次元正則化を用いた有限温度NJL模型におけるメソンの振舞い

(Meson properties in the NJL model with dimensional regularization at finite temperature)

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1. Introduction

- Nambu--Jona-Lasinio (NJL) model contains 4-fermion interactions. Since 4-fermion interaction is a dimension 6 operator, the model is not renormalizable in 4 space-time dimensions.

  → In a nonrenormalizable model, most of the physical quantities depend on the regularization method.

- In order to regularize fermion loop integrals, one usually introduces a momentum scale Λ to cut off integration momenta higher than Λ.
  e.g. Y. Nambu, G. Jona-Lasinio, Phys. Rev, 122, 345 (1961), U. Vogl, W. Weise, Prog. Part. Nucl. Phys. 27, 195 (1991), S.P. Klevansky, Rev. Mod. Phys. 64, 649 (1992), T. Hatsuda, T. Kunihiro, Phys. Rep. 247, 221 (1994), ...

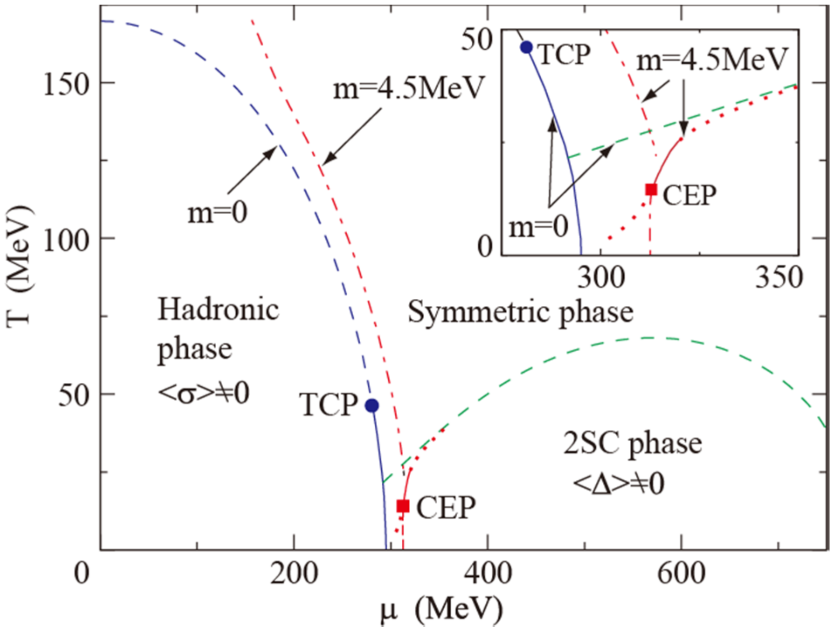
- Other methods are also studied in NJL type models:
  smooth cutoff, R.S. Plant, M.C. Birse, NPA 628, 607 (1998),
  T. Hell, S. Roessner, M. Cristoretti, W. Weise PRD79,014022 (2009)
  Pauli- Villars, A.A. Osipov, H. Hansen, B. Hiller, NPA 745, 81 (2004)
  Schwinger proper-time method, T. Inagaki, D.K., T. Murata, PTP111, 371 (2004)
  dimensional regularization, S. Krewald, K. Nakayama, Ann. Phys. 216, 201 (1992),
  T. Inagaki, T. Kouno, T. Muta Int. J. Mod. Phys. A10, 2241, (1995),
  R.G. Jafarov, E.V. Rochev, Russ. Phys. J. 49, 364 (2006)

- In the previous study we discussed *T*-μ phase structure of 2-flavor extended NJL model by using the momentum cutoff and the dimensional regularization.

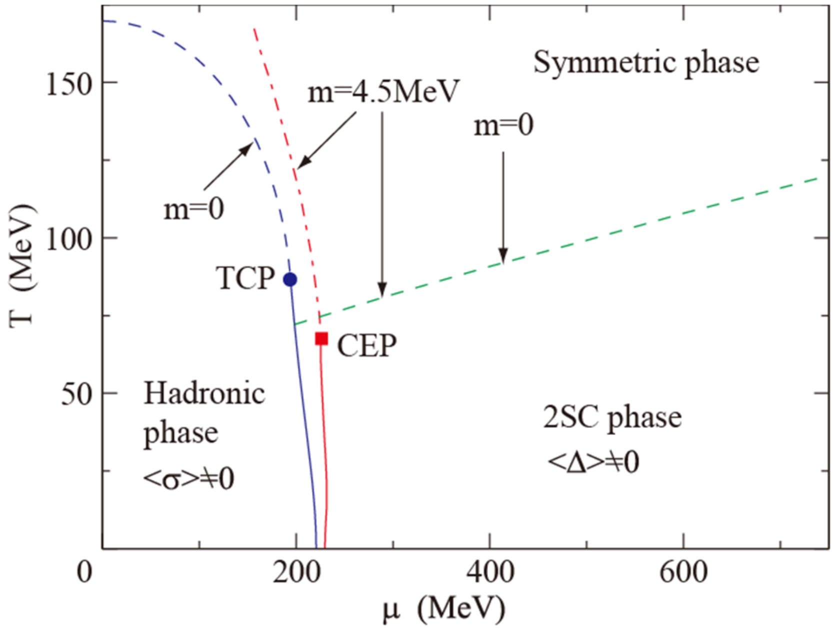
chiral phase transition: $\langle\sigma\rangle = 0$ for $m = 0$

T. Fujihara, D.K., T. Inagaki, A. Kvinihidze, PRD79, 096008 (2009)

quark number susceptibility : maximum of $\chi_q = \dfrac{\partial^2}{\partial \mu^2} V_{\rm eff}(\langle\sigma\rangle, \langle\Delta\rangle)$, for $m = 4.5$MeV



cut-off regularization Λ = 720MeV



dimensional regularization

- In this talk we consider meson properties on 3-flavor NJL model at finite *T* .

dimensional regularization: ○ Poincare invariance, $m_{\eta'}$=958MeV > Λ (η' decay ×)

△ 4 dimensional space-time meaning

2. 3-flavor NJL model

SU(3)$_L$ × *SU(3)$_R$* / *SU(3)$_V$*

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m_i)\psi + G_S \sum_{a=0}^{8} [(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}\gamma_5\lambda^a\psi)^2] \quad \leftarrow \text{dim 6 operator}$$

$$-K[\det_{i,j} \bar{\psi}_i(1-\gamma_5)\psi_j + \det_{i,j} \bar{\psi}_i(1+\gamma_5)\psi_j] \quad \leftarrow \text{dim 9 operator}$$

where *i,j* (=*u,d,s*) denote flavor indices, λ^a (a = 0,1,2,...8) are the Gell-Mann matrices $\lambda^0 = \sqrt{2/3}I_3,\ m_i = \mathrm{diag}(m_u, m_d, m_s),\ m_d = m_u.$

Third term represents 6-fermion interaction, it breaks $U(1)_A$ symmetry.

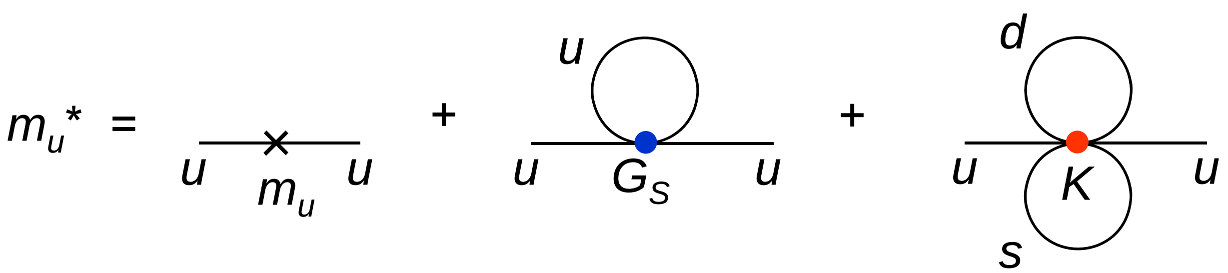
M. Kobayashi, T. Maskawa, PTP 44, 1422 (1970), G. ‘t Hooft, PRD 14, 3432 (1976)

In the leading order of the 1/N_c expansion gap equation is given by

$$m_i^* = m_i + 4G_S i\mathrm{tr}S^i + 2K i\mathrm{tr}S^j \cdot i\mathrm{tr}S^k \qquad G_S N_c, K N_c^2 \simeq O(1)$$

where trace takes color, spinor indices, $i\mathrm{tr}S^u(m_u^*) = \int \frac{d^D k}{(2\pi)^D} \mathrm{tr}\frac{i}{\not{k} - m_u^* + i\epsilon}$

$$= -\langle \bar{u}u \rangle M_0^{D-4}.$$



Parameter fixing

T. Inagaki, D.K., H. Kohyama, A. Kvinihidze, PRD83, 034005 (2011)

To evaluate the meson properties, we fix the model parameters,

Cutoff regularization: m_u , m_s , G_S , K, Λ $\quad \int^{\Lambda} d^3k$

Dimensional regularization: m_u , m_s , G_S , K, D, M_0 $\quad M_0^{4-D}\int^{\infty} dk^D$

by using the following values, m_u =3-6MeV,

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$$m_\pi = 138\mathrm{MeV},\ f_\pi = 92\mathrm{MeV},\ m_K = 495\mathrm{MeV},\ m_{\eta'} = 958\mathrm{MeV}$$

In dimensional regularization case, we take additional one quantity, topological susceptibility χ (=(170MeV)^4) or m_η .

- Meson mass: In the leading order of the $1/N_c$ expansion, pseudo-scalar meson propagator is given by,

$$S_{ab}(p^2) = \gamma_5 T_a \frac{2K_a^+}{1-2K_a^+\Pi_{ps}(p^2)}\gamma_5 T_b \quad \rightarrow\ 1-2K_a^+\Pi_{ps}(p^2=m_{ps}^2)=0$$

where Π_{ps} are the meson self-energies, K_a^+ correspond to each effective coupling channel, $K_a^+ \propto G_S + K\cdot i\mathrm{tr}S^i$

- Topological susceptibility

In NJL model axial current becomes

$$\partial_\mu J_5^\mu = 2N_f\underline{[-iK\{\det\bar{\psi}(1-\gamma_5)\psi + \text{h.c.}\}]} + 2i\bar{\psi}m\gamma_5\psi$$

c.f. in QCD axial current is

$$\partial_\mu J_5^\mu = 2N_f\underline{Q(x)} + 2i\bar{\psi}m\gamma_5\psi,\ \ Q(x) = \frac{g^2}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$$

Comparing these expression, one can read the topological density,

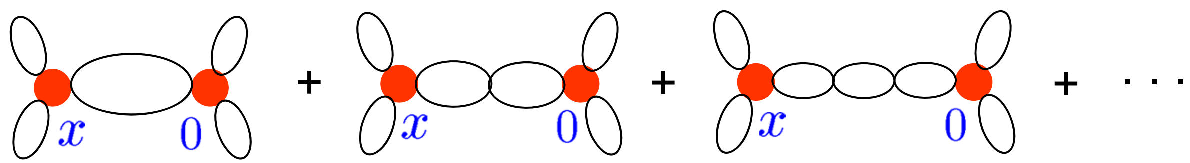
T.Hatsuda, T.Kunihiro, Phys.Rep. 247,221(1994), K.Fukushima, K.Ohnishi, K.Ohta, PRC63,04503 (2001)

$$Q(x) = -iK\{\det\bar{\psi}(1-\gamma_5)\psi + \text{h.c.}\}$$

and topological susceptibility is defined as

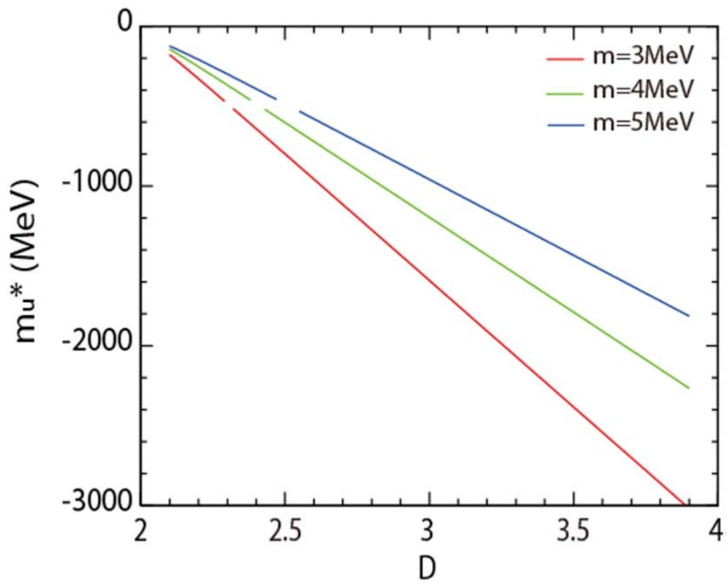
$$\chi = \int d^4x\langle 0|TQ(x)Q(0)|0\rangle_{\text{connected}}$$

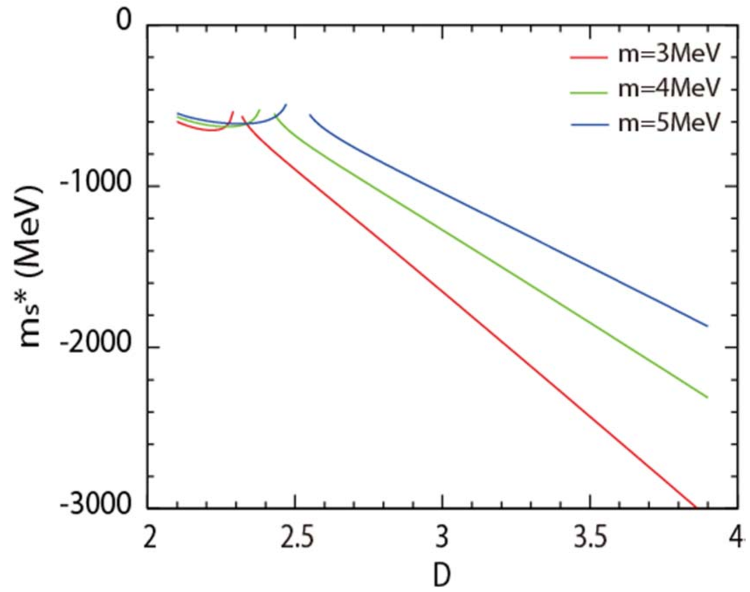
$\simeq$



- Solutions of the gap equations m_i^* as a function of dimension D

$$m_i^* = m_i^*(m_u, m_\pi, m_K, m_{\eta'}, D) \qquad i = u, s$$





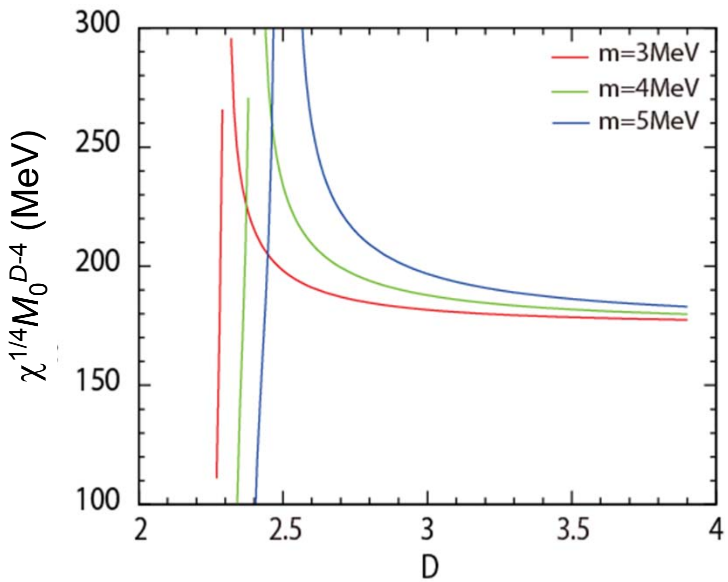
There are no solutions at D~2.5. In is region self-energy Π is divergent,

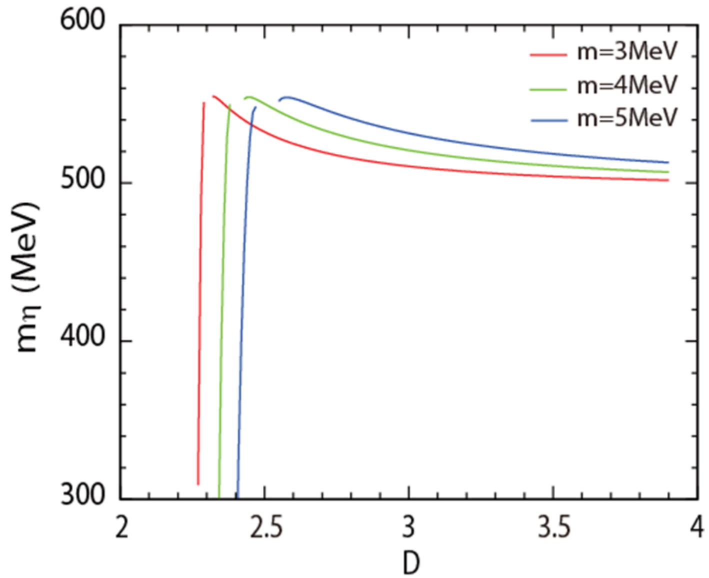
$$\Pi_{ii}(p^2 = m_{\eta'}^2) \propto \int \frac{d^D k}{i(2\pi)^D} \frac{1}{(k^2 - m_i^{*2})\{(k - m_{\eta'})^2 - m_i^{*2}\}}$$

$$\ni 1/\sqrt{4m_i^{*2} - m_{\eta'}^2}^{\,2-D/2} \to \infty \qquad \text{for } (2m_i^*)^2 \to m_{\eta'}^2 = (958\text{MeV})^2$$

We have defined $m_{\eta'}$ as the real part of the pole for η’ meson propagator.

- Topological susceptibility χ and m_η as a function of dimension D





- Obtained variables for cutoff and dimensional regularizations (input value)

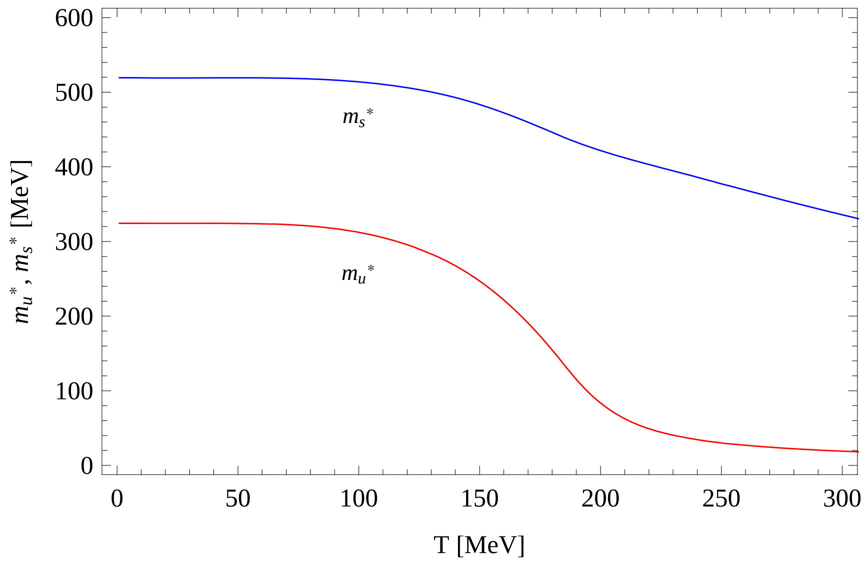
	regularization	m_u	m_s	m_η	$\chi^{1/4}$	$-\langle\bar{u}u\rangle^{1/3}$	D
a)	cutoff	(5.5)	136	482	163	245	4
b)	dimensional	(5.5)	150	473	(170)	247	2.47
	dimensional	(5.5)	148	(548)	224	246	2.78
c)	dimensional	(3.0)	84.9	(548)	244	301	2.29
	exp./empirical	3.4-6.8	94.5-176	548	170-179	228-287	4

3. Meson properties at finite temperature

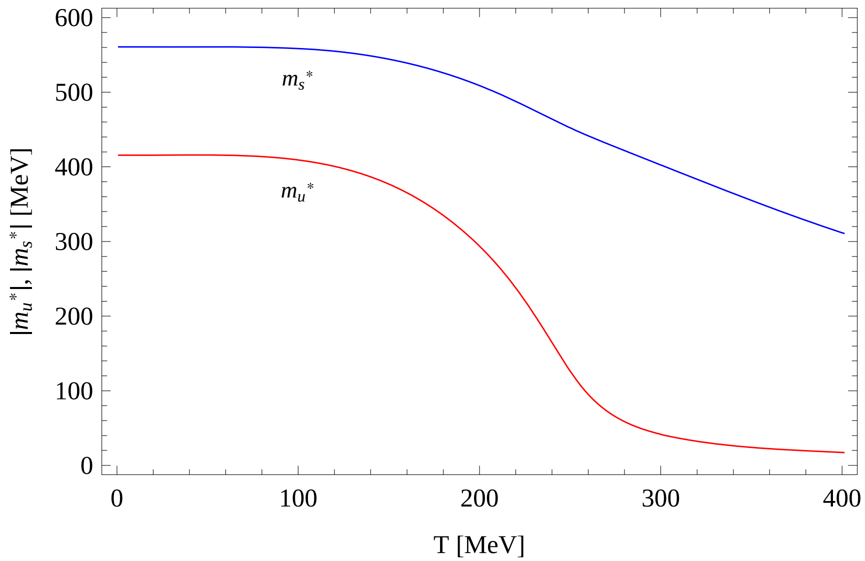
We introduce temperature T and chemical potential μ
using the imaginary time formalism $(t \to -ix_4,\ 0 \le x_4 \le 1/T)$.
The quark propagator is modified as, $k_0 \to i\omega_n + \mu,\ \omega_n = (2n+1)\pi T,\ n \in \mathbf{Z}$

$$-\langle \bar{u}u \rangle M_0^{D-4} = T \sum_{n=-\infty}^{\infty} \int \frac{d^{D-1}\boldsymbol{k}}{(2\pi)^{D-1}} \mathrm{tr} \frac{1}{\boldsymbol{k}\cdot\boldsymbol{\gamma} - (\omega_n - i\mu)\gamma_4 + m_u^* - i\epsilon}$$

- Behavior of the gap equations as a function of T and T_c T_c: maximum of $\frac{\partial \langle \bar{u}u \rangle}{\partial T}$

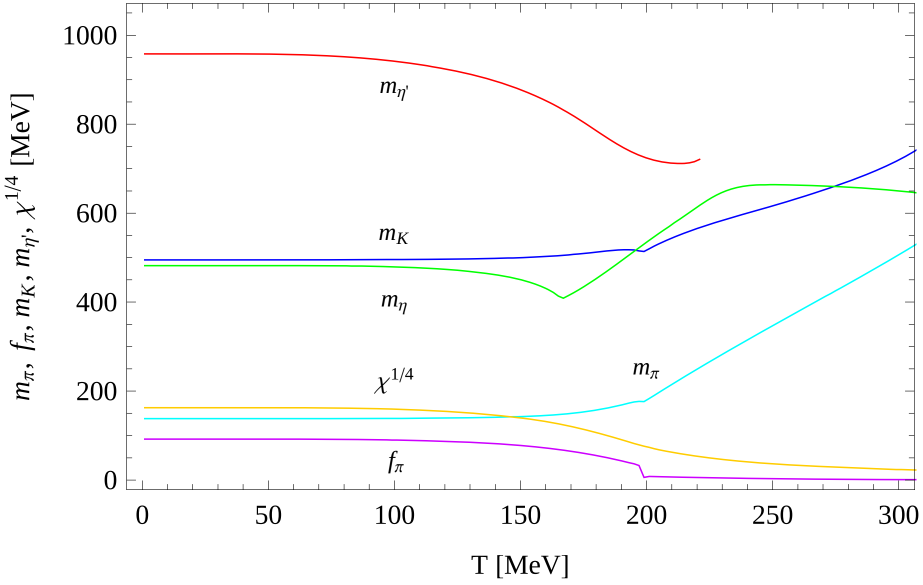


a) cutoff regularization T_c = 183MeV
μ = 0

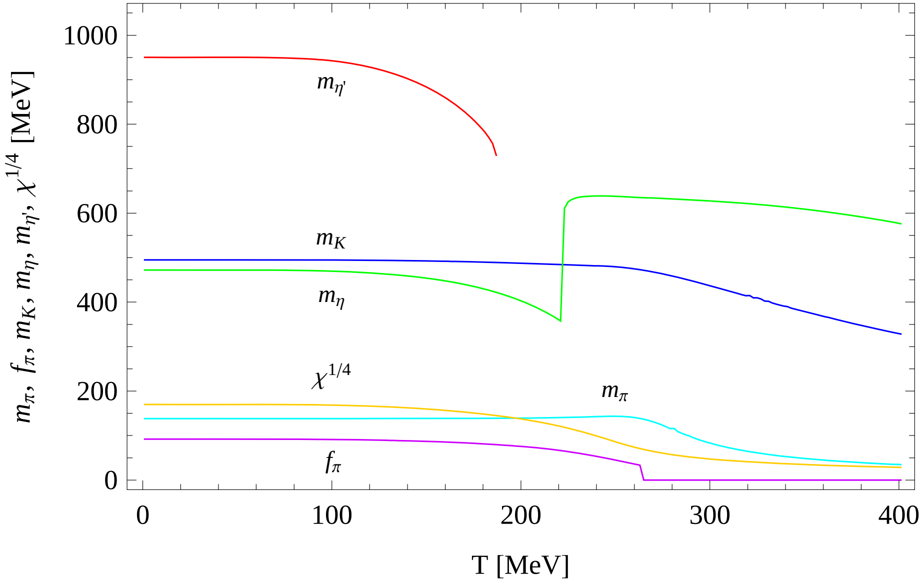


b) dimensional regularization T_c = 240MeV
μ = 0

- Behavior of meson masses, f_π and χ as a function of T



a) cut-off regularization



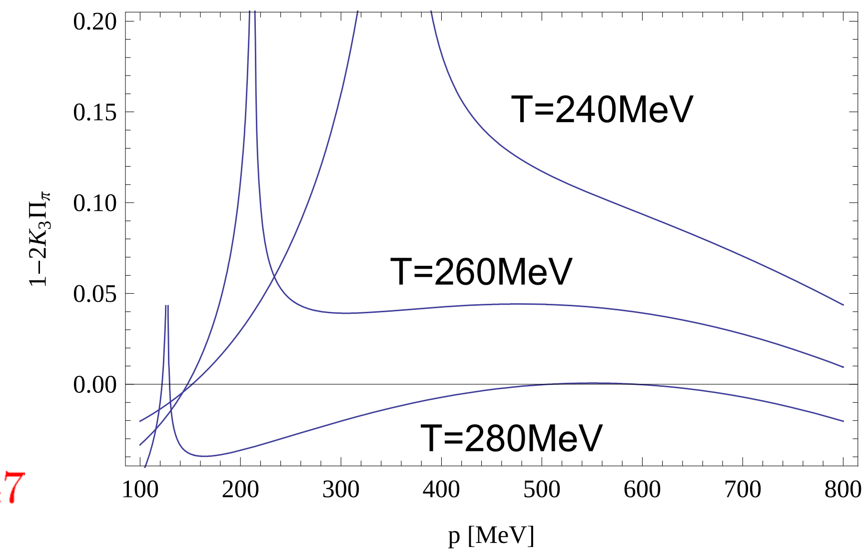
b) dimensional regularization

- Denominator of the pion (kaon) propagator, $1 - 2\,K_3\,\Pi_\pi$

These peaks come from the divergence of $\Pi_\pi\ (p^2)$

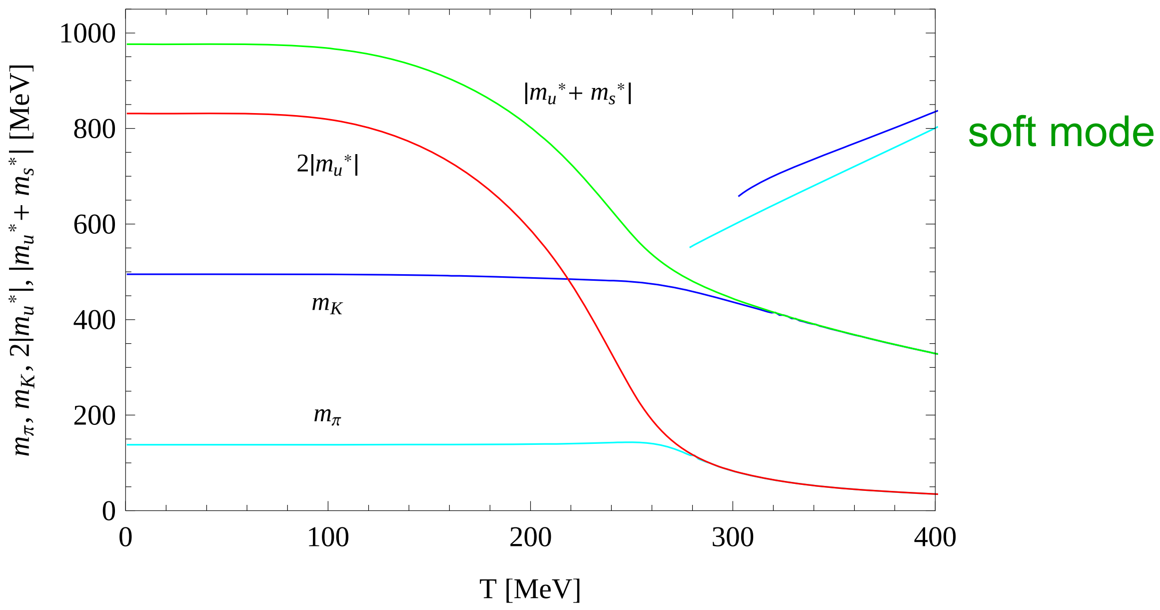
$$\Pi_\pi(p^2) \ni 1/\sqrt{4m_u^{*2} - p^2}^{\,2-D/2} \to \infty$$

$$\text{for } p^2 \to (2m_u^*)^2,\ D = 2.47$$



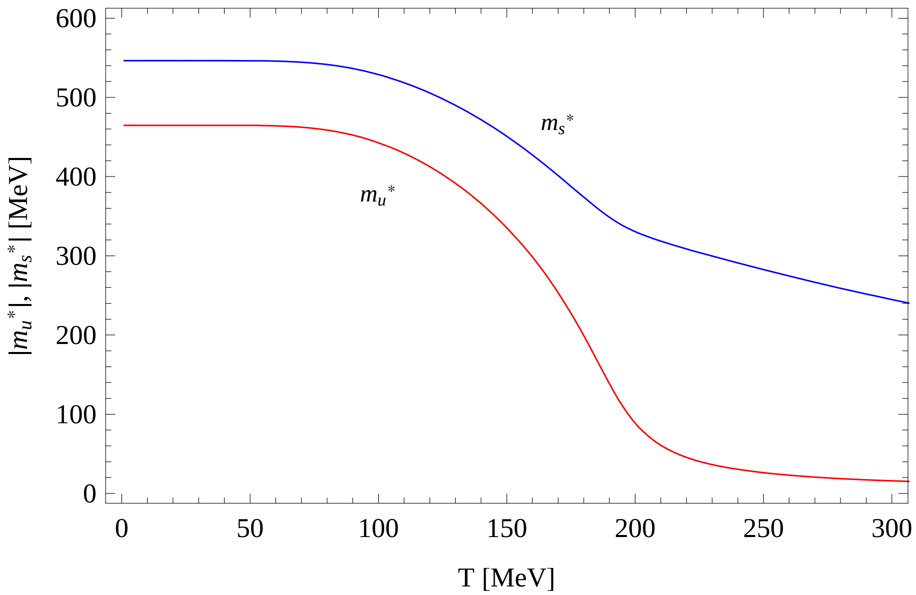
- Behavior of meson masses (soft mode), $m_i^* + m_j^*$ as a function of T

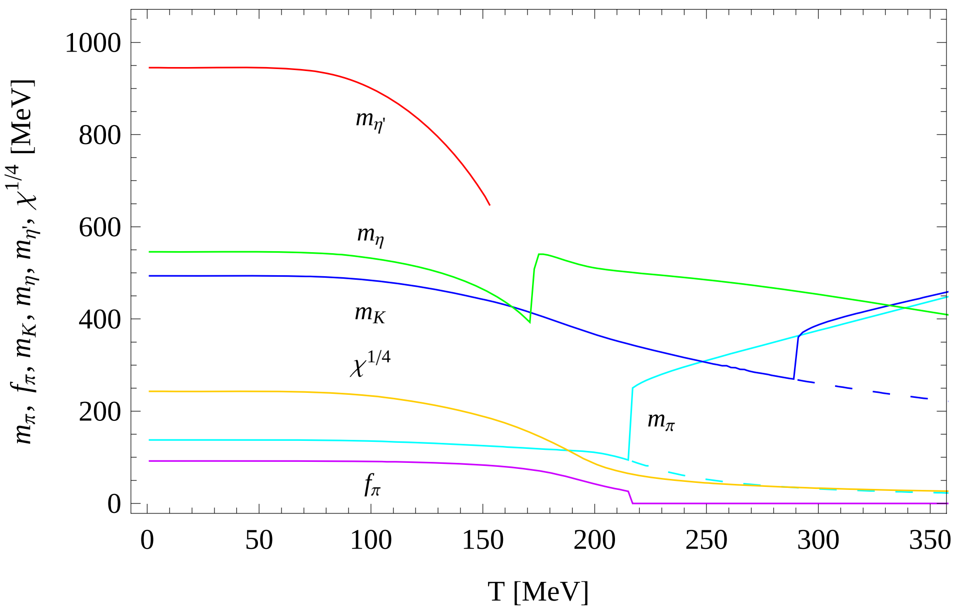
There are no positive values for the denominator of the meson propagator except the divergence of Π_{ps}. In these region we take maxim of the denominator for the meson propagator (soft mode).



Here, $2m_u^*$ and $m_u^* + m_s^*$ for $T < T_c$ correspond to σ, $\kappa(K_0^*)$ meson masses, respectively.

- Dimensional regularization, case c)





solution of the gap equations

meson masses, f_π and χ

In this case, critical temperature is an appropriated value, T_c = 184MeV.

4. Summary

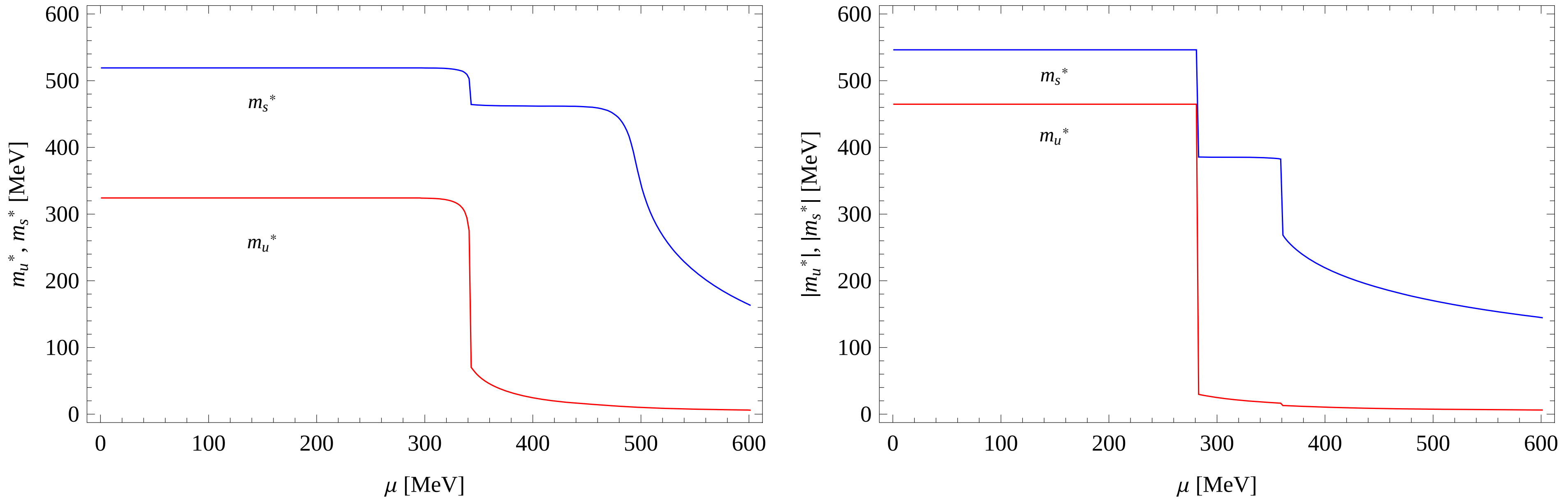
- We have fixed the model parameters with meson masses, pion decay constant and(or) topological susceptibility. Using the obtained parameters, we calculated m_s , m_η , χ , etc. with cutoff and dimensional regularizations. These results with the dimensional method almost reproduce the experimental/empirical values as well as those of the cut-off method.

- We have evaluated the behavior of the meson properties at finite *T*. In dimensional regularization case b), T_c is higher than the well-known value. However, in dimensional regularization case c), T_c is appropriated value. The results by the dimensional regularization have similar meson properties to the cutoff ones, where unwanted divergence of Π_{ps} appear near T_c and meson behaviors are irregular in this region.

Future works

- Evaluation of the meson properties as a function of μ (almost finished).
- We plan to evaluate the phase structure of T-μ plane.
- Other regularization schemes: Pauli-Villars regularization, Schwinger proper-time method
- Calculation which does not depend on the regularizations

- Behavior of the gap equations as a function of μ T = 10MeV



a) cutoff regularization

c) dimensional regularization