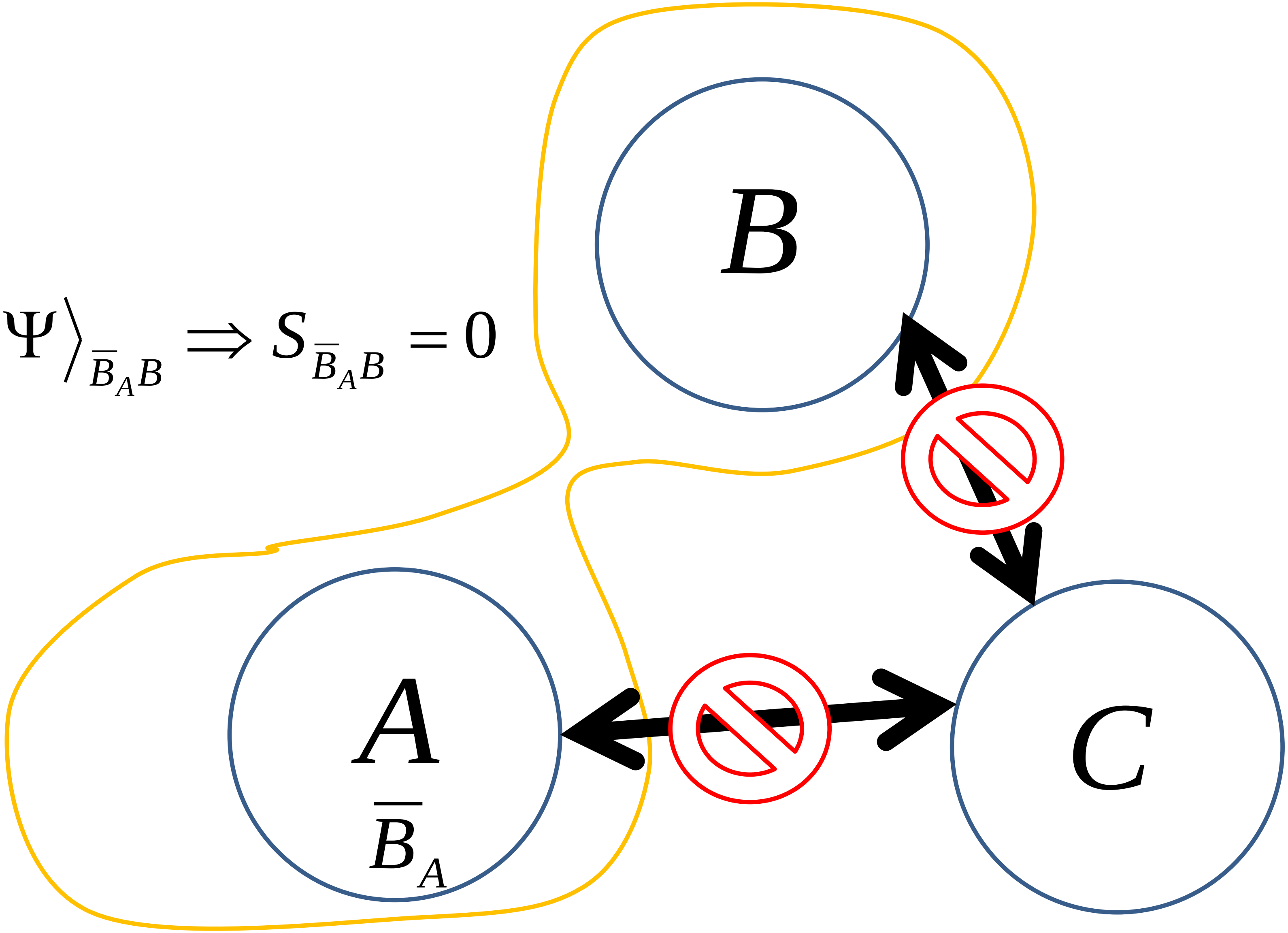
$$\boxed{I(A \| B) = 0}$$

No correlation between A and B.

Monogamy conflict arises between A,B, and C!

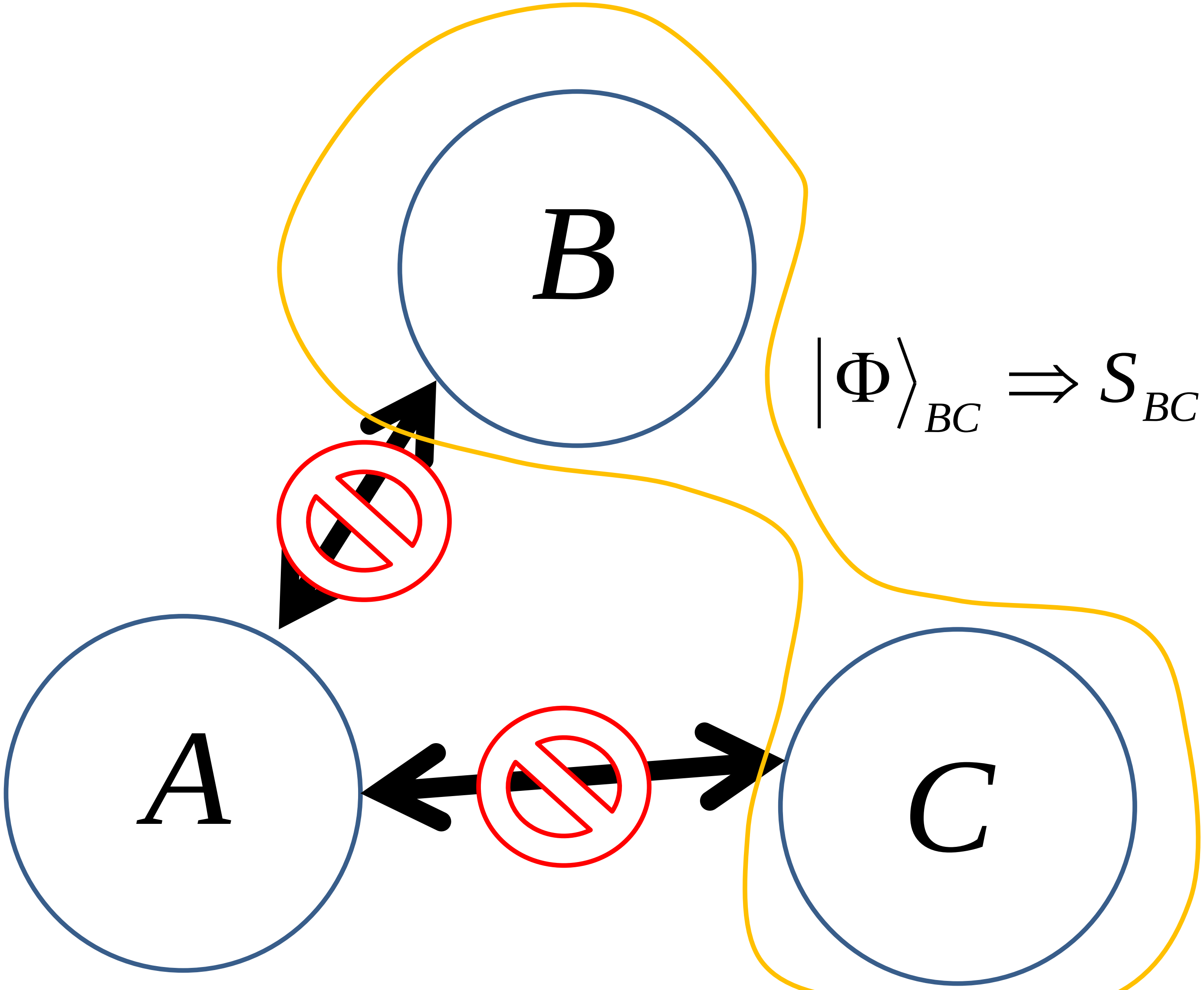
Typical-State Condition of A and B:

$$\left|\Psi\right\rangle_{\overline{B}_A B} \Rightarrow S_{\overline{B}_A B} = 0$$



Quantum monogamy

No-Drama Condition across Horizon:

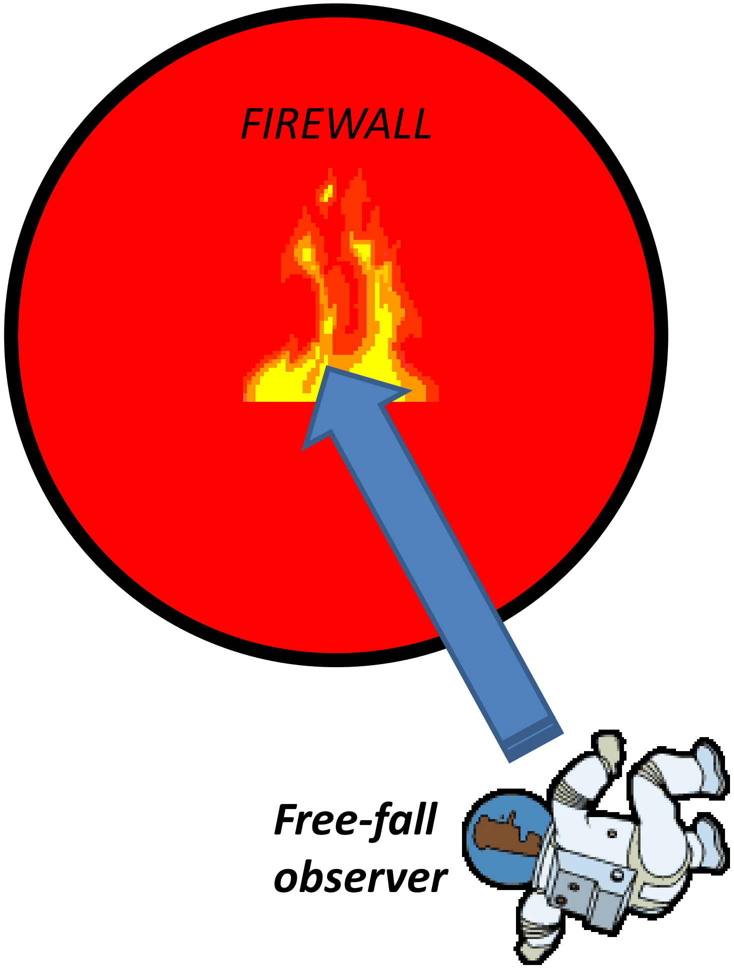


$$|\Phi\rangle_{BC} \Rightarrow S_{BC} = 0$$

Quantum monogamy

At least, one of the assumptions must be wrong!

Mathur and AMPS conjectured that no-drama condition does not hold and firewalls appear on the horizon!



THE DRAMA ON THE HORIZON

They argue that there does not exist the interior of BH!

Another bold remark by Harlow and Hayden:

"Different Unitary Quantum Mechanics for Different Observers."

For Outside Observers,

Typical-State Condition of A and B:

$$\boxed{I(A\|B) >> 1}$$

B is almost maximally entangled with A.

$$\boxed{I(B\|C) = 0}$$

No correlation between B and C.

For Infalling Observers,

No-drama Condition across Horizon:

$$\boxed{I(B\|C) >> 1}$$

B is highly entangled with C in a pure state.

$$\boxed{I(A\|B) = 0}$$

No correlation between A and B.

Strong Complementarity Conjecture

Our Argument:

M.H., Jiro Matsumoto and Ken Funo, arXiv:1306.5057

Typical-State Condition does not hold:

High-energy entanglement structure is modified so as to yield the correct description of low-energy effective field theory.

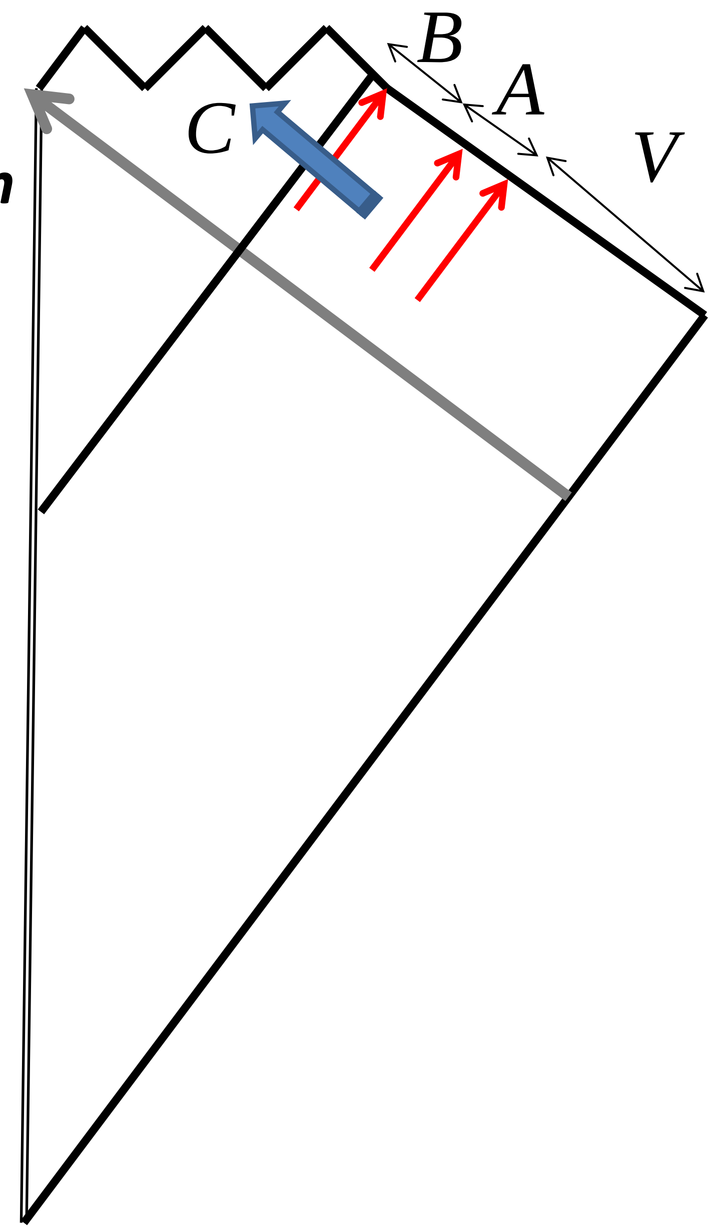
$$\boxed{S_{\overline{B}_A B} \neq 0}$$

No-Drama condition does not imply purity of BC-system state:

BC system is actually entangled with both A and zero-point fluctuation V.

Main contribution comes from V.

$$\boxed{S_{BC} \neq 0}$$

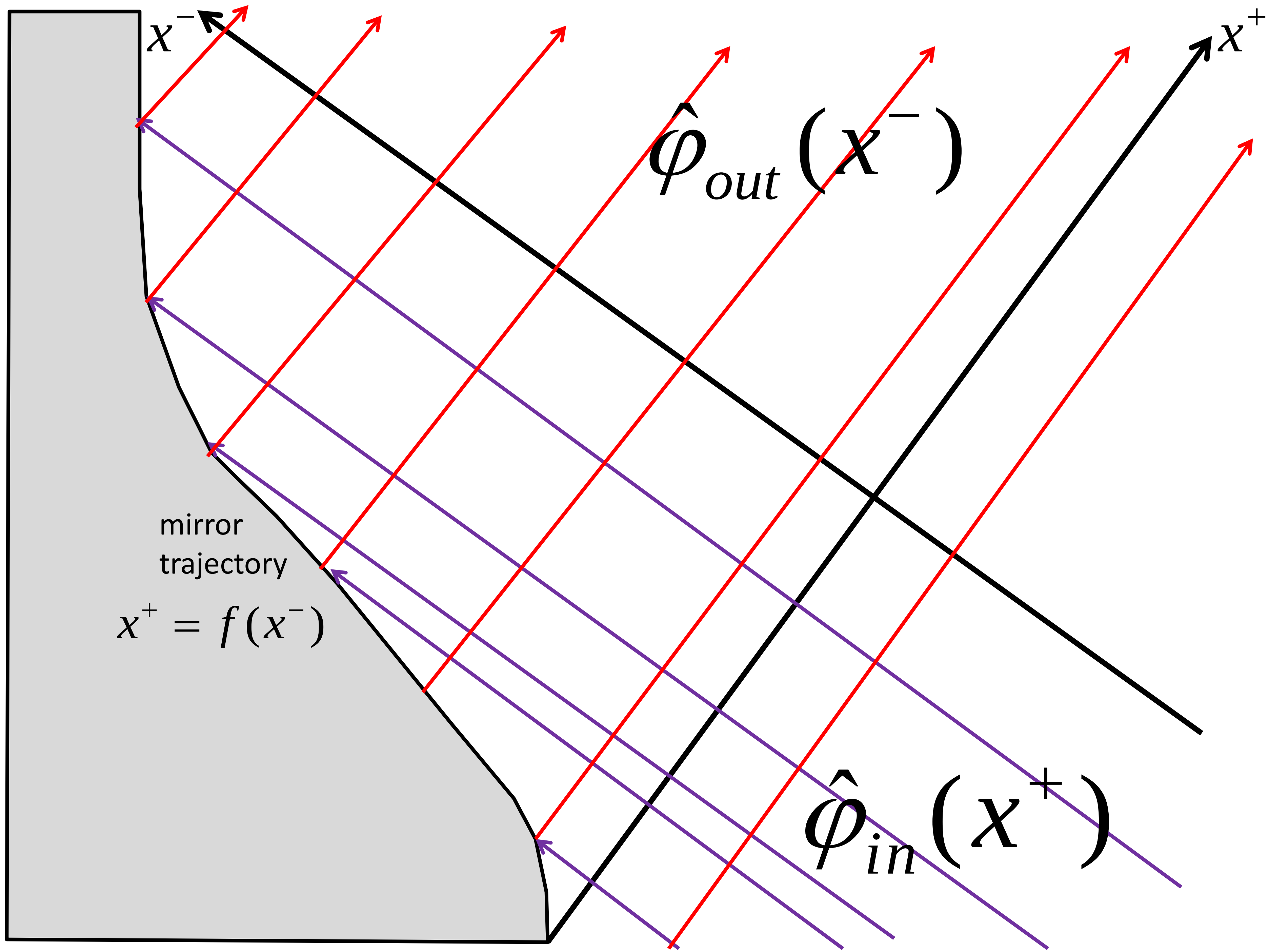


No Strong Subadditivity Paradox!

In order to make our argument concrete, let us consider a 1+1 dim. Moving Mirror Model.

The model mimics gravitational collapse of spherical matter shell and really generates Hawking radiation. Besides, it is completely unitary!

⇒This unitary model is one of the best quantum systems for checking the reasoning of Mathur and AMPS in quantum black holes.



x^-
x^+
$\hat{\varphi}_{out}(x^-)$
mirror
trajectory
$x^+ = f(x^-)$
$\hat{\varphi}_{in}(x^+)$

Mirror Trajectory:

$$x^{+} = f(x^{-}) \qquad \left(x^{\pm} = t \pm x\right)$$

Boundary Condition:

$$\hat{\varphi}\,|_{x^{+} = f(x^{-})} = 0$$

Solution:

$$\hat{\varphi}\left(x, t\right) = \hat{\varphi}_{in}(x^{+}) - \hat{\varphi}_{in}(f(x^{-}))$$

Standard quantization:

$$\hat{\varphi}_{in}\left(x^{+}\right)=\int_{0}^{\infty}\left(\hat{a}^{(in)}{}_{\omega}u_{\omega}(x^{+})+\hat{a}^{(in)}{}_{\omega}{}^{\dagger}u_{\omega}(x^{+})^{*}\right)d\omega$$

$$u_{\omega}(x^{-})=\frac{1}{\sqrt{4\pi\omega}}\exp\left(-i\omega x^{-}\right)$$

The in-vacuum state:

$$\hat{a}^{(in)}{}_{\omega}\left|0_{in}\right\rangle=0$$

Out-field is quantized as

$$\hat{\varphi}_{out}\left(x^{-}\right)=\int_0^{\infty}\left(\hat{a}^{(out)}{}_{\omega}u_{\omega}(x^{+})+\hat{a}^{(out)}{}_{\omega}{}^{\dagger}u_{\omega}(x^{+})^{*}\right)d\omega$$

$$u_{\omega}(x^{+})=\frac{1}{\sqrt{4\pi\omega}}\exp\left(-i\omega x^{+}\right)$$

The out-field can be described by in-field operators via $\hat{\varphi}_{out}(x^{-})=\hat{\varphi}_{in}(f(x^{-}))$ ***.***

$$\hat{\varphi}_{out}\left(x^{-}\right)=\int_0^{\infty}\left(\hat{a}^{(in)}{}_{\omega}v_{\omega}(x^{-})+\hat{a}^{(in)}{}_{\omega}{}^{\dagger}v_{\omega}(x^{-})^{*}\right)d\omega$$

Scattered-Wave Mode Function: $$\boxed{v_{\omega}(x^{-})=\frac{1}{\sqrt{4\pi\omega}}\exp\left(-i\omega f(x^{-})\right)}$$

Moving Mirror Model in 1+1 dim. mimics 3+1 dim. spherical gravitational collapse.

$$f(x^-) = -\frac{1}{\kappa}\ln\left(1+e^{-\kappa x^-}\right)$$

$$f(x^- \approx -\infty) \approx x^-$$

The mirror does not move in the past.

$$f(x^- \approx \infty) \approx -\frac{1}{\kappa}\exp\left(-\kappa x^-\right)$$

The mirror accelerates and approaches the light trajectory,
$x^+ = 0.$

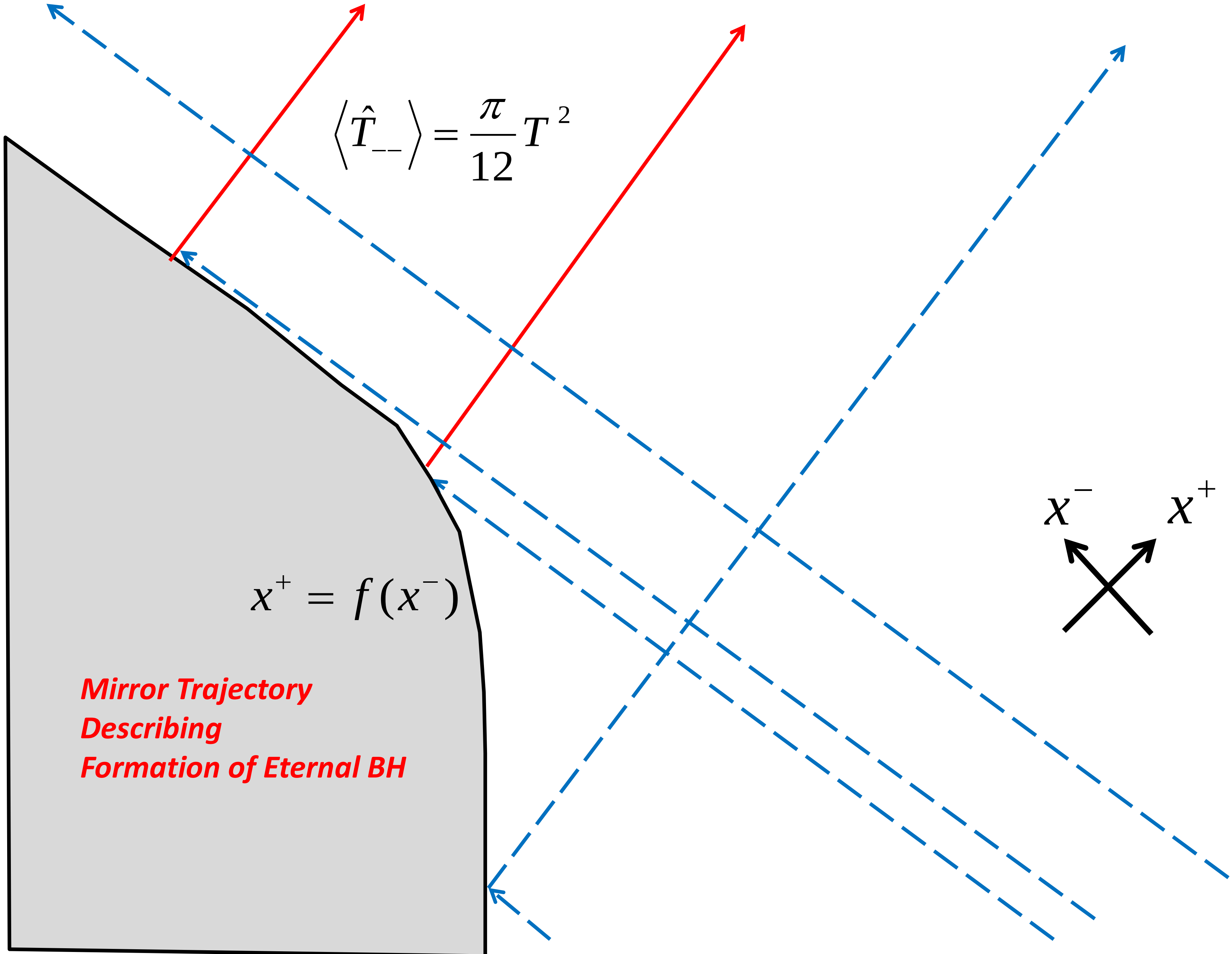
acceleration

The mirror emits thermal flux in the late time!

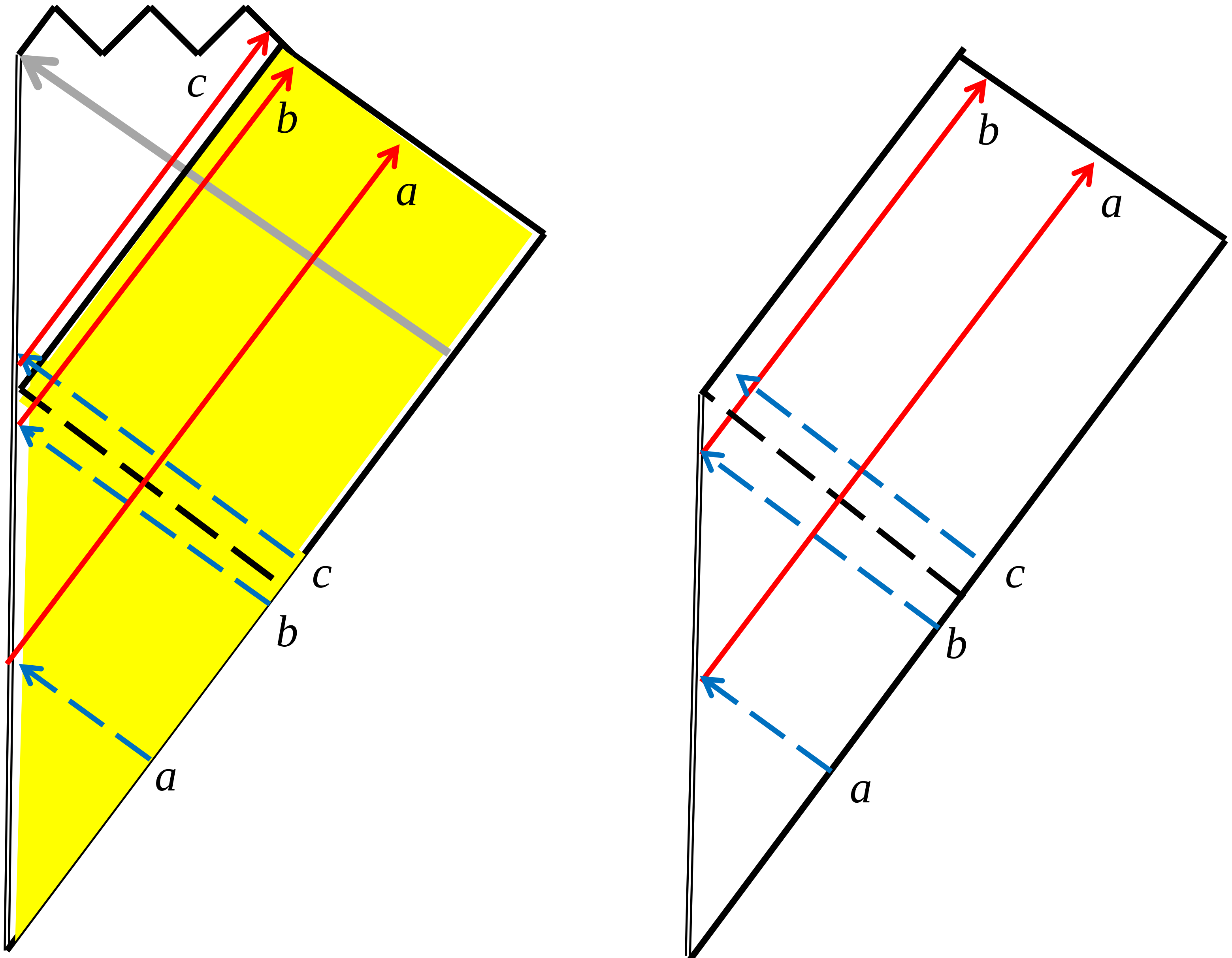
$$\boxed{\left\langle 0_{in} \left| \hat{T}_{--}(x^{-} >> 1/\kappa) \right| 0_{in} \right\rangle = \frac{\pi}{12} T^{2}}$$

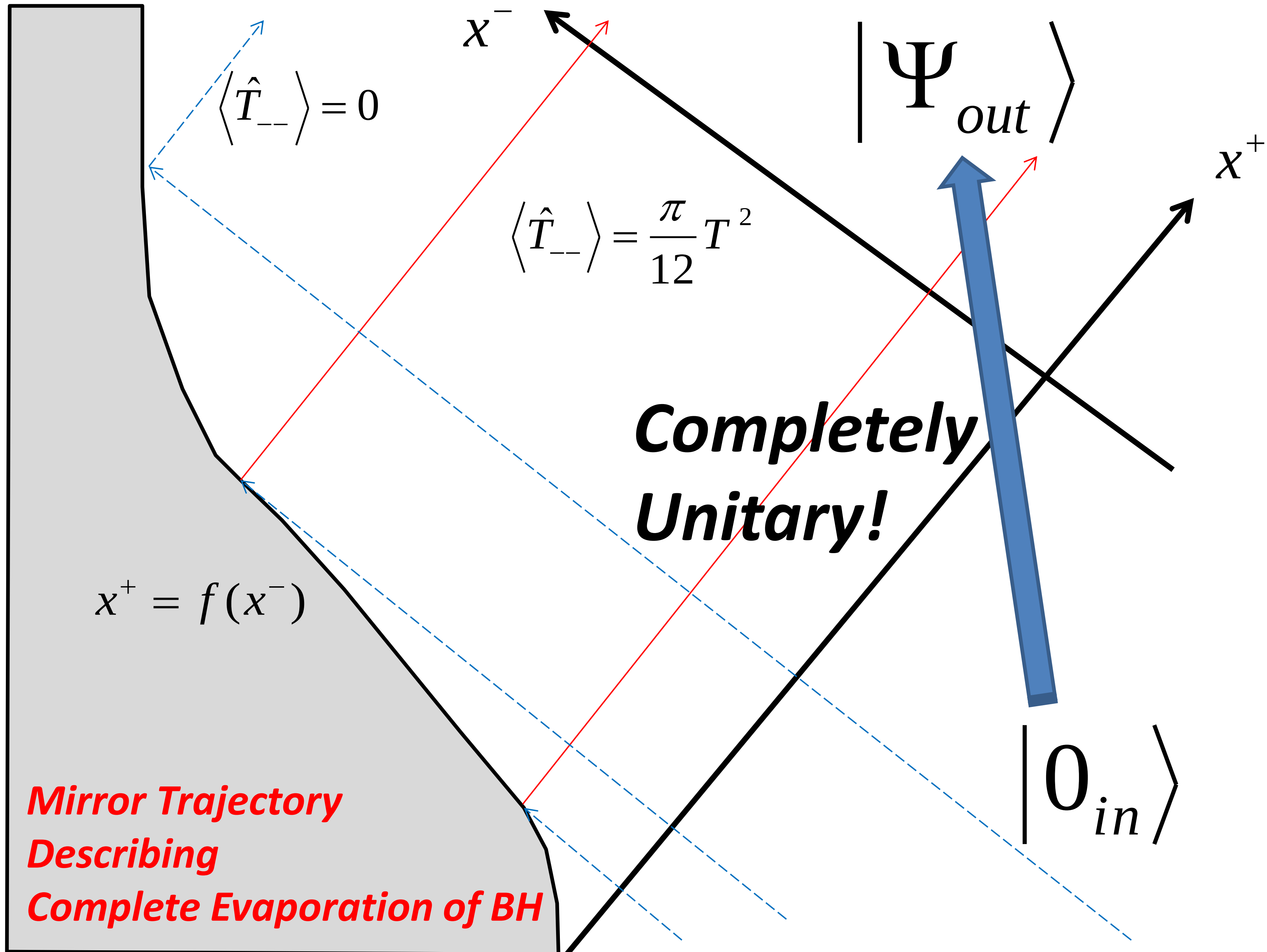
Temperature: $T = \dfrac{\kappa}{2\pi}$ ***acceleration***

$$\left\langle 0_{in} \left| \hat{a}^{(in)}{}_{\omega}{}^{\dagger} \hat{a}^{(in)}{}_{\omega} \right| 0_{in} \right\rangle \propto \frac{1}{\exp\left(\dfrac{\omega}{T}\right) - 1}$$


$\langle \hat{T}_{--} \rangle = \frac{\pi}{12} T^2$
$x^+ = f(x^-)$
x^-
x^+
Mirror Trajectory
Describing
Formation of Eternal BH

Moving Mirror Model in 1+1 dim. mimics 3+1 dim. spherical gravitational collapse.





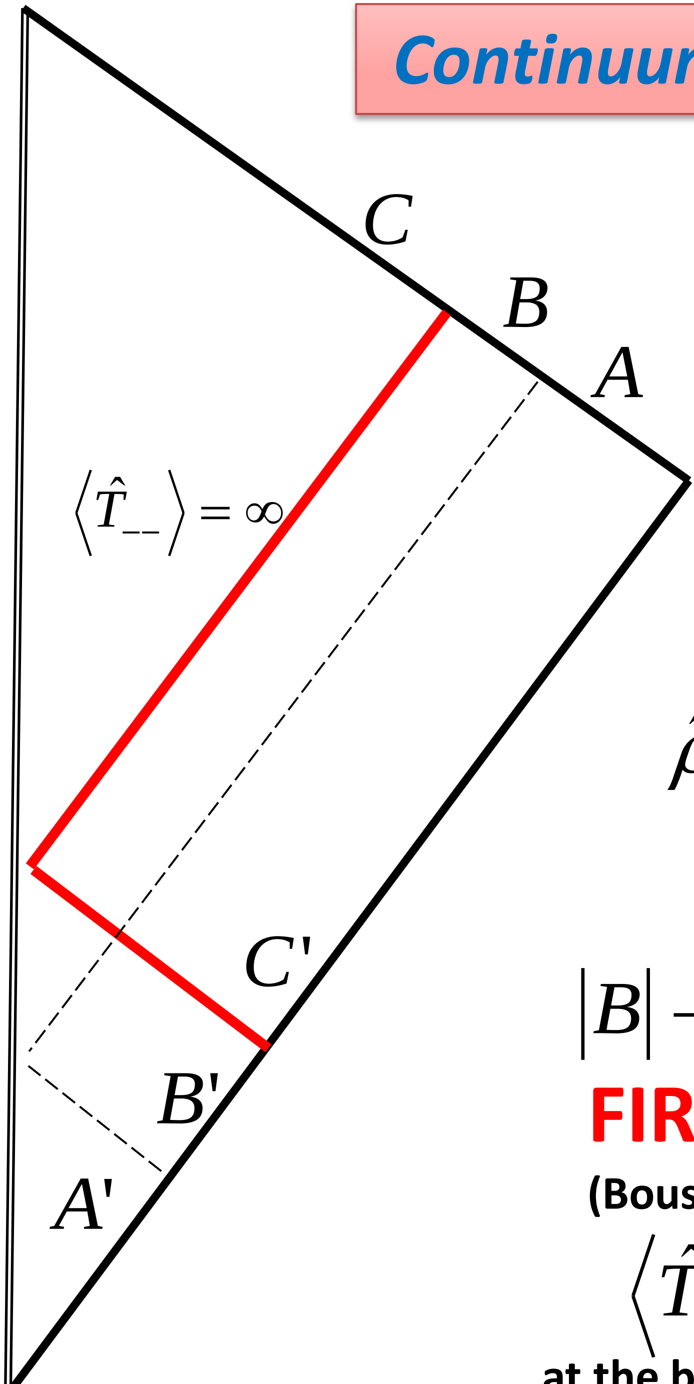
x^-
x^+
$\left|\Psi_{out}\right\rangle$
$\left\langle \hat{T}_{--} \right\rangle = 0$
$\left\langle \hat{T}_{--} \right\rangle = \frac{\pi}{12} T^2$
Completely Unitary!
$x^+ = f(x^-)$
$\left|0_{in}\right\rangle$
Mirror Trajectory Describing Complete Evaporation of BH

Our Argument (1):

Typical-state condition should be modified in order to reproduce a correct description of low-energy field theory.

$$\boxed{S_{\bar{B}_A B} \neq 0}$$

Continuum Limit of Typical-State Condition?



$$S_{\bar{B}_A B} = 0$$

$$|0_{in}\rangle \propto \left(\sum_{n=1}^{|B|} |n\rangle_B |n\rangle_{\bar{B}_A} \right) \left(\sum_{\gamma=1}^{|C|} |\gamma\rangle_C |\gamma\rangle_{\bar{C}_A} \right)$$

$$\hat{\rho}_{BC} \propto \frac{1}{|B|} \hat{I}_B \otimes \frac{1}{|C|} \hat{I}_C$$

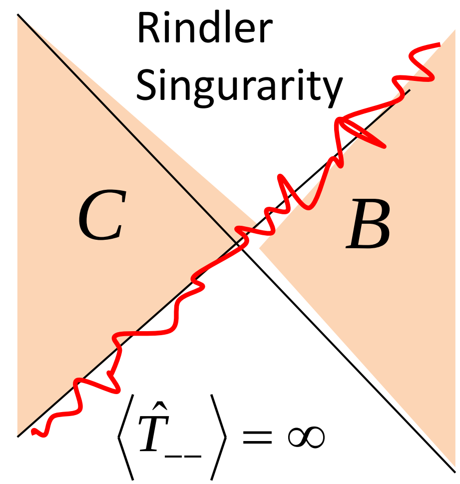
$$|B| \to \infty, \; |C| \to \infty$$

FIREWALLS

(Bousso, Marolf&Polchinski)

$$\langle \hat{T}_{--} \rangle = \infty$$

at the boundary of B and C.



This picture can be reproduced by a limit with a bad regularization!

$$\left|0_{in}\right\rangle \propto \prod_{\omega}\left[\sum_{n}\exp\left(-\frac{n\pi\omega}{\kappa}\right)\left|n,\omega\right\rangle_{A}\left|n,\omega\right\rangle_{BC}\right].$$

The scenarios of Mathur , AMPS and Bousso correspond to regularization without scale and translational symmetries !

$$\left|0_{in}\right\rangle \propto \lim_{\kappa\to\infty}\prod_{\omega}\left[\sum_{n=1}^{|B||C|}\exp\left(-\frac{n\pi\omega}{\kappa}\right)\left|n,\omega\right\rangle_{A}\left|n,\omega\right\rangle_{BC}\right]$$

$$=\prod_{\omega}\left[\sum_{n=1}^{|B||C|}\left|n,\omega\right\rangle_{A}\left|n,\omega\right\rangle_{BC}\right]$$

← ***(Would-be) typical state!***

$$\hat{\rho}_{BC}=\prod_{\omega}\left(\sum_{m=1}^{|B||C|}\left|n,\omega\right\rangle_{BC}\left\langle n,\omega\right|_{BC}\right)=\frac{1}{|B|}\hat{I}_{B}\otimes\frac{1}{|C|}\hat{I}_{C}$$

⇒ Singluarity on Rindler horizon = FIREWALL

However,
for regularization with scale and translational invariance,

$$\left|0_{in}\right\rangle \propto \prod_{\omega}\left[\sum_{n}\exp\left(-\frac{n\pi\omega}{\kappa}\right)\left|n,\omega\right\rangle_{A}\left|n,\omega\right\rangle_{BC}\right]$$

$$\omega'=\frac{\omega}{\kappa},\ a'_{\omega'}=\sqrt{\kappa}a_{\omega}$$

$$\left|0_{in}\right\rangle \propto \prod_{\omega}\left[\sum_{n}\exp\left(-n\pi\omega\right)\left|n',\omega\right\rangle_{A}\left|n',\omega\right\rangle_{BC}\right]$$

$$\hat{\rho}_{BC}=\frac{1}{Z_{R}}\exp\left(-2\pi\hat{H}_{R}\right)\neq\frac{1}{|B|}\hat{I}_{B}\otimes\frac{1}{|C|}\hat{I}_{C}$$

$$\boxed{I(B\|C)\neq 0}$$

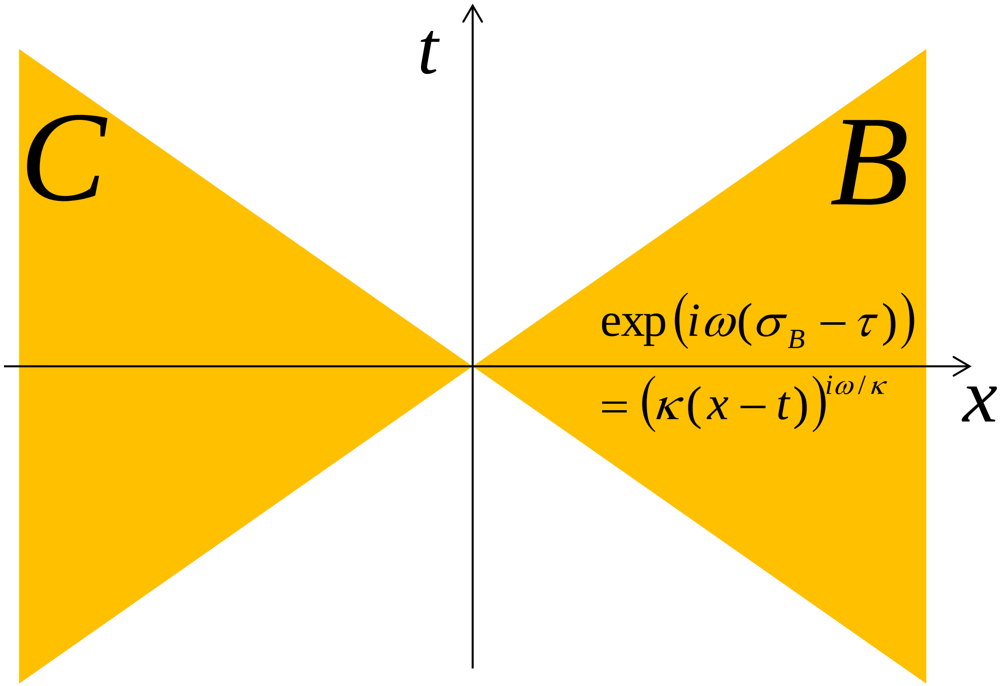
$$\boxed{S_{\overline{B}_{A}B}\neq 0}$$

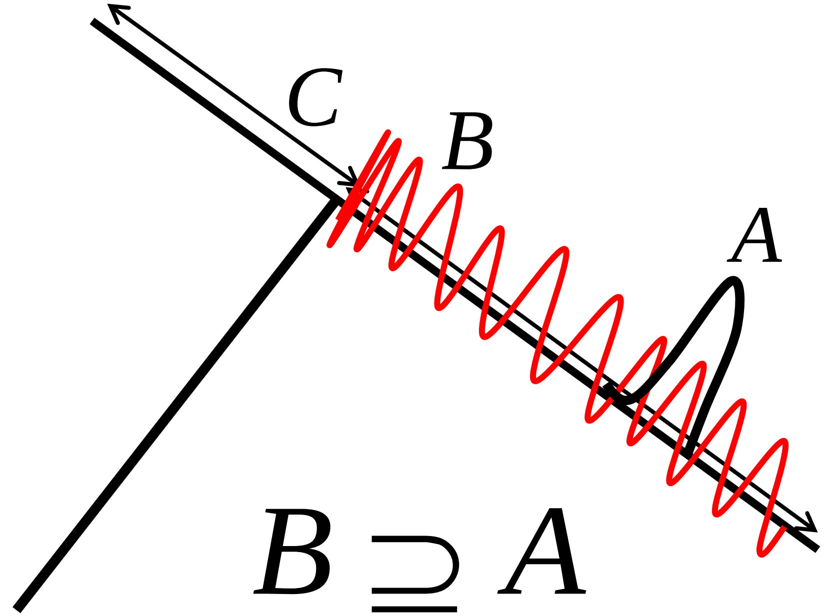
Our Argument (2):

No-drama condition does not imply the purity of BC system, if local independence between A and B holds.

$$\boxed{S_{BC} \neq 0}$$

No-Drama Condition across Horizon:





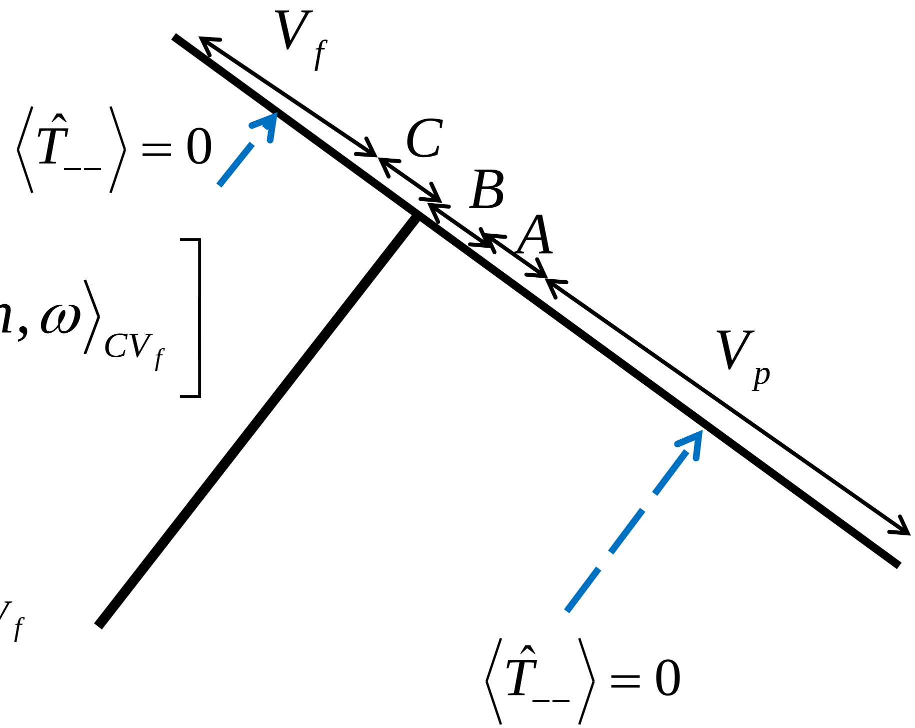
$$|0_{in}\rangle \propto \prod_{\omega}\left[\sum_{n}\exp\left(-\frac{n\pi\omega}{\kappa}\right)|n,\omega\rangle_B |n,\omega\rangle_C\right]$$ **←AMPS purity of BC system**

The support of Rindler mode functions are not localized!

Overlap of A and B cannot be neglected.

Note: Strict localization cannot be attained by superposing one-particle states. (J. Knight 1961)

In order to impose strict localization of A, B and C, local vacuum regions must be introduced.



$$|0_{in}\rangle \propto \prod_{\omega}\left[\sum_{n}\exp\left(-\frac{n\pi\omega}{\kappa}\right)|n,\omega\rangle_{V_pAB}|n,\omega\rangle_{CV_f}\right]$$

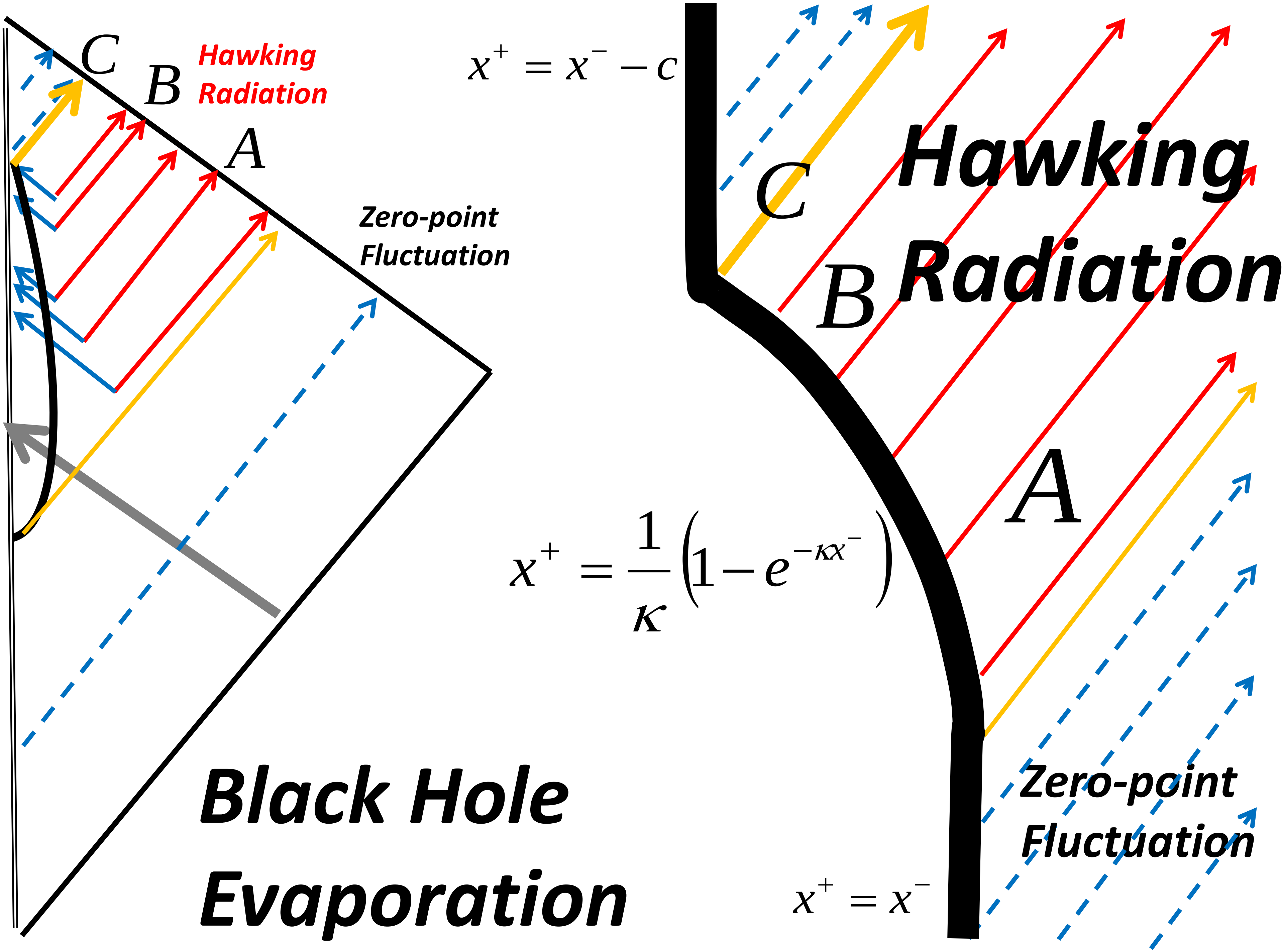
$$\sum_{n}\exp\left(-\frac{n\pi\omega}{\kappa}\right)|n,\omega\rangle_{V_pAB}|n,\omega\rangle_{CV_f}$$
$$\neq |\psi(\omega)\rangle_{V_pA}|\Psi(\omega)\rangle_{BC}\,|\phi(\omega)\rangle_{V_f}$$

Localized BC system is actually entangled with A and zero-point fluctuations: $S_{BC} > 0.$

Our Argument (3):

<<Information Loss Problem>>

It may be possible that main entangled partner of Hawking radiation is zero-point fluctuations in local vacuum regions.

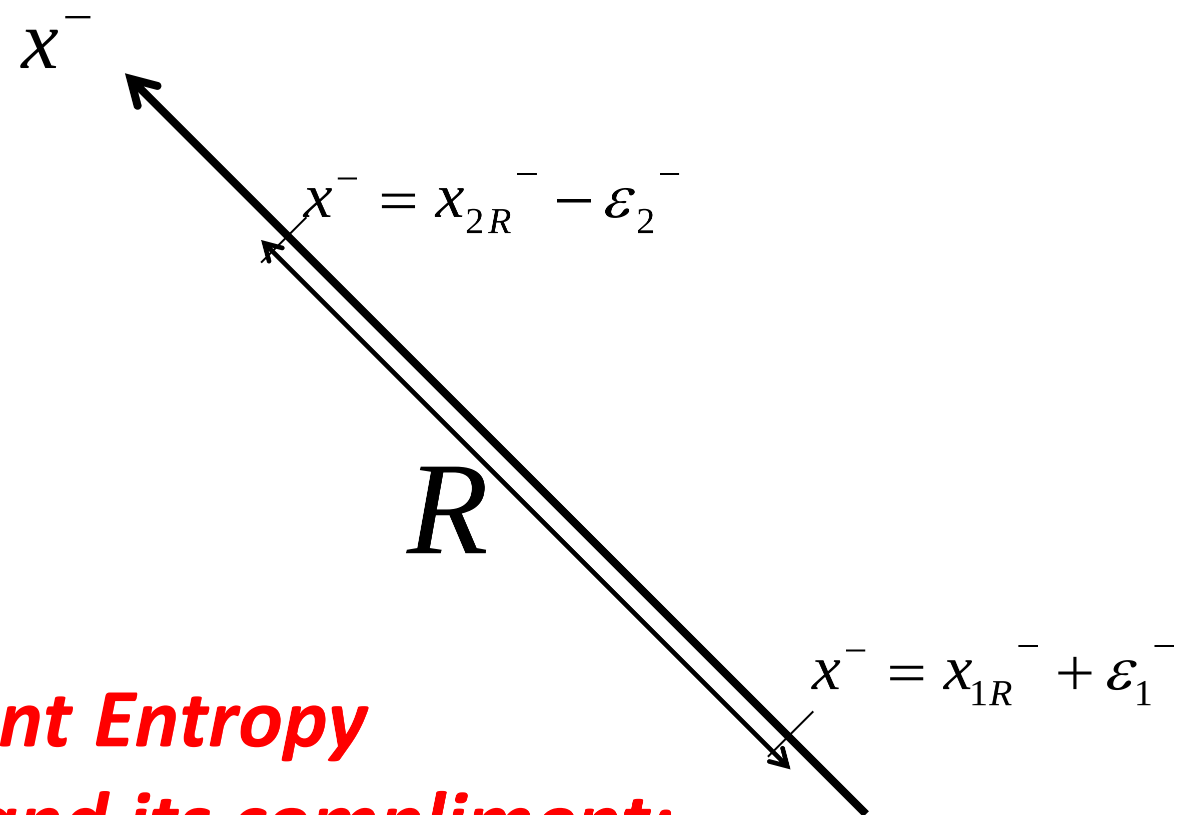

C
B
A
Hawking
Radiation
Zero-point
Fluctuation
Black Hole
Evaporation
$x^+ = x^- - c$
$x^+ = \frac{1}{\kappa}\left(1 - e^{-\kappa x^-}\right)$
$x^+ = x^-$
C
B
A
Hawking
Radiation
Zero-point
Fluctuation

What is the entangled partner of Hawking radiation?

Is it the final informative shock waves emitted by BH burst?

$$\boxed{C\,?}$$

No!!

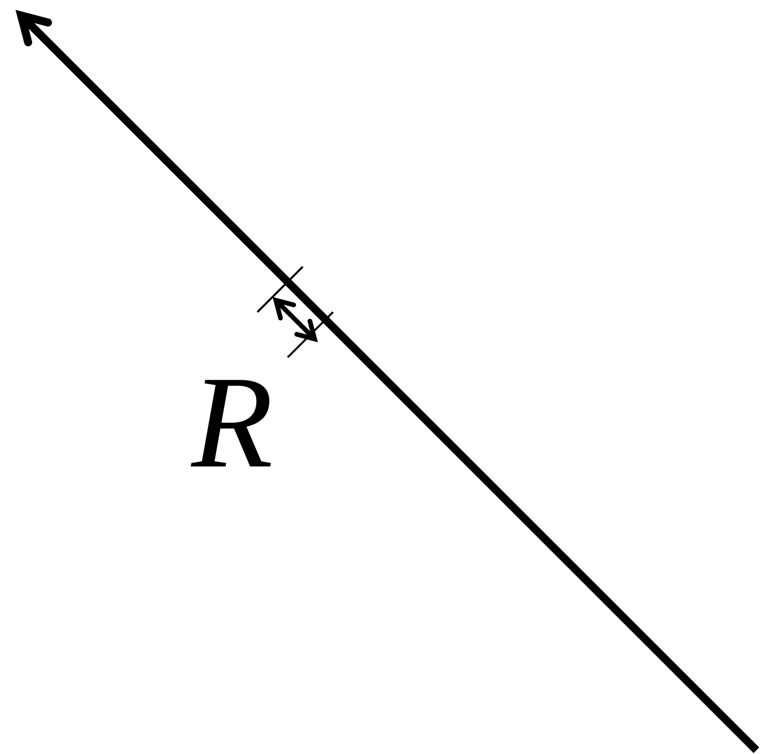


Entanglement Entropy between R and its compliment:

$$S_{EE}(R,\overline{R}) = \frac{1}{12}\ln\left(\frac{\left(f(x_{2R}{}^-) - f(x_{1R}{}^-)\right)^2}{\partial_- f(x_{2R}{}^-)\partial_- f(x_{1R}{}^-)\varepsilon_2{}^-\varepsilon_1{}^-}\right)$$

(Holzhey-Larsen-Wilczek)

This formula shows that shock waves, which are confined in a very narrow space, are not entangled with anything!



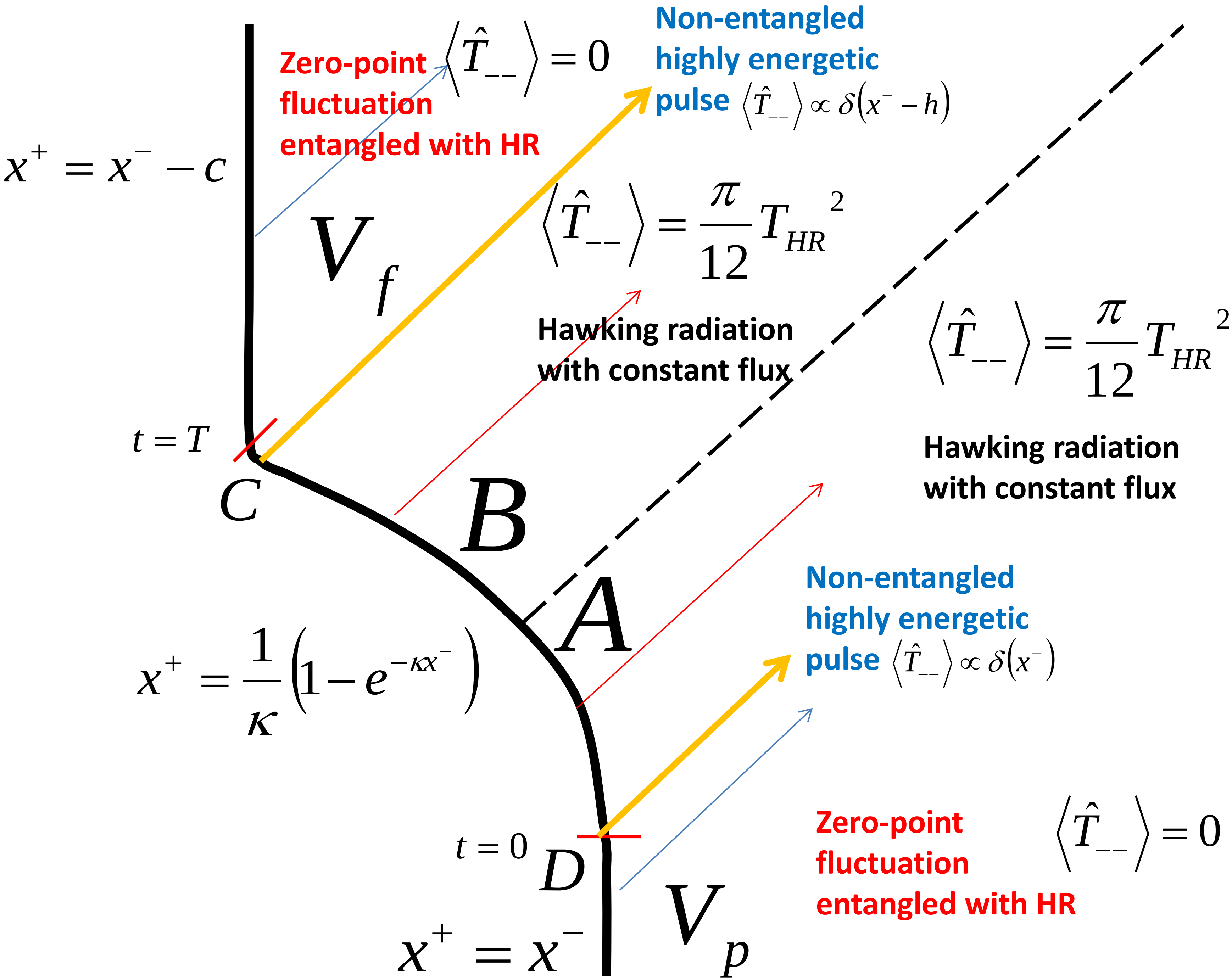
$$S_{EE}(R,\overline{R}) = \frac{1}{12}\ln\left(\frac{\left(f(x_{2R}{}^{-}) - f(x_{1R}{}^{-})\right)^2}{\partial_- f(x_{2R}{}^{-})\partial_- f(x_{1R}{}^{-})\varepsilon_2{}^{-}\varepsilon_1{}^{-}}\right) \approx 0$$

Gosh!
What is actually the entangled partner of Hawking radiation?

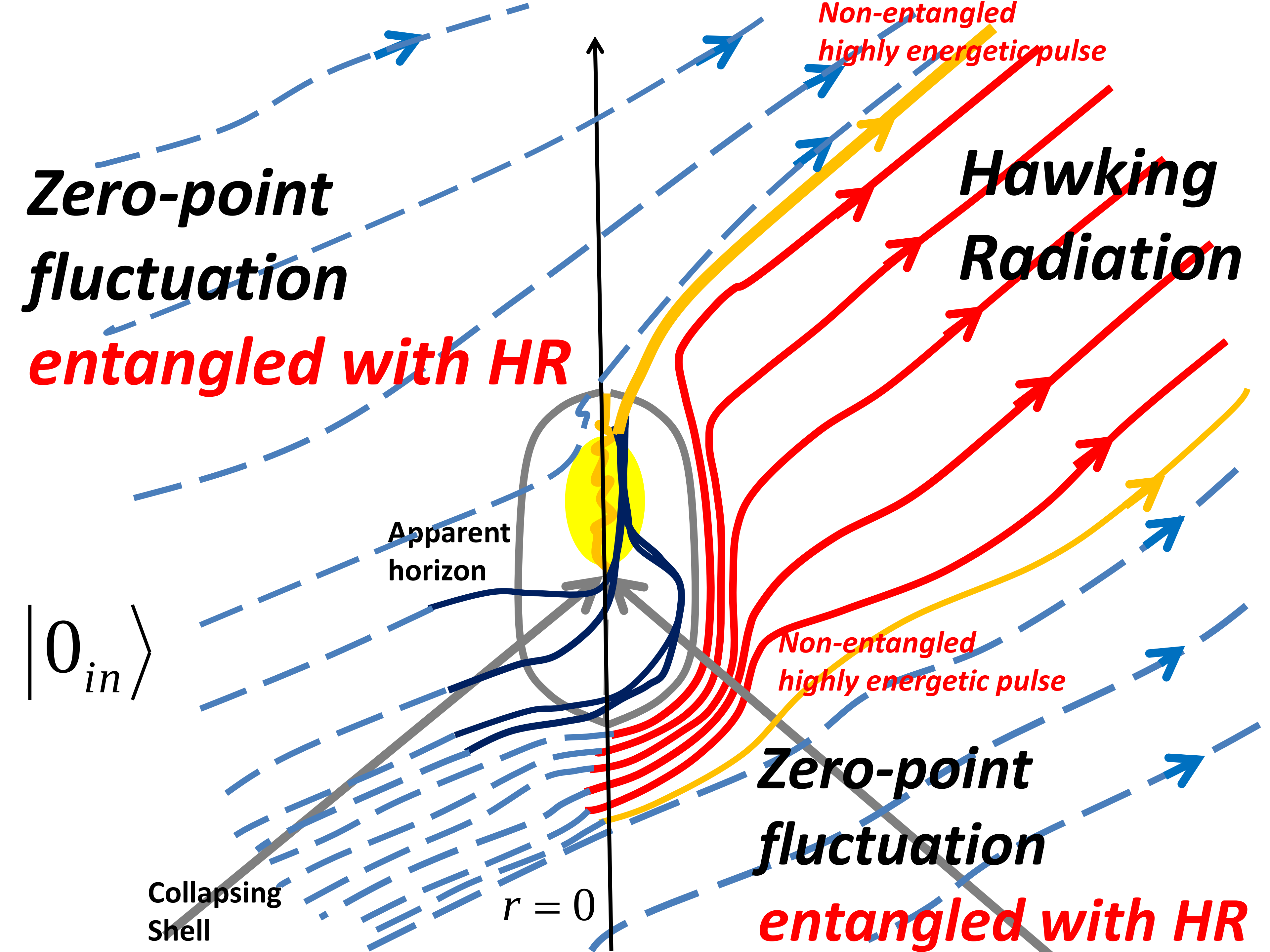
⇒ *Zero-point fluctuations in local vacuum regions*

(Wilczek 1992, Hotta-Matsumoto-Funo 2013)

Entanglement without energy cost!



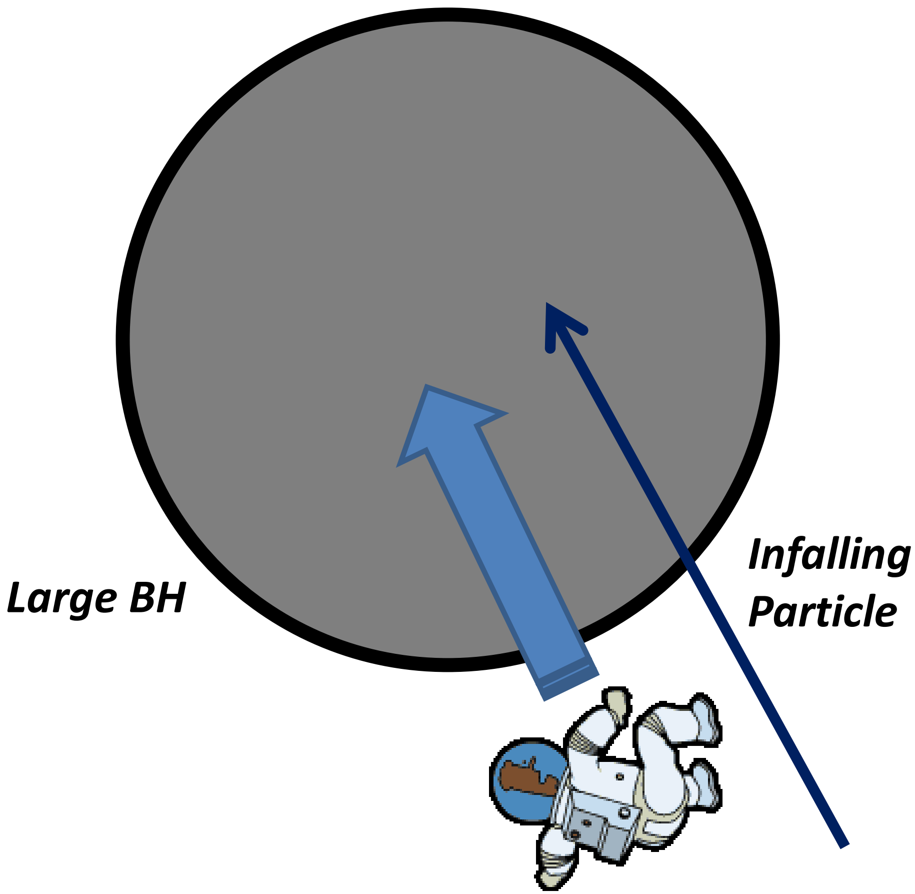
Non-entangled
highly energetic
pulse $\langle \hat{T}_{--} \rangle \propto \delta(x^- - h)$
Zero-point
fluctuation
entangled with HR
$\langle \hat{T}_{--} \rangle = 0$
$x^+ = x^- - c$
V_f
$\langle \hat{T}_{--} \rangle = \frac{\pi}{12} T_{HR}{}^2$
Hawking radiation
with constant flux
$\langle \hat{T}_{--} \rangle = \frac{\pi}{12} T_{HR}{}^2$
Hawking radiation
with constant flux
$t = T$
C
B
A
$x^+ = \frac{1}{\kappa}\left(1 - e^{-\kappa x^-}\right)$
Non-entangled
highly energetic
pulse $\langle \hat{T}_{--} \rangle \propto \delta(x^-)$
$t = 0$
D
Zero-point
fluctuation
entangled with HR
$\langle \hat{T}_{--} \rangle = 0$
$x^+ = x^-$
V_p

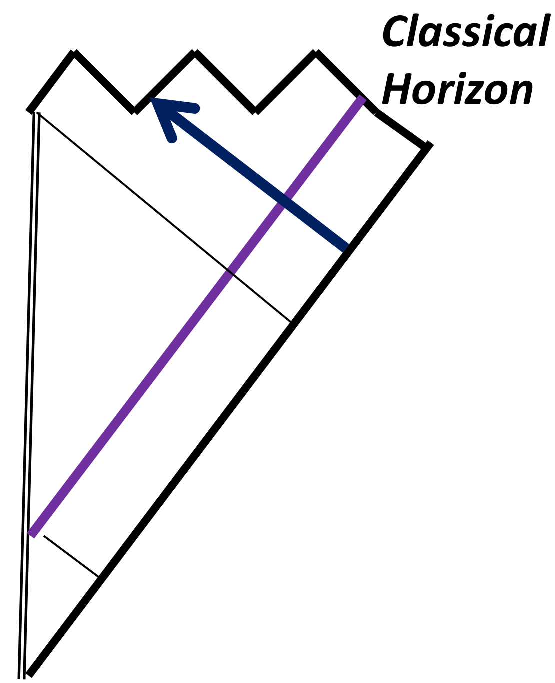


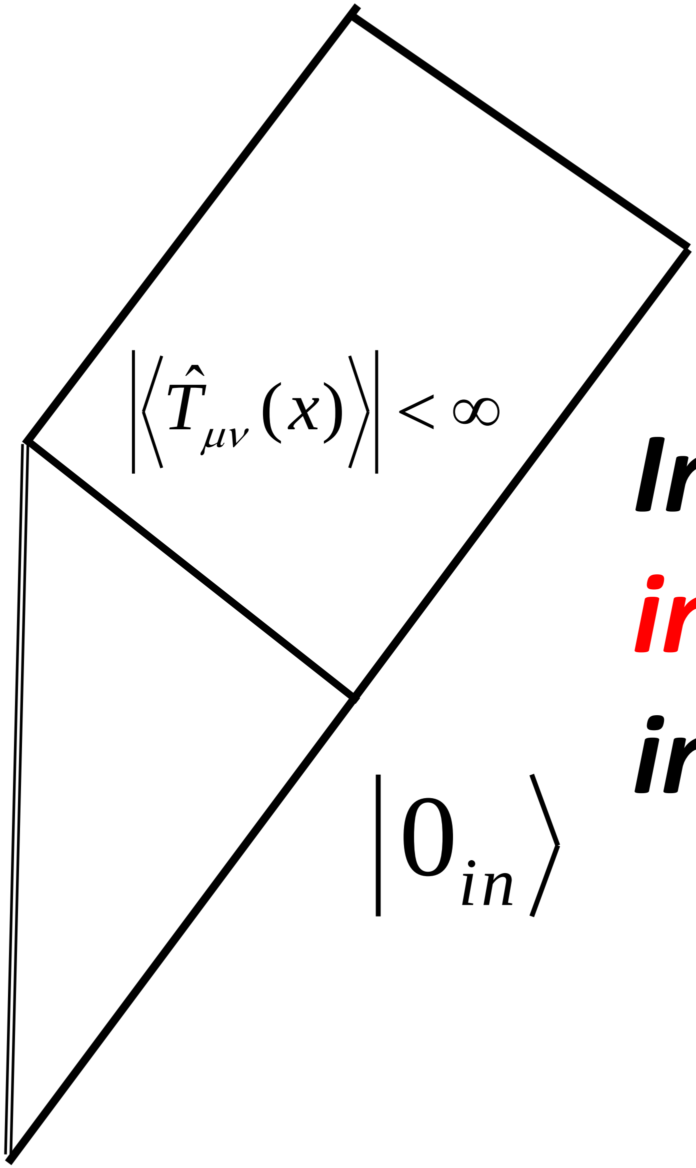
Non-entangled highly energetic pulse
Zero-point fluctuation
entangled with HR
Hawking Radiation
Apparent horizon
$|0_{in}\rangle$
Non-entangled highly energetic pulse
Zero-point fluctuation
entangled with HR
Collapsing Shell
$r = 0$

Firewall paradox of AMPS is resolved in this model!

Free-fall observers do not encounter firewalls when come across event horizon!



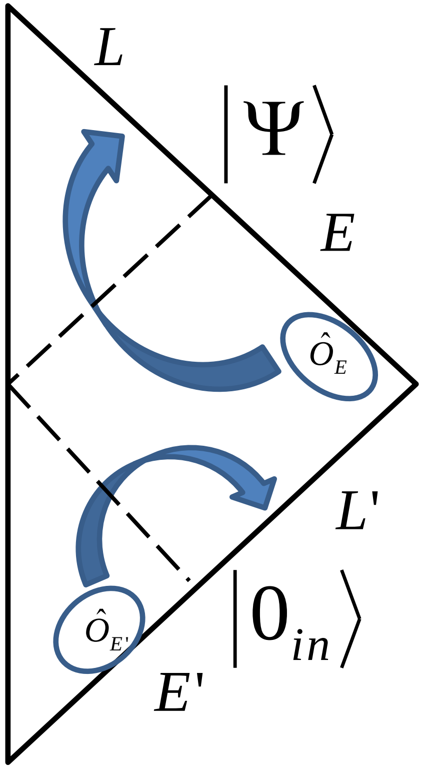




In fact, NO FIREWALLS
in an average meaning
in moving mirror models.

However, we have another firewall paradox in the moving mirror model.

The point is Reeh-Schlieder theorem in quantum field theory.



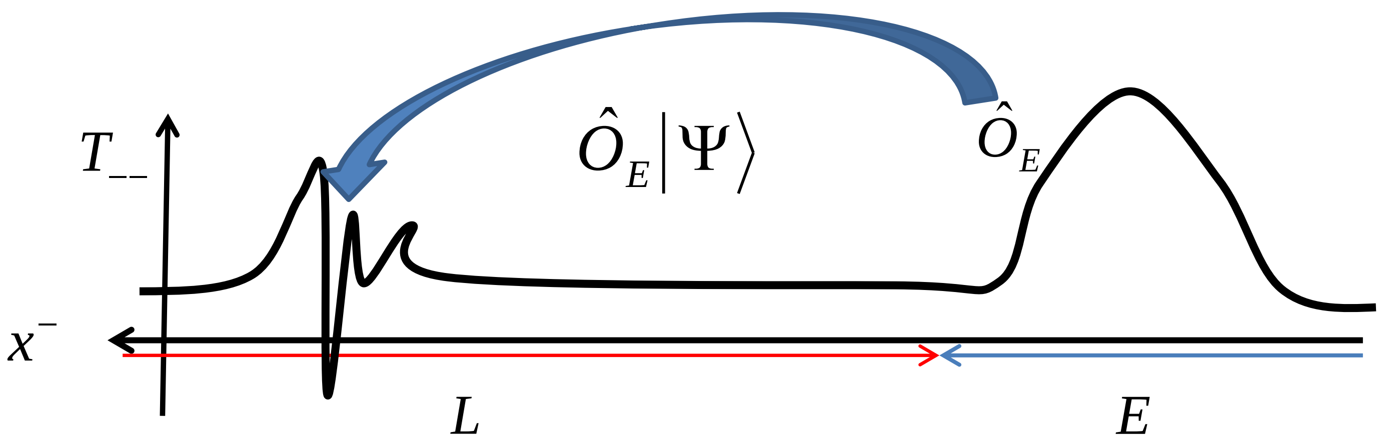
Reeh-Schlieder theorem:

The set of states generated from $|0_{in}\rangle$ by the polynomial algebra of local operators in any bounded spacetime region is dense in the total Hilbert space of the field. Thus, in principle, any state of L' can be arbitrarily closely reproduced by acting a polynomial of local operators of E' on $|0_{in}\rangle$.

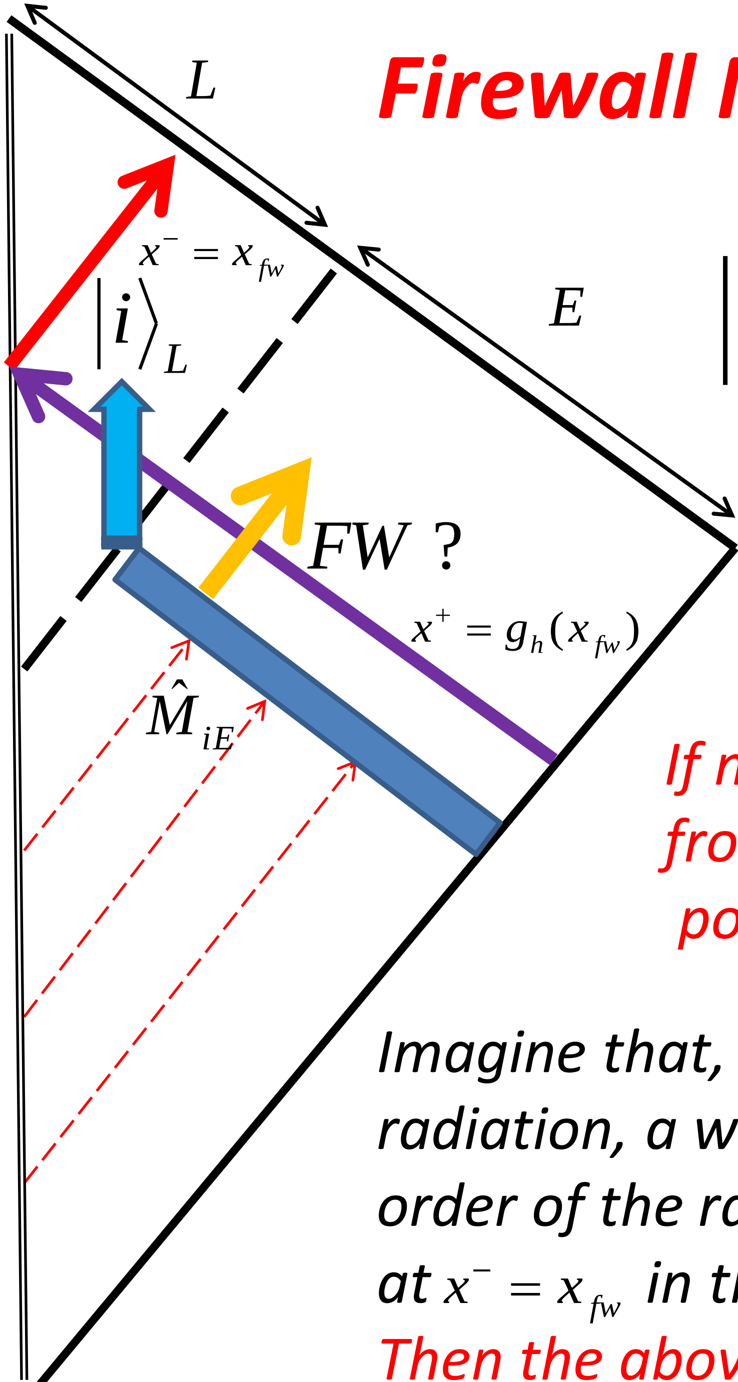
This property is maintained in the time evolution of

$$|0_{in}\rangle \rightarrow |\Psi_{out}\rangle.$$

The Reeh-Schlieder property ensures much entanglement of the system in the final state of the moving-mirror model.



Firewall Measurement Paradox:



$$|\Psi\rangle = \sum_i |\psi_{iE}\rangle |i_L\rangle$$

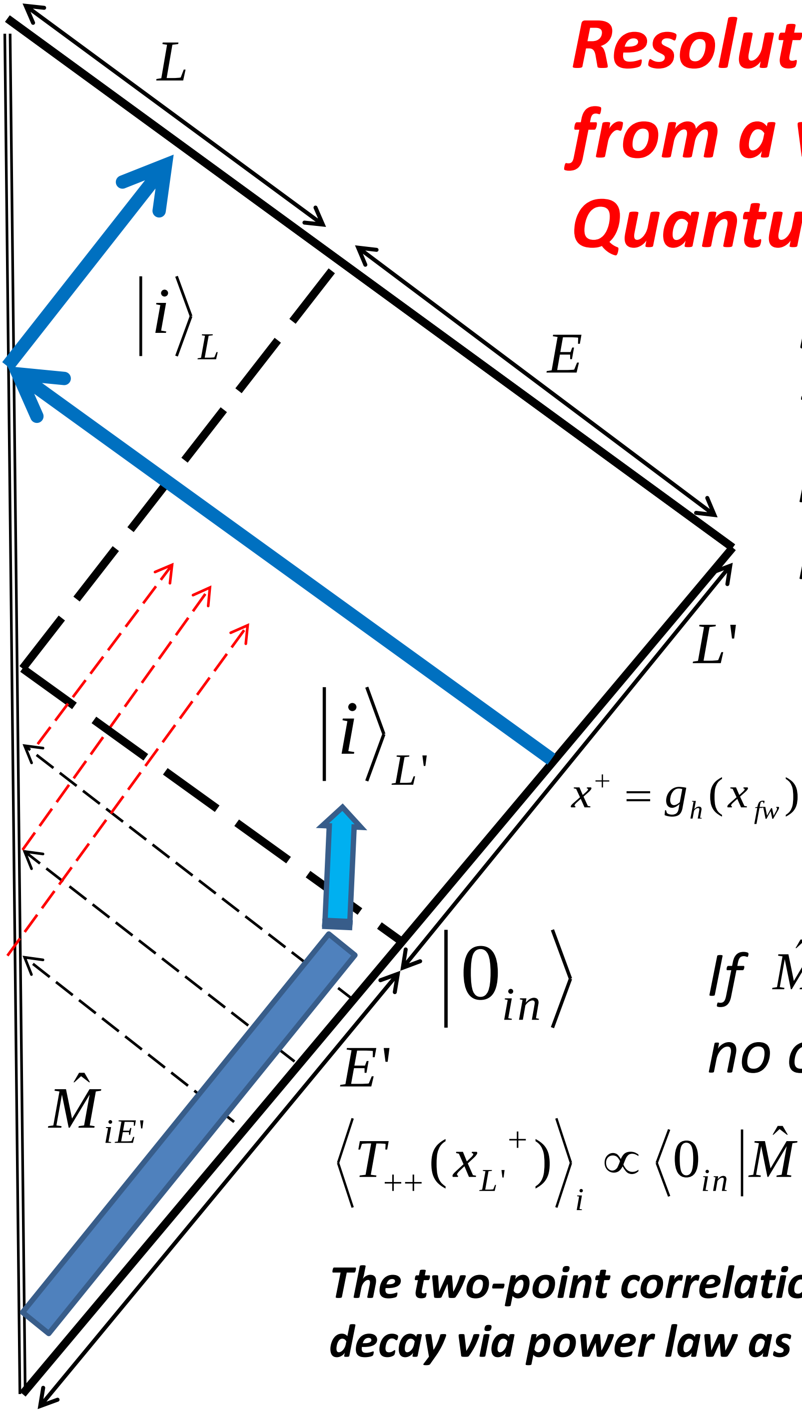
measured

post-measurement state of L

If measurement operator of E is constructed from Reeh-Schlieder operation, an arbitrary post-measurement state of L can emerge.

Imagine that, besides the background Hawking radiation, a wave packet with positive energy of the order of the radiation temperature appears at $x^- = x_{fw}$ in the post-measurement state $|i\rangle_L$ of L.
Then the above firewall (FW) appears at $x^+ = g_h(x_{fw})$.

Resolution of the Paradox from a viewpoint of Quantum Measurement Energy Cost



Because the mirror merely stretches the modes of the field, the future measurement is equivalent to a past measurement for the in-vacuum state.

$$\hat{M}_{iE} \Leftrightarrow \hat{M}_{iE'}$$

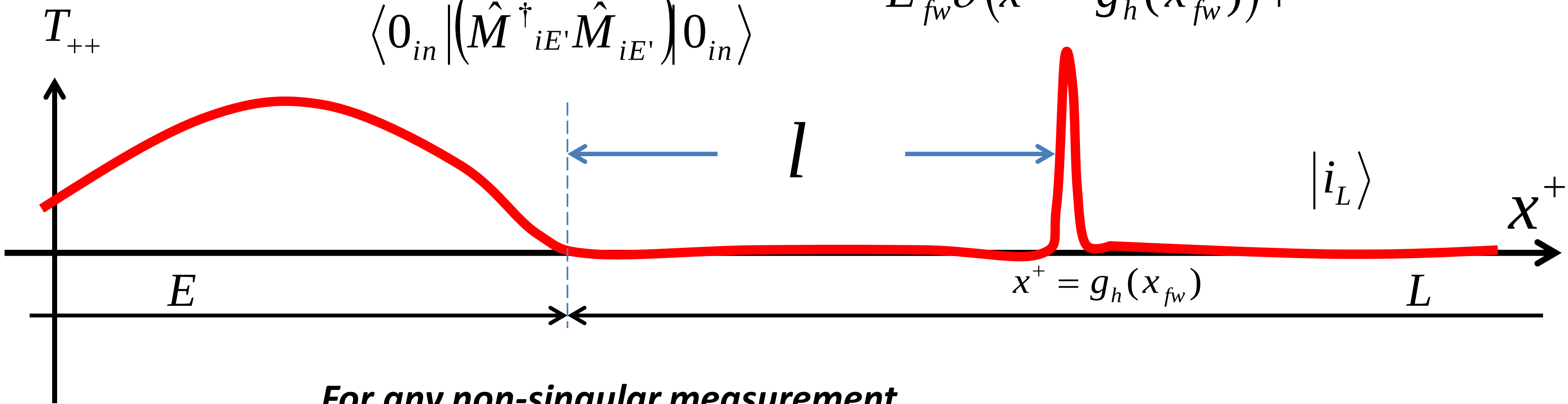
$$\sum_i \hat{M}^\dagger{}_{iE} \hat{M}_{iE} = I \qquad \sum_i \hat{M}^\dagger{}_{iE'} \hat{M}_{iE'} = I$$

If $\hat{M}^\dagger{}_{iE'} \hat{M}_{iE'}$ *is not singular,*
no outstanding peak of energy flux appears.

$$\left\langle T_{++}(x_{L'}{}^+) \right\rangle_i \propto \left\langle 0_{in} \right| \hat{M}^\dagger{}_{iE'} T_{++}(x_{L'}{}^+) \hat{M}_{iE'} \left| 0_{in} \right\rangle = \left\langle 0_{in} \right| T_{++}(x_{L'}{}^+) \hat{M}^\dagger{}_{iE'} \hat{M}_{iE'} \left| 0_{in} \right\rangle$$

The two-point correlation functions for non-singular measurements simply decay via power law as a function of the distance. ***⇒ No Firewalls!***

$$\frac{\left\langle 0_{in}\middle|T_{++}(x^{+})\left(\hat{M}^{\dagger}{}_{iE'}\hat{M}_{iE'}\right)\middle|0_{in}\right\rangle}{\left\langle 0_{in}\middle|\left(\hat{M}^{\dagger}{}_{iE'}\hat{M}_{iE'}\right)\middle|0_{in}\right\rangle} = E_{fw}\delta\left(x^{+} - g_{h}(x_{fw})\right) + \cdots$$



For any non-singular measurement,

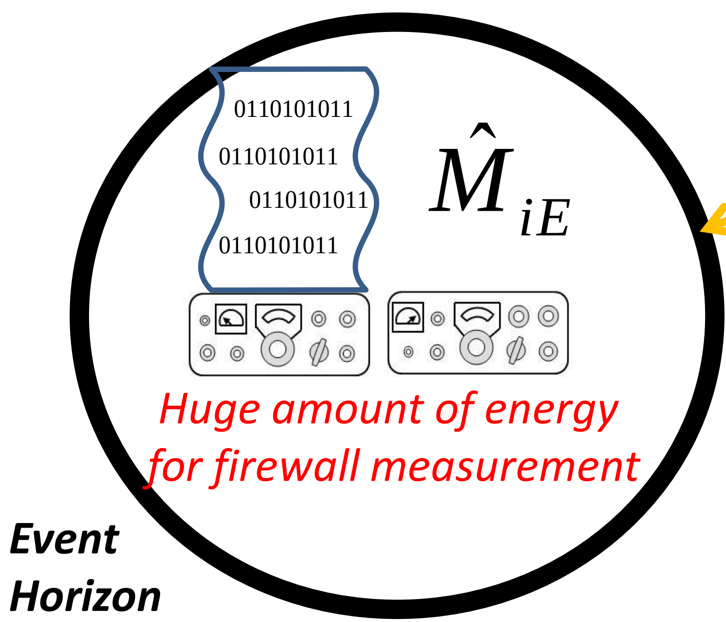
$$E_{fw} < \frac{1}{12\pi r l} << E_{planck}$$

$\Rightarrow$ ***No Firewall appears!***

$$r = \frac{\left\langle 0_{in}\middle|\hat{M}^{\dagger}{}_{iE'}\hat{M}_{iE'}\middle|0_{in}\right\rangle}{\left\langle 0_{in}\middle|\left(I - \hat{M}^{\dagger}{}_{iE'}\hat{M}_{iE'}\right)\middle|0_{in}\right\rangle} = O(1)$$

The local measurements generally inject energy on average to the system in $|0_{in}\rangle$ owing to its passivity property (Pusz and Woronowicz). Thus the measurements always require an energy cost. Though the Reeh-Schlieder theorem is mathematically correct, it does not guarantee that the measurement energy to create $|i_L\rangle$ is finite.

***Singular measurements*, which yield firewalls, generally require preparation of a divergent amount of energy in the measurement region before the measurement is performed and this energy is expected to provide a large back reaction to the spacetime. The effect may cause formation of a new black hole in the measurement region and enclose the measurement device within the event horizon before it outputs results.**



Black hole is formed in the measurement region during the preparation of huge energy for the firewall measurement

$\Rightarrow$ *Firewall Information Censorship*

(Hotta-Matsumoto-Funo,2013)

Summary

○ *Strong subadditivity paradox is a superficial one.*
If the models which allow correct continuum limit to low-energy field theory, typical-state condition does not hold.
If strict localization of subsystems (A,B and C), no-drama condition does not imply purity of BC system. Acutually, both A and zero-point fluctuation are entangled with the BC system.
No firewall is required by the entanglement monogamy argument.

○ *Reeh-Schlieder theorem rises a measurement-based firewall problem. However, the amount of measurement energy of firewalls becomes divergent. The effect may cause formation of a new black hole in the measurement region and enclose the measurement device within the event horizon before it outputs results.* ⇒ *Firewall Information Censorship*