QCD和則と最大エントロピー法を用いた有限温度におけるクォークコニウムのスペクトル解析

（Spectral analysis of quarkonium from QCD sum rules and the maximum entropy method)

P. Gubler and M. Oka, Prog. Theor. Phys. **124**, 995 (2010).

P. Gubler, K. Morita and M. Oka, Phys. Rev. Lett. **107**, 092003 (2011).

熱場の量子論とその応用

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- The method: QCD Sum Rules and the Maximum Entropy Method
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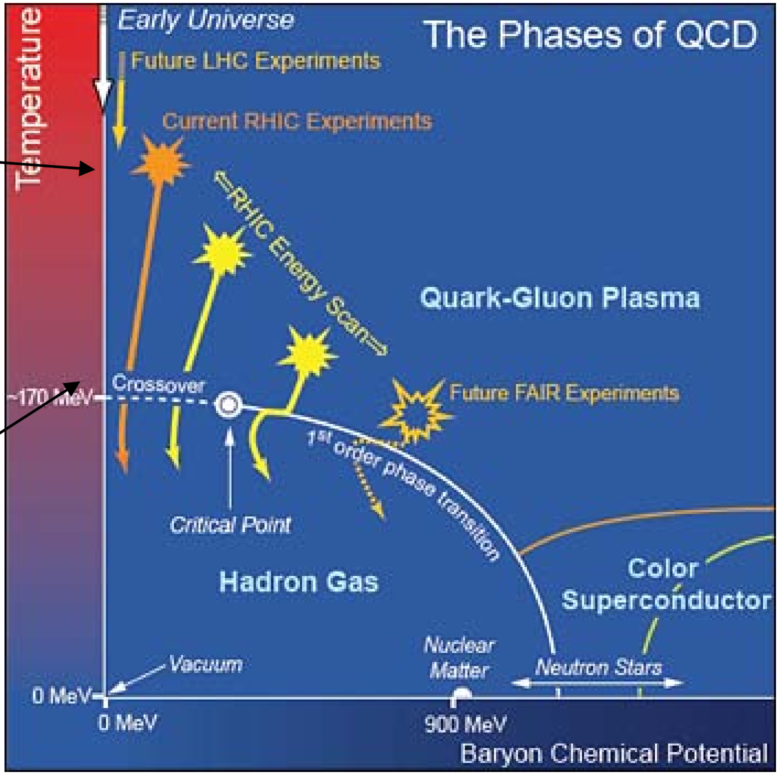
Introduction: Quarkonia

General Motivation: Understanding the behavior of matter at high T.

Υ, η_b . . .

J/ψ, η_c . . .



- Phase transition:

  QGP ($T>T_c$) ↔ confining phase ($T<T_c$)

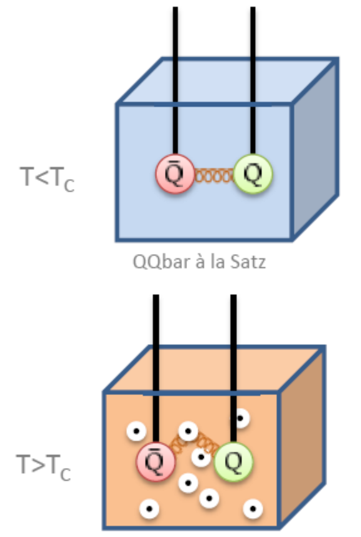
- Currently investigated at RHIC and LHC

- Heavy Quarkonium: clean probe for experiment

Why are quarkonia useful?

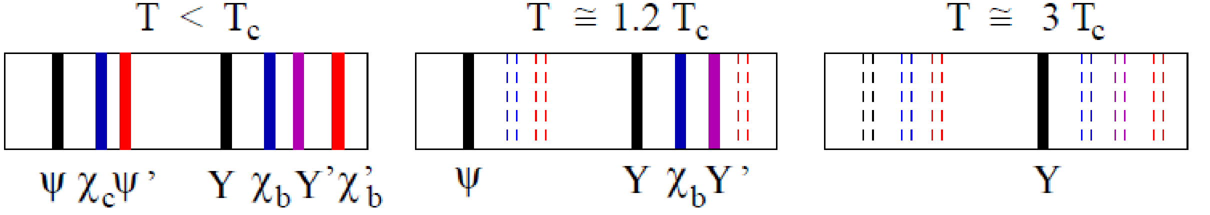
Prediction of J/ψ suppression above T_c due to Debye screening:

T. Matsui and H. Satz, Phys. Lett. B **178**, 416 (1986).
T. Hashimoto et al., Phys. Rev. Lett. **57**, 2123 (1986).



Lighter quarkonia melt at low T, while heavier ones melt at higher T

→ Thermometer of the QGP



Results from lattice QCD

- During the last 10 years, a picture has emerged from studies using lattice QCD (and MEM), where J/ψ survives above T_c, but dissolves below 2 T_c.

M. Asakawa and T. Hatsuda, Phys. Rev. Lett. 92 012001 (2004).

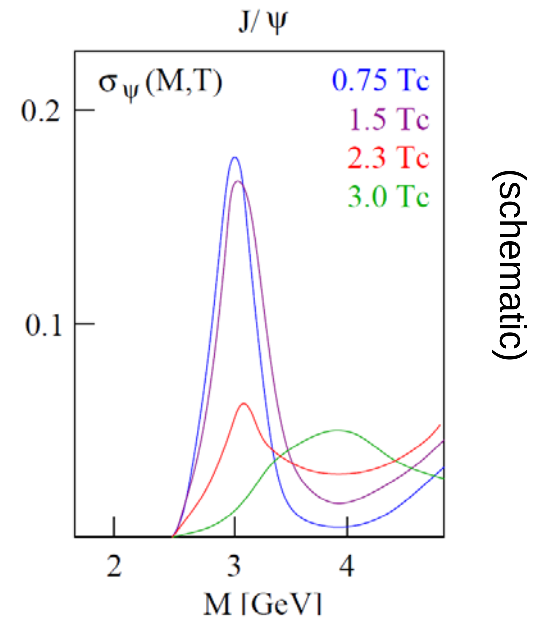
S. Datta *et al*, Phys. Rev. D69, 094507 (2004).

T. Umeda *et al*, Eur. Phys. J. C39, 9 (2004).

A. Jakovác *et al*, Phys. Rev. D75, 014506 (2007).

G. Aarts *et al*, Phys. Rev. D 76, 094513 (2007).

H.-T. Ding *et al*, PoS LAT2010, 180 (2010).



Taken from
H. Satz, Nucl.Part.Phys. **32**, 25 (2006).

- However, there are also indications that J/ψ survives up to 2 T_c or higher.

H. Iida *et al*, Phys. Rev. D 74, 074502 (2006).

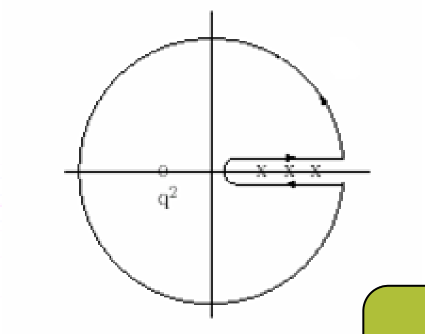
H. Ohno *et al*, PoS LAT2008, 203 (2008).

QCD sum rules

M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147, 385 (1979); B147, 448 (1979).

In this method the properties of the two point correlation function is fully exploited:

$$\Pi(q^2) = i \int d^4x e^{iqx} \langle 0 | T\{\chi(x)\overline{\chi}(0)\} | 0 \rangle$$



$$\rightarrow \Pi(q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im}\Pi(s)}{s - q^2 - i\epsilon}$$

is calculated "perturbatively", using OPE

spectral function of the operator χ

After the Borel transformation:

$$G_{OPE}(M) = \frac{1}{\pi} \int_0^\infty ds \frac{1}{M^2} e^{-\frac{s}{M^2}} \text{Im}\Pi(s)$$

The Maximum Entropy Method

→ Bayes' Theorem

$$P[\rho|G,I] = \frac{P[G|\rho,I]P[\rho|I]}{P[G|I]}$$

<u>likelihood function</u>

$$P[G|\rho,I] = \frac{1}{Z_L}e^{-L[\rho]}$$

$$L[\rho] = \frac{1}{2(M_{\max}-M_{\min})}\int_{M_{\min}}^{M_{\max}} dM \frac{\left[G_{OPE}(M) - G_{\rho}(M)\right]^2}{\sigma^2(M)}$$

Corresponds to ordinary
χ^2-fitting.

<u>prior probability</u>

$$P[\rho|I] = \frac{1}{Z_s}e^{\alpha S[\rho]}$$

$$S[\rho] = \int_0^{\infty} d\omega \left[\rho(\omega) - m(\omega) - \rho(\omega)\log\left(\frac{\rho(\omega)}{m(\omega)}\right)\right]$$

(Shannon-Jaynes entropy)

"default model"

M. Jarrel and J.E. Gubernatis, Phys. Rep. 269, 133 (1996).
M.Asakawa, T.Hatsuda and Y.Nakahara, Prog. Part. Nucl. Phys. 46, 459 (2001).

The charmonium sum rules at finite T

The application of QCD sum rules has been developed in:

A.I. Bochkarev and M.E. Shaposhnikov, Nucl. Phys. B 268, 220 (1986).

T.Hatsuda, Y.Koike and S.H. Lee, Nucl. Phys. B 394, 221 (1993).

$$M(\nu) = \int_0^\infty e^{-\nu t} \rho(4m_c^2 t) dt \qquad (\nu \equiv \frac{M^2}{4m_c^2})$$

$$M(\nu) = A(\nu)\Big[1 + a(\nu)\alpha_s(\nu) + b(\nu)\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle_T}{m_c^4} + c(\nu)\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle_{T,2}}{m_c^4} + d(\nu)\frac{\langle g^3 G^3\rangle_T}{m_c^6}\Big]$$

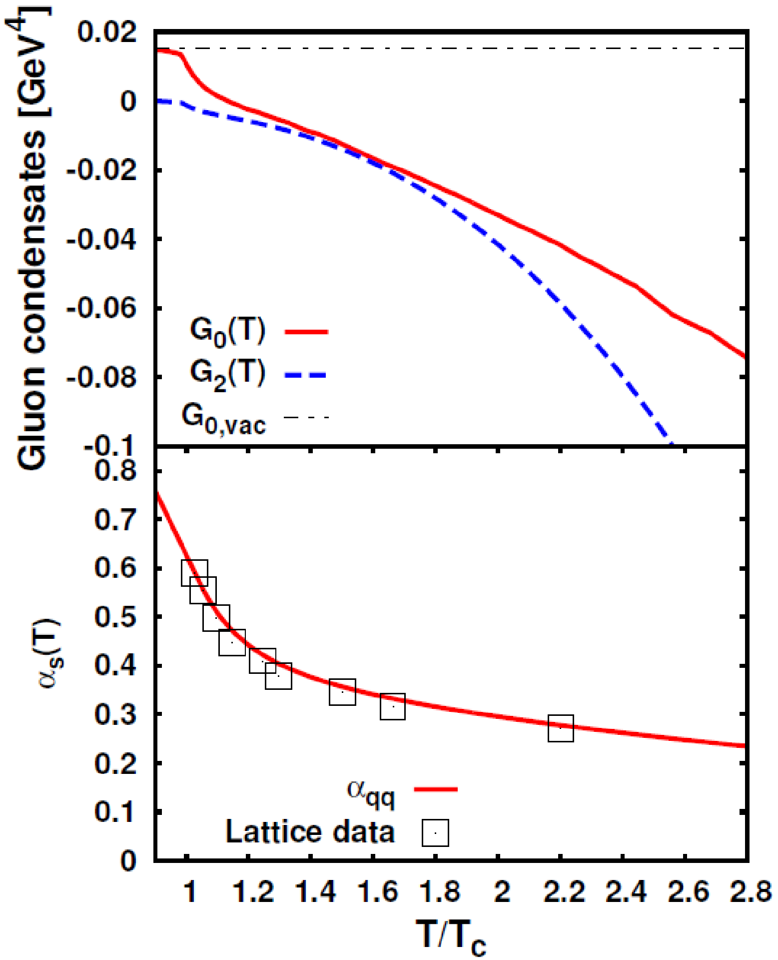
depend on T

Compared to lattice:

No reduction of data points that can be used for the analysis, allowing a direct comparison of T=0 and T≠0 spectral functions.

The T-dependence of the condensates

K. Morita and S.H. Lee, Phys. Rev. Lett. 100, 022301 (2008).



taken from:

K. Morita and S.H. Lee, Phys. Rev. D82, 054008 (2010).

Considering the trace and the traceless part of the energy momentum tensor, one can show that in tht quenched approximation, the T-dependent parts of the gluon condensates by thermodynamic quantities such as energy density ε(T) and pressure p(T).

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle_T = \langle \frac{\alpha_s}{\pi} G^2 \rangle_{\text{vac.}} - \frac{8}{11}(\epsilon - 3p)$$

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle_{T,2} = -\frac{\alpha_s(T)}{\pi}(\epsilon + p)$$

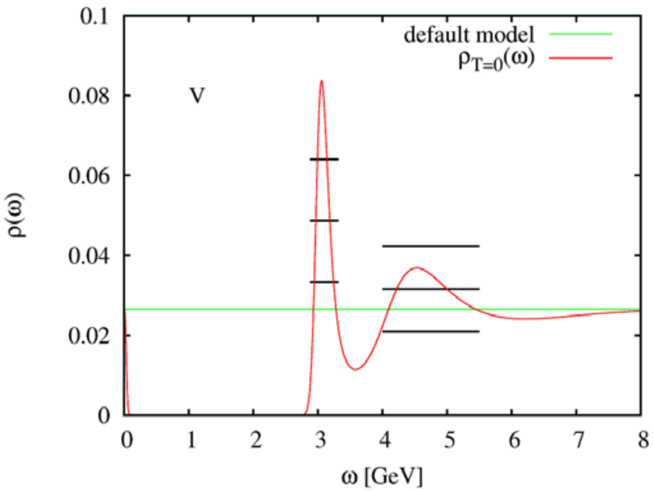
The values of ε(T) and p(T) are obtained from quenched lattice calculations:

G. Boyd *et al*, Nucl. Phys. B 469, 419 (1996).

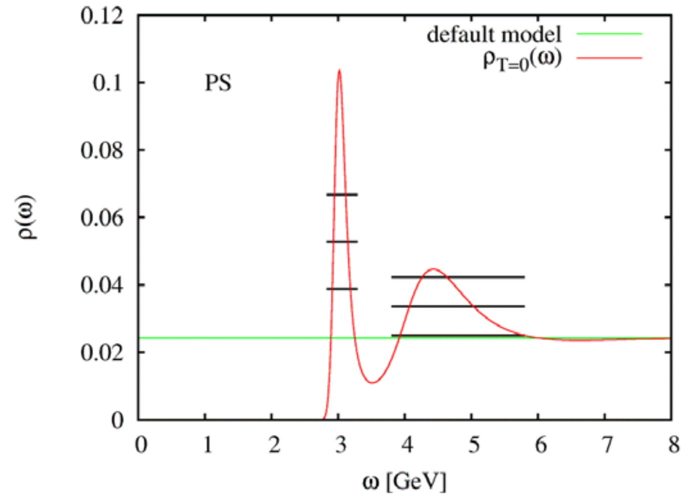
O. Kaczmarek *et al*, Phys. Rev. D 70, 074505 (2004).

MEM Analysis at T=0

S-wave

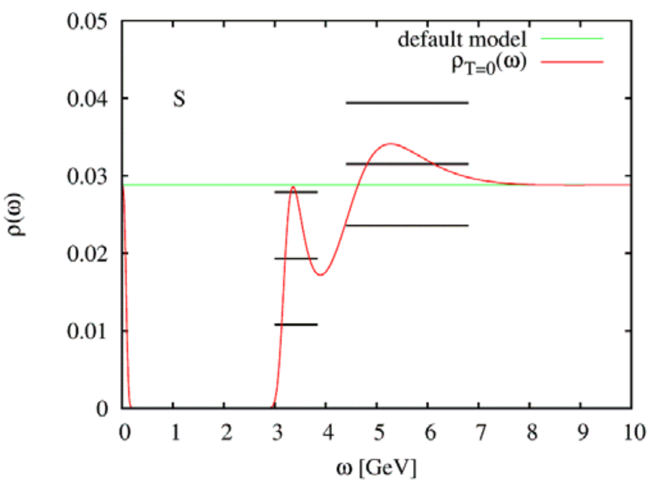


$m_{J/\psi}$=3.06 GeV (Exp: 3.10 GeV)

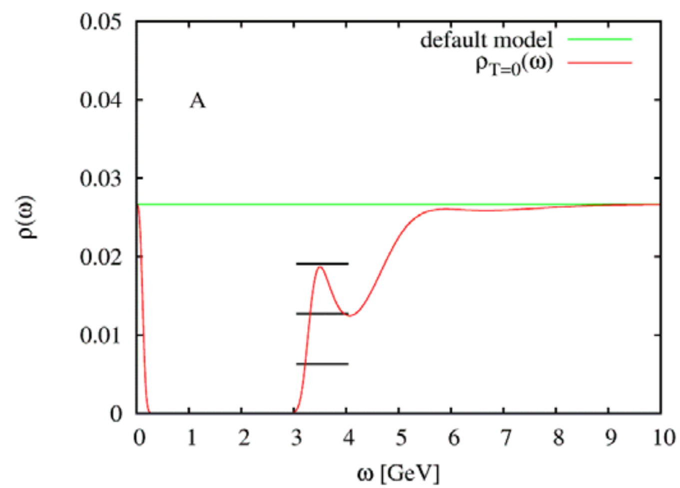


$m_{\eta c}$=3.02 GeV (Exp: 2.98 GeV)

P-wave



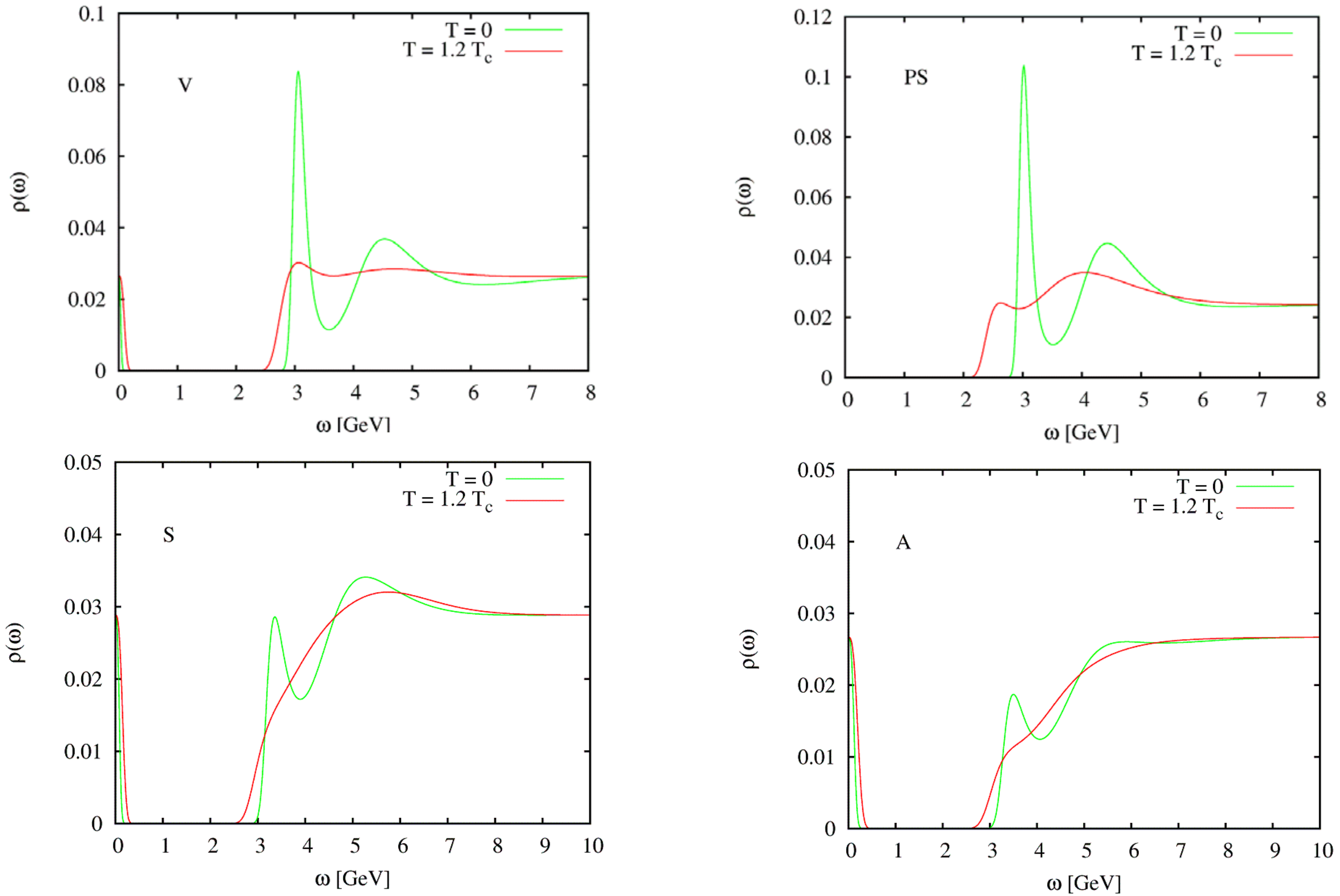
$m_{\chi 0}$=3.36 GeV (Exp: 3.41 GeV)



$m_{\chi 1}$=3.50 GeV (Exp: 3.51 GeV)

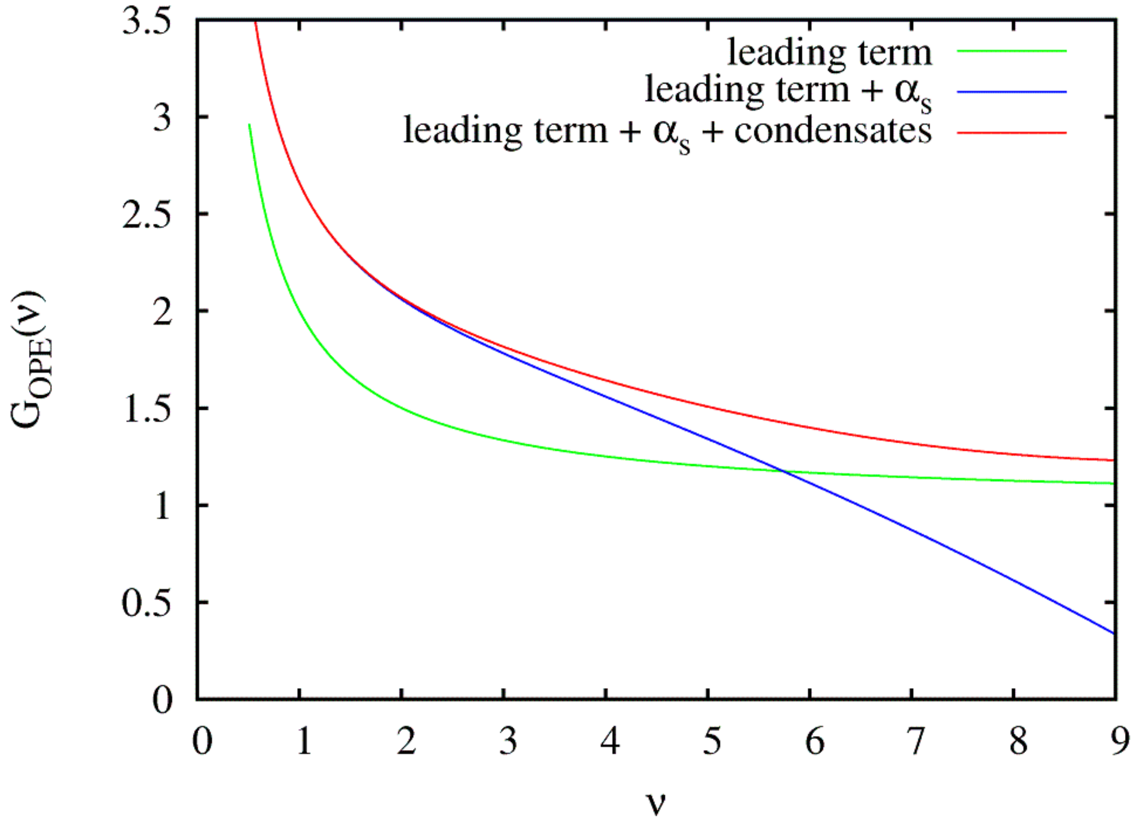
$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012 \pm 0.0036 \text{ GeV}^4$$
$$m_c = 1.277 \pm 0.026 \text{ GeV}$$

The charmonium spectral function at finite T



What is going on behind the scenes ?

The OPE data in the Vector channel at various T:



$T=0.2\ T_c$

cancellation between α_s and condensate contributions

Conclusions

- We have shown that MEM can be applied to QCD sum rules
- We could observe the melting of the S-wave and P-wave charmonia using finite temperature QCD sum rules and MEM
- Both J/ψ, η_c, χ_{c0}, χ_{c1} melt between T ~ 1.0 T_C and T ~ 1.2 T_c, which is below the values obtained in lattice QCD

Outlook

- Bottomium (see poster of K. Suzuki)
- Including higher orders (α_s, twist)
- Extending the method to investigations of other particles (D, …)

Backup slides

The basic problem to be solved

$$G_{OPE}(M) = \frac{1}{M^2}\int_0^\infty ds e^{-\frac{s}{M^2}} \rho(s)$$

given
(but only incomplete and with error)

"Kernel"

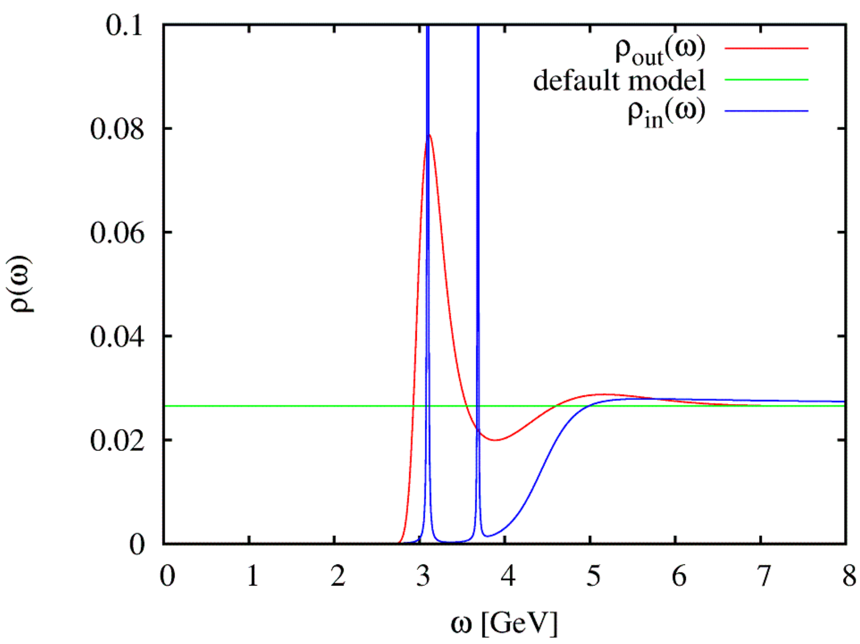
?

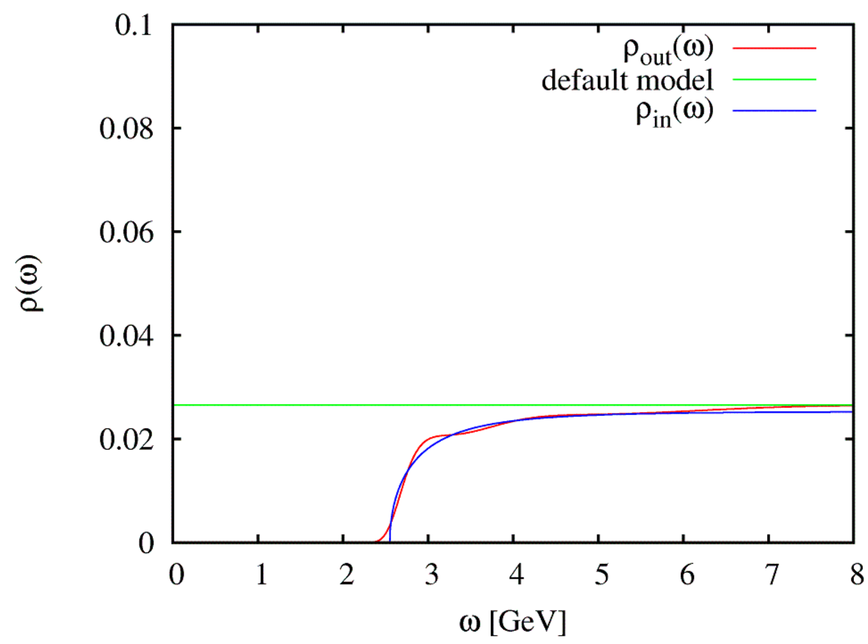
This is an ill-posed problem.

But, one may have additional information on ρ(ω), which can help to constrain the problem:

- Positivity: $\rho(\omega) \geq 0$
- Asymptotic values: $\rho(\omega = 0), \rho(\omega = \infty)$

A first test: mock data analysis



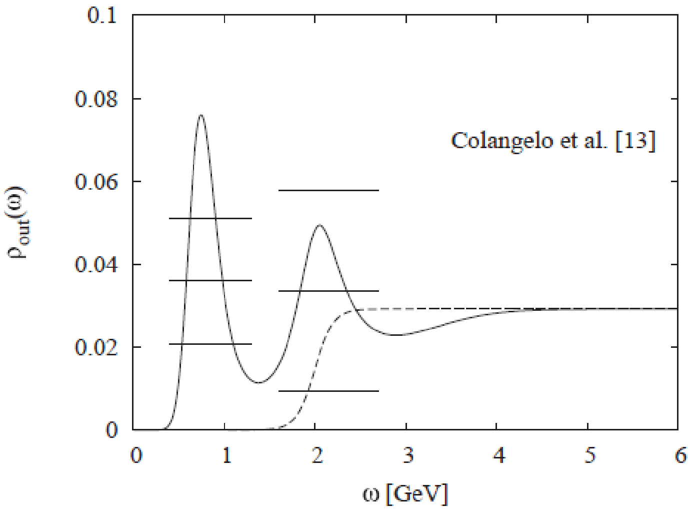


Both J/ψ and ψ' are included into the mock data, but we can only reproduce J/ψ.

When only free c-quarks contribute to the spectral function, this should be reproduced in the MEM analysis.

First applications in the light quark sector

ρ-meson channel

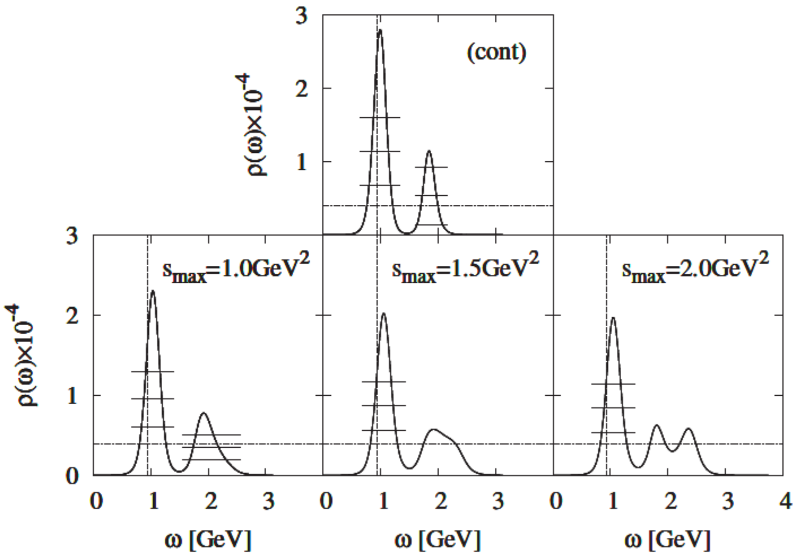


$$m_\rho = 0.76 \pm 0.07 \text{ GeV}$$
$$F_\rho = 0.174 \pm 0.039 \text{ GeV}$$

Experiment:
m_ρ= 0.77 GeV
F_ρ= 0.141 GeV

PG and M. Oka, Prog. Theor. Phys. **124**, 995 (2010).

Nucleon channel



$$m_N = 0.99 \pm 0.13 \text{ GeV}$$

Experiment:
m_N= 0.94 GeV

K. Ohtani, PG and M. Oka, arXiv:1104.5577 [hep-ph].