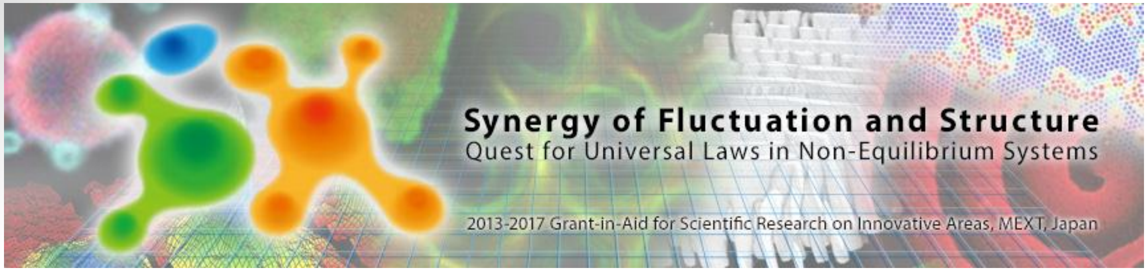


Replica Symmetry Breaking in trajectories of a driven Brownian particle

Shin-ichi Sasa and Masahiko Ueda

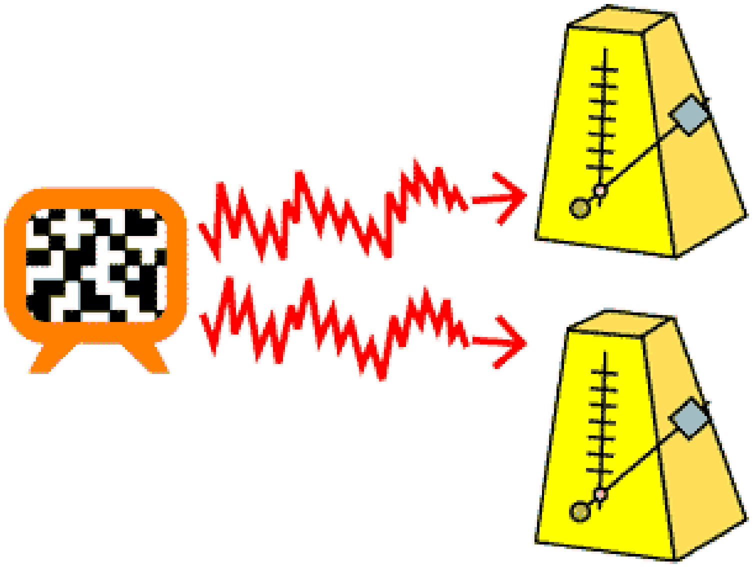
2015/08/13@Kyoto

PART I

Motivation

Common-noise induced synchronization

Teramae-Tanaka, PRL, 2004



http://www-waka.ics.es.osaka-u.ac.jp/~teramae/nis.html

What is this ?

Attraction of two independent trajectories by the common noise

Consider a limit-cycle oscillator + noise

and its Replica (with a slightly different initial condition)

Dynamical system theory:

Nagative Lyapunov exponent (separation ratio of nearby trajectories)

Statistical mechanics of trajectories:

common noise = quenched disorder

One stable (dominant) trajectory (=configuration in time)

No-independent noise ⇔ T=0

Finite temperature T >0

T >0 ⇔ Adding independent noises

- **no** synchronization
- **power-law** behavior in T →0

(Teramae-Tanaka, PRL, 2004:
Nakao, Arai,Kawamura,PRL, 2007)

We ask a possibility of the two extensions

common noise = quenched disorder

T=0

one dominant (irregular) trajectory (=configuration in time)

Several dominant (irregular) trajectories (=configuration in time)

Clustering in the trajectory space!

T>0

fragile of "T=0" behavior against adding independent noises

Robust of "T=0" behavior against adding independent noises

This is

Replica **S**ymmetry **B**reaking
in trajectories

Question

Is it possible ?

In this talk,

We study a simple model :

a Brownian particle driven by a KPZ field

We provide evidence

of RSB in trajectories of the single particle

See Ueda-Sasa, arXiv:1411.1816

to appear in Phys. Rev. Lett. soon

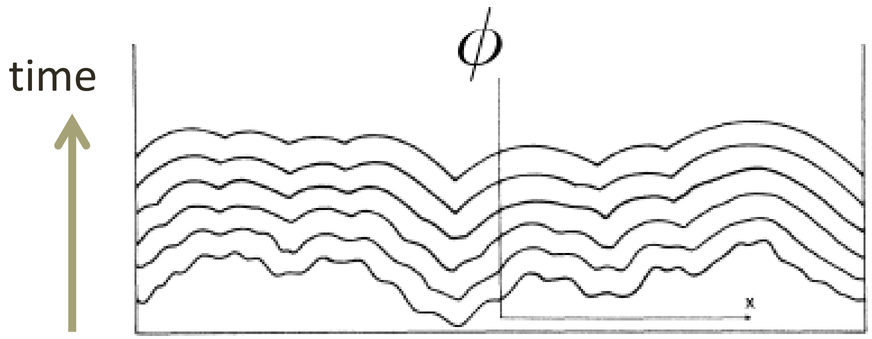
PART II

MODEL and OBSERVATION

Particle driven by a KPZ field

$$\frac{\partial \phi}{\partial t} = \nu \frac{\partial^2 \phi}{\partial x^2} + \frac{1}{2}\left(\frac{\partial \phi}{\partial x}\right)^2 + v(x,t)$$

$$\langle v(x,t) v(x',t') \rangle = 2B\delta(x-x')\delta(t-t')$$

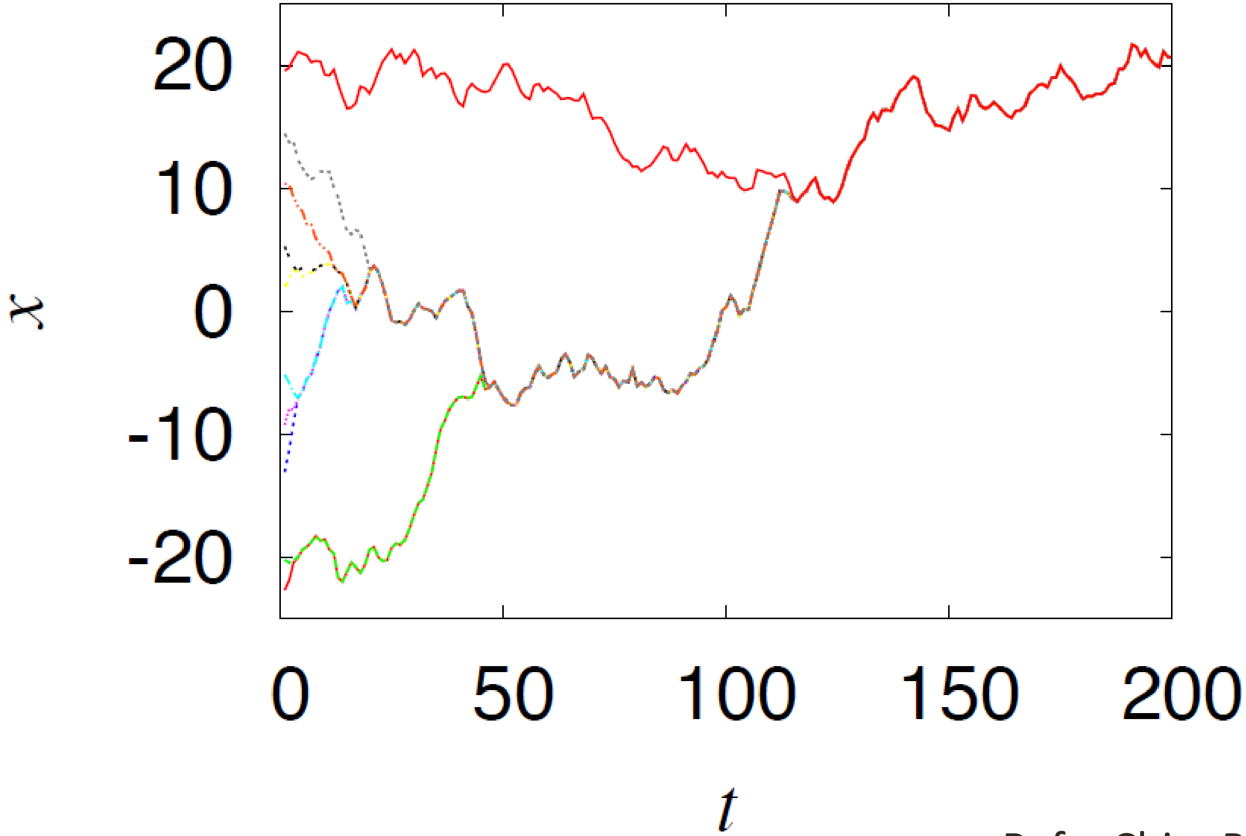


$$u(x,t) = -\frac{\partial \phi}{\partial x}(x,t)$$

$$\dot{x}(t) = u(x(t),t)$$

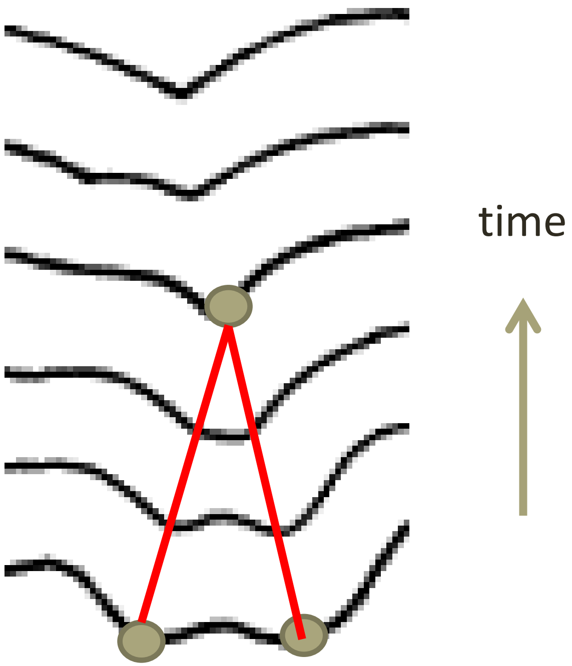
Ref: Chin, Phys. Rev. E, 2002

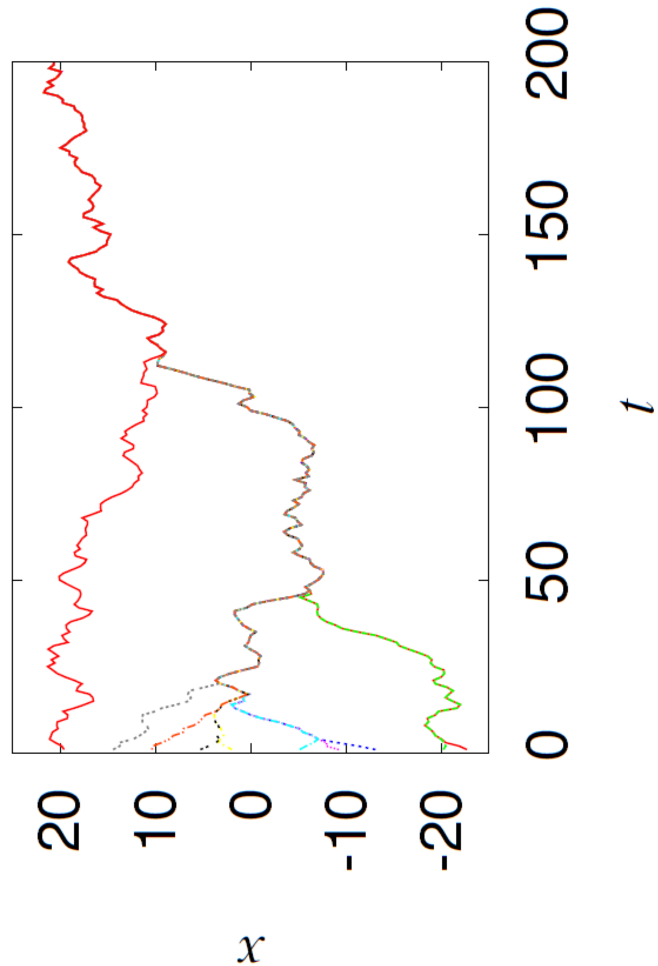
Ten independent particles driven by the KPZ field



Ref: Chin, Phys. Rev. E, 2002

Mechanism





Adding independent noises

$$\frac{\partial \phi}{\partial t} = \nu \frac{\partial^2 \phi}{\partial x^2} + \frac{1}{2}\left(\frac{\partial \phi}{\partial x}\right)^2 + v(x,t)$$

$$\langle v(x,t) v(x',t') \rangle = 2B\delta(x-x')\delta(t-t')$$

$$u(x,t) = -\frac{\partial \phi}{\partial x}(x,t)$$

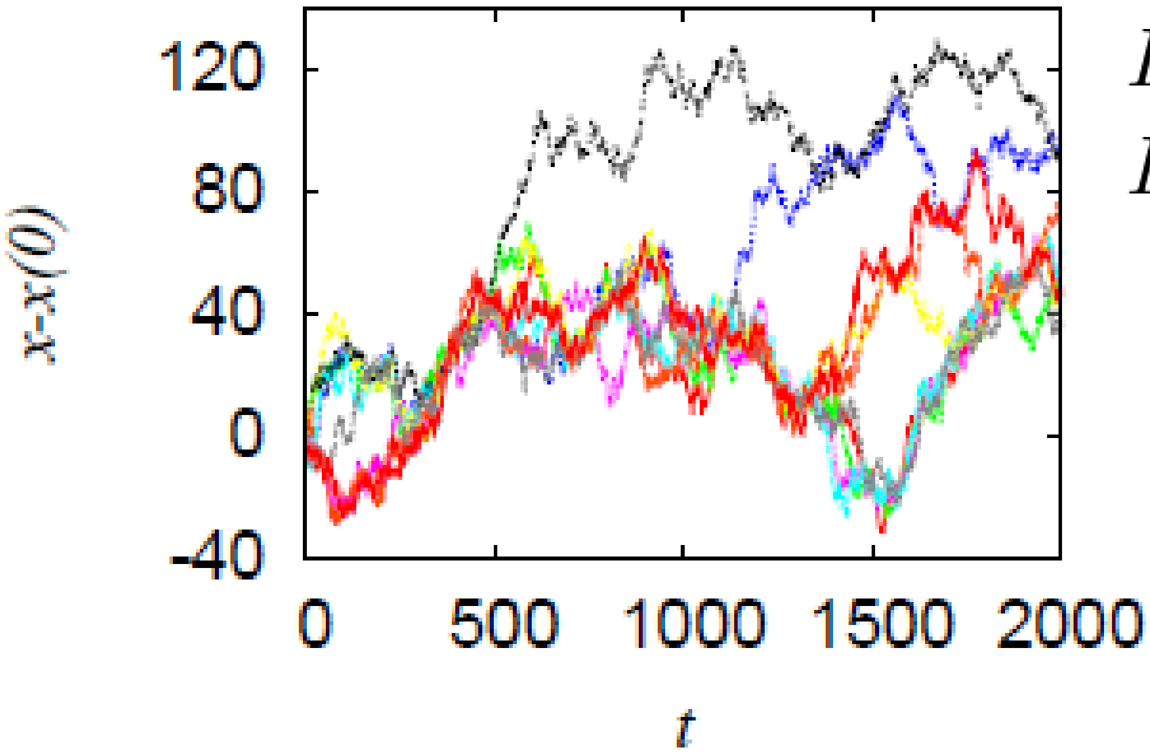
$$\dot{x}(t) = u(x(t),t) + \underline{\xi(t)}$$

$$\underline{\langle \xi(t)\xi(t') \rangle = 2D\delta(t-t')}$$

$$D = \nu$$

Ref: Drossel and Karder, Phys. Rev. B, 2002

Ten trajectories from the same initial position



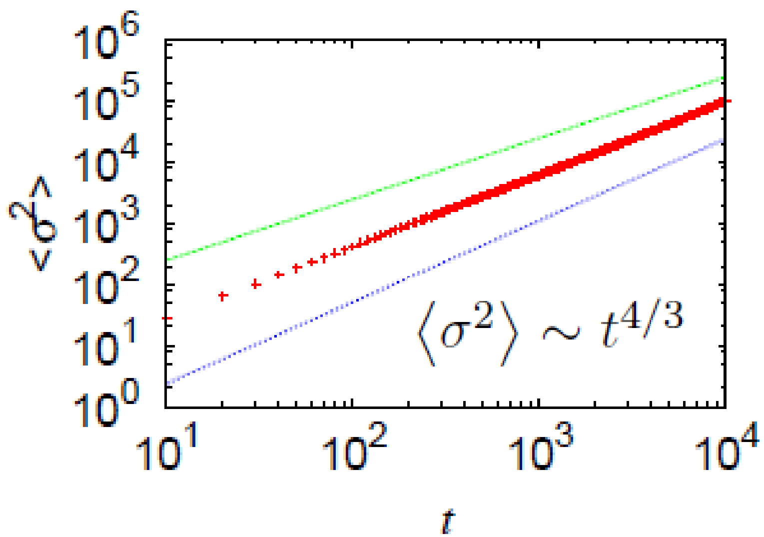
$D = \nu = 1$

$B = 2.5$

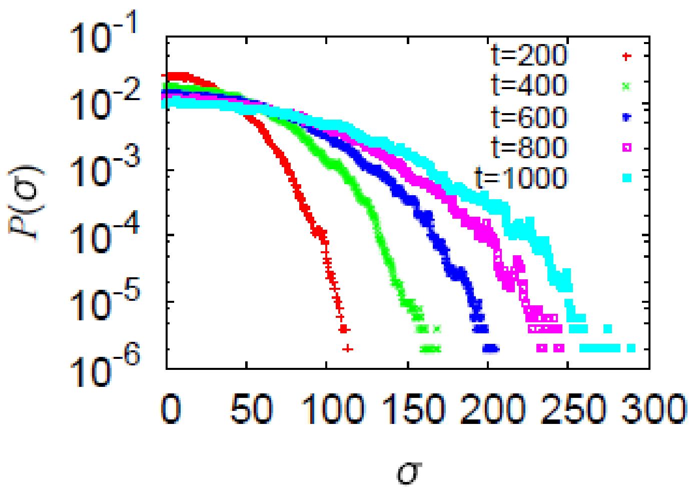
Single particle diffusion

Displacement $\sigma \equiv |x(t) - x(0)|$

Mean-squared displacement



Distribution of displacement



Super-diffusion

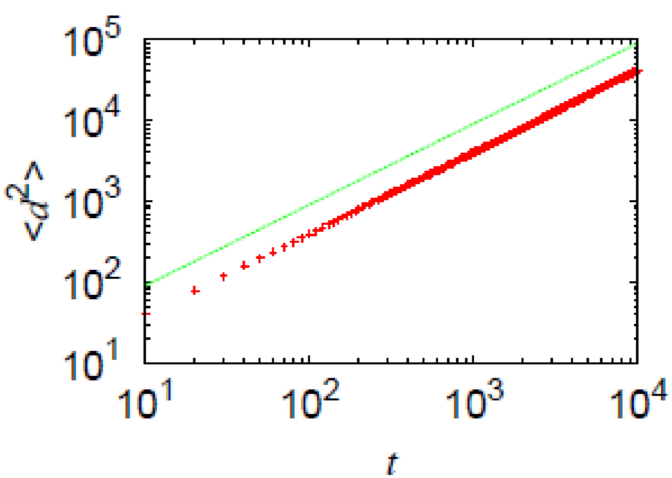
Gaussian

Ref: Drossel and Karder, Phys. Rev. B, 2002

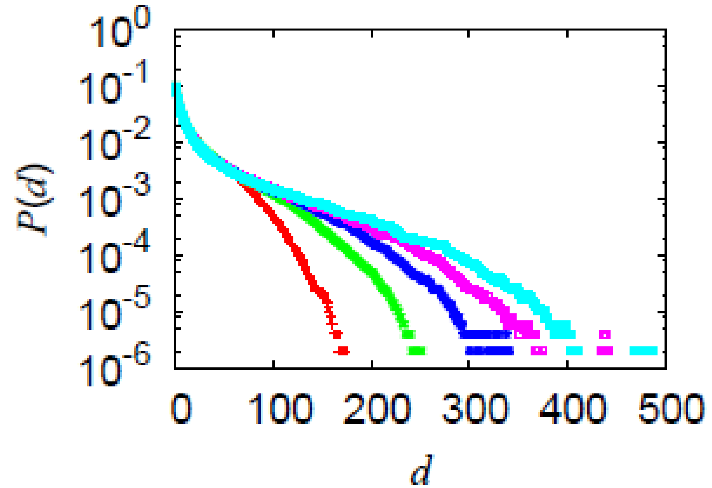
Relative diffusion

Relative distance $d \equiv \left| x^{(1)} - x^{(2)} \right|$

Mean-squared relative distance



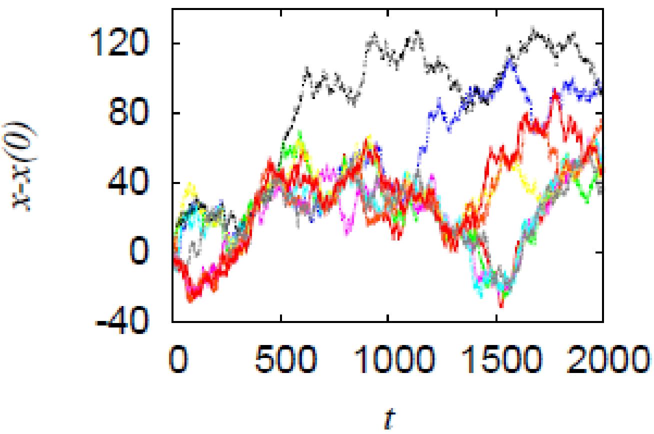
Distribution of relative distance



normal-diffusion

Non-Gaussian

How to detect RSB



overlap

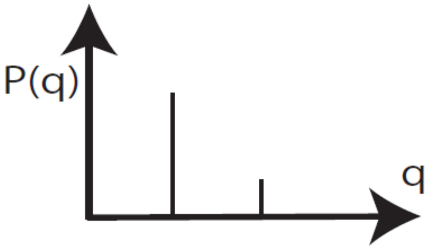
$$q \equiv \frac{1}{M}\sum_{j=1}^{M}\theta\left(\ell - \left|x^{(1)}(j\Delta t) - x^{(2)}(j\Delta t)\right|\right)$$

$\ell = 5$ Localization length

Mean field spin glass model
(e.g. 3-SK model)

$$P(q) \equiv \mathbb{E}\left[\frac{1}{Z^2}\sum_{\boldsymbol{\sigma},\boldsymbol{\sigma}'} e^{-\beta H(\boldsymbol{\sigma})-\beta H(\boldsymbol{\sigma}')}\delta\left(q - q_{\boldsymbol{\sigma},\boldsymbol{\sigma}'}\right)\right]$$

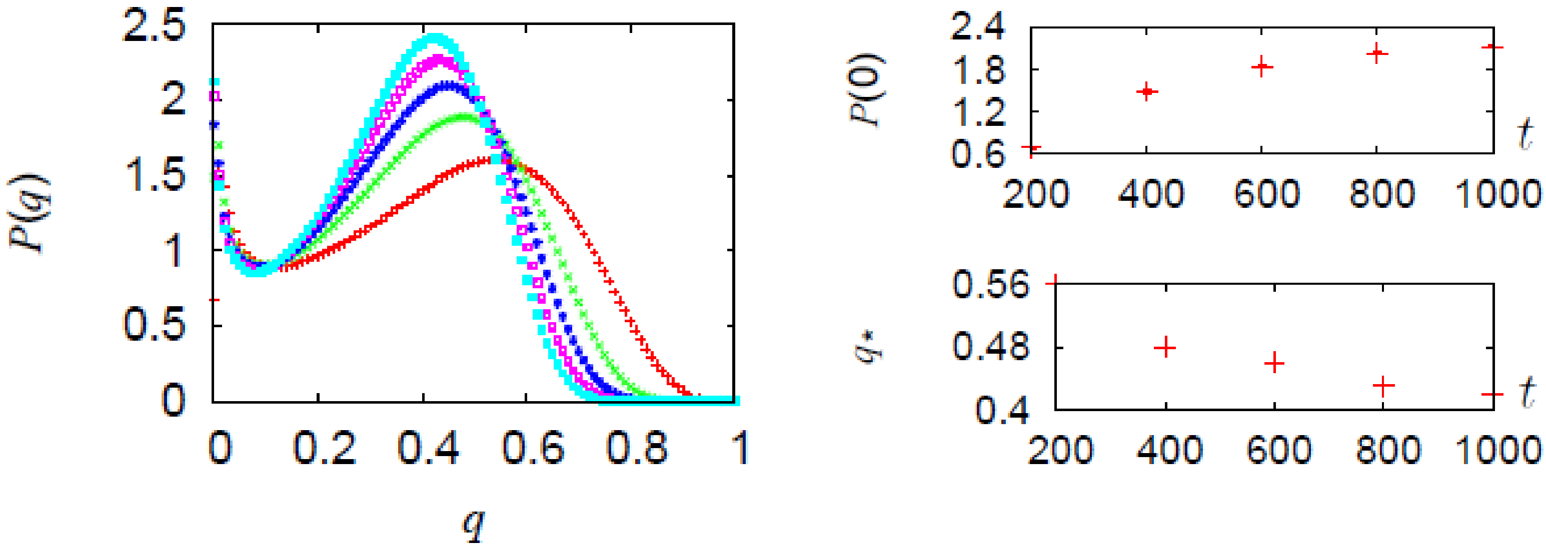
$$q_{\boldsymbol{\sigma},\boldsymbol{\sigma}'} \equiv \frac{1}{N}\sum_{i=1}^{N}\sigma_i\sigma_i'$$



(1-RSB)

P(q) for trajectories

$$q \equiv \frac{1}{M}\sum_{j=1}^{M}\theta\left(\ell - \left|x^{(1)}(j\Delta t) - x^{(2)}(j\Delta t)\right|\right)$$



Clear two peaks!

1-RSB ?

Comments

This may be a numerical artifact.....

This may come from

a finite size effect.....

PART III

Theoretical analysis

Statistical mechanics of trajectories

Probability measure of trajectory x

$$\mathcal{P}[x|x(0)=x_0,\phi]=\frac{1}{Z_0}e^{-\frac{1}{4D}\int_0^\tau dt\left[\dot{x}(t)+\frac{\partial\phi}{\partial x}(x(t),t)\right]^2+\frac{1}{2}\int_0^\tau dt\,\frac{\partial^2\phi}{\partial x^2}(x(t),t)}$$

$A(x)$ Quantity depending on trajectory

$$\langle A\rangle_\phi \equiv \int \mathcal{D}x\mathcal{P}(x|x_0,\phi)A(x)$$

average over trajectories

$$\langle A\rangle = \int \mathcal{D}\phi\mathcal{P}(\phi)\left\langle A\right\rangle_\phi$$

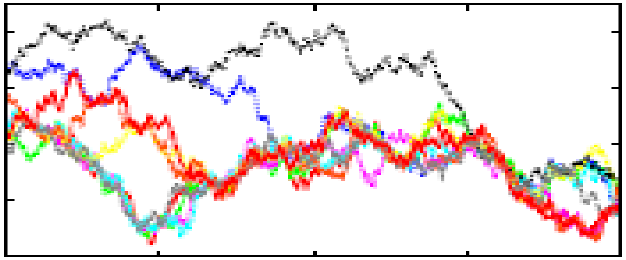
average over the velocity field

Trick –I

$$\mathcal{P}[x|x(0)=x_0,\phi] = \frac{1}{Z_0} e^{-\frac{1}{4D}\int_0^\tau dt\left[\dot{x}(t)+\frac{\partial\phi}{\partial x}(x(t),t)\right]^2 + \frac{1}{2}\int_0^\tau dt\,\frac{\partial^2\phi}{\partial x^2}(x(t),t)}$$

Modification

$$\mathcal{P}[x|x(\tau)=x_0,\phi] = \frac{1}{Z_0} e^{-\frac{1}{4D}\int_0^\tau dt\left[\dot{x}(t)+\frac{\partial\phi}{\partial x}(x(t),t)\right]^2 - \frac{1}{2}\int_0^\tau dt\,\frac{\partial^2\phi}{\partial x^2}(x(t),t)}$$



$$t=\tau$$

Trick-II

$$\mathcal{P}\left[x|x(\tau)=x_0,\phi\right]=\frac{1}{Z_0}e^{-\frac{1}{4D}\int_0^\tau dt\left[\dot{x}(t)^2+2\frac{d}{dt}\phi(x(t),t)-2\frac{\partial\phi}{\partial t}(x(t),t)\right]}$$

$$\times e^{-\frac{1}{4D}\int_0^\tau dt\left\{\left[\frac{\partial\phi}{\partial x}(x(t),t)\right]^2+2D\frac{\partial^2\phi}{\partial x^2}(x(t),t)\right\}}$$

$$=\frac{1}{Z_0}e^{-\frac{1}{2D}[\phi(x(\tau),\tau)-\phi(x(0),0)]-\frac{1}{4D}\int_0^\tau dt\left[\dot{x}(t)^2-2v(x(t),t)\right]}$$

$$=\frac{1}{Z_0}e^{-\frac{1}{2D}[\phi(x_0,\tau)-\mathrm{const.}]-\frac{1}{4D}\int_0^\tau dt\left[\dot{x}(t)^2-2v(x(t),t)\right]}$$

Trick-II

$$\mathcal{P}\left[x|x(\tau)=x_0,\phi\right]=\frac{1}{Z_0}e^{-\frac{1}{4D}\int_0^\tau dt\left[\dot{x}(t)^2+2\frac{d}{dt}\phi(x(t),t)-2\frac{\partial\phi}{\partial t}(x(t),t)\right]}$$

$$\times e^{-\frac{1}{4D}\int_0^\tau dt\left\{\left[\frac{\partial\phi}{\partial x}(x(t),t)\right]^2+2D\frac{\partial^2\phi}{\partial x^2}(x(t),t)\right\}}$$

$$=\frac{1}{Z_0}e^{-\frac{1}{2D}[\phi(x(\tau),\tau)-\phi(x(0),0)]-\frac{1}{4D}\int_0^\tau dt\left[\dot{x}(t)^2-2v(x(t),t)\right]}$$

$$=\frac{1}{Z_0}e^{-\frac{1}{2D}[\phi(x_0,\tau)-\mathrm{const.}]-\frac{1}{4D}\int_0^\tau dt\left[\dot{x}(t)^2-2v(x(t),t)\right]}$$

$$\boxed{\text{Equilibrium Statistical Mechanics of Directed Polymer in RP!}}$$

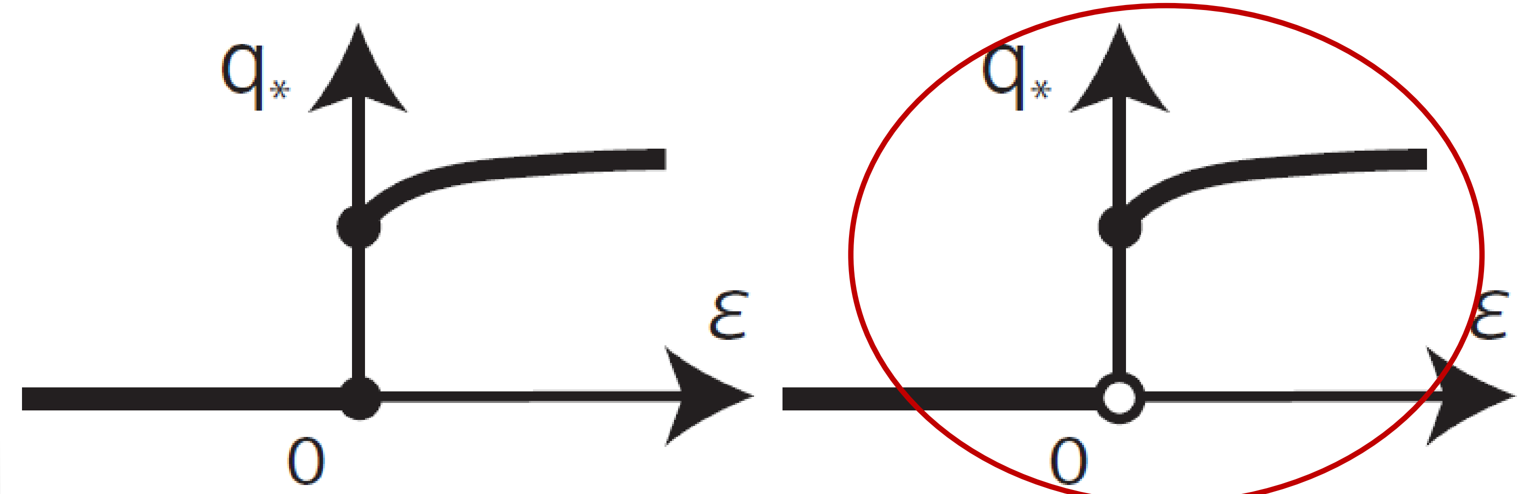
A Study on DP in RP

G. Parisi, J. Phys. France 51 1595-1606 (1990)

On the replica approach to random directed polymers in two dimensions

$$\psi(\epsilon) \equiv \lim_{\tau\to\infty} \tau^{-1} \log \langle e^{\tau\epsilon q}\rangle$$ c-generating function of overlap

$$\langle q\rangle_\epsilon \equiv \frac{\partial}{\partial\epsilon}\psi(\epsilon)$$ Expected value in the biased ensemble



1-RSB weakly RSB

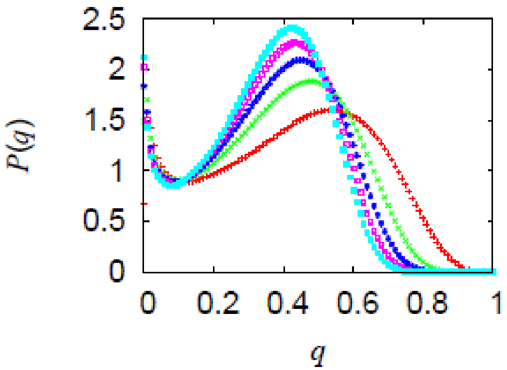
Numerical Study on DP in RP

M. Mezard, J. Phys. France 51 1831-1846 (1990)

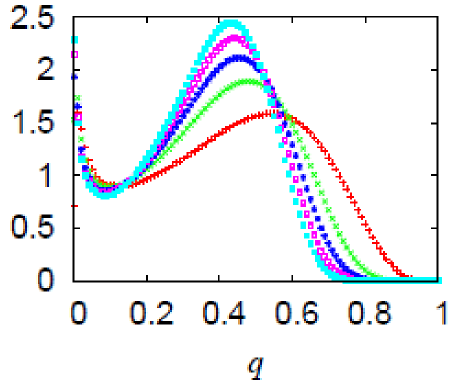
On the glassy nature of random directed polymers in two dimensions

It supported Parisi's result , but for the discretized model

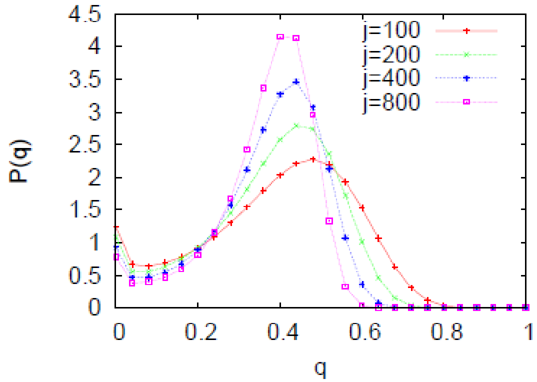
We carefully studied



BP by the KPZ
1-RSB ?



Directed polymer
1-RSB ?



Discretized directed polymer:
Weakly RSB
consistent with Parisi-Mezard

Tentative conclusion
- conjecture -

Equilibrium ensemble of directed polymer
is equivalent to
the path ensemble of a Brownian particle by the KPZ

The independence of the boundary condition (in the time axis)

Equilibrium ensemble of directed polymer
exhibits 1-RSB,
while its discretized version exhibits weakly RSB

PART IV

Concluding Remarks

Summary of my talk

We have studied a simple model :

a Brownian particle driven by a KPZ field

We have provided evidence

of RSB in trajectories of the single particle

See Ueda-Sasa, arXiv:1411.1816

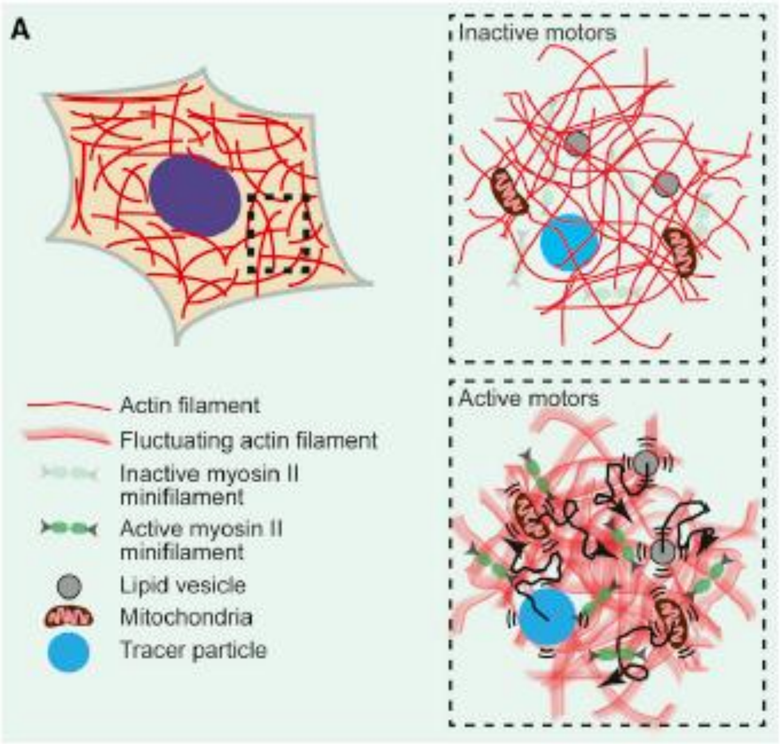
to appear in Phys. Rev. Lett. soon

Next obvious questions

Other examples?

Experiments?

Diffusion
in active environments



Cell 158 822-832 (2014)
Probing the Stochastic, Motor-Driven Properties of the Cytoplasm Using Force Spectrum Microscopy, M Guo et al

Statistical mechanics of trajectories

Dynamical free energy

= cumulant generating function for a quantity X

X : activity, current, Lyapunov exponent, or overlap

Non-analytic (singular) behavior of the dynamical free energy

= one class of non-equilibrium phase transitions

Classification?

The concept of **universality class** ?

Last message

The concept of RSB:

born in spin glass problems

Application to molecular glasses, jamming..

Application to information science

Application to **time-series analysis of fluctuation!**