Thermal Phase Transition in Gauge-Higgs Unification in Warped Spacetime

(work in progress)

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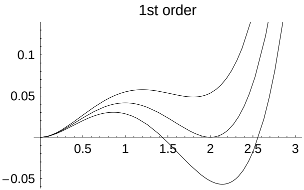
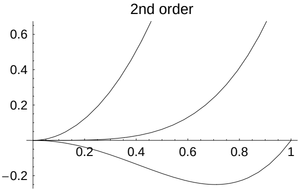
Outline

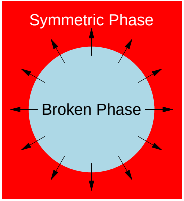
1. Introduction
2. Formulation of Effective potential at finite temperature with non-periodic KK modes
3. Numerical Study of $SO(5) \times U(1)$ GHU model [preliminary]

Electroweak phase transition and thermal effects

- Our universe : baryon asymmetric
- Baryogenesis - Sakhalov's three conditions
  1. B-violation process
  2. C and CP symmetry is broken
  3. Out of thermal equilibrium
- Electroweak baryogenesis - the 3rd condition requires the first-order phase transition and the expanding bubbles (inside : broken phase)







- For EWSB, we study
  1. the order of the phase transition - 1st order or 2nd order
  2. (when 1st-order) we check the Shaposhnikov's "non-washing-out condition":

$$\frac{\varphi_c}{T_c} \gtrsim 1, \qquad (1)$$

$$T_c : \text{ critical temperature} \quad \varphi_c : \text{VEV at } T = T_c \qquad (2)$$

- Phase transition @high-T for SM and other Models
  1. Standard Model - 2nd order
  2. SUSY(MSSM) - 1st order for some models [Cline etal,Farrar etal, Losada, Bodeker etal,Carena etal, Funakubo]
  3. 2HDM, extended Higgs - conditions for 1st order obtained [Kanemura etal]
  4. Little Higgs - Symmetry non-restoration [Espinosa, etl.al.(2004), Aziz, et.al.(2009)]
     - Attempts to get EWPT [Ahriche, et.al. (2009-)]
  5. GHU in flat-ExD (Hosotani mechanism, flat ExD)
     - - 1st order [Ho-Hosotani (1990), Panico, et.al.('05)]
     - - φ_c/T_c [Maru-Takenaga('05,'06)]

Gauge-Higgs unification (GHU)

- Gauge-Higgs unification (GHU) [N.S.Manton (1983), Hosotani('83),...,HH-Inami-Lim,]:
  - extra-dimensional component of the gauge field = the Higgs field

$$A_M = (A_\mu, A_y = h) \tag{3}$$

  - gauge symmetry is spontaneously broken by nonzero $\langle A_y \rangle$
  - Effective potential and the Higgs-mass is finite, thanks to the gauge symmetry in the higher-dimensional spacetime
    $\rightarrow$ solve the gauge hierarchy (fine-tuning) problem
  - GHU on RS [Oda-Weiler,Falkowski, Hosotani,...,HH, and "holographic models"(agaghe, contino)]
    - fermion mass hierarchy, hierarchy between E_{EW} and E_{Planck} are explained. Sufficiently large higgs mass is obtained.
    - complicated KK mass structure : we cannot apply the Poisson sum formula to the KK-series

Effctive Potential at Finite Temperature in Higher-Dimensional Spacetime

FT Effects with non-periodic KK tower

- 1-loop effective potential (per field degrees of freedom) at temperature T with Kaluza-Klein mass m_ℓ:

$$V_{\rm eff}^{1-{\rm loop}} = \frac{(-1)^{2\eta}}{2}\frac{1}{\beta}\int\frac{d^3p}{(2\pi)^3}\sum_{m=-\infty}^{\infty}\sum_{n}\ln\left[\left(\frac{2\pi(m+\eta)}{\beta}\right)^2+\vec{p}^2+M_n^2\right],$$
$$\eta = 0(\text{boson}),\ \tfrac{1}{2}(\text{fermion}),\quad \beta\equiv 1/T. \tag{4}$$

  - When the extra dimension is compactified on S^1 (radius R),

$$M_n^2 = \left(\frac{2\pi n+\theta}{2\pi R}\right)^2 + M^2,\quad M:\text{bulk mass} \tag{5}$$

    $\rightarrow$ one may make use of many tricks (Poisson sum formula, etc...)
  - For non-periodic KK modes (e.g. warped compactification) we needs another way of summation.

- Poisson re-summation only for Matsubara modes gives

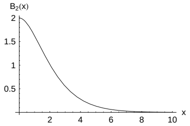
$$V_{\rm eff}^{1-{\rm loop}} = -\frac{(-)^{2\eta}}{2}\sum_n \frac{\Gamma(-2)}{(4\pi)^2}|M_n|^4 +2(-1)^{2\eta-1}\sum_n\sum_{\tilde{m}=1}^{\infty}(-)^{2\eta\tilde{n}}\frac{(\tilde{m}\beta|M_n|)^2K_2(\tilde{m}\beta|M_n|)}{(\sqrt{2\pi}\tilde{m}\beta)^4} \quad (6)$$

  - the 1st term turns out to be the zero-templerature effective potential (dimensional regularization is used inversely)

$$\frac{(-)^{2\eta}}{2}\int\frac{d^4k}{(2\pi)^4}\ln(p^2+M_n^2)\equiv V_{\rm eff}(T=0), \quad (7)$$

    As for GHU on RS, Effective potential for $T=0$ can be calculated [Falkowski, Oda-Weiler, ..., HH]

  - $x^dK_d(x)\equiv B_d(x)$ is a super-convergent function of x:



$\rightarrow$ Finite correction to $V_{\rm eff}$ is obtained numerically with desired accuracy.

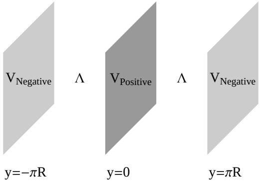
Numerical Study of $SO(5) \times U(1)_X$ GHU model [preliminary]

Field Theory in the Randall-Sundrum space-time

- non-factorizable metric:

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2, \quad k : AdS_5 \text{ curvature} \tag{8}$$

- circle with identification : $y \to -y$ fundamental region : $[0, \pi R]$ fixed points : $y_0 = 0, \quad y_1 = \pi R$



- Hierarchy
  1. UV (hidden brane) scales : Λ, M_5, k, R
  2. IR (visible brane) scales : $m_{KK} = \pi k e^{-kR\pi} \frac{1}{1-e^{-\pi k R}}$
  3. $kR \simeq 12 \to e^{kR\pi} \simeq M_{\text{Planck}}/M_{\text{Weak}}$

In the coordinate $(x^\mu, z \equiv e^{ky})$,
In the KK expansion $\Phi(x,z) = \sum_n \phi_n(x) f_n(z)$, a KK wave function $f_n(z)$ is written in terms of Bessel functions:

$$f_n(z) = z^\beta [A J_\alpha(m_n z/k) + B Y_\alpha(m_n z/k)], \quad (9)$$

and the kk spectrum is determined by boundary conditions at $z = z_i$ $(z_0 = 1,\ z_1 = e^{kL})$

$$f_n(z_i) = 0, \quad \text{Dirichlet} \quad (10)$$

$$(z^\gamma f_n(z))'|_{z=z_i} = 0, \quad \text{Neumann} \quad (11)$$

- We cannot express a m_n as a explicit function of n but we have only implicit conditions $F(m_n) = 0$.

$SO(5) \times U(1)$ GHU model

As an application, we study the finite-temperature effect on the $SO(5) \times U(1)_X$ GHU model. [Hosotani etal]

- outlines
  - orbifold breaking $SO(5) \times U(1)_X \to SO(4) \times U(1)_X$
    - $SO(4) \sim SU(2)_L \times SU(2)_R$: custodial symmetry
    - 4 $SO(5)/SO(4)$ broken generators ($\to SU(2)_L$ Higgs doublet)
  - $SU(2)_R$ is broken by scalar on the Planck brane
  - fermions : $SO(5)$-5's(vector) on bulk and $\boldsymbol{SU(2)_L}$-doublets on Planck brane $\to$ up- and down- sector fermions with desired masses from one Wilson-line phase, and others remain heavy states

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- outcomes
  - m_h: $70\mathrm{GeV} \sim 140\mathrm{GeV}$
  - The model have "H-parity"
    - $P_H = -1$ is assigned for h and $+1$ for other SM fields
    - All P_H-odd interactions ($hWW, hZZ, h\bar{f}f, hhh$) vanish. $\to$ the model can avoid the LEP constraint ($m_h \leq 114\mathrm{GeV}$)
    - h is stable and can be the candidate of dark matter (higgs dark-matter).

Effective potential

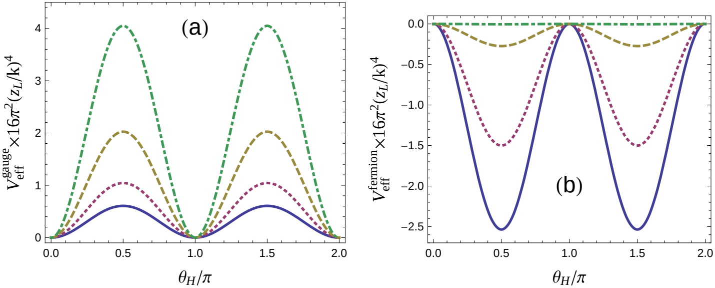

Figure: [preriminary] For $z_L = 10^{15}$, (a) V_{eff}^{gauge} (b) $V_{eff}^{fermion}$ with solid ($\tilde{T} = 0.1$), dotted (0.5), dashed (1.0), dot-dashed (2.0)

- As T grows, $V_{eff}^{fermion}$ quickly shrinks [c.f. Ho-Hosotani(1990)]

Phase Transition

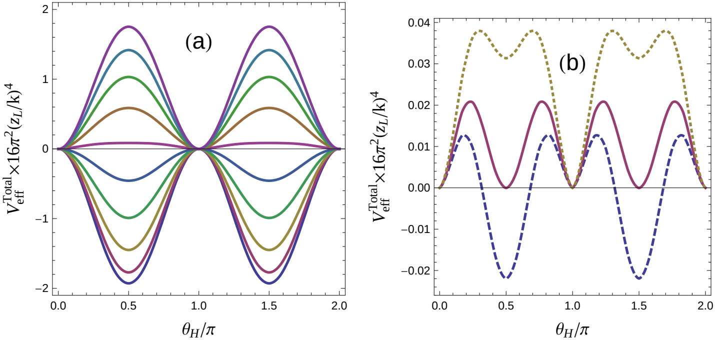

Figure: [preliminary] Plots of $V_{eff}^{Total}(\theta_H, T)$ ($z_L = 10^{15}$) (a) plots for $\tilde{T} = 0.1, ..., 1.0$ (from bottom to top), (b) for $\tilde{T} = 0.58$ (dashed), 0.5841 (solid) and 0.59 (dotted).

Results for various z_L

Table: [preliminary] Critical temperature T_c, the ratio φ_c/T_c, the height of the potential barrier $V_{\rm barrier} \equiv V^{\rm Total}(\pi/4, T_c)$ for various value of $z_L = e^{\pi k R}$

z_L	10^5	10^7	10^{10}	$10^{12.15}$	10^{15}	10^{17}
T_c[GeV]	158	188	224	246	273	290
φ_c/T_c	1.6	1.3	1.1	1.0	0.90	0.85
$V_{\rm barrier}$ $[10^6 {\rm GeV}^4]$	3.0	3.8	4.9	5.5	6.3	6.8
$V_{\rm barrier}^{1/4}/T_c$	0.26	0.24	0.21	0.20	0.18	0.18

- first-order phase transition with very shallow potential

$$V_{\rm barrier}/T_c^4 \sim (0.2)^4 \tag{12}$$

- Shaposhnikov's criteria (aka "non washing-out condition")
  - $\varphi_c = \varphi_0 \simeq 246{\rm GeV}$ in this model
  - The condition $\varphi_c/T_c \gtrsim 1$ is satisfied for $z_L \lesssim 10^{12}$

Summary

- We Numerically studied the $SO(5) \times U(1)_X$ GHU model on RS at finite-temperature.
  - finite-temperature corrections are obtained by summing up Kaluza-Klein masses and dual Matsubara modes.
  - We obtained critical temperature and the height of the potential wall of the model.

Perspective

- This method can by applied to study other models of extra dimension at finite temperature.
- Spharelon process in higher-dimensional space-time
- Flavor mixing, CP violation phase in GHU

Backup Slides

[Trodden, RMP71-1463]

- quantum scalar field theory:
  - @ tree level

$$V(\varphi) = -\frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4, \quad \varphi \equiv \sqrt{\phi^\dagger \phi}, \tag{13}$$

  - @ 1-loop zero temperature

$$V_{\rm eff}^{(1)}(\varphi) = -\frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4 + \frac{1}{64\pi^2}(3\lambda\varphi^2 - \mu^2)^2 \ln\left(\frac{3\lambda\varphi^2 - \mu^2}{2\mu^2}\right) + \frac{21\lambda\mu^2}{64\pi^2}\varphi^2 - \frac{27\lambda^2}{128\pi}\varphi^4, \tag{14}$$

  - SM, @ 1-loop, finite temperature

$$V_{\rm eff}^{(1)}(\varphi;T) = \left(\frac{3g^2}{32} + \frac{\lambda}{4} + \frac{m_t^2}{4v^2}\right)(T^2 - T_*^2)\varphi^2 - \frac{3g^2}{32\pi}T\varphi^3 + \frac{\lambda}{4}\varphi^4, \tag{15}$$

v : usual Higgs VEV

    - cubic φ-term : $\rightarrow$: first order phase transition

Effective Potential

- Effective potential consists bosonic and fermionic parts:

$$V_{\rm eff}^{\rm Total}(\theta_H, T) \quad = \quad V_{\rm eff}^{\rm gauge}(\theta_H, T) + V_{\rm eff}^{\rm fermion}(\theta_H, T), \tag{16}$$

  - $V^{\rm gauge}$ contains the loop contribution from W, Z, 3 Higgs bosons and their KK excitations
  - $V_{\rm fermion}$ contains the loop contribution from top-quark and its KK excitations
  - In the following, other contributions are neglected.
- each parts are decomposed zero-temperature parts and finite-temperature corrections

$$V_{\rm eff}^{f}(\theta_H, T) \quad = \quad V_{\rm eff,T=0}^{f}(\theta_H) + \Delta V_{\rm eff}^{f}(\theta_H, T), \quad (f = \text{gauge, fermon}) \tag{17}$$

Zero temperature parts: [c.f. Hosotani, etal, PRD**78**-096002, **82**-115024]

$$V_{\mathrm{eff,T=0}}^{\mathrm{gauge}}(\theta_H) = \underbrace{4I[\frac{1}{2}Q_0(q,\tfrac{1}{2},\theta_H)]}_{W-tower} + \underbrace{2I[\frac{1}{\cos^2\theta_W}Q_0(q,\tfrac{1}{2},\theta_H)]}_{Z-tower} + \underbrace{3I[Q_0(q,\tfrac{1}{2},\theta_H)]}_{Higgs-tower}, \quad (18)$$

$$V_{\mathrm{eff,T=0}}^{\mathrm{fermion}}(\theta_H) = \underbrace{-4\cdot 3I[\frac{1}{2}Q_0(q,c_{\mathrm{tb}},\theta_H)]}_{top-tower}, \quad (19)$$

where

$$I[Q(q;c,\theta_H)] \equiv \frac{\tilde{k}^4}{(4\pi)^2}\int_0^\infty dq\, q^3 \ln[1+Q(q;c,\theta_H)], \quad \tilde{k} \equiv k/z_L, \quad (20)$$

$$Q_0 \equiv \frac{z_L}{q^2}\frac{\sin^2\theta_H}{\hat{F}_{c-\frac{1}{2},c-\frac{1}{2}}(q/z_L,q)\hat{F}_{c+\frac{1}{2},c+\frac{1}{2}}(q/Z_L,q)}, \quad (21)$$

$$\hat{F}_{\alpha,\beta}(u,v) \equiv I_\alpha(u)K_\beta(v) - e^{-i(\alpha-\beta)\pi}K_\alpha(u)I_\beta(v), \quad (22)$$

and c_{tb} is determined to obtain the correct top-quark mass for z_L.

non-zero temperature corrections:

$$\Delta V_{\rm eff}^{\rm gauge}(\theta_H,T) = 4S[P_W(\lambda,\theta_H),T,0]+4S[P_Z(\lambda,\theta_H),T,0] +4S[P_H(\lambda,\theta_H),T,0] \quad (23)$$

$$\Delta V_{\rm eff}^{\rm fermion}(\theta_H,T) = 12S[P_T(\lambda,\theta_H),T,1/2], \quad (24)$$

$$S[P(\lambda,\theta_H),T,\eta] \equiv -(-)^{2\eta}\frac{\tilde{T}^4\tilde{k}^4}{2\pi^2}\sum_{\tilde{m}=1}^{\infty}\frac{(-)^{2\tilde{m}\eta}}{\tilde{m}^4}\sum_{n=0}^{\infty}B_2(\frac{\tilde{m}\tilde{\lambda}_n}{\tilde{T}}), \quad (25)$$

$$\tilde{T}\equiv T/\tilde{k}, \quad B_2(x)\equiv x^2K_2(x), \quad (26)$$

where $\tilde{\lambda}_n$ is the n-th smallest root of P_f : $P_f(\tilde{\lambda}_n/z_L,\theta_H)=0$,

$$P_W(\lambda,\theta_H) = 2S(1;\lambda)C'(1;\lambda)+\lambda\sin^2\theta_H, \quad \text{[for } W\text{-boson KK tower]} \quad (27)$$

$$P_Z(\lambda,\theta_H) = 2S(1;\lambda)C'(1;\lambda)+\lambda\frac{\sin^2\theta_H}{\cos^2\theta_W}, \quad \text{[for } Z\text{-boson KK tower]} \quad (28)$$

$$P_H(\lambda,\theta_H) = 2S(1;\lambda)C'(1;\lambda)+\lambda\sin^2\theta_H, \quad \text{[for higgs-boson KK tower]} \quad (29)$$

$$P_T(\lambda,\theta_H) = 2S_L(1;\lambda,c_{tb})S_R(1;\lambda,c_{tb})+\sin^2\theta_H, \quad \text{[for top-quark KK tower]} \quad (30)$$

with

$$S(z;\lambda)\equiv+\frac{\pi}{2}\lambda zz_LF_{1,0}(\lambda z,\lambda z_L), \quad C(z;\lambda)\equiv-\frac{\pi}{2}\lambda zz_LF_{1,1}(\lambda z,\lambda z_L), \quad (31)$$

$$S_{L/R}(z;\lambda,c)=\mp\frac{\pi}{2}\lambda\sqrt{zz_L}F_{c-\frac{1}{2},c-\frac{1}{2}}(\lambda z,\lambda z_L), \quad (32)$$

- In this model, we have very few free parameter: z_L
  1. Compare the W-mass $80.4\mathrm{GeV}$ and the first zero of (27) at $\theta_H = \pi/2$
     $\rightarrow$ Determine the normalization of k ($\tilde{k}$)
  2. Compare the top mass $173\mathrm{GeV}$ and the first zero of (30) at $\theta_H = \pi/2$
     $\rightarrow$ determine the value of c_{top}
     - we can obtain the correct value of c_{tb} only when $z_L \gtrsim 3.4$.
  3. Then, calculate $V_{eff}(\theta_H, T)$ with z_L and c_{tb}
- we have summed up 100 dual Matsubara modes and 200 Kaluza-Klein modes in each part of ΔV_{eff}, to calculate V_{eff} with desired accuracy.