# Phase Transition due to Quantum Dissipation

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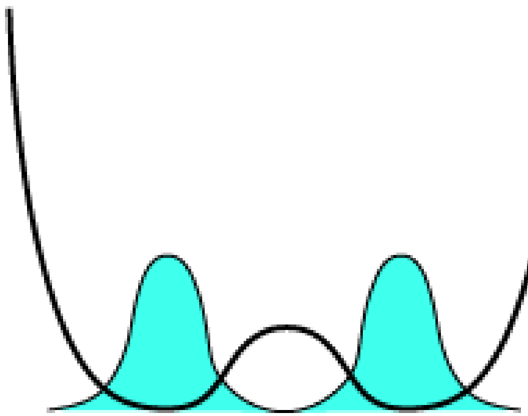
# Quantum-Classical phase transition

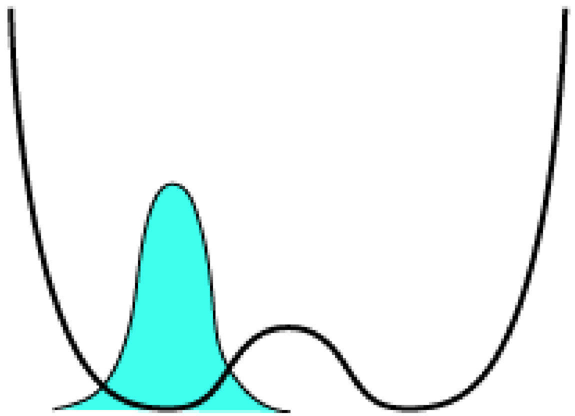
weak dissipation
- tunneling effect
- Rabi Oscillation
- coherent
- Quantum
- symmetric

$\eta_c$

Critical dissipation

strong dissipation
- no-tunneling effect
- localization
- decoherent
- Classical
- symmetry broken





double well $V(x) = -\frac{1}{2}x^2 + \lambda x^4$

# Caldeira-Leggett model

$$S[\,q,\{x_\alpha\}] = \int dt \left\{ \frac{1}{2}M\dot{q}^2 - V_0(q) + \sum_\alpha \left[ \frac{1}{2}m_\alpha \dot{x}_\alpha^2 - \frac{1}{2}m_\alpha\omega_\alpha^2 x_\alpha^2 - qC_\alpha x_\alpha \right] \right\}$$

target environment

path integrate out environmental degrees of freedom

- non-local effective action

$$\Delta S_{\mathrm{NL}} = \frac{\eta}{4\pi}\int \mathrm{d}s\mathrm{d}\tau \frac{(q(s)-q(\tau))^2}{|s-\tau|^p}$$

- Simple case : Effective Ising spin model

In particular ,long range Ising model has a long history itself and it is known that phase transition exist for $1 < p \leq 2$.

seek for critical coupling constant $\eta_c$

→ Discretizing time and taking only two values $\pm v$ for $q$, the system in Euclidean path integral formalism is equivalent to 1-D Ising model with long range interactions

$$H = \sum_{i,n} K_n \sigma_i \sigma_{i+n} \qquad K_n = \frac{\eta}{n^p}$$

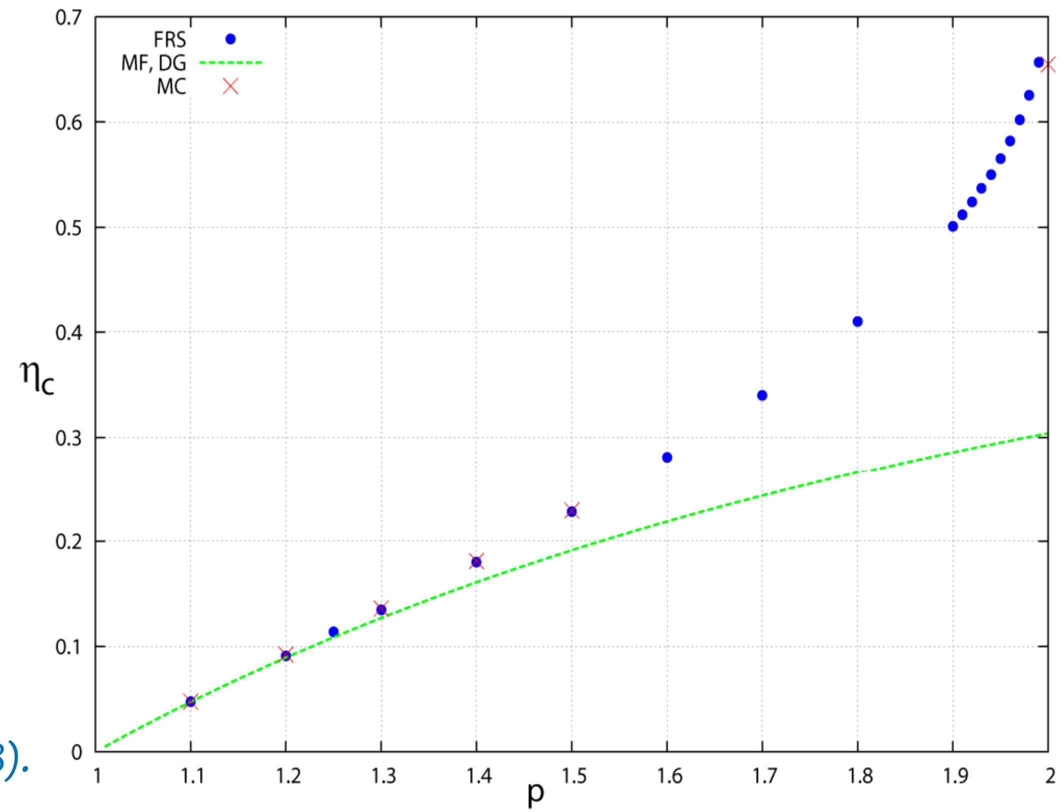
# Our scheme to approach infinitely long range interactions

- **BDRG(Block Decimation Renormalization Group)**
  calculation for the finite range interactions
- **FRS（Finite Range Scaling method)**
  Evaluation of the criticality

<mathematical phsics>
F. J. Dyson, Commun. Math. Phys. **12**, 91 (1969) and R. B. Griffiths, Commun. Math. Phys. **6**, 121 (1967)

*<Lattice Simulation>*
*E. Luijten and H. Mesingfeld, Phys. Rev. Lett. 86 ,509(2001) and E. Luijten and H. W. J. Blote, Phys. Rev. B **56**, 8945(1997).*

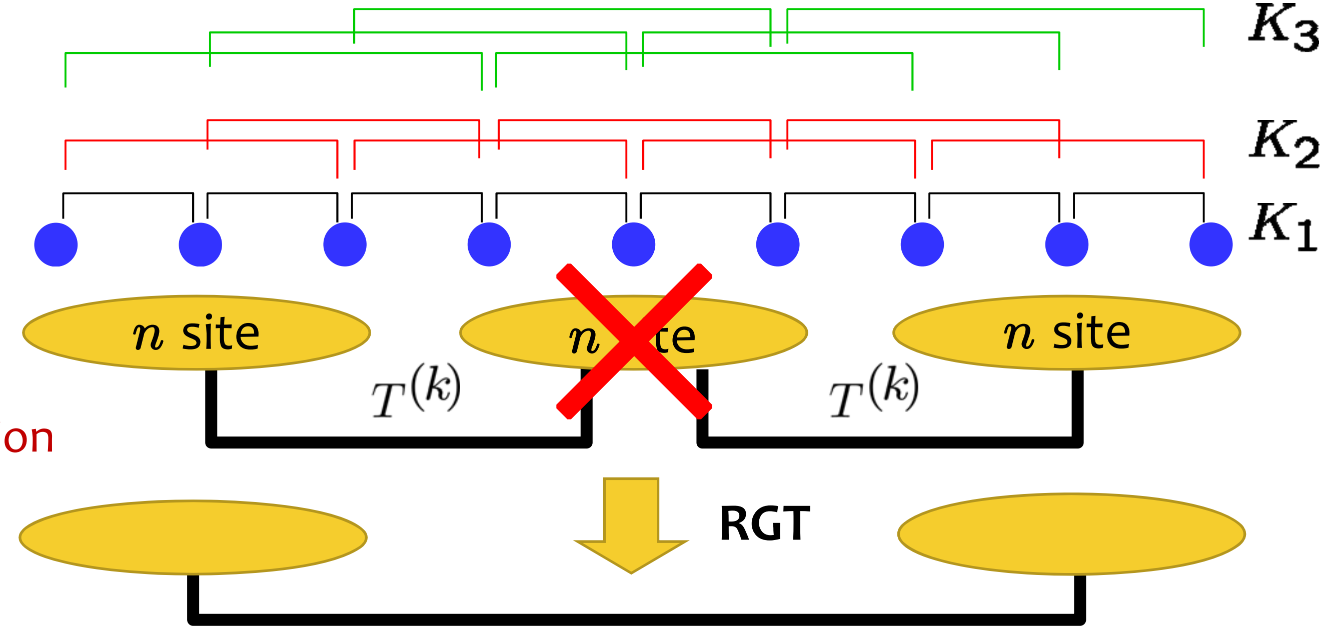
*<BDRG&FRS>*
*A Finite-Range Scaling Method to Analyze Systems with Infinite-Range Interactions, Ken-Ichi Aoki, Tamao Kobayashi and Hiroshi Tomita, Prog.Theor.Phys. **119**-3, 509-514 (2008).*



# BDRG(Block Decimation Renormalization Group)

- Non-nearest interactions are not easily treated by the original DRG because it requires the interaction space of infinite dimension.
- We define BDRG, an extended DRG to fit long range (but finite) interactions.
- We take the maximal range of interaction $n$ and treat the long range interactions $K_n$ .

**<example $n=3$>**




The $k$-th RG transformation

$$T^{(k+1)} = T^{(k)}T^{(k)}$$

- There are only nearest neighbor inter-block interactions. The system is regarded as a nearest neighbor multi-state model.
- $T$ (transfer) matrices represent the interactions between neighboring blocks.

  In the case of Ising model, the dimension of $T$ matrix is $2^n \times 2^n$.
- BDRG is able to calculate finite-range system exactly.

# Quantum Mechanical BDRG

We restrict interaction range $n$, then **1-block involves $n$ sites**.
Note that a state is infinite dimensional in quantum mechanics.

$$\boxed{x_1 \cdots x_n}\ \boxed{y_1 \cdots y_n}\ \boxed{z_1 \cdots z_n} \quad \Rightarrow \quad \boxed{a_1 \cdots a_n}\ \boxed{b_1 \cdots b_n}\ \boxed{c_1 \cdots c_n}$$

$$e^{-W(x_1 \cdots x_n, y_1 \cdots y_n)} \quad e^{-W(y_1 \cdots y_n, z_1 \cdots z_n)} \qquad e^{-\tilde{W}(a_1 \cdots b_n, b_1 \cdots b_n)} \quad e^{-\tilde{W}(b_1 \cdots b_n, c_1 \cdots c_n)}$$

bi-local potential $W$ and $T$ matrix

$$T = e^{-W(x_1 \cdots x_n,\ y_1 \cdots y_n)} = \langle x_1 x_2 \cdots x_n \mid \hat{U} \mid y_1 y_2 \cdots y_n \rangle$$

completeness: $$\int dy_1 \cdots dy_n \mid y_1 \cdots y_n \rangle \langle y_1 \cdots y_n \mid = 1$$
$$= \sum_{a_1 \cdots a_n} \mid a_1 \cdots a_n \rangle \langle a_1 \cdots a_n \mid$$

**change of base**

*We need to choose a proper set of states.

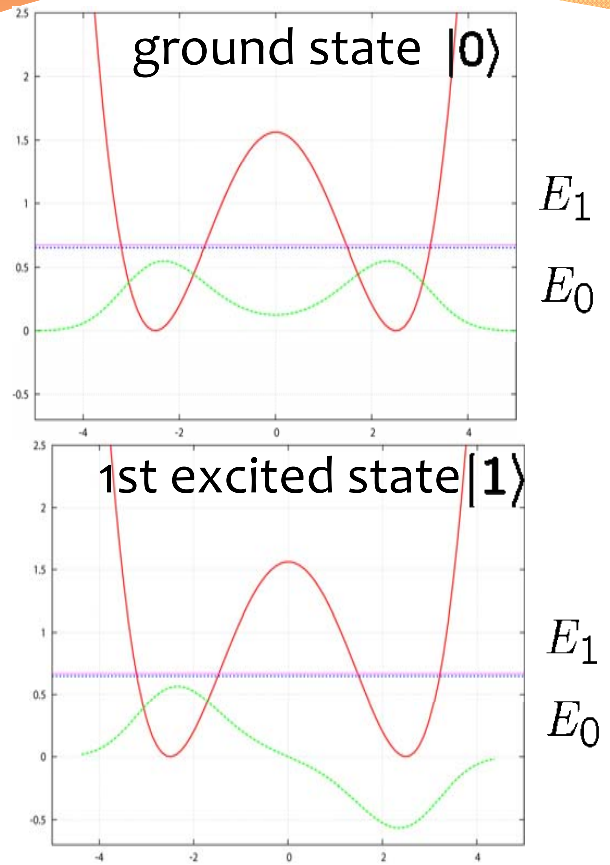
$$\begin{aligned} T &= e^{-\tilde{W}(a_1 \cdots a_n, b_1 \cdots b_n)} = \langle a_n \cdots a_n \mid \hat{U} \mid b_1 \cdots b_n \rangle \\ &= \int dx_1 \cdots dx_n dy_1 \cdots dy_n \langle a_1 \cdots a_n \mid x_1 \cdots x_n \rangle \langle x_1 \cdots x_n \mid \hat{U} \mid y_1 \cdots y_n \rangle \langle y_1 \cdots y_n \mid b_1 \cdots b_n \rangle \\ &= \int dx_1 \cdots dx_n dy_1 \cdots dy_n \psi^*_{a_1}(x_1) \cdots \psi^*_{a_n}(x_n) e^{-W(x_1 \cdots x_n, y_1 \cdots y_n)} \psi_{b_1}(y_1) \cdots \psi_{b_n}(y_n) \end{aligned}$$

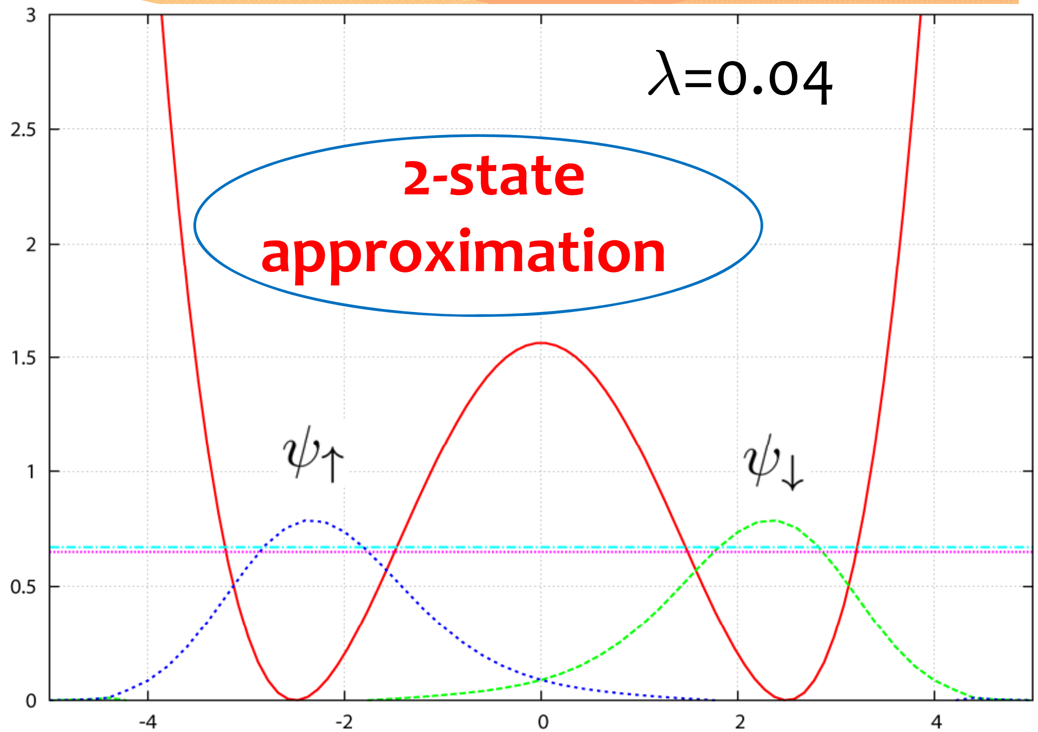
$\Rightarrow$ States$\{\mid a_n \rangle\}$ need to be restricted.

# Ground state approximation

We take the linear combination of ground state and 1st excited state of double well without dissipation, $\eta = 0$. It is one of 2-state approximations and consistent to the procedure in the CL model.

$$\psi_{\uparrow,\downarrow} = (|0\rangle \pm |1\rangle)/\sqrt{2}$$




Our aim is to evaluate the plausibility of our BDRG & FRS in the effective Ising model with the ground state approximation.

# For example, $n=2$

$$\{\psi_{a_n}(x)\} = \{\psi_\uparrow(x)\ ,\ \psi_\downarrow(x)\}$$ ground state approximation

$$T \;=\; e^{-W(a_1a_2,\ b_1b_2)} = \int dx_1dx_2dy_1dy_2\ \psi^*_{a_1}(x_1)\psi^*_{a_2}(x_2)\psi_{b_1}(y_1)\psi_{b_2}(y_2)$$

$$\times \exp\Bigg[-\frac{m(x_1-x_2)^2}{4\epsilon} - \frac{m(x_2-y_1)^2}{2\epsilon} - \frac{m(y_1-y_2)^2}{4\epsilon}$$ :kinetic term

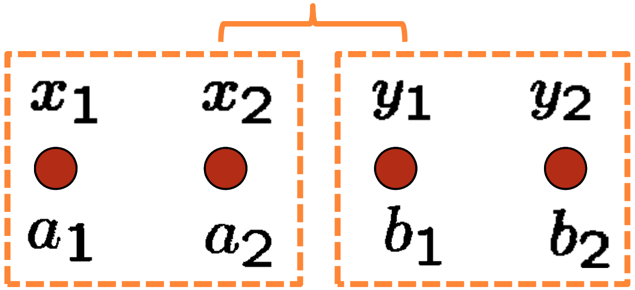
$$-\frac{\epsilon}{2}\left(V(x_1)+V(x_2)+V(y_1)+V(y_2)\right)\Bigg]$$ :potential term

$$-\frac{\eta}{2\pi}\epsilon^{2-p}\left[\frac{1}{2}(x_1-x_2)^2+(x_2-y_1)^2+\frac{1}{2}(y_1-y_2)^2\right.$$

$$\left.\frac{1}{2^p}\left((x_1-y_1)^2+(x_2-y_2)^2\right)\right]$$ :dissipation term

*$\epsilon$ : discretization step

$$T = e^{-W(x_1x_2,y_1y_2)}$$




- $2^4$ integrations of 4-dimension
- In the ground state approximation, $2^{2n}$ integrations of $2n$-dimensions are necessary to get the initial $T$ matrix of BDRG. To make highly multi-dimensional integrations we adopt the Monte Carlo method.

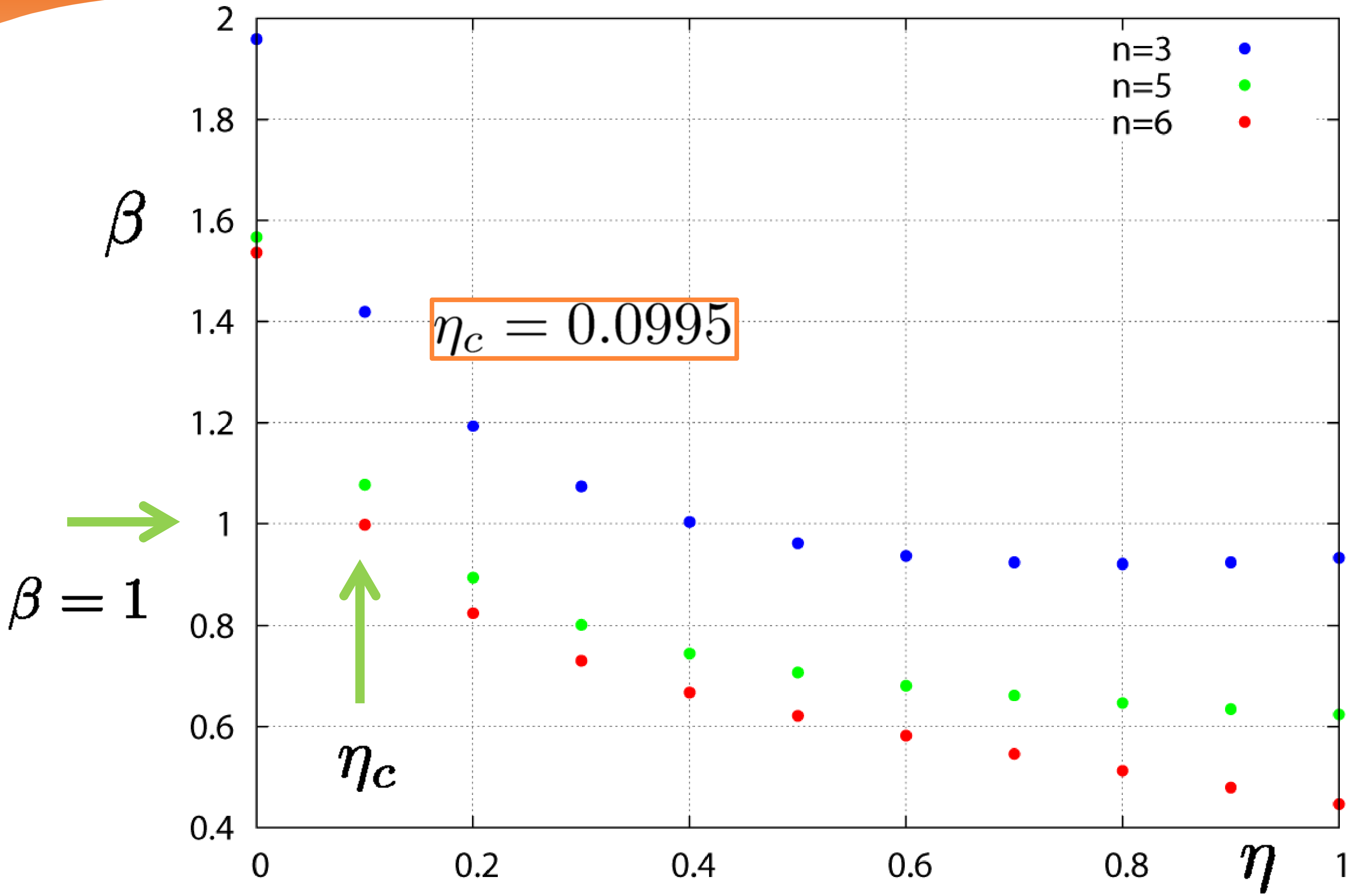
# Finite Range Scaling (FRS) method

- This method can estimate infinite range information from finite range information to determine the critical point quantitatively.
- First, the finite range susceptibility $\chi(n)$ is calculated exactly by BDRG.
- Next, assuming that the variation of the susceptibility with respect to range $n$ satisfies the following scaling relation, we find the scaling exponent $\beta$ (FRS exponent) ,

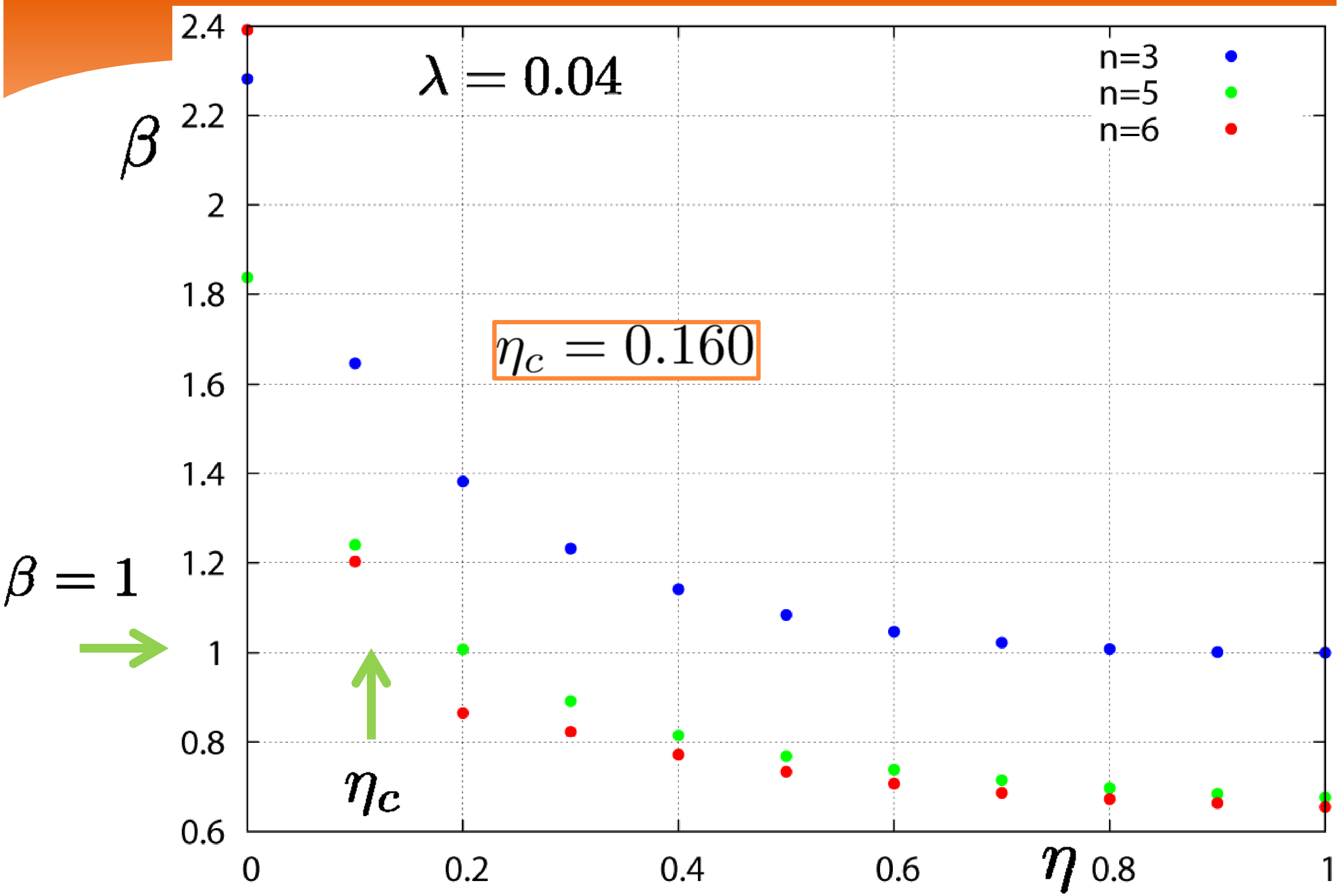
$$\log \chi(n) - \log \chi(n-1) \underset{n\to\infty}{\Rightarrow} \left(\frac{1}{n}\right)^{\beta(p,\eta)}$$

- Then, the infinite n behavior of $\chi$ (n) is controlled by the zeta function $\zeta(\beta)$, which has a pole singularity at $\beta$=1.
- Finally, the critical $\eta$ is determined by the condition $\beta$ = 1.
- Hereafter we search for the point $\boxed{\beta(p,\ \eta) = 1}$ .

# Behavior of $\beta$ $\lambda = 0.03$




$p = 1.7$ , $\epsilon = 1.2$ , $N_{\mathrm{MC}} = 10^6$ (number of configurations )




$p = 1.7\ ,\ \epsilon = 1.2\ ,\ N_{\rm MC} = 10^6$

# Comparison with other approaches

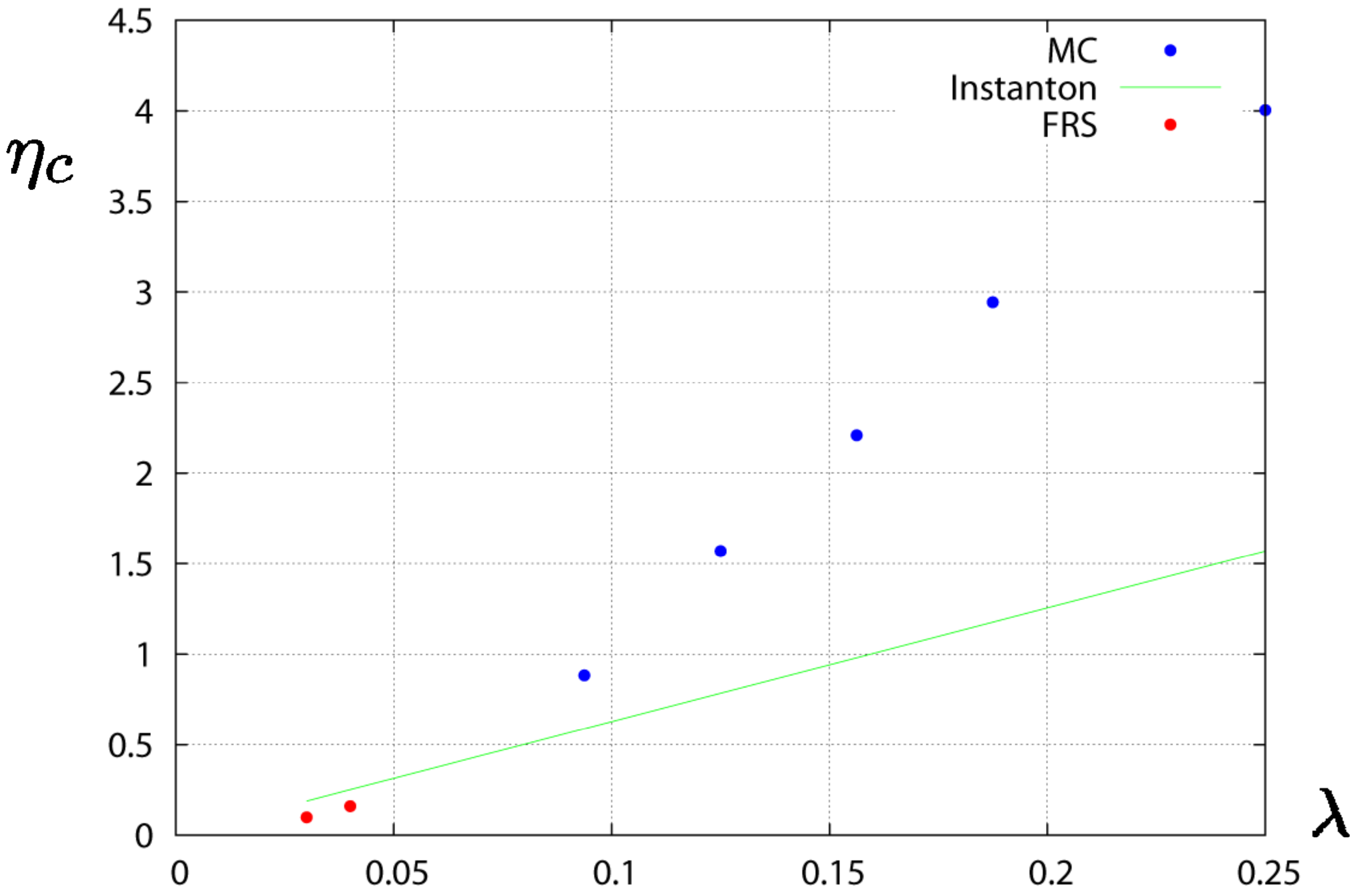
<Earlier researches about dissipative double well quantum mechanics>

Monte Carlo simulation :

Takeshi MATSUO, Yuhei NATSUME and Takeo KATO, J. Phys. Soc. Jpn. 75 (2006) 1-4.

Other approach by using non-perturbative Renormalization Group method :

Ken-Ichi Aoki and Atsushi Horikoshi, Phys.Lett. A 314 (2003) 177-183.



# Summary

- We study the Classical-Quantum phase transition in the quantum dissipative double well system, where the Caldeira-Leggett effective long range interactions cause the dissipative effects.
- Discretizing the time and taking the 2-state approximation with the ground state, we convert the system into an effective long range Ising model. We apply our BDRG & FRS method, which has been proven successful in the simple long range Ising model, to this effective Ising model.
- Highly multi-dimensional integrations are necessary to calculate effective Ising interactions and we adopt the Monte Carlo method, the validity of which has been confirmed by comparing our MC results with the continuum limit $\epsilon \to 0$ in case of no-dissipation.
- Then applying our FRS method, we get the critical dissipation by the condition, the FRS exponent $\beta\,(\eta_c)=1$.
- We compared our results with those obtained by other approaches.
- Further calculations are required to reduce possible systematic errors.