Anomaly-induced charges in nucleons

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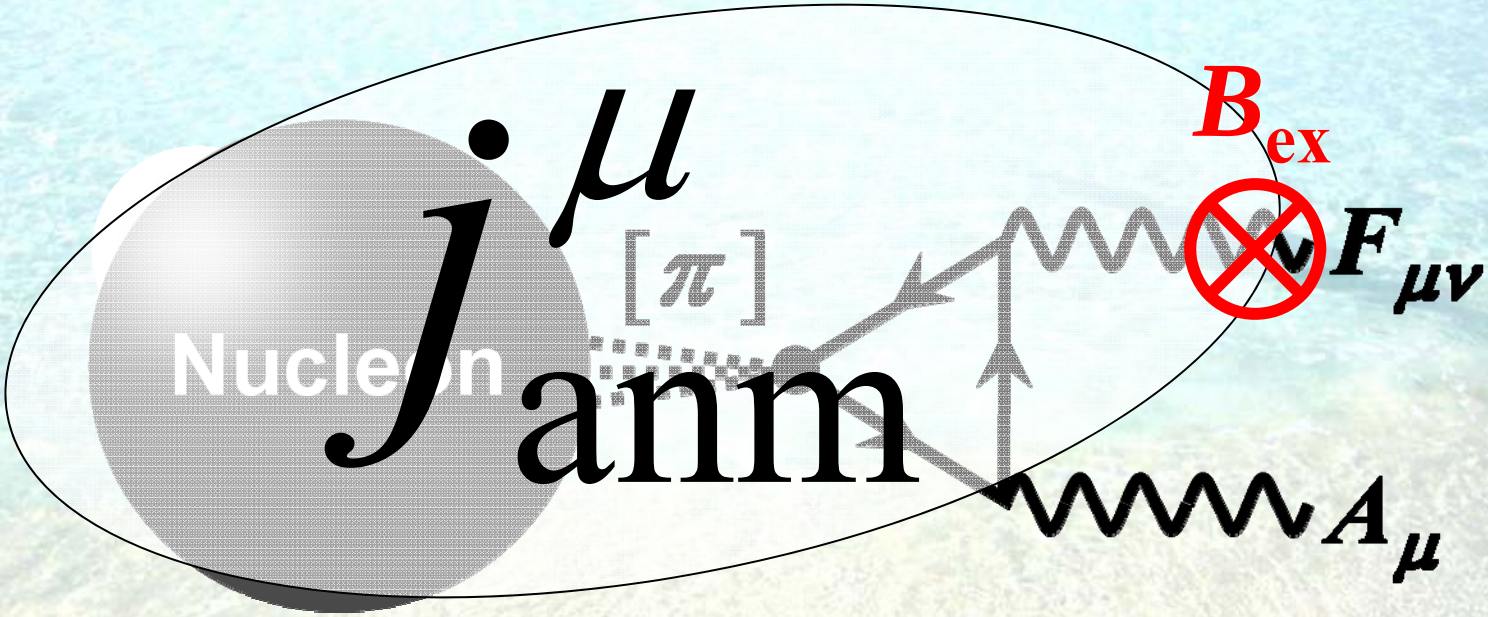


Skyrme model + **WZW** action

→ **Electric charge** and quadrupole moment induced under **external magnetic field**

Eto, Hashimoto, Iida, Ishii, Y.M. arXiv: 1103.5443

Skyrme model ($N_f = 2$)

Nucleon (Skyrmion) = Topological soliton of pions

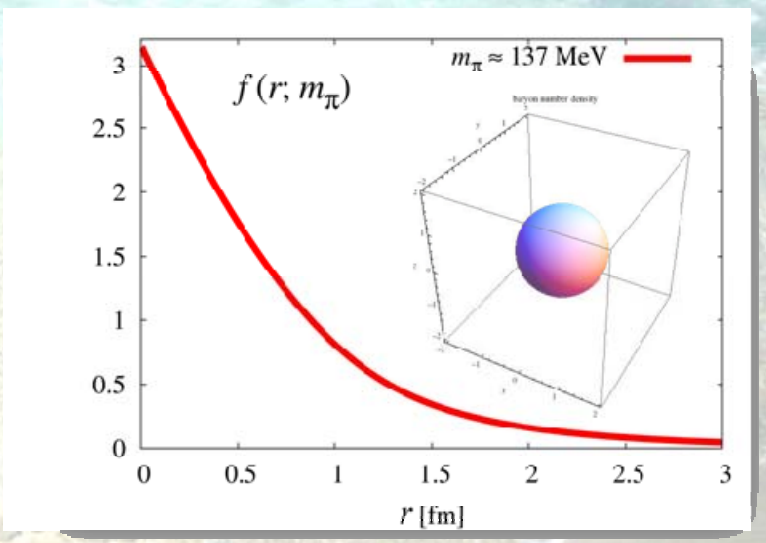
$$S_{\rm SKM} = \int d^4x (\mathcal{L}_{\rm kin} + \mathcal{L}_{\rm mass})$$

$$\mathcal{L}_{\rm kin} = -\frac{F_\pi^2}{16}{\rm tr}(R_\mu R^\mu) + \frac{1}{32e_s^2}{\rm tr}([R_\mu, R_\nu][R^\mu, R^\nu]) \qquad R_\mu = \partial_\mu U U^\dagger$$

$$\mathcal{L}_{\rm mass} = \frac{m_\pi^2 F_\pi^2}{16}{\rm tr}(U + U^\dagger - 2)$$

Pion field (hedgehog ansatz)

$$U = e^{2i\pi\cdot\tau/F_\pi} = e^{if(r)\hat{\mathbf{x}}\cdot\tau}$$



Reproduce several quantities (F_π, g_A, …) within 30% errors

[Adkins-Nappi-Witten, 1983]

Wess-Zumino-Witten action (Nf = 2)

$$S_{\rm wzw}[A_\mu] = -\int d^4x \, \frac{e}{2} \, j_B^\mu \, A_\mu$$

$$P_\sigma \equiv i{\rm tr}[\tau_3(U^\dagger \partial_\sigma U + \partial_\sigma U U^\dagger)]$$

$$j_B^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}{\rm tr}[R_\nu R_\rho R_\sigma] - \frac{e}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\partial_\nu\left(A_\rho P_\sigma\right)$$

baryon number — **chiral anomaly in QCD**

$$\propto \pi_0 \vec{E}\cdot\vec{B} + \mathcal{O}(F_\pi^{-3})$$

- Decomposition: $A_\mu = \bar{A}_\mu + a_\mu$ (background + fluctuation)

EM current : gauge invariant & conserved

$$j^\mu_{\rm em,wzw} = \delta S_{\rm wzw}/\delta a_\mu$$

$$= \epsilon^{\mu\nu\rho\sigma}\left[\frac{e}{48\pi^2}{\rm tr}[R_\nu R_\rho R_\sigma] + \frac{e^2}{32\pi^2}\partial_\sigma(\bar{A}_\rho P_\nu)\right] + j^\mu_{\rm anm}$$

$$j^\mu_{\rm anm} = -\frac{e^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}(\partial_\nu \bar{A}_\rho)P_\sigma$$ **: anomaly current**

Total charge

$$Q = \int d^3x \, (j^0_V + j^0_{\mathrm{em,wzw}}) = e\left(I_3 + \frac{1}{2}N_B\right) + \frac{1}{2}Q_{\mathrm{anm}}$$

"Gell-Mann-Nishijima formula"

Anomaly-current with Skyrmion

$$P_\sigma \equiv i\mathrm{tr}[\tau_3(U^\dagger \partial_\sigma U + \partial_\sigma U U^\dagger)]$$

Static solution w/o EM field

$$U = e^{if(r)\hat{\mathbf{x}}\cdot\tau} \quad \Rightarrow \quad j^\mu_{\mathrm{anm}} = -\frac{e^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}(\partial_\nu \bar{A}_\rho)P_\sigma$$

Soliton quantization: "slowly rotating" in moduli space

$$U(x) \to g(t)U(x)g^\dagger(t), \qquad g(t) \in \mathrm{SU}(2_{\mathrm{F}}) \simeq S^3$$

Anomaly-current in external magnetic field ($2\boldsymbol{B}^{\mathrm{i}} = \varepsilon^{\mathrm{ijk}}\boldsymbol{F}_{\mathrm{jk}}$)

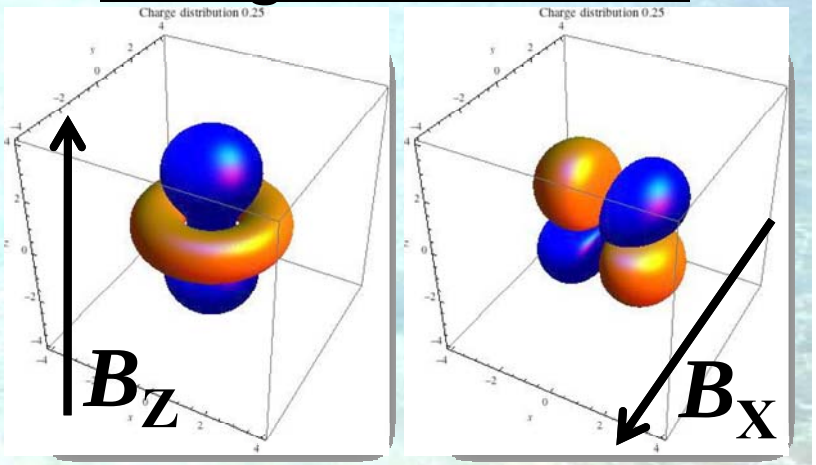
$$\langle j^0_{\mathrm{anm}}\rangle_{I_3,S_3} = \frac{iN_c}{96\pi^2}\, e^2 B_i \langle P_i\rangle_{I_3,S_3}, \quad \langle j^i_{\mathrm{anm}}\rangle = 0$$

$$\langle P_0\rangle_{I_3,S_3} = 0, \quad \langle P_i\rangle_{I_3,S_3} = -\frac{16i}{3}(e_s F_\pi)\, I_3 S_3 \left[\left(f' - \frac{\sin(2f)}{2r}\right)\hat{x}_i x_3 + \frac{\sin(2f)}{2r}\delta_{i3}\right]$$

Induced charge by external magnetic field

$$\langle j^0_{\rm anm}\rangle_B \neq 0\,, \quad \langle j^i_{\rm anm}\rangle_B = 0$$

Charge distribution



Anomalous charge

$$Q_{\rm anm} = \int d^3x \langle j^0_{\rm anm}\rangle$$

$$= e\frac{4N_c}{27\pi} I_3 S_3 \frac{eB_Z}{(e_s F_\pi)^2} c_0$$

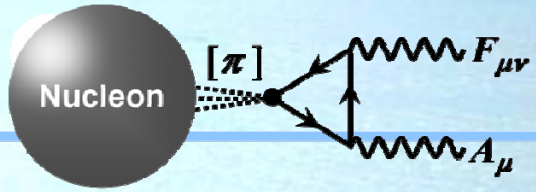
Numerical coefficient : $c_0 = \int_0^\infty dr[r^2 f' + \sin(2f)] \sim -10.2$

Dipole moment = 0, Quadrupole mom. ≠ 0

Magnitude $Q_{\rm anm}/e \sim 10^{-20}[\mathrm{G}^{-1}] I_3 S_3 \times B\ [\mathrm{G}]$

Magnetor (nutron star): B = 10^{15} [G], Heavy-ion collision: B = 10^{17} [G]

Where the charge comes from?



Pion expansion

$$j^\mu_{\rm anm} A_\mu \sim {\rm tr}[\tau_3 U^\dagger \partial U] A\, F$$

$$U = e^{2i\pi\cdot\tau/F_\pi} = e^{if(r)\hat{\mathbf{x}}\cdot\tau}$$

$$\sim \underline{\partial\pi_0 AF} + \underline{\pi\pi\partial\pi AF} + \cdots$$

$\pi_0 \to 2\gamma$ — Multi-pion effect (pion cloud)

Contribution from $\pi_0 \to 2\gamma$ to $Q_{\rm anm}$: $\pi_0 \sim f(r)\hat{x}_3$

$$\int d^3x \partial\pi_0 = \frac{4\pi}{3}\int_0^\infty dr[r^2 f' + 2rf] = \frac{4\pi}{3}\left[r^2 f\right]_0^\infty = 0$$ **: vanish**

Induced charge carried by **pion cloud** surrounding nucleons

c.f.) Kharzeev-Yee-Zahed, 1104.0998

$\pi_0 \to 2\gamma$ Lagrangian: calculation $j_{\rm anm}$

$$\mathcal{L}_{\rm anm} \sim e^2 N_c \pi^0 \vec{E}\cdot\vec{B}$$

Induced charge = 0
Quadrupole ≠ 0

Anomaly-induced charges by external mag. filed

Can you believe?? artificial problems

- Uniform background magnetic field?

  Magnetic field should close → Circular ele. current (poster)

- Cancelation due to deformation of Skyrmion?

  Leading-order calc. up to O(eB) → No cancelation in pertavative discussion (poster)

  back reaction (future)

- Skyrme model and WZW action?

  Reproduce nucleon and chiral anm. → e.g.) Nucleon in Lattice QCD with mag. field (future)

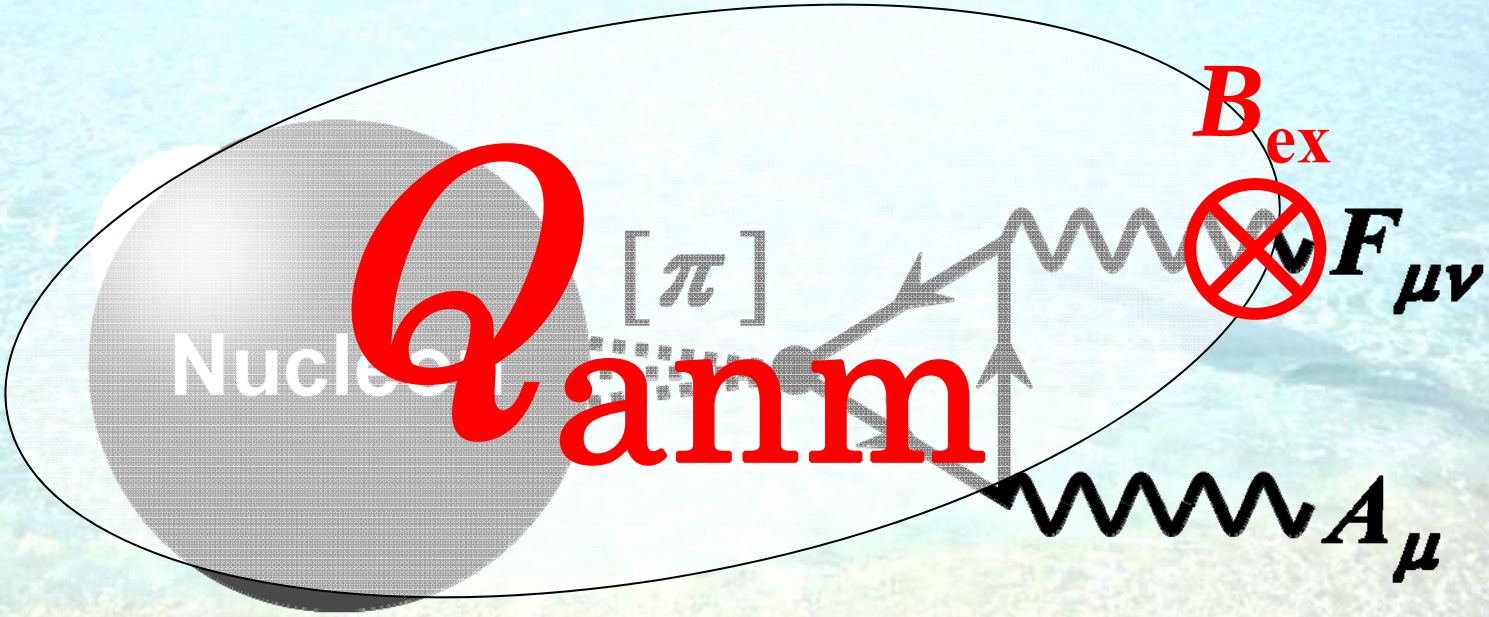
- Charge conservation violated?

  Gauge inv. j_{anm} used in static manner

  What's happen when ex. field is turned on?

  → Time evolution of quantized Skyrmion with gauge field…

Anomalous charge induced!



THANK YOU

FOR YOUR ATTENTION!

Consistency with experiments?

- electric charge: $(q_P+q_e) < 10^{-21}e$ (from PDG)
  from an experiment using SF_6
  … assuming $Q_{tot}=N_{SF6} \times 70 \times (q_P+q_e)$

$\mid q_p+q_e \mid /e$			References	History since 1990
See DYLLA 1973 for a summary of experiments on the neutrality of matter. See also "*n* CHARGE" in the neutron Listings.				
VALUE	***DOCUMENT ID***		***COMMENT***	
$<1.0\times10^{-21}$	[1] DYLLA	73	Neutrality of SF_6	
* * * We do not use the following data for averages, fits, limits, *etc.* * * *				
$<3.2\times10^{-20}$	[2] SENGUPTA	00	binary pulsar	
$<0.8\times10^{-21}$	MARINELLI	84	Magnetic levitation	

[1] Assumes that $q_n = q_p+q_e$.
[2] SENGUPTA 2000 uses the difference between the observed rate of of rotational energy loss by the binary pulsar PSR B1913+16 and the rate predicted by general relativity to set this limit. See the paper for assumptions.

PREVIOUS NEXT

… Our case: anomalouscharge depends on S_3 and I_3
→ cancel out!

- electric dipole: no evidence for finite electric dipole
  (C.A.Baker et al., Phys.Rev.Lett.97, 131801, 2006)

no contradiction

No cancelation

- Two possible sources of cancellation
  1. Modification of electric current itself (operator)
  2. Modification of wave functions

1. Skyrme model: $U(1)_{em}$ = part of the isonspin

   Therefore, I_3 is defined as

$$I_3 = \frac{i}{2}\left[a_0\frac{\partial}{\partial a_3} - a_3\frac{\partial}{\partial a_0} - a_1\frac{\partial}{\partial a_2} + a_2\frac{\partial}{\partial a_1}\right] \qquad g(t) = \sum_{i=0-3} a_i\tau_i \in \mathrm{SU}(2)$$

… irrelevant to eB

- Modification of wave functions

EM: SU(2) → U(1) $\quad S = 2\lambda\sum_{i=0}^{3}\left[(\dot{a}_i)^2 - eB_3V(a)\right] \qquad g(t) = \sum_{i=0-3} a_i\tau_i$

$$\Rightarrow \quad |1/2\rangle = |1/2\rangle_0 + eB_3\sum_{n=1}^{\infty}\frac{V_{1/2,n+1/2}}{E_{1/2} - E_{n+1/2}}|n+1/2\rangle_0 + \mathcal{O}((eB)^2)$$

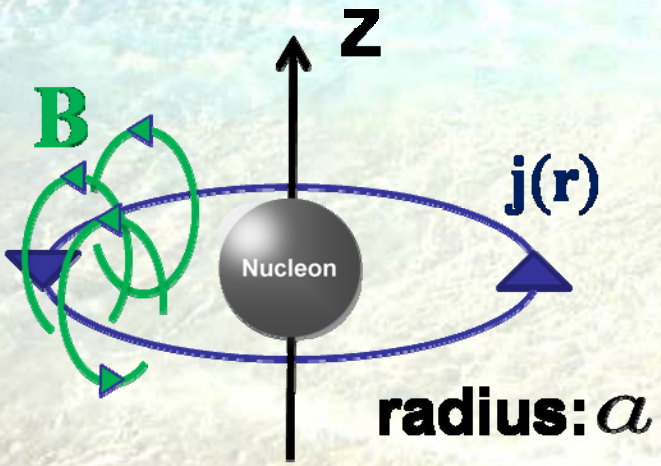
Thus $\;{}_0\langle n+1/2|I_3|1/2\rangle_0 = 0 \;\Rightarrow\; \langle I_3\rangle = \langle I_3\rangle_0 + \mathcal{O}((eB)^2)$

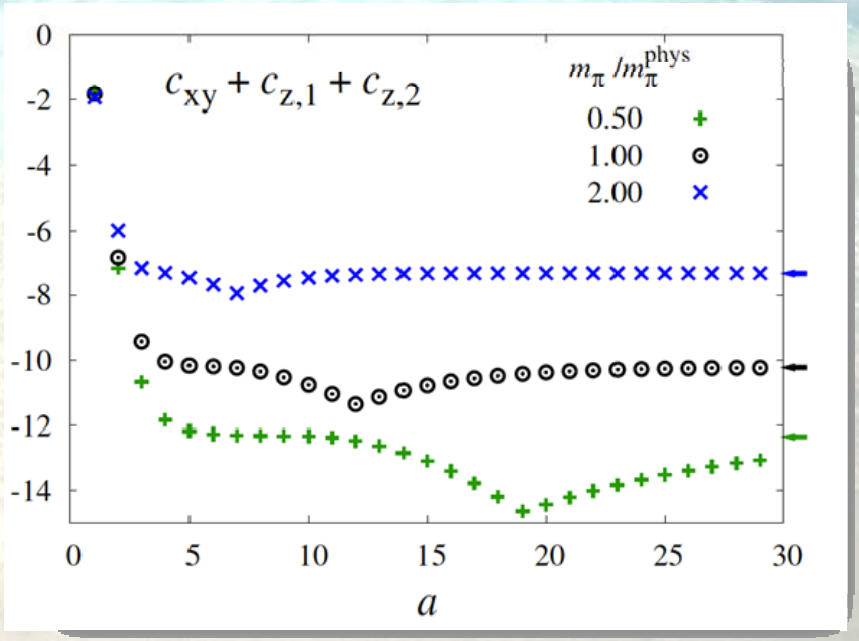
Uniform magnetic field … ill defined?

Induced charge in circular electric current

$$\mathbf{j}(\mathbf{r}) \equiv \frac{j_0 a}{2\pi}\delta(z)\delta(\sqrt{x^2+y^2}-a)(-\sin\zeta, \cos\zeta, 0)$$

$$Q_{\rm anm} = \frac{4eN_c}{27\pi}(I_3 S_3)\frac{ej_0}{(e_s F_\pi)^2}(c_{xy}+c_{z,1}+c_{z,2})$$

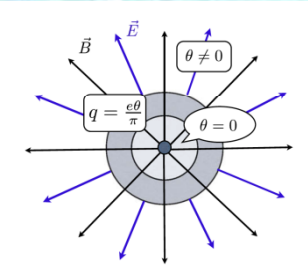


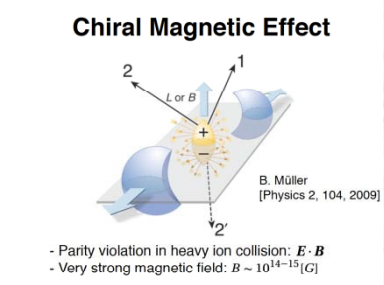


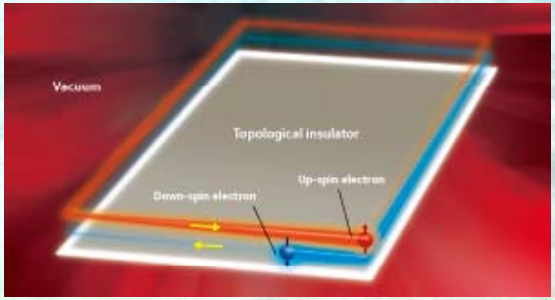
… Q_{anm} induced for circular current

Relation between other effects

Witten effect, chiral magnetic effect, topological insulator…







… described by $F\tilde{F}$ term

cf) CME: $\theta F_{\mu\nu}\tilde{F}^{\mu\nu} \longrightarrow -\epsilon^{\mu\nu\rho\sigma}(\partial_\mu\theta)A_\nu(\partial_\rho A_\sigma)$

$$j^\nu = -(\partial_\mu\theta)\tilde{F}^{\mu\nu} \xrightarrow{\theta=\theta(t)} j^i = -(\partial_0\theta)B^i$$

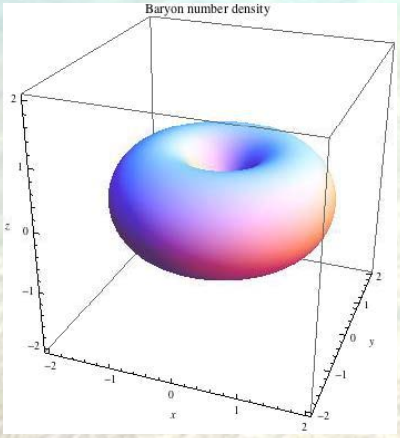
- **In our case:** $-\frac{e^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}P_\mu a_\nu(\partial_\rho\bar{A}_\sigma)$ **… described by similar term**

Multi-baryon systems

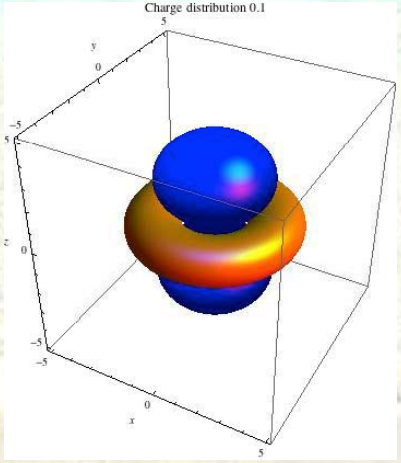
- $j^0_{\rm anm}(x)$ for multi-baryon systems in classical level

Rational map ansatz
(C.J.Houghton et al., Nucl.Phys.B510 (1998) 508.)

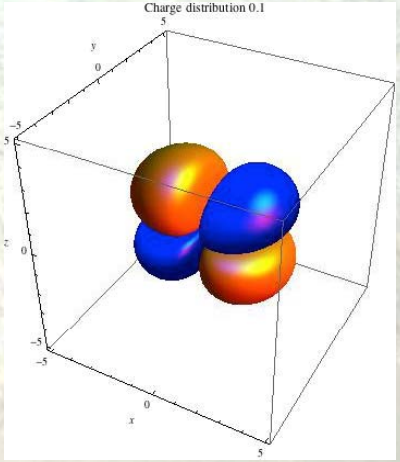
B=2



Baryon number density



$j^0_{\rm anm}(x)$ **under Bz**



$j^0_{\rm anm}(x)$ **under Bx**

Multi-baryon systems

B=3 4 5 6 7 17