格子ゲージ理論による２次の輸送係数の測定

Lattice simulation of second order transport coefficients

熱場の量子論とその応用

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Background

Heavy ion collision @ RHIC

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Low viscous QCD matter

↑

Relativistic ideal hydrodynamic models

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Heavy ion collision @ RHIC

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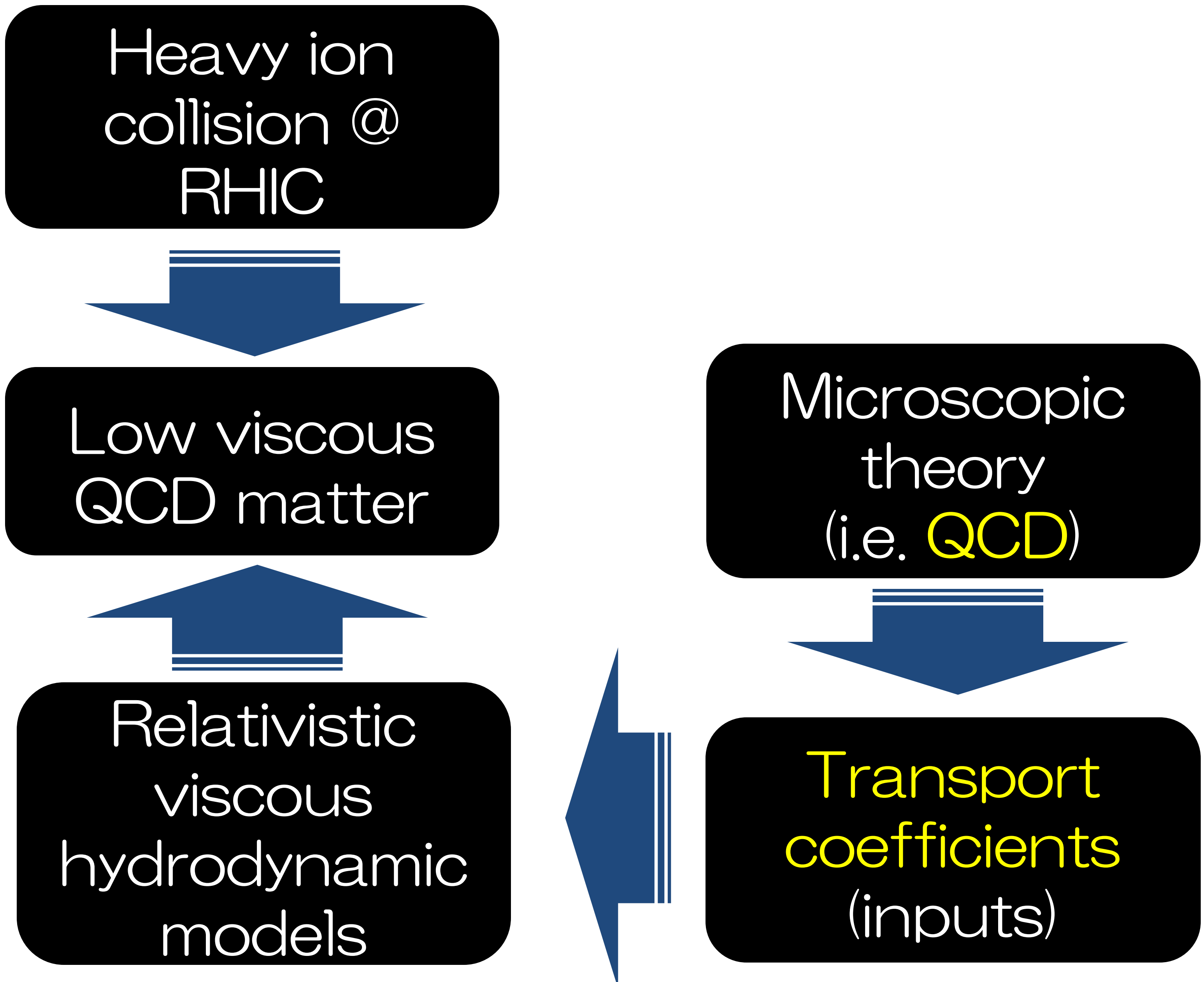
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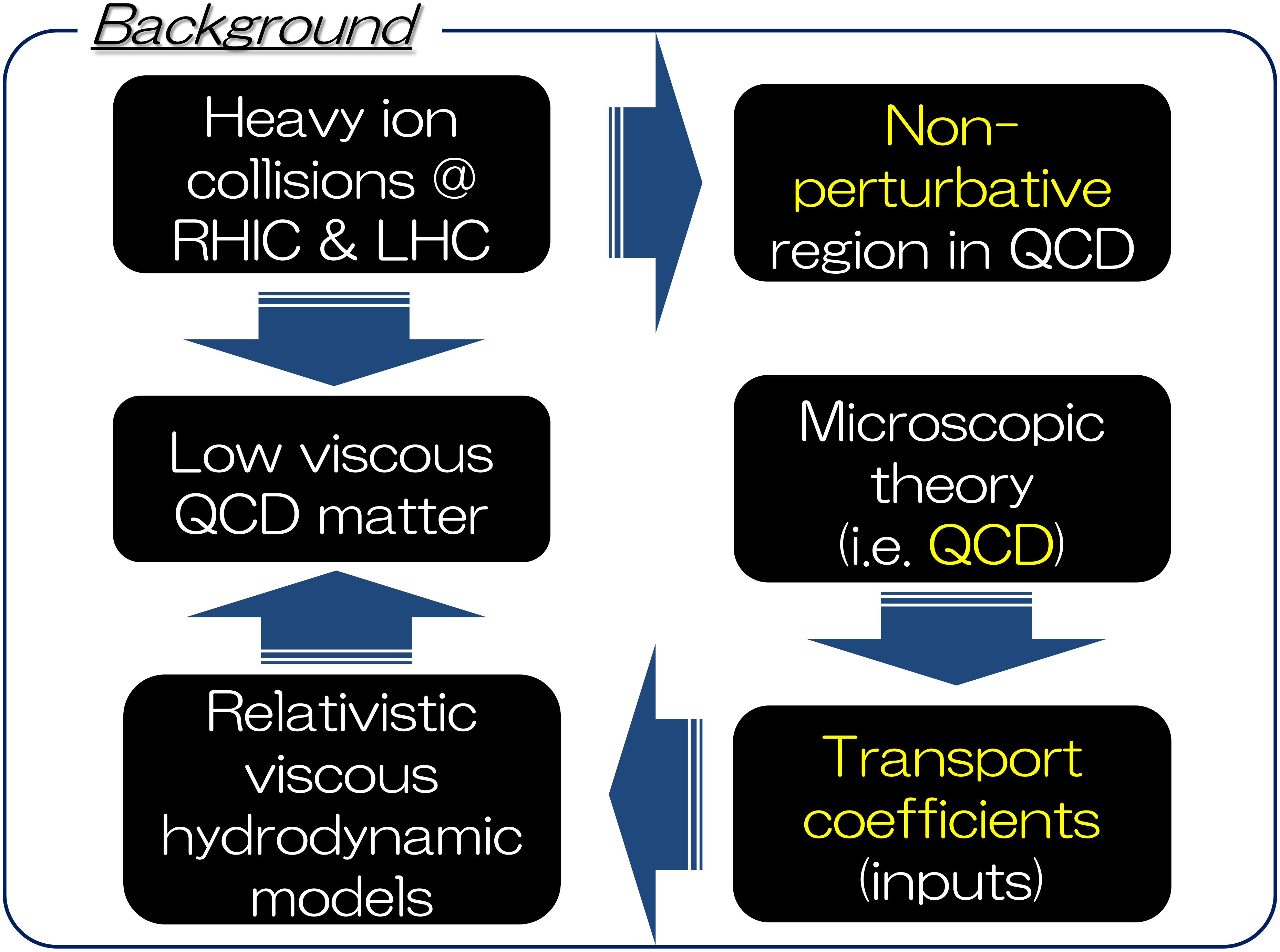
Low viscous QCD matter
Relativistic viscous hydrodynamic models
Microscopic theory (i.e. QCD)
Transport coefficients (inputs)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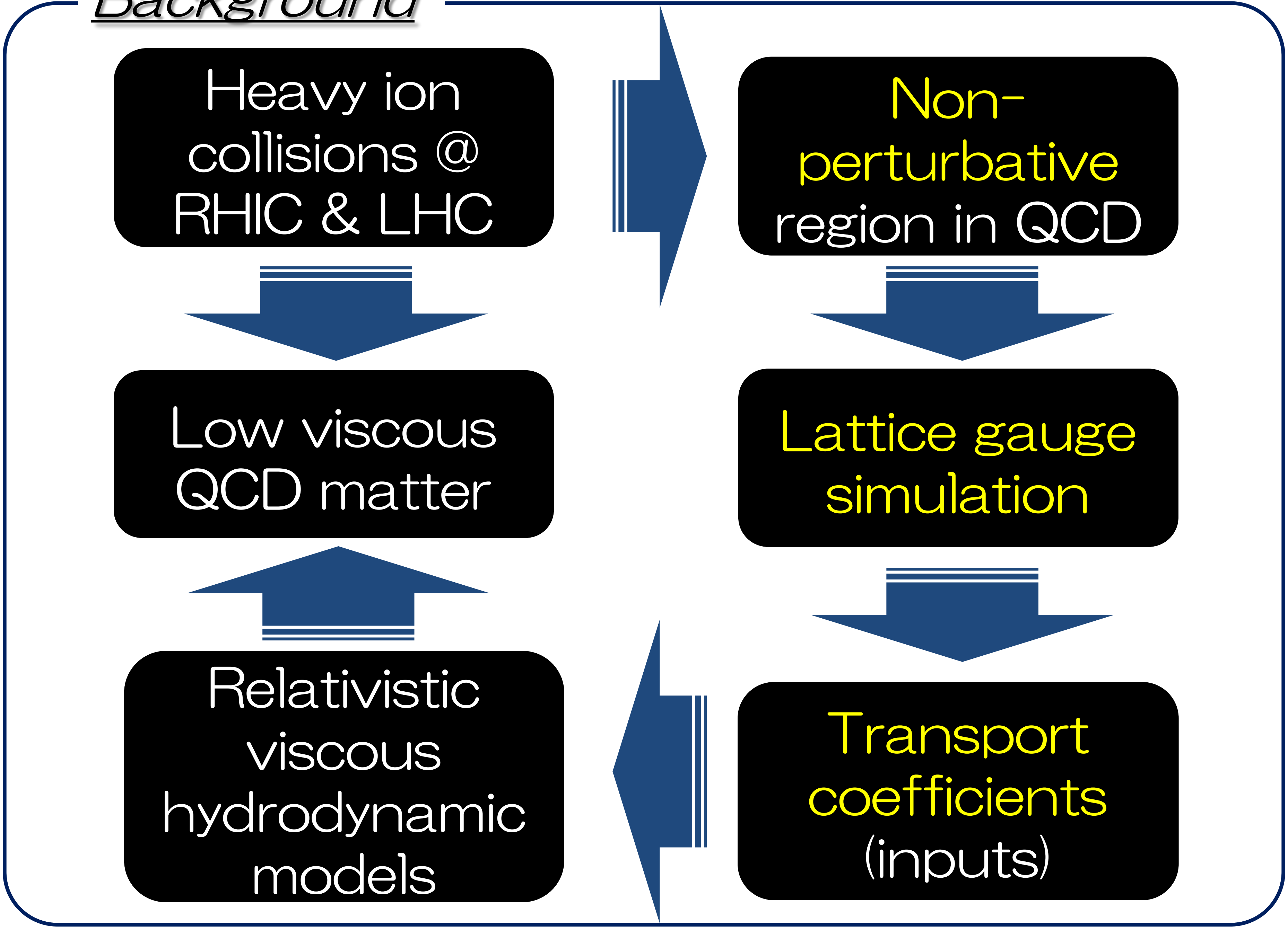


Heavy ion collisions @ RHIC & LHC
Non-perturbative region in QCD
Low viscous QCD matter
Lattice gauge simulation
Relativistic viscous hydrodynamic models
Transport coefficients (inputs)

Purpose

We attempt to evaluate the transport coefficients in 2nd order viscous hydrodynamics by SU(3) lattice gauge simulation.

※In the following, we focus on "a shear viscosity to relaxation time ratio".

Relativistic viscous hydrodynamics

☆ Navier-Stokes theory (1st order theory)→Causality ×

$$\pi^{\mu\nu} = 2\eta\nabla^{<\mu}u^{\nu>}$$

$\pi^{\mu\nu}$: shear stress tensor , η: shear viscosity , u^{μ} : fluid velocity

C. Eckart, Phys. Rev. **58**, 919(1940).
L. D. Landau and E. M. Lifshitz, *Fluid Mechanics*(Pergamon, New York, 1959).
W. A. Hiscock and L. Lindblom, Ann. Phys. (N.Y.) **151**, 466(1983).
;Phys. Rev. D **31**, 725 (1985).

☆ Israel-Stewart theory (2nd order theory)→Causality ○

$$\dot{\pi}^{\mu\nu} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\nabla^{<\mu}u^{\nu>} + \pi^{\mu\nu}\eta T\partial_\rho\left(\frac{\tau_\pi u^\rho}{2\eta T}\right)\right]$$

※The relaxation time τ_π does not appear in 1st order theory and makes 2nd order theory causal.

W.Israel and J.M.Stewart, Ann. Phys. (N.Y.) **118**, 341(1979).

Shear viscosity to relaxation time ratio

☆ The shear viscosity η by Kubo formula,

$$\eta = i \int_{t>0} dt\, t \int d^3x < [\pi_{12}(t,\vec{x}), \pi_{12}(0,\vec{0})] >$$

☆ Identity : $<[A,B]> = i\int_0^{1/T} d\lambda < \dot{A}(-i\lambda)B >$

λ : Imaginary time

☆ Relaxation-time approximation :

$$< \int d\lambda\, \pi_{12}(t-i\lambda)\pi_{12}(0) > \approx e^{-t/\tau_\pi} < \int d\lambda\, \pi_{12}(-i\lambda)\pi_{12}(0) >$$

☆ Local rest frame : $\pi_{12} = T_{12}$

$$\frac{\eta}{\tau_\pi} = \int_0^{1/T} d\lambda \int d^3x < T_{12}(\lambda,\vec{x}) T_{12}(0,\vec{0}) >$$

$$\equiv \int d^4x < T_{12}(x) T_{12}(0) >^E$$

Regularization

☆ In the quantum field theory, vacuum expectation values diverge in the short distance limit.

Example : Propagator of scalar field

$$\left\langle T\phi(x)\phi(y)\right\rangle = \int \frac{d^4p}{(2\pi)^4}\frac{i}{p^2-m^2}e^{-p(x-y)} \xrightarrow{x\to y} \infty$$

Regularize by T=0 subtraction.

$$\left.\frac{\eta}{\tau_\pi}\right|_{T-0} \equiv \int d^4x < T_{12}(x)T_{12}(0) >^{E}_{T\neq 0} - \int d^4x < T_{12}(x)T_{12}(0) >^{E}_{T=0}$$

However, this quantity is unphysical. Why?

OPE & Contact term

☆ Operator-product expansion (OPE)

→ $\langle T_{12}(x) T_{12}(0) \rangle$ in the short distance

$$< T_{12}(x) T_{12}(0) >^E \sim \frac{C}{x^8} + \sum_\mu C_\mu < T_{\mu\mu}(0) >^E \delta^4(x) + \cdots$$

T-independent

Contact term (T-dependent)

☆Leading-order OPE result

$$\sum_\mu C_\mu < T_{\mu\mu}(0) >^E = G^E(q_4 \to \infty, \vec{q} = \vec{0}) = \frac{2}{3} < T_{00} >^E + \frac{1}{6} < F^2 >^E$$

Fourier transform of $\langle T_{12} T_{12} \rangle$

Gluon condensate

S. Caron-Huot, Phys. Rev. D**79** 125009(2009).

T. Springer, C. Gale, S. Jeon, & S. H. Lee, Phys. Rev. D**82** 106005(2010).

OPE & Contact term

☆ What we calculate is

$$\left.\frac{\eta}{\tau_\pi}\right|_{phys} = \int d^4x \langle T_{12}(x) T_{12}(0) \rangle^E_{phys}$$

$$\cong \int d^4x \langle T_{12}(x) T_{12}(0) \rangle^E_{T-0} - \left[\frac{2}{3} \langle T_{00} \rangle^E_{T-0} + \frac{1}{6} \langle F^2 \rangle^E_{T-0} \right]$$

Contact term

※ $\langle T_{00} \rangle$ & $\langle F^2 \rangle$ are related to thermodynamic quantities

$$\langle T_{00} \rangle^E_{T-0} = -\langle T^{00} \rangle^M_{T-0} = -\frac{\varepsilon + P}{2} \ , \ \langle F^2 \rangle^E_{T-0} = \langle F^2 \rangle^M_{T-0} = -(\varepsilon - 3P)$$

They are well-studied by lattice simulation.

G. Boyd, J. Engels, F. Karsch, E. LAermann, C. Legeland, M. Lutgemeruer, and B. Petersson, Nucl. Phys. B **469**:419-444(1996).

Setup

- SU(3) pure gauge (with Wilson action)
- 1HB + 4OR for each update
- Traceless definition for $T_{\mu\nu}$
- Clover type plaquette for $F_{\mu\nu}$
- Isotropic lattice
- Lattice sizes

  β=6.499(a=0.049fm) : $32^3 \times 4,6,8,32$

  β=6.205(a=0.074fm) : $32^3 \times 4,6,8,32$

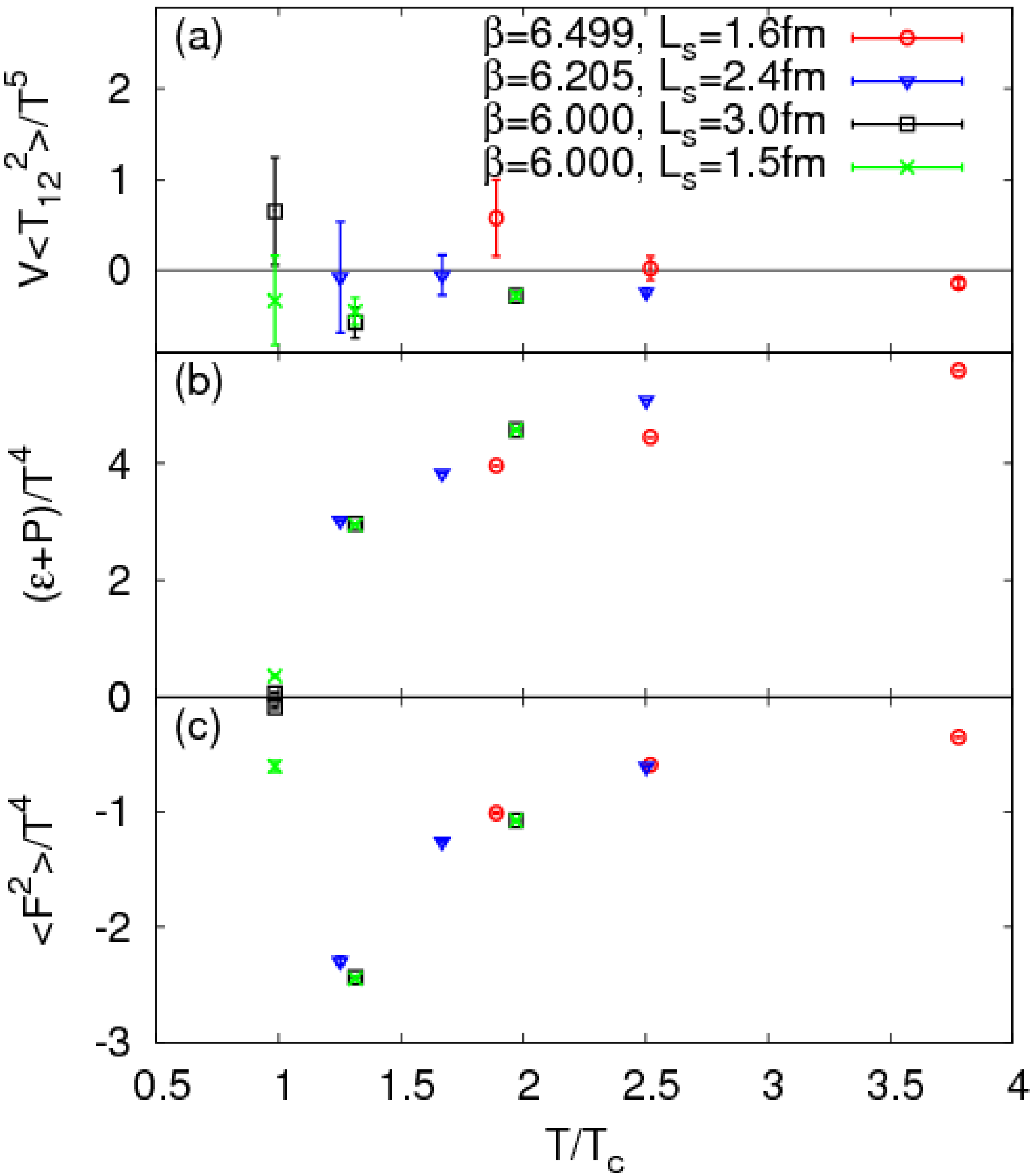
  β=6.000(a=0.094fm) : $32^3 \times 4,6,8,16$

  β=6.000(a=0.094fm) : $16^3 \times 4,6,8,16$

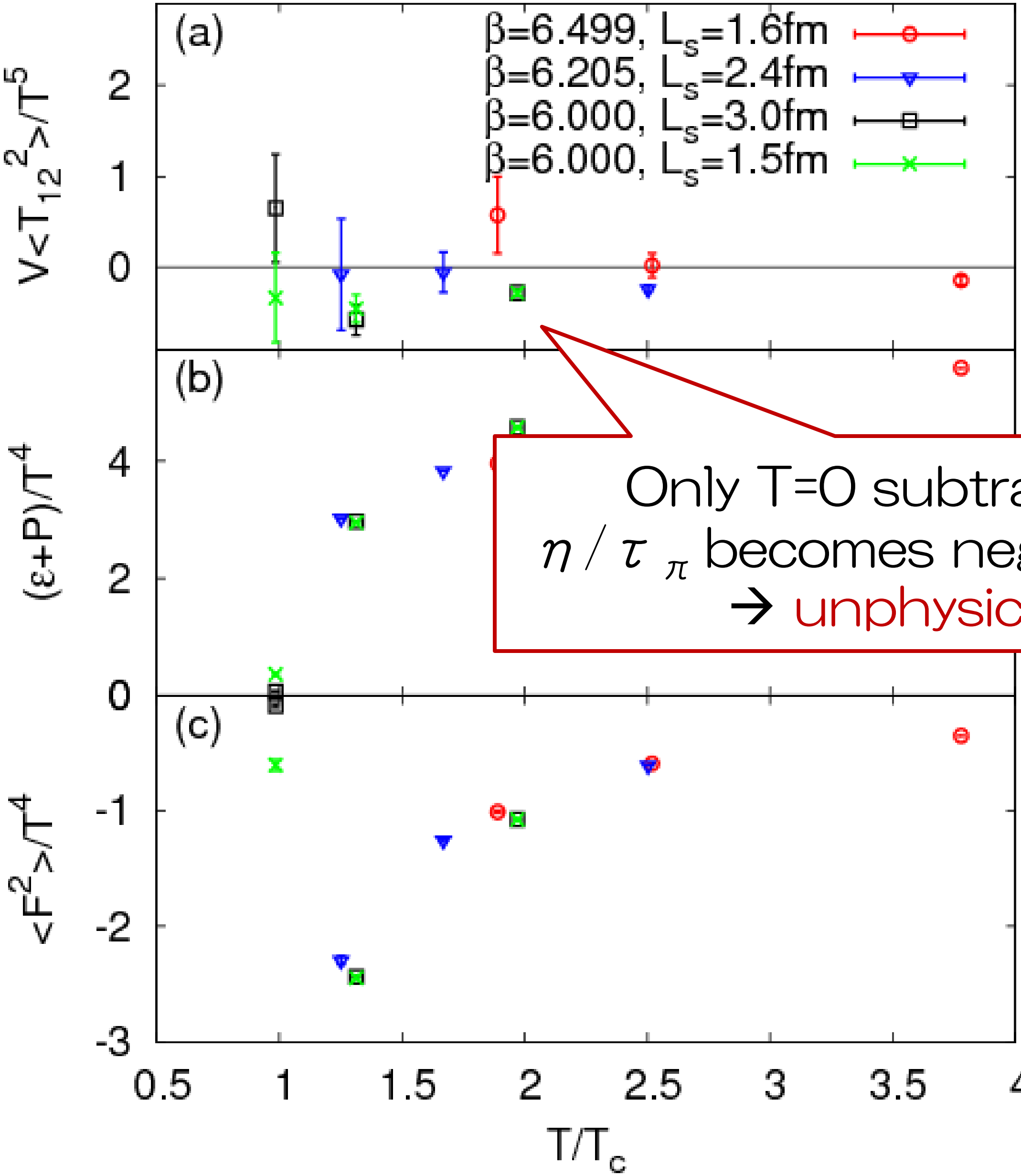
  lattice spacing dependence (first three lattices)

  volume dependence (last two lattices)
- Number of configurations : 300,000-2,000,000
- Error estimation by jackknife method (bin size=50-1000)

Results : $(\eta / \tau_{\pi})_{T\text{-}O}$ & contact term

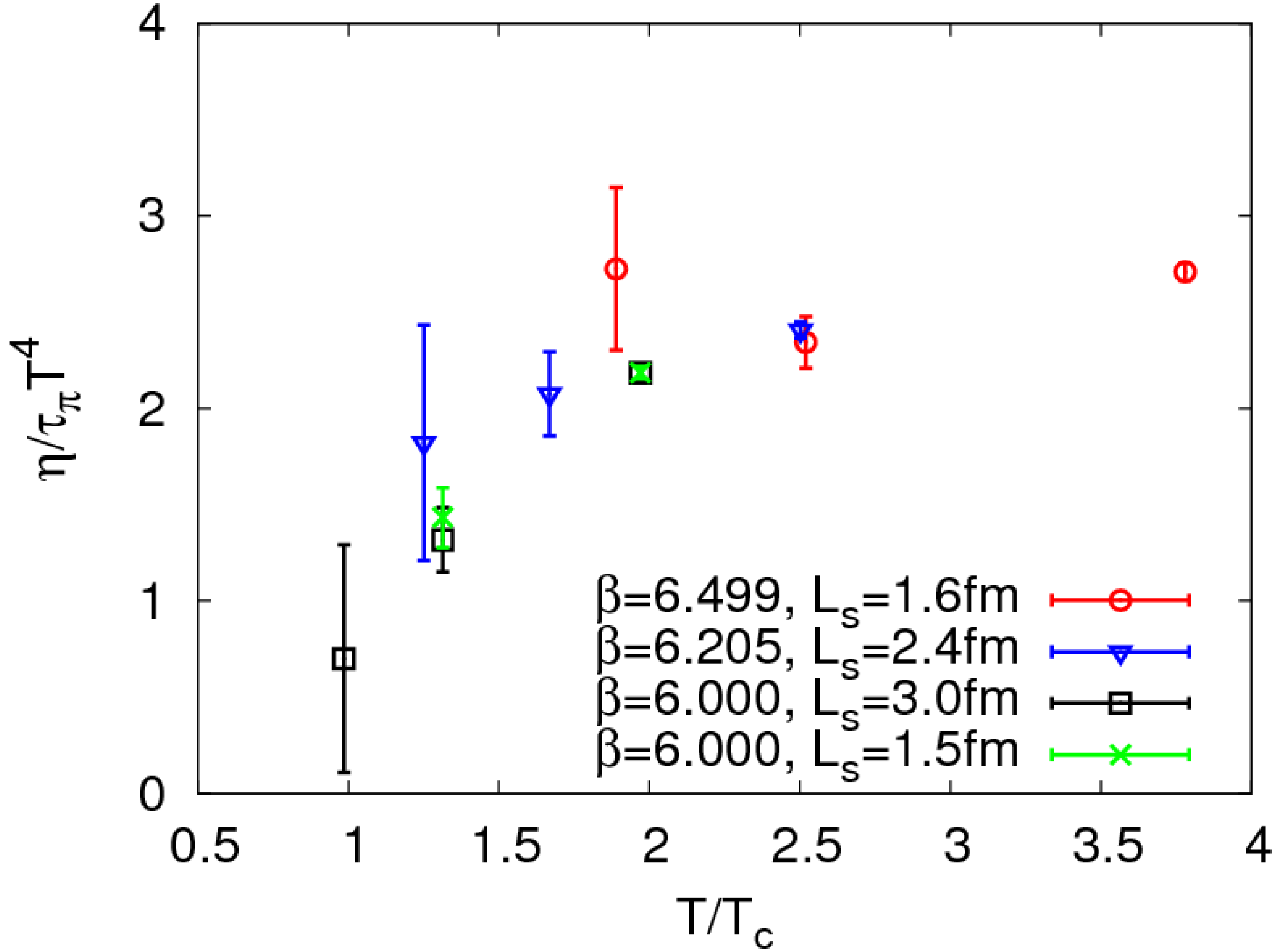


Results : $(\eta / \tau_\pi)_{T-0}$ & contact term



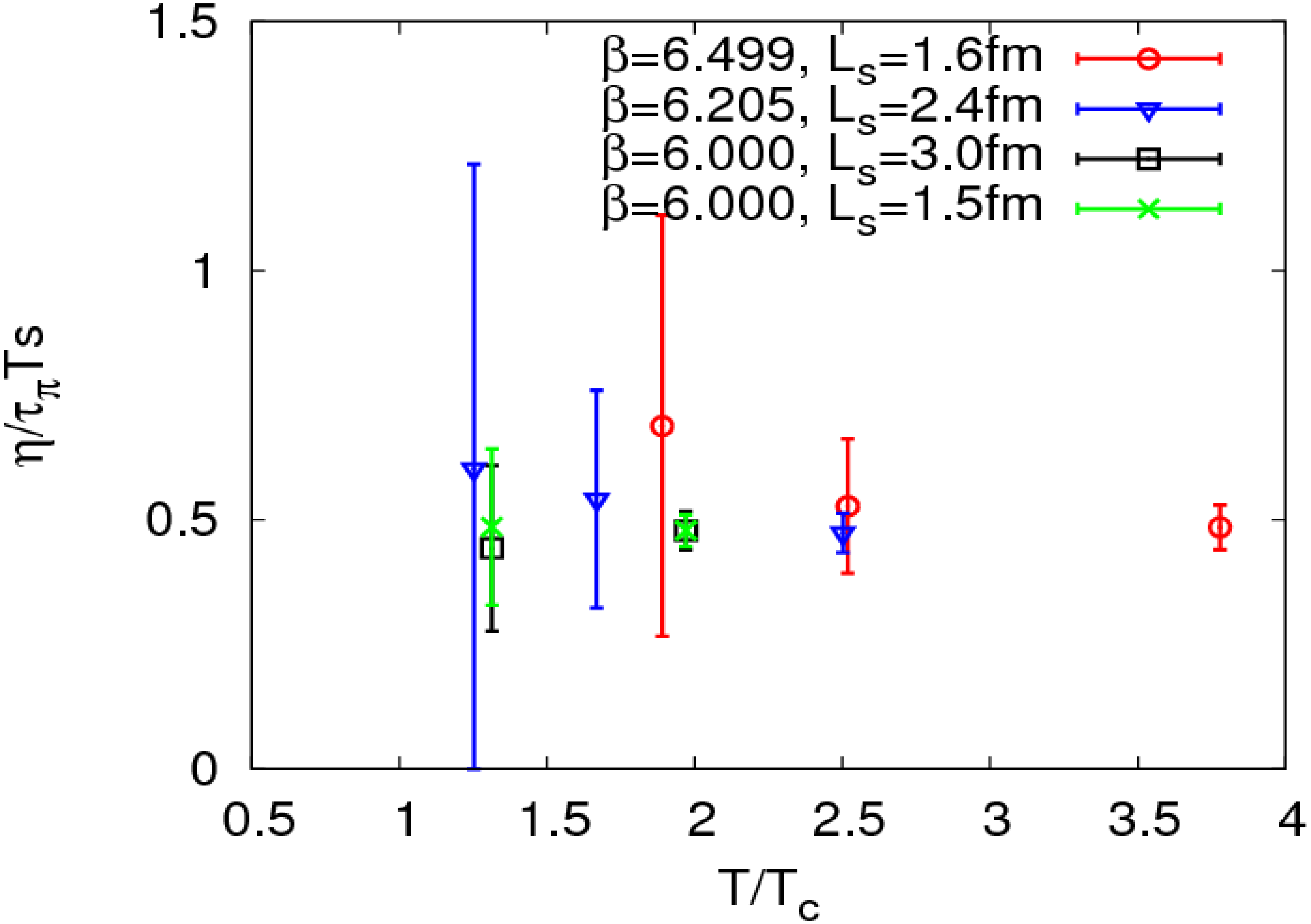
Only T=0 subtraction,
η / τ_π becomes negative …?
→ unphysical.

Results : $(\eta / \tau_\pi)_{phys}$



①Lattice spacing & volume dependences : small.
②Non-negative.

Results : signal speed



$$v_T^2 = \frac{\eta}{\tau_\pi(\varepsilon + P)} = \frac{\eta}{\tau_\pi Ts}$$

Summary

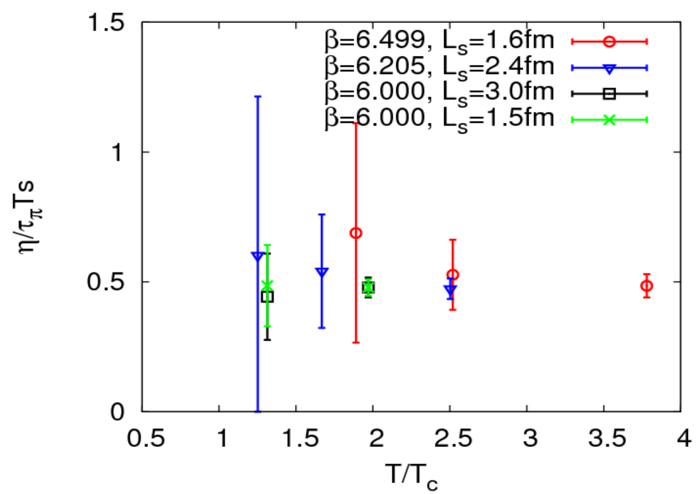
☆ We evaluated the 2nd order transport coefficient η/τ_π by SU(3) lattice gauge simulation for the temperature range realized at the LHC.

☆ There was extra contribution from the contact term which cannot be removed by T=0 subtraction.

☆ We obtained η/τ_π from direct measurement of lattice observables ($\int d^4x\langle T_{12}T_{12}\rangle$, $\langle T_{00}\rangle$ and $\langle F^2\rangle$).

☆ Signal speed ≒ 0.5 → Causality ◎.

☆ Future works
①High statistics near Tc.
②Bulk & heat channels.



Appendix

Relativistic viscous hydrodynamics

☆ Basic equations & constraint

Energy-momentum conservation

Charge number conservation

+ 2nd law of thermodynamics

$$\partial_\mu T^{\mu\nu} = 0 \quad , \quad \partial_\mu N^\mu = 0$$

$$\partial_\mu S^\mu \geq 0$$

5 equations for 14 variables

Additive 9 equations

$T^{\mu\nu} = T^{\mu\nu}_{eq.} + \delta T^{\mu\nu}$: non-equilibrium energy-momentum tensor

$N^\mu = N^\mu_{eq.} + \delta N^\mu$: non-equilibrium particle current

$S^\mu = S^\mu_{eq.} + \delta S^\mu$: non-equilibrium entropy current

W. Israel and J. M. Stewart, Ann. Phys. (N.Y.) **118**, 341(1979).

Additive 9 equations

☆ Step1 : The entropy current S^{μ}

$$S^{\mu} = S^{\mu}_{eq.} + \Delta S^{\mu}(\Pi, q^{\mu}, \pi^{\mu\nu})$$

dissipative fluxes

Π : bulk viscous pressure , q : heat flow , π : stress tensor

☆ Step2 : 2nd law of thermodynamics

$$T\partial_{\mu} S^{\mu} = (Flux) \times (Force) \geq 0$$

T : temperature

☆ Step3 : Linear response

$$(Flux) \propto (Force)$$

9 equations for dissipative fluxes

1st order viscous hydrodynamics

☆Step1：The entropy current S^{μ}

$$S^{\mu} = S^{\mu}_{eq.} + \frac{q^{\mu}}{T}$$

☆Step2：2nd law of thermodynamics

$$T\partial_{\mu} S^{\mu} = \Pi X - q_{\mu} X^{\mu} + \pi_{\mu\nu} X^{\mu\nu} \geq 0$$

$q_{\mu} q^{\mu} < 0$

X : thermodynamic forces

☆Step3：Linear response

$$\Pi = \varsigma X \quad , \quad q^{\mu} = \lambda X^{\mu} \quad , \quad \pi^{\mu\nu} = 2\eta X^{\mu\nu}$$

ζ : bulk viscosity , λ : heat conductivity , η : shear viscosity

1st order transport coefficients (Non-negative)

C. Eckart, Phys. Rev. **58**, 919(1940).

L. D. Landau and E. M. Lifshitz, *Fluid Mechanics*(Pergamon, New York, 1959).

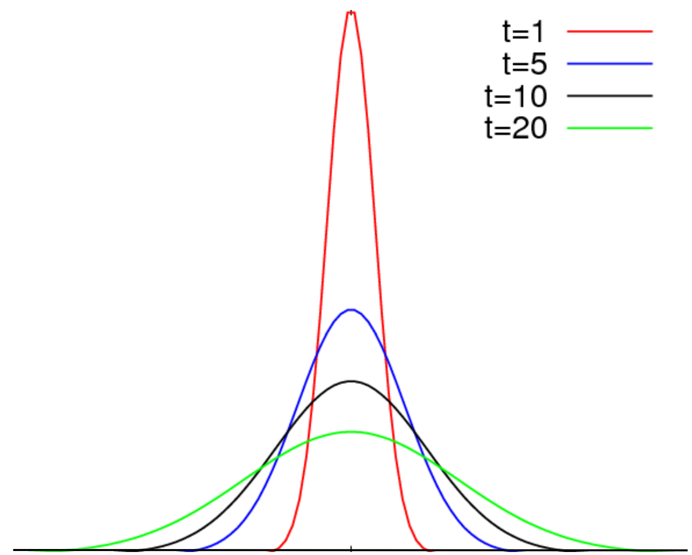
1st order viscous hydrodynamics

☆ We obtained additive 9 eqs. for dissipations.
→ But 1st order hydrodynamics violates causality.

Example : Heat conduction of 1D fluid

Parabolic

$$\frac{\partial T}{\partial t} = D \frac{\partial^2 T}{\partial x^2}$$



W. A. Hiscock and L. Lindblom, Ann. Phys. (N.Y.) **151**, 466(1983).
;Phys. Rev. D **31**, 725 (1985).

2^{nd} order viscous hydrodynamics

☆ Step1 : The entropy current S^{μ}

$$S^{\mu} = S^{\mu}_{eq.} + \frac{q^{\mu}}{T} - (\beta_0 \Pi^2 - \beta_1 q_{\nu} q^{\nu} + \beta_2 \pi_{\nu\lambda} \pi^{\nu\lambda}) \frac{u^{\mu}}{2T}$$

u^{μ} : fluid velocity , β : 2^{nd} order transport coefficients (≧0)

☆ Step2 : 2^{nd} law of thermodynamics

$$T\partial_{\mu} S^{\mu} = \Pi X - q_{\mu} X^{\mu} + \pi_{\mu\nu} X^{\mu\nu} \geq 0$$

X : thermodynamic forces

☆ Step3 : Linear response

$$\frac{dA}{dt} = -\frac{1}{\tau_A}[A + \cdots], \ \beta_0 \equiv \frac{\tau_{\Pi}}{\varsigma}, \ \beta_1 \equiv \frac{\tau_q}{\lambda T}, \ \beta_2 \equiv \frac{\tau_{\pi}}{2\eta}$$

A = Π,q,π : dissipative fluxes , τ_A : relaxation times
ζ : bulk viscosity , λ : heat conductivity , η : shear viscosity

※ τ_A are characteristic times that fluxes relax to steady values.

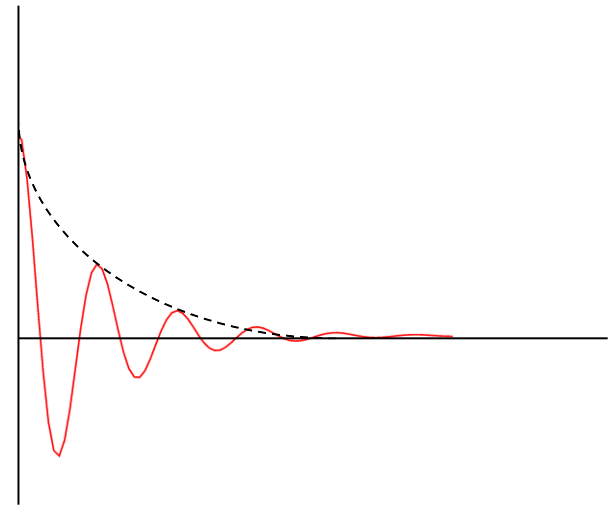
2nd order viscous hydrodynamics

☆ 2nd order hydrodynamics can be causal.

Example : Heat conduction of 1D fluid

Hyperbolic

$$\frac{\partial^2 T}{\partial t^2} = c^2 \frac{\partial^2 T}{\partial x^2}$$



※ If relaxation times take quite small value,
2nd order theory violates causality.

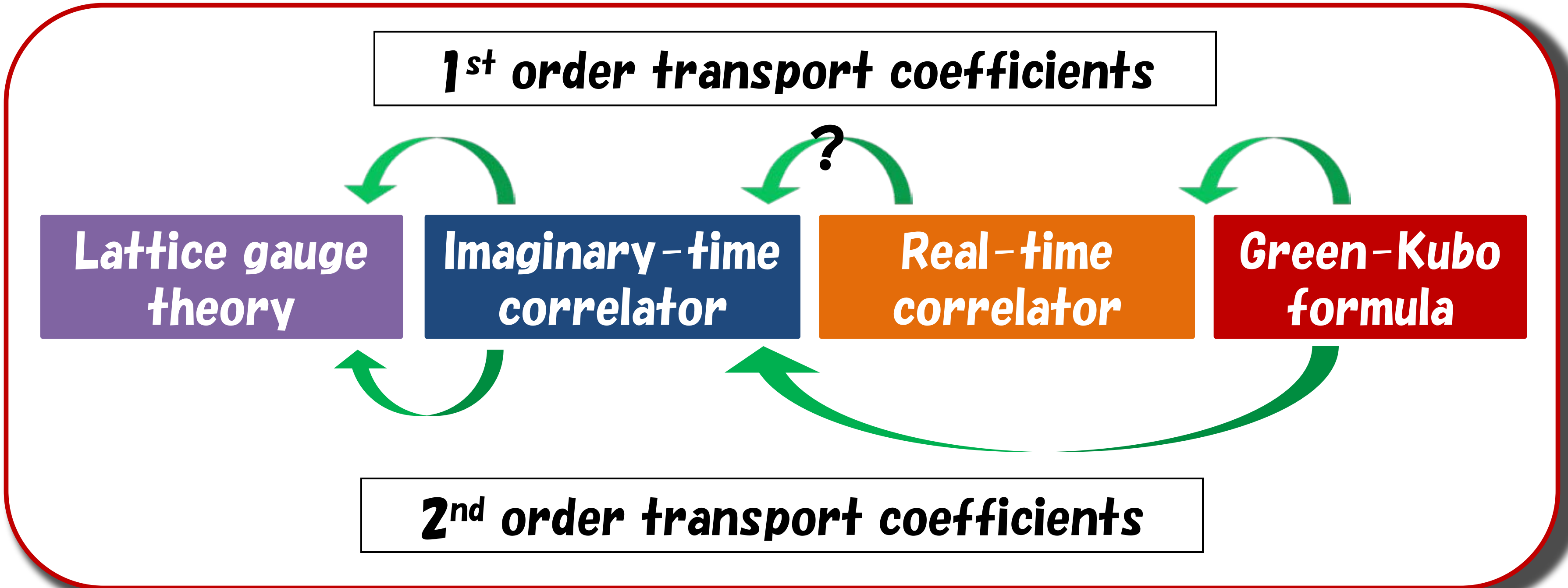
A. Muronga, Phys. Rev. C **69**:034903 (2004).

U. Heinz, H. Song, and A. K. Chaudhuli, Phys. Rev. C **73**:034904(2006).

Advantage

☆ Eta/tau_pi can be obtained from lattice observable.

$$\frac{\eta}{\tau_\pi} = \int d^3x \int_0^{1/T} d\lambda \left\langle T_{12}(\lambda, \vec{x}) T_{12}(0, \vec{0}) \right\rangle$$



F. Karsch and H.W. Wyld, Phys. Rev. D **35**:2518(1987).

A. Nakamura and S. Sakai, Phys. Rev. Lett. 94: 072305(2005).

H. B. Meyer, Phys. Rev. D**76**:101701(2007).

D. Kharzeev and K. Tuchin, JHEP 0809, 093(2008).

OPE

☆ Operator-Product Expansion (OPE)

Behavior of composite operator in the short distance

$$A(x)B(y) \xrightarrow{x \to y} \sum_i C_i(x-y) O_i\left(\frac{x+y}{2}\right)$$

A,B,O : local operator , C : c-number function

The coefficients C_i behave as

$$C_i(x-y) \xrightarrow{x \to y} \left(\frac{1}{x-y}\right)^{d_A + d_B - d_i}$$

d_A, d_B, d_i : dimensions of each operator

→Lower order terms in d_i are dominance in OPE.

K. G. Wilson, Phys. Rev. **179,** 1499(1969).