QCD相図の実験的検証における陽子およびバリオン数ゆらぎの役割について

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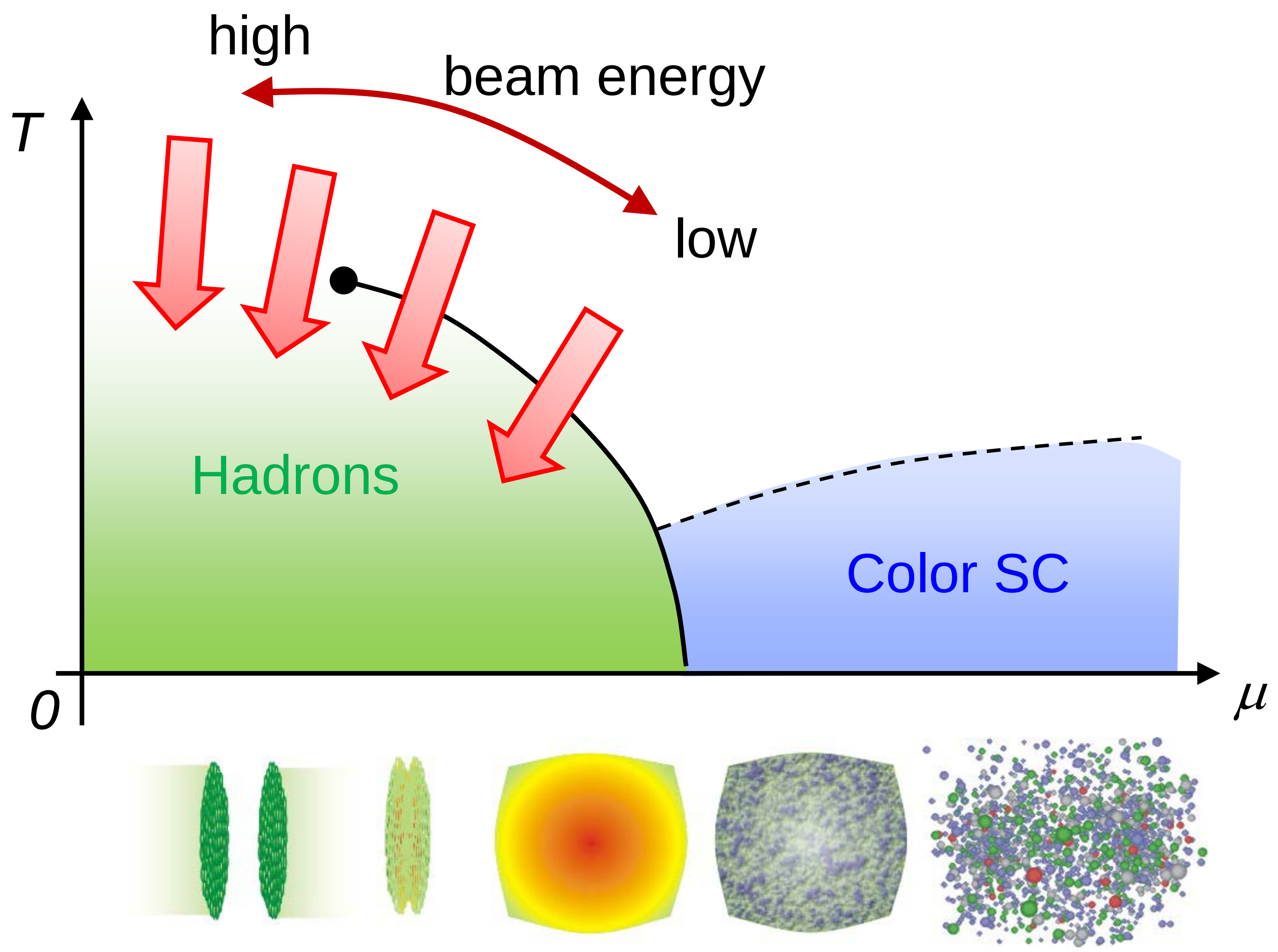
(阪大)

MK, M. Asakawa, arXiv:1107.2755[nucl-th]

Energy Scan Program @ RHIC



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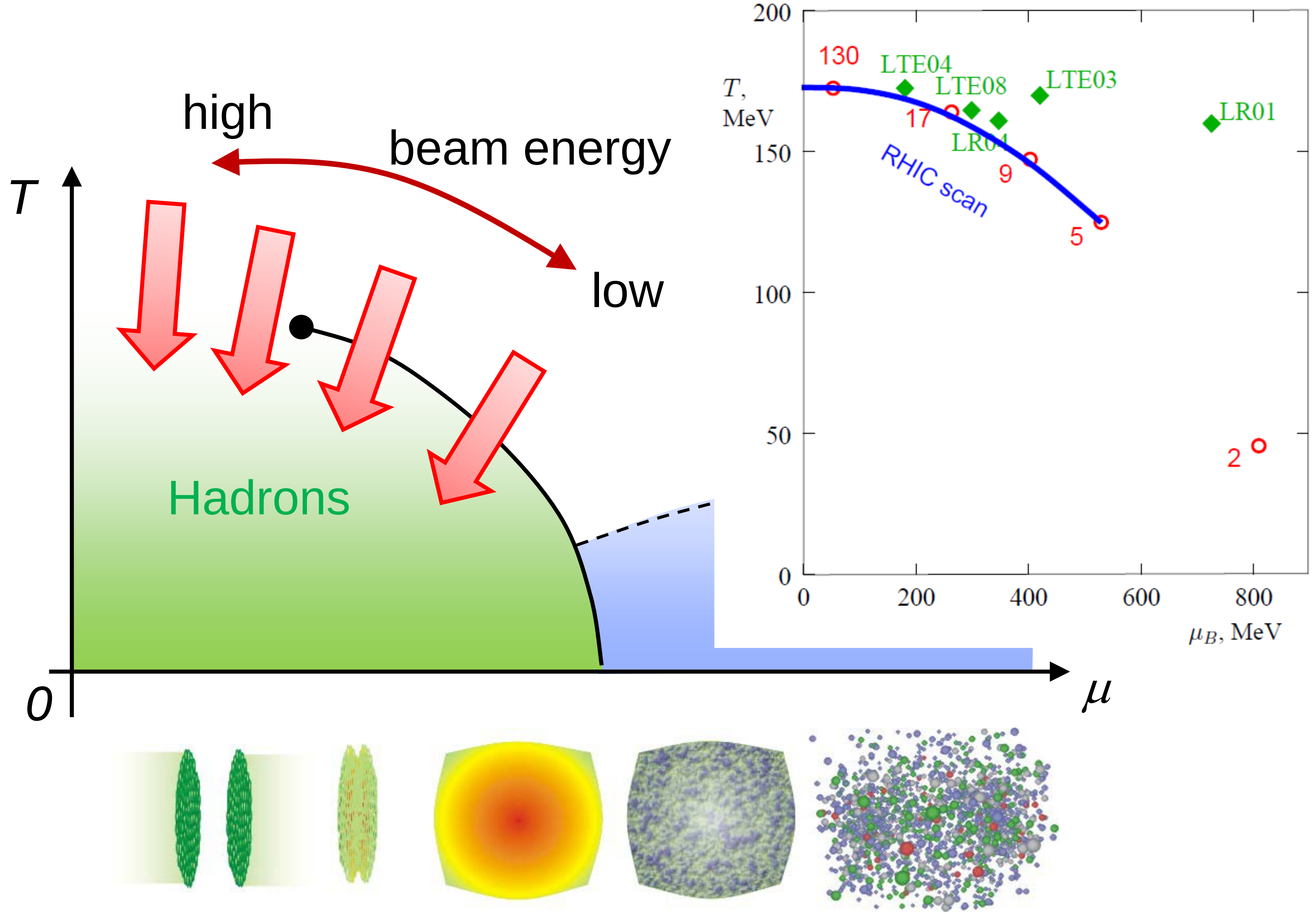
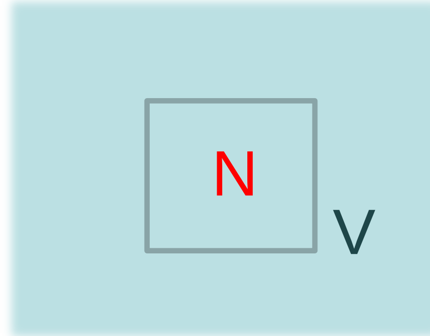
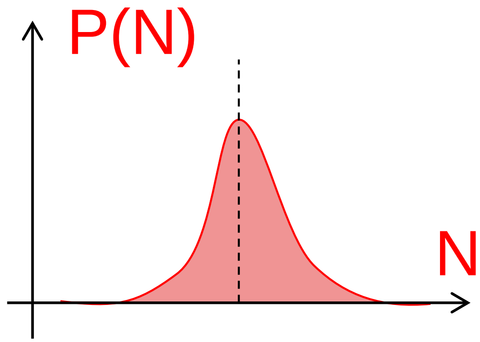


Fluctuations

平衡状態において、
物理量はゆらいでいる。

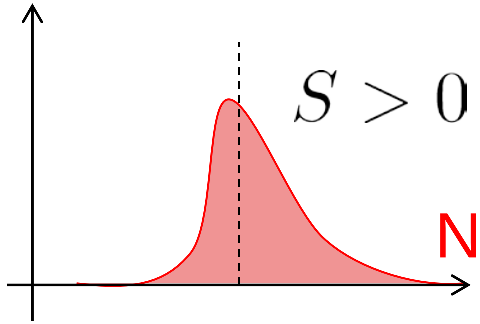




ゆらぎを特徴づける量

$$\delta N = N - \langle N \rangle$$

- Variance: $\langle \delta N^2 \rangle = V\chi_2 = \sigma^2$
- Skewness: $S = \dfrac{\langle \delta N^3 \rangle}{\sigma^3}$
- Kurtosis: $\kappa = \dfrac{\chi_4}{\chi_2 \sigma^2}$ ⇐ $V\chi_4 = \langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2$
- And much higher…



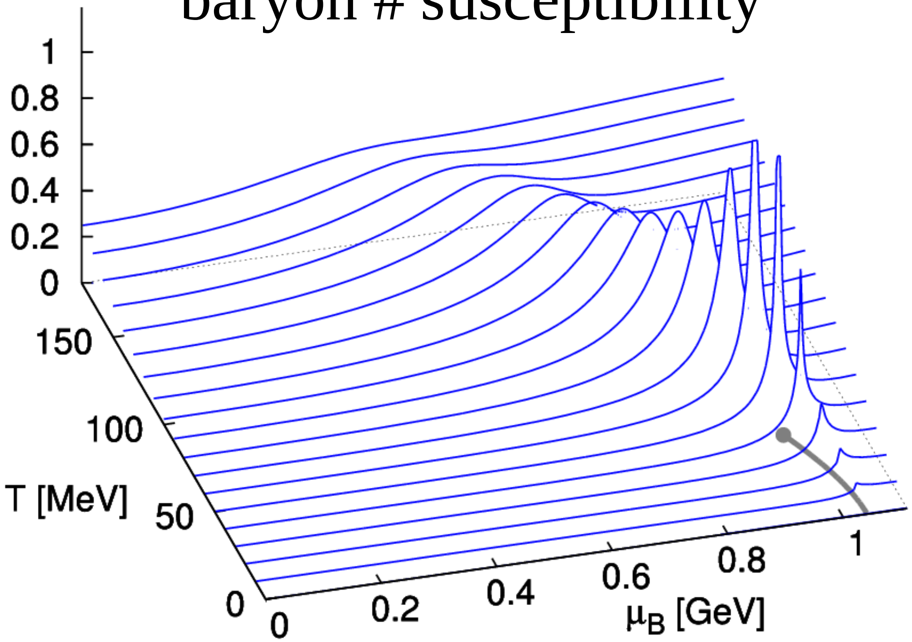
Fluctuations at QCD Critical Point

Stephanov, Rajagopal, Shuryak '98,'99

- 2^{nd} order phase transition at the CP.

- divergences in fluctuations of
  - p_T distribution
  - freezeout T
  - baryon number, proton, chage, …

baryon # susceptibility



- Higher order moments has stronger ξ dep near the CP. Stephanov, '09

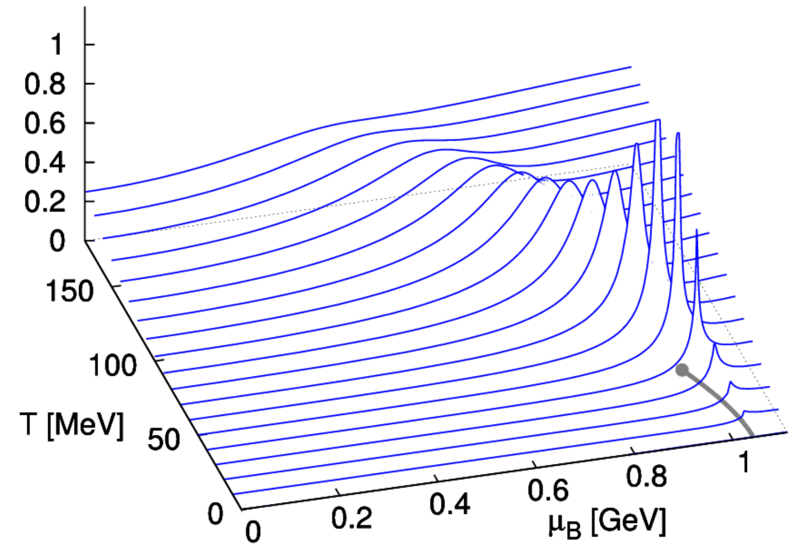
$$\langle \delta N^2 \rangle \sim \xi^2 \quad \langle \delta N^3 \rangle = \xi^{4.5} \quad \langle \delta N^4 \rangle_c = \xi^7$$

Baryon # 3rd Moment

Asakawa, Ejiri, MK, PRL103 (2009)

- χ_B has an edge along the phase boundary

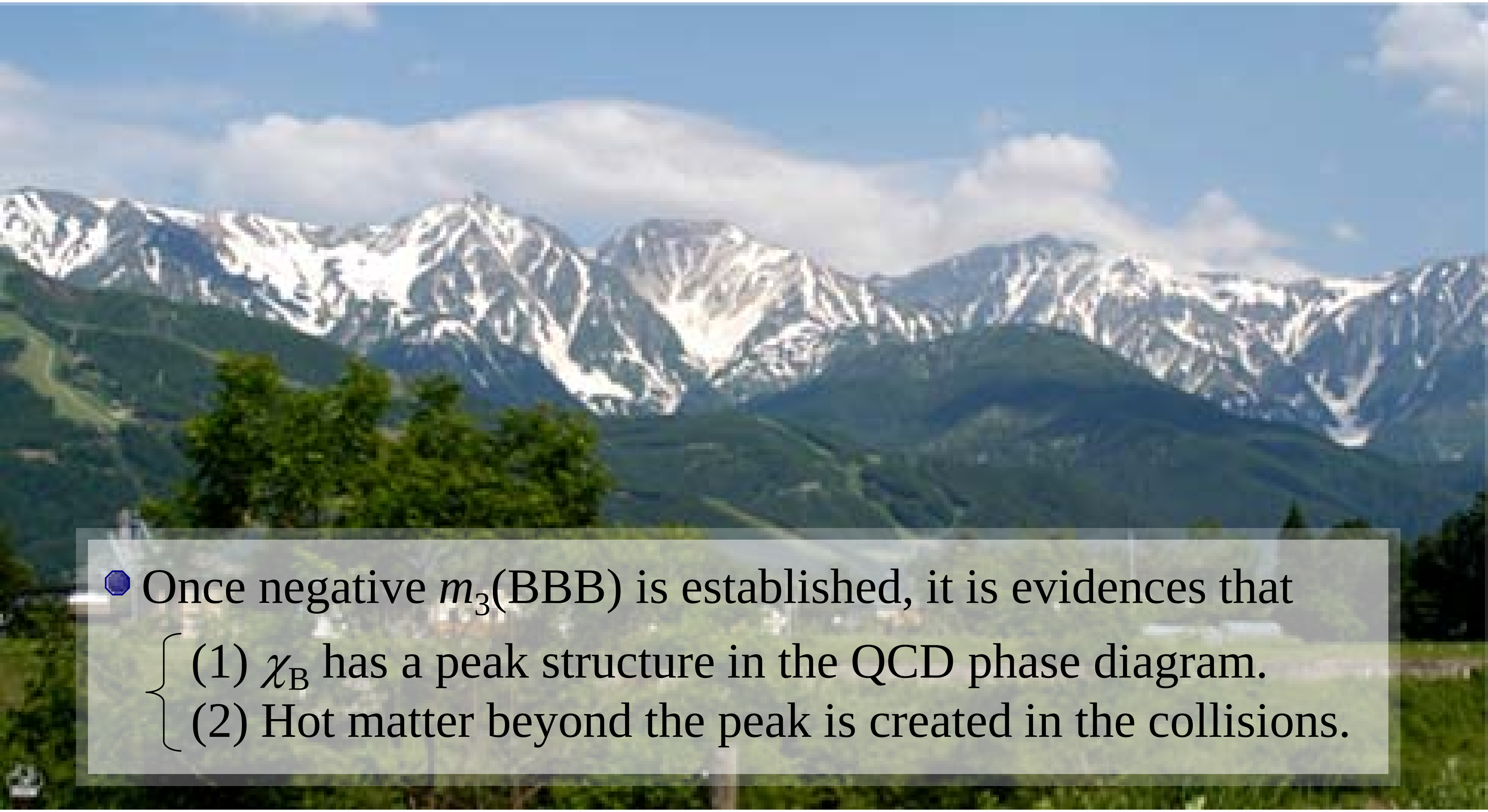
$\dfrac{\partial \chi_B}{\partial \mu_B}$ changes the sign at QCD phase boundary!



$$\chi_B = -\frac{1}{V}\frac{\partial^2 \Omega}{\partial \mu_B^2} = \frac{\left\langle (\delta N_B)^2 \right\rangle}{VT} \quad \Rightarrow \quad \frac{\partial \chi_B}{\partial \mu_B} = -\frac{1}{V}\frac{\partial^3 \Omega}{\partial \mu_B^3} = \frac{\left\langle (\delta N_B)^3 \right\rangle}{VT^2}$$

Third moment changes the sign at the QCD phase boundary!

Impact of Negative Third Moments



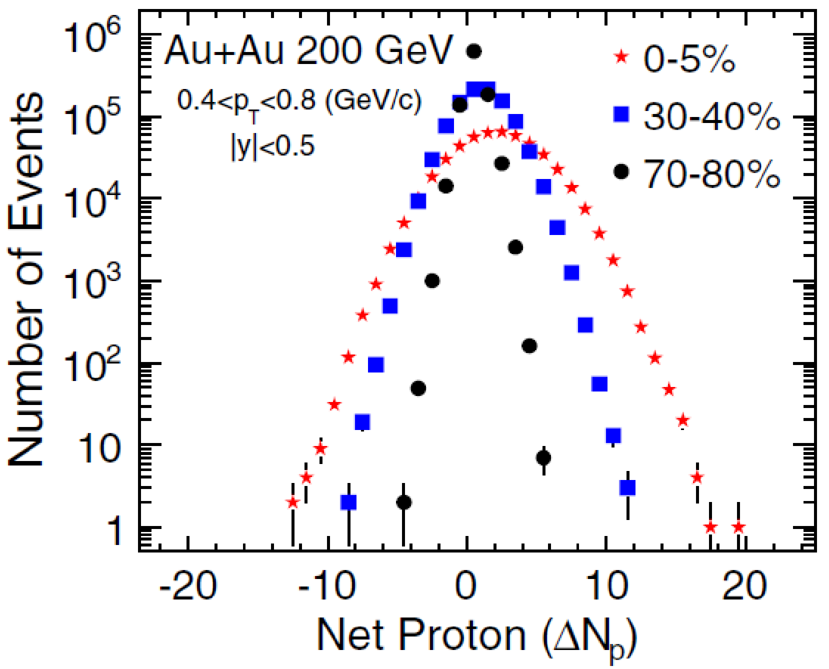
- Once negative m_3(BBB) is established, it is evidences that
  - (1) χ_B has a peak structure in the QCD phase diagram.
  - (2) Hot matter beyond the peak is created in the collisions.

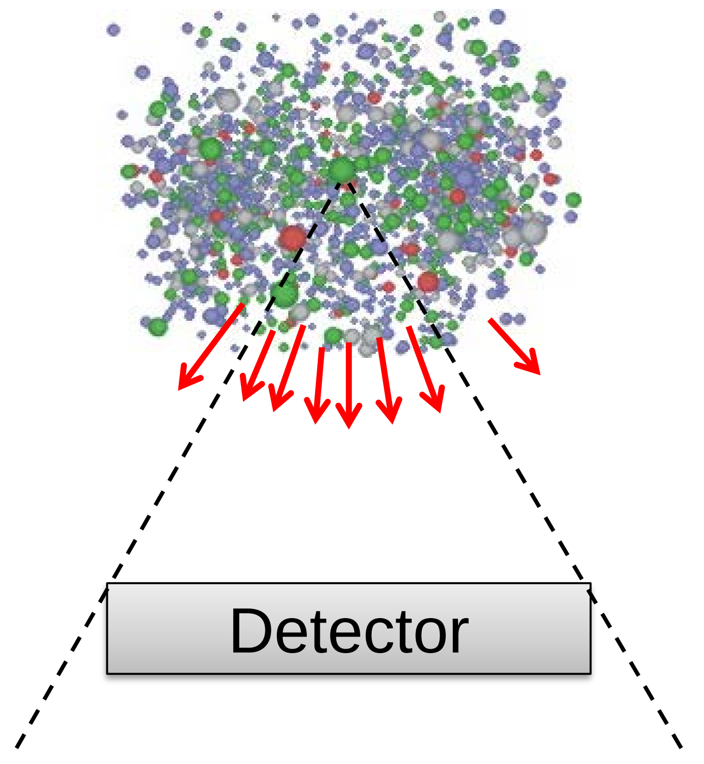
- •**No** dependence on any specific models.
  •Just the sign! **No** normalization (such as by N_{ch}).

Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in experiments.

STAR, PRL**105** (2010)



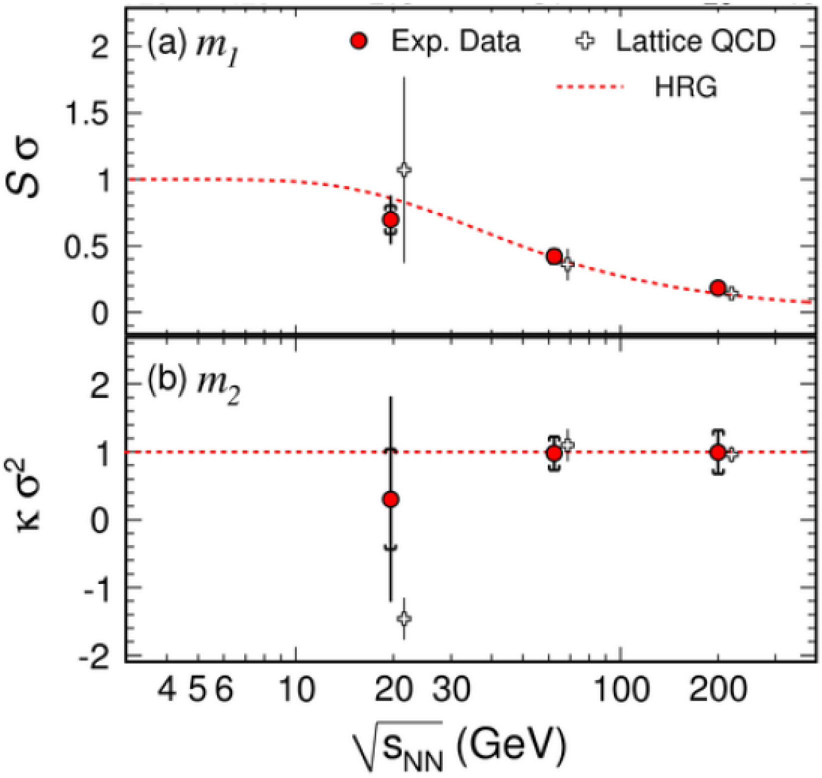


Variation of N_Q in a rapidity range is small for conserved charges.

Asakawa, et al., ’00; Jeon, Koch, ’00; Shuryak, Stephanov, ’02

Proton # Fluctuations @ STAR

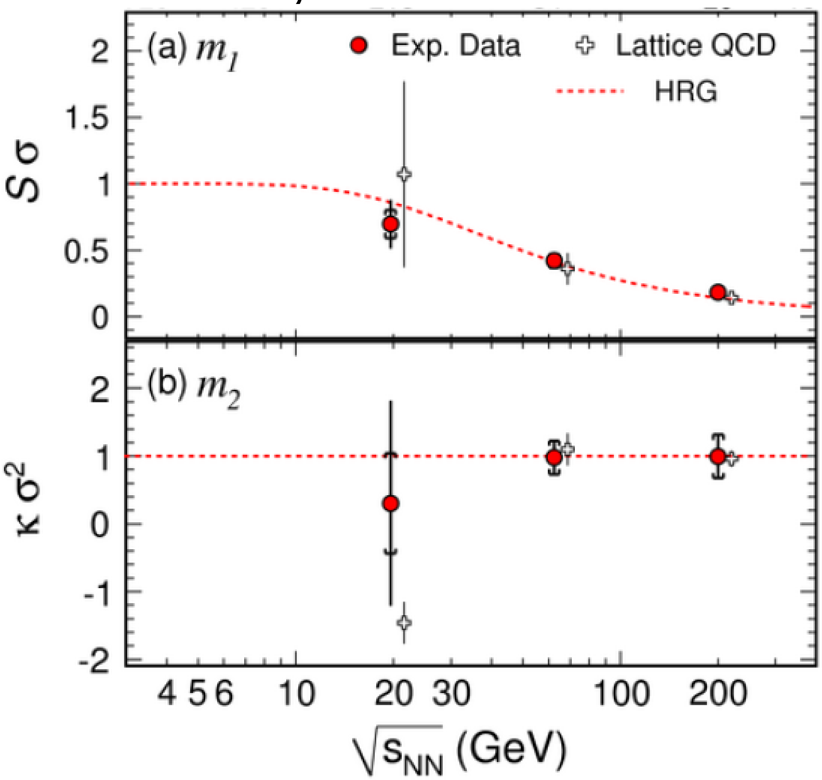
STAR, 2010



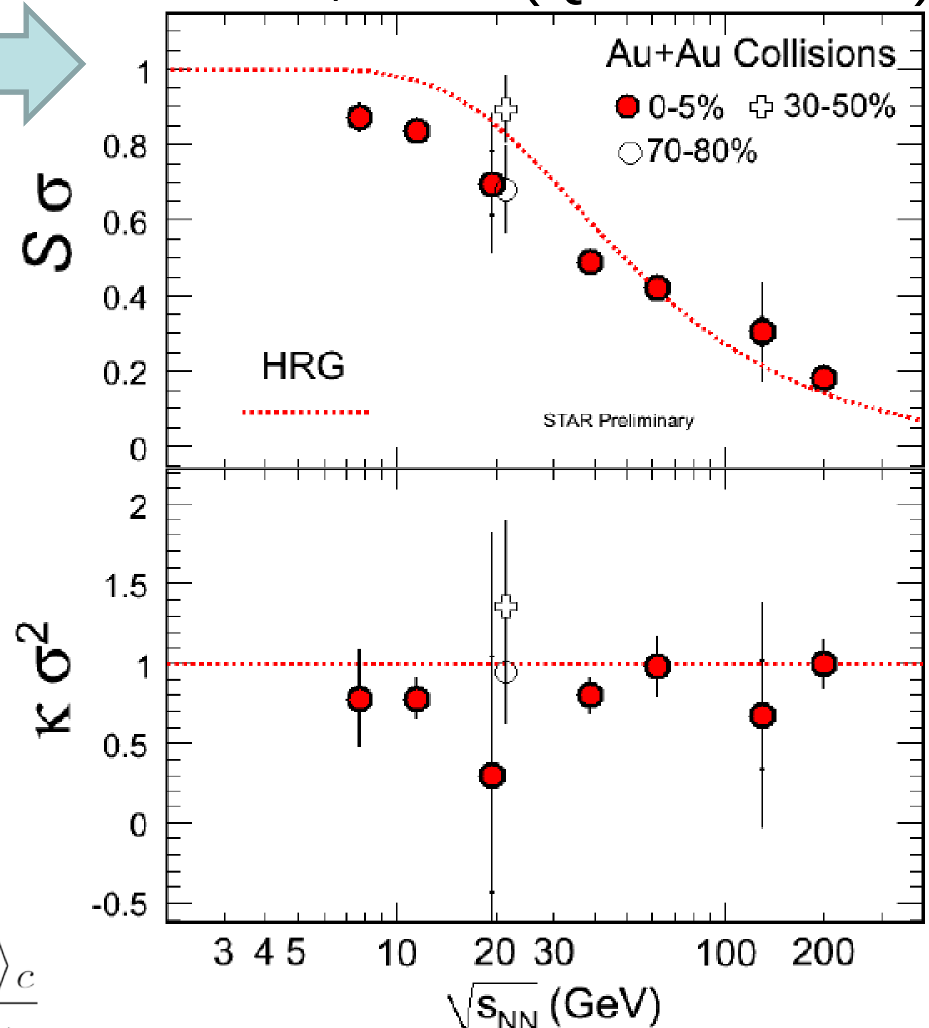
$$S\sigma = \frac{\langle(\delta N_p^{(\mathrm{net})})^3\rangle}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle}; \quad \kappa\sigma^2 = \frac{\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle}$$

Proton # Fluctuations @ STAR

STAR, 2010



STAR, 2011 (Quark Matter)



$$S\sigma = \frac{\langle(\delta N_p^{(\mathrm{net})})^3\rangle}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle}, \quad \kappa\sigma^2 = \frac{\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle}$$

high μ

low μ

Proton # Fluctuations @ STAR

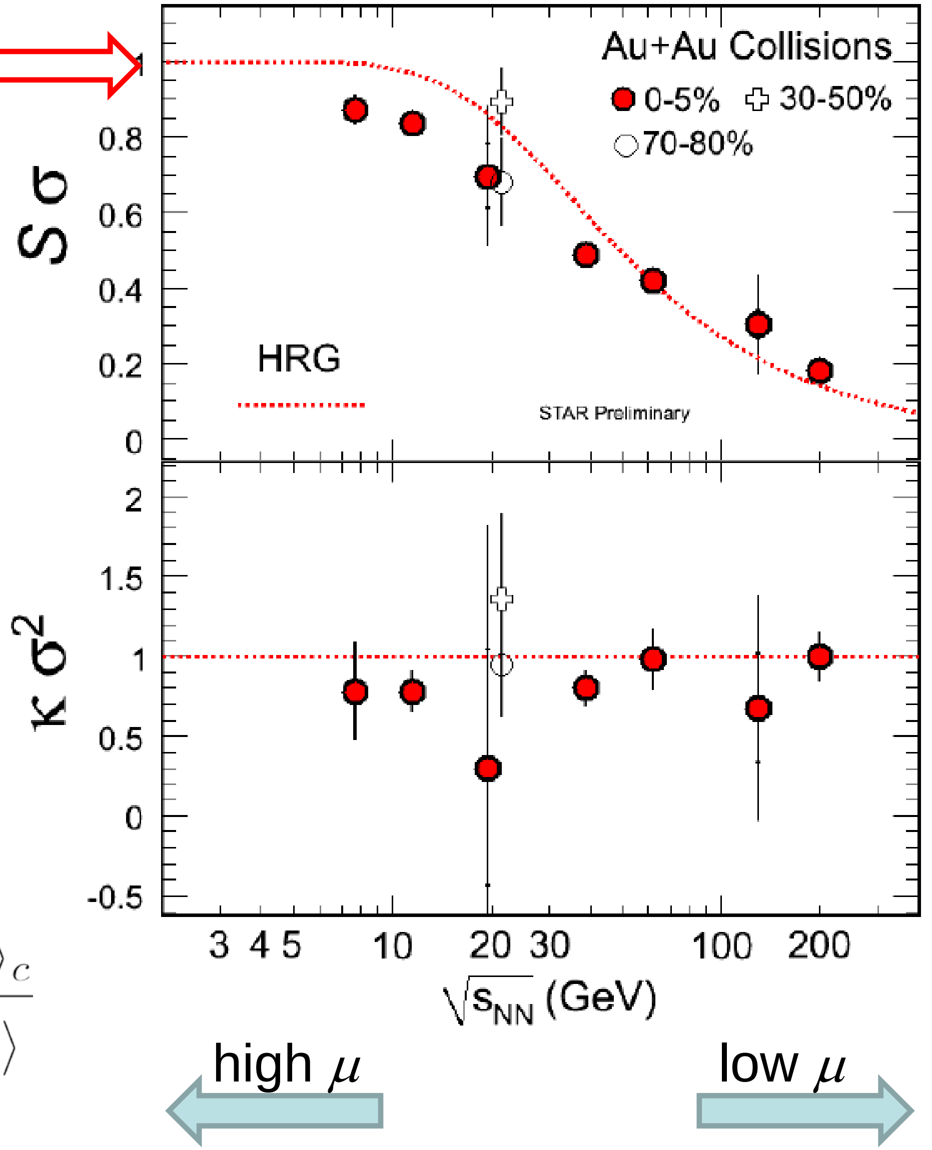
HRG:
Hadron Resonance Gas

||

Free gas composed of all hadrons & resonances

Poisson distribution for hadrons since $m_H \gg T, \mu$

STAR, 2011 (Quark Matter)



$$S\sigma = \frac{\langle (\delta N_p^{(\mathrm{net})})^3 \rangle}{\langle (\delta N_p^{(\mathrm{net})})^2 \rangle}, \quad \kappa\sigma^2 = \frac{\langle (\delta N_p^{(\mathrm{net})})^4 \rangle_c}{\langle (\delta N_p^{(\mathrm{net})})^2 \rangle}$$

$$\langle \delta N_{\mathrm{B}}^{n} \rangle \overset{?}{\simeq} 2\langle \delta N_{p}^{n} \rangle$$

How do these cumulants look like?
How are they different?

Baryon # fluctuations are desirable! Since they can

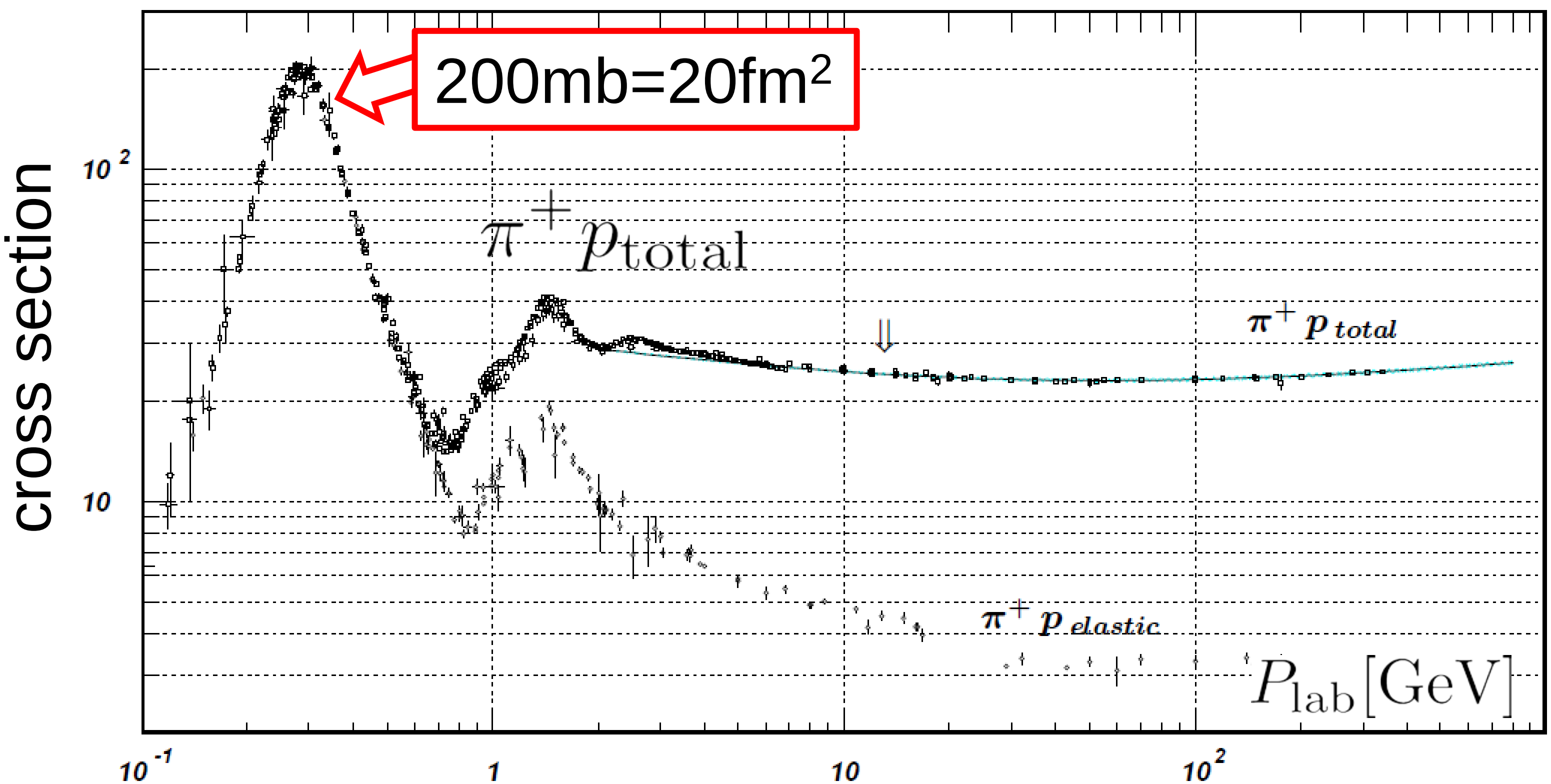
- remember fluctuations generated in earlier stages.
- clearly reflect signals of phase transitions.

Variation of Proton # in Hadronic Phase

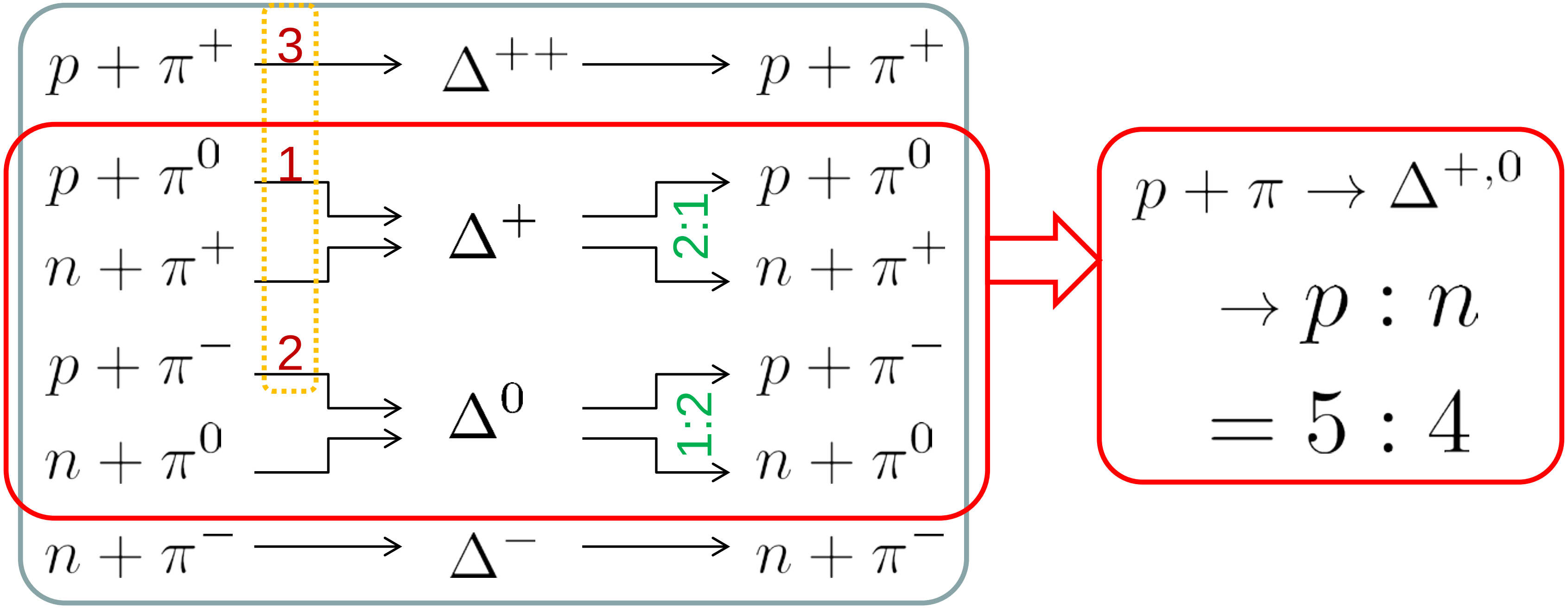
- Proton # varies even after chemical freezeout via charge exchange reactions mediated by Δ(1232):

$$p, n \to \Delta(1232) \to p, n \qquad \pi \to \Delta(1232) \to \pi$$

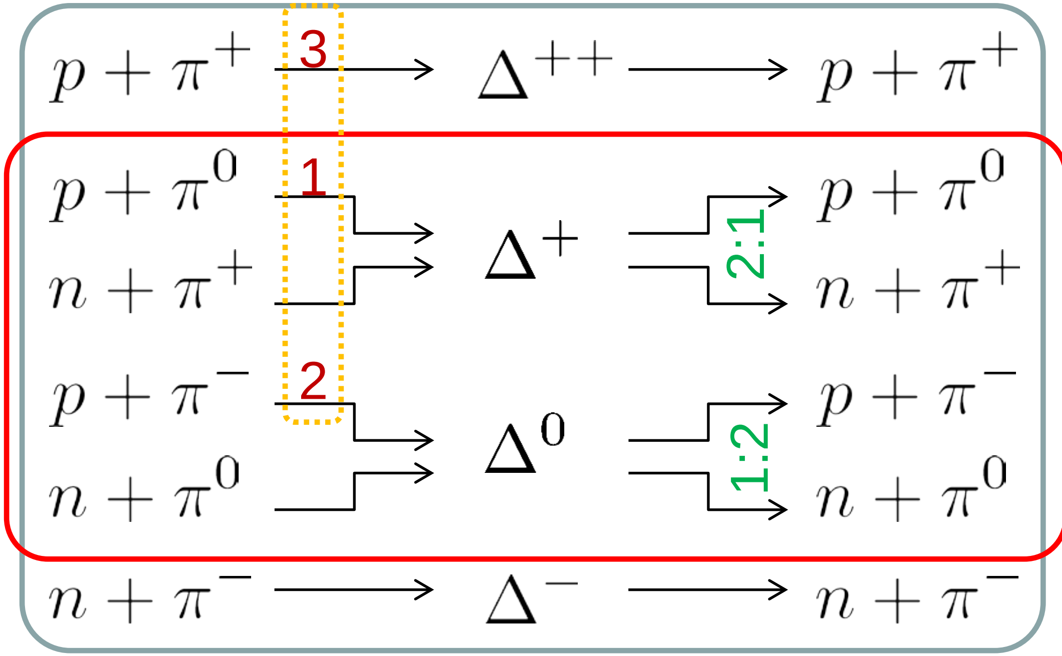
$$I = 3/2 \qquad \Gamma \simeq 1.8 \text{ [fm]}$$



Δ(1232)



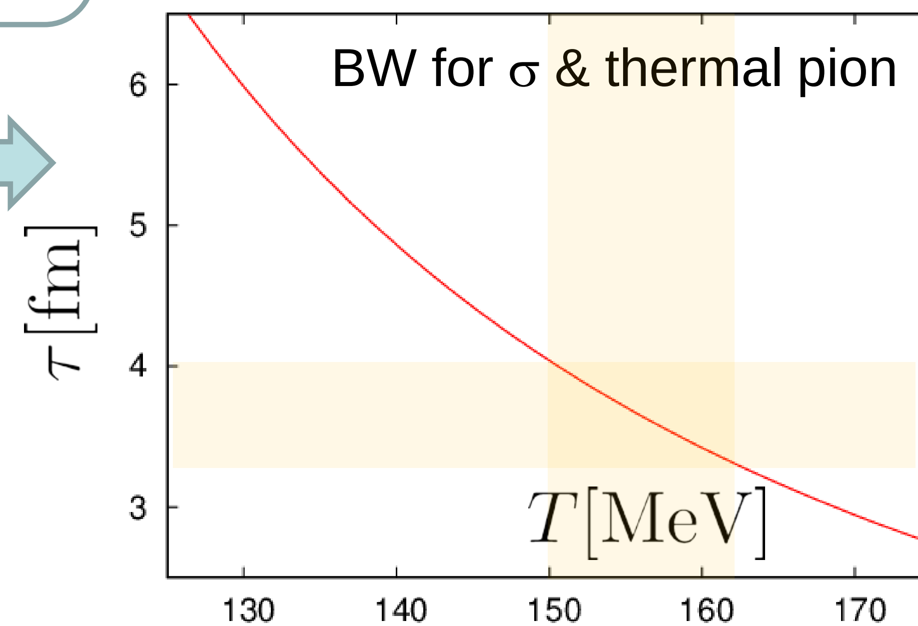
Δ(1232)



$$p+\pi \rightarrow \Delta^{+,0}$$
$$\rightarrow p:n$$
$$=5:4$$

Lifetime to create Δ^+ or Δ^0

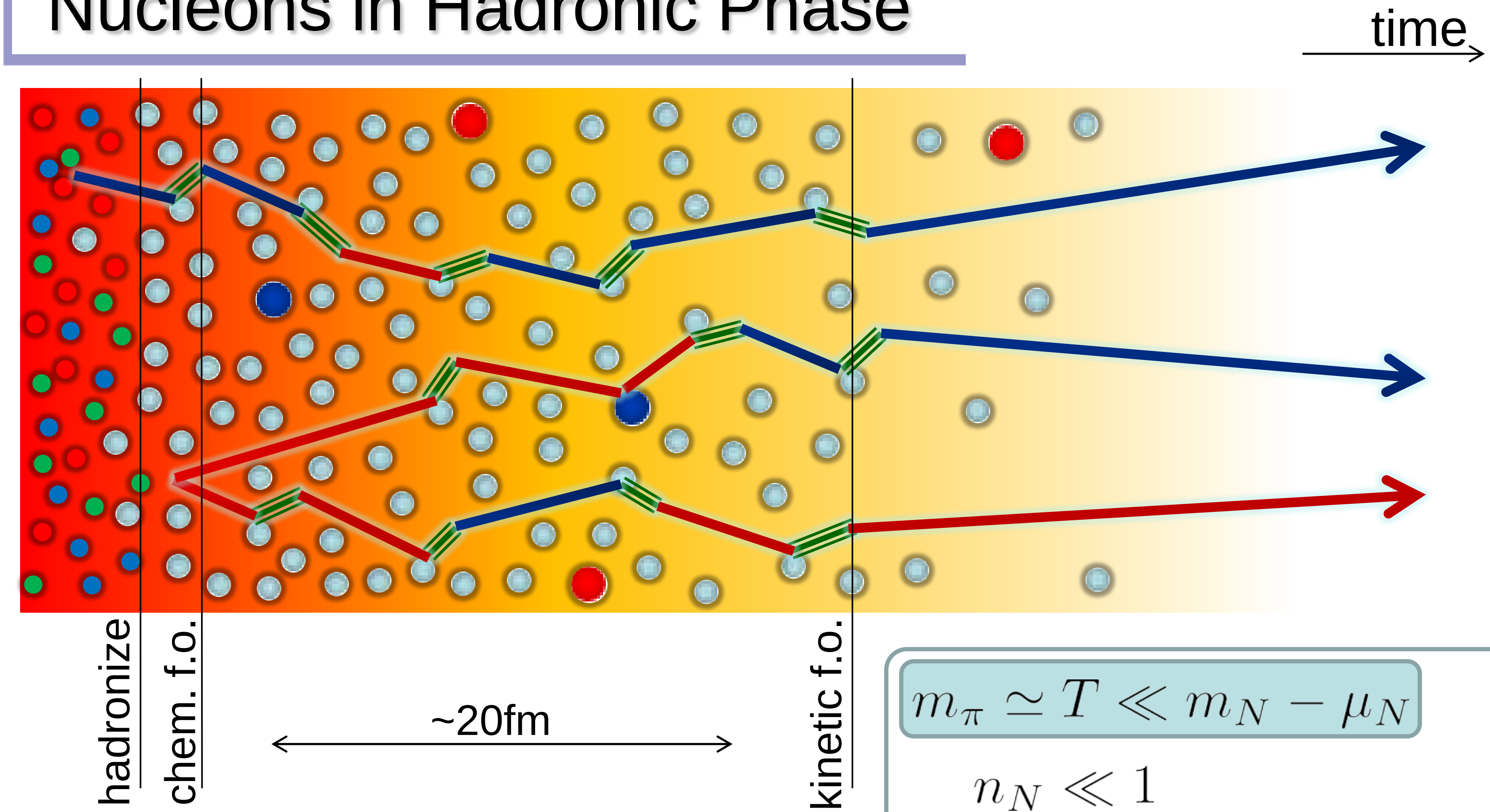
$$\tau^{-1} = \int \frac{d^3k_\pi}{(2\pi)^3}\sigma(E_{\rm cm})v_\pi n(E_\pi)$$

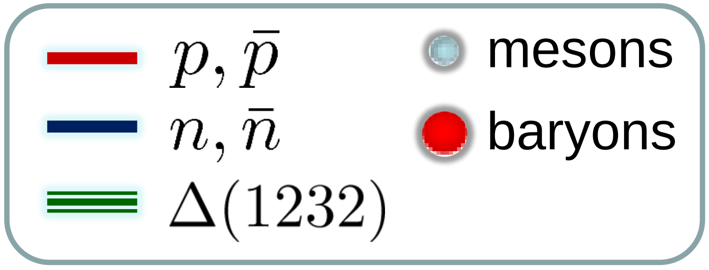


$$\tau \ll (\text{freezeout time}) \simeq 20[\text{fm}]$$

c.f.) Nonaka, Bass, 2007

Nucleons in Hadronic Phase





$$m_\pi \simeq T \ll m_N - \mu_N$$

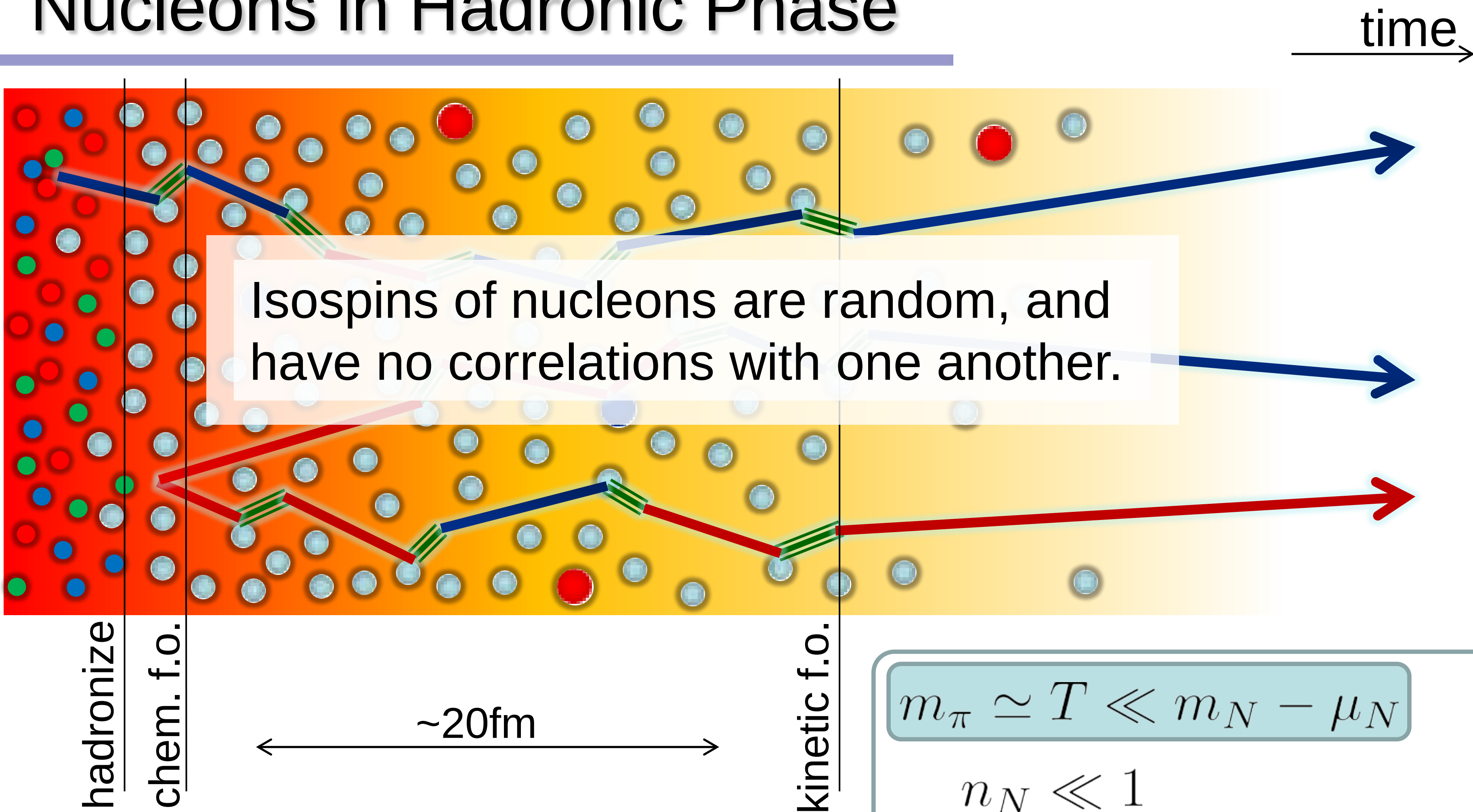
$n_N \ll 1$

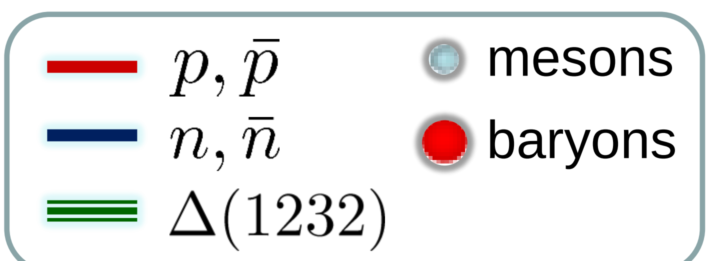
- rare NN collisions
- no quantum corr.

$n_N \ll n_\pi$

- so many pions

Nucleons in Hadronic Phase





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- rare NN collisions
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$n_N \ll n_\pi$

- so many pions

Probability Distribution to find particles in each event

$$P(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$$
$$= F(N_N, N_{\bar{N}}) B(N_p; N_N) B(N_{\bar{p}}; N_{\bar{N}})$$

$$B(k; N) = \frac{1}{2^N} \frac{N!}{k!(N-k)!}$$

binomial distribution func.

$$\begin{cases} N_N = N_p + N_n \\ N_{\bar{N}} = N_{\bar{p}} + N_{\bar{n}} \end{cases}$$

NOTE:

- The factorization is applied to distributions in any phase space in the final state.
- $F(N_N, N_N)$ may carry fluctuations in early stage.

Proton & Baryon # Fluctuations

$$\begin{cases} N_p^{(\rm net)} = N_p - N_{\bar{p}} \\ N_p^{(\rm tot)} = N_p + N_{\bar{p}} \end{cases}$$

N_p

$$\langle (\delta N_{\rm N}^{(\rm net)})^2 \rangle = 4 \langle (\delta N_p^{(\rm net)})^2 \rangle - 2 \langle N_p^{(\rm tot)} \rangle$$

$$\langle (\delta N_p^{(\rm net)})^2 \rangle = \frac{1}{4} \langle (\delta N_{\rm N}^{(\rm net)})^2 \rangle + \frac{1}{4} \langle N_{\rm N}^{(\rm tot)} \rangle$$

$N_{\rm N}$

Similar formulas up to any order!

For free gas

$$\langle (\delta N_p^{(\rm net)})^2 \rangle = \frac{1}{2} \langle (\delta N_{\rm N}^{(\rm net)})^2 \rangle$$

Strange Baryons

Decay Rates:

Λ $m_\Lambda \simeq 1116[\mathrm{MeV}]$

$$\Rightarrow p : n \simeq 1.6 : 1$$

Σ $m_\Sigma \simeq 1190[\mathrm{MeV}]$

$$\Rightarrow p : n \simeq 1 : 1.8$$

Decay modes:

$\Lambda \to p + \pi^-$ 64%
$\Lambda \to n + \pi^0$ 36%

$\Sigma^+ \to p + \pi^0$ 52%
$\Sigma^+ \to n + \pi^+$ 48%

$\Sigma^0 \to \Lambda \to p + \pi^-$ 64%
$\Sigma^0 \to \Lambda \to n + \pi^0$ 36%

$\Sigma^- \to n + \pi^-$

Regarding these ratios even, protons from these decays is incorporated into the binomial distribution. Then, $N_N \to N_B$

An Extreme Example

(1) $N_B^{(\mathrm{net})} = N_B - N_{\bar{B}}$ reflects primordial fluctuations.

(2) $N_B, N_{\bar{B}}$ are Poissonian.

$$
\begin{cases}
2\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c = \frac{1}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \frac{7}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_{c,\mathrm{free}}
\end{cases}
$$

what we want

noise

For free gas

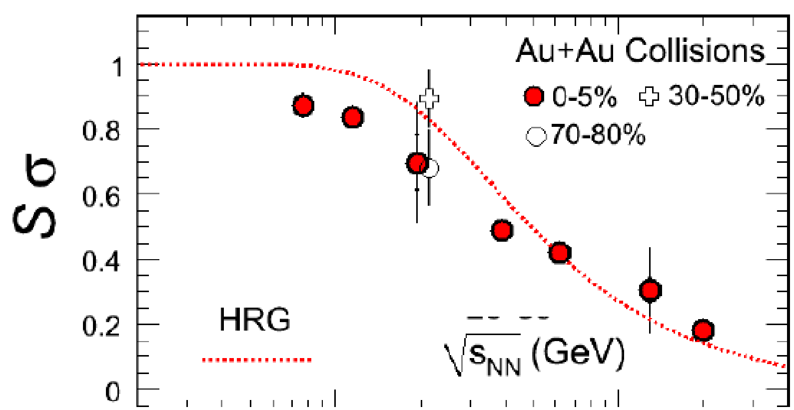
$$2\langle(\delta N_p^{(\mathrm{net})})^n\rangle_c = \langle(\delta N_{\mathrm{N}}^{(\mathrm{net})})^n\rangle_c$$

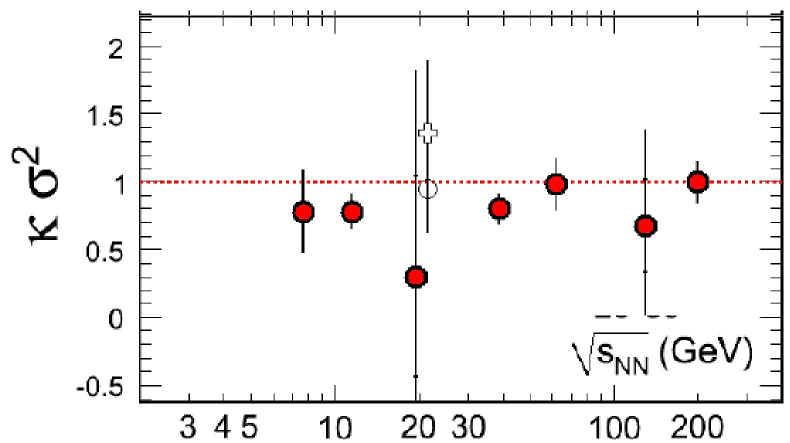
An Extreme Example

(1) $N_B^{(\mathrm{net})} = N_B - N_{\bar{B}}$ reflects primordial fluctuations.

(2) $N_B, N_{\bar{B}}$ are Poissonian.

$$\begin{cases} 2\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^2\rangle_\mathrm{free} \\ 2\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{4}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^3\rangle_\mathrm{free} \\ 2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c = \frac{1}{8}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^4\rangle_c + \frac{7}{8}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^4\rangle_{c,\mathrm{free}} \end{cases}$$





Summary

- Baryon and proton # fluctuations are different.

- We obtained formulas to relate baryon # cumulants with experimental observales.

- Experimental analysis of baryon # fluctuations may verify
  - signals of QCD phase transition
  - speed of baryon number diffusion in the hadronic stage.

Future Work

- Refinement of the formulas to include
  nonzero isospin density / low beam energy region

Proton # Cumulants

$$\begin{aligned}
\langle N_{\mathrm{B}}^{(\mathrm{net})}\rangle =& 2\langle N_p^{(\mathrm{net})}\rangle, \\
\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle =& 4\langle(\delta N_p^{(\mathrm{net})})^2\rangle - 2\langle N_p^{(\mathrm{tot})}\rangle, \\
\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle =& 8\langle(\delta N_p^{(\mathrm{net})})^3\rangle - 12\langle\delta N_p^{(\mathrm{net})}\delta N_p^{(\mathrm{tot})}\rangle \\
&+ 6\langle N_p^{(\mathrm{net})}\rangle, \\
\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c =& 16\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c - 48\langle(\delta N_p^{(\mathrm{net})})^2\delta N_p^{(\mathrm{tot})}\rangle \\
&+ 48\langle(\delta N_p^{(\mathrm{net})})^2\rangle + 12\langle(\delta N_p^{(\mathrm{tot})})^2\rangle \\
&- 26\langle N_p^{(\mathrm{tot})}\rangle,
\end{aligned}$$

$$\begin{aligned}
\langle N_p^{(\mathrm{net})}\rangle =& \frac{1}{2}\langle N_{\mathrm{B}}^{(\mathrm{net})}\rangle, \\
\langle(\delta N_p^{(\mathrm{net})})^2\rangle =& \frac{1}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle + \frac{1}{4}\langle N_{\mathrm{B}}^{(\mathrm{tot})}\rangle, \\
\langle(\delta N_p^{(\mathrm{net})})^3\rangle =& \frac{1}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{8}\langle\delta N_{\mathrm{B}}^{(\mathrm{net})}\delta N_{\mathrm{B}}^{(\mathrm{tot})}\rangle, \\
\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c =& \frac{1}{16}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \frac{3}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\delta N_{\mathrm{B}}^{(\mathrm{tot})}\rangle \\
&+ \frac{3}{16}\langle(\delta N_{\mathrm{B}}^{(\mathrm{tot})})^2\rangle - \frac{1}{8}\langle N_{\mathrm{B}}^{(\mathrm{tot})}\rangle,
\end{aligned}$$

Manipulations

$$\langle N_p^{(\mathrm{net})} \rangle = \sum_{N_{\{p,n,\bar{p},\bar{n}\}}} (N_p - N_{\bar{p}}) P(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$$

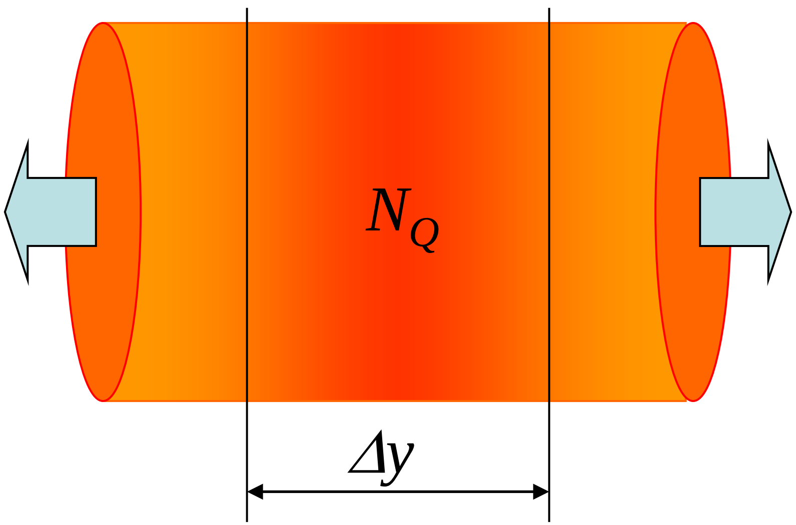
$$= \sum_{N_{\mathrm{N}}, N_{\bar{\mathrm{N}}}} F(N_{\mathrm{N}}, N_{\bar{\mathrm{N}}}) \sum_{N_p, N_{\bar{p}}} (N_p - N_{\bar{p}}) B(N_p; N_{\mathrm{N}}) B(N_{\bar{p}}; N_{\bar{\mathrm{N}}})$$

$$= \sum_{N_{\mathrm{N}}, N_{\bar{\mathrm{N}}}} F(N_{\mathrm{N}}, N_{\bar{\mathrm{N}}}) \left(\frac{N_N}{2} - \frac{N_{\bar{N}}}{2} \right)$$

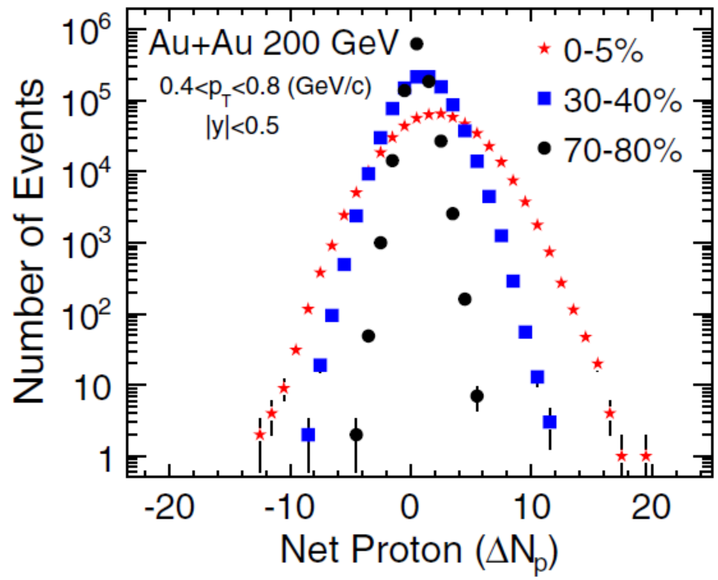
$$= \frac{1}{2} \langle N_N^{(\mathrm{net})} \rangle$$

Event-by-Event Analysis @ HIC

観測可能粒子のゆらぎ、及び高次のモーメントは、
event-by-event解析で「観測」できる。



STAR, PRL**105** (2010)

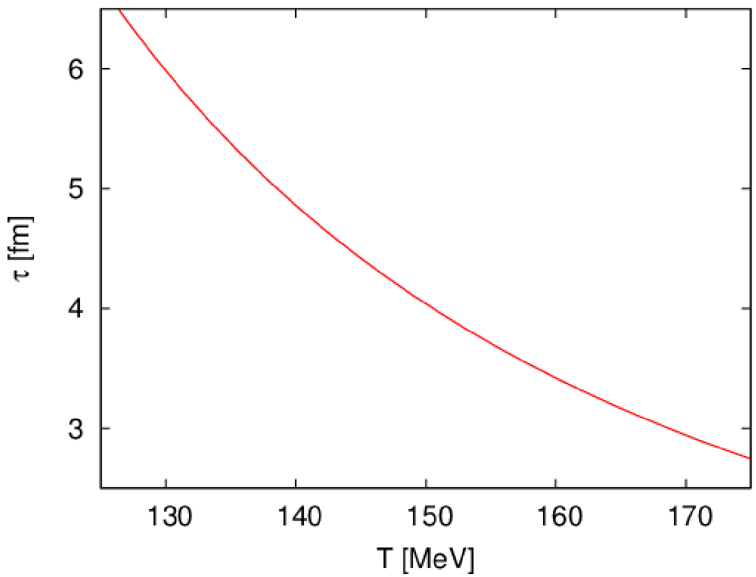


FreezeoutからのN_Qの変化は、
保存量であれば小さいと期待できる。

Asakawa, et al., ’00, Jeon, Koch, ’00
Shuryak, Stephanov, ’02

Nucleon Time Scales in Fireballs

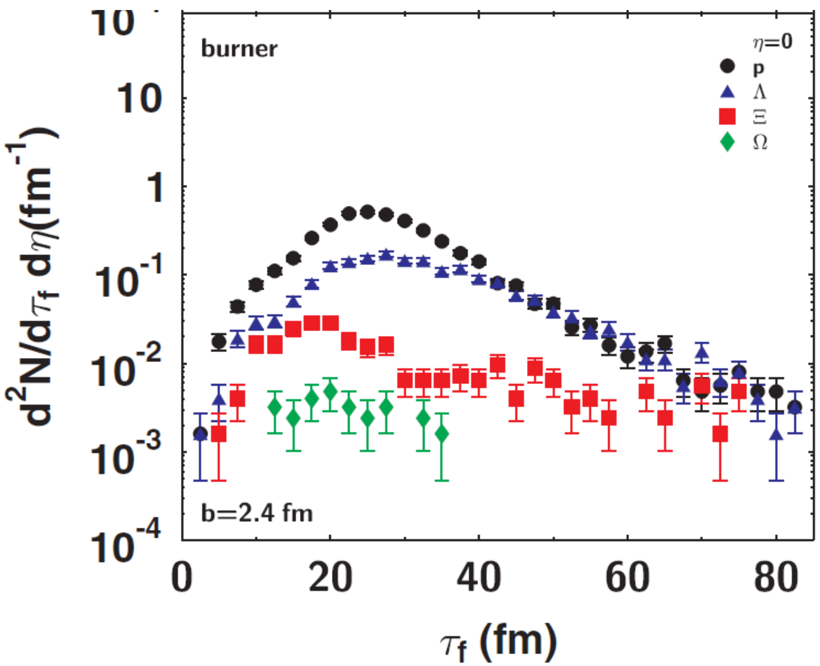
Mean time to create $\Delta^{+,0}$



$$\tau = 3 \sim 4[\text{fm}]$$

Freeze-out time

Nonaka, Bass, 2007



$$\tau_{\text{f.o.}} > 20[\text{fm}]$$