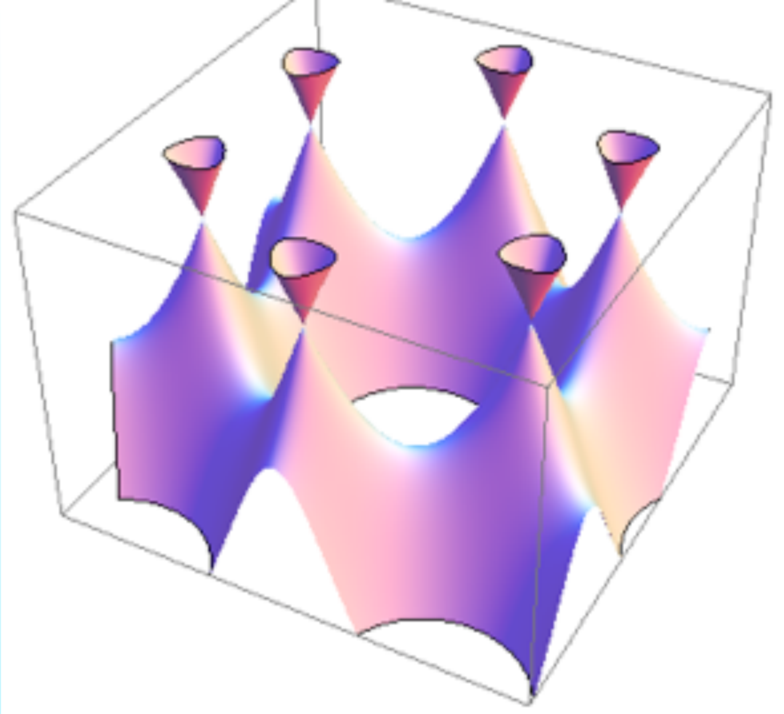
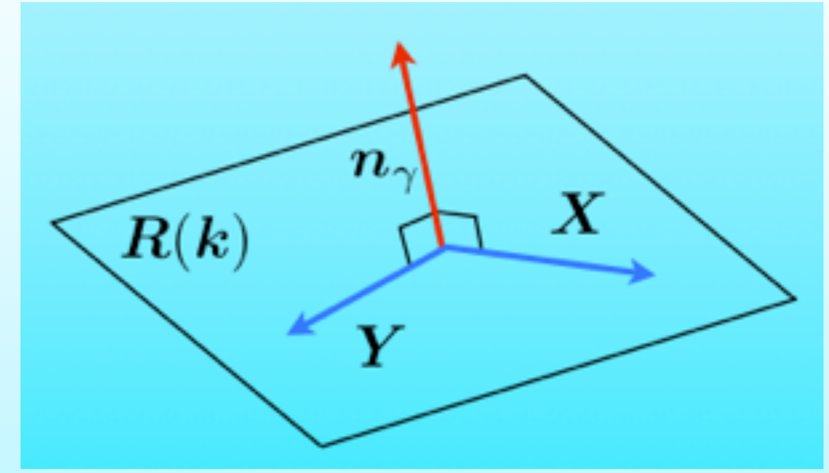
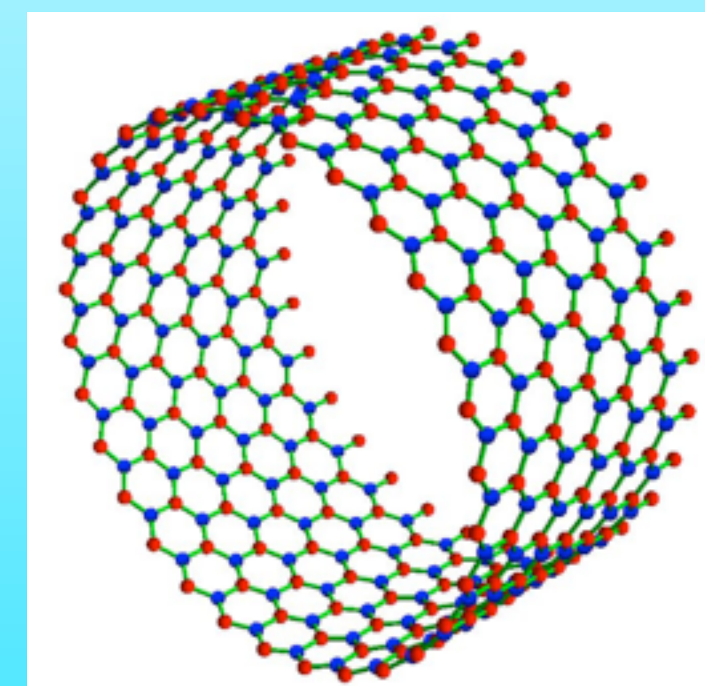
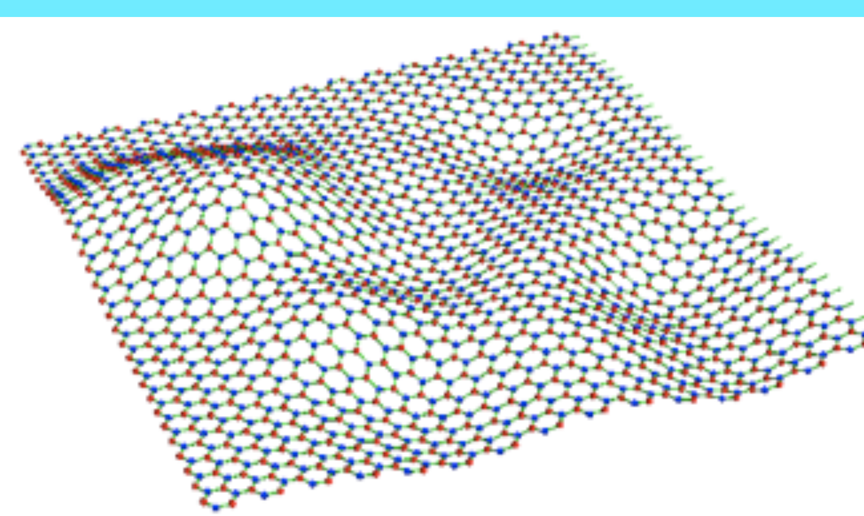




Anomalous properties of graphene & chiral symmetry

Institute of Physics
University of Tsukuba
JAPAN
Yasuhiro Hatsugai





$$\sigma_{xy} = \frac{e^2}{h}\frac{1}{2\pi i}\int \mathrm{Tr}\, dA$$

University of Tsukuba
筑波大学

Related works have been done and in progress with

H. Aoki, Univ. Tokyo
T. Kawarabayashi, Toho Univ.
Y. Hamamoto, Tsukuba-Univ.
T. Fukui, Ibaragi Univ.
M. Arikawa, Univ. Tsukuba
M. Arai, NIMS
T. Morimoto, Univ. Tokyo
H. Watanabe, Univ. Tokyo
S. Ryu, UC Berkeley
M. Kohmoto, Univ. of Tokyo
Y.S.Wu, Univ. Utah
X.G.Wen, MIT
Y. Morita, Gunma, Univ.

5 October 2010

Congratulation!

The Nobel Prize in Physics 2010

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics for 2010 to

Andre Geim and **Konstantin Novoselov**

University of Manc

More than new material

"for groun … l material graphene"

The Nobel Prize in Physics 2010
Andre Geim, Konstantin Novoselov

The Nobel Prize in Physics 2010

Andre Geim

Konstantin Novoselov

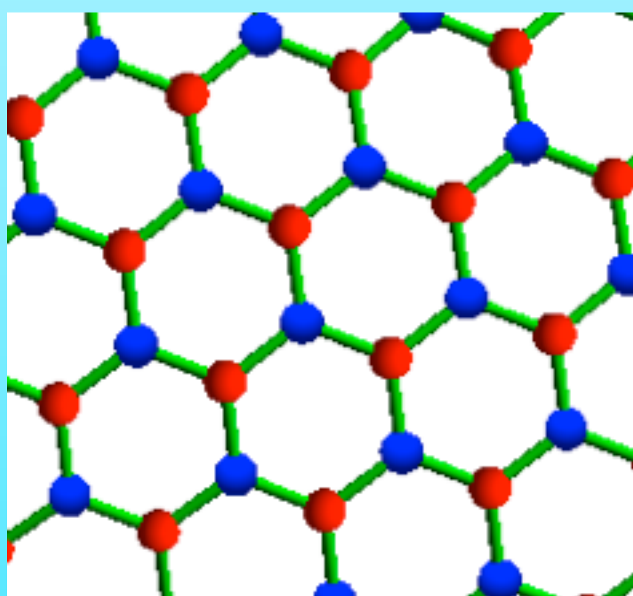
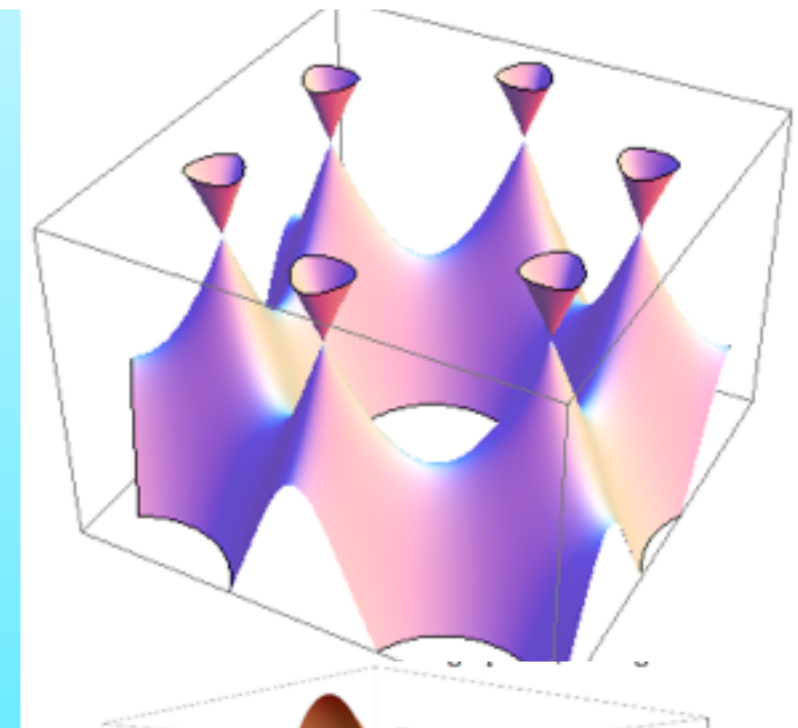
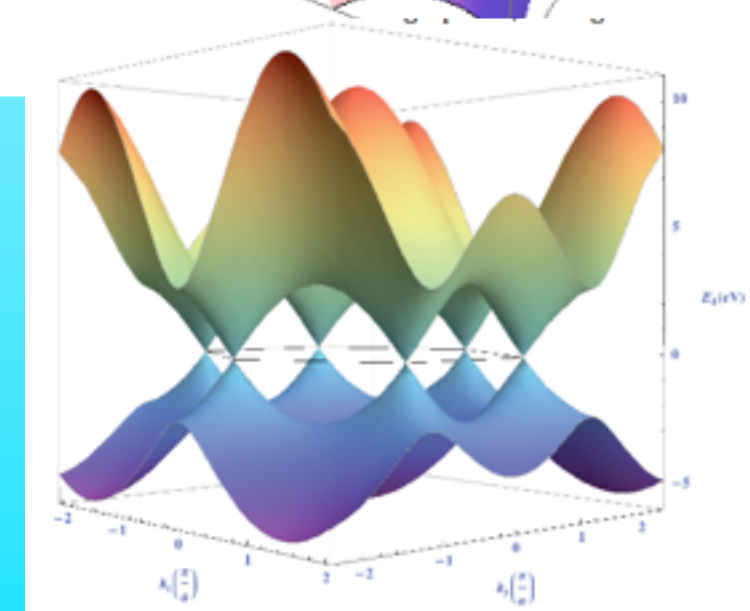


Photo: Sergeom, Wikimedia





Talk today :focus on chiral symmetry

Topological Stability of Massless Dirac Cones

- Effective mass approximation to the Dirac fermion
- Zero gap semiconductor with chiral symmetry
- Fermion doubling as of the 2D Nielsen-Ninomiya theorem

Topological aspects of graphene : Bulk

- Berry connection of the filled Dirac sea
- Lattice gauge fields in a parameter space

Topological aspects of graphene : Edge

- Zero modes at the zigzag & bearded edges
- Bulk-Edge correspondence for Dirac sea in a magnetic field
- Bulk–Edge correspondence for other phenomena

Chiral symmetry and particle-particle interaction

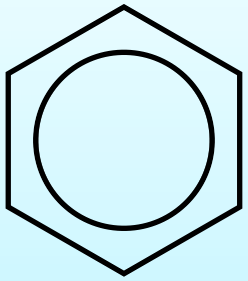
- Mean field approximation & bond ordering
- ...

Topological Stability of Massless Dirac Cones

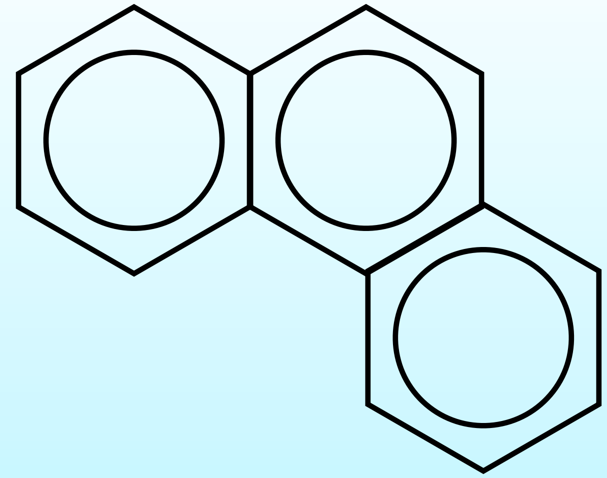
- Effective mass approximation to the Dirac fermion
- Zero gap semiconductor with chiral symmetry
- Fermion doubling as of the 2D Nielsen-Ninomiya theorem

Graphene??

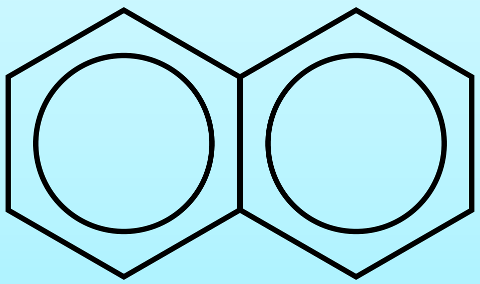
- π-electron systems



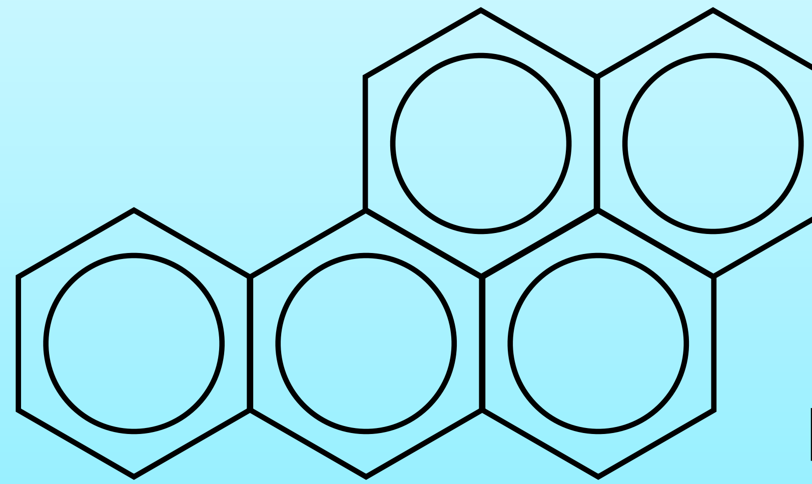
benzene



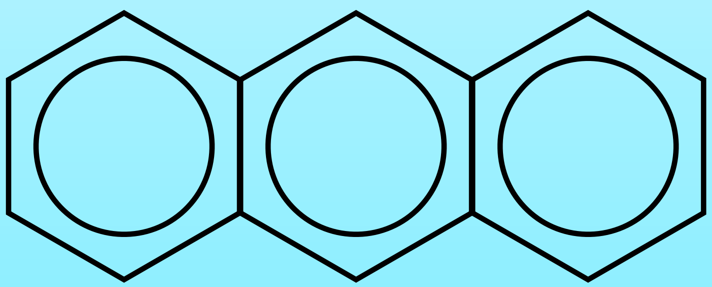
phenanthrene



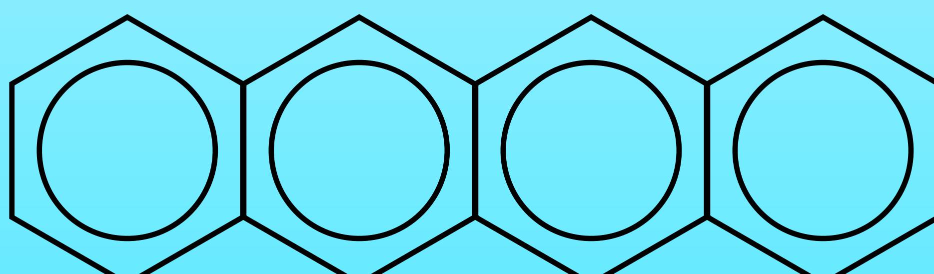
naphthalene



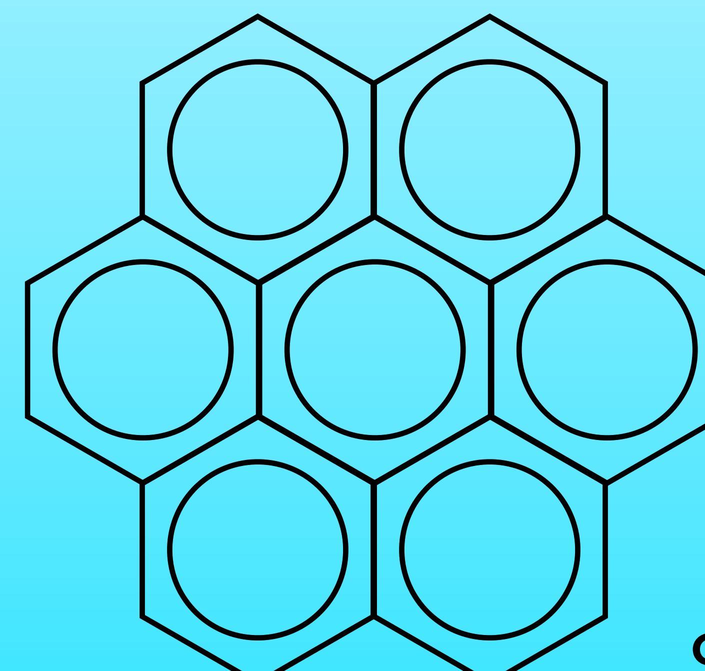
benzopyrene



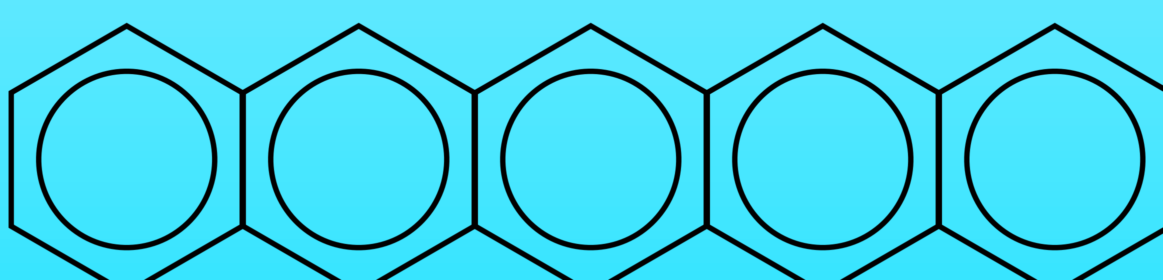
anthracene



tetracene



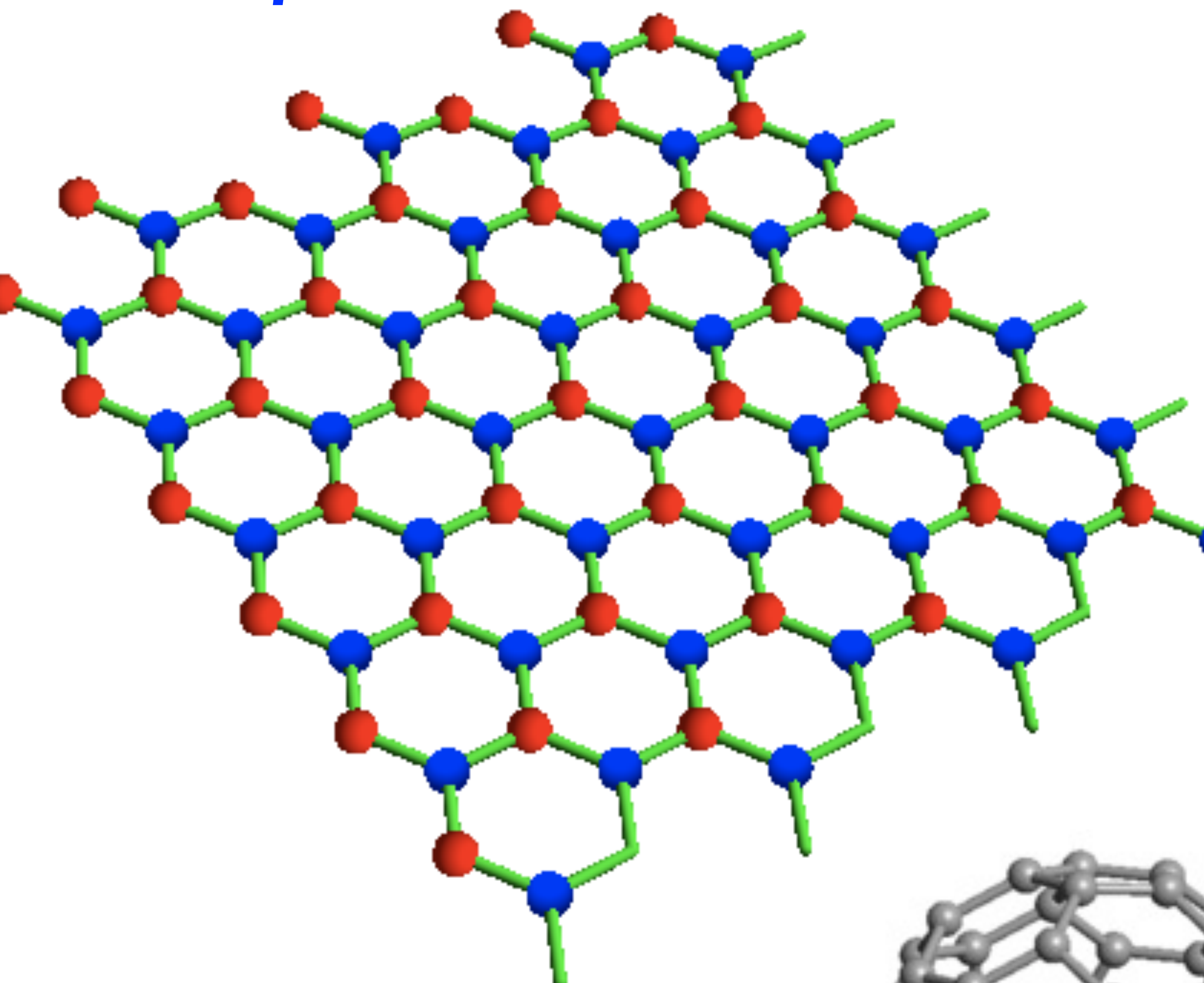
coronene



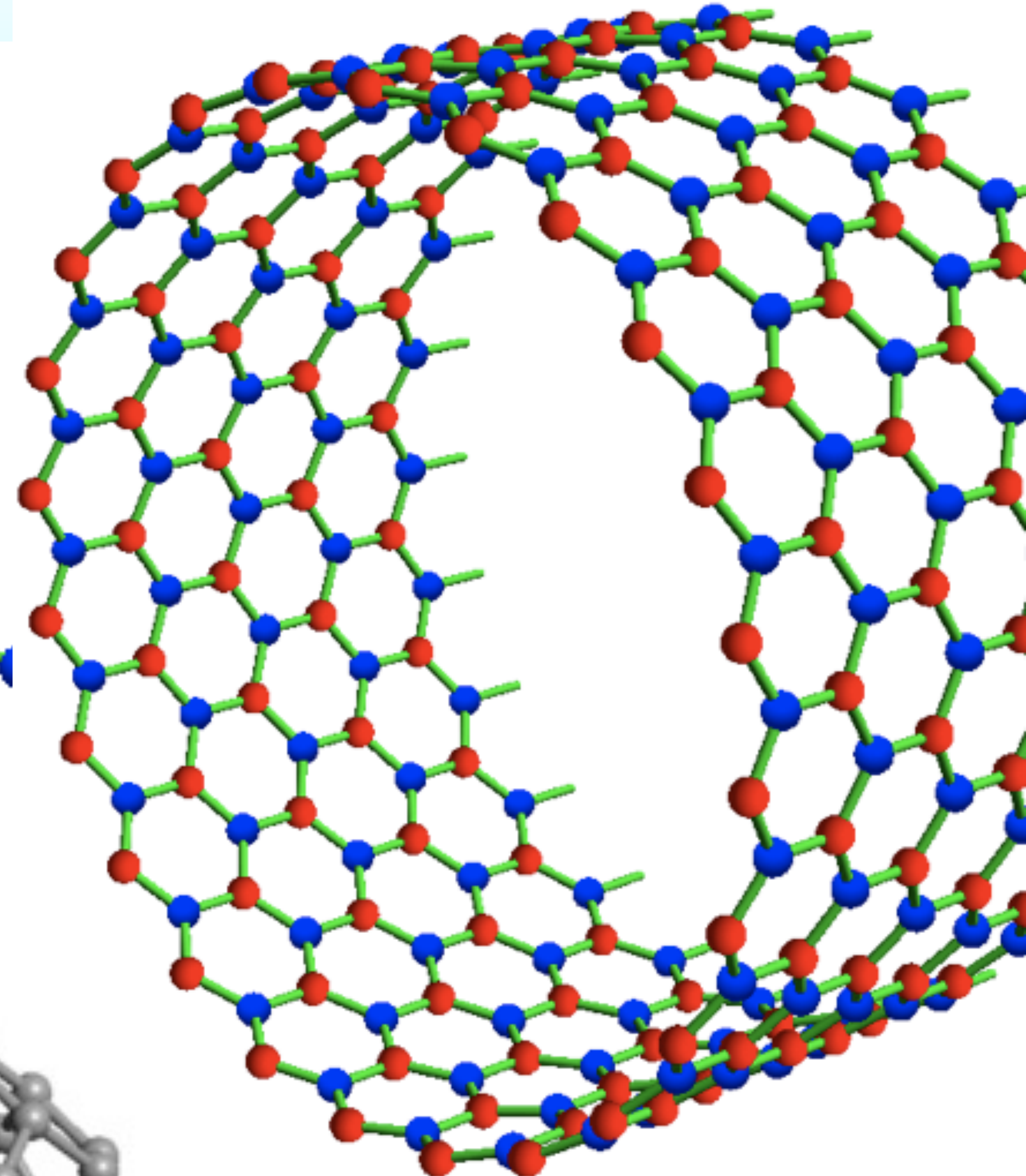
pentacene

Graphene??

Graphene



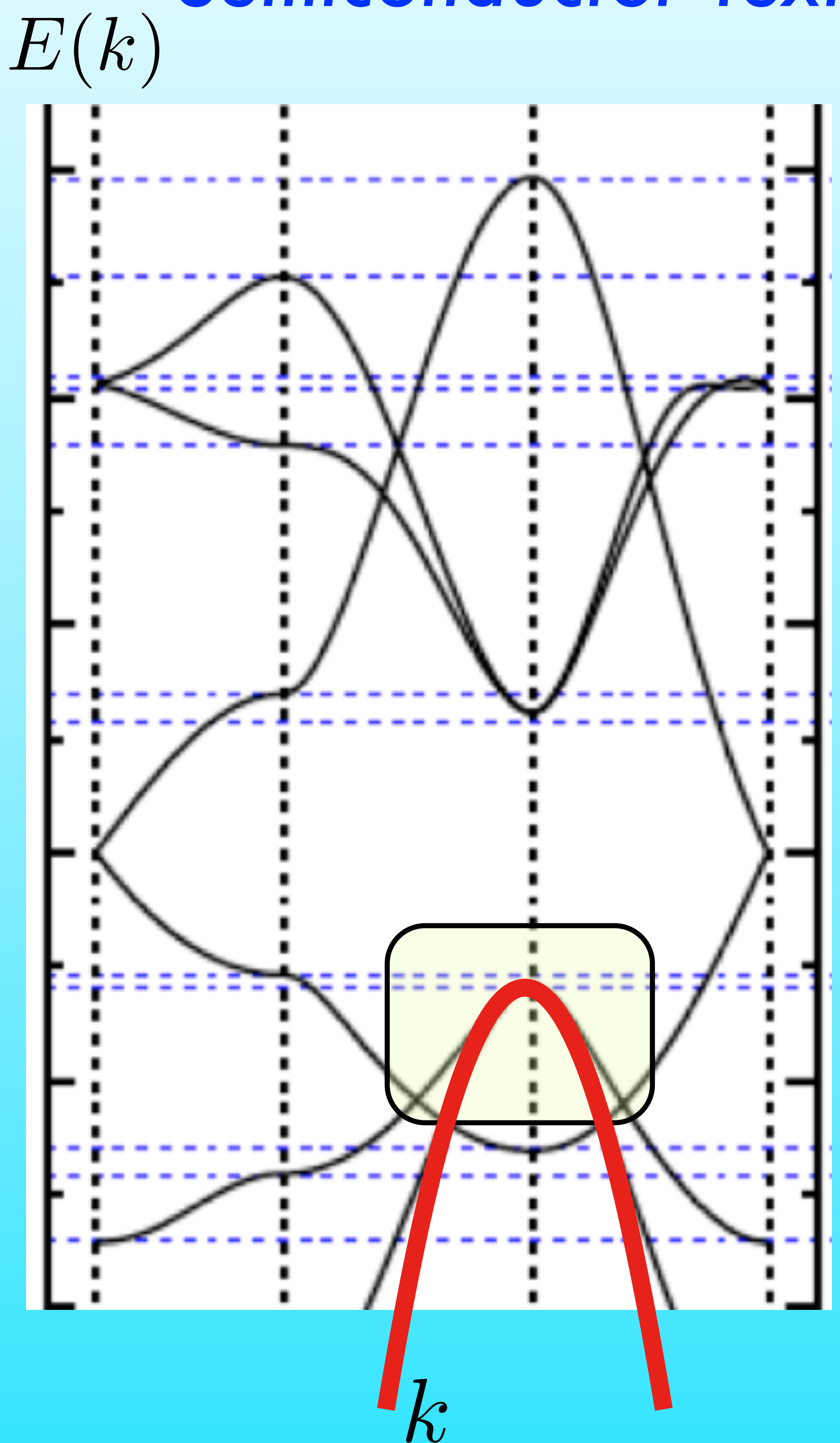
Carbon nanotube



Fullerene

Effective mass approximation for semiconductor

Semiconductor Text book



approximate by parabolic dispersion

$$E(k) = \frac{\hbar^2 k^2}{2m^*}$$

Effective Theory:

Schrodinger Equation
with effective mass

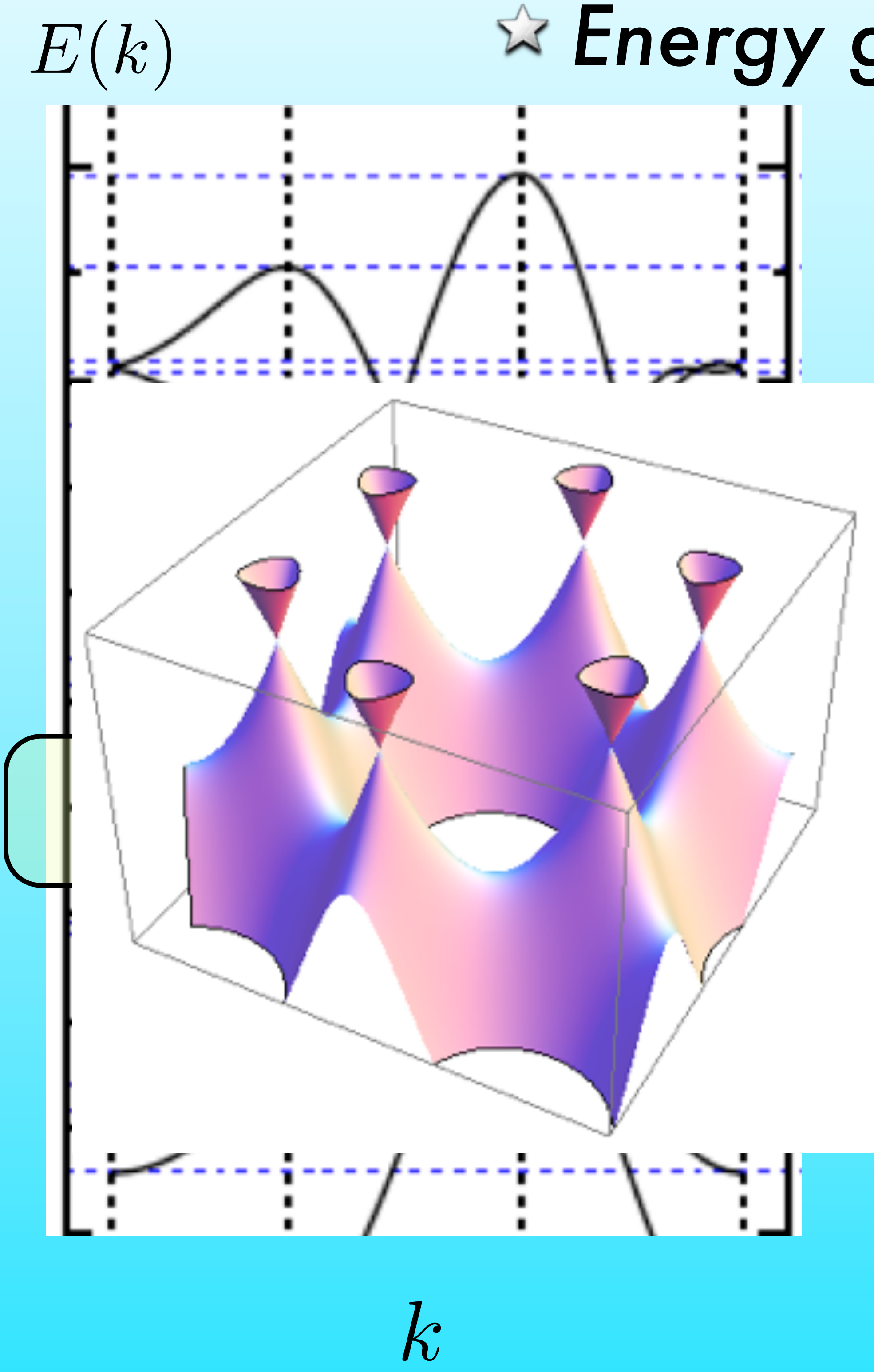
$$H\psi = E\psi$$
$$H = \frac{\boldsymbol{p}^2}{2m^*}$$

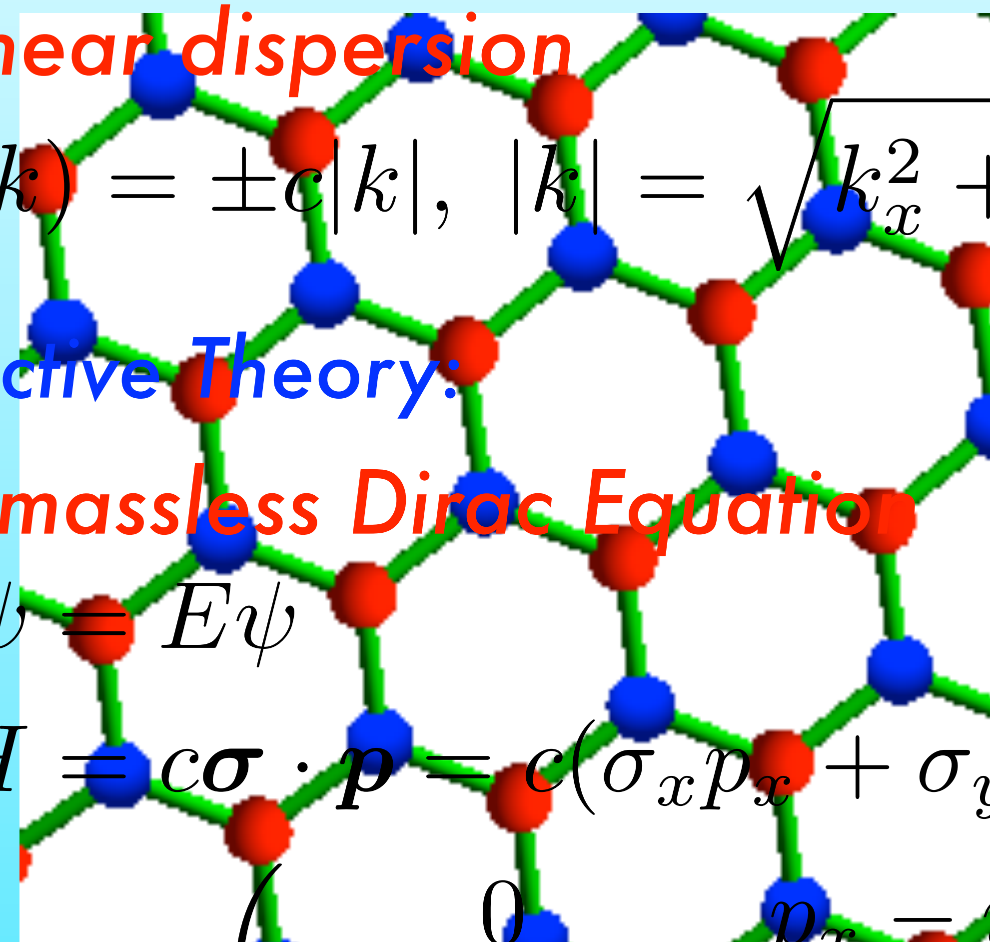
$$\boldsymbol{p} = -i\hbar\nabla = -i\hbar\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$$

Single parameter m^* (effective mass)
characterizes the low energy physics

What is special for the graphene

- ☆ 2Dimensional Semiconductors P. Wallace '47
- ☆ Energy gap is zero! Zero gap Semiconductor





linear dispersion

$$E(k) = \pm c|k|, \ |k| = \sqrt{k_x^2 + k_y^2}$$

Effective Theory:

massless Dirac Equation

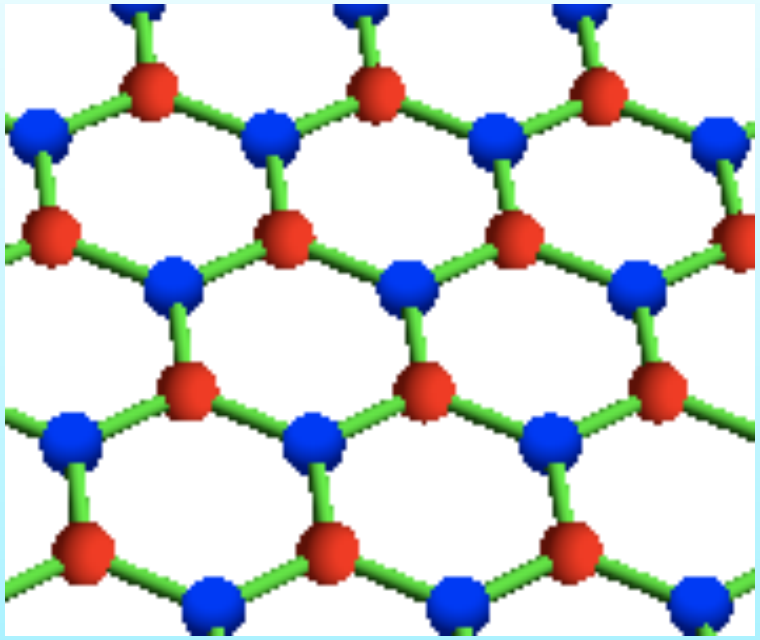
$$H\psi = E\psi$$

$$H = c\boldsymbol{\sigma}\cdot\boldsymbol{p} = c(\sigma_x p_x + \sigma_y p_y)$$

$$= c\begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix}$$

Condensed matter realization of relativistic particles

Realization of massless Dirac fermions



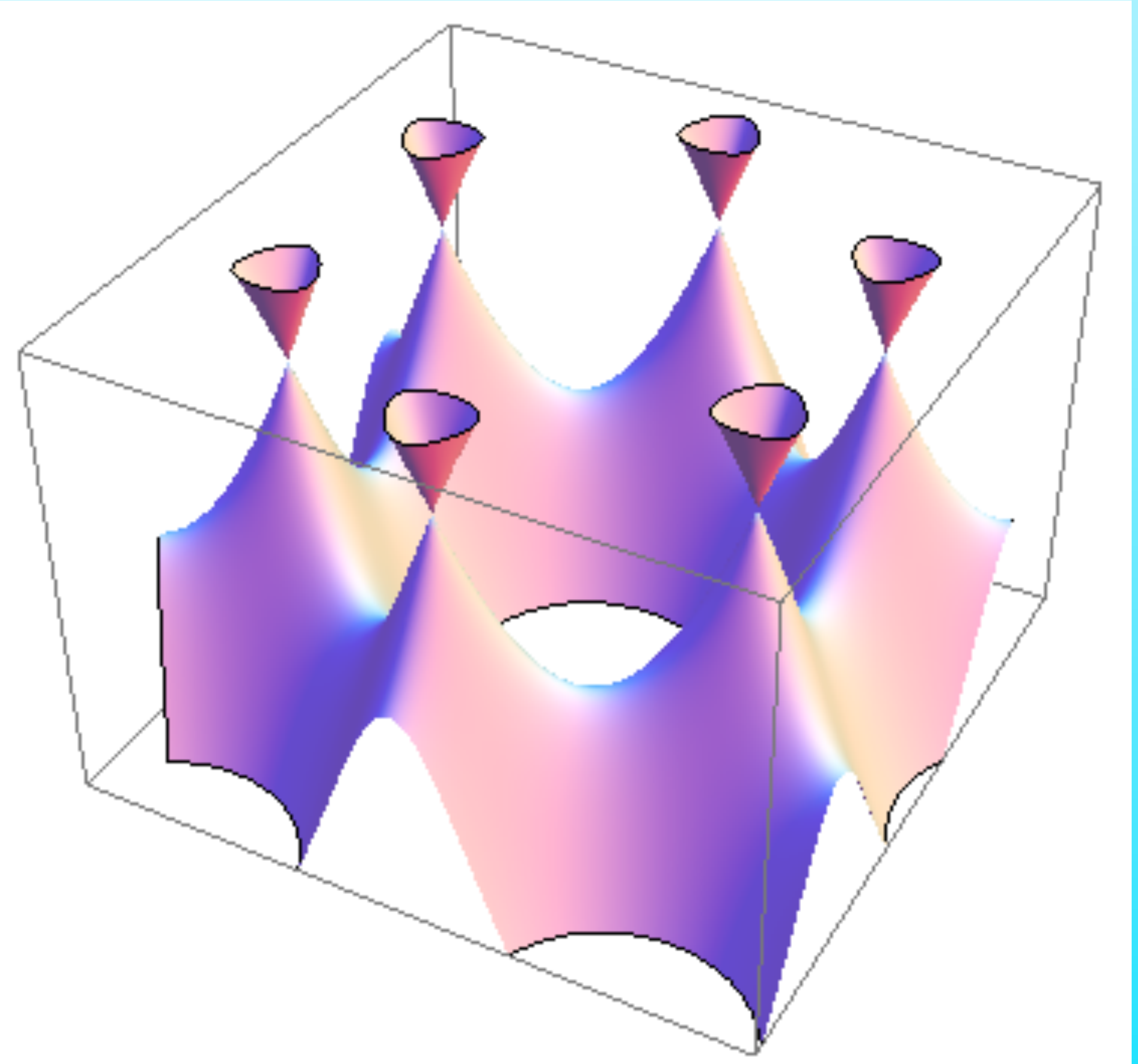
linear dispersion

$$E(k) = \pm c|k|, \ |k| = \sqrt{k_x^2 + k_y^2}$$

Effective Theory: massless
Dirac Fermions

☆ On a honeycomb lattice

without any regularization



Wallace '47
Semenoff '85
Haldane '88

chiral symmetric   doubling

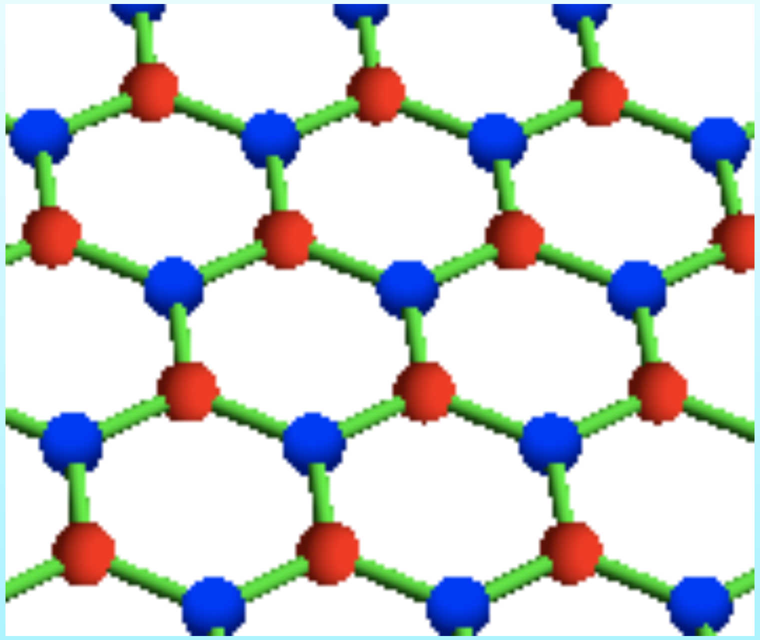
$$\{H, {}^{\exists}\gamma\} = 0, \quad \gamma^2 = 1$$

Nielsen-Ninomiya '81

Fermion doubling

2D analogue of
Nielsen-Ninomiya theorem
in 4D lattice Gauge theory
Topological !

Chiral symmetry ?



Wallace '47
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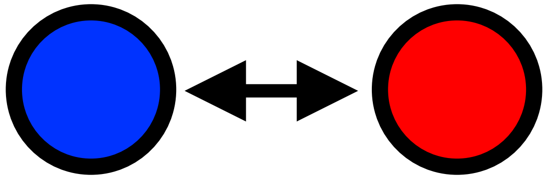
chiral symmetric   doubling

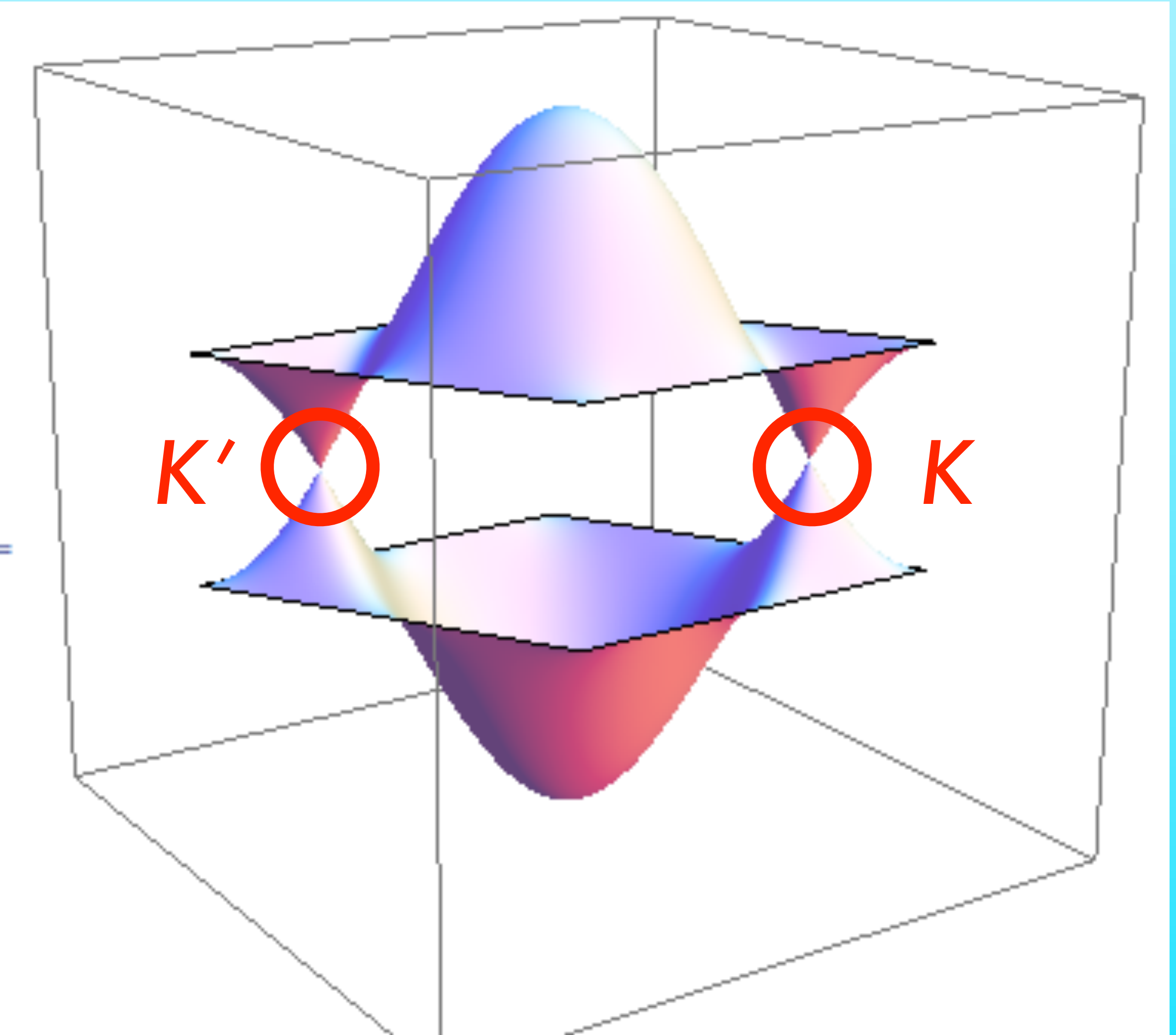
$$\{H, {}^{\exists}\gamma\} = 0, \quad \gamma^2 = 1$$

Nielsen-Ninomiya '81

Chiral Symmetry

Equivalence of
Hopping between





honeycomb lattice: Bipartite

Fermion doubling

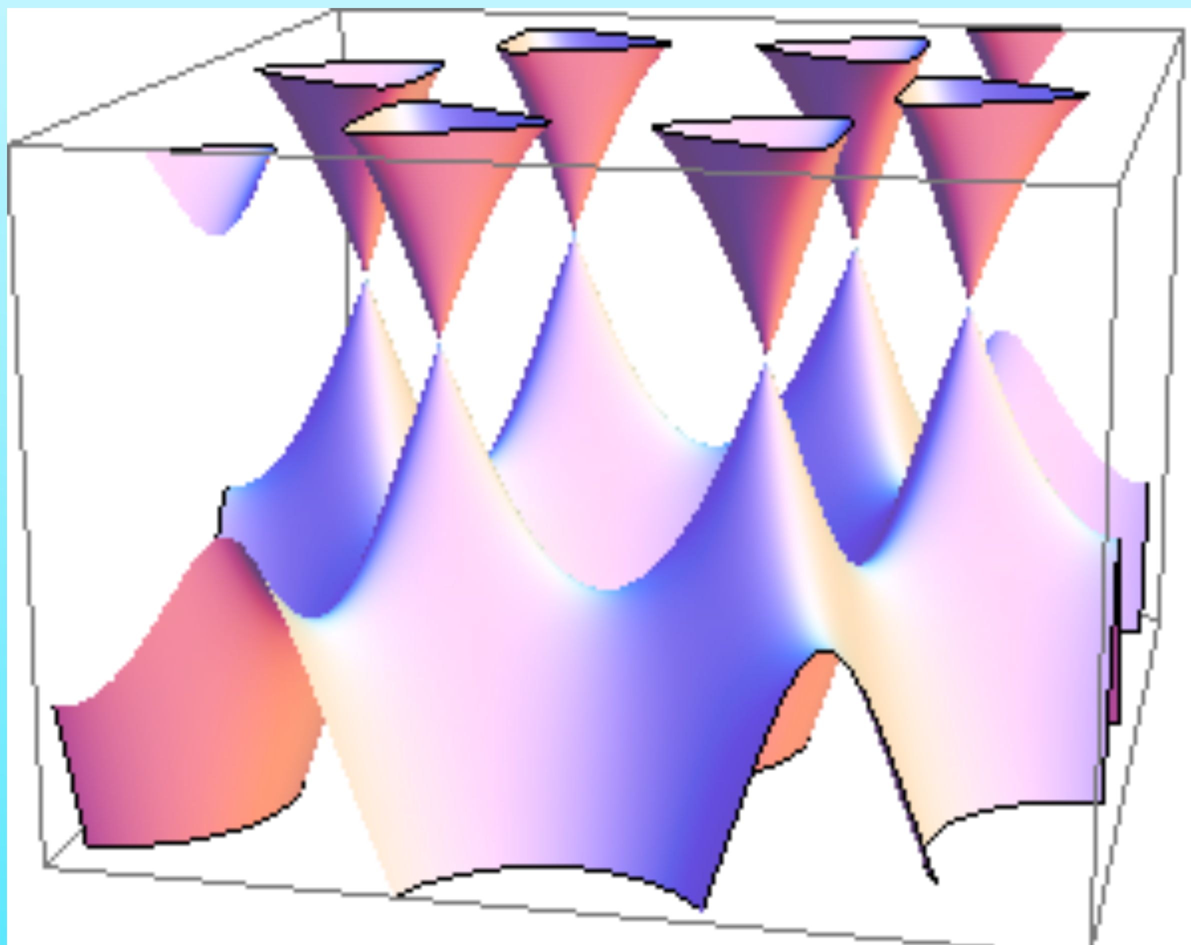
2D analogue of
Nielsen-Ninomiya theorem
in 4D lattice Gauge theory
Topological !

Dirac Cones are Stable!

- ☆ The Dirac Cornes are not accidental
- ☆ It has topological stability

Hatsugai, Fukui, Aoki, '06

extended BZ

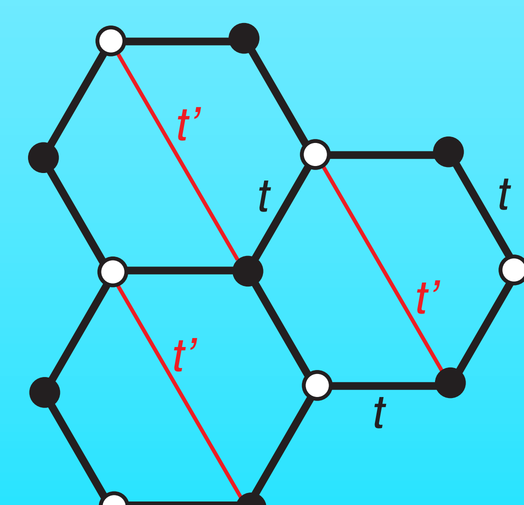


Chiral Symmetry

$$\{H, {}^{\exists}\gamma\} = 0, \quad \gamma^2 = 1$$

Doubled Dirac Cones

- Dirac Cones are stable for small but finite perturbation
- It can be gapped, if it's large.



chiral symmetric perturbation

respect chiral symmetry

Geometrical meaning of Chiral symmetry

$$H(\boldsymbol{k}) = \boldsymbol{R}(\boldsymbol{k})\cdot\boldsymbol{\sigma} = \begin{pmatrix} R_z & R_x - iR_y \\ R_y + iR_y & -R_z \end{pmatrix}$$

$E = \pm|\boldsymbol{R}(\boldsymbol{k})|$

3D (R_x, R_y, R_z)

$$\{H_{\text{eff}}, {}^{\exists}\gamma\} = H_{\text{eff}}\gamma + \gamma H_{\text{eff}} = 0 \qquad \gamma^2 = 1$$

$$\gamma = \begin{cases} \sigma_z & \text{: bipartite lattice \& hopping between them} \quad R_z = 0 \\ \sigma_y & \text{: real } H_{\text{eff}} \text{ : Time reversal \& Inversion} \quad R_y = 0 \end{cases}$$

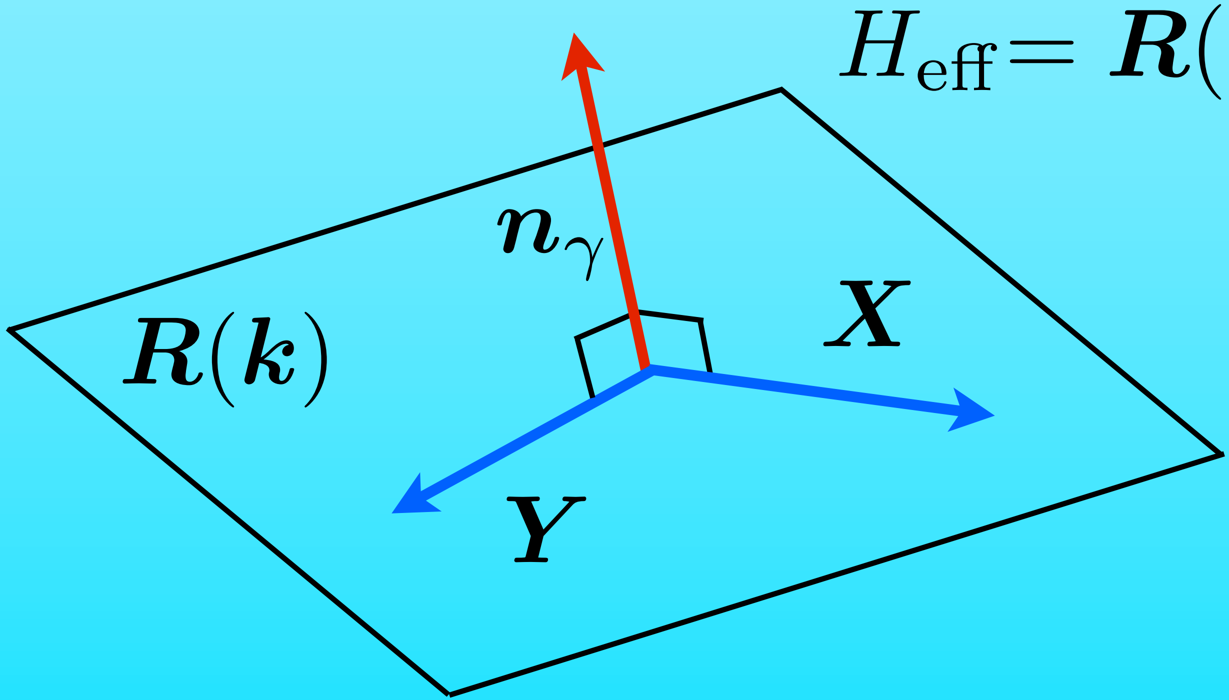
Generically

$$\gamma = \boldsymbol{n}_\gamma\cdot\boldsymbol{\sigma} \quad \{H_{\text{eff}}, \gamma\} = 0 \rightleftarrows \boldsymbol{n}_\gamma \perp \boldsymbol{R}$$

Zero gap condition

$H_{\text{eff}} \to 0,\ \boldsymbol{k} \to \boldsymbol{k}_0$ expand by $\delta\boldsymbol{k} = \boldsymbol{k} - \boldsymbol{k}_0$

$$H_{\text{eff}} = \boldsymbol{R}(\boldsymbol{k})\cdot\boldsymbol{\sigma} \approx (\boldsymbol{X}\cdot\boldsymbol{\sigma})\delta k_x + (\boldsymbol{Y}\cdot\boldsymbol{\sigma})\delta k_y$$



$$\boldsymbol{X} = \partial_{k_1}\boldsymbol{R},\ \boldsymbol{Y} = \partial_{k_2}\boldsymbol{R}$$

chirality

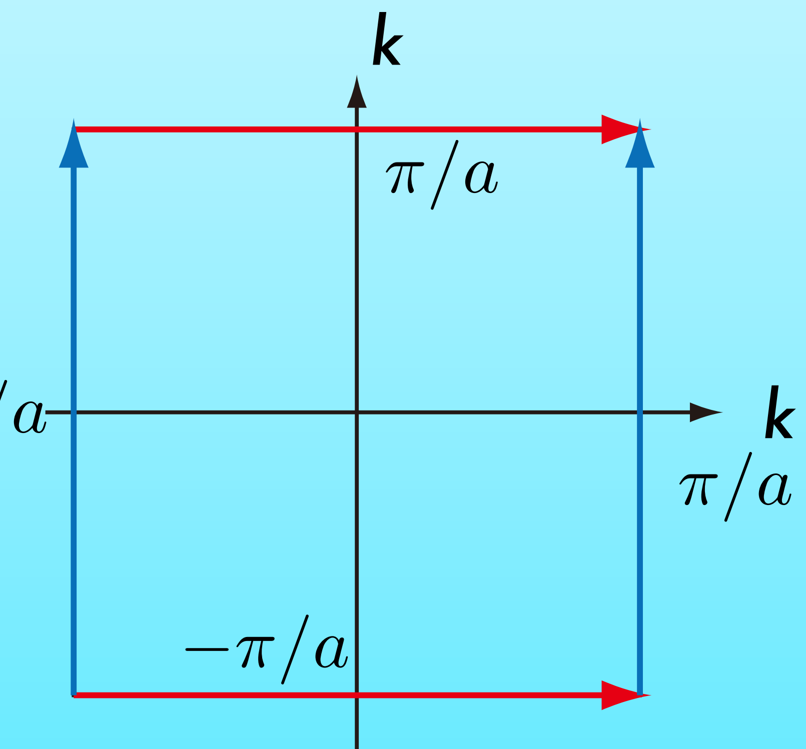
$$(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{n}_\gamma) = \begin{cases} \text{right handed} & \chi = +1 \\ \text{left handed} & \chi = -1 \end{cases}$$

$$\boldsymbol{X}, \boldsymbol{Y} \rightleftarrows m^*$$

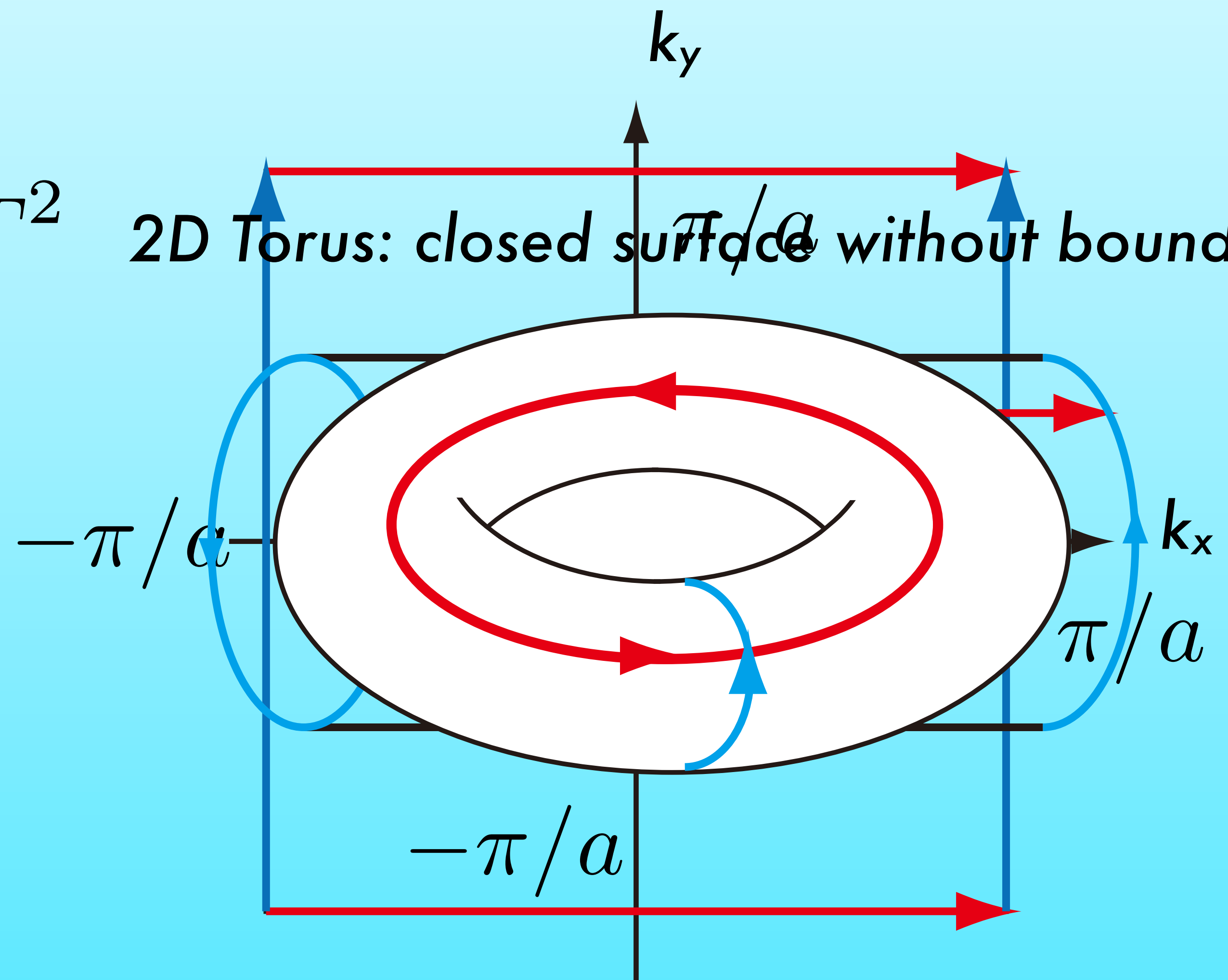
Topological stability of the Doubled Dirac cones

c.f. 4D graphene & chiral symmetry, M. Creutz '08
also with TR inv. 5D YH, '10

2D Brillouin zone :periodic in k_x & k_y



T^2 2D Torus: closed surface without boundaries



Topological stability of the Doubled Dirac cones

c.f. 4D graphene & chiral symmetry, M. Creutz '08
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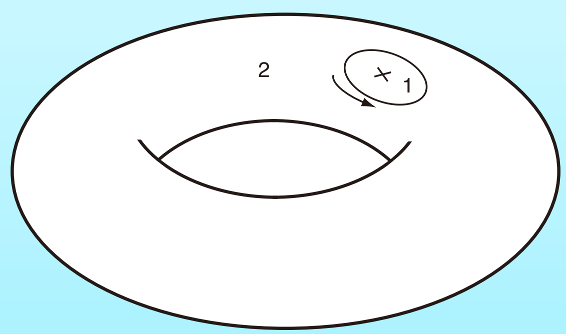
$$H(\boldsymbol{k}) = \boldsymbol{R}(\boldsymbol{k}) \cdot \boldsymbol{\sigma} = \begin{pmatrix} R_z & R_x - iR_y \\ R_y + iR_y & -R_z \end{pmatrix}$$

3D (R_x, R_y, R_z)

2D Brillouin zone :periodic in k_x & k_y

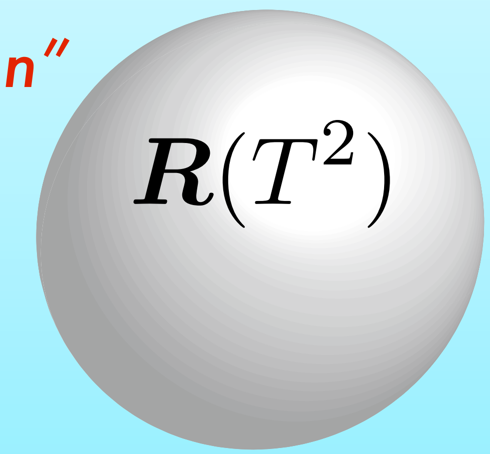
2D Torus T^2



map

Generically 2-D closed surface in 3D

"balloon"



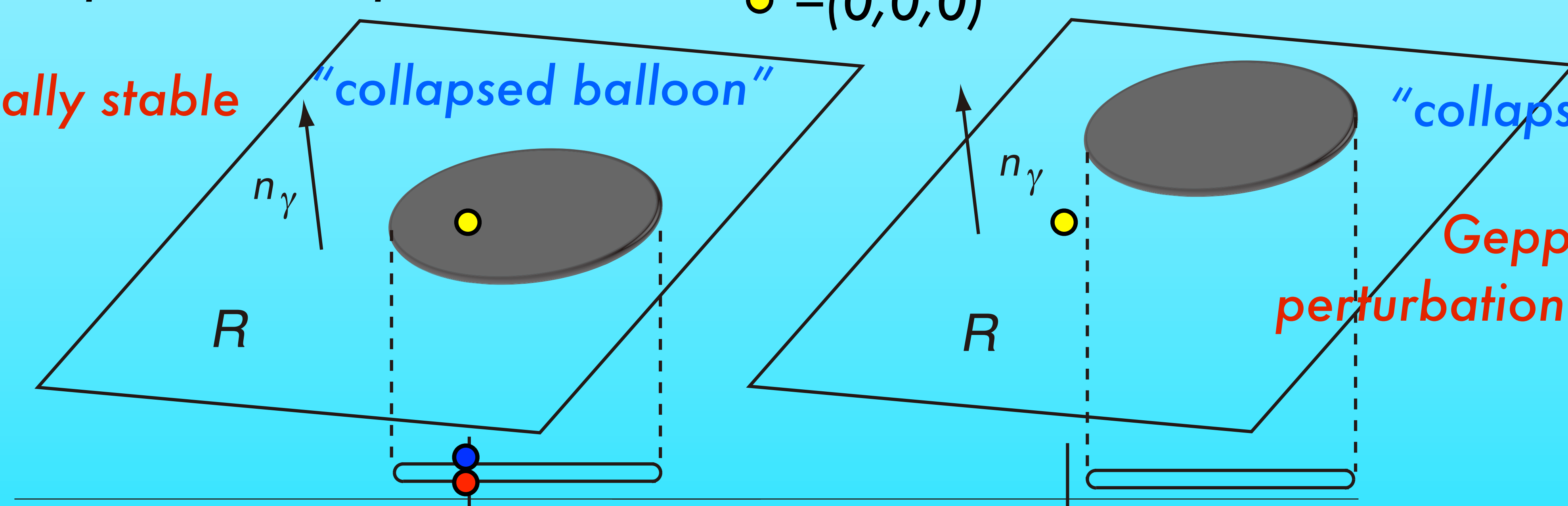
Chiral symmetry $\{H, \gamma\} = 0 \rightleftarrows \boldsymbol{n}_\gamma \perp \boldsymbol{R}$

$\gamma = \boldsymbol{n}_\gamma \cdot \boldsymbol{\sigma}$ $\boldsymbol{R}(\boldsymbol{k})$ is on a plane normal to $\boldsymbol{n}_\gamma$

$\boldsymbol{R}(T^2)$ is collapsed on the plane

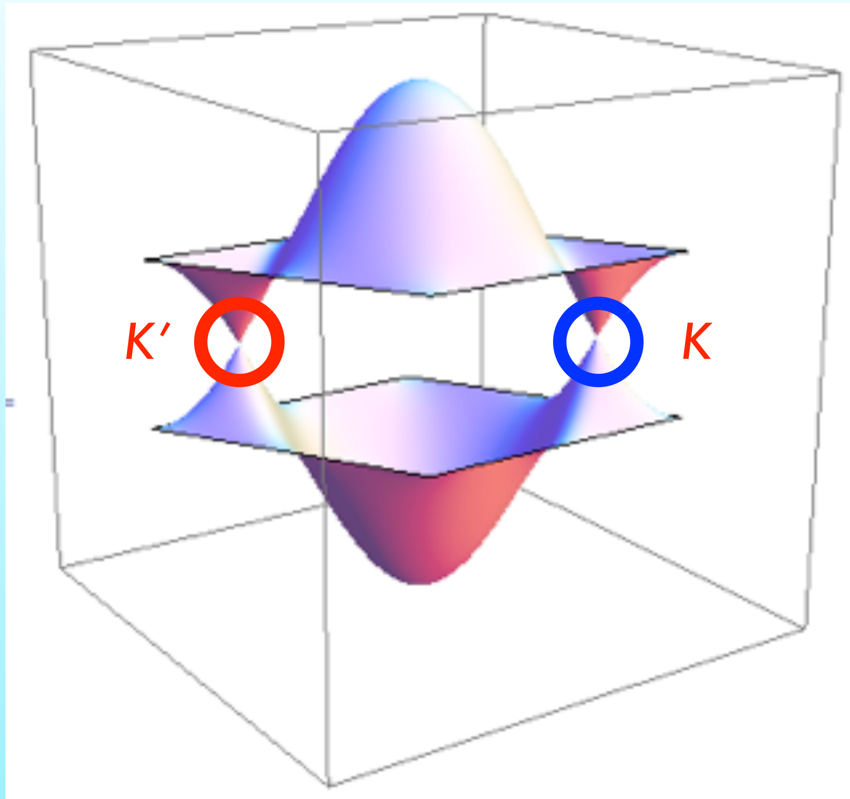
=(0,0,0)

Topologically stable



doubled Dirac cones

chirality is reversed among the Doubled Dirac cones

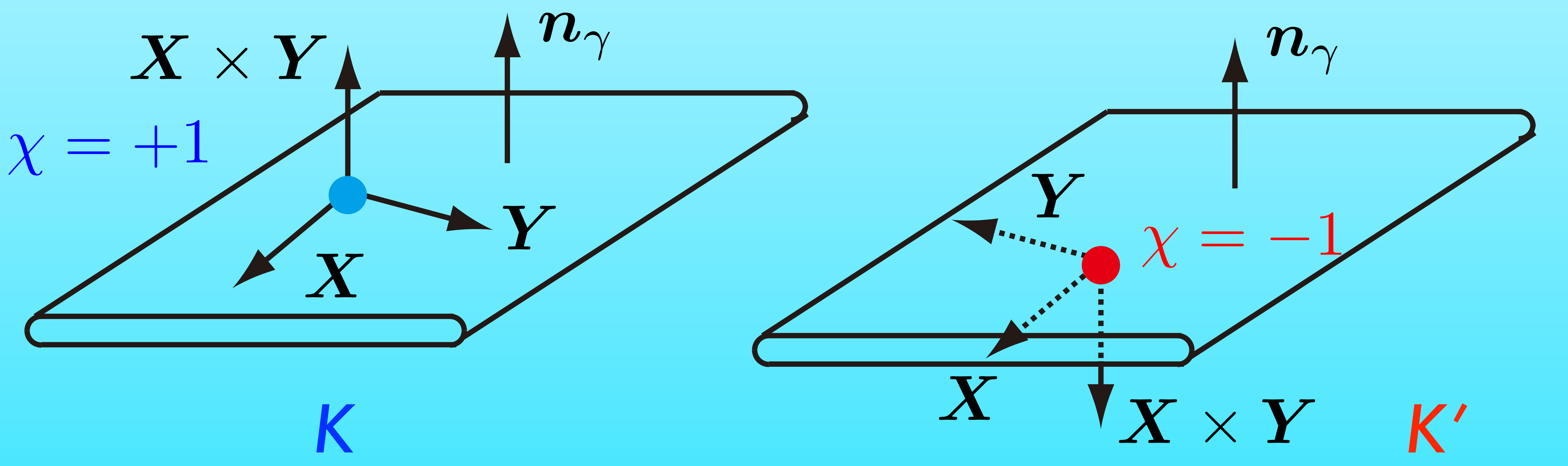


$$\{H,\gamma\}=0 \;\rightleftarrows\; \boldsymbol{n}_\gamma \perp \boldsymbol{R}$$

$$\gamma = \boldsymbol{n}_\gamma \cdot \boldsymbol{\sigma}$$

$$H_{=}(\boldsymbol{X}\cdot\boldsymbol{\sigma})\delta k_x + (\boldsymbol{Y}\cdot\boldsymbol{\sigma})\delta k_y$$

$$(\boldsymbol{X},\boldsymbol{Y},\boldsymbol{n}_\gamma) = \begin{cases} \text{right handed} & \chi=+1 \\ \text{left handed} & \chi=-1 \end{cases}$$



$$\boldsymbol{X}\times\boldsymbol{Y} = \chi|\boldsymbol{X}\times\boldsymbol{Y}|\boldsymbol{n}_\gamma$$

Chiral symmetric Dirac fermion and n = 0 LL

$$\{H,\gamma\}=0 \quad \gamma=\boldsymbol{n}_\gamma\cdot\boldsymbol{\sigma} \qquad H=(\boldsymbol{X}\cdot\boldsymbol{\sigma})\delta k_x+(\boldsymbol{Y}\cdot\boldsymbol{\sigma})\delta k_y$$

$$\boldsymbol{X}\times\boldsymbol{Y}=\chi|\boldsymbol{X}\times\boldsymbol{Y}|\boldsymbol{n}_\gamma$$

$$\hbar\delta\boldsymbol{k}\to\boldsymbol{\pi}=-i\hbar\boldsymbol{\nabla}-e\boldsymbol{A}$$

$$H=\hbar^{-1}\Big[(\boldsymbol{X}\cdot\boldsymbol{\sigma})\pi_x+(\boldsymbol{Y}\cdot\boldsymbol{\sigma})\pi_y\Big] \qquad \boldsymbol{B}=\mathrm{rot}\,\boldsymbol{A}$$

$$H^2 = v^2\left(\boldsymbol{\pi}^\dagger\boldsymbol{\Xi}\,\boldsymbol{\pi}\right)-\chi\frac{\hbar\omega_C}{2}\boldsymbol{\gamma} \qquad \det\boldsymbol{\Xi} = 1$$

anisotropic Landau level — cancel the zero point energy for the chirality χ

$$\hbar\omega_C(n+1/2)$$

$$\omega_C=2eBc^2$$

$$v^2\equiv|\boldsymbol{X}\times\boldsymbol{Y}|/\hbar^2$$ "fermi velocity"

$$\epsilon_n=\pm v\sqrt{2neB\hbar}$$ n=0 Landau level has a fixed chirality

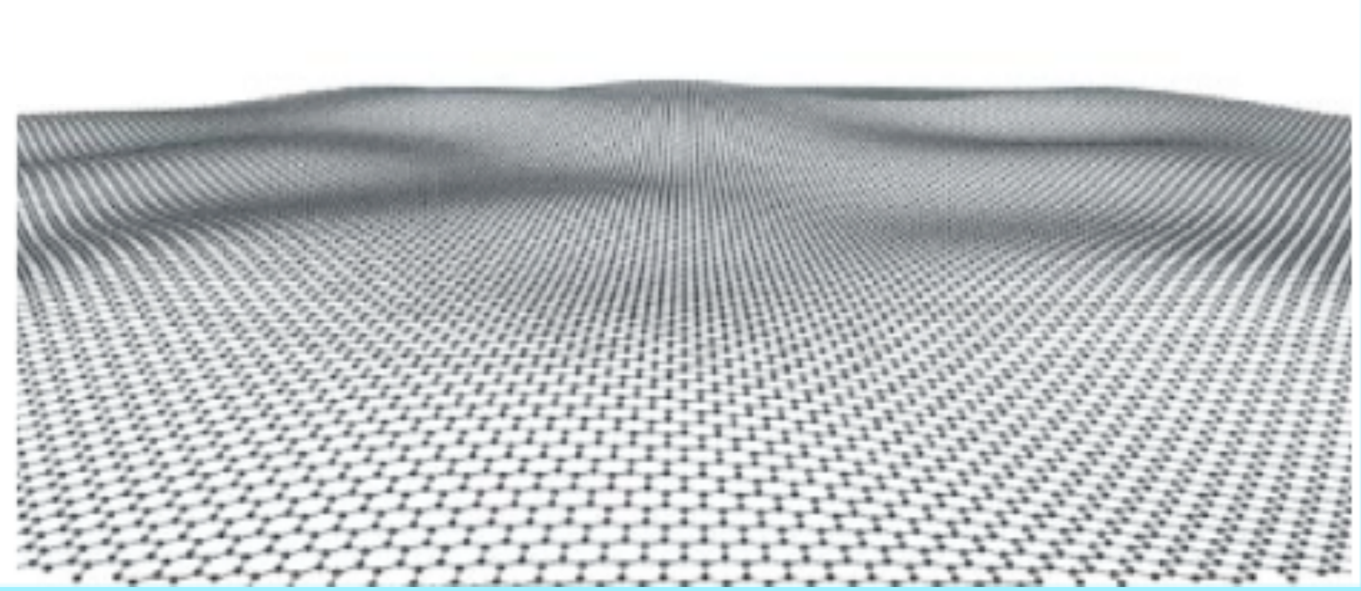
$$\gamma\Psi_0=\chi\Psi_0$$

This n=0 L.L. has topological stability protected by the index theorem & Aharonov-Casher argument

chiral symmetry needed   Aharonov-Casher '79

Ripples of graphene

☆ Ripples as random gauge field in free standing graphene



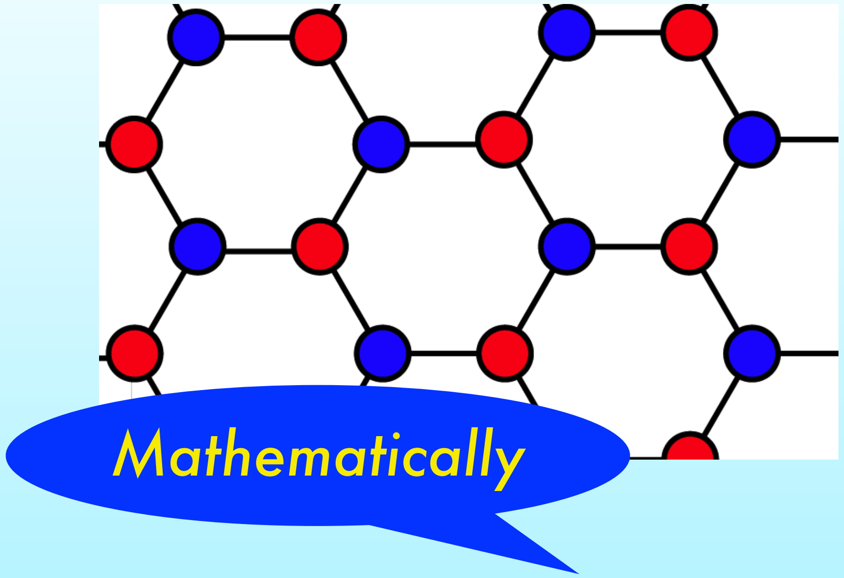
(Meyer, Geim et al, Nature 2007)

Gauge field fluctuation

Neto-Guinea-Peres-Novoselov-Geim '09

Random hopping model on a honeycomb lattice
(phase)
with spatial correlation

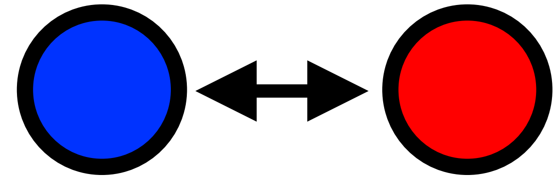
Chiral Symmetry of Graphene



Chiral Symmetry

Equivalence of

Hopping between



Hamiltonian anti-commutes with $^{\exists}\Gamma$

$$\{\Gamma, H\} = \Gamma H + H\Gamma = 0\ ,\ \Gamma^2 = 1$$

$$H = \begin{pmatrix} \boldsymbol{O} & \boldsymbol{D} \\ \boldsymbol{D}^\dagger & \boldsymbol{O} \end{pmatrix} \qquad \Gamma = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{O} \\ \boldsymbol{O}^\dagger & -\boldsymbol{I} \end{pmatrix}$$

Perturbations

Uniform site energy

Staggered site energies

Antiferromagnetic order

Bond order

Random potential

Random hopping

Ripples ?

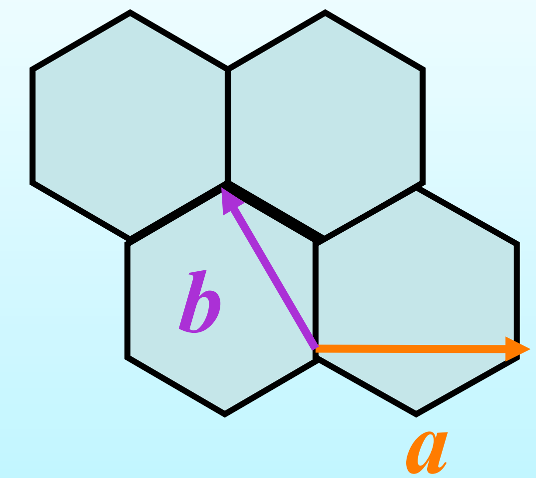
Model

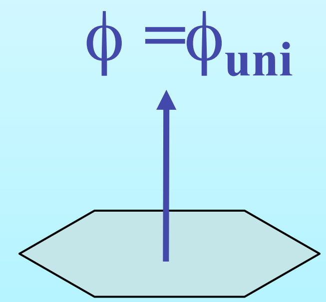
c.f. Dirac fermions on square lattice
T. Kawarabayashi, Y.Hatsugai and H.Aoki,, PRL 103, 156804 (2009)

2D Honeycomb Lattice in disordered hopping amplitude

$$H = \sum_{<r,r'>} t_{r,r'} e^{i\theta_{rr'}} c_r^\dagger c_{r'}$$

$$t_{r,r'} = t + \delta t_{r,r'}$$





$$\sum_{\hexagon} \theta_{r,r'} = -2\pi\phi_{\rm uni}/\phi_0$$

Disordered components δt

$$P(\delta t) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp(-\delta t^2/2\sigma_t^2)$$

$$W_{\delta t} = \sigma_t\sqrt{12}$$ **effective width**

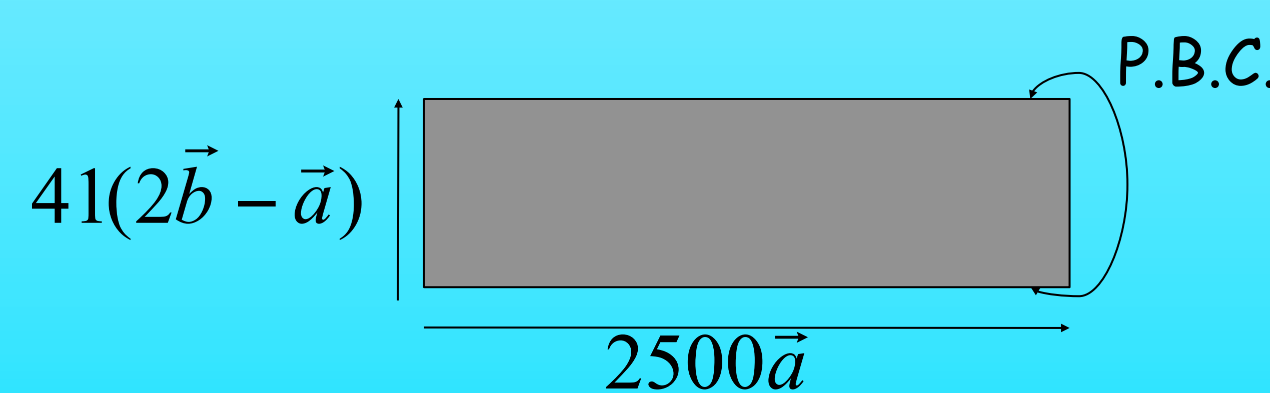
$$\left\langle \delta t_{r_i} \delta t_{r_j} \right\rangle = \sigma_t^2 \exp(-|r_i - r_j|^2/4\eta_t^2)$$

η_t : correlation of random hopping

Density of states

The Green function method
Schweitzer, Kramer, MacKinnon (1984)

$$\rho(E) = -\frac{1}{\pi}\left\langle {\rm Im} G_{r,r}(E + i\gamma)\right\rangle_r$$



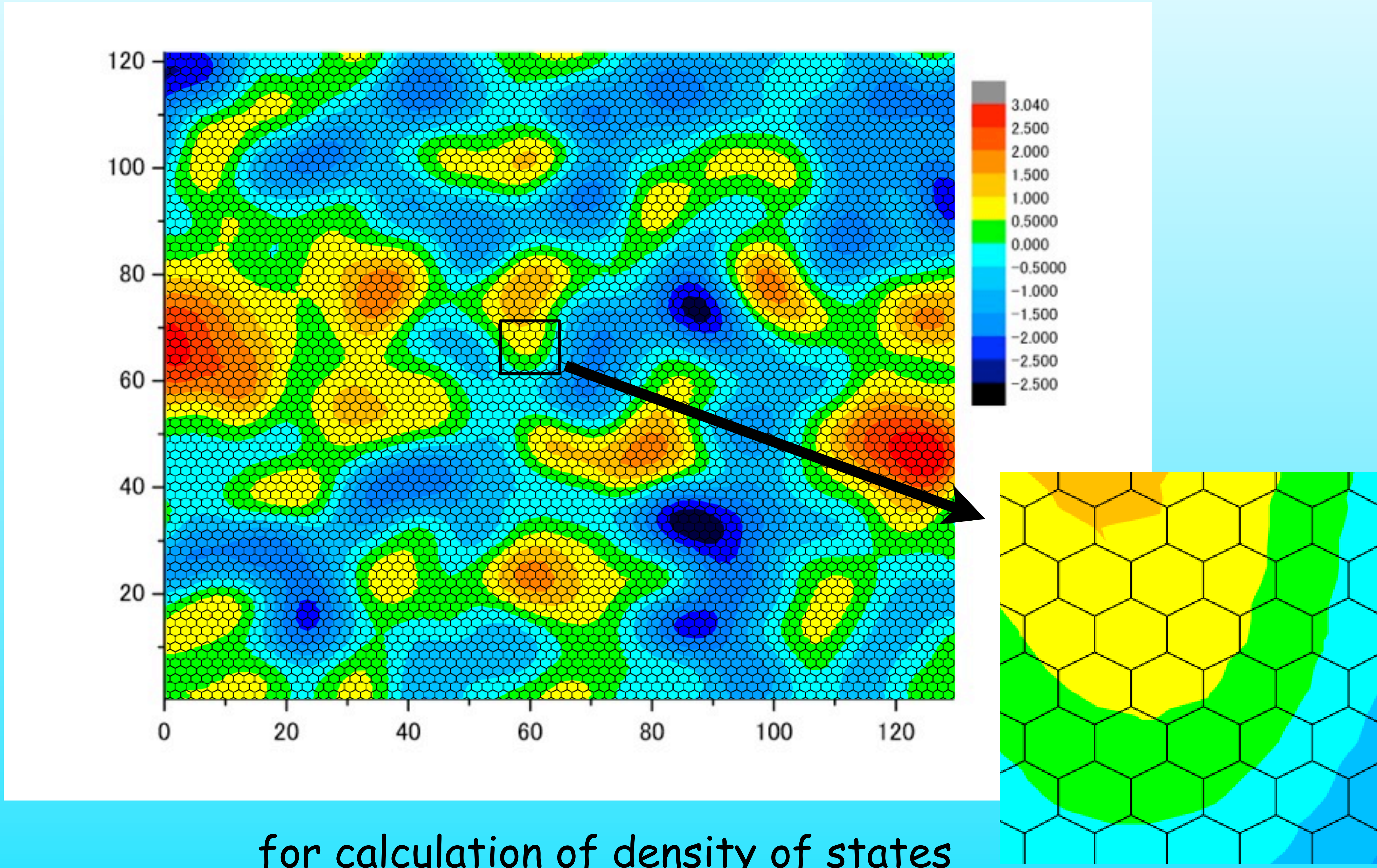
Correlated Random Hopping

T. Kawarabayashi, Y.Hatsugai and H.Aoki,, PRL 103, 156804 (2009)

Correlated Random Hopping

(distribution of gauge field) $\sqrt{3}\eta_t / |\vec{a}| = 5.0$

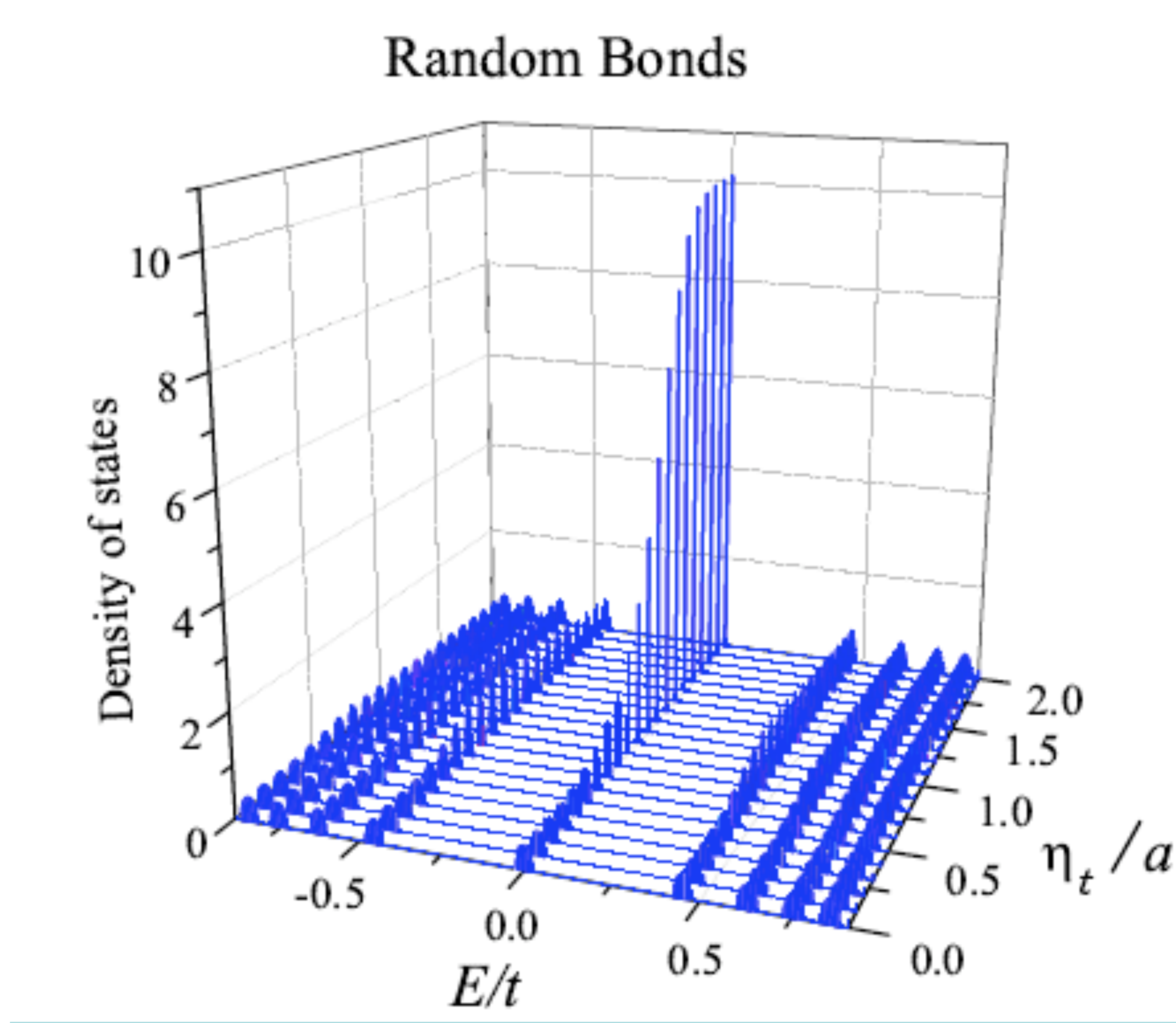
Landscape of hopping amplitude $W_{\delta t}/t = 2.0$



for calculation of density of states

Effect of spatial correlation

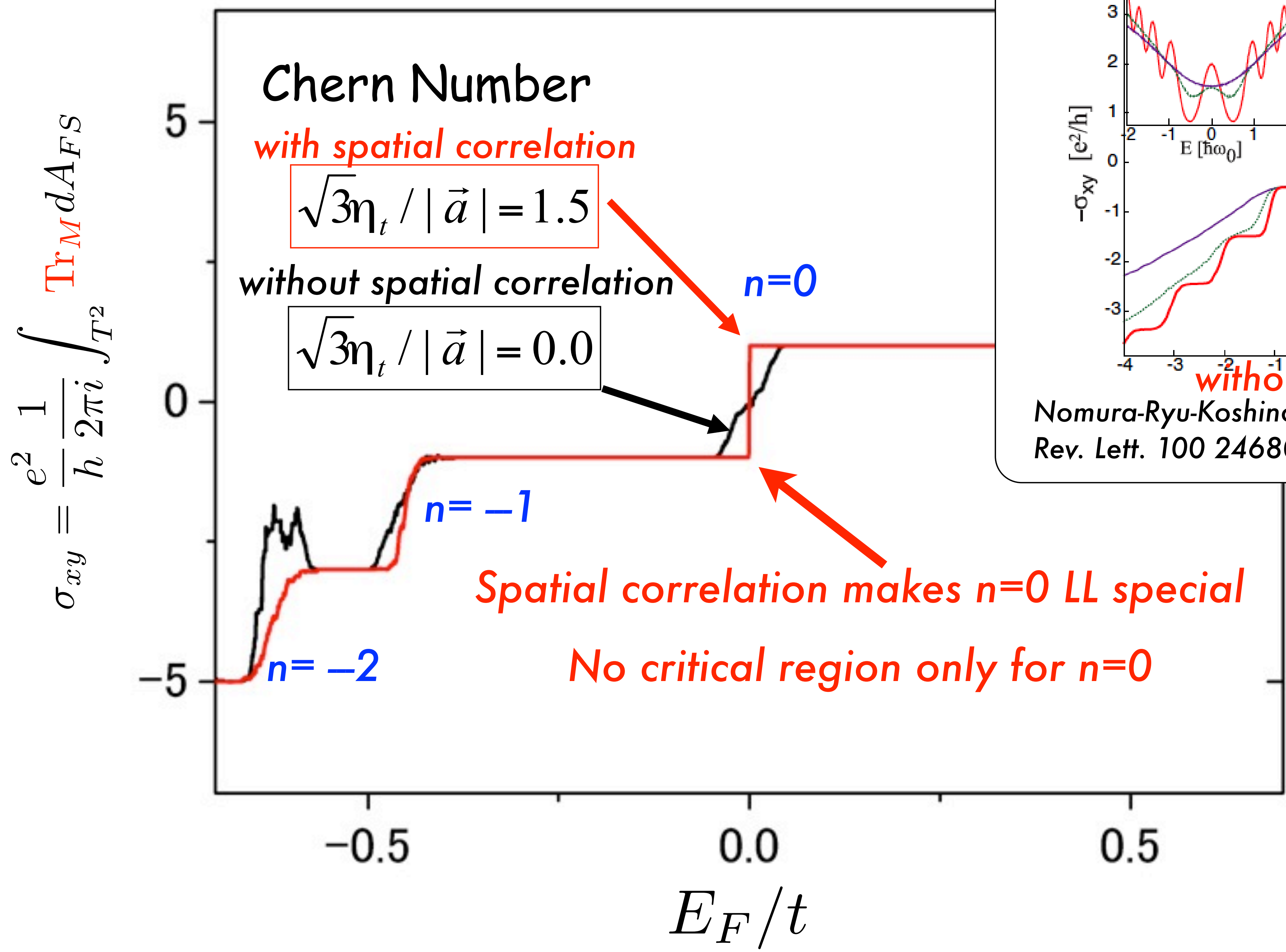
T. Kawarabayashi, Y.Hatsugai and H.Aoki, PRL 103, 156804 (2009)



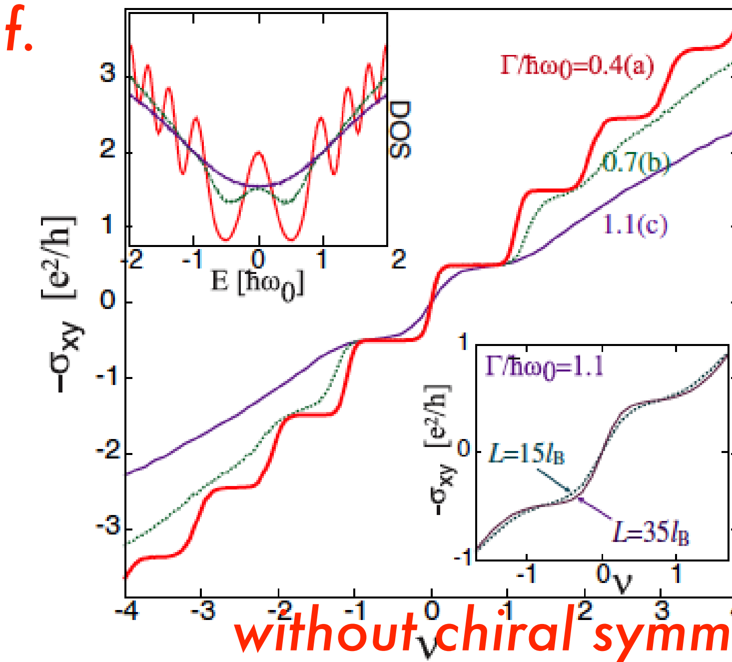
spatial correlation length

almost no broadening when the correlation exceeds lattice constant

$W_{\delta t}/t = 0.4$ $\phi_{uni}/\phi_0 = 1/50$ System size 20×20



c.f.



without chiral symmetry

Nomura-Ryu-Koshino-Mudry-Furusaki, Phys. Rev. Lett. 100 246806 (2008)

100 samples

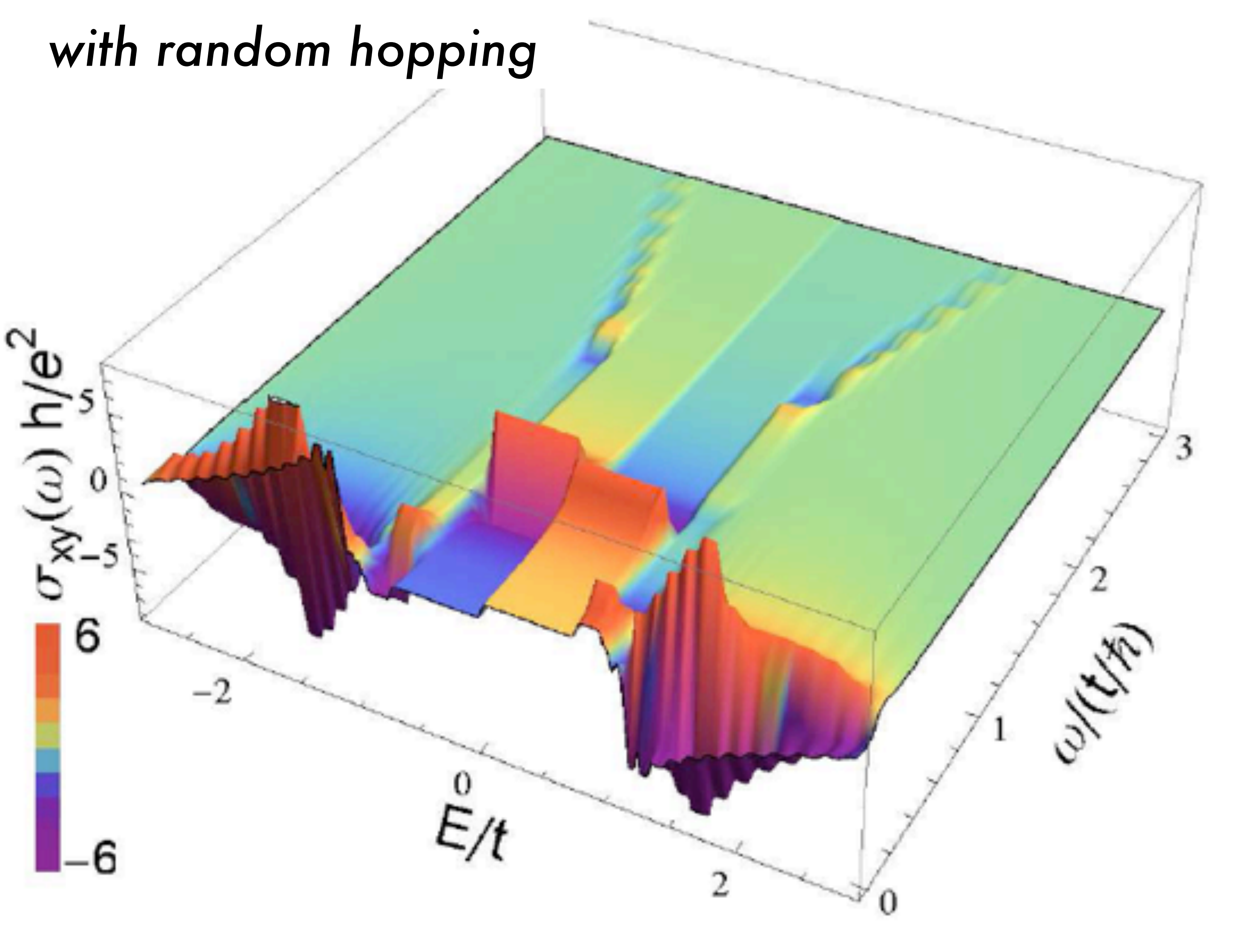
T. Kawarabayashi, Y. H. and H. Aoki, PRL 103, 156804 (2009)

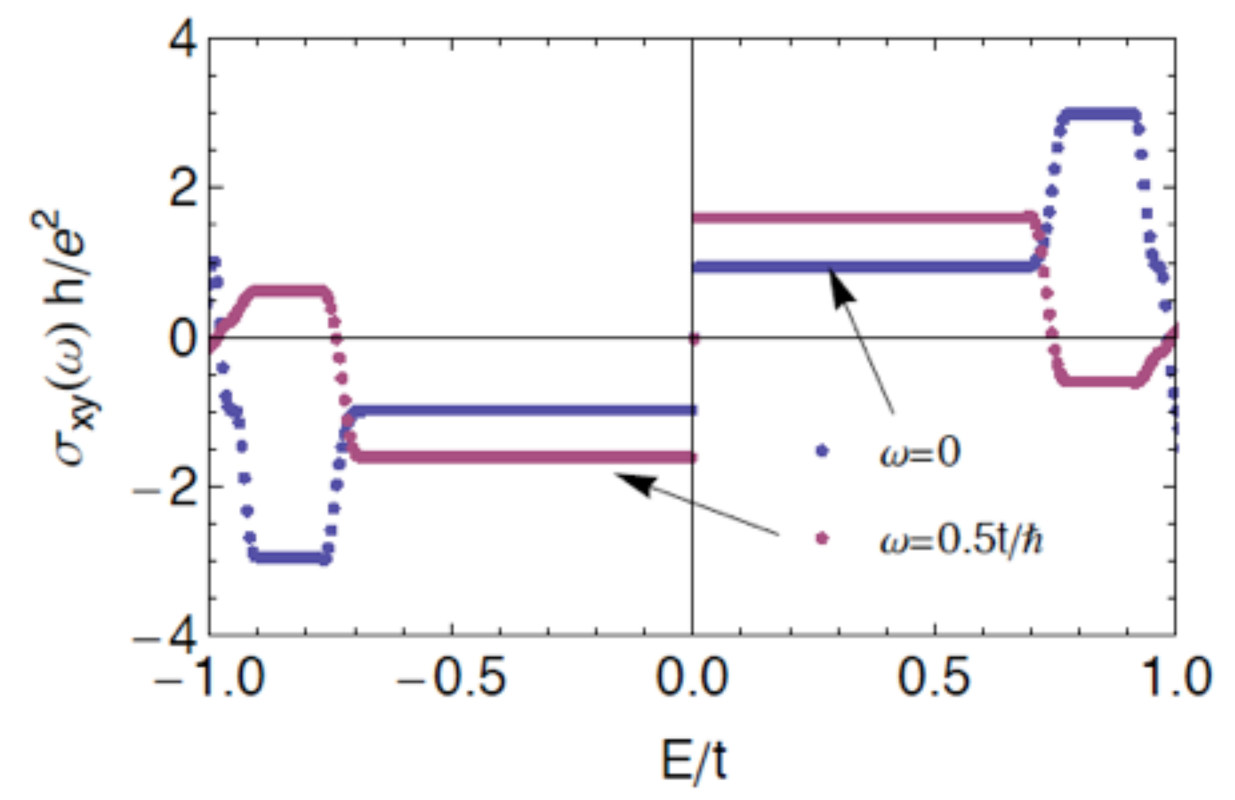
T. Morimoto, Y. H. and H. Aoki, Phys. Rev. Lett. 103, 116803 (2009).

Optical Hall conductivity

(only ripples as randomness in free standing graphene)

with random hopping





Anormaly at n=0 LL

T. Kawarabayashi, T. Morimoto,Y.Hatsugai and H.Aoki, submitted

Topological aspects of graphene : Bulk

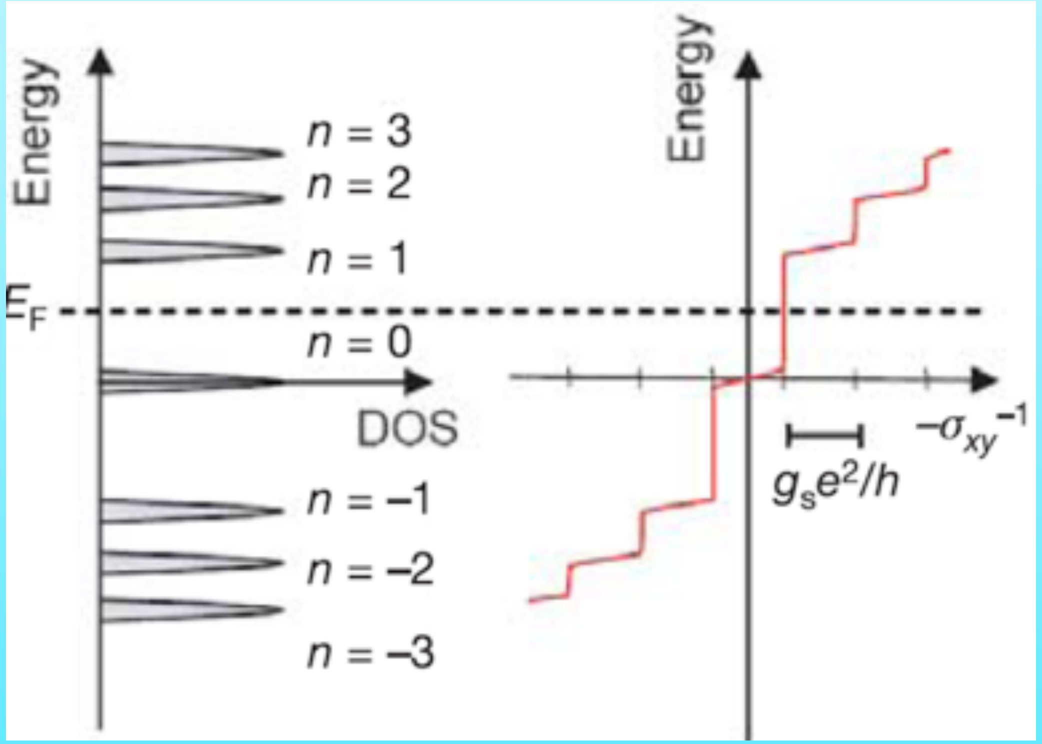
- Berry connection of the filled Dirac sea
- Lattice gauge fields in a parameter space

Observation of Anomalous QHE in Graphene

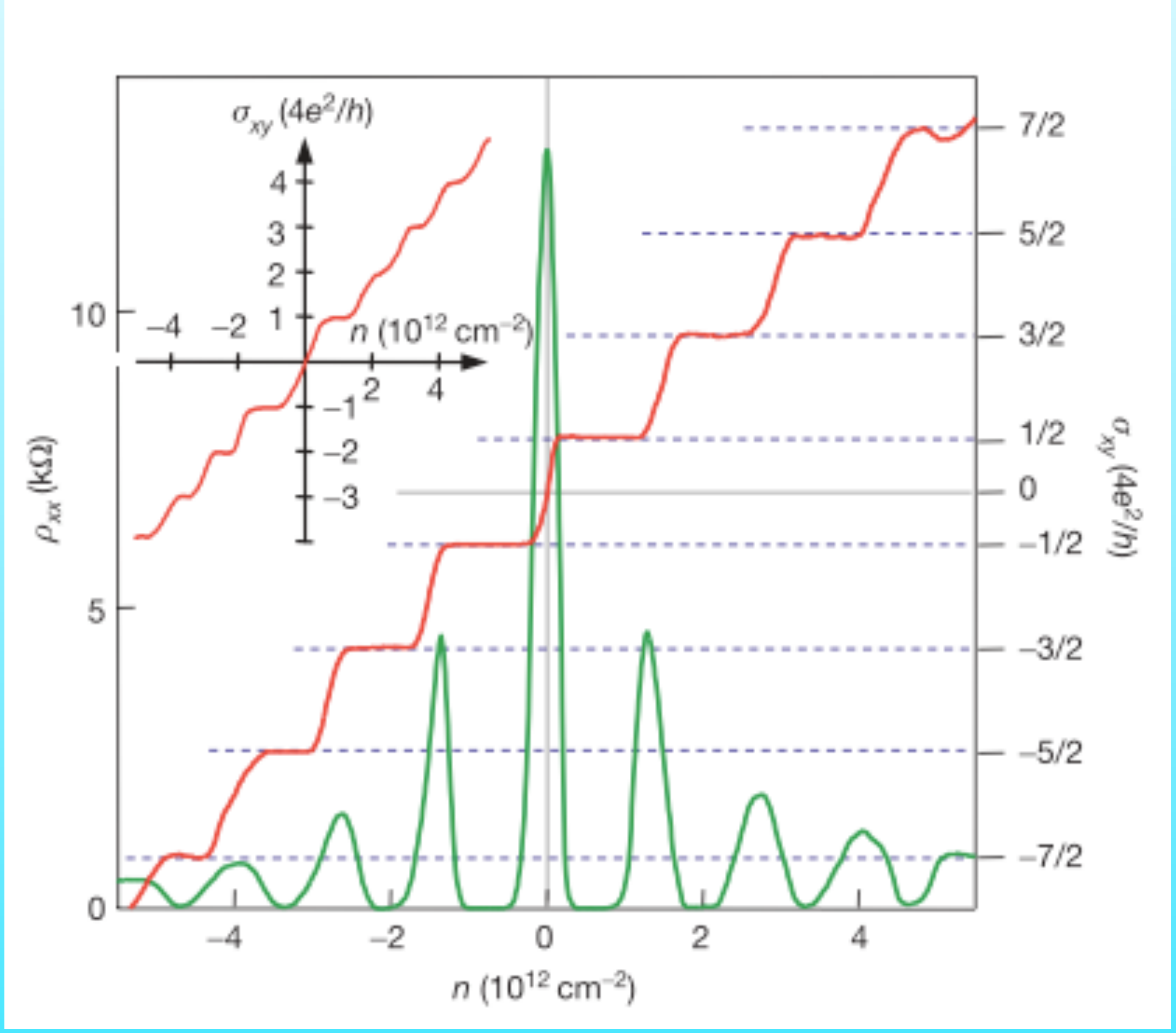
- Anomalous QHE of gapless Dirac Fermions

$$\sigma_{xy} = \frac{e^2}{h}(2n+1), \ n = 0, \pm 1, \pm 2, \cdots$$

$$= 2\frac{e^2}{h}(n + \frac{1}{2})$$



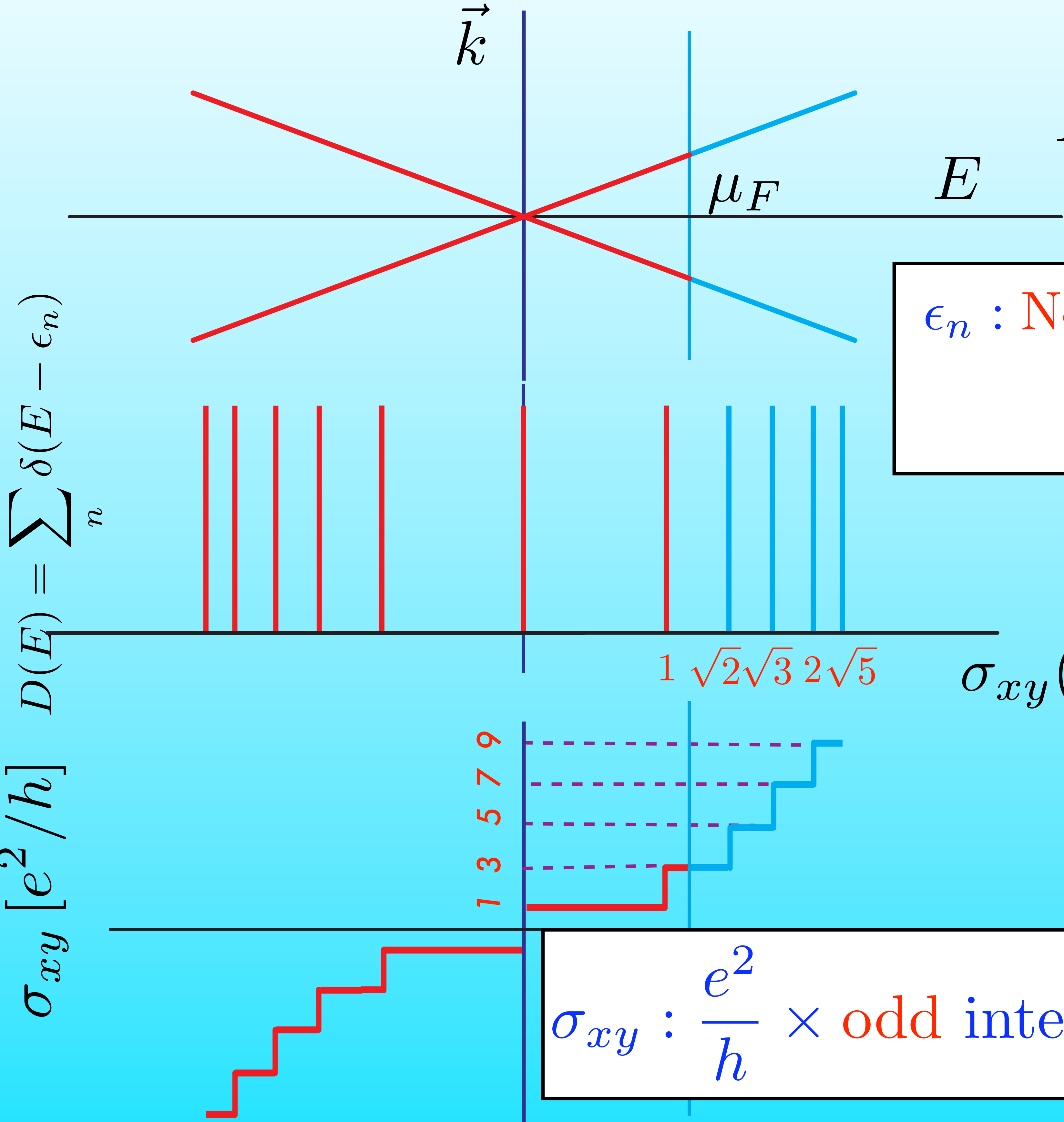
Zhang et al. Nature 2005



Novoselov et al. Nature 2005

QHE of Graphene (Gapless Semiconductor)

☆ Landau Level of Doubled Dirac Fermions



$$\sigma_{xy} : \frac{e^2}{h} \times \text{odd integer}$$

$$E(B=0) = \pm \hbar c |\vec{k}|$$

ϵ_n : No zero point energy shift

$$\epsilon_n = \pm C\sqrt{Bn}$$

McClure, 1956

$$\sigma_{xy}(\mu_F) = \frac{e^2}{\hbar}(2n+1)$$

$$n = 0, 1, 2, 3, \cdots$$

$$\epsilon_{n-1} < \mu_F < \epsilon_n$$

Zheng-Ando 2002

Gusynin-Sharapov, 2005

Peres-Guinea-Neto 2006

...

Theoretical Background

Why the QHE of graphene is interesting ?

Topological Insulators of Dirac sea

Topological Insulators

Gapped Quantum Liquids

Featureless !!

Use Geometrical Phases of the Quantum states

To characterize the topological insulators

- ✓ Berry phases
- ✓ Chern numbers
  1st, 2nd, ...

multi-component : non Abelian

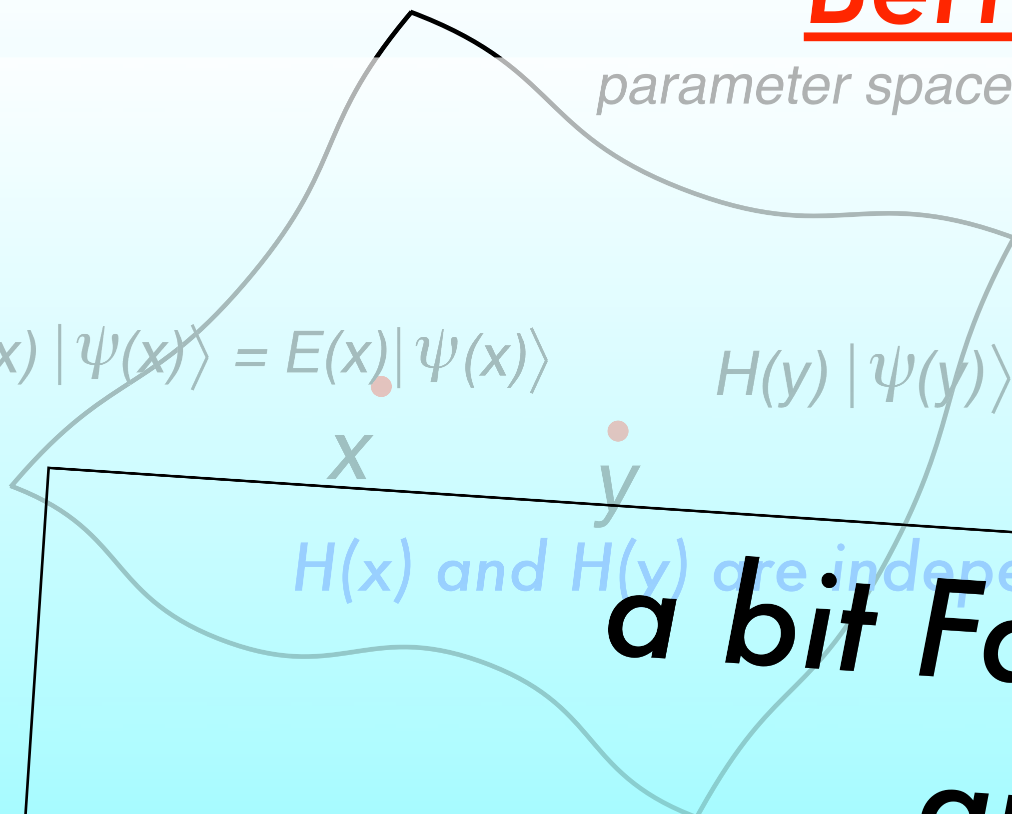
✓ Berry connections

$$A = \Psi^\dagger d\Psi = \Psi^\dagger \partial_\mu \Psi \, dx^\mu$$

$$\Psi = (|\psi_1\rangle, \cdots, |\psi_M\rangle)$$

Berry Connection?

Berry '84



Eigenvectors (space) with Parameters

$H(x)\,|\psi(x)\rangle = E(x)|\psi(x)\rangle$ $H(y)\,|\psi(y)\rangle = E(y)|\psi(y)\rangle$

H(x) and H(y) are independent

$H(x)\,\psi(x) = E(x)\,\psi(x)$

(Abelian)

a bit Fancy topology
appear in
condensed matter physics & graphene

Quantum interference between states nearby Fiber Bundle

Berry connection : $A_\psi = \langle\psi|d\psi\rangle = \langle\psi|\frac{d}{dx}\psi\rangle dx.$

Gauge Transformation $|\psi(x)\rangle = |\psi'(x)\rangle e^{i\Omega(x)}$ gauge potential

$A_\psi = A'_\psi + id\Omega = A'_\psi + i\frac{d\Omega}{dx}dx$

Geometrical quantities

$i\gamma_C(A_\psi) = \int_C A_\psi$: Berry phase :gauge dependent $\bmod 2\pi$

Quantum interference

Gauge Transformation

Geometrical quantities

Bulk σ_{xy} of Filled Fermi sea & Dirac Sea

- Integration of the Manybody Berry Connection of the "Fermi Sea" & "Dirac Sea"

Niu-Thouless-Wu'84

$$\mathcal{A} = \langle \Psi | d\Psi \rangle \quad \sigma_{xy} = \frac{e^2}{h}\frac{1}{2\pi i}\int_{T^2} d\mathcal{A}$$

$|\Psi\rangle$: manybody state

Technology 1

When non-interacting, $|\Psi\rangle = (\boldsymbol{c}^\dagger \boldsymbol{\psi}_1)\cdots(\boldsymbol{c}^\dagger \boldsymbol{\psi}_M)|0\rangle$ Filled Dirac sea

Collect M states below the Fermi level

$$\mathcal{A} = \langle \Psi | d\Psi \rangle = \mathrm{Tr}\, \boldsymbol{A}_D$$

many body — one body matrix valued

$$\boldsymbol{A}_D \equiv \Psi^\dagger d\Psi = \begin{pmatrix} \langle \psi_1^\dagger | d\psi_1 \rangle & \cdots & \langle \psi_1^\dagger | d\psi_M \rangle \\ \vdots & \ddots & \vdots \\ \langle \psi_M^\dagger | d\psi_1 \rangle & \cdots & \langle \psi_M^\dagger | d\psi_M \rangle \end{pmatrix}$$

$$\Psi = (|\psi_1\rangle, \cdots, |\psi_M\rangle)$$

TKNN '82 Chern #'s of one body states

$$\sigma_{xy} = \frac{e^2}{h}\frac{1}{2\pi i}\int_{T^2} \mathrm{Tr}_M dA_D$$

Non Abelian extension for the Chern numbers

YH '04

We just care sum of the Chern numbers not each of them

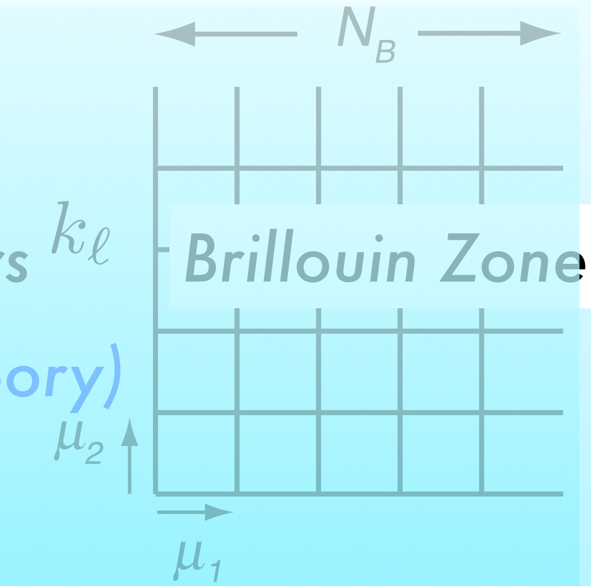
Numerical advantage for graphene : L.L. of Filled Dirac sea

Numerical Technique from the Lattice gauge theory

☆ Topological Invariant on Discretized Lattice

Lattice in k space (discretization for the integral)

Technical Advantage for large Chern Numbers



$$\sigma_{xy} = \frac{e^2}{h}\frac{1}{2\pi i}\sum F_{1234}$$

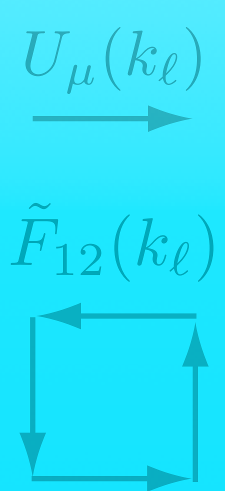
Luscher '82 (Lattice Gauge Theory)
gauge invariant

$$F_{1234} = \mathrm{Im}\log U_{12}U_{23}U_{34}U_{41}$$

$$U_{mn} = \det_j \Psi_m^\dagger \Psi_n, \quad \Psi_n = (\psi_1(k_n), \cdots, \psi_j(k_n))$$

Fermi Sea of j filled bands

Technology 2
Chern number extension of the KSV formula for polarization



$$U_\mu(k_\ell) \equiv \langle n(k_\ell)|n(k_\ell+\hat\mu)\rangle/\mathcal{N}_\mu(k_\ell)$$

$$\mathcal{N}_\mu(k_\ell) = |\langle n(k_\ell)|n(k_\ell+\hat\mu)\rangle|$$

$$\tilde F_{12}(k_\ell) \equiv \ln U_1(k_\ell)U_2(k_\ell+\hat 1)U_1(k_\ell+\hat 2)^{-1}U_2(k_\ell)^{-1}$$

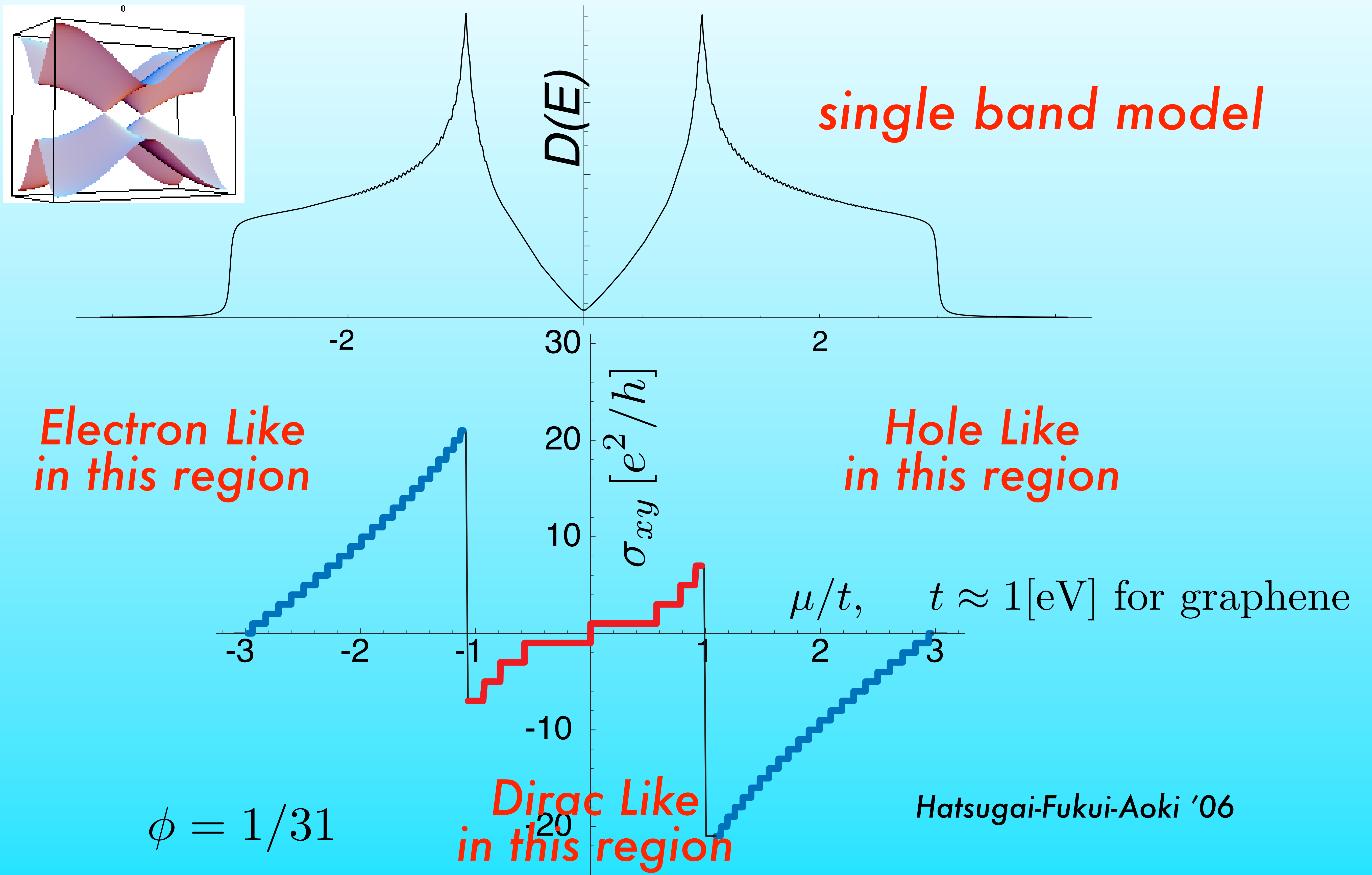
$$-\pi < \tilde F_{12}(k_\ell)/i \le \pi$$ *(principal value)*

Berry phase formula for polarization: King-Smith & Vanderbilt '93

Analogue for the Chern numbers
Fukui-Hatsugai-Suzuki 2005

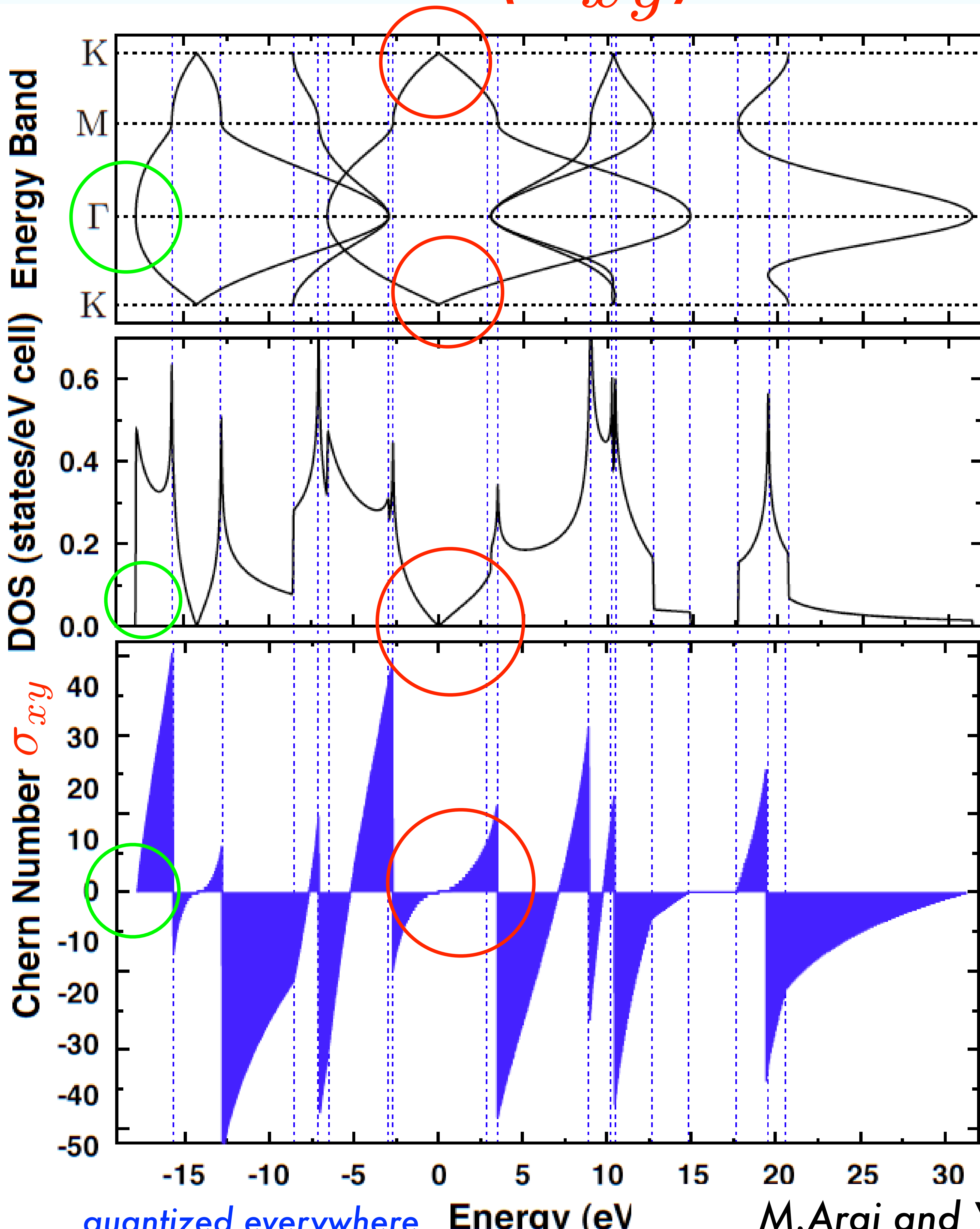
Hall Conductace vs chemical potential

- Accurate Hall conductance over whole spectrum



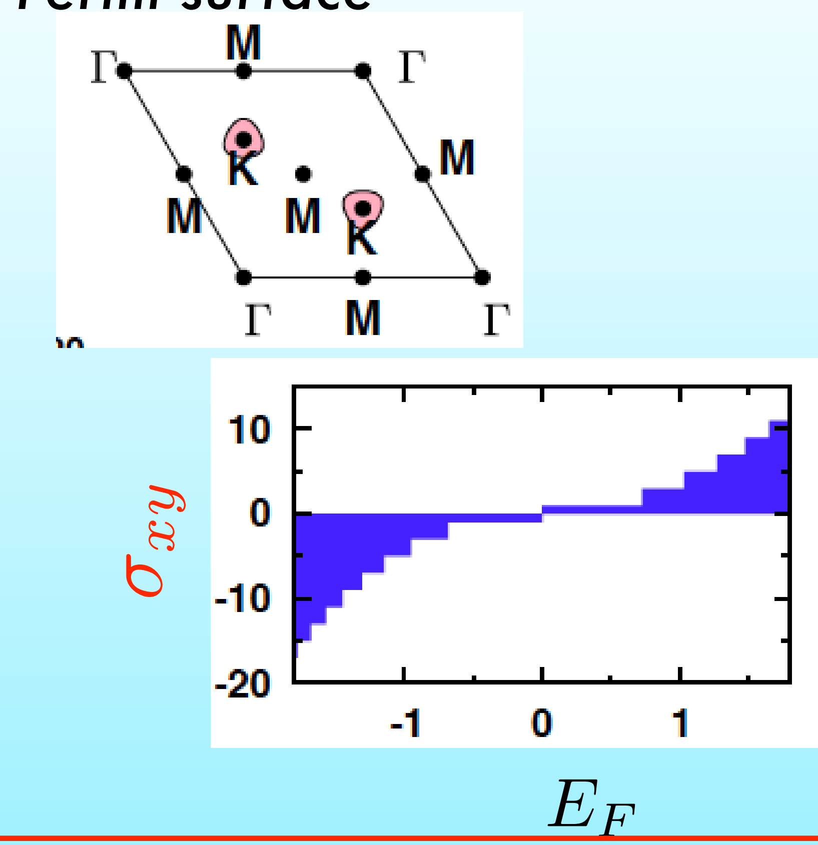
Hatsugai-Fukui-Aoki '06

Chern numbers (σ_{xy}) based on Realistic Band Calc.

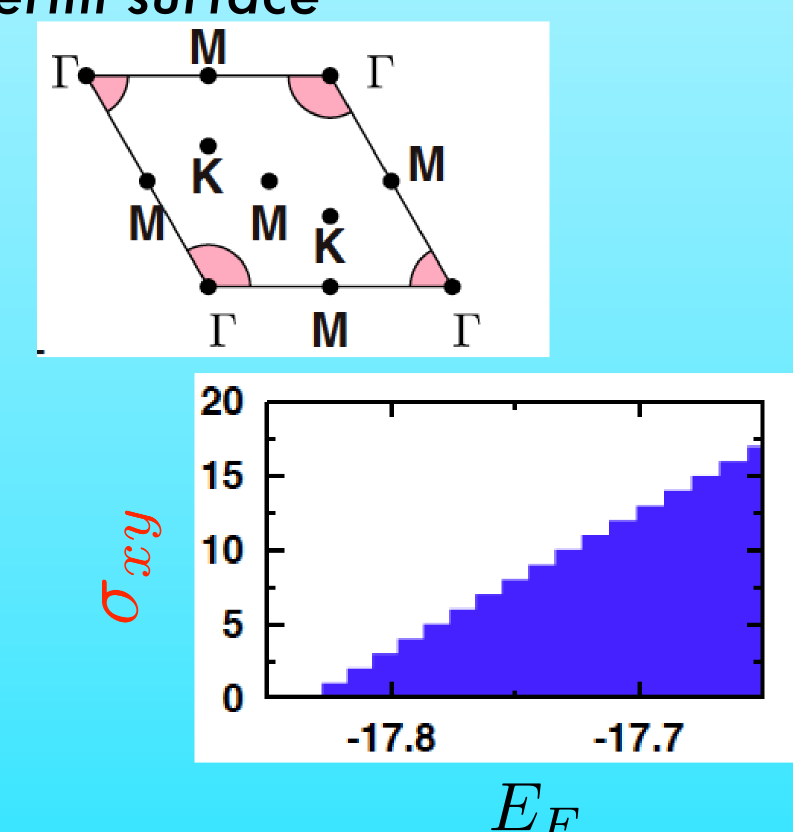


quantized everywhere

Fermi surface



Fermi surface



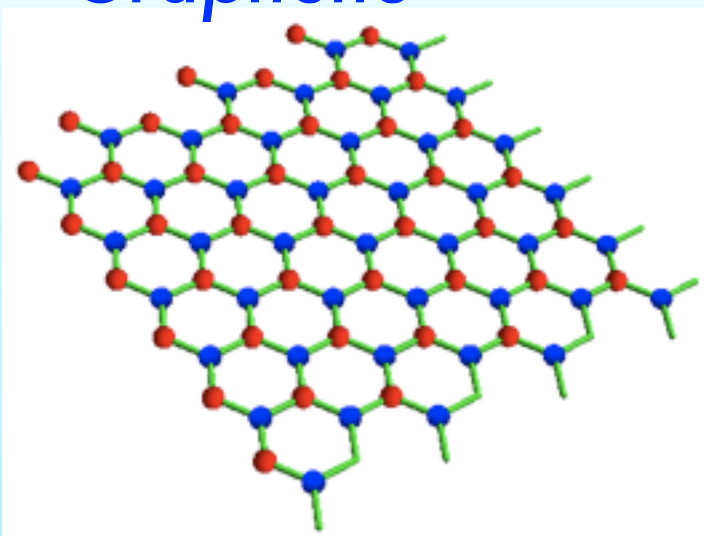
M.Arai and Y.Hatsugai, Phys.Rev. B79, 075429 (2009)

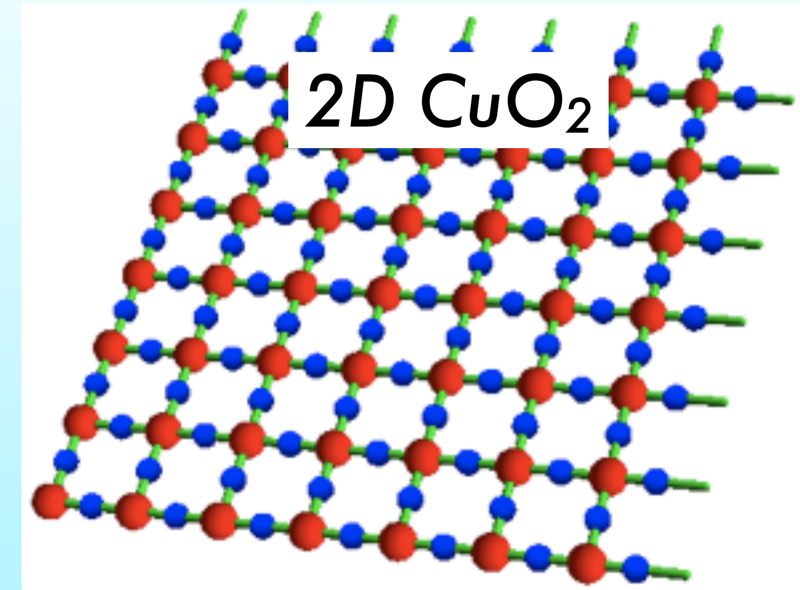
Topological aspects of graphene : Edge

- Zero modes at the zigzag & bearded edges
- Bulk-Edge correspondence for Dirac sea in a magnetic field
- Bulk–Edge correspondence for other phenomena

Topological Universality for zero modes of Dirac Fermions

Graphene



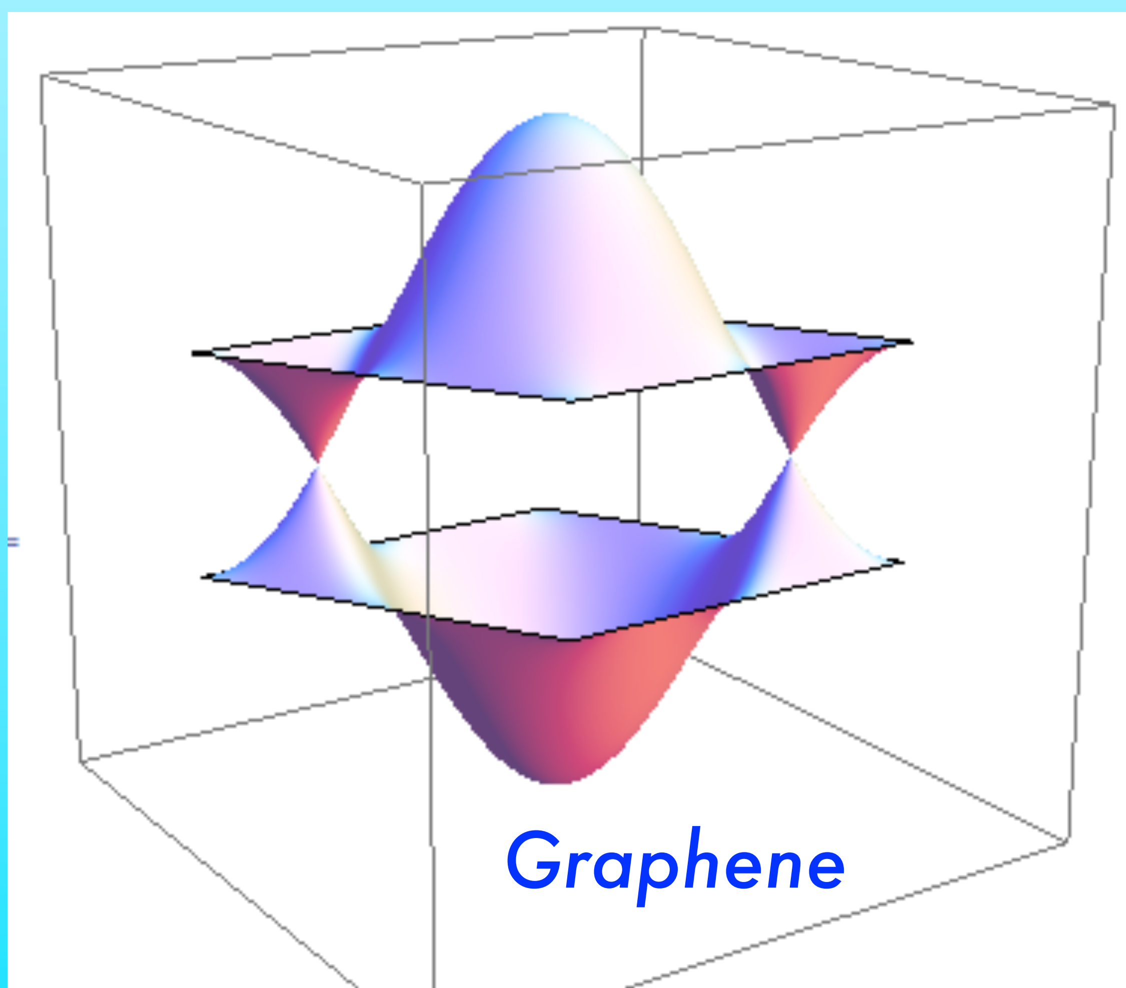


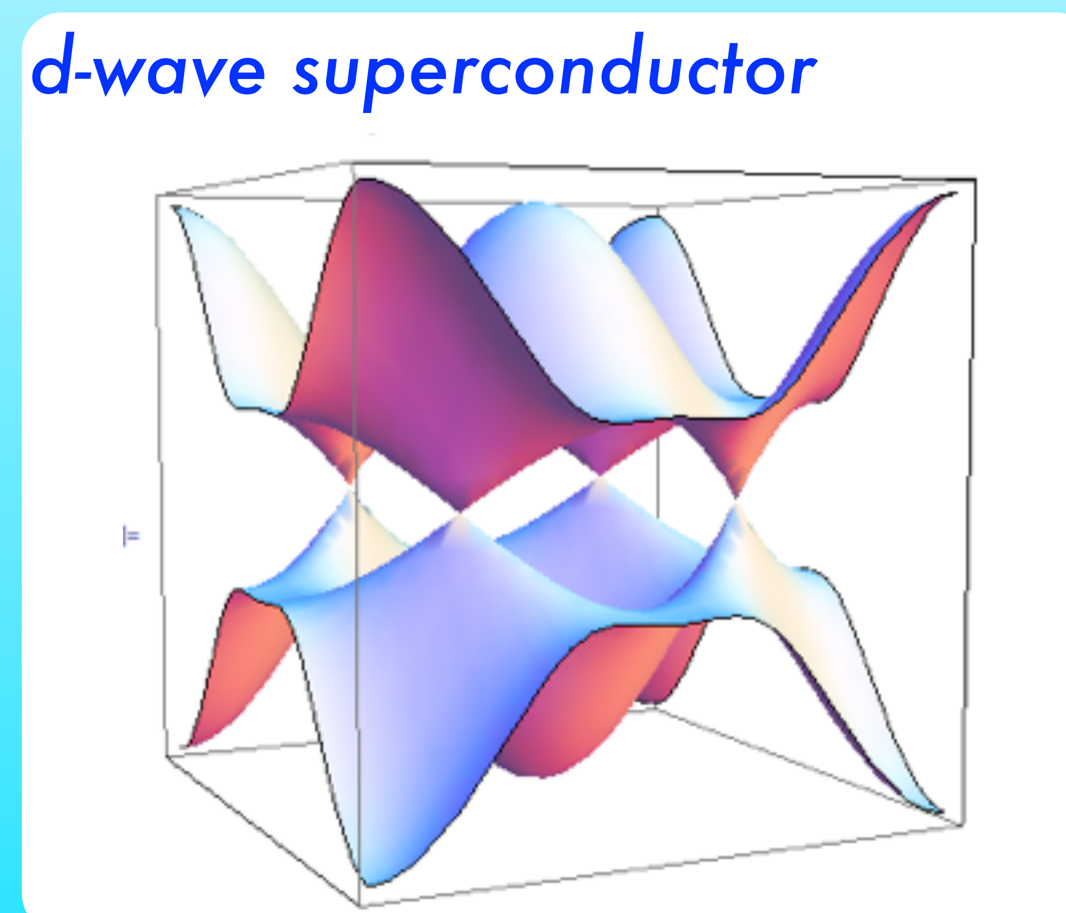
2D Dirac fermions :

Edge States

Zero mode localized states

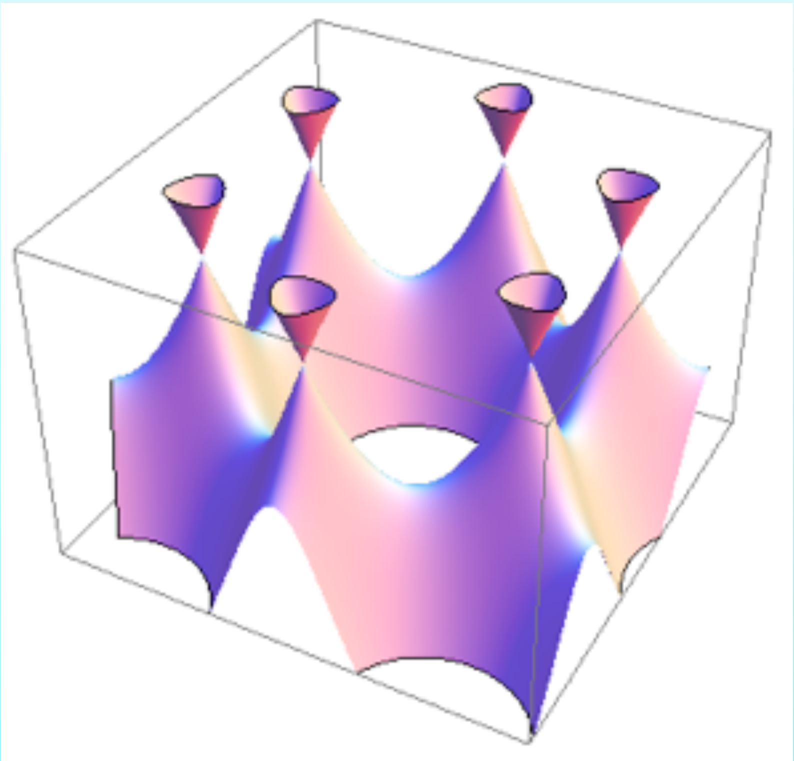
YH, '09 (review)



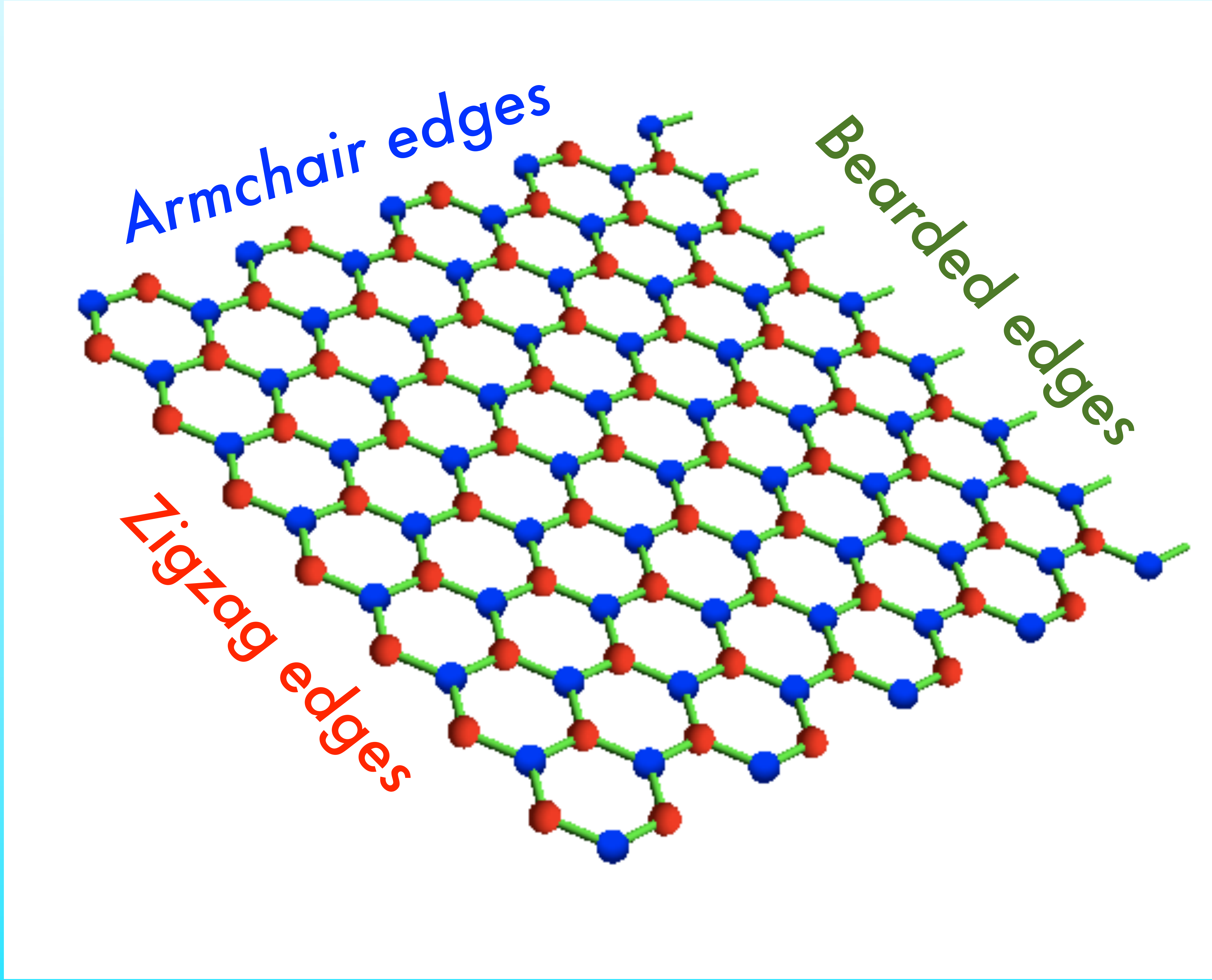


Zero mode localized states ??

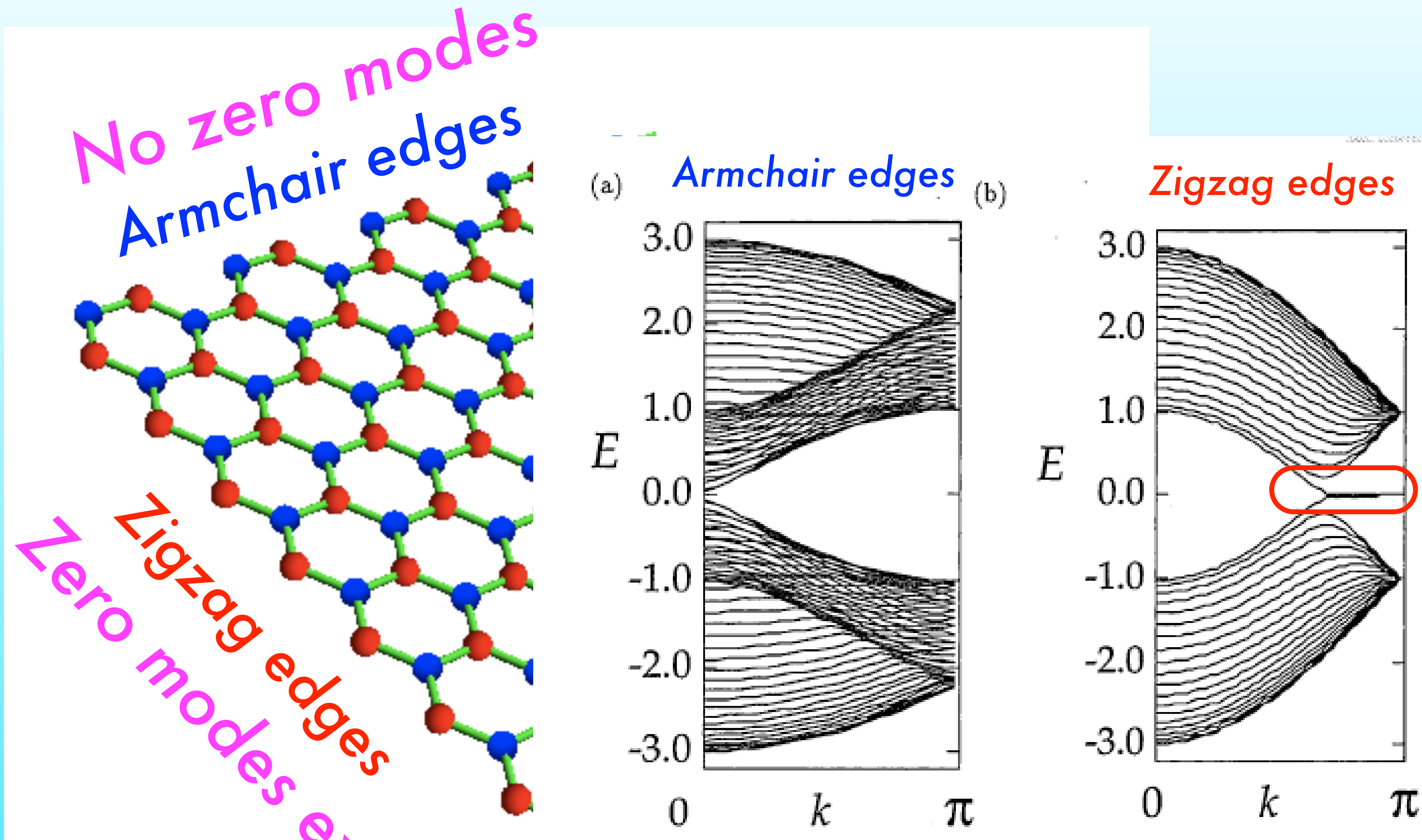
Graphene



Several types in edges



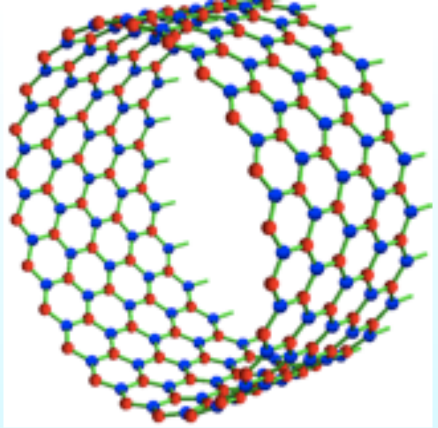
Zero mode localized states ??



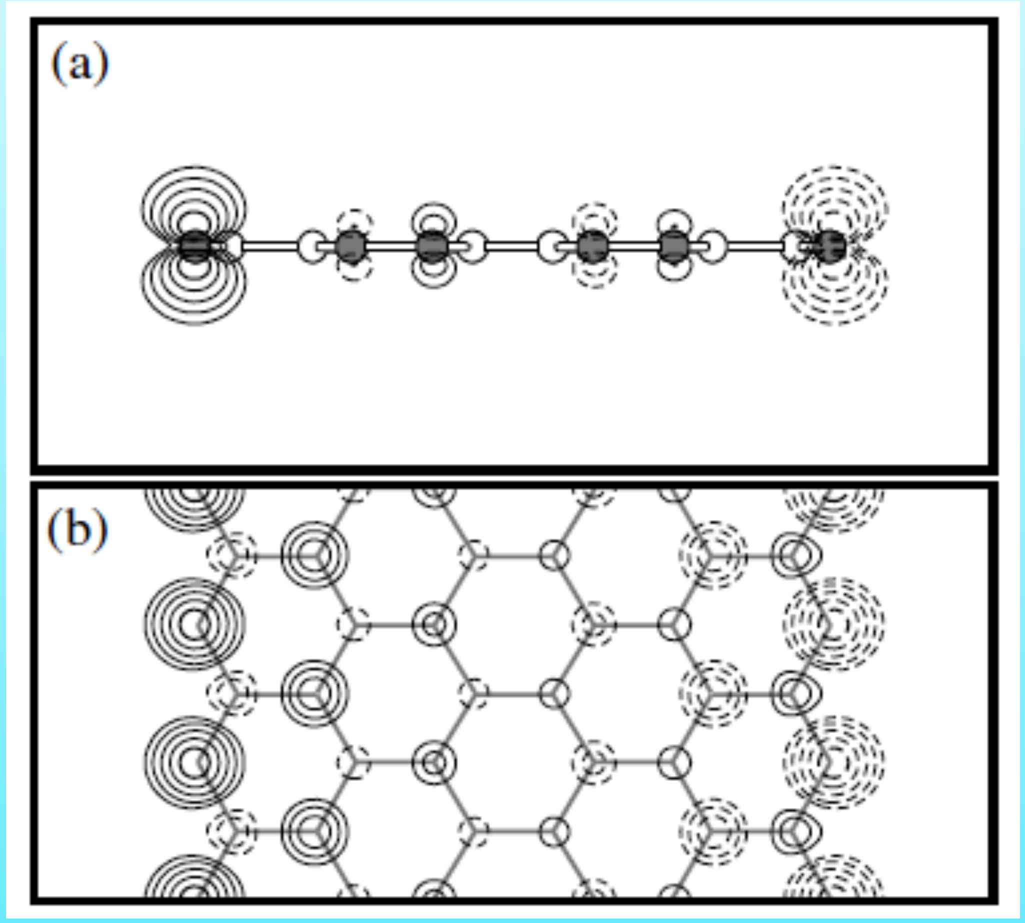
Fujita et al., J. Phys. Soc. Jpn. 65, 1920 (1996)

Fujita et al.,'96

It's real !

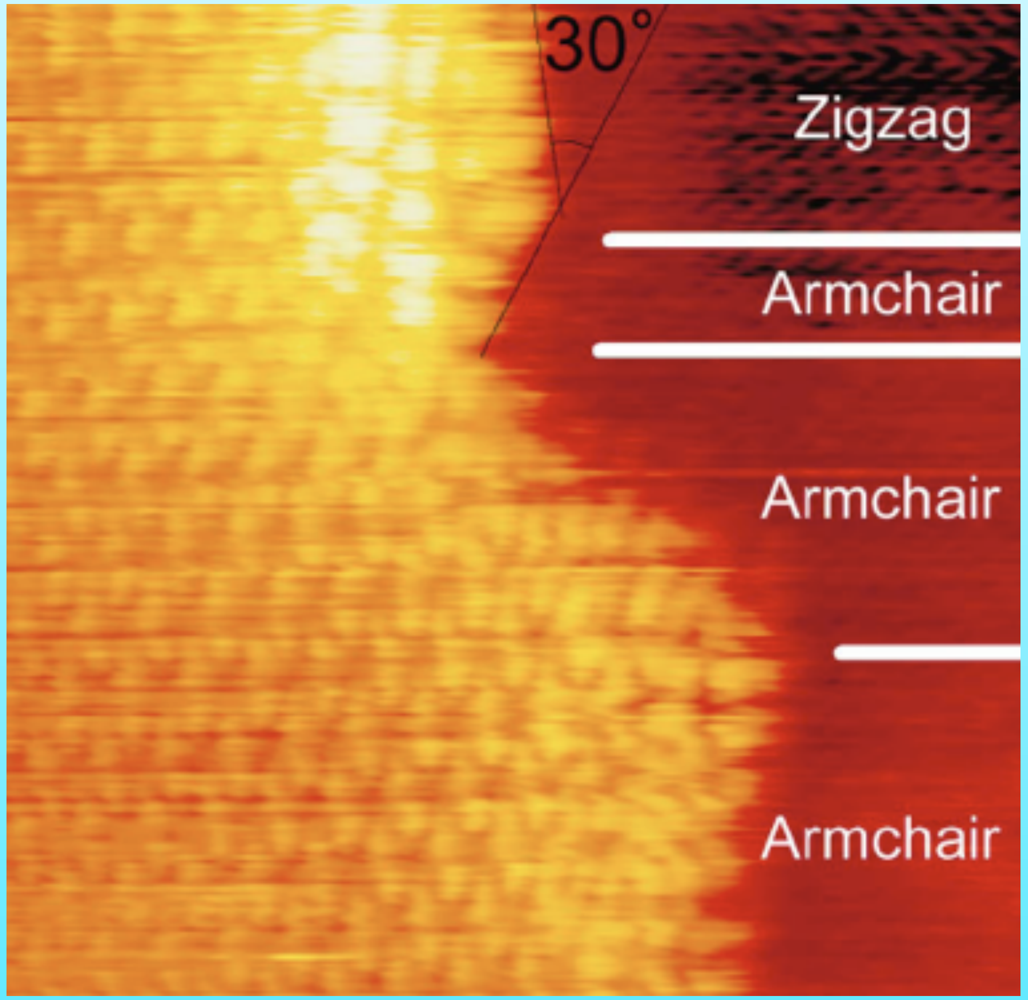


First principle calculation



Okada and Oshiyama,
Phys. Rev. Lett. 87, 146803 (2001)

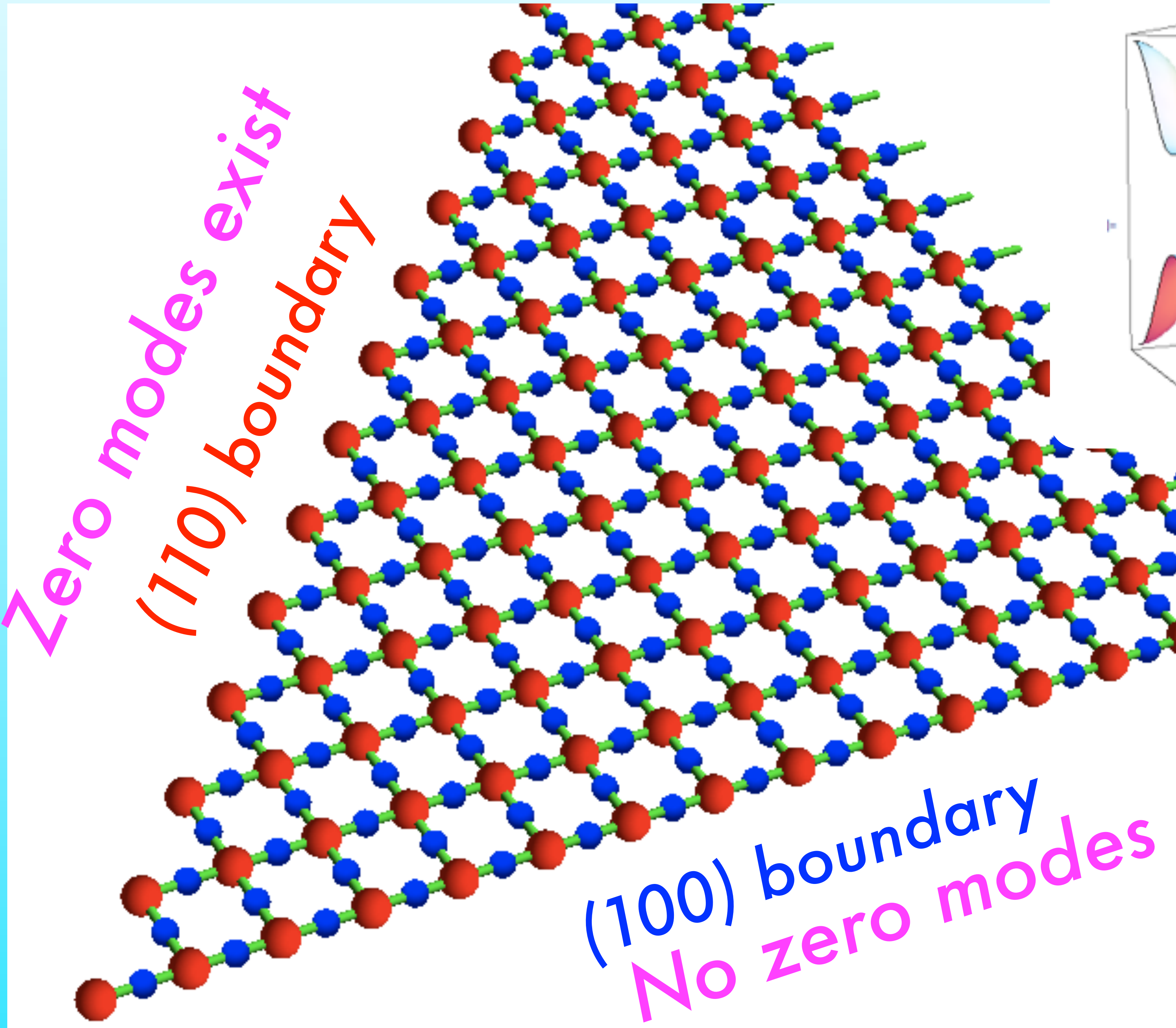
STM image



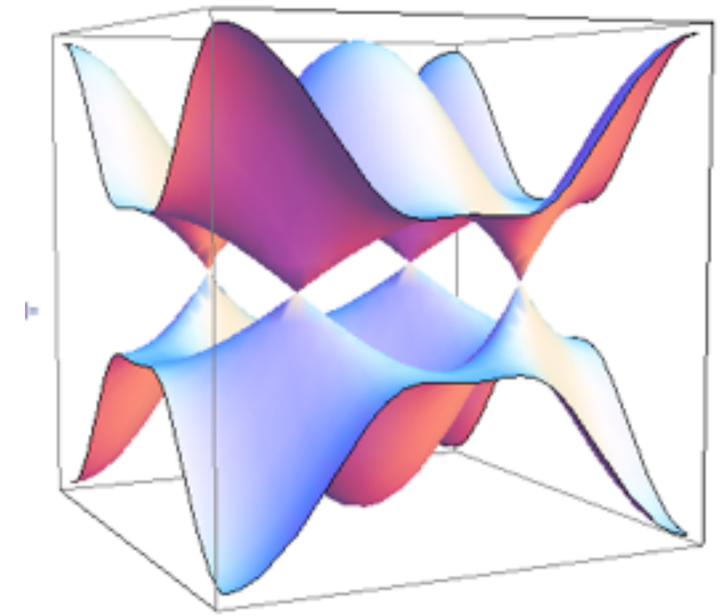
Kobayashi et al,
Phys. Rev. B71, 193406 (2005)

Zero mode localized states as Andreev bound state

d-wave superconductor



d-wave superconductor



Andreev bound states

Hu, '94

Zero Bias Conductance Peak in Anisotropic Superconductivity

d-wave superconductivity

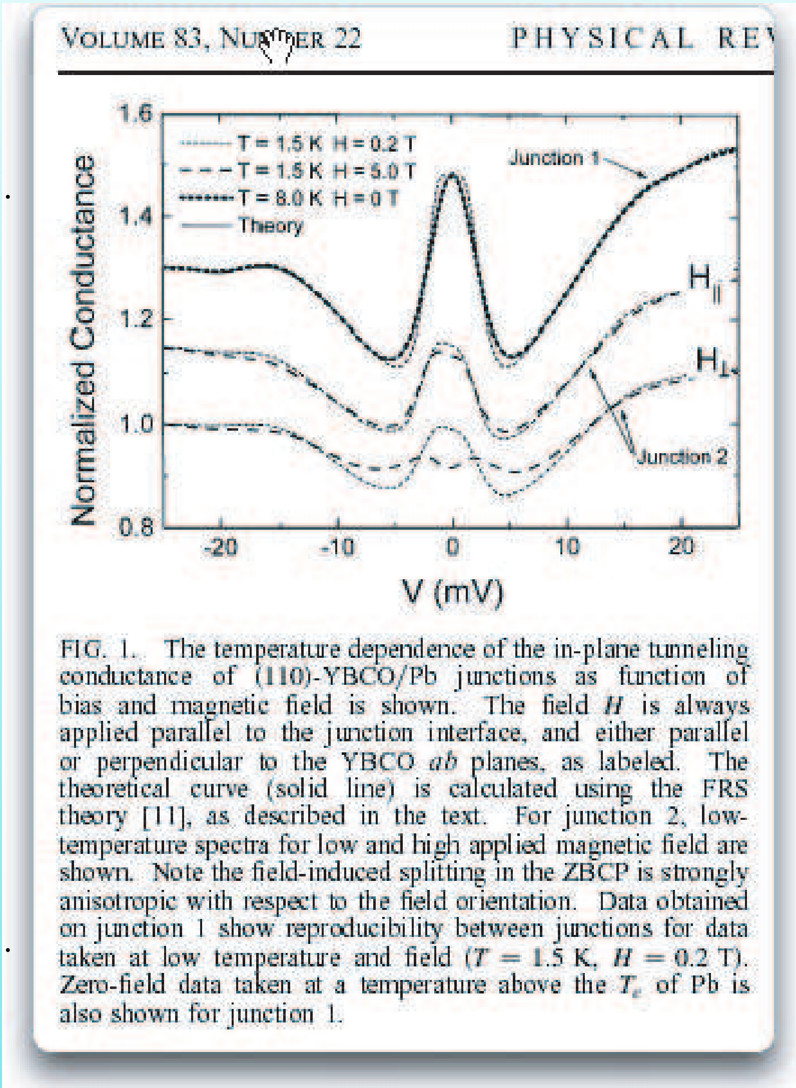


FIG. 1. The temperature dependence of the in-plane tunneling conductance of (110)-YBCO/Pb junctions as function of bias and magnetic field is shown. The field H is always applied parallel to the junction interface, and either parallel or perpendicular to the YBCO ab planes, as labeled. The theoretical curve (solid line) is calculated using the FRS theory [11], as described in the text. For junction 2, low-temperature spectra for low and high applied magnetic field are shown. Note the field-induced splitting in the ZBCP is strongly anisotropic with respect to the field orientation. Data obtained on junction 1 show reproducibility between junctions for data taken at low temperature and field ($T = 1.5$ K, $H = 0.2$ T). Zero-field data taken at a temperature above the T_c of Pb is also shown for junction 1.

Zero Energy Boundary States of Anisotropic Superconductivity

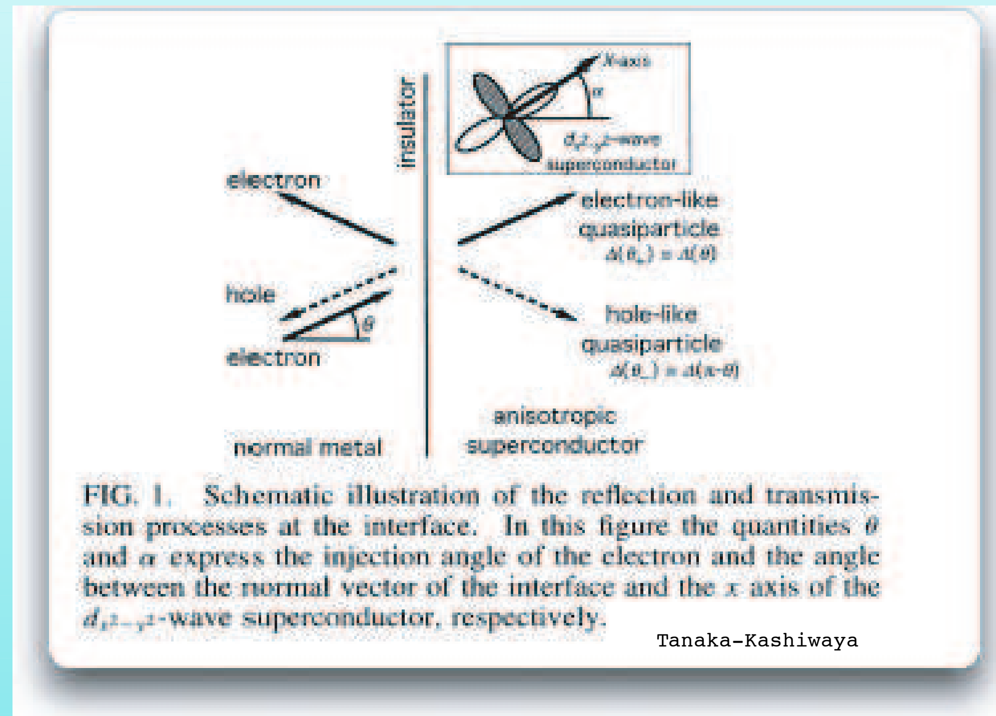


FIG. 1. Schematic illustration of the reflection and transmission processes at the interface. In this figure the quantities θ and α express the injection angle of the electron and the angle between the normal vector of the interface and the x axis of the $d_{x^2-y^2}$-wave superconductor, respectively.

L. J. Buchholtz,G. Zwicknagl, Phys. Rev. B 23, 5788 (1981) (p wave)

C.-R. Hu, Phys. Rev. Lett. 72, 1526 (1994) (d wave)

S. Kashiwaya, Y. Tanaka, Phys. Rev. Lett. 72, 1526 (1994)

M. Matsumoto and H. Shiba, JPSJ, 1703 (1995)

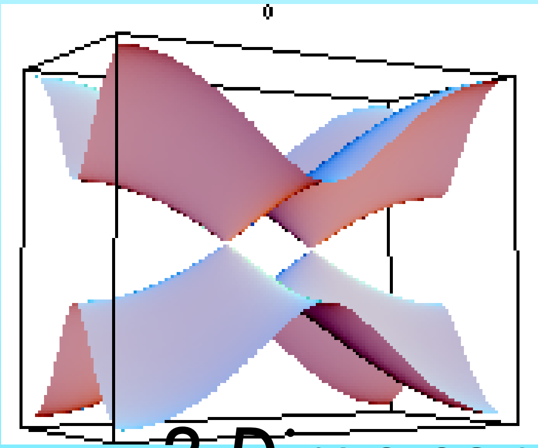
(fig.) M. Aprili, E. Badica, and L. H. Greene,Phys. Rev. Lett. 83, 4630 (1999)

Universality of Zero Energy Edge States

'02–'04 S. Ryu & YH

- ☑ Zero energy edge states of graphene
- ☑ Andreev bound states of d-wave superconductors

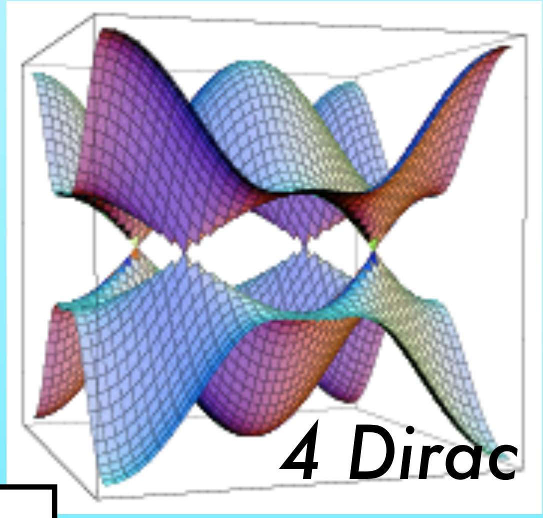
graphene



2 Dirac cones

These 2 systems are topologically equivalent

d-wave superconductor



4 Dirac cones

Symmetry protected
Zero modes of Dirac fermions
:1D Flat Band of edge states

Γ :Bipartite

(A-B sublattice symmetry)

Γ :Time Reversal

(Real Order parameter)

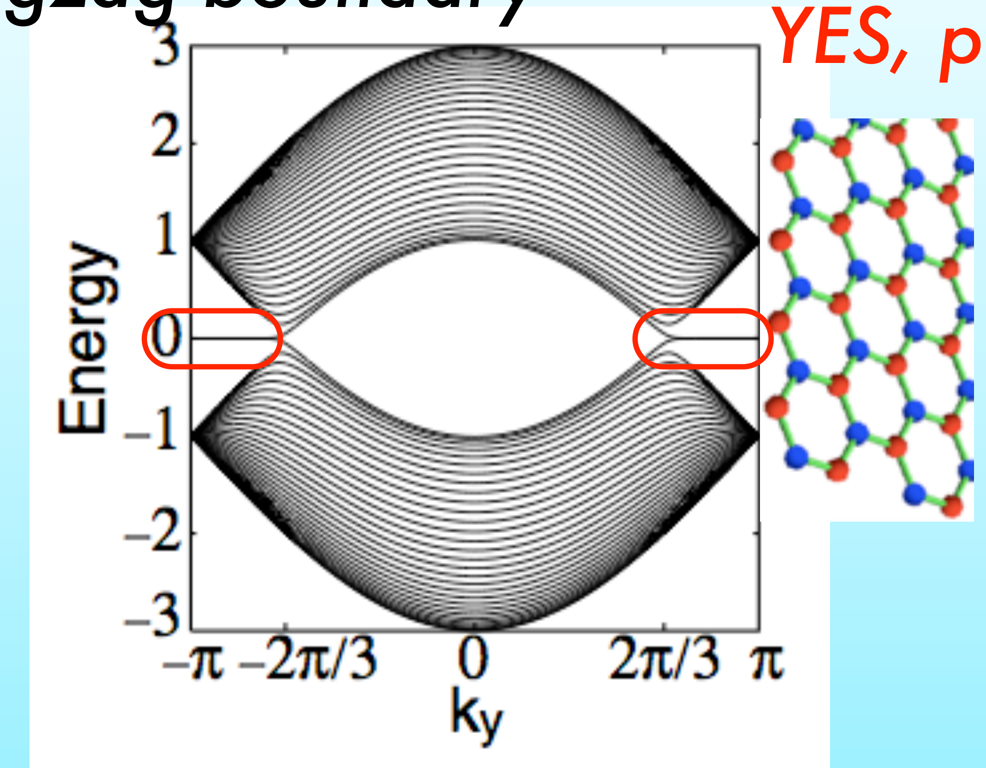
$^{\exists}\Gamma$ chiral symmetry

$$\{\Gamma, H\} = \Gamma H + H\Gamma = 0\ ,\ \Gamma^2 = 1$$

Are there zero mode edge states ? YES or NO

Graphene

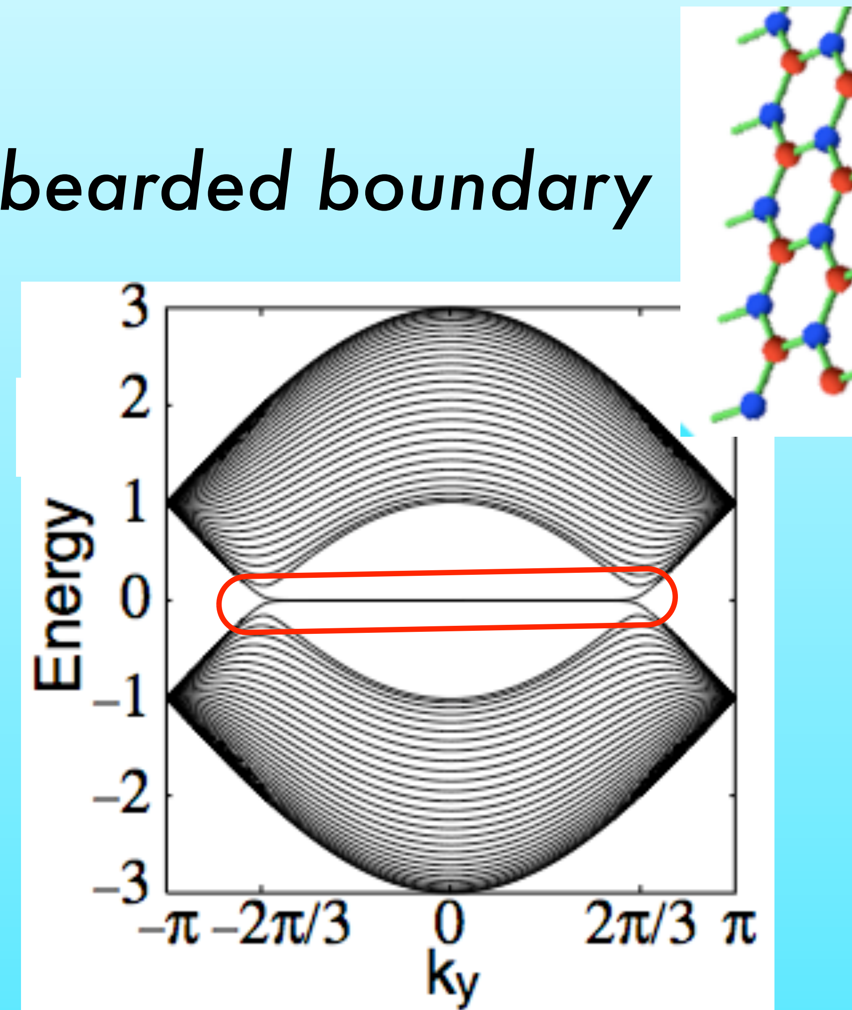
zigzag boundary



YES, partially

$$\frac{2\pi}{3} < k_y < \frac{4\pi}{3}$$

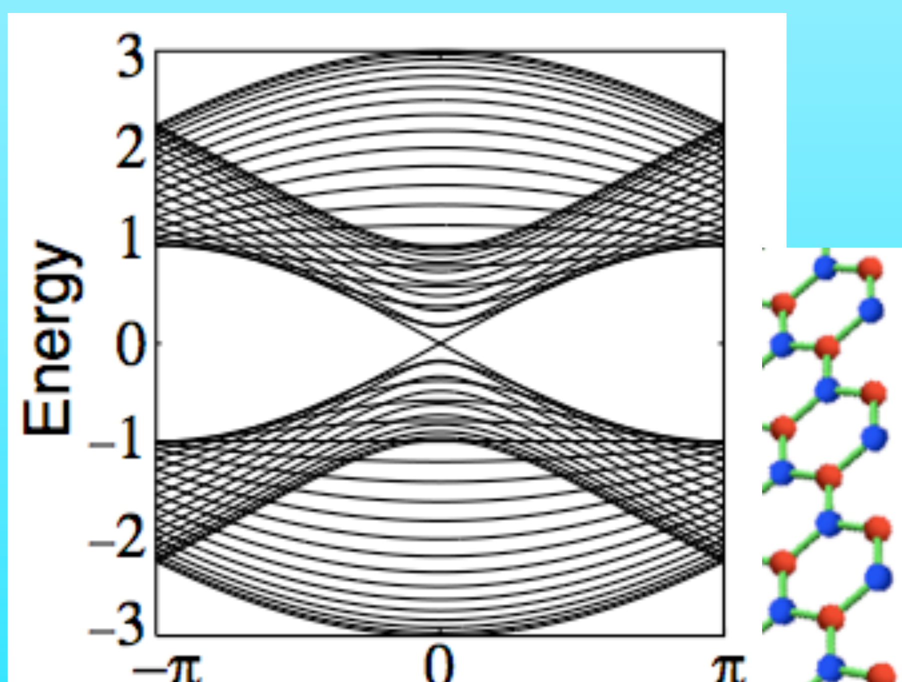
bearded boundary



YES, partially

$$|k_y| < \frac{2\pi}{3}$$

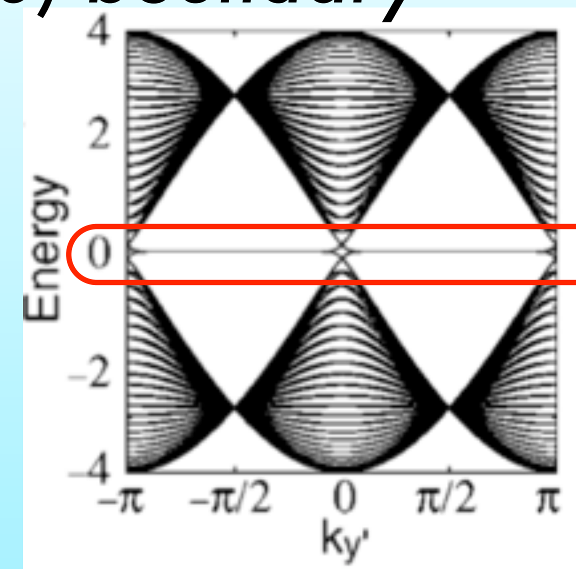
Armchair boundary



No

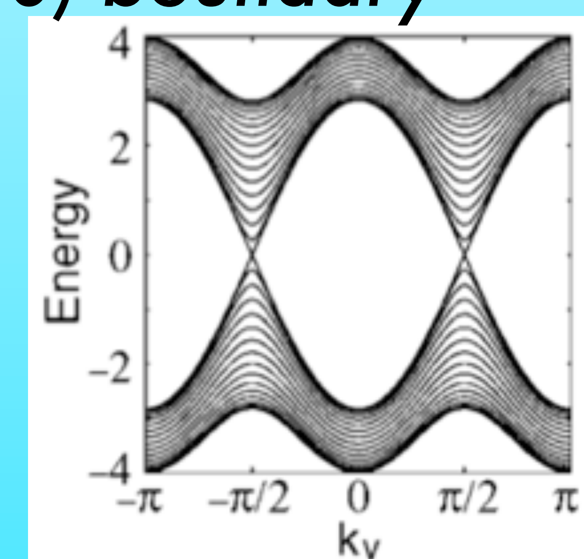
d-wave superconductivity

(110) boundary



YES always

(100) boundary



No

Z_2 Berry phases determine the zero modes

Lattice analogue of Witten's SUSY QM

S.Ryu & Y.Hatsugai, '02
YH'06

Berry phase for each k_y

$$\gamma(k_y) = \int_{k_y:\mathrm{fixed}} dk_x\, A(k_x, k_y) \qquad A = \langle \psi(k) | \nabla_k \psi(k) \rangle$$

Zak

Require Local Chiral Symmetry
(ex. bipartite)

$$\{\Gamma, H\} = \Gamma H + H\Gamma = 0$$

Z_2 quantization 1D

$$\gamma(k_y) = \begin{cases} \pi \\ 0 \end{cases}$$

c.f. Z_2 in 3D TR inv. case

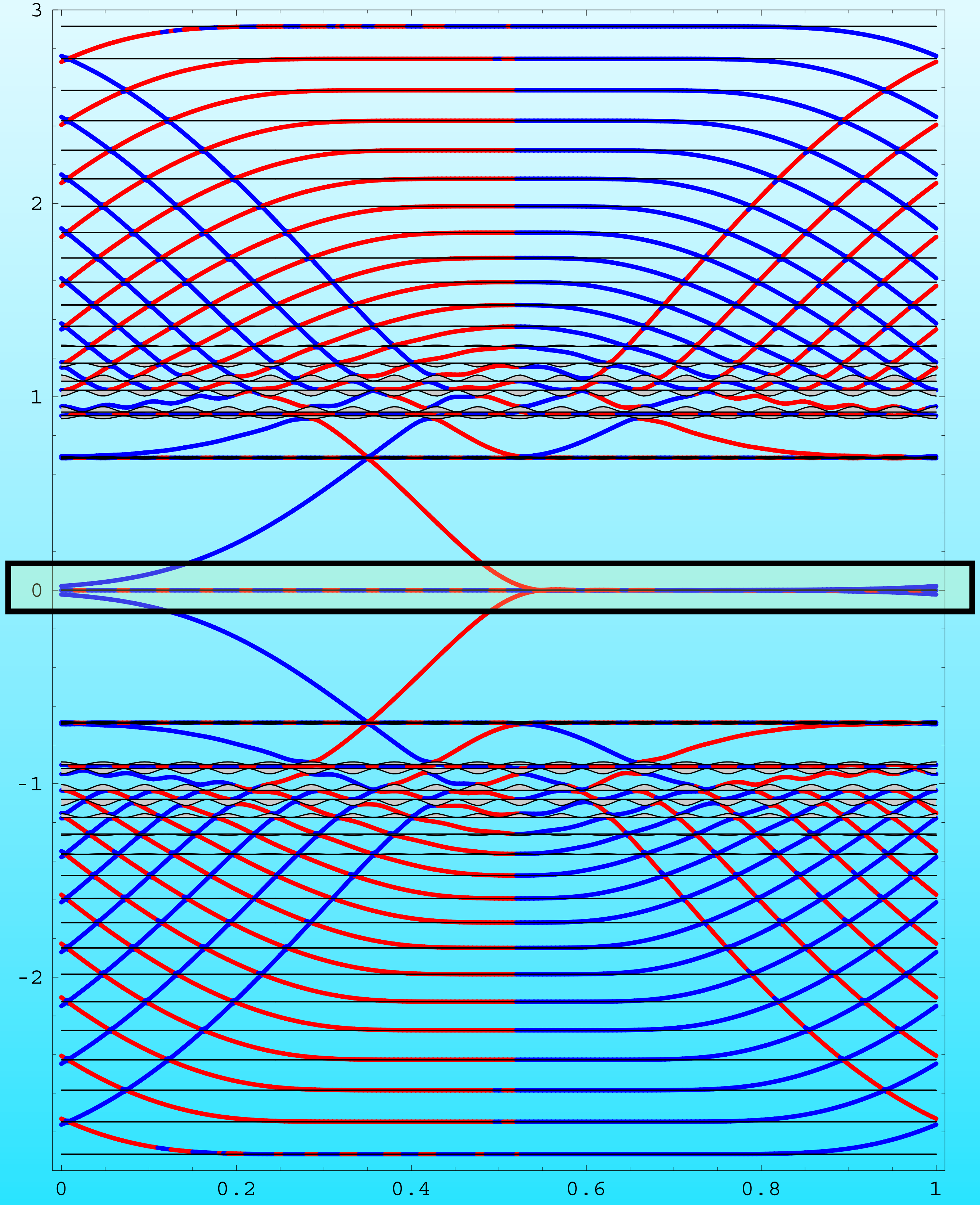
$\gamma(k_y) = \pi$ ⇒ **Zero energy localized states EXIST**

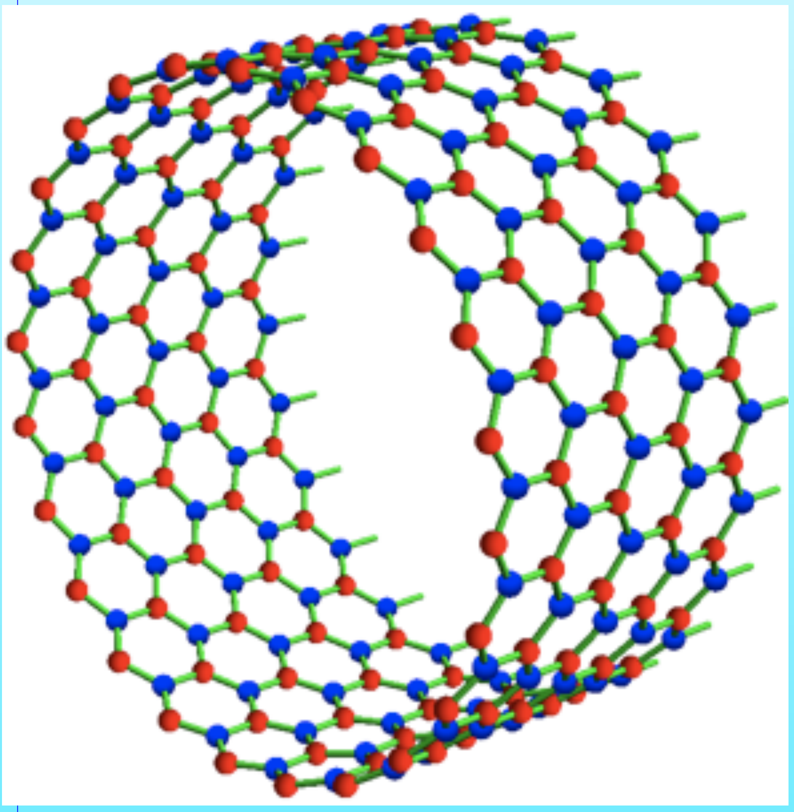
: There exists odd number of zero modes

Bulk-edge correspondence: "Bulk determines the edges"

With magnetic field

M. Arikawa, H. Aoki & YH, Phys. Rev. B79, 075429 (2009)





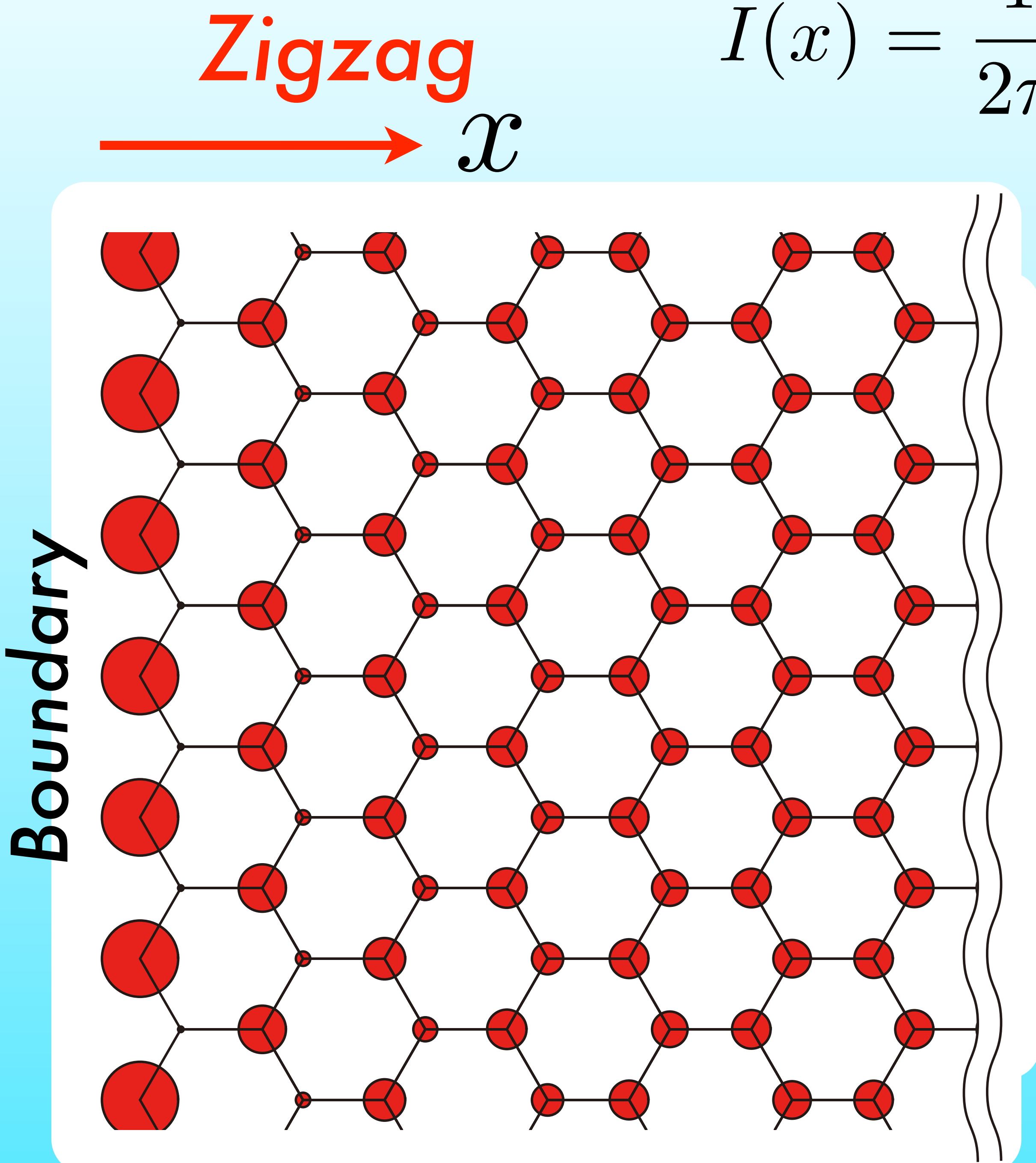
Landau Levelsof graphene with edges

LDOS around E=0 (with Landau Level)

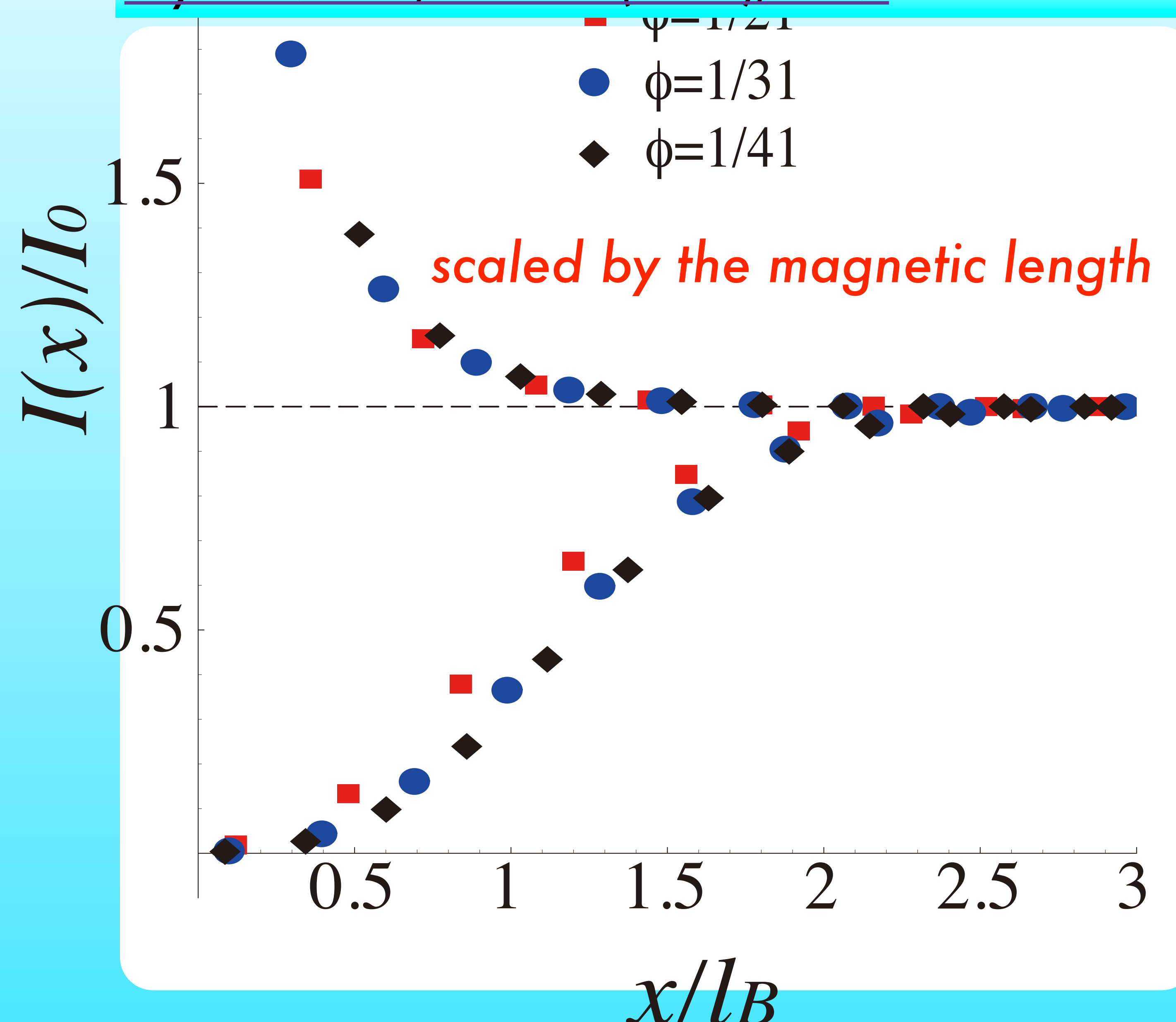
$$I(x) = \frac{1}{2\pi}\int_{-E_C}^{E_C} dE \int_{-\pi}^{\pi} dk_y \, |\Psi(x, k_y, E)|^2$$

M. Arikawa, H. Aoki & YH

Phys. Rev. B79, 075429 (2009),arXiv: & 0806.2429



STM observable



Strong enhancement near the edge

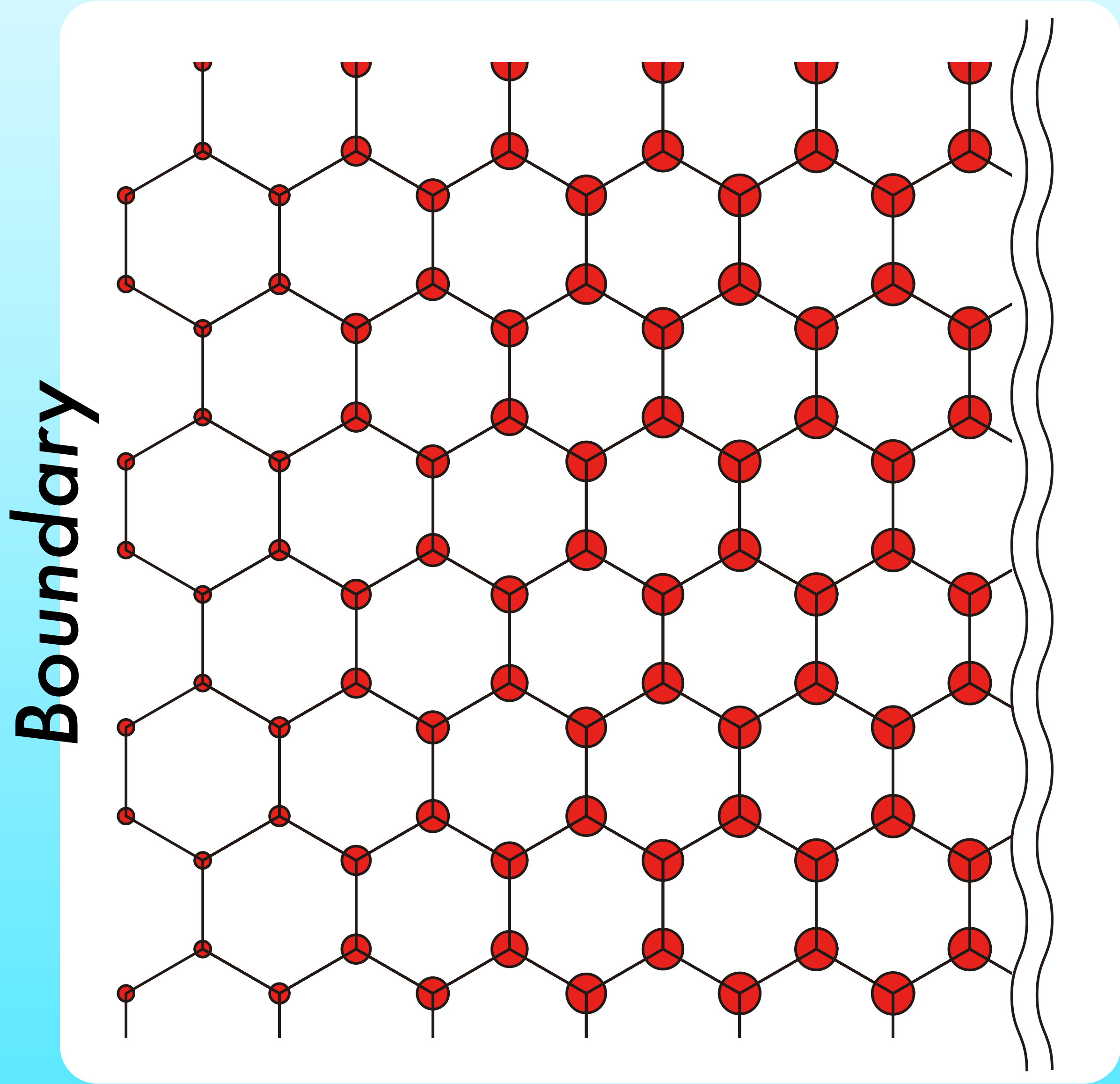
Characteristic feature of the Graphene Zigzag edges!

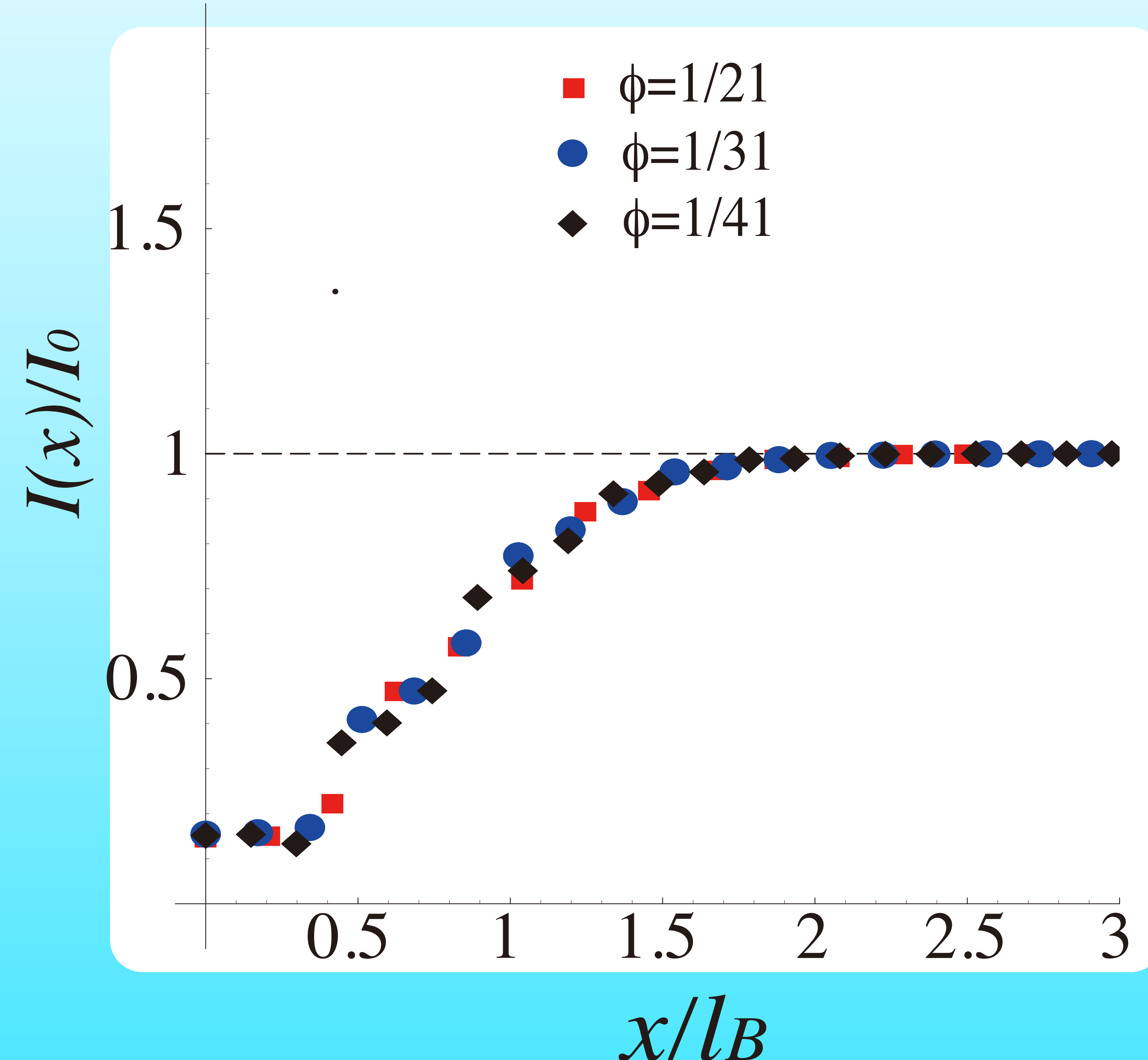
LDOS around E=0 (with Landau Level)

$$I(x) = \frac{1}{2\pi}\int_{-E_C}^{E_C} dE \int_{-\pi}^{\pi} dk_y\, |\Psi(x, k_y, E)|^2$$

Armchair

x

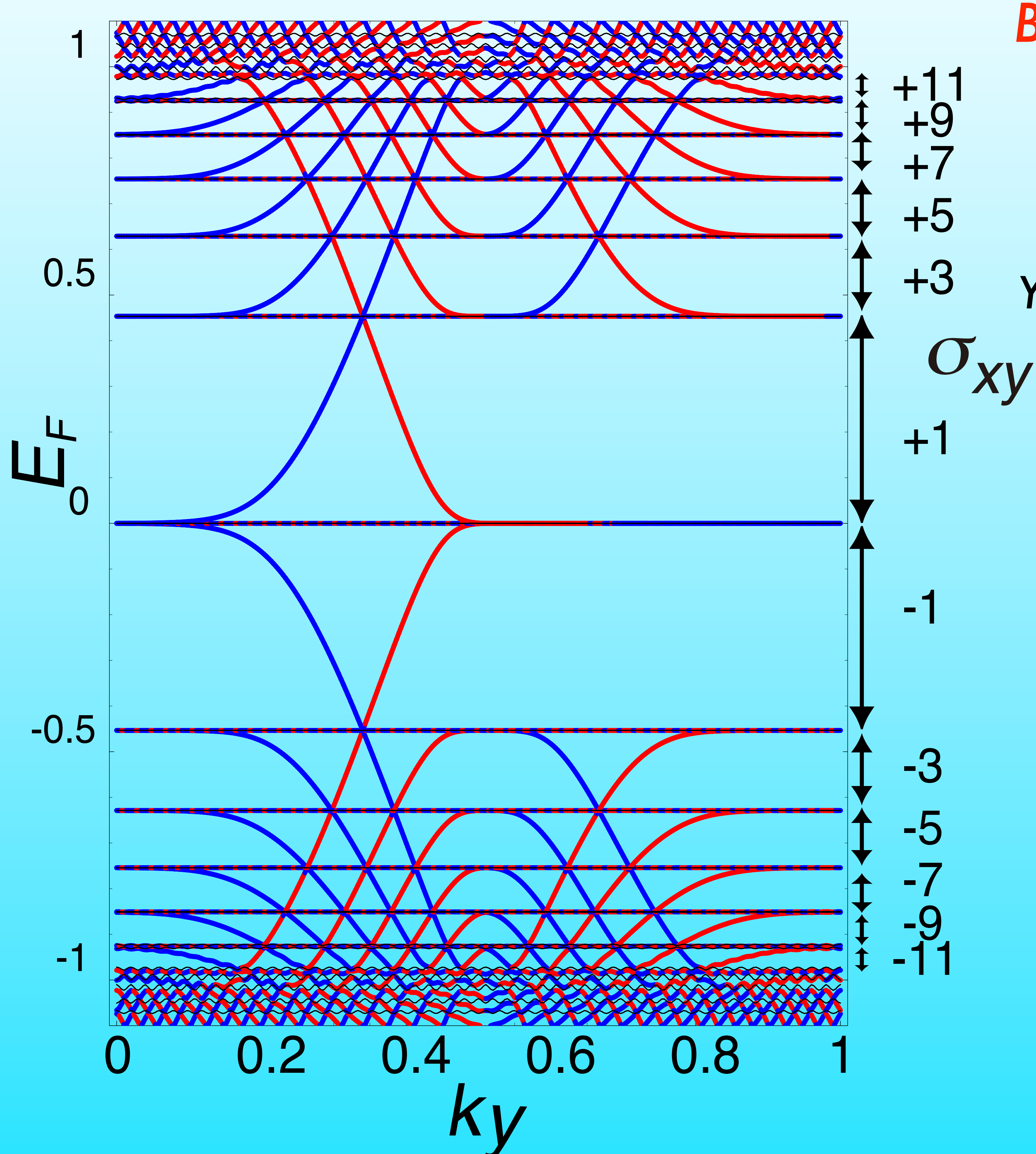




Suppression near the edge

Standard behavior due to edge potential

QH edge states of graphene (classic)

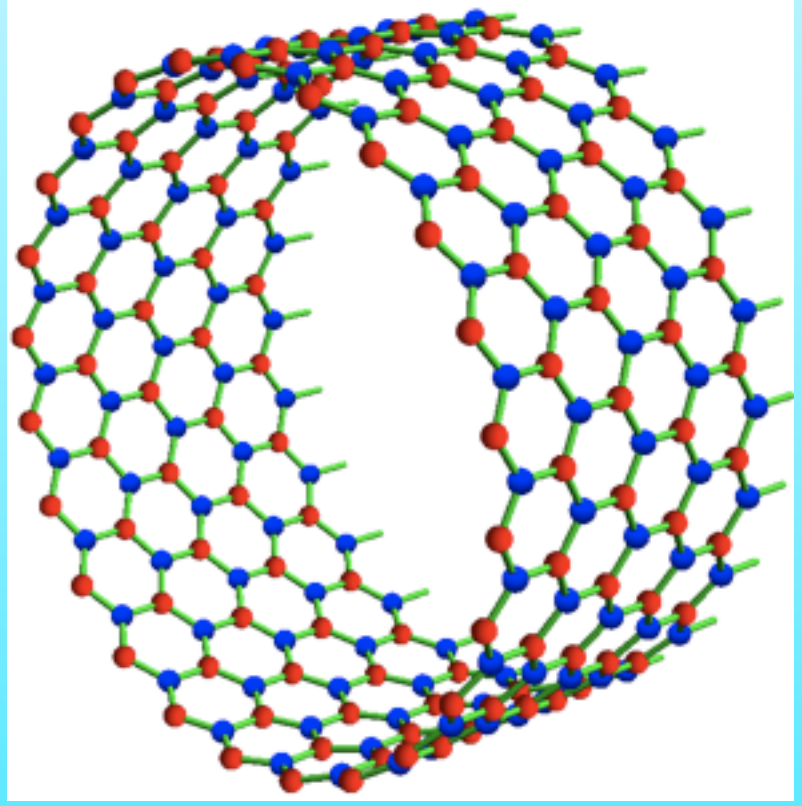


Bulk — Edge Correspondence

$$\sigma_{xy}^{\mathrm{bulk}} = \sigma_{xy}^{\mathrm{edge}}$$

Y. Hatsugai, PRL 71, 3691 (1993)

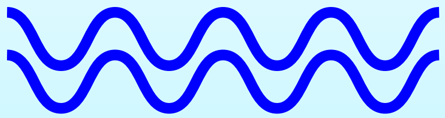
YH Fukui Aoki, PRB 74, 205414 (2006)

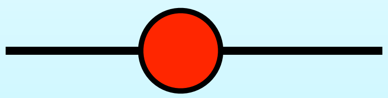


Landau Levels of graphene with edges

Bulk-Edge correspondence

Universality





Bulk state (scattering state) Bulk Gap Non trivial Vacuum	Control with each other ⟷	Edge state (Bound state) Particles in the gap

Y. Hatsugai, PRL 71, 3691 (1993)

Bulk determines the edges

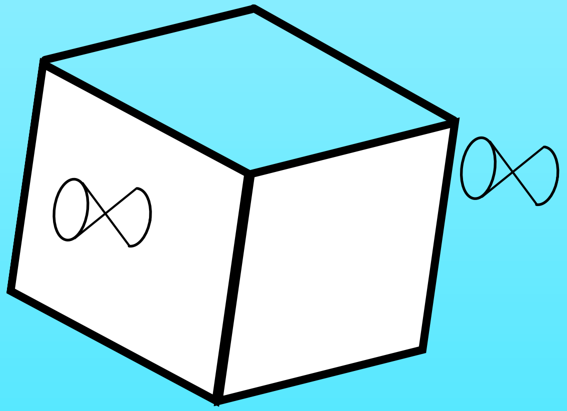
Edge characterizes the bulk

c.f. holographic principle (AdS-CFT)

Spin Hall edge states

2D

3D



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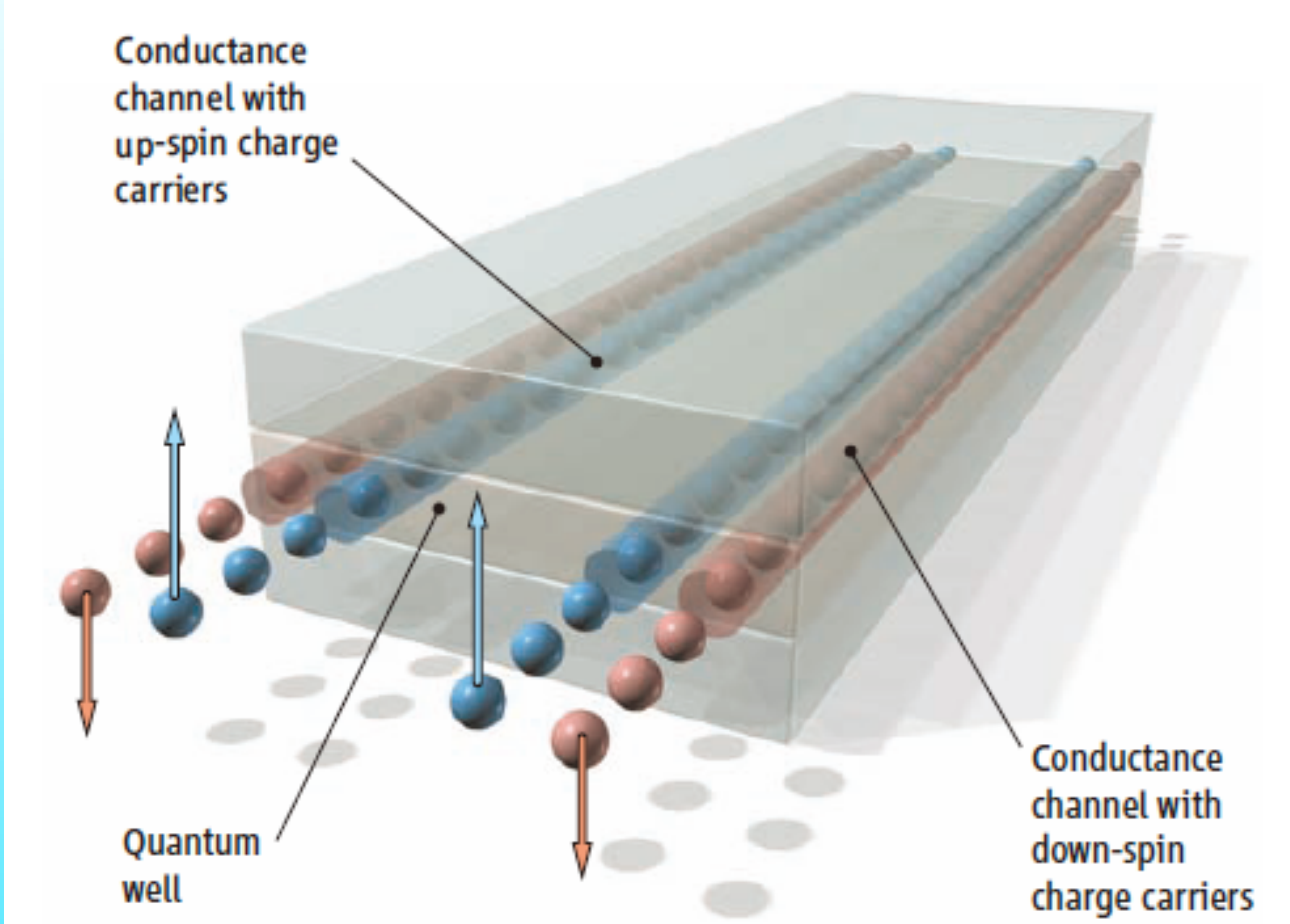


Schematic of the spin-polarized edge channels in a quantum spin Hall insulator.

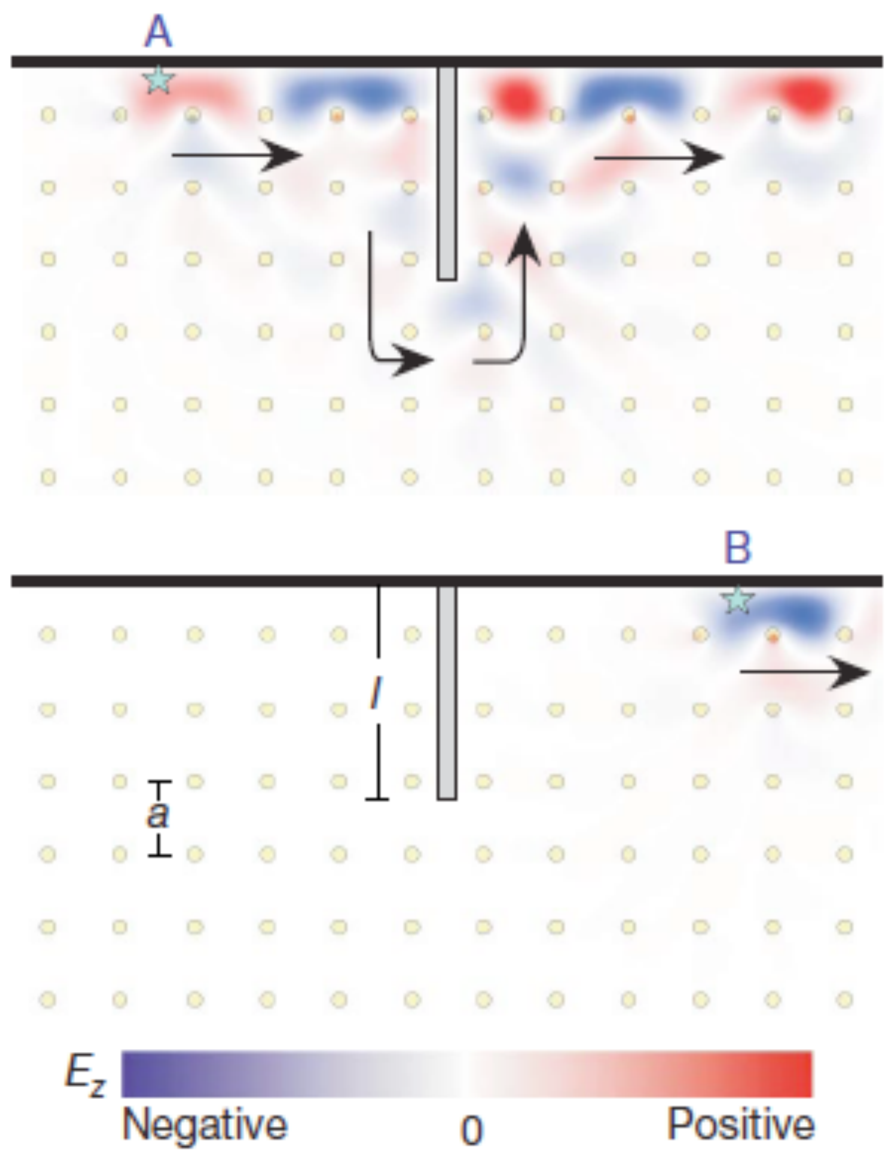
Konig, Wiedmann, Brüne, Roth, Hartmut Buhmann, Molenkamp,Qi and Zhang, Science 318, 776 (2007)

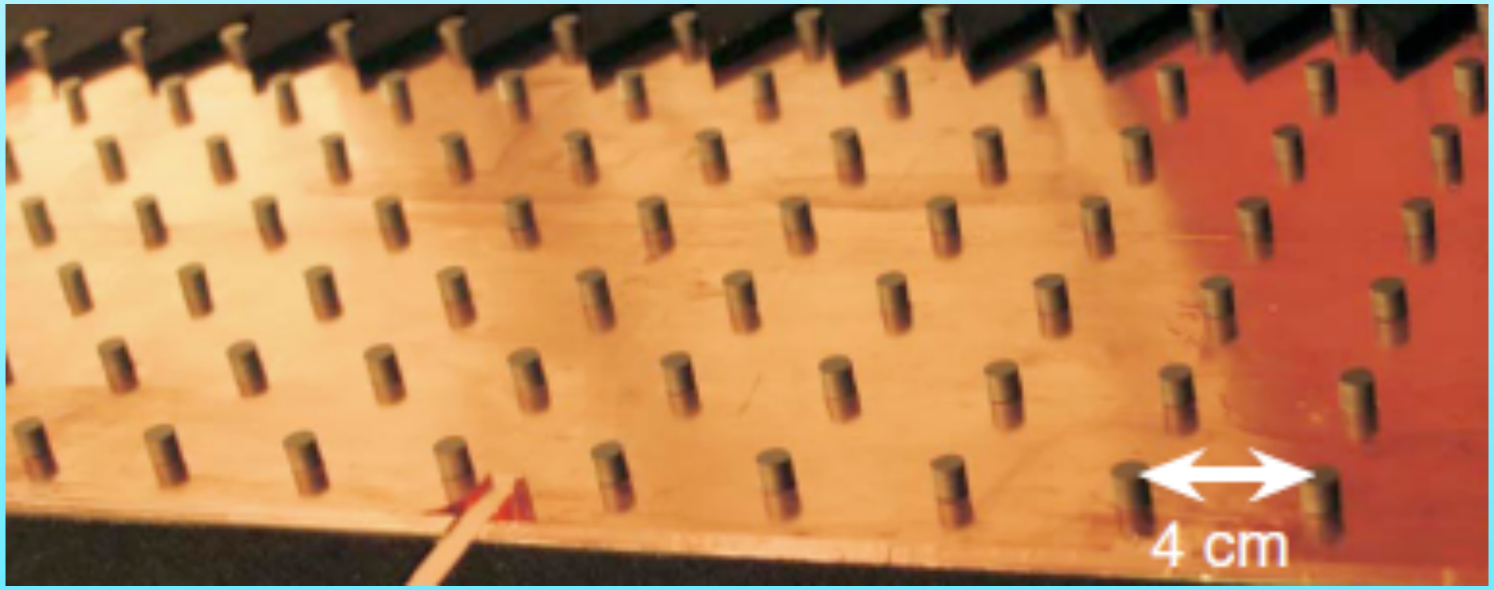
One way mode in photonic crystals

PRL **100**, 013905 (2008) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2008

Reflection-Free One-Way Edge Modes in a Gyromagnetic Photonic Crystal

Zheng Wang, Y. D. Chong, John D. Joannopoulos, and Marin Soljačić
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA





Observation of unidirectional backscattering-immune topological electromagnetic states

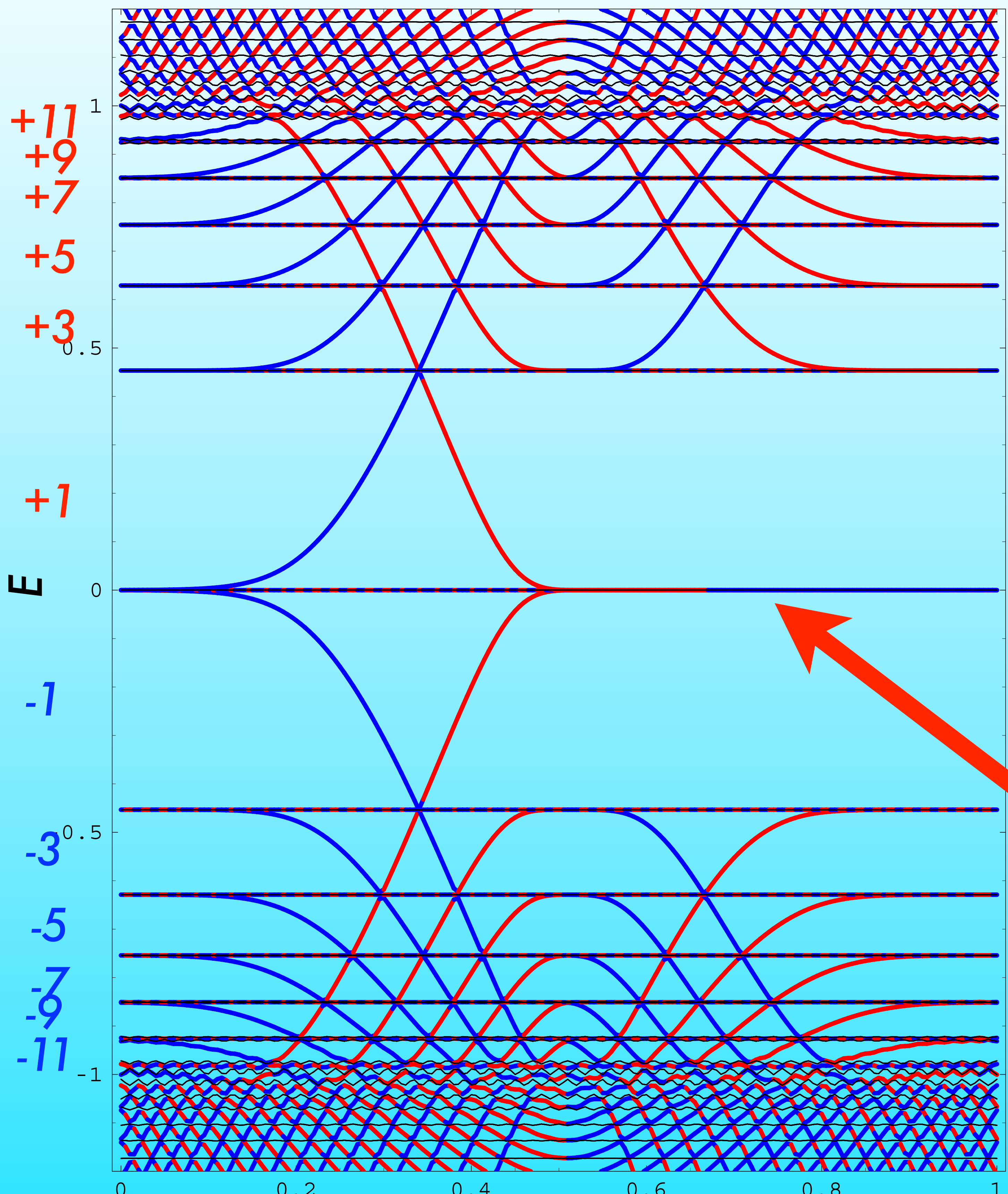
Z. Wang, Y. Chong, J.D.Joannopoulos, M. Solijacic

Nature 461 , 772 (2009)

Chiral symmetry and particle-particle interaction

- *Mean field approximation & bond ordering*
- ...

N=0 Landau Level & Chiral Symmetry



$\phi = 1/51$ Zigzag Edges

$$\exists \gamma, \quad \gamma^\dagger = \gamma^{-1} = \gamma$$

$$\{\boldsymbol{H}, \gamma\} = \boldsymbol{H}\gamma + \gamma\boldsymbol{H} = 0$$

$(-E, E)$: chiral pair $E \neq 0$

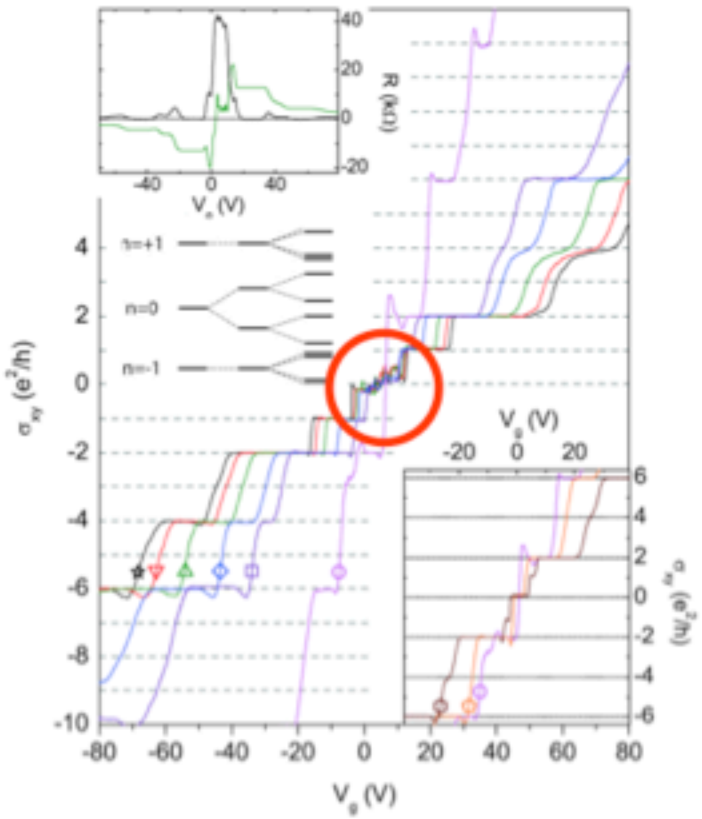
or

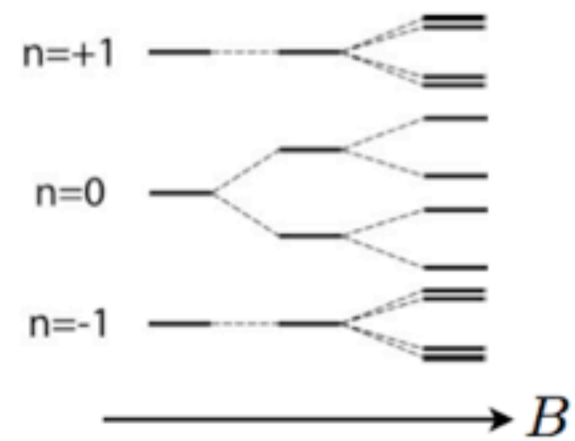
zero modes $(E = 0)$

E=0 Landau Level is special

Electron correlation effect

Gap opening of the *n*=0 Landau Level at high magnetic fields





Zhang et al., PRL (2006)
Jiang et al., PRL (2007)

Theoretical studies

Nomura and MacDonald, Phys. Rev. Lett. (2006)
Gusynin, Miransky, Sharapov, and Shovkovy, Phys. Rev. B (2006) *etc.*

Chiral condensates

Drut and Lähde, PRB (2009); PRL (2009)
Araki and Hatsuda, Phys. Rev. B (2010)
Araki, Annals Phys. 326, 1408 (2011)

Summary

Topological Stability of Massless Dirac Cones

- ☆ Effective mass approximation to the Dirac fermion
- ☆ Zero gap semiconductor with chiral symmetry
- ☆ Fermion doubling as of the 2D Nielsen-Ninomiya theorem

Topological aspects of graphene : Bulk

- ☆ Berry connection of the filled Dirac sea
- ☆ Lattice gauge fields in a parameter space

Topological aspects of graphene : Edge

- ☆ Zero modes at the zigzag & bearded edges
- ☆ Bulk-Edge correspondence for Dirac sea in a magnetic field
- ☆ Bulk–Edge correspondence for other phenomena

Chiral symmetry and particle-particle interaction

- ☆ Mean field and Bond ordering
- ☆ Beyond MF in progress

Bulk-Edge correspondence

NON TRIVIAL BULK IMPLIES EDGE STATES

Topology is everywhere
in
condensed matter

Thank you