UMR Gulliver

Effets Collectifs & Matière Molle

Shear Modulus and Dilatancy Softening above Jamming

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Overview

- Introduction

  Elasticity of Jammed materials

- Probing jamming in a system of photo-elastic (soft) discs

  Preparing a jammed granular glass

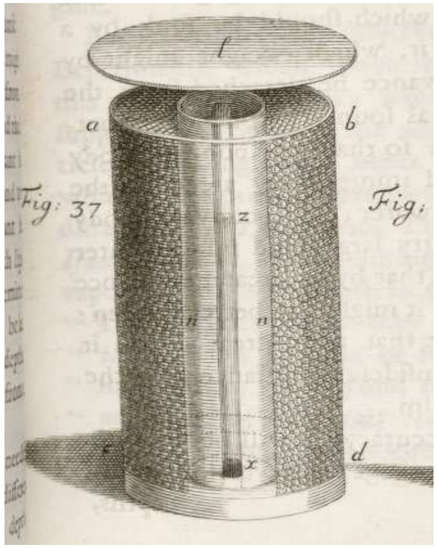
  Exploring the vicinity of point J

- Mechanical response to a point like disturbance

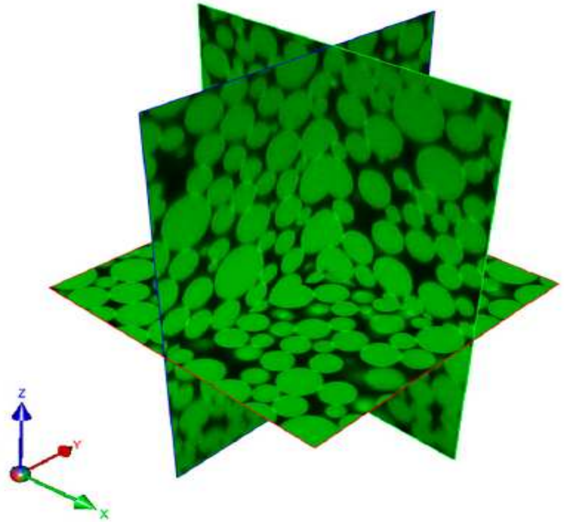
  Dilatancy and Shear Softening around an Inflater

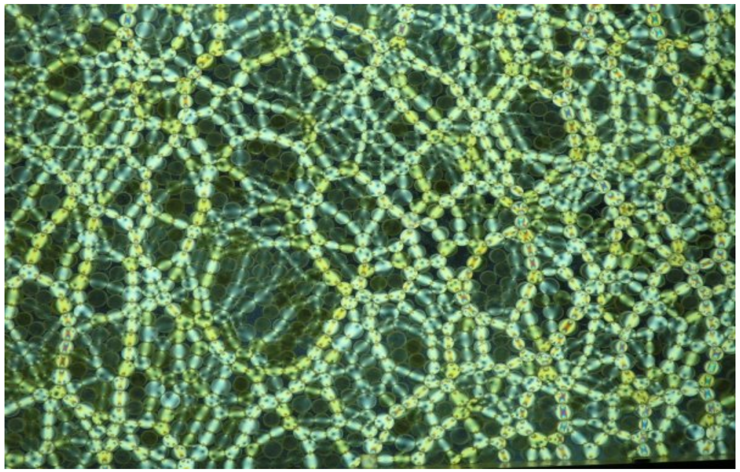
Experimental samples of jammed materials

Green peas, Hales, 1727

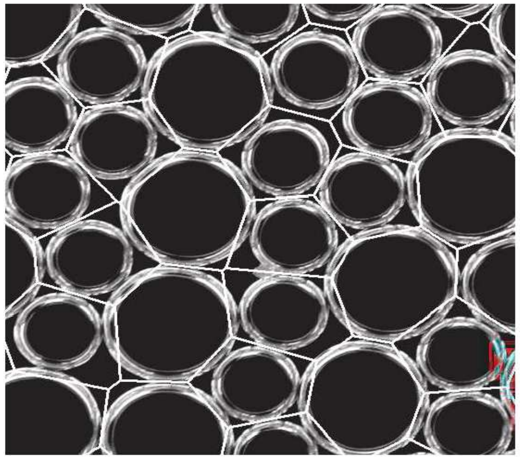


Emulsion, Jorjadze et al., 2011





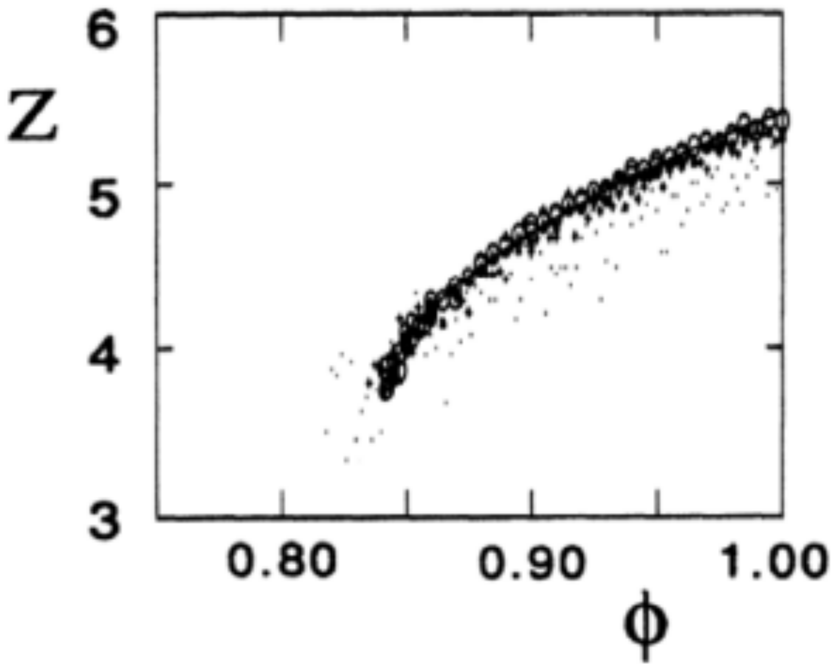
Grains, Behringer

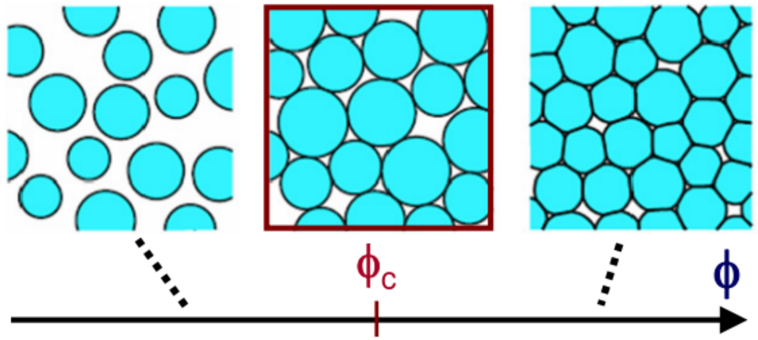


Foam, Katgert et van Hecke, 2010

A paradigm: Jamming of soft spheres at T=0

- A well defined concept,
  (*O'Hern et al. (2002)*)





A geometrical transition

- # of contacts jumps to $z_J = z_{iso}$
- $\delta z = z - z_{iso} \sim (\Phi - \Phi_J)^{1/2}$
- $g(r=d)$ = delta function

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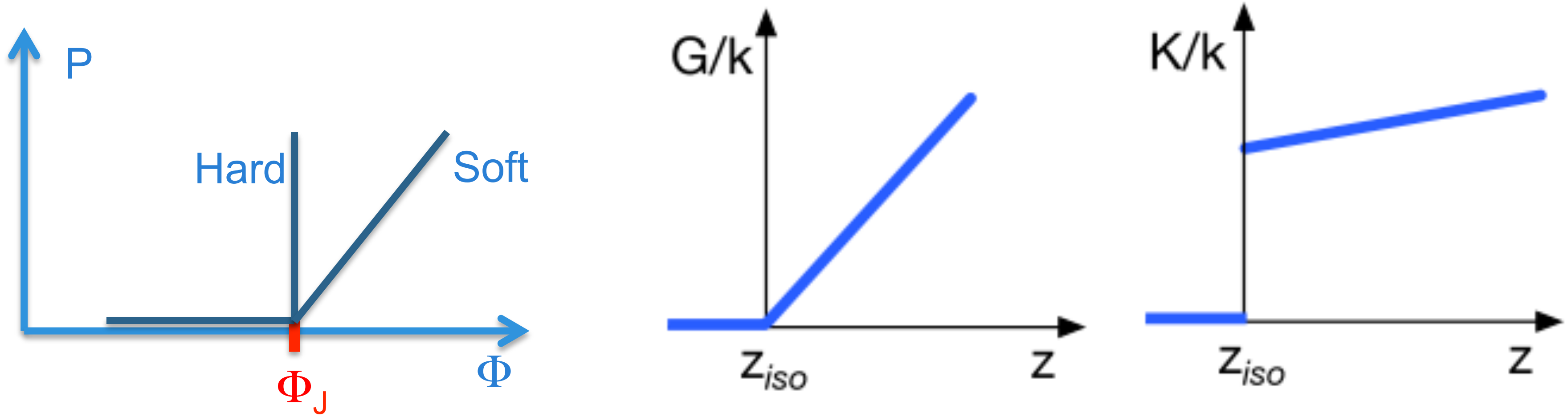
Rigidity transition of frictionless packings



A mechanical transition

- Pressure scales as prescribed by elasticity of the individual grains P ~ $\delta\phi^{\mu}$
- Elastic moduli scaling K/k ~ δz^{0}; G/k ~ δz ~ $\delta\phi^{0.5}$
- NB : k is the stiffness of the individual grains k ~ $\delta\phi^{\mu-1}$
- In all cases G/K ~ $\delta\phi^{0.5}$

Length scales

- Two length scales associated with the jamming transition

Lerner, E. et al (2014). Soft Matter, 10(28), 5085–5092.

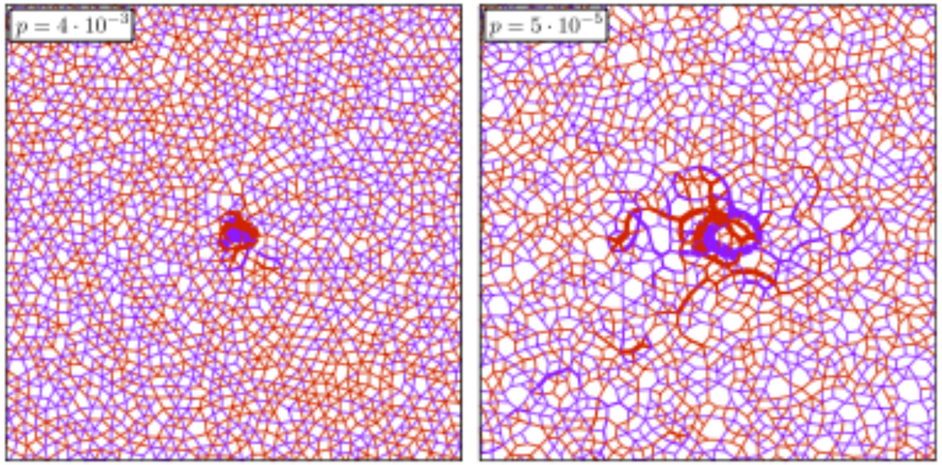
  - $l^* \sim 1/\delta z$: length below which stability feels boundaries

    can be seen as a point to set lengthscale

  - $l_c \sim 1/\delta z^{0.5}$: length of 2-points correlations , or response to point perturbation

    also the length scale beyond wich continuum elasticity holds

- However numerical study $l_c \sim 1/\delta z$ Ellenbroek, et al (2006).PRL *97*(25), 258001



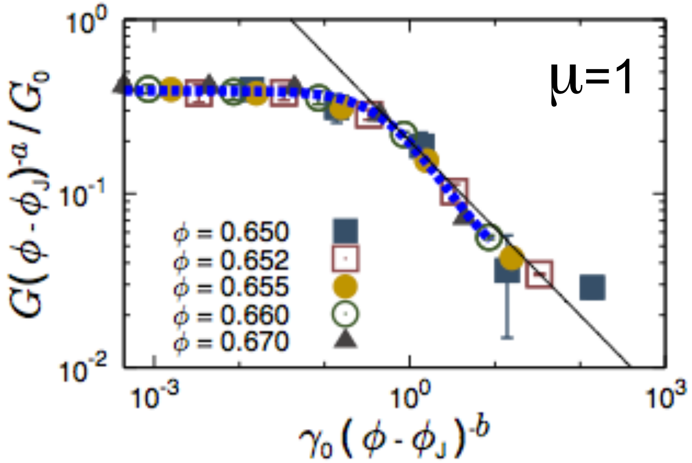
Finite size effects

- Validity of linear elasticity close to jamming : a long standing debate

- Shear softening for strain $\gamma > \gamma_c$

Hayakawa H. et al. PRE ***90,*** *042202 (2014)*

  - $G \sim \delta\phi^{\mu-1/2}\,\gamma^{-1/2}$
  - $\gamma_c \sim \delta\phi^{\mu}$



- More Recent finite size scaling analysis

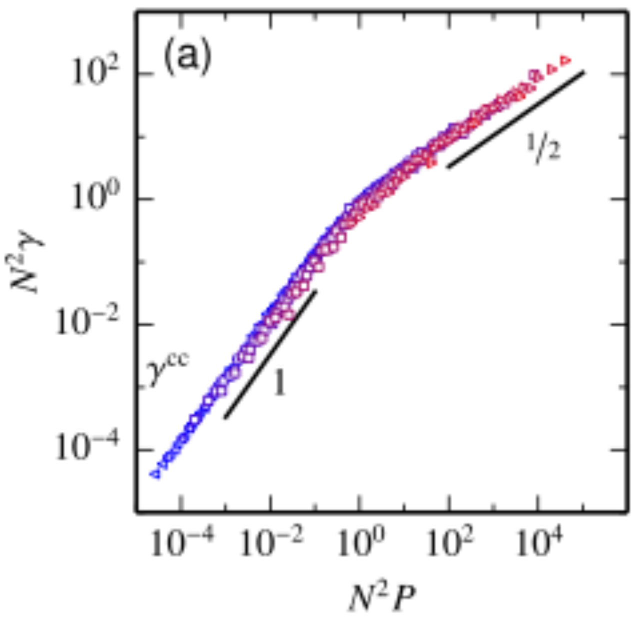
Goodrich et al (2014). *PRE*, *90*(2), 022138;

van Deen et al (2014). *PRE*, *90*(2), 020202

  - $\gamma_c \sim P$ for $N^2 P \ll 1$
  - $\gamma_c \sim P^{1/2} / N$ for $N^2 P \gg 1$

NB : $P \sim \delta\phi^{\mu}$

the closer to jamming, the smaller the critical strain

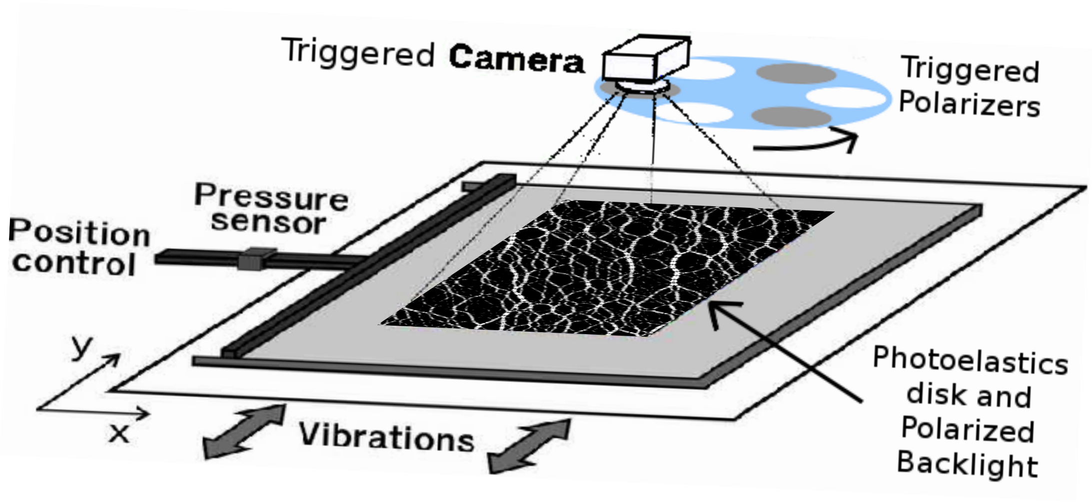


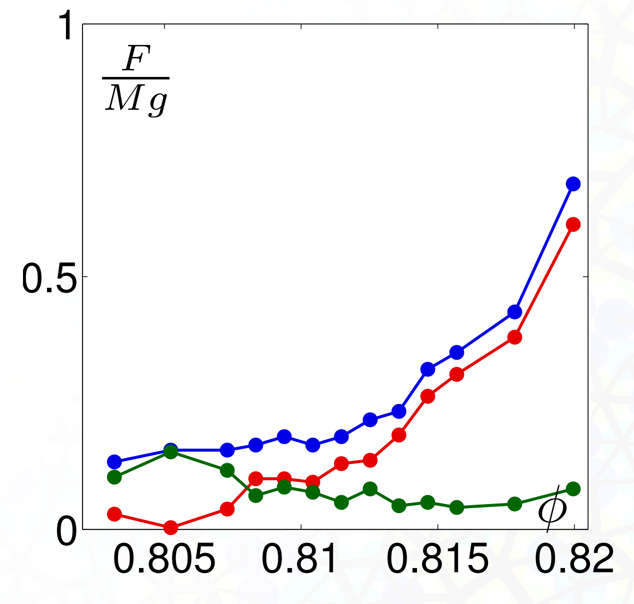
The present experimental study :

- Prepare a jammed granular glass
- Perform the inflater experiment
- Identify the linear and non linear regime for the response
- Qualify the scaling with the distance to jamming

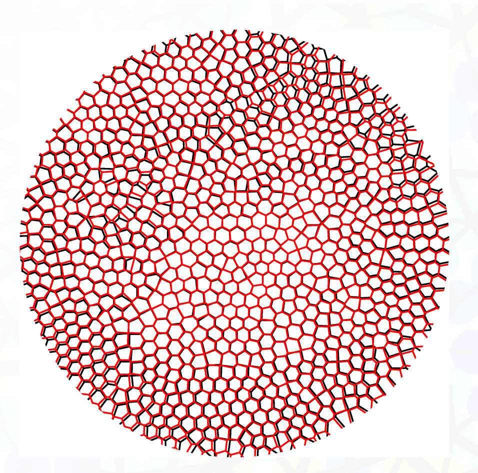
Preparing a jammed granular glass

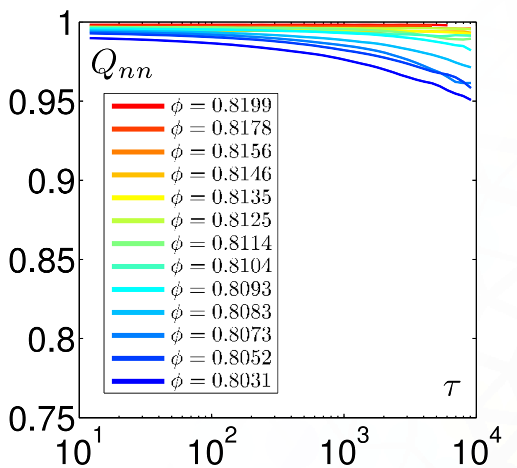
- A bi-disperse 2d packing $\mu \approx 1.7 \pm 0.1$





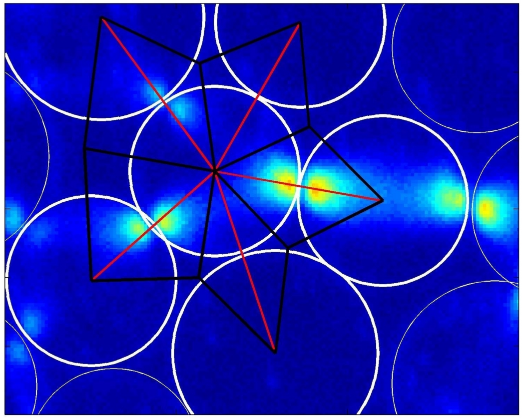
- A frozen structure





Signature of jamming within contacts …

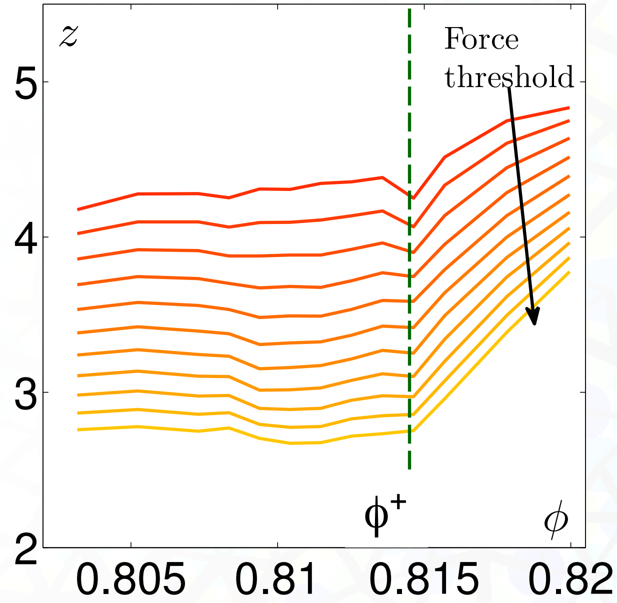
Interparticle force measurement

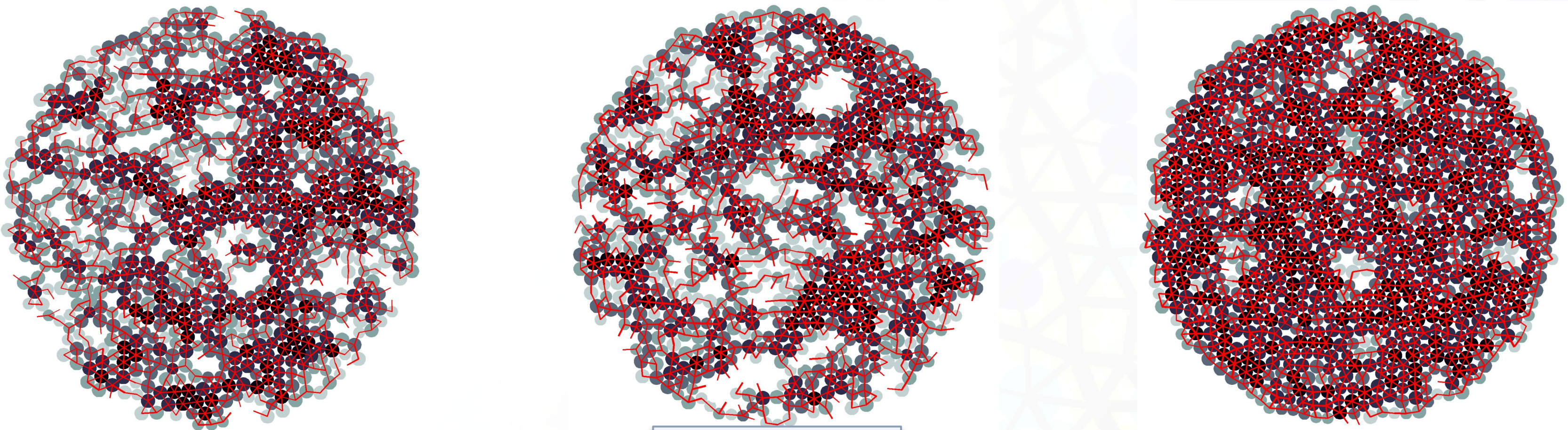


thresholding
gap $< \epsilon$

$\rightarrow$

thresholding
force $> f_0$





$\phi < \phi^+$

$\phi^+ = 0.814$

$\phi > \phi^+$

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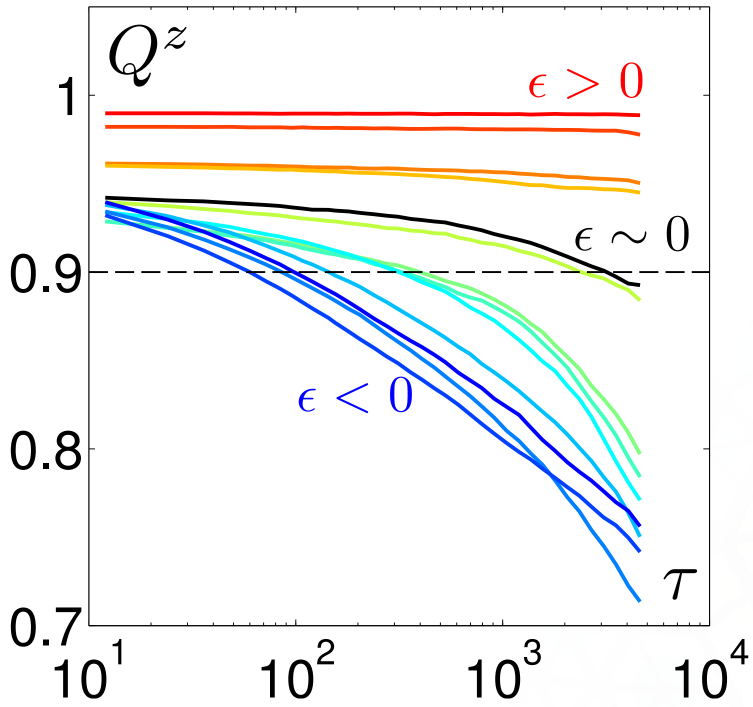
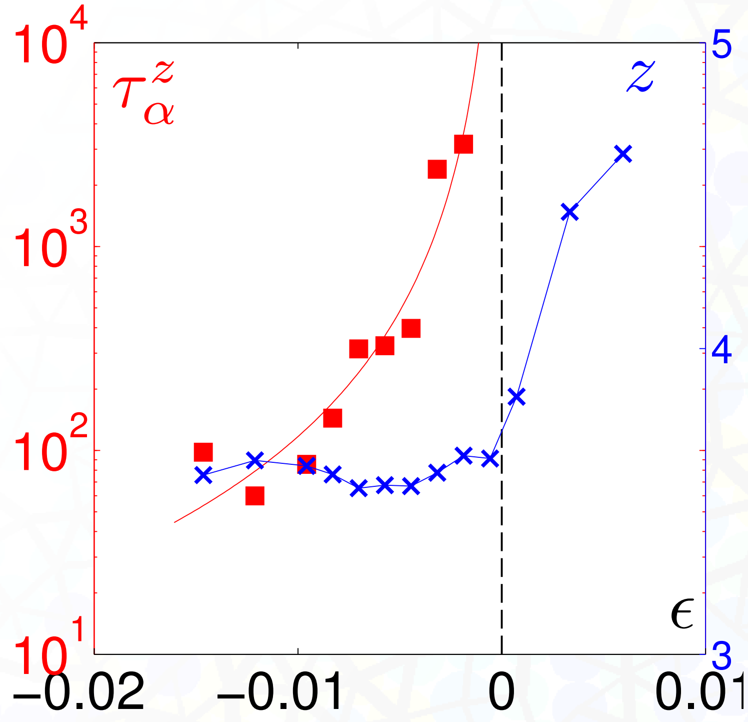
… and contact dynamics

$$Q^z(t,\tau) = \frac{1}{N}\sum_i Q_i^z(t,\tau) \quad \text{where } Q_i^z(t,\tau) = \begin{cases} 1 \text{ if } |z_i(t+\tau) - z_i(t)| \leq 1 \\ 0 \text{ if } |z_i(t+\tau) - z_i(t)| > 1 \end{cases}$$

$$Q_z(\tau) = \langle Q^z(t,\tau)\rangle_t$$



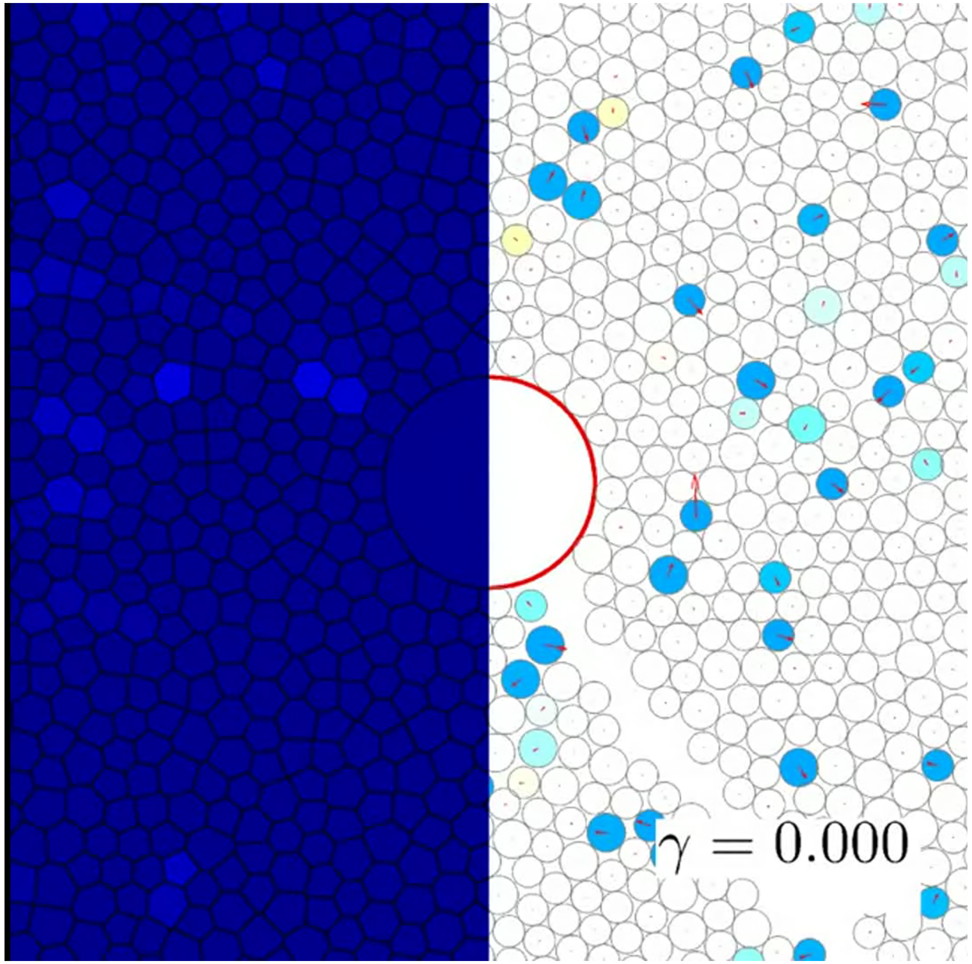


Probing elasticity : set up

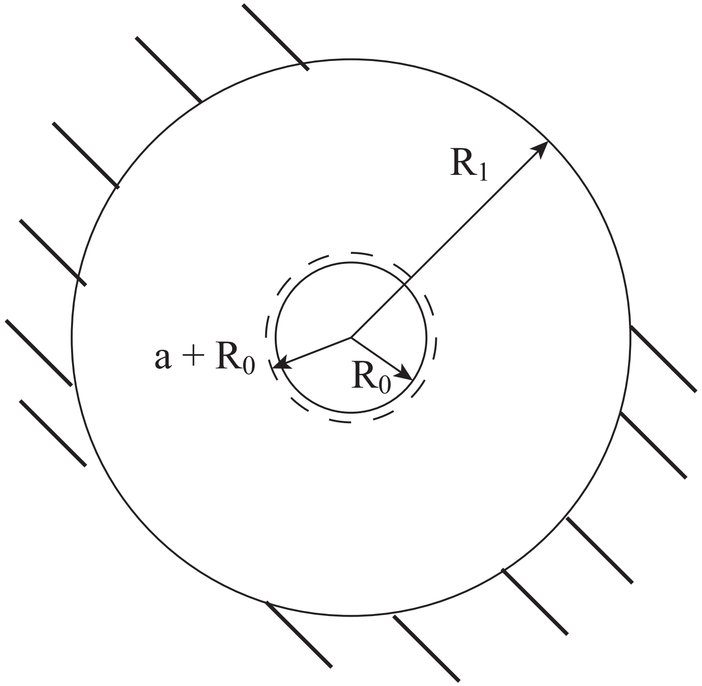
- Prepare the system at large packing fraction (under vibration)
- Inflate an intruder in the center (the vibration is stopped)
- Decrease the packing fraction (under vibration)
- iterate

R0 - > R0 + a

γ = a/R0



Probing elasticity : the linear elastic framework



$$A = \frac{R_0^2}{\left(R_1^2 - R_0^2\right)} \qquad B = \frac{R_1^2}{\left(R_1^2 - R_0^2\right)}$$

$$div\left(\underline{\underline{\sigma}}\right) = 0$$

$$U(R_0) = a$$
$$U(R_1) = 0$$

$$\underline{\underline{\sigma}} = \frac{1}{2} Tr(\underline{\underline{\sigma}})\underline{\underline{1}} + \underline{\underline{\tau}}$$

$$\underline{\underline{\sigma}} = K Tr(\underline{\underline{\varepsilon}})\underline{\underline{1}} + 2G\underline{\underline{\gamma}}$$

$$\underline{\underline{\varepsilon}} = \frac{1}{2}\left[\underline{\underline{\nabla U}} + {}^{t}\underline{\underline{\nabla U}}\right] = \frac{1}{2} Tr(\underline{\underline{\varepsilon}})\underline{\underline{1}} + \underline{\underline{\gamma}}$$

$$\delta \equiv Tr\left(\underline{\underline{\varepsilon}}\right) = -2\frac{a}{R_0}A$$

$$\gamma \equiv J_2\left(\underline{\underline{\gamma}}\right) \equiv \sqrt{\frac{1}{2}\underline{\underline{\gamma}} \circ \underline{\underline{\gamma}}} = \frac{a}{R_0} B \left(\frac{R_0}{r}\right)^2$$

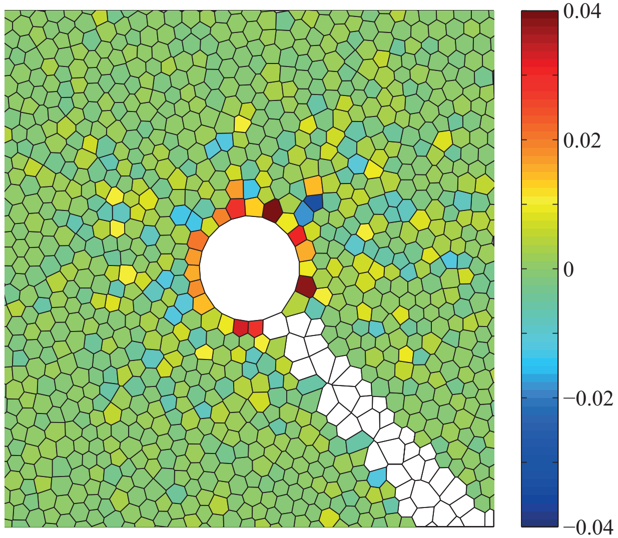
$$P \equiv Tr\left(\underline{\underline{\sigma}}\right) = K Tr\left(\underline{\underline{\varepsilon}}\right)$$

$$\tau \equiv J_2\left(\underline{\underline{\tau}}\right) = \sqrt{\frac{1}{2}\underline{\underline{\tau}} \circ \underline{\underline{\tau}}} = 2G J_2\left(\underline{\underline{\gamma}}\right)$$

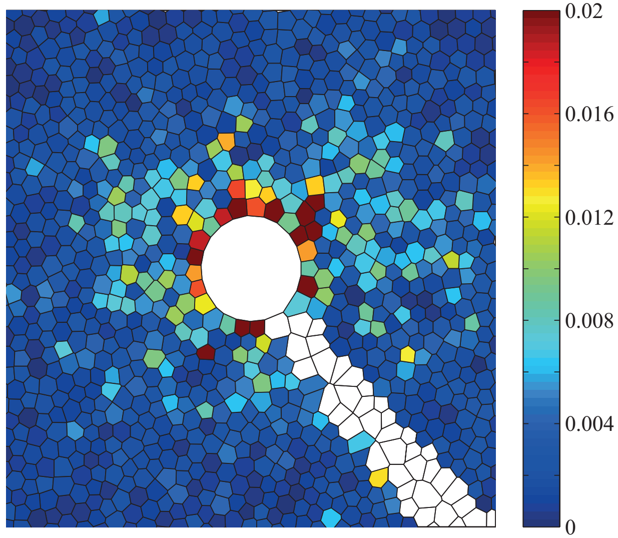
- Nota Bene : In the limit of large R_1, A->0, B->1 : this is a shear test!

For each packing fraction and each a/R_0

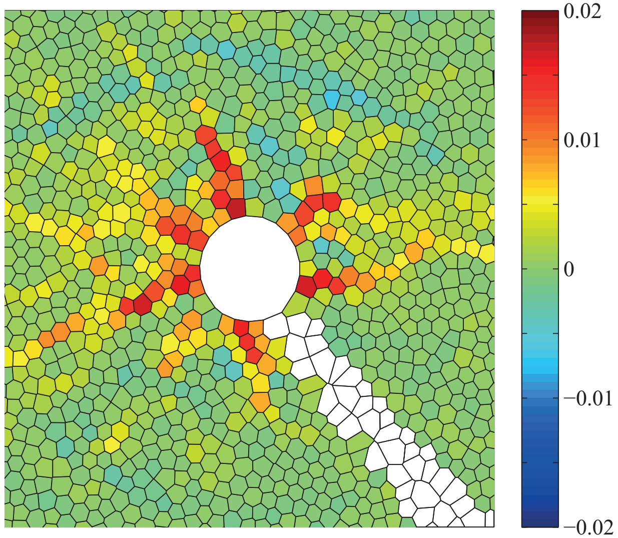
$\delta=Tr(\varepsilon)$



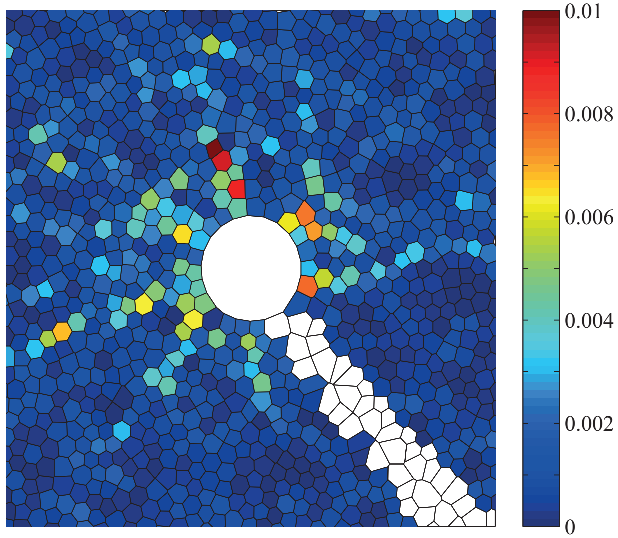
$\gamma=J_2(\varepsilon)$



$P=Tr(\sigma)$

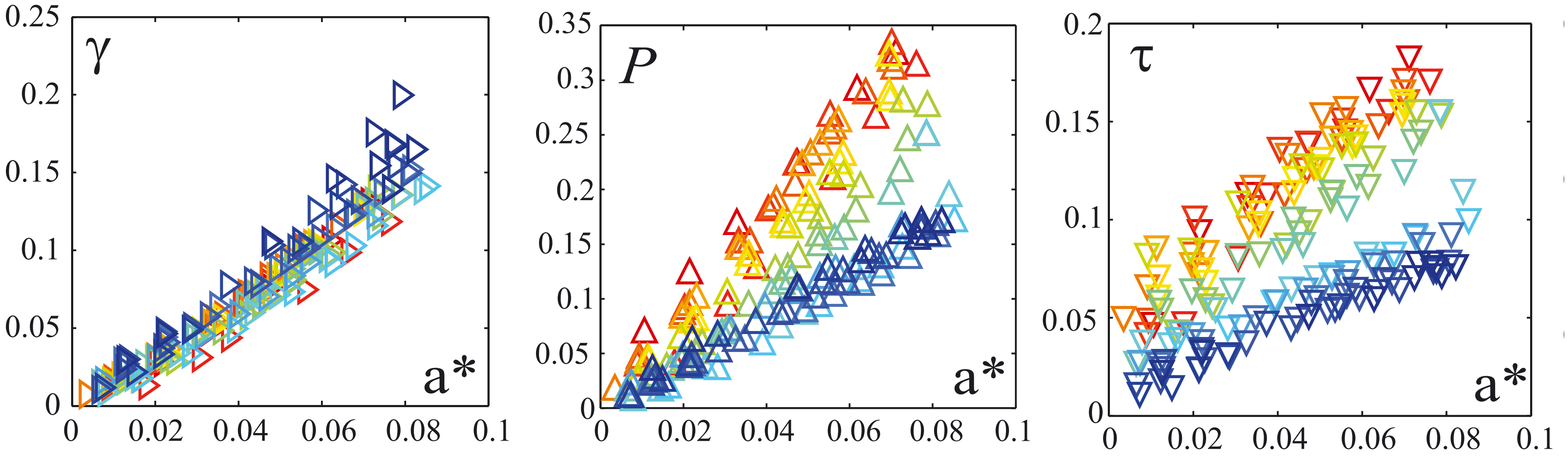


$\tau=J_2(\sigma)$



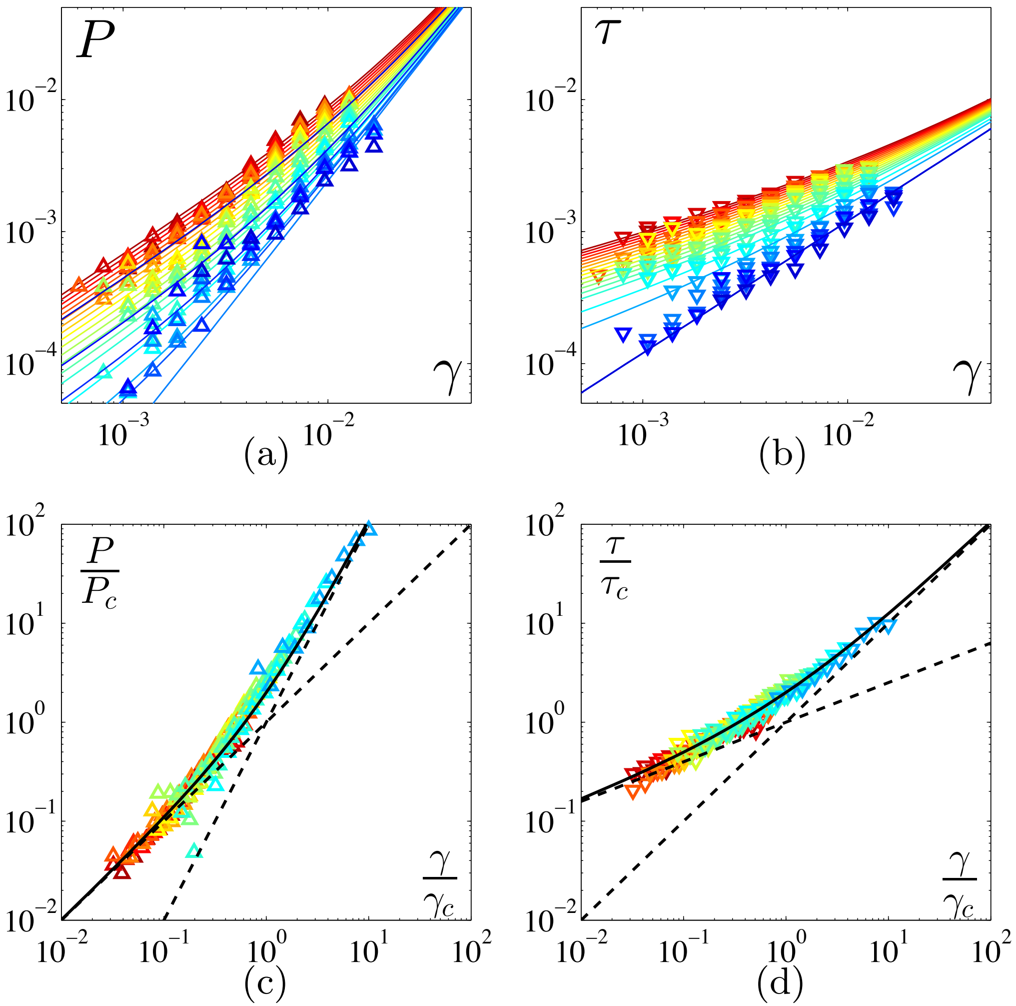
Spatially averaged quantities

$$a^* = a / R_0$$



- The average shear strain entirely controled by a*
- The stresses are non linear and depend on packing fraction

Parametric plot of local stress vs. local strain



$$P = [R_0 + R_{nl}(\Delta\phi, \gamma)]\, \gamma^2$$
$$\tau = 2\,[G_0 + G_{nl}(\Delta\phi, \gamma)]\, \gamma$$

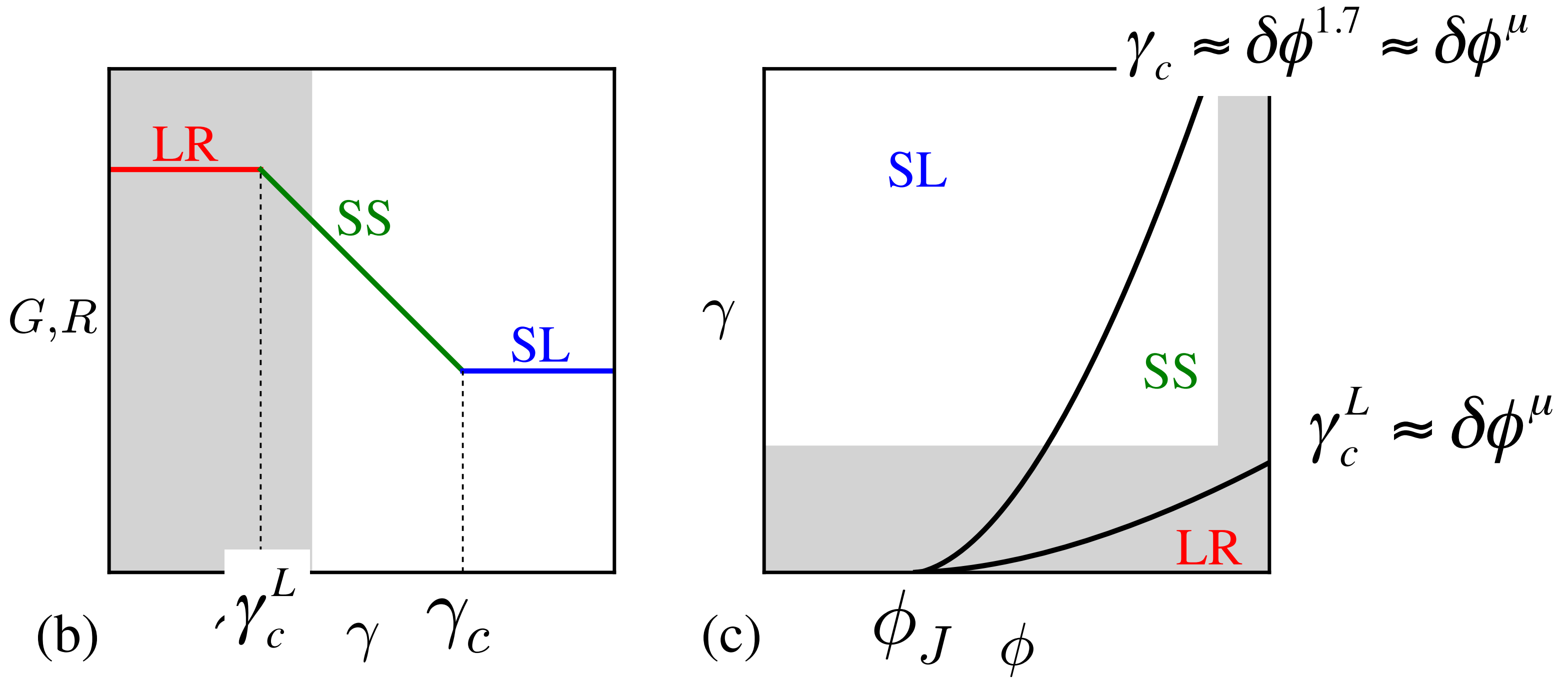
$$R_{nl}(\Delta\phi, \gamma) = \begin{cases} 0 & \text{for } \phi < \phi_J \\ a\Delta\phi^{\mu}\gamma^{\alpha-2} & \text{for } \phi > \phi_J \end{cases}$$
$$G_{nl}(\Delta\phi, \gamma) = \begin{cases} 0 & \text{for } \phi < \phi_J \\ b\Delta\phi^{\nu}\gamma^{\beta-1} & \text{for } \phi > \phi_J \end{cases}$$

- $\mu = 1.7$ $\alpha = 1.0 < 2$
- $\nu = 1.0$ $\beta = 0.4 < 1$

=> softening at small γ !

$\gamma_c \sim \Delta\phi^{\zeta}$, ζ=1.7

Softening regime in the (ϕ, γ) plane



- We recover the softening regime observed by Hayakawa et al.
- The coupling to dilatancy is a key ingredient role as anticipated

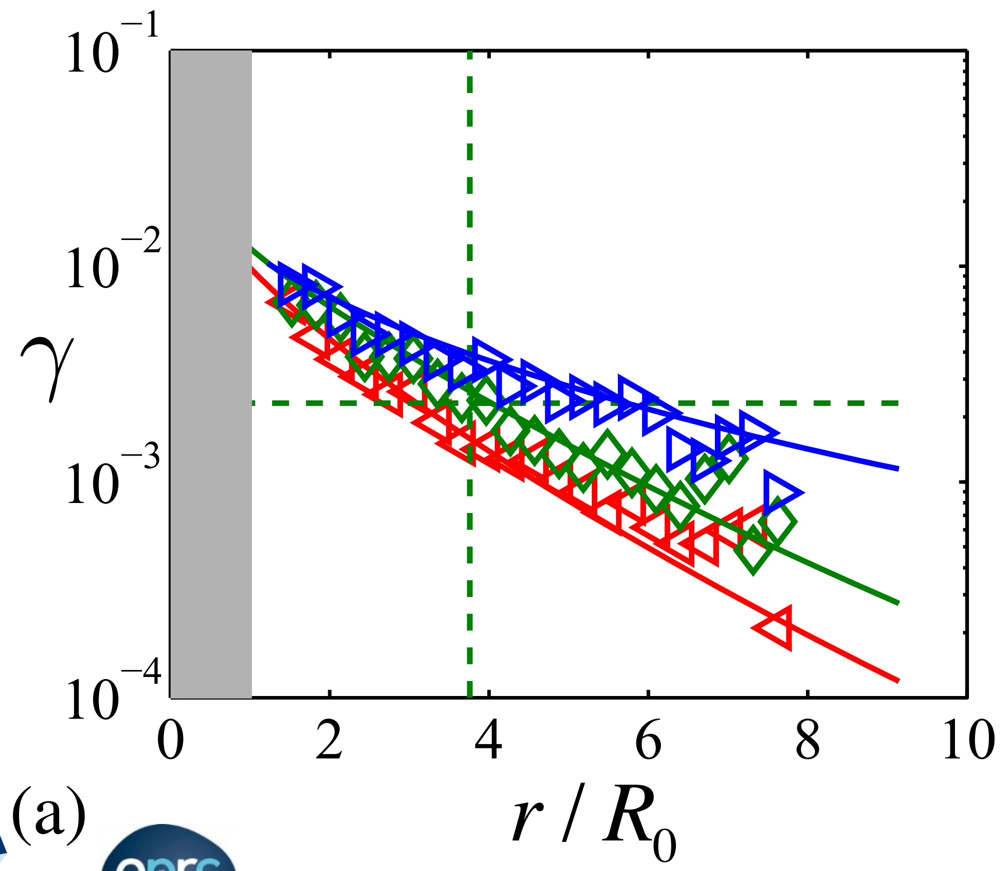
Tighe B. Granular Matter (2014) 16:203–208

- What is captured here is the transition to a saturated linear regime

  => different from linear elasticity

Radial profiles

- From $div\left(\underline{\underline{\sigma}}\right)=0$ and the NL constitutive equation,
- Assuming azimuthal invariance,
- = > radial strain profiles γ(r), to be compared with direct observation



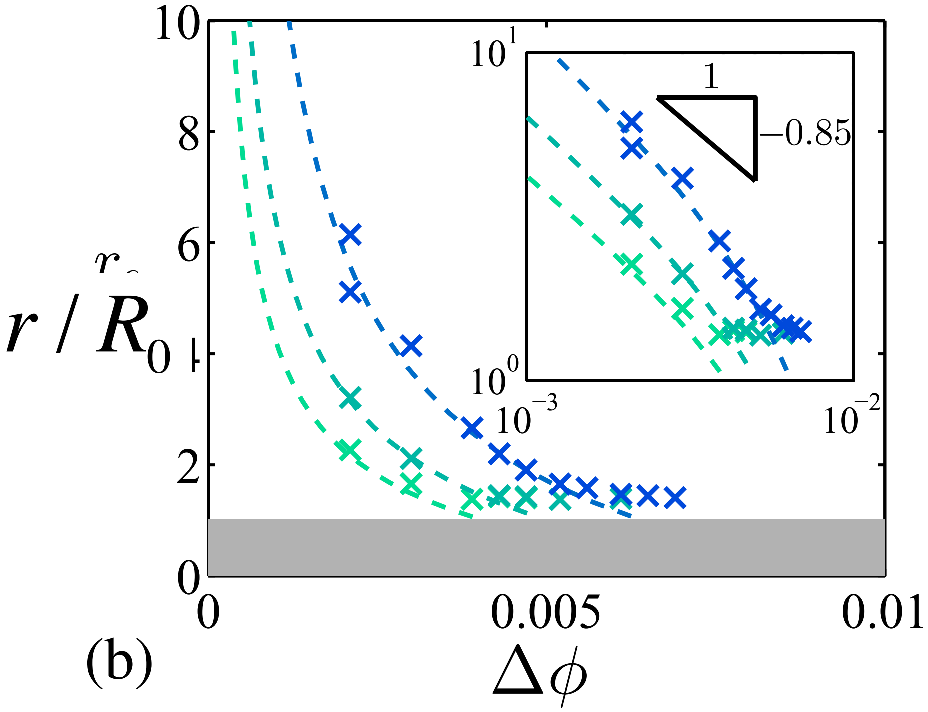
$\gamma > \gamma_c \Leftrightarrow r < r_c$

saturated linear regime

$\gamma < \gamma_c \Leftrightarrow r > r_c$

genuine non linear regime

Critical length diverging at jamming



$$\frac{r_c}{R_0} = \left(\frac{a^*}{\gamma_c}\right)^{1/2} \exp\left[\frac{R_0}{2G_0} a^* \left(1 - \frac{\gamma_c}{a^*}\right)\right].$$

$$\boxed{r_c \sim \gamma_c^{-1/2} \sim \Delta\phi^{-0.85}}$$

- Remember the 2 lengths : $l^* \sim 1/\delta z \sim 1/\delta\phi^{1/2}$ and $l_c \sim 1/\delta z^{0.5} \sim 1/\delta\phi^{1/4}$

  => none of these length scale; also here potential dependent

- Close to jamming $r_c > l^* > l_c$

Conclusion

- Probing experimentally the mechanical response to a point shear perturbation, we have shown that
  - We could not access linear response in the range of ϕ and strain explored here
  - It saturates to an affective linear regime at large strain
  - The critical strain scales like $\delta\phi^{\mu}$, where μ characterizes the grains stiffness.
  - In the non linear regime both shear modulus and dilatancy soften like strain$^{1/2}$
  - The critical distance from intruder length-scale above which non-linear effects can be seen diverges like $\delta\phi^{-\mu/2}$
- Thank you!

Further readings :

- **Soft Matter, 2014, 10, 1519**
- **PRL, 2014, 113 198001**