(Towards) Spectral functions from 2PI effective action

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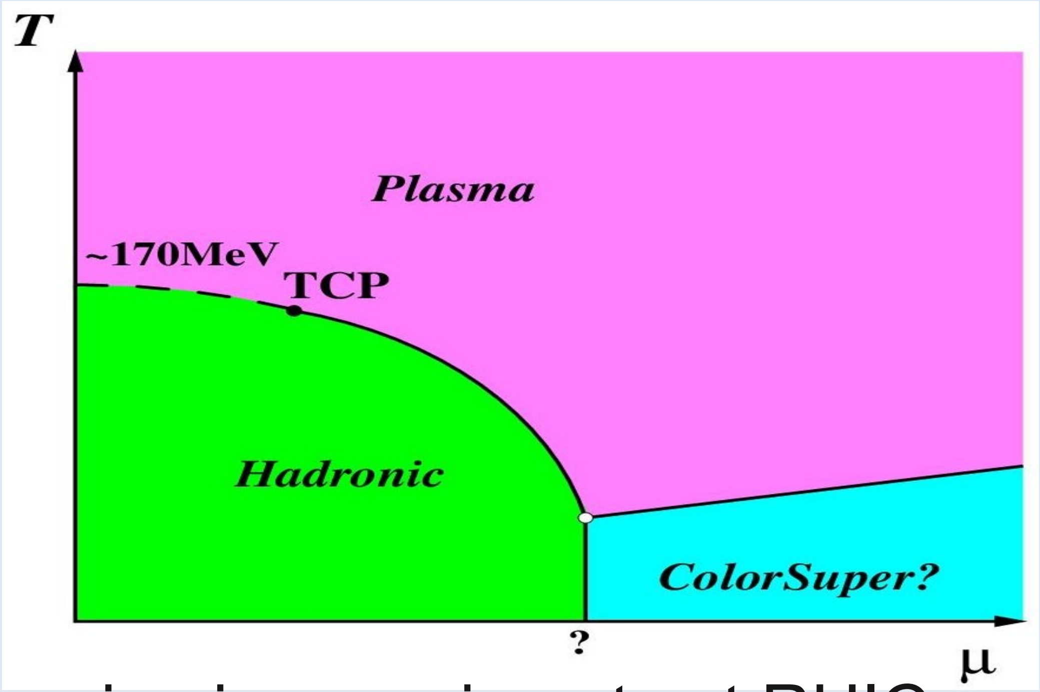
- Motivation – QCD critical point
- 2PI effective action at a critical point
- Application to NJL model
- Outlook

Motivation – QCD critical point

- Schematic phase diagram of QCD from chiral models
  - Phase boundary btw different symmetry realizations
  - Tri-critical pt in chiral limit → the **QCD CP** at nonzero *m*
  - Simple models and big extrapolation → large quantitative uncertainties



- Search for a critical point ongoing in experiments at RHIC

Motivation – QCD critical point

- Critical point at nonzero m was suggested by chiral models
  - NJL; Asakawa-Yazaki, QCD-like; Barducci et al., ChRM; Halasz et al.
- Serious discussions on exp'tal signatures by Stephanov-Rajagopal-Shuryak ('98, '99) – the cornerstone on Diagram
  - Diverging susceptibilities of baryon number, heat, ...
  - Microscopic calculation of suscept's – Hatta-Ikeda (03), …
  - Continued discussions on higher moments, Kurtosis, …
- Which mode is responsible for the QCD-CP? – softening 'hydro'
  - Microscopic calculation – HF-Ohtani (03, 04)
  - Macroscopic argument for 'model H' – HF-Ohtani, Son-Stephanov (04)
  - Mode coupling analysis is in parallel w/ liq-gas CP Minami-Kunihiro (11)

Motivation – QCD critical point

- Aim of this work
  - (Dynamic Universality is conjectured to be model H)
  - Derive the critical mode in a microscopic calculation beyond mean-field approximation, $\omega \sim k^z$
  - Compute the scaling property to check if it is model H

- This talk
  - shows a use of 2PI formalism at a critical point
  - presents the equations to be solved in NJL model

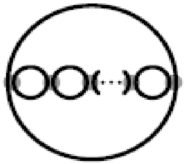
2PI effective action formalism

- Generally applicable for a system in and out-of equilibrium
- Here study O(N) ϕ4 theory at a critical point as an example

2PI formalism at a critical point

2PI effective action (EA) as a func of ϕ and ***G***

$$\Gamma_{2\mathrm{PI}}[\phi, G] = S[\phi] + \frac{1}{2}\mathrm{Tr}\ln G^{-1} + \frac{1}{2}\mathrm{Tr}G_0^{-1}G + \overline{\Gamma}_2[\phi, G]$$

- to NLO in 1/N, $\overline{\Gamma}_2^{\mathrm{LO}}[G] =$  $\overline{\Gamma}_2^{\mathrm{NLO}}[G] =$ 
- variation w.r.t. ***G*** gives a self-consistent (SD, KB) eqn

$$G_{ab}^{-1}(\boldsymbol{x}, \boldsymbol{y}) = G_{0,ab}^{-1}(\boldsymbol{x}, \boldsymbol{y}) - \Sigma_{ab}[\phi, G(\boldsymbol{x}, \boldsymbol{y})]$$

$$\Sigma_{ab}[\phi, G(x, y)] = -2\frac{\delta\overline{\Gamma}_2[\phi, G]}{\delta G_{ba}(y, x)}$$

- One can fix the exponent η at O(1/N), appliying the scaling form at the critical point (ϕ=0)

$$\tilde{G}(p) = \frac{1}{p^2}\left(\frac{p}{\Lambda}\right)^{\eta}$$

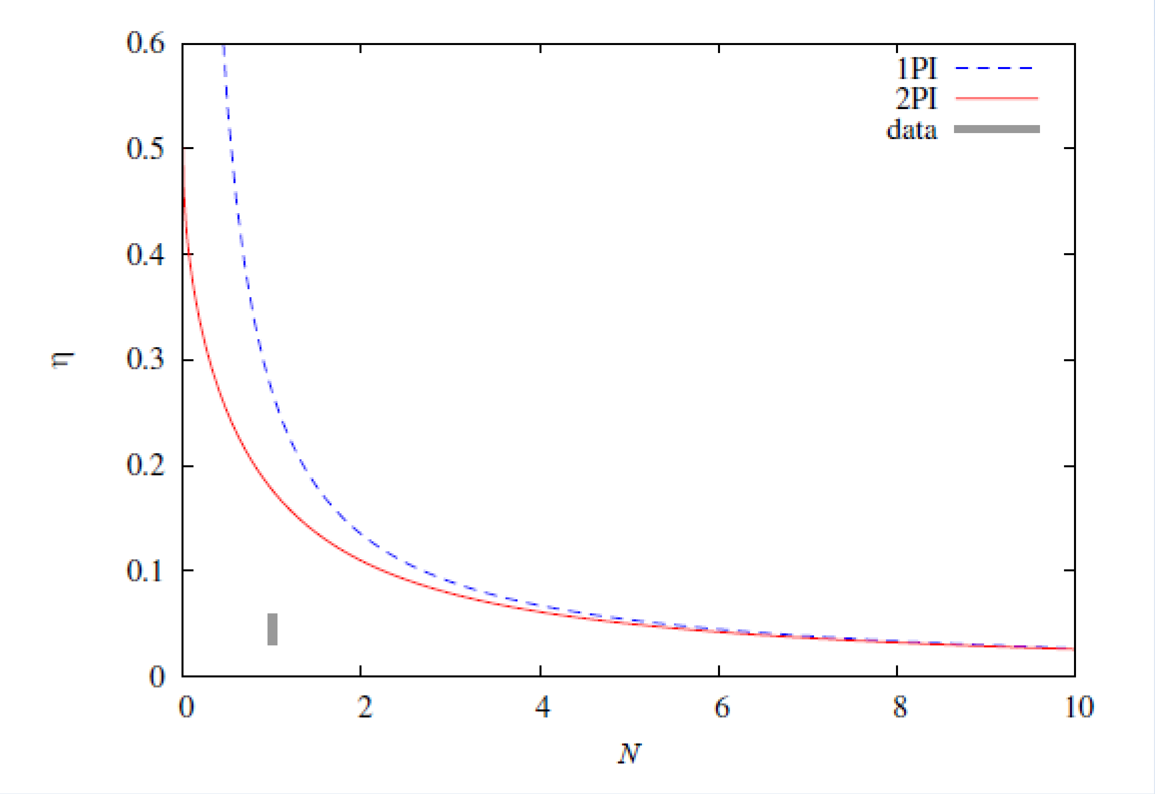
See also Basili'ev, and Gracey

2PI formalism at a critical point

Alford-Berges-Cheyne (04)

Exponent η for O(N) ϕ4 theory at O(1/N)

comparison between 1PI ($8/3\pi^2 N$, strict 1/N) and 2PI results



2PI formalism at a critical point

2PI effective action (EA) as a func of ϕ and ***G***

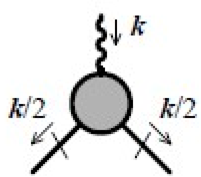
$$\Gamma_{2\mathrm{PI}}[\phi,G] = S[\phi] + \frac{1}{2}\mathrm{Tr}\ln G^{-1} + \frac{1}{2}\mathrm{Tr}G_0^{-1}G + \overline{\Gamma}_2[\phi,G]$$

- Differentiating the eqn for ***G*** w.r.t m^2 yields an eqn for $\Gamma^{(2,1)}$:

$$\Gamma^{(2,1)}(x,y;z) = \Gamma_0(x,y;z) + \int d^3x'd^3y' D(x,y;x',y')\Gamma^{(2,1)}(x',y';z)$$

- Pluging ***G*** into *D* and solving $\Gamma^{(2,1)}$ upto 1/N, we can fix the exponent ν at O(1/N)

$$\tilde{\Gamma}^{(2,1)}\left(\frac{k}{2},\frac{k}{2};k\right) =$$



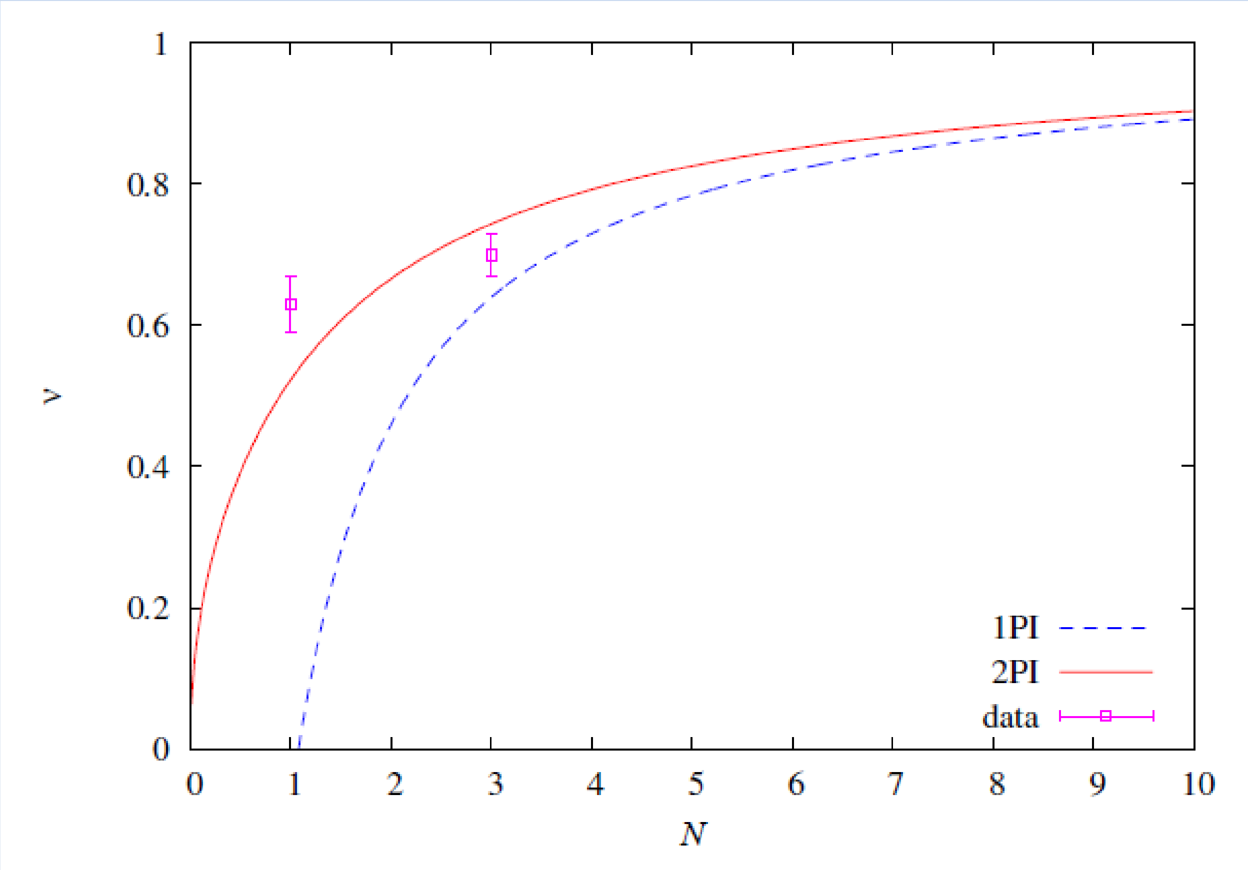
$$\sim k^{2-\eta-1/\nu}$$

2PI formalism at a critical point

Saito-HF-Itakura-Morimatsu (11)

Exponent ν for φ4 theory at O(1/N)

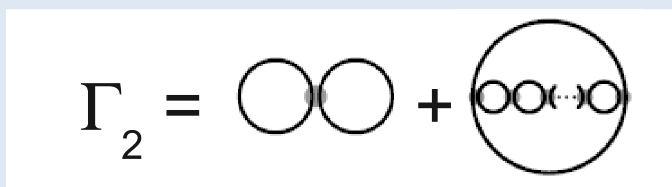
comparison between 1PI ($1-3\pi^2/32N$) and 2PI results



Lesson: 2PI EA is useful to get improved estimate for η,ν

2PI formalism for NJL

- Our interest is in dynamic modes near the QCD-CP
  - NJL – the same global symmetry as QCD
  - Several 1/N calculations done before (e.g., Oertel et al.)
  - use here a real-time path, which avoids analytic continuation (necessary if imaginaty-time used)



- Eq. of Motion

$$D^{R^{-1}}(\omega,k) = (i\not{k} - m) - \Sigma^R(\omega,k)$$

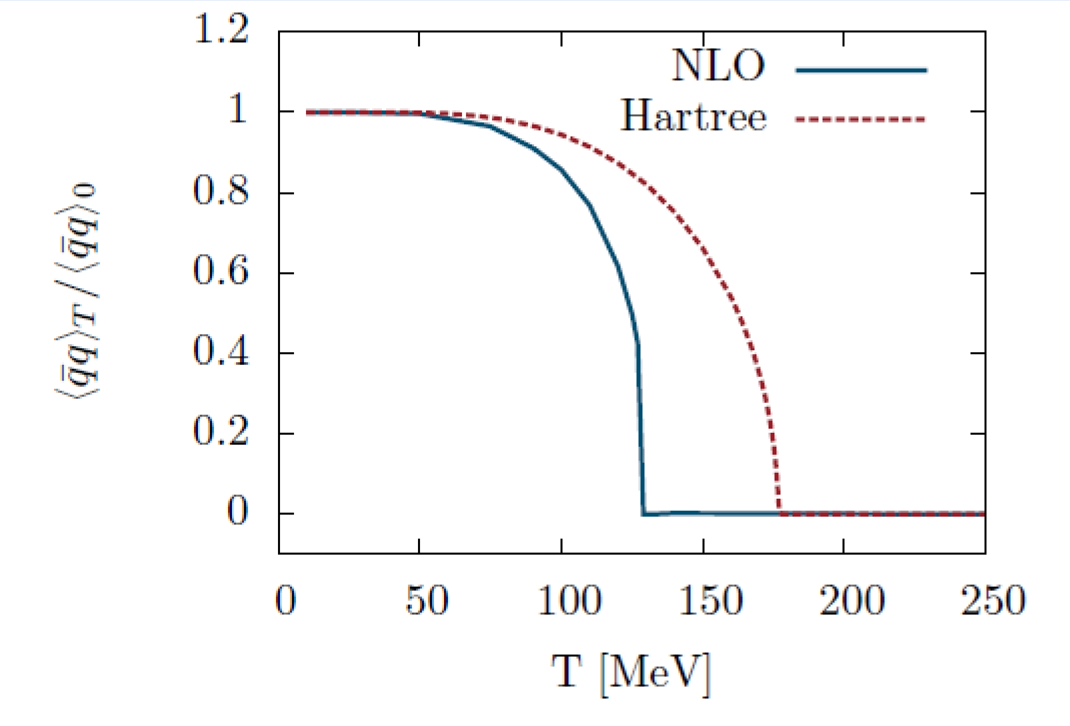
$$n^F(\omega)\Sigma_\rho(\omega) = -i\int \frac{d\omega'}{2\pi} n^F(\omega-\omega')n^B(\omega')I_\rho(\omega-\omega')\rho(\omega')$$

I is a chain of bubble diagrams

2PI formalism for NJL

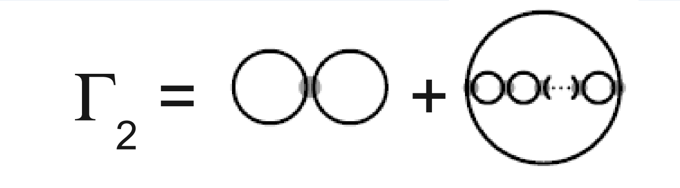
- The same approximation in imaginary time formalism, yields a second-order transition at finite T

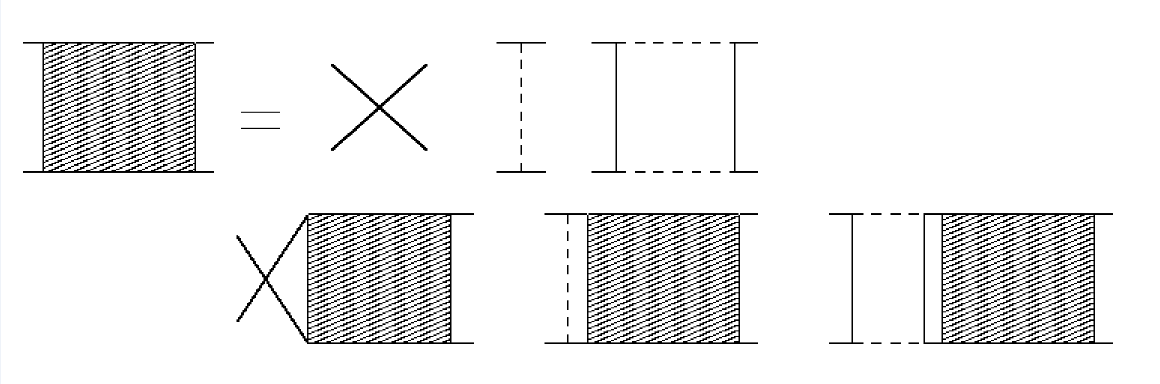
Muller et al., PRD81, ('10)



2PI formalism for NJL

- Bosonic modes are obtained as response to external fields $s(x)$, $p^a(x)$, $\mu(x)$, …, etc. – 2nd derivatives 2PI EA
- Should be consistent w/ the Goldstone theorem
- Kernel for BSE is obtained by cutting two lines in Γ_2





- Given this non-local kernel, it'd be tough to solve this BSE.

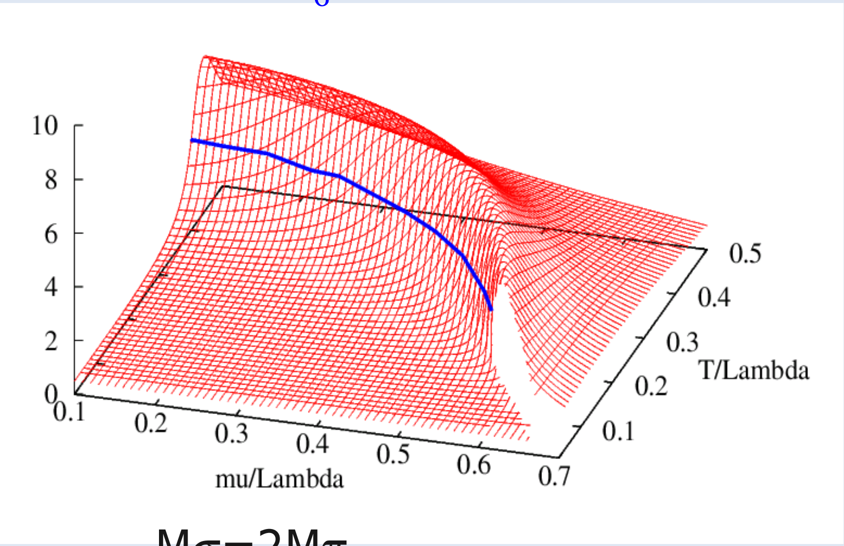
Outlook

- We explained importance to study the spectral function near the critical point
- We proposed the 2PI effective action approach to study the critical dynamics
  - It's useful to compute the static exponents
- Numerically tjough as it is, we plan to solve the BSE at small (ω,k) in order to study scaling property at critical points; $\omega \sim k^z$
- Future perspective: work towards link/foundation of dynamic universality from microscopic models/theories

Leading order calculation in NJL

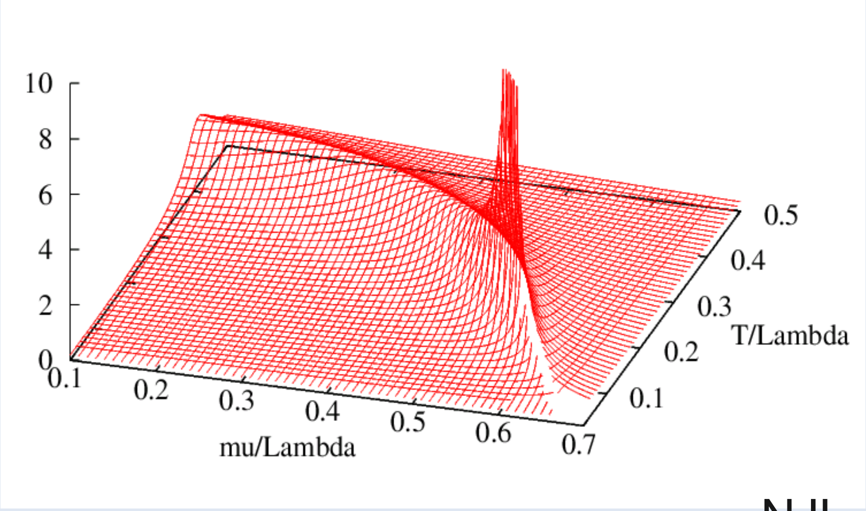
- Chiral phase boundary vs a critical point

$1/M_\sigma^2$ [a.u.]



Mσ=2Mπ

χ_{scalar} [a.u.]



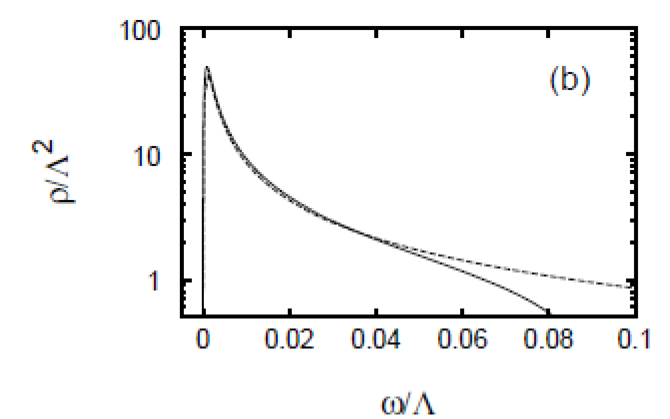
NJL

Leading order calculation in NJL

- Spectral function near the critical point
  - Sigma mode is massive
  - Critical mode appear in the space-like region = fluctuations of quark distribution function

$$\chi_{mm}(\omega,\mathbf{q}) \sim \frac{\mathrm{Re}\Pi_{mm}(0,\mathbf{q})}{-\mathrm{i}2g\mathrm{Im}\Pi_{mm}(\omega,\mathbf{q}) + (1-2g\mathrm{Re}\Pi_{mm}(0,\mathbf{q}))} = \frac{1}{-\mathrm{i}\frac{\omega}{\lambda(\mathbf{q})} + \chi^{-1}_{mm}(\mathbf{q})} = \frac{\lambda(\mathbf{q})}{-\mathrm{i}\omega + \omega_c(\mathbf{q})}$$

$$\omega_c(\mathbf{q}) = \chi^{-1}_{mm}(\mathbf{q})\lambda(\mathbf{q})$$



QCD 臨界点；*well-known (2)*

- 感受率の発散はどこからくるのか？
  - スカラー、ベクトルモードのスペクトル関数(NJL)
    - 空間的運動量領域に鋭いピーク