Towards thermalization in heavy-ion collisions

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- 2PI formalism
- Nonequilibrium evolution equations
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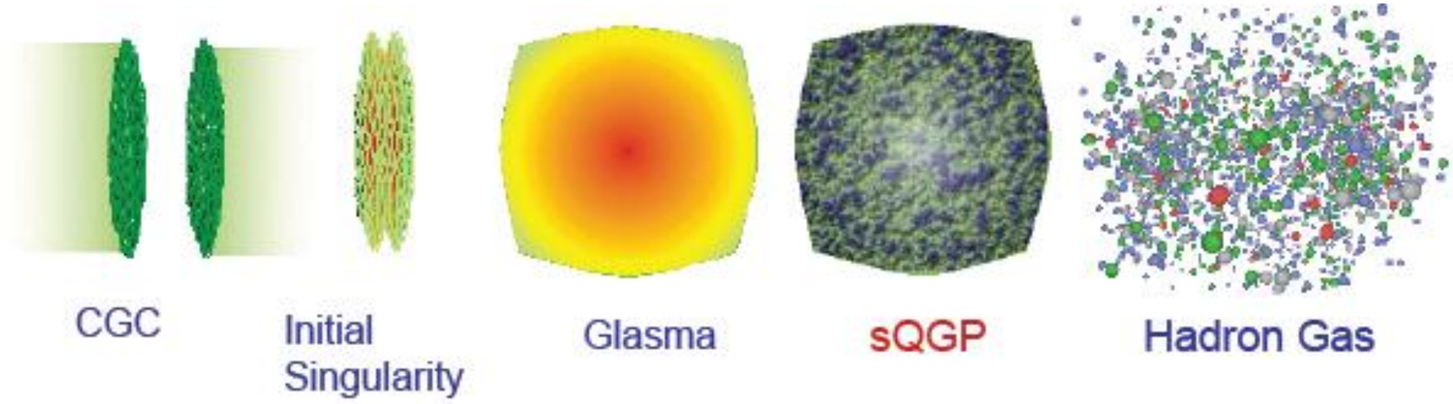
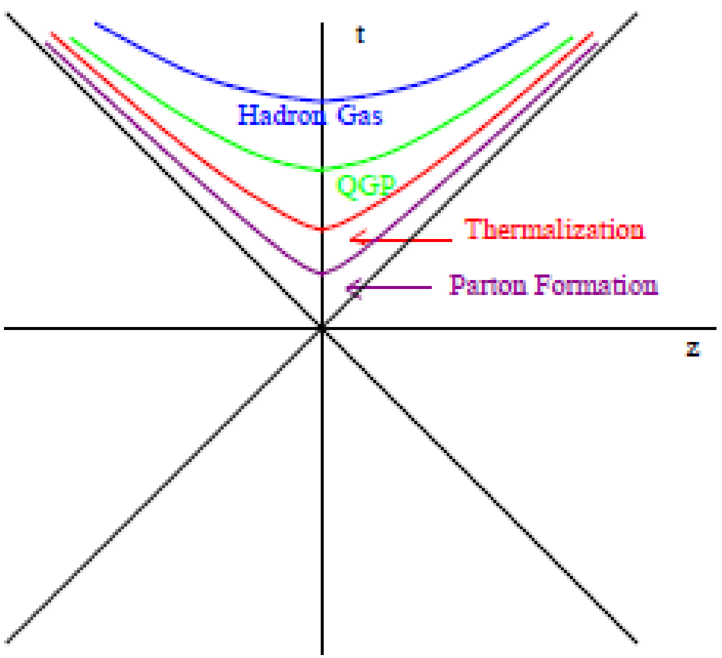
Early stage of HIC





Hydro works at late times

Earliest times Color Glass Condensate (CGC)

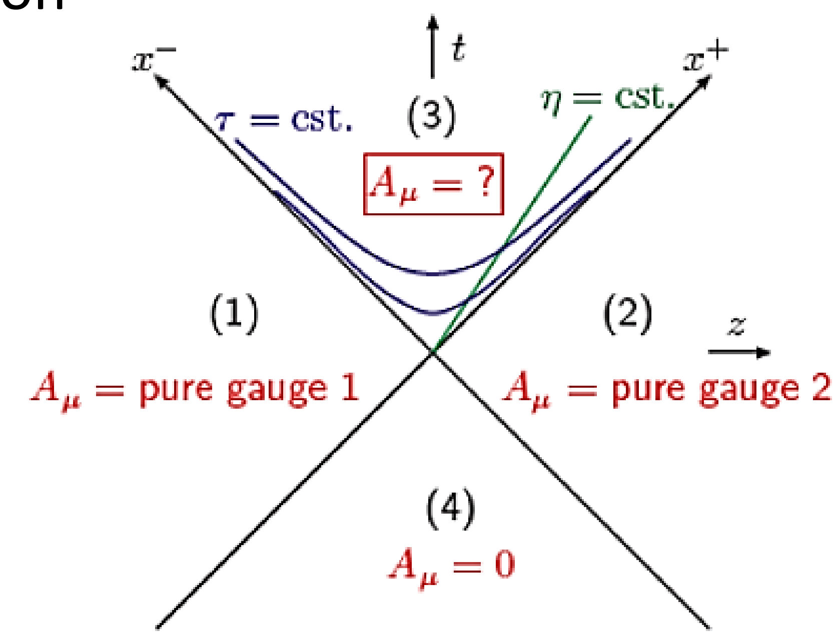
Intermediate times difficult.
Very fast equilibration ?

Gluodynamics in the $\tau - \eta$ coordinates

Proper time $\tau = \sqrt{t^2 - (x^3)^2}\,,$ Rapidity $\eta = \tanh^{-1}\frac{x^3}{t}$

Solve the classical Yang—Mills equation in the gauge $A^\tau = 0$

$$-\frac{1}{\tau}\partial_\tau\left(\frac{1}{\tau}\partial_\tau A_\eta\right) + \frac{1}{\tau^2}D_i F_{i\eta} = 0\,,$$
$$-\frac{1}{\tau}\partial_\tau\left(\tau\partial_\tau A_i\right) + \left(\frac{1}{\tau^2}D_\eta F_{\eta i} + D_j F_{ji}\right) = 0$$



The initial condition

$$A_i = \mathcal{A}_i^1 + \mathcal{A}_i^2\,, \qquad A_\eta = 0\,,$$
$$\tau\partial_\tau A_i = 0\,, \qquad \frac{1}{\tau}\partial_\tau A_\eta^a = -g f_{abc}\mathcal{A}_i^{1b}\mathcal{A}_i^{2c}$$

Kovner, McLerran, Weigert (1995)

Thermalization in CGC picture

Classical YM eq. alone is insufficient for the problem of thermalization.
Need quantum fluctuations

Romatschuke, Venugopalan (2006);
Fukushima, Gelis, McLerran (2007)

Present status: Classical statistical approach
Quantum fluctuations resummed to all orders (see later)

Dusling, Epelbaum, Gelis, Venugopalan, (2010)

In this work we propose to apply the 2PI formalism
to the problem of thermalization in HIC

2PI formalism

Berges, AIP conf. proc. 739, 3 (2005)

- First principle calculation in field theories out of equilibrium.
- Based on the CJT effective action

$$\Gamma[A,G] = S_{YM}[A] + \frac{i}{2}\mathrm{tr}\ln G^{-1} + \frac{i}{2}\mathrm{tr}\, G_0^{-1}[A]G + \Gamma_2[A,G]$$

Classical field   Quantum fluctuation $G = \langle aa \rangle$ 2PI diagrams

- Achieves quantum thermal equilibrium (Bose-Einstein distribution) starting from far-from-equilibrium initial conditions

Effective action to three-loops

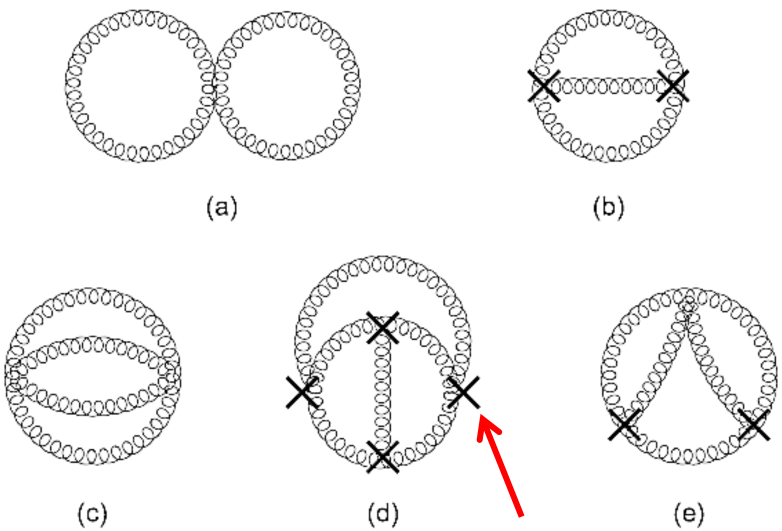
Equation of motion

$$\frac{\delta\Gamma}{\delta A} = 0 \qquad \frac{\delta\Gamma}{\delta G} = 0 \qquad (\, G = G_0 + G_0 \Pi G \,)$$

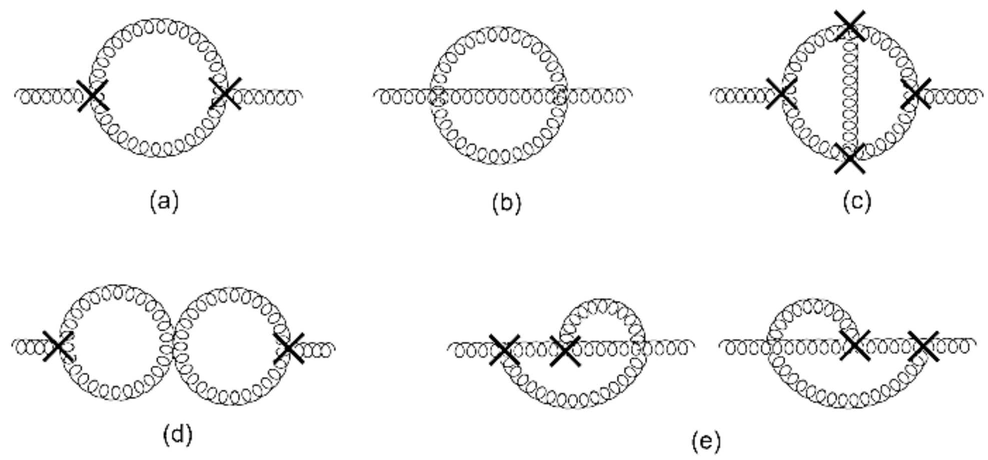
$$G(x, x') = \langle \mathrm{T}_C\{a(x)a(x')\}\rangle = \mathcal{F} - \frac{i}{2}(\theta_C(\tau - \tau') - \theta_C(\tau' - \tau))\,\rho$$

Statistical function

Spectral function



$D = \partial - igA$



Getting rid of the metric factors

Spatial metric $\gamma_{\alpha\beta} \equiv \mathrm{diag}\left(\tau^2, 1, 1\right) \qquad x^\alpha = (\eta, x_\perp)$

$$S_{YM} = \int d\tau d\eta d^2x_\perp \sqrt{\gamma}\left[\frac{1}{2}\gamma^{\alpha\beta}\partial_\tau A_\alpha \partial_\tau A_\beta - \frac{1}{4}\gamma^{\alpha\beta}\gamma^{\gamma\delta}F_{\alpha\gamma}F_{\beta\delta}\right]$$

$$\mathcal{L}_3 + \mathcal{L}_4 = -g f_{abc}\gamma^{\alpha\gamma}\gamma^{\beta\delta}(D_\alpha a_\beta)^a a^b_\gamma a^c_\delta - \frac{g^2}{4} f_{abc} f_{ab'c'}\gamma^{\alpha\gamma}\gamma^{\beta\delta} a^b_\alpha a^c_\beta a^{b'}_\gamma a^{c'}_\delta$$

Rescale $\zeta \equiv \tau\eta \qquad A_\eta = \tau A_\zeta\,, \qquad a_\eta = \tau a_\zeta\,, \qquad \partial_\eta = \tau\partial_\zeta$

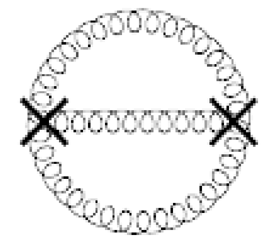
in the interaction terms, but not in the kinetic term.

In the new coordinates $x^I = (\zeta, x_\perp)$

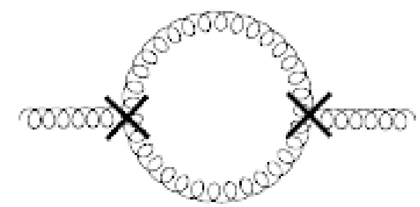
Feynman rules are formally the same as in the flat metric case.

Results at two-loops

$$\left(\partial_\tau^2 A_I + \frac{1}{\tau}\partial_\tau A_I - \delta_{I\zeta}\frac{A_I}{\tau^2} - D_J F_{JI}\right)^a$$
$$= g f_{abc}\left(D^{be}_{xI}\mathcal{F}^{ec}_{JJ}(x,y) + D^{be}_{xJ}\left(\mathcal{F}^{ec}_{JI}(x,y) - 2\mathcal{F}^{ec}_{IJ}(x,y)\right)\right)_{y=x}$$
$$+\frac{ig^2}{2}C_{ab,cd}\int_{\tau_0}^{\tau} d^4y\, V^y_{lmn,LMN}\Big[\rho^{bl}_{IL}(x,y)\mathcal{F}^{cm}_{JM}(x,y)\mathcal{F}^{dn}_{JN}(x,y)$$
$$+(\mathcal{F}\rho\mathcal{F}) + (\mathcal{F}\mathcal{F}\rho) - \frac{1}{4}(\rho\rho\rho)\Big]$$



$$\left[\left(\partial_\tau^2 + \frac{1}{\tau}\partial_\tau - \frac{1}{\tau^2}\delta_{I\zeta}\right)\delta_{IJ} - \left(D^2\delta_{IJ} - D_I D_J - 2igF_{IJ}\right)\right]^{ab}\mathcal{F}^{bc}_{JK}(x,y)$$
$$+g^2\left(C_{ad,be}\mathcal{F}^{de}_{IJ}(x,x) + \frac{1}{2}C_{ab,de}\mathcal{F}^{de}_{MM}(x,x)\delta_{IJ}\right)\mathcal{F}^{bc}_{JK}(x,y)$$
$$= -\int_{\tau_0}^{\tau} d^4z\,\Pi_\rho(x,z)^{ab}_{IJ}\mathcal{F}^{bc}_{JK}(z,y) + \int_{\tau_0}^{\tau'} d^4z\,\Pi_{\mathcal{F}}(x,z)^{ab}_{IJ}\rho^{bc}_{JK}(z,y)\,,$$



$$\Pi_{\mathcal{F}}(x,y)^{ab}_{IJ} = \frac{1}{2}\overrightarrow{V}^{x}_{alm,ILM}\left(\mathcal{F}^{ll'}_{LL'}\mathcal{F}^{mm'}_{MM'} - \frac{1}{4}\rho^{ll'}_{LL'}\rho^{mm'}_{MM'}\right)_{xy}\overleftarrow{V}^{y}_{bl'm',JL'M'}$$

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Remarks

- Transform covariantly under the residual (τ -independent) gauge transformation

$$A_I \to U A_I U^\dagger + \frac{i}{g} U \partial_I U^\dagger \qquad a_I^a \to (U a_I U^\dagger)^a = U^{ab} a_I^b$$

- Challenging to solve numerically if the background is inhomogeneous. First try the homogeneous case

- The initial condition for $\mathcal{F}$ nontrivial

Dusling, Gelis, Venugopalan (2011)

Comparison with previous approaches

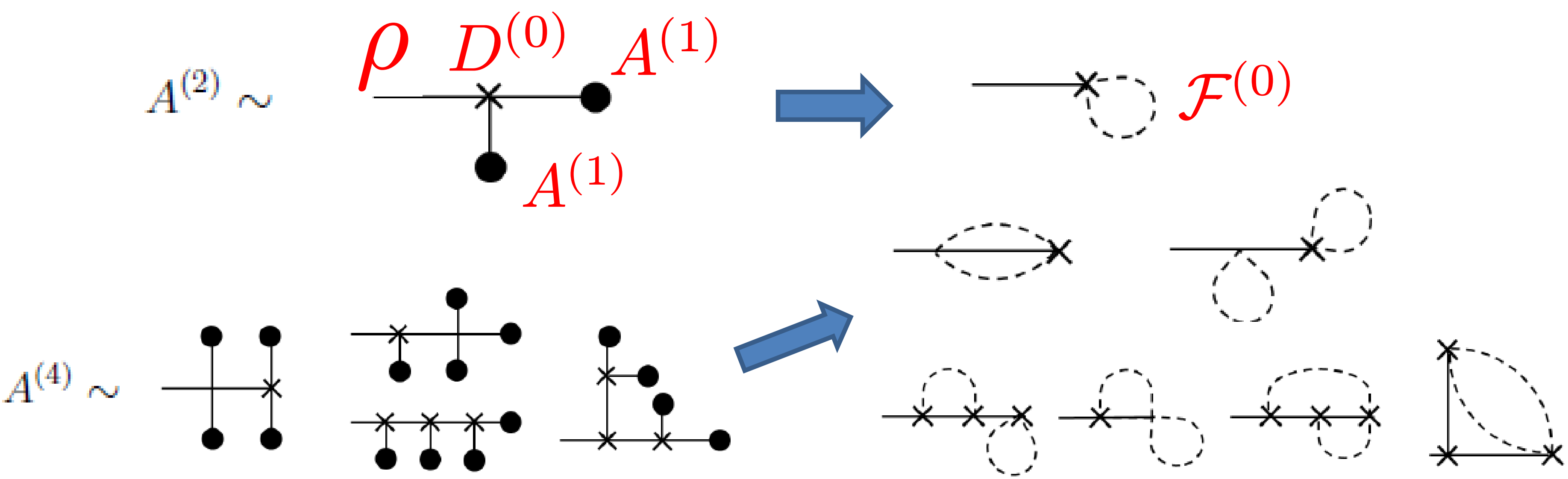
Son (1996) ; Dusling, Epelbaum, Gelis, Venugopalan, (2010)

Solve the YM equation perturbatively $A_I = A_I^{(0)} + A_I^{(1)} + A_I^{(2)} + \cdots$

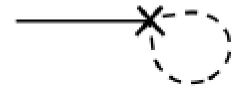
classical YM linearized around YM

with the initial condition $A(\tau_0) = A^{(0)}(\tau_0) + A^{(1)}(\tau_0)$

Identify $\mathcal{F}^{(0)}(x,y) \equiv A^{(1)}(x)A^{(1)}(y)$



Diagrammatic interpretation

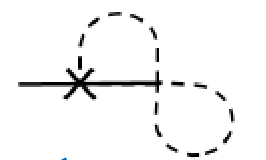


$$\left(\partial_\tau^2 A_I + \frac{1}{\tau}\partial_\tau A_I - \delta_{I\zeta}\frac{A_I}{\tau^2} - D_J F_{JI}\right)^a$$

$$= g f_{abc}\left(D^{bc}_{xI}\mathcal{F}^{ec}_{JJ}(x,y) + D^{bc}_{xJ}\left(\mathcal{F}^{ec}_{JI}(x,y) - 2\mathcal{F}^{ec}_{IJ}(x,y)\right)\right)_{y=x}$$

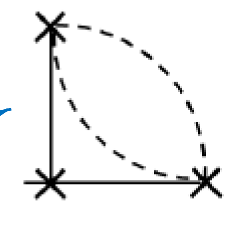
$$+\frac{ig^2}{2}C_{ab,cd}\int_{\tau_0}^{\tau} d^4y\, V^y_{lmn,LMN}\Big[\rho^{bl}_{IL}(x,y)\mathcal{F}^{cm}_{JM}(x,y)\mathcal{F}^{dn}_{JN}(x,y)$$

$$+(\mathcal{F}\rho\mathcal{F}) + (\mathcal{F}\mathcal{F}\rho) - \frac{1}{4}(\rho\rho\rho)\Big]$$

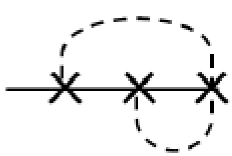


$$\left[\left(\partial_\tau^2+\frac{1}{\tau}\partial_\tau - \frac{1}{\tau^2}\delta_{I\zeta}\right)\delta_{IJ} - (D^2\delta_{IJ} - D_ID_J - 2igF_{IJ})\right]^{ab}\mathcal{F}^{bc}_{JK}(x,y)$$

$$+g^2\left(C_{ad,be}\mathcal{F}^{de}_{IJ}(x,x) + \frac{1}{2}C_{ab,de}\mathcal{F}^{de}_{MM}(x,x)\delta_{IJ}\right)\mathcal{F}^{bc}_{JK}(x,y)$$



$$= -\int_{\tau_0}^{\tau} d^4z\,\Pi_\rho(x,z)^{ab}_{IJ}\mathcal{F}^{bc}_{JK}(z,y) + \int_{\tau_0}^{\tau'} d^4z\,\Pi_{\mathcal{F}}(x,z)^{ab}_{IJ}\rho^{bc}_{JK}(z,y)\,,$$



$$\Pi_{\mathcal{F}}(x,y)^{ab}_{IJ} = \frac{1}{2}\overrightarrow{V}^x_{alm,ILM}\left(\mathcal{F}^{ll'}_{LL'}\mathcal{F}^{mm'}_{MM'} - \frac{1}{4}\rho^{ll'}_{LL'}\rho^{mm'}_{MM'}\right)_{xy}\overleftarrow{V}^y_{bl'm',JL'M'}$$

Classical statistical approximation

$$\mathcal{F}\mathcal{F} \gg \rho\rho \to 0$$

In free scalar theory,

$$\mathcal{F}(t-t',p) = \frac{\cos(t-t')p}{p}\left(n(p)+\frac{1}{2}\right) \qquad \rho(t-t',p) = \frac{\sin(t-t')p}{p}$$

Valid only at low momentum $p \ll T$

Classical thermal equilibrium $n(p) = \dfrac{T}{p} - \dfrac{1}{2}$

In the 2PI formalism the Bose-Einstein distribution is guaranteed.

Boltzmann (exponential) distribution is an unmistakable feature in HIC !

Conclusions

- CGC meets the 2PI formalism
- Complementary to the classical statistical approximation.
- In principle, 2PI can describe the evolution from right after the collision to the late quantum regime in a single framework.