Replica Symmetry Breaking in trajectories of a driven Brownian particle

Shin-ichi Sasa and Masahiko Ueda

2015/08/13@Kyoto
PART I

Motivation
Common-noise induced synchronization

Teramae-Tanaka, PRL, 2004

http://www-waka.ics.es.osaka-u.ac.jp/~teramae/nis.html
What is this?

Attraction of two independent trajectories by the common noise

Consider a limit-cycle oscillator + noise and its Replica (with a slightly different initial condition)

Dynamical system theory:
Negative Lyapunov exponent (separation ratio of nearby trajectories)

Statistical mechanics of trajectories:
common noise = quenched disorder
One stable (dominant) trajectory (=configuration in time)
No-independent noise $\iff T=0$
Finite temperature $T > 0$

$T > 0 \iff$ Adding independent noises
- no synchronization
- power-law behavior in $T \to 0$

(Teramae-Tanaka, PRL, 2004:
We ask a possibility of the two extensions

common noise = quenched disorder

\( T=0 \)
- one dominant (irregular) trajectory (=configuration in time)

\( T>0 \)
- Robust of “\( T=0 \)” behavior against adding independent noises

\( T>0 \)
- Several dominant (irregular) trajectories (=configuration in time)

Clustering in the trajectory space!
This is

Replica Symmetry Breaking in trajectories
Question

Is it possible?
In this talk,

We study a simple model:

a Brownian particle driven by a KPZ field

We provide evidence of RSB in trajectories of the single particle

PART II
MODEL and OBSERVATION
Particle driven by a KPZ field

\[
\frac{\partial \phi}{\partial t} = \nu \frac{\partial^2 \phi}{\partial x^2} + \frac{1}{2} \left( \frac{\partial \phi}{\partial x} \right)^2 + v(x, t)
\]

\[
\langle v(x, t)v(x', t') \rangle = 2B\delta(x - x')\delta(t - t')
\]

\[
u(x, t) = -\frac{\partial \phi}{\partial x}(x, t)
\]

\[
\dot{x}(t) = u(x(t), t)
\]

Ref: Chin, Phys. Rev. E, 2002
Ten independent particles driven by the KPZ field

Ref: Chin, Phys. Rev. E, 2002
Mechanism
Adding independent noises

\[ \frac{\partial \phi}{\partial t} = \nu \frac{\partial^2 \phi}{\partial x^2} + \frac{1}{2} \left( \frac{\partial \phi}{\partial x} \right)^2 + v(x, t) \]

\[ \langle v(x, t)v(x', t') \rangle = 2B \delta(x - x') \delta(t - t') \]

\[ u(x, t) = -\frac{\partial \phi}{\partial x}(x, t) \]

\[ \dot{x}(t) = u(x(t), t) + \xi(t) \]

\[ \langle \xi(t)\xi(t') \rangle = 2D \delta(t - t') \]

\[ D = \nu \]

Ten trajectories from the same initial position

\[ D = \nu = 1 \]
\[ B = 2.5 \]
Single particle diffusion

Displacement

\[ \sigma \equiv |x(t) - x(0)| \]

Mean-squared displacement

\[ \langle \sigma^2 \rangle \sim t^{4/3} \]

Distribution of displacement

Super-diffusion

Gaussian

Relative diffusion

Relative distance

\[ d \equiv |x^{(1)} - x^{(2)}| \]

Mean-squared relative distance

Distribution of relative distance

normal-diffusion

Non-Gaussian
How to detect RSB

overlap

\[ q \equiv \frac{1}{M} \sum_{j=1}^{M} \theta \left( \ell - \left| x^{(1)}(j\Delta t) - x^{(2)}(j\Delta t) \right| \right) \]

\[ \ell = 5 \quad \text{Localization length} \]

Mean field spin glass model
(e.g. 3-SK model)

\[ P(q) \equiv \mathbb{E} \left[ \frac{1}{Z^2} \sum_{\sigma,\sigma'} e^{-\beta H(\sigma) - \beta H(\sigma')} \delta \left( q - q_{\sigma,\sigma'} \right) \right] \]

\[ q_{\sigma,\sigma'} \equiv \frac{1}{N} \sum_{i=1}^{N} \sigma_i \sigma_i' \]

(1-RSB)
P(q) for trajectories

\[ q \equiv \frac{1}{M} \sum_{j=1}^{M} \theta \left( \ell - \left| x^{(1)}(j\Delta t) - x^{(2)}(j\Delta t) \right| \right) \]

Clear two peaks! 1-RSB?
Comments

This may be a numerical artifact.....

This may come from a finite size effect.....
PART III

Theoretical analysis
Statistical mechanics of trajectories

Probability measure of trajectory $\mathcal{X}$

$$\mathcal{P}[x|x(0) = x_0, \phi] = \frac{1}{Z_0} e^{-\frac{1}{4D} \int_0^\tau dt \left( \dot{x}(t) + \frac{\partial \phi}{\partial x}(x(t), t) \right)^2 + \frac{1}{2} \int_0^\tau dt \frac{\partial^2 \phi}{\partial x^2}(x(t), t)}$$

$A(x)$ Quantity depending on trajectory

$$\langle A \rangle_\phi \equiv \int \mathcal{D}x \mathcal{P}(x|x_0, \phi) A(x)$$

average over trajectories

$$\langle A \rangle = \int \mathcal{D}\phi \mathcal{P}(\phi) \langle A \rangle_\phi$$

average over the velocity field
Trick – I

\[ \mathcal{P}[x|x(0) = x_0, \phi] = \frac{1}{Z_0} e^{-\frac{1}{4D} \int_0^\tau dt \left[ \dot{x}(t) + \frac{\partial \phi}{\partial x}(x(t),t) \right]^2} + \frac{1}{2} \int_0^\tau dt \frac{\partial^2 \phi}{\partial x^2}(x(t),t) \]

Modification

\[ \mathcal{P}[x|x(\tau) = x_0, \phi] = \frac{1}{Z_0} e^{-\frac{1}{4D} \int_0^\tau dt \left[ \dot{x}(t) + \frac{\partial \phi}{\partial x}(x(t),t) \right]^2} - \frac{1}{2} \int_0^\tau dt \frac{\partial^2 \phi}{\partial x^2}(x(t),t) \]

\[ t = \tau \]
Trick-II

\[ P[x|x(\tau) = x_0, \phi] = \frac{1}{Z_0} e^{-\frac{1}{4D} \int_0^\tau dt \left[ \dot{x}(t)^2 + 2 \frac{d}{dt} \phi(x(t),t) - 2 \frac{\partial \phi}{\partial t} (x(t),t) \right]}
\times e^{-\frac{1}{4D} \int_0^\tau dt \left\{ \left[ \frac{\partial \phi}{\partial x} (x(t),t) \right]^2 + 2D \frac{\partial^2 \phi}{\partial x^2} (x(t),t) \right\}} \]

\[ = \frac{1}{Z_0} e^{-\frac{1}{2D} \left[ \phi(x(\tau),\tau) - \phi(x(0),0) \right] - \frac{1}{4D} \int_0^\tau dt \left[ \dot{x}(t)^2 - 2v(x(t),t) \right]} \]

\[ = \frac{1}{Z_0} e^{-\frac{1}{2D} \left[ \phi(x_0,\tau) - \text{const.} \right] - \frac{1}{4D} \int_0^\tau dt \left[ \dot{x}(t)^2 - 2v(x(t),t) \right]} \]
Trick-II

\[ \mathcal{P} [x | x(\tau) = x_0, \phi] = \frac{1}{Z_0} e^{- \frac{1}{4D} \int_0^\tau dt [\dot{x}(t)^2 + 2 \frac{d}{dt} \phi(x(t), t) - 2 \frac{\partial \phi}{\partial t}(x(t), t)]} \]

\[ \times e^{- \frac{1}{4D} \int_0^\tau dt \left\{ \left[ \frac{\partial \phi}{\partial x}(x(t), t) \right]^2 + 2D \frac{\partial^2 \phi}{\partial x^2}(x(t), t) \right\}} \]

\[ = \frac{1}{Z_0} e^{- \frac{1}{2D} [\phi(x(\tau), \tau) - \phi(x(0), 0)] - \frac{1}{4D} \int_0^\tau dt [\dot{x}(t)^2 - 2v(x(t), t)]} \]

\[ = \frac{1}{Z_0} e^{- \frac{1}{2D} [\phi(x_0, \tau) - \text{const.}] - \frac{1}{4D} \int_0^\tau dt [\dot{x}(t)^2 - 2v(x(t), t)]} \]
A Study on DP in RP


On the replica approach to random directed polymers in two dimensions

\[ \psi(\epsilon) \equiv \lim_{\tau \to \infty} \tau^{-1} \log \left< e^{\tau \epsilon q} \right> \]

c-generating function of overlap

\[ \langle q \rangle_\epsilon \equiv \frac{\partial}{\partial \epsilon} \psi(\epsilon) \]

Expected value in the biased ensemble

1-RSB

weakly RSB
Numerical Study on DP in RP


On the glassy nature of random directed polymers in two dimensions

It supported Parisi’s result, but for the discretized model

We carefully studied

BP by the KPZ 1-RSB?

Directed polymer 1-RSB?

Discretized directed polymer: Weakly RSB consistent with Parisi-Mezard
Tentative conclusion
- conjecture -

Equilibrium ensemble of directed polymer is equivalent to the path ensemble of a Brownian particle by the KPZ.

The independence of the boundary condition (in the time axis)

Equilibrium ensemble of directed polymer exhibits 1-RSB, while its discretized version exhibits weakly RSB.
PART IV

Concluding Remarks
We have studied a simple model: a Brownian particle driven by a KPZ field.

We have provided evidence of RSB in trajectories of the single particle.

Next obvious questions

Other examples?

Experiments?
Diffusion in active environments

Cell 158 822-832 (2014)
Probing the Stochastic, Motor-Driven Properties of the Cytoplasm Using Force Spectrum Microscopy, M Guo et al
Statistical mechanics
of trajectories

Dynamical free energy
= cumulant generating function for a quantity $X$
$X$: activity, current, Lyapunov exponent, or overlap

Non-analytic (singular) behavior of the dynamical free energy
= one class of non-equilibrium phase transitions

Classification?
The concept of **universality class**?
Last message

The concept of RSB:

born in spin glass problems

Application to molecular glasses, jamming..

Application to information science

Application to time-series analysis of fluctuation!