Effects of Chern-Simon term on scalar,spin density in 3-d gauge theory

Thermal quantum field theory and its appllication,PanasonicYukawa Hall, August 31(2010)Yuichi Hoshino,Kushiro National College of Technology


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1 Thermal D-S equation

S-D equation for fermion mass at finite T QED.Imaginary part of the propagator.

$$M(p_0, |\mathbf{p}|) = -\mathbf{e}^2 \int \frac{d^4q}{(2\pi)^4} M(q_0, |\mathbf{q}|)$$

$$\times (P \frac{1}{q^2 - M^2(q_0, |\mathbf{q}|)} \Re(2D^T(p-q) + D^L(p-q))$$
$$\coth(\frac{\beta|p_0 - q_0|}{2})$$

$$+\Im(2D^T(p-q)+D^L(p-q))\delta(q^2-M^2(q_0,|\mathbf{q}|)\tanh(\frac{\beta|q_0|}{2})). \quad (1)$$

1st term:on-shell photon in the lowest approximation, infrared divergent.2nd term:on-shell fermion but mass is not fixed-> mass changing effect.by Hiroshima,Nara Group.First term drops in instantaneous approximation.

2 spectral function in 3-d QED

1 Old attemt by Bloch-Nordsieck near $p^2 = m^2$,

$$S_F(p) \simeq \frac{\gamma \cdot p + m}{m^2(1 - p^2/m^2)^{1-D}}, D = \frac{\alpha(d-3)}{2\pi}, \alpha = \frac{e^2}{4\pi}.$$

2 3-dim-th suggests validity in whole region.we evaluate $F(x)$

$$S'_F(x) = S^0_F(x) \exp(F). \tag{2}$$

3 QED: infrared divergences with massless photon determine anomalous dimension as KT as $\ln(\mu|x|)$.

4 TMG:Chern-Simon and Instanton effects modify aomalous dimension.

5 $\left\langle \overline{\psi}\psi \right\rangle_{\pm} \propto \theta$ or ∞ for 2 and 4-spinor,$Z_2^{-1} = 0$.

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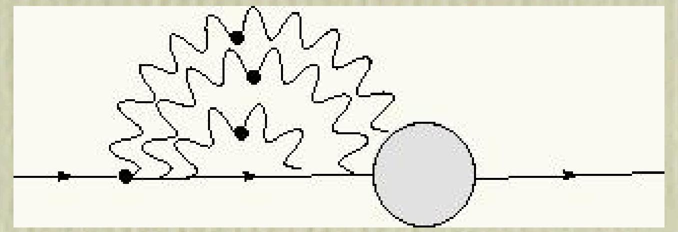


2.1 Soft-photon summation

Photon attached with external line is most singular by low-energy theorem

$$T_1 = -ie\frac{(r+k)\cdot\gamma + m}{(r+k)^2 - m^2}\gamma_\mu \epsilon^\mu(k,\lambda) \times \exp(i(k+r)\cdot x)U(r,s). \quad (3)$$

$O(e^2)$ spectral function F is given

$$F = \int \frac{d^3k}{(2\pi)^2}\delta(k^2)\theta(k_0)\exp(ik\cdot x)\sum_{\lambda,S} T_1\overline{T_1}. \quad (4)$$

Model independent form

$$\sum_{\lambda,s} T_1\overline{T}_1 = -e^2(\frac{\gamma\cdot r}{m} + 1)[\frac{m^2}{(r\cdot k)^2} + \frac{1}{(r\cdot k)} + \frac{d-1}{k^2}].$$

2.2 Evaluation of F

μ :infrared cut-off.

$$D_F^{(0)}(x)_+ = \int \frac{d^3k}{i(2\pi)^2}\delta(k^2-\mu^2)\theta(k^0)\,\mathrm{exp}(ik\cdot x) = \frac{\mathrm{exp}(-\mu\,|x|)}{8\pi i\,|x|}, \tag{5}$$

$$F = ie^2m^2\int_0^\infty \alpha d\alpha D_F(x+\alpha r) - e^2\int_0^\infty d\alpha D_F(x+\alpha r) - ie^2(d-1)\frac{\partial}{\partial\mu^2}D_F(x). \tag{6}$$

In quenced case for finite μ, F is written as

$$F = -\frac{e^2}{8\pi}\left(\frac{\mathrm{exp}(-\mu\,|x|)}{\mu} - |x|\,\mathrm{Ei}(\mu\,|x|)\right) - \frac{e^2}{8\pi\sqrt{r^2}}\,\mathrm{Ei}(\mu\,|x|) -(d-1)\frac{e^2}{16\pi\mu}\,\mathrm{exp}(-\mu\,|x|), r^2=m^2, \tag{7}$$

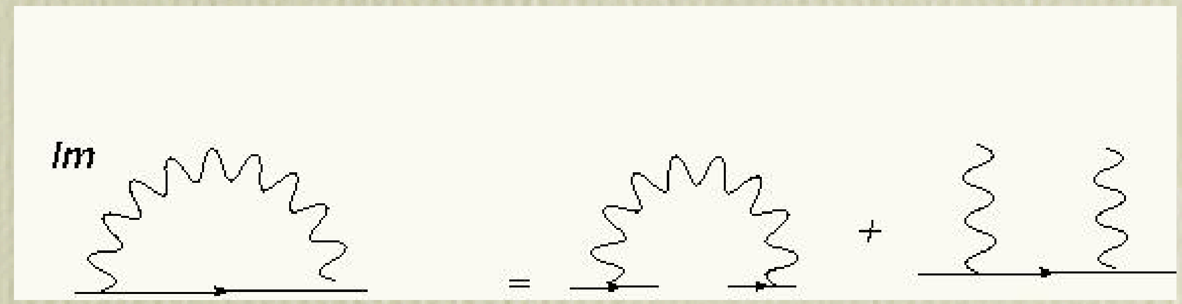
where

$$\mathrm{Ei}(z) = \int_1^\infty \frac{\exp(-zt)}{t} dt, \quad (8)$$

$$\mathrm{Ei}(\mu\,|x|) = -\gamma - \ln(\mu\,|x|) + (\mu\,|x|) + O(\mu^2). \quad (9)$$

For the leading order in μ we have at short distance

$$F = \frac{(1+d)e^2}{16\pi\mu} + \frac{e^2\gamma}{8\pi m} + \frac{e^2}{8\pi m}\ln(\mu\,|x|) + \frac{e^2}{8\pi}\,|x|\ln(\mu\,|x|) - \frac{e^2}{16\pi}\,|x|\,(d+1-2\gamma). \quad (10)$$



where γ is Euler's constant and m is a physical mass.

$$m\bar{\rho}(x) = \frac{m\exp(-m\,|x|)}{4\pi\,|x|}\exp(F) \quad (11)$$

There is mass shift and its log correction

$$\Delta m|x| = \frac{e^2}{8\pi}|x| \ln(\mu\,|x|) - \frac{e^2}{16\pi}|x|(d + 1 - 2\gamma). \quad (12)$$

exp(F) is parametrized in the following form

$$\exp(F) = A(\mu\,|x|)^{D+C|x|}, \quad (13)$$

$$A = \exp(\frac{\gamma e^2}{8\pi m} + \frac{e^2}{16\pi\mu}(d+1)),$$

$$D = \frac{e^2}{8\pi m}, C = \frac{e^2}{8\pi}.\text{Gauge invariant.} \quad (14)$$

F **acts to change power of** $|x|$ **and mass.If** $D = 1, S_F(0) = finite. \left\langle \overline{\psi}\psi \right\rangle \propto \mu$.Unquenched case.Using massive fermion vacuum polarization.

$$\exp(F(x,\mu) \to \int \rho_\gamma(s) \exp(F(x,\sqrt{s})ds, \quad (15)$$

$$\rho_\gamma(s) = \delta(s) + \frac{1}{\pi}\,\mathrm{Im}\,\frac{1}{-s + \Pi(s)}. \quad (16)$$

Good agreement with S-D equation with vertex correction and vacuum polarization.

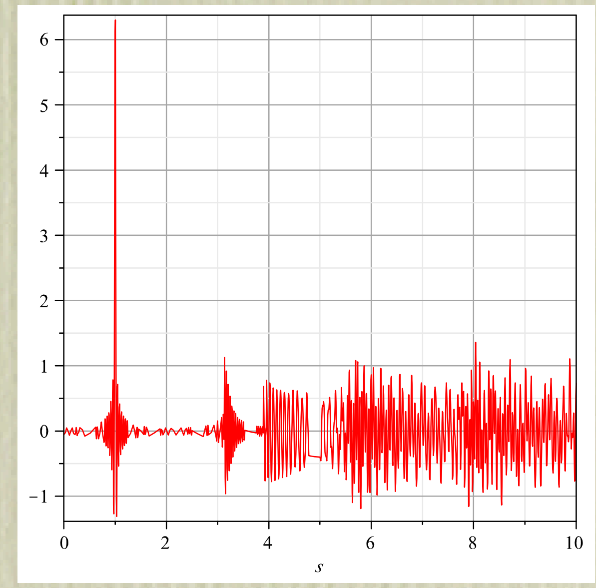
3 Minkowski region

$$p^2/m^2 = s, \rho(s) = \frac{1}{2\pi m^2}\int_{-\infty}^{\infty} dx e^{-i(s-1)x} \exp(\widetilde{F}(\frac{x}{m^2})) \tag{17}$$

There is an infrared cut-off effect.

$$\int_0^{1/\mu} \frac{dx}{\pi}(\mu x)\cos(sx) = \frac{-\mu + \sqrt{s^2+\mu^2}\cos(s/\mu+\delta)}{s^2}, \tag{18}$$

where $\tan\delta = \mu/s$.



$\rho(s)$ for $N = 1$ in unit e^2

4 Renormalization constant

In the beginnings we assume the asymptotic field ;$\psi(x)_{t\to+\infty,-\infty}$ $\sqrt{Z_2}\psi(x)_{out,in}$.If we assume spectral function,we have

$$
\begin{aligned}
S(p) &= \int ds \frac{\rho_1(s)p\cdot\gamma+\rho_2(s)}{p^2+s}, \\
m_0 Z_2^{-1} &= \lim_{p\to\infty} tr(p^2 S(p)) = \int ds\rho_2(s) = 0, \\
Z_2^{-1} &= \lim_{p\to\infty} tr(\gamma\cdot pS(p)) = \int ds\rho_1(s) = 0,
\end{aligned}
\tag{19}
$$

provided

$$S(p)_{p\to\infty} \propto \frac{1}{p^4}. \tag{20}$$

This may show that non-pole contribution is not positive for spectral function in the case of comoposite operator insertion by K.Nishijima[11].

5 Chern-Simon QED,QCD

$$
\begin{aligned}
L &= \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}\theta\epsilon^{\mu\nu\rho}F_{\mu\nu}A_\rho + \overline{\psi}(i\gamma\cdot(\partial - ieA) - m)\psi \\
&\quad + \frac{1}{2d}(\partial\cdot A)^2, \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
L &= \frac{1}{4g^2}tr(F_{\mu\nu}F^{\mu\nu}) - \frac{\theta}{4g^2}\epsilon^{\mu\nu\rho}tr(F_{\mu\nu}A_\rho - \frac{2}{3}A_\mu A_\nu A\rho) \\
&\quad + \overline{\psi}(i\gamma\cdot(\partial - ieA) - m)\psi + i\partial^\mu\overline{C}\cdot D_\mu C \\
&\quad + \frac{1}{2g^2d}(\partial\cdot A)^2 \qquad (22)
\end{aligned}
$$

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}, g_{\mu\nu} = diag(1, -1, -1)$$

$$
\begin{aligned}
\gamma_0 &\equiv \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, \gamma_{1,2} \equiv -i\begin{pmatrix} \sigma_{1,2} & 0 \\ 0 & -\sigma_{1,2} \end{pmatrix}, \\
\gamma_4 &\equiv \gamma^4 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \gamma_5 \equiv \gamma_0\gamma_1\gamma_2\gamma_3 = \begin{pmatrix} 0 & -iI \\ iI & 0 \end{pmatrix}, \\
\gamma_{45} &= \gamma^{45} = -i\gamma_4\gamma_5, \gamma_{\mu 4} = i\gamma_\mu\gamma_4, \gamma_{\mu 5} = i\gamma_\mu\gamma_5. \qquad (23)
\end{aligned}
$$

There are two redundant matrices which anticommutes with other three γ matrices.

There exists two kinds of chiral transformation $\psi \to \exp(i\alpha\gamma_4)\psi, \psi \to \exp(\alpha\gamma_5)\psi$,

for massless theory invariant.$U(2)$ symmetry is generated by $\{I_4, \gamma_4, \gamma_5, \gamma_{45}\}$.

Mass term breaks $\{\gamma_4, \gamma_5\}$ symmetry down to

$U(1) \times U(1)$ generated by $\{I_4, \gamma_{45}\}$.

$$\tau \equiv \gamma_{45} = \frac{i}{2}[\gamma_4, \gamma_5] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \tau_\pm = \frac{1 \pm \tau}{2}. \tag{24}$$

Ordinary mass $m_e\overline{\psi}\psi$ breaks chiral symmetry.Parity violating mass $m_o\overline{\psi}\tau\psi$ is parity odd but singlet under chiral transformation.Parity transform is $x' = (x^0, -x^1, x^2)$.Here $\overline{\psi}\tau\psi$ is a spin density.

$$\psi^+\frac{i}{2}[\gamma_1, \gamma_2]\psi = \overline{\psi}\tau\psi = n_\uparrow(x) - n_\downarrow(x). \tag{25}$$

Chiral representation with mass $m_\pm = m_e \pm m_o$

$$S_F(p) = \frac{1}{m_\epsilon I + m_O\tau - \gamma \cdot p} = \frac{(\gamma \cdot p + m_+)\tau_+}{p^2 - m_+^2 + i\epsilon} + \frac{(\gamma \cdot p + m_-)\tau_-}{p^2 - m_-^2 + i\epsilon} \quad (26)$$

5.0.1 two-component spinor (τ_+)

quenched case θ is an intrinsic cut-off and we have no infrared divergences.θ is assumed to modify the anomalous dimension of fermion.

$$D_F^0(k) = -i(\frac{g_{\mu\nu} - k_\mu k_\nu/k^2 - i\theta\epsilon_{\mu\nu\rho}/k^2}{k^2 - \theta^2 + i\epsilon}) - id\frac{k_\mu k_\nu}{k^4}, \quad (27)$$

Converting partial fraction

$$\rho^e(s) = \delta(s - \theta^2), \rho^O(s) = \frac{1}{\theta}[(\delta(s - \theta^2) - \delta(s)]. \quad (28)$$

$$\sum_{\lambda,S} T_1 \overline{T}_1 = \frac{-e^2(\gamma\cdot p+m)}{2m}[\frac{m^2}{(p\cdot k)^2}+\frac{1}{p\cdot k}+\frac{(d-1)}{k^2}] - \frac{\gamma\cdot p}{m}\frac{e^2}{4\theta}\frac{m\tau}{p\cdot k}. \quad (29)$$

In $O(e^2)$ we have $1/p\cdot k$

$$. - \left\langle \frac{1}{p\cdot k} \right\rangle \sim \frac{\gamma + \ln(\theta|x|)}{8\pi m}(\theta|x| \ll 1). \quad (30)$$

As in QED_3 we set anomalous dimension $D \geq 1$ for finite vacuum expectation value

$$D = \frac{e^2}{8\pi m}+\frac{e^2}{32\pi\theta} \geq 1, m = e^2/8\pi/(1-\frac{e^2}{32\pi\theta}), \quad (31)$$

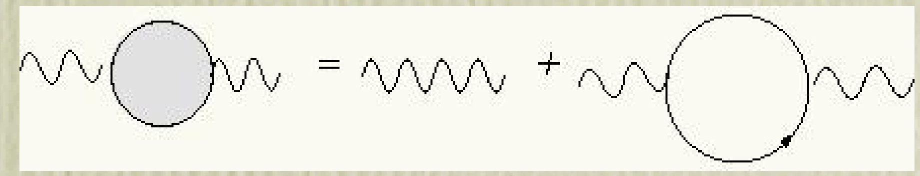
which may be consistent with the Laddar Schwinger-Dyson eq.by T.Matsuyama & H.Nagahiro.It has been shown that $\left\langle \overline{\psi}\psi \right\rangle \propto \theta$ in [6].θ dependence of mass m may be small.For infinitesimal θ linear approximation holds,we have $m = e^2/8\pi$ as in QED.

poster

In QCD_3 instanton effects is

$$\begin{aligned} \theta &= ng^2/4\pi, n(0, \pm 1, \pm 2, ...), \\ D &= \frac{e^2}{8\pi m} + \frac{1}{8n} \geq 1, m = e^2/8\pi/(1 - \frac{1}{8n}), \\ n &\neq 0, \left\langle \overline{\psi}\psi \right\rangle = finite. \end{aligned} \quad (32)$$

unquenched case



This approximation holds for $\theta \geq 2m$.Including vacuum polarization function that is written by parity even and odd piece

$$\Pi_{\mu\nu}(k) = (g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})\Pi^e(k) + i\theta\epsilon_{\mu\nu\rho}k_\rho\Pi^O(k). \quad (33)$$

The exact propagator is given by

$$\begin{aligned} D_{\mu\nu}(k) &= (g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} - i\mathcal{M}(k)\frac{\epsilon_{\mu\nu\rho}k_\rho}{k^2}) \\ &\quad \times \frac{-i}{Z(k)(k^2 - M_R^2(k))}, \end{aligned} \tag{34}$$

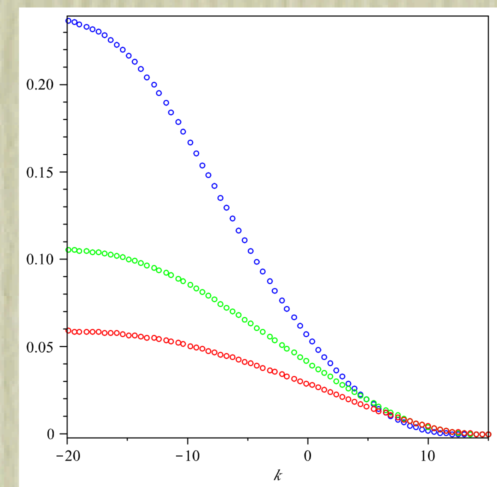
$$Z(k) = 1 - \Pi^e(k)/k^2, \tag{35}$$

$$M_R(k) = \frac{\theta(1 - \Pi^O(k)/k^2)}{1 - \Pi^e(k)/k^2}. \tag{36}$$

Property of $M_R(p)$.Not a pole but zeromomentum mass.In the weak coupling limit $M_R(p) = \theta$ is a pole.the Following Coleman's theorem for Chern-Simon QED and Toplogical Ward-Identity for Topological QCD,$M_R(0) = g^2 n/4\pi$,we may use renormalized parameter $M_R(0)$ in place of θ.Then we have a spectral function for unquenched case

$$\rho(s) = \frac{1}{\pi}\Im\frac{-1}{Z(s)(s - M_R^2(s))}. \tag{37}$$

$$\exp(\tilde{F}(x)) = \int ds\rho(s)\exp(F(x,s)). \tag{38}$$



$4S_F(x)$ for $N = 1\tilde{}3, D = 1$ in unit of e^2.

Thus the phase structures are the same with that of quenched case. C-S QED:large N,only broken phase.

Topologically massive QCD: large N_c,only broken phase.

R.Pisarsky& S.Rao discussed the infrared behaviour of Π at higher order and point out the above results in perturbative sense.

5.0.2 4-component spinor

It is easy to derive the anomalous dimension in the 2-component spinor case.

$$m_{\pm} = m_e \pm m_O \tag{39}$$

$$\begin{aligned}\sum_{\lambda,S} T_1 \overline{T}_1 &= \frac{-e^2(\gamma\cdot p + m_+)}{2m_+}[\frac{m_+^2}{(p\cdot k)^2} + \frac{1}{p\cdot k} + \frac{(d-1)}{k^2}] \\ &-\frac{\gamma\cdot p}{m_+}\frac{e^2}{4\theta}\frac{m_+\tau_+}{p\cdot k} + (m_+\tau_+ \to m_-\tau_-).\end{aligned} \tag{40}$$

$$\begin{aligned}S_F(p)_{p\to\infty} &\sim \frac{(\gamma\cdot p + m_+)\tau_+}{m_+^2(p^2/m_+^2 - 1 + i\epsilon)^{1+D_+}} \\ &+\frac{(\gamma\cdot p + m_-)\tau_-}{m_-^2(p^2/m_-^2 - 1 + i\epsilon)^{1+D_-}}\end{aligned} \tag{41}$$

There is no extra constraints for

$$D_{\pm} = \frac{e^2}{8\pi m_{\pm}} \pm \frac{e^2}{32\pi\theta}. \tag{42}$$

6 Summary

QED,$U(2)->U(1)\times U(1)$

C-S QED can be understood by our mehod.

2-spinor: $\left\langle\overline{\psi}\psi\right\rangle_+ = finite.$

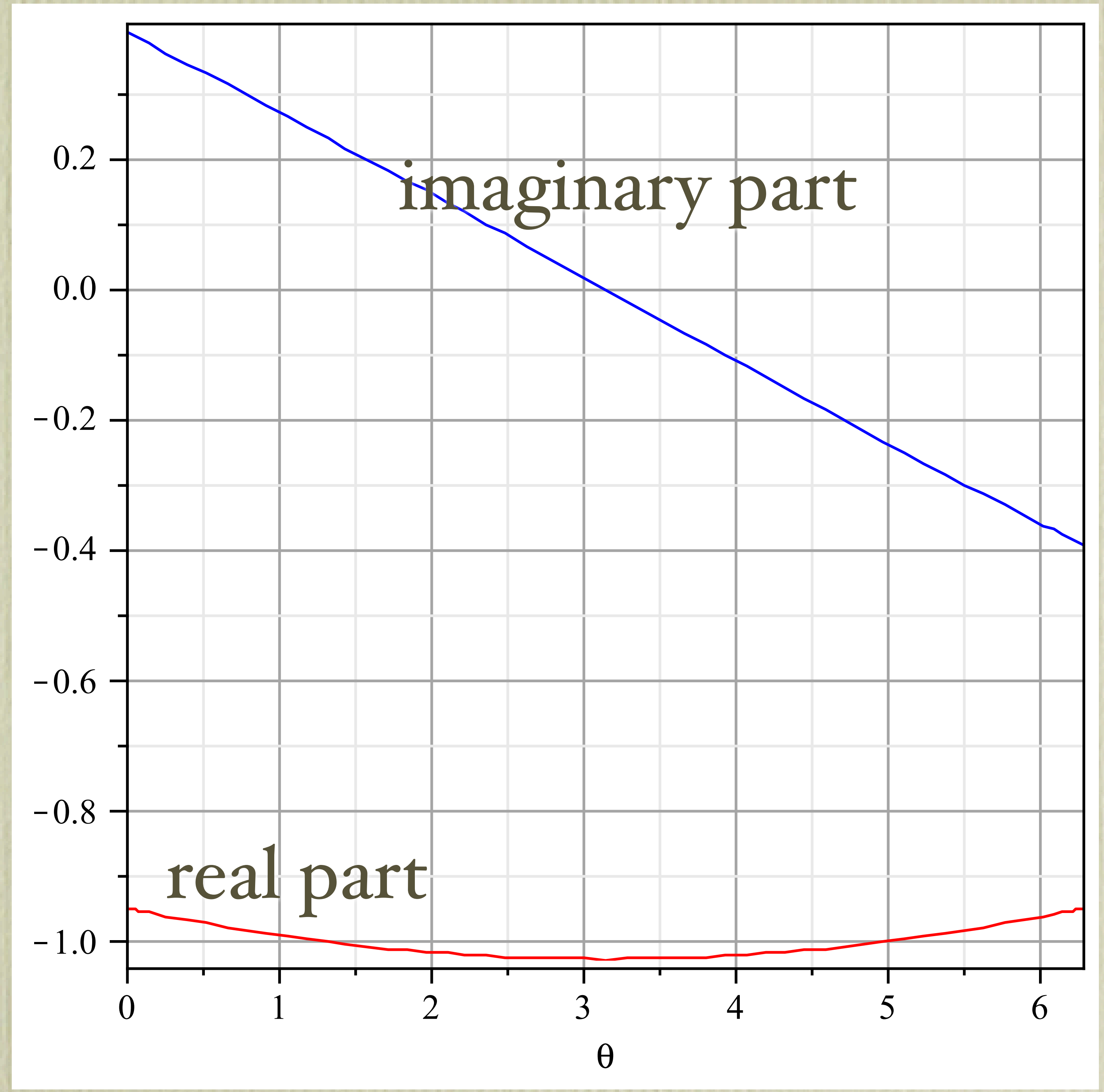
4-spinor: Both of $\left\langle\overline{\psi}\psi\right\rangle_\pm$ can be finite ?.

What can be done for Topologically massive QCD ?

Summation for n.

$$\sum_{n=-\infty}^{\infty} \exp(in\theta)$$

Mass (theta)



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