Sang Wook Kim
(Pusan N. Univ)

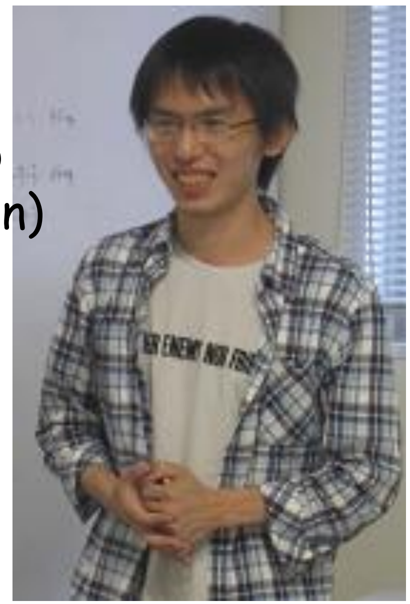
Information

heat engine in quantum world

Kyoto (2015.7.31)

Collaborators

Simone de Liberato
(Univ. of Southampton)



Takahiro Sagawa
(Univ. of Tokyo)

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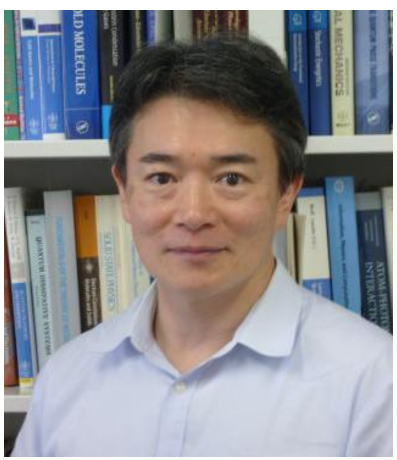
(PNU)

Jung Jun Park
(Singapore Natl. U.)



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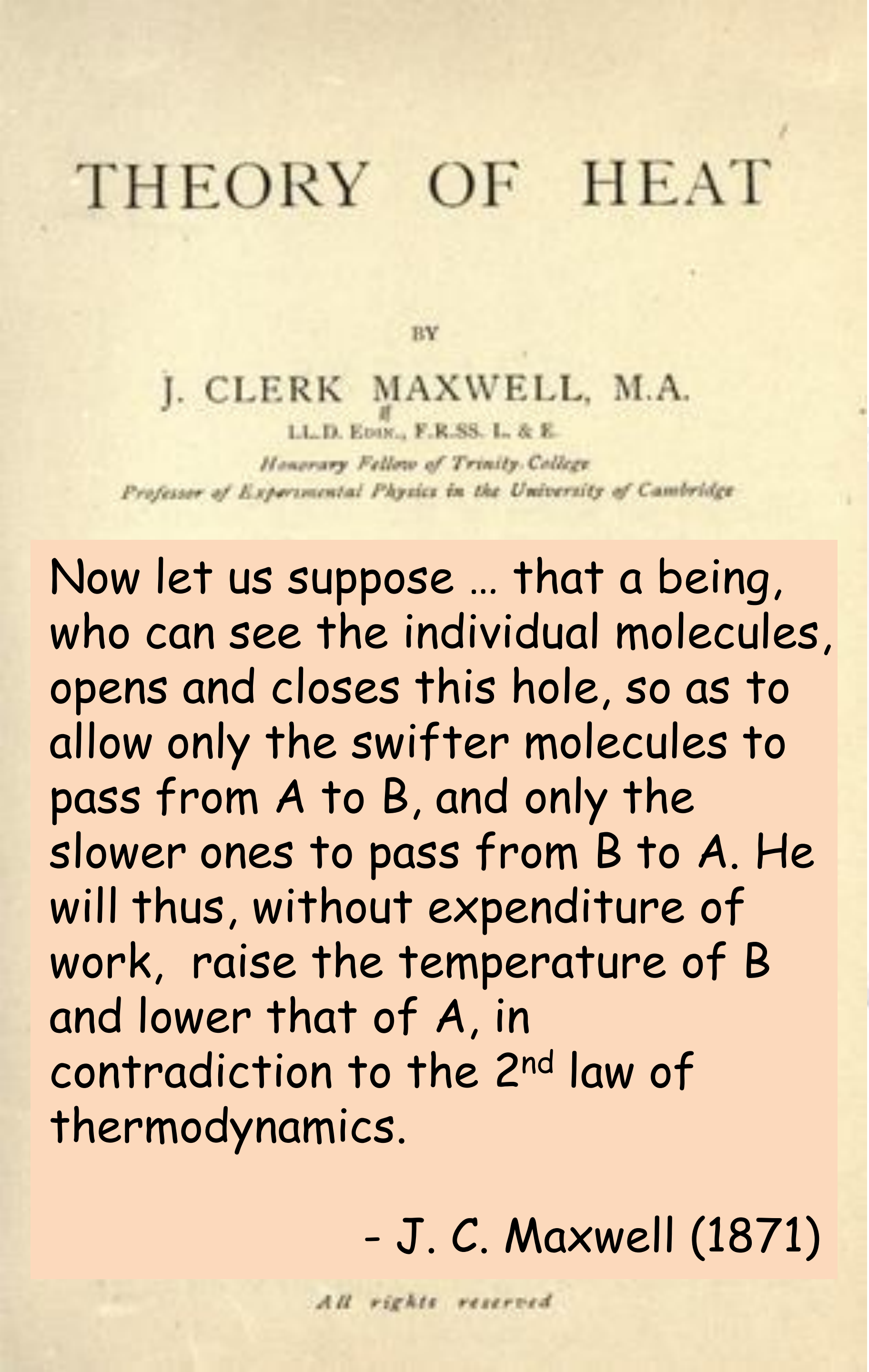
Maxwell's Demon



THEORY OF HEAT

BY

J. CLERK MAXWELL, M.A.

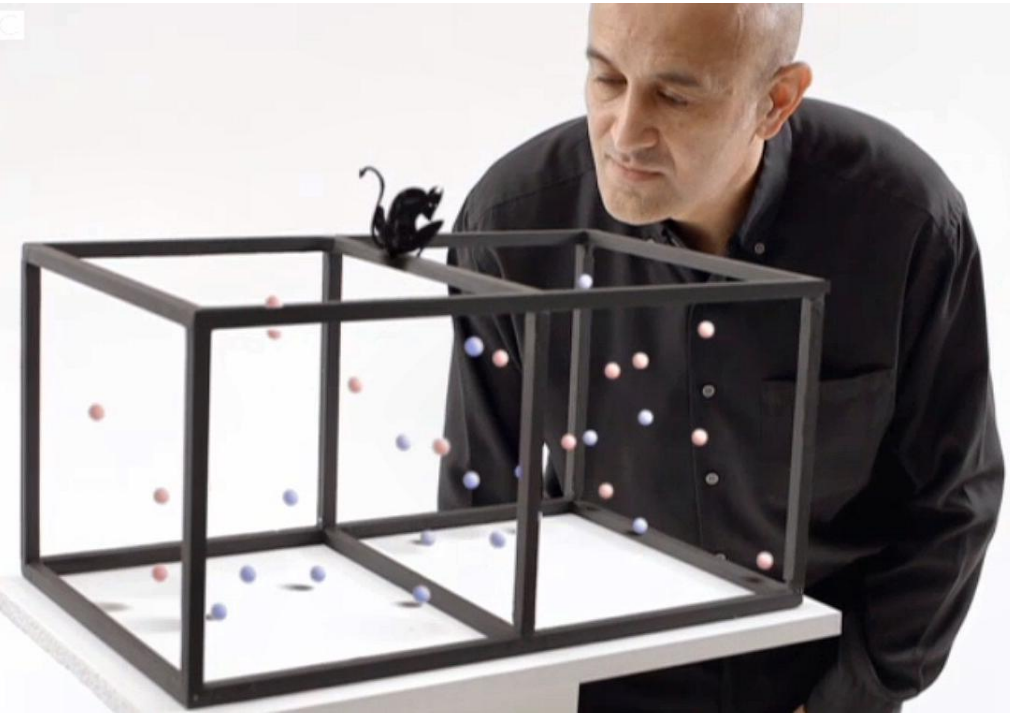
LL.D. EDIN., F.R.SS. L. & E.

Honorary Fellow of Trinity College

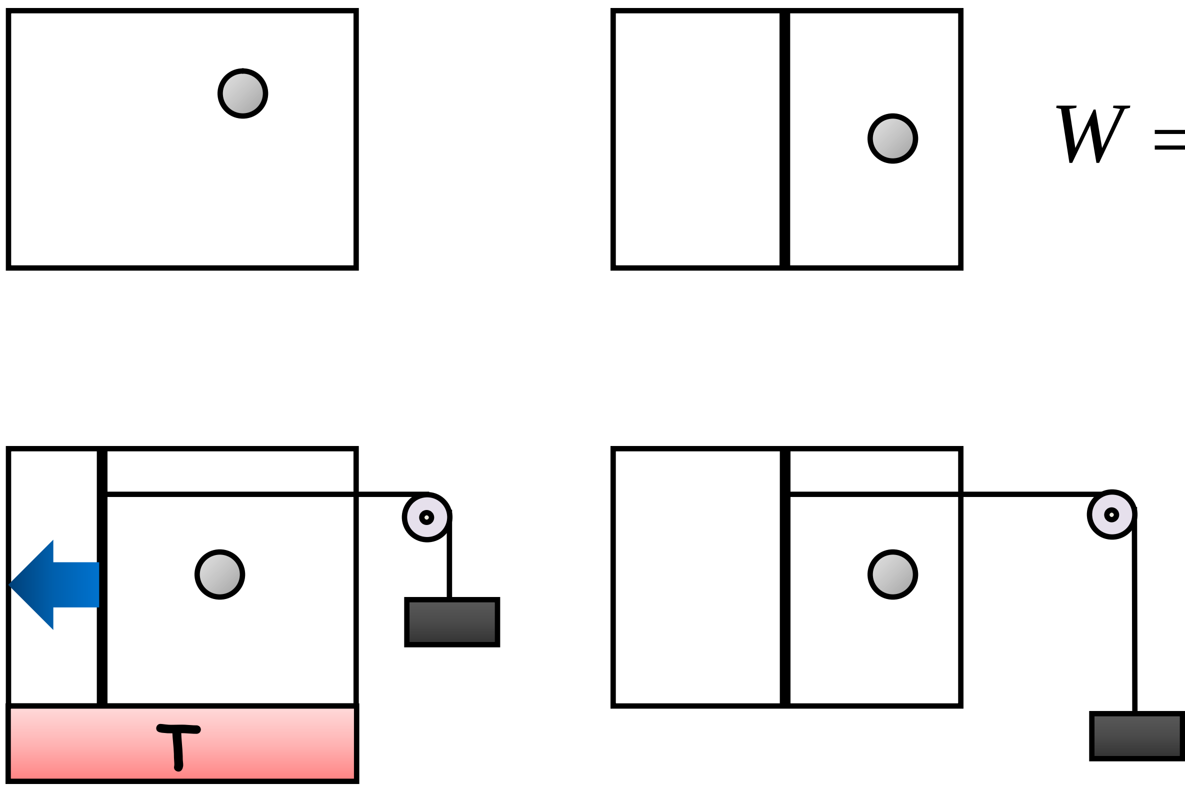
Professor of Experimental Physics in the University of Cambridge

Now let us suppose … that a being, who can see the individual molecules, opens and closes this hole, so as to allow only the swifter molecules to pass from A to B, and only the slower ones to pass from B to A. He will thus, without expenditure of work, raise the temperature of B and lower that of A, in contradiction to the 2nd law of thermodynamics.

- J. C. Maxwell (1871)

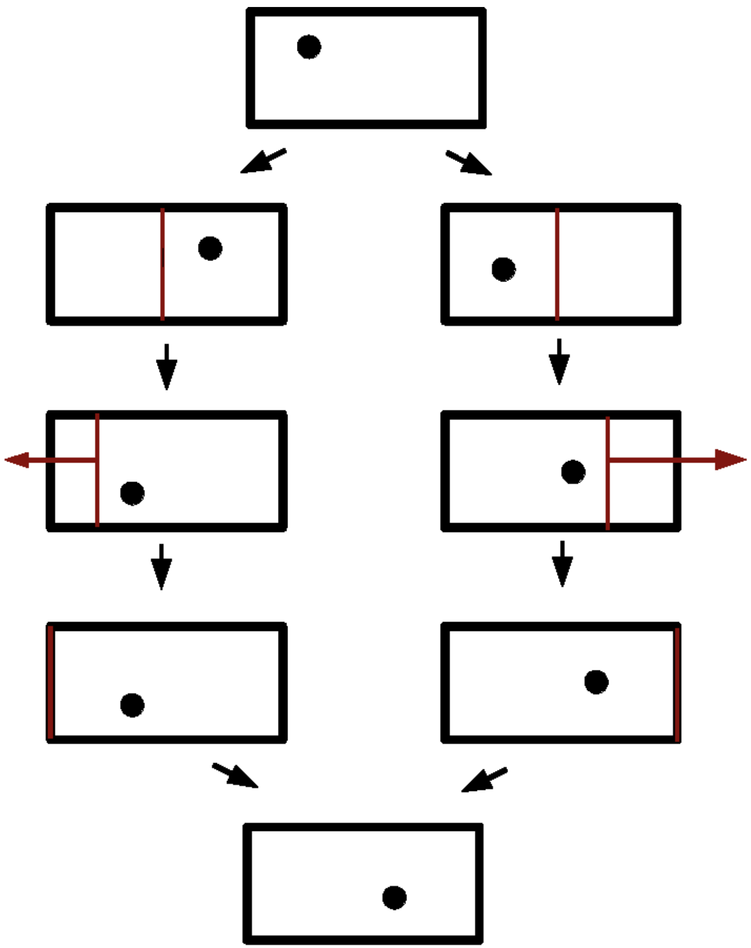


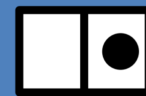
Szilard's engine (1929)



$$W = \int p\,dV$$

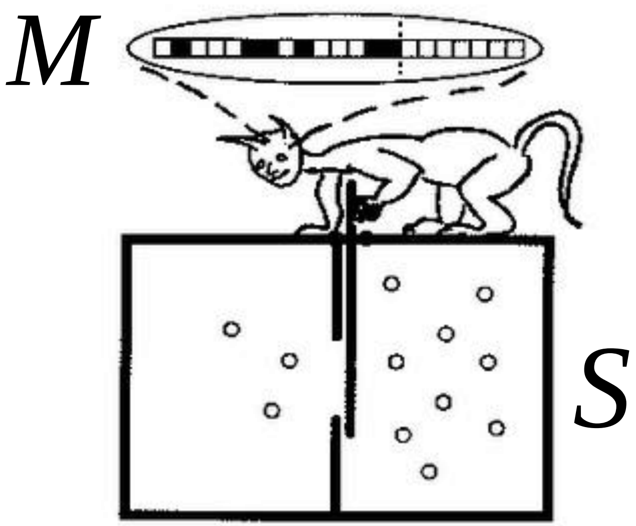
Flow of entropy



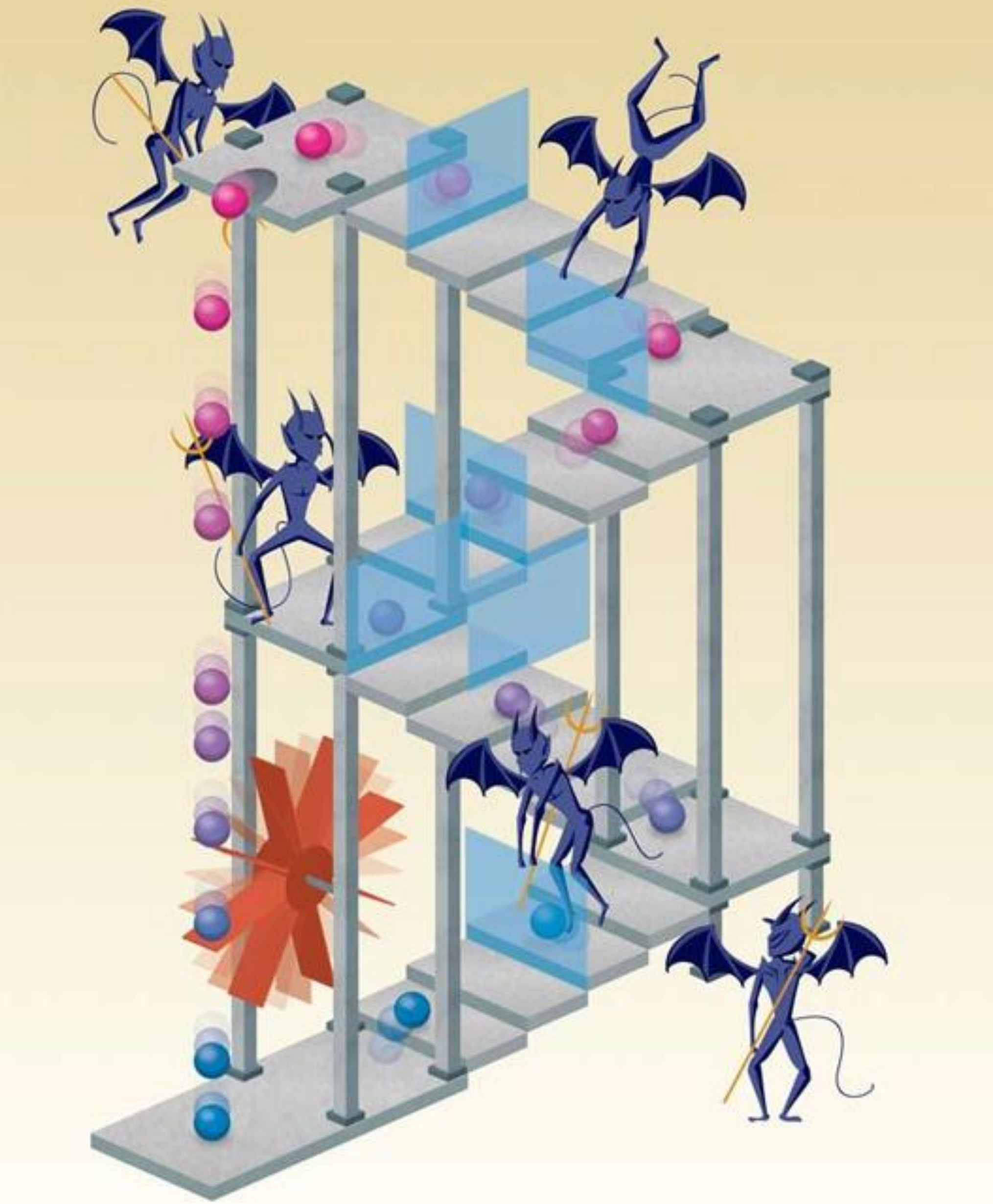
		
$\ln 2$	0	S_0
$\boxed{\ln 2}$		S_0
increase	$\ln 2$	**decrease**
$\ln 2$	$\ln 2$	$S_0 - \ln 2$
$\ln 2$	0	S_0

Information heat engine

$$W \leq -\Delta F + kTI(S:M)$$



Sagawa & Ueda, PRL (2008)

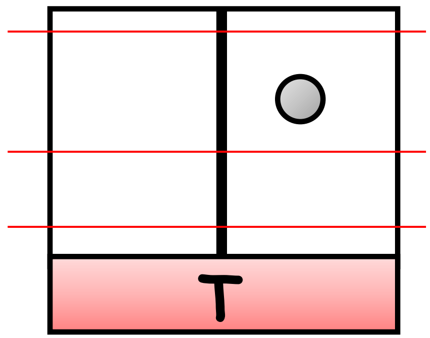


Toyabe, Sagawa, Ueda, Muneyuki & Sano, Nature Phys. (2010)

Quantum dynamical demon?

Thermodynamic work in Q-world

$$H\psi_n = E_n\psi_n$$



$$\sum_n E_n dP_n = dU - \sum_n P_n dE_n$$

$$TdS = dU + dW$$

$$S = -k\sum_n P_n \ln P_n$$

$$U = \sum_n E_n P_n$$

$$P_n = \frac{e^{-\beta E_n}}{Z}$$ in equilibrium

partition function

$$dU = \sum_n (E_n dP_n + P_n dE_n)$$

q-thermodynamic heat

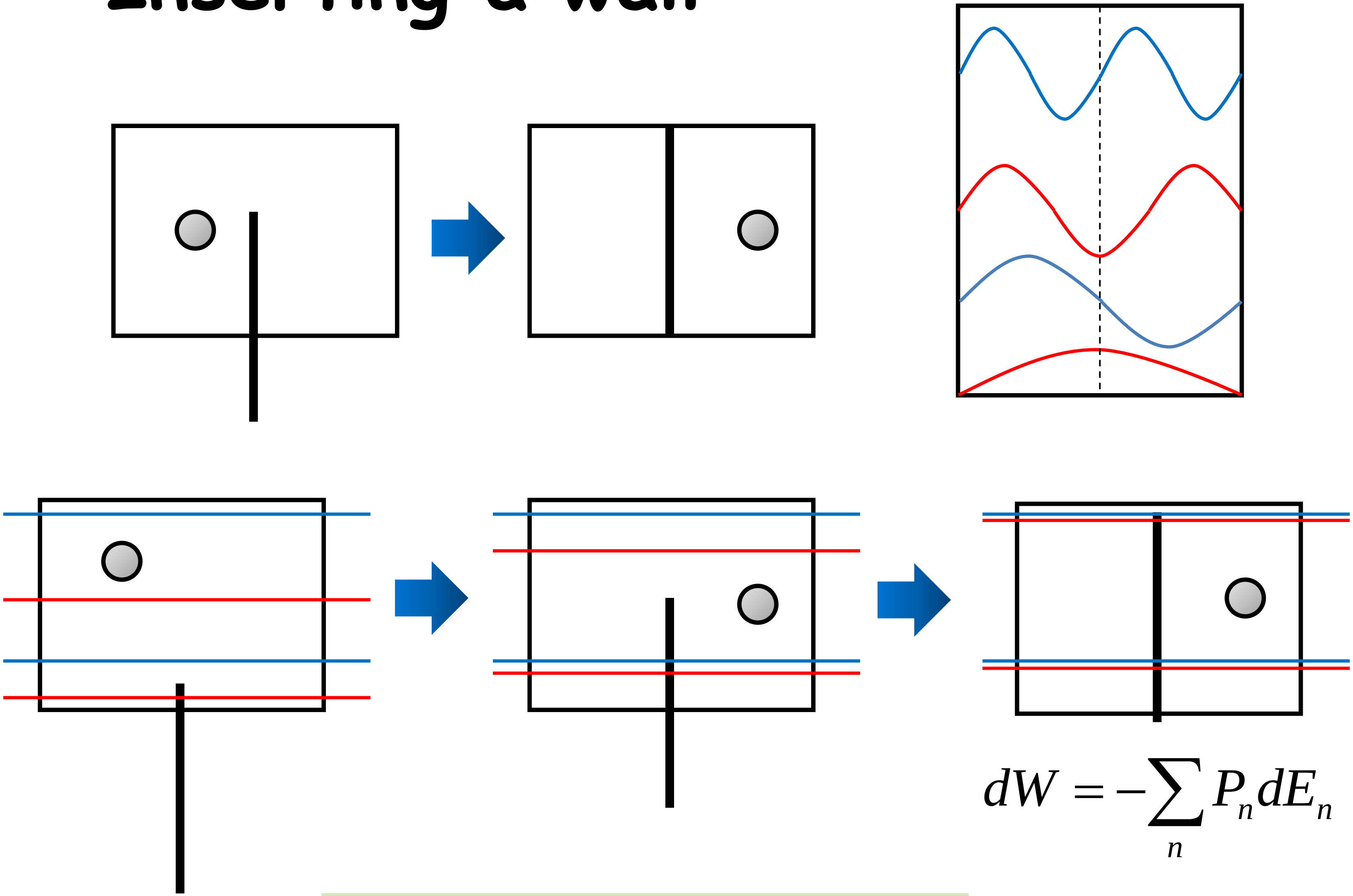
$$dQ = \sum_n E_n dP_n$$

q-thermodynamic work

$$dW = -\sum_n P_n dE_n$$

Kieu, PRL (2004)

Inserting a wall



$$dW = -\sum_n P_n dE_n$$

Inserting a wall is considered
as an isothermal process.

Adiabatic process for inserting a wall

$$dQ = \sum_n E_n dP_n = 0$$

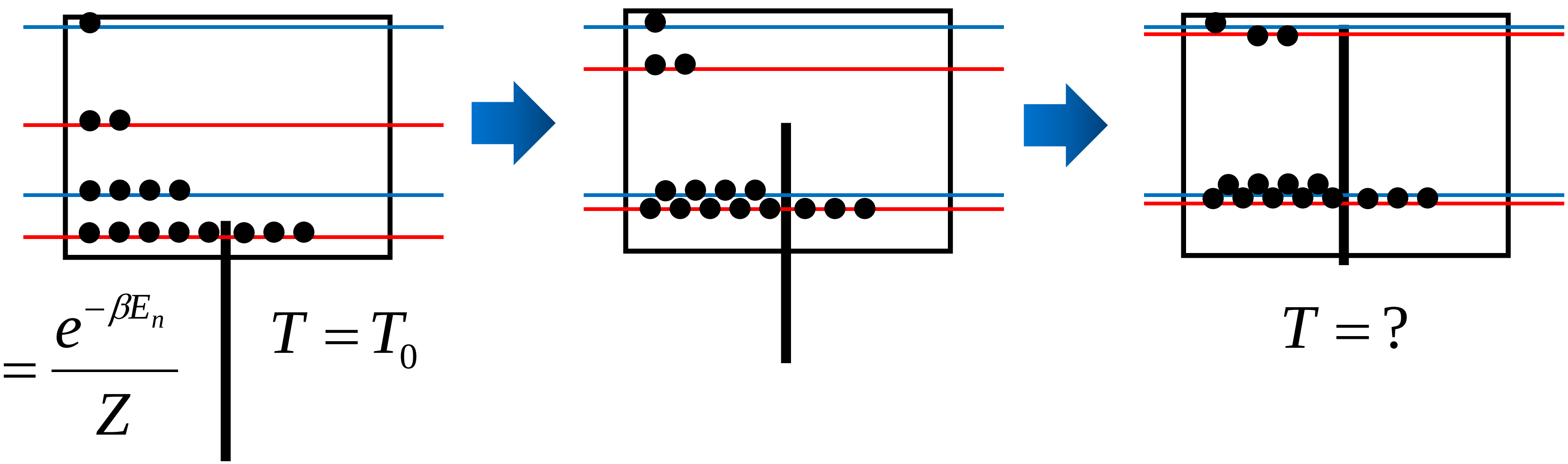
$$dP_n = 0$$

quantum adiabatic

$$dP_n \neq 0$$

$$dQ = \sum_n E_n dP_n = 0$$

T should be changed



$$P_n = \frac{e^{-\beta E_n}}{Z}$$

$$T = T_0$$

$$T = ?$$

The final state should be in non-equilibrium, so that the irreversible process inevitably occurs in isothermal expansion.

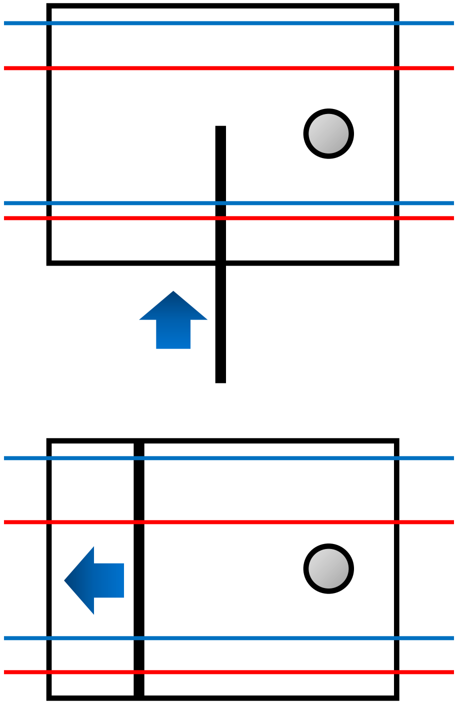
Q-work in an isothermal process

$$W = -\sum_n \int_{X_1}^{X_2} \frac{e^{-\beta E_n}}{Z} dE_n$$

$$= kT \sum_n \int_{X_1}^{X_2} \frac{\partial \ln Z}{\partial E_n} \frac{\partial E_n}{\partial X} dE$$

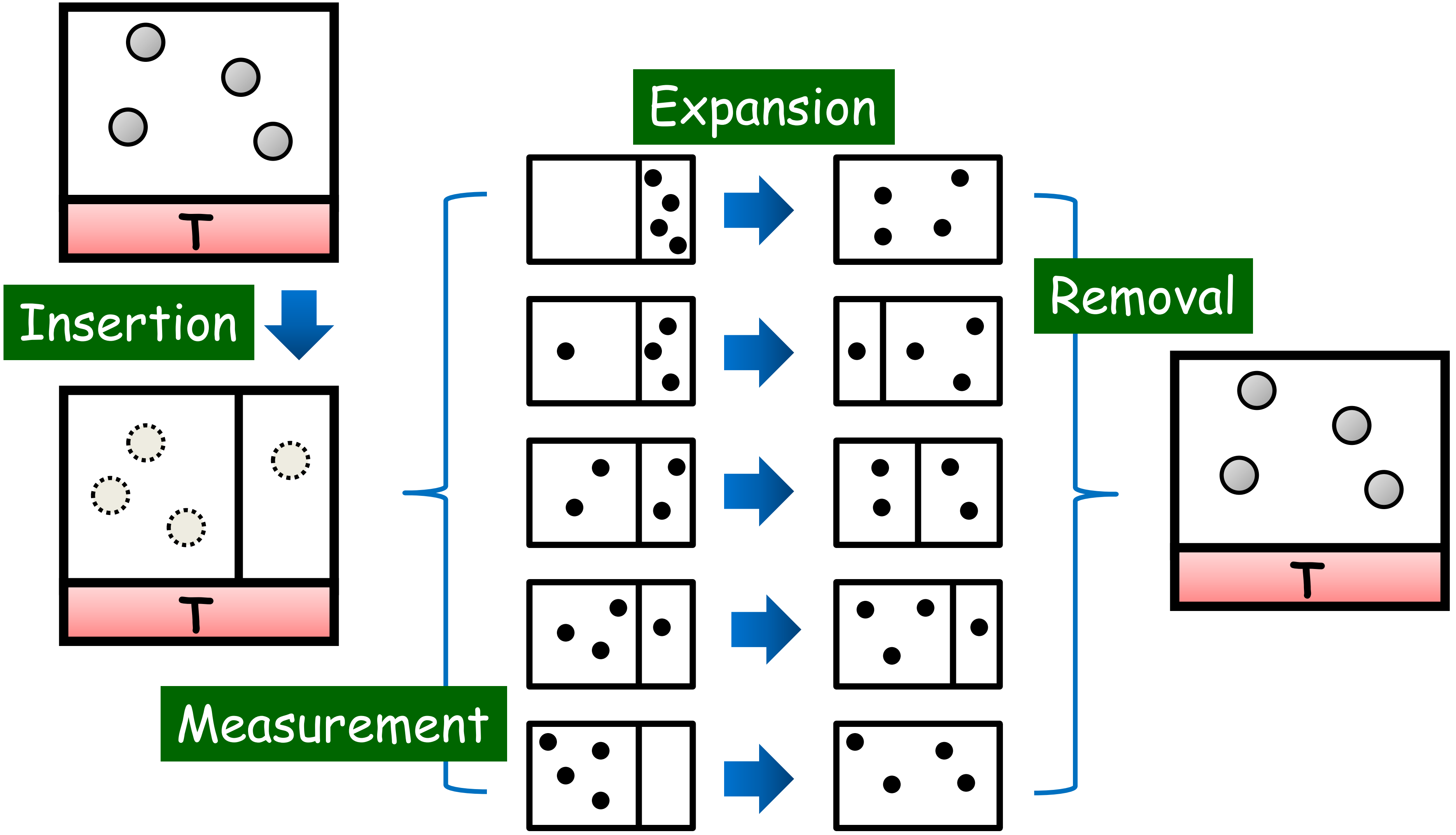
$$= kT\left[\ln Z(X_2) - \ln Z(X_1)\right]$$

Helmholtz free energy difference



(Note) Due to isothermal process, we don't have to consider a full density matrix.

Thermodynamic process



Q-work of q-Szilard engine

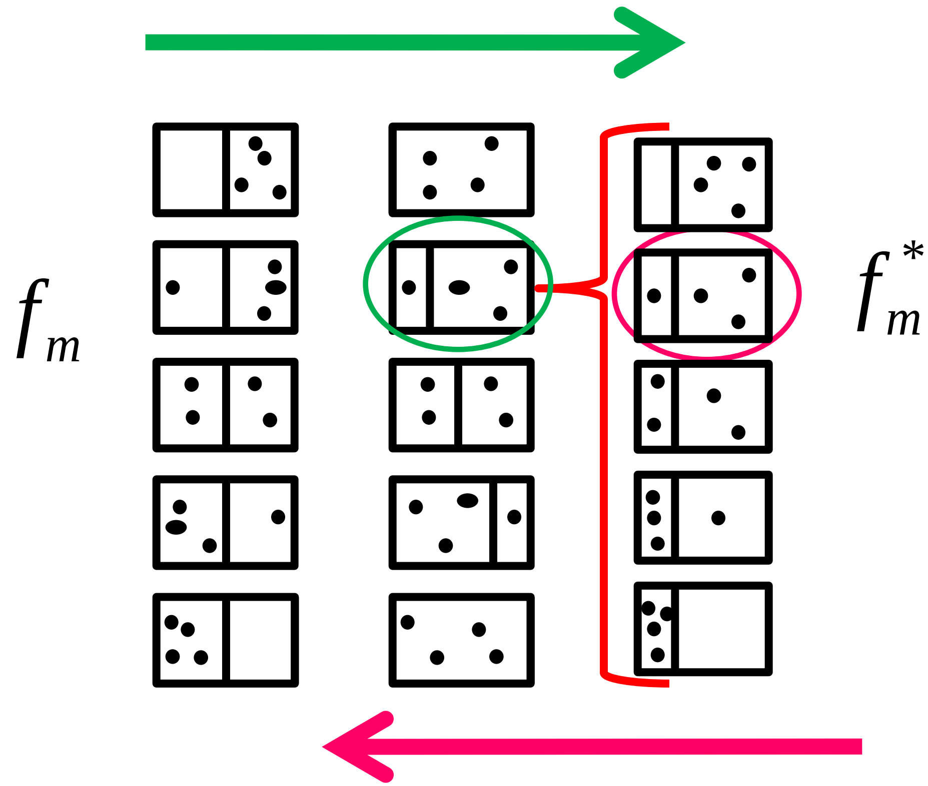
$$W_{tot} = W_{ins} + W_{\exp} + W_{rem}$$

$$= -kT \sum_{m=0}^{N} f_m \ln\left(\frac{f_m}{f_m^*}\right)$$

$$f_m^* = \frac{Z_m(l_{eq}^m)}{Z(l_{eq}^m)}$$

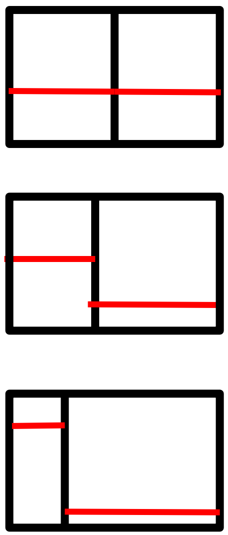
$$Z(l_{eq}^m) = \sum_{n=0}^{N} Z_n(l_{eq}^m)$$

$$\sum_{m=1}^{N} f_m^* \neq 1$$

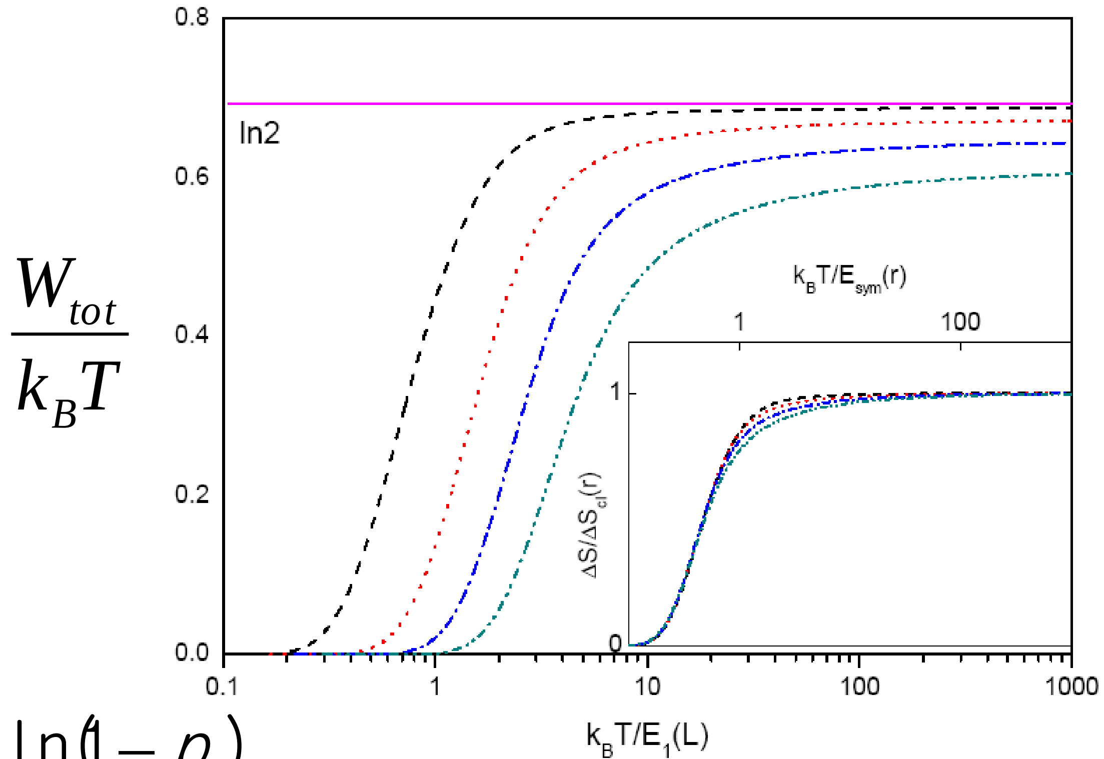


SWK, Sagawa, De Liberato & Ueda, PRL (2011)

Single particle q-Szilard engine



p $\quad$ $1-p$



$$\frac{W_{tot}}{k_B T} = -p \ln p - (1-p)\ln(1-p)$$

The 3rd law of thermodynamics

$$S \to 0 \quad \text{as } T \to 0$$

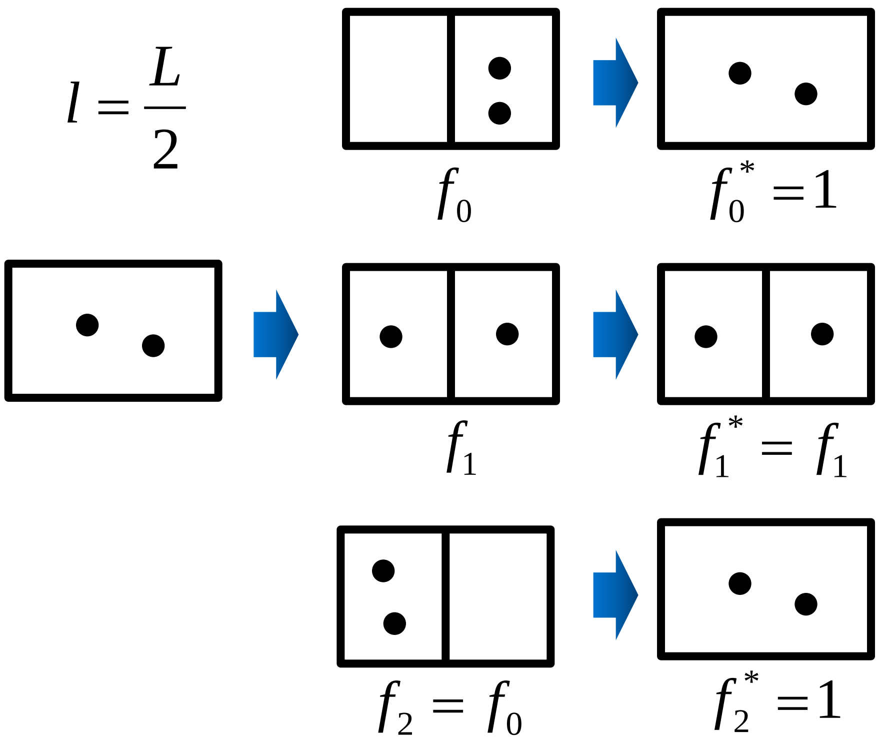
K.-H. Kim & SWK, J. Korean Phys. Soc. (2012)

Two particle q-Szilard engine I

$$W_{tot} = -kT\sum_{m=0}^{N} f_m \ln\left(\frac{f_m}{f_m^*}\right)$$

$$W_{tot} = -2kTf_0 \ln f_0$$

$$l = \frac{L}{2}$$



$$f_0 = \frac{z^2(l,\beta) + z(l,2\beta)}{2[2z^2(l,\beta) + z(l,2\beta)]}$$

$$z(a,b) = \sum_n e^{-bE_n(a)}$$

$$E_n(a) = \frac{h^2}{8m}\frac{n^2}{a^2}$$

Two particle q-Szilard engine II

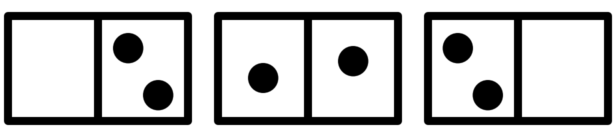
$$W_{tot} = -2kTf_0 \ln f_0$$

Bosons

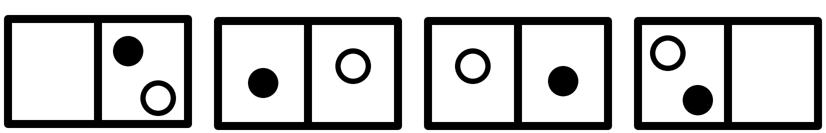
$$W_{tot} \approx kT\left(\frac{2}{3}\right)\ln 3 \qquad \text{for } T \to 0$$

$$W_{tot} \approx kT\ln 2 \qquad \text{for } T \to \infty$$

$$f_0 \to \frac{1}{3} \quad \text{as } \ T \to 0$$



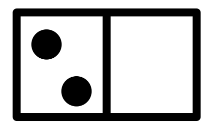
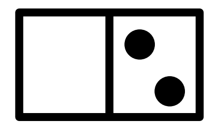
$$f_0 \to \frac{1}{4} \quad \text{as } \ T \to \infty$$



Fermions (spinless)

$$W_{tot} \approx 0 \qquad \text{for } T \to 0$$

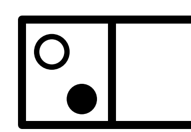
$$W_{tot} \approx kT\ln 2 \qquad \text{for } T \to \infty$$

Both  and 

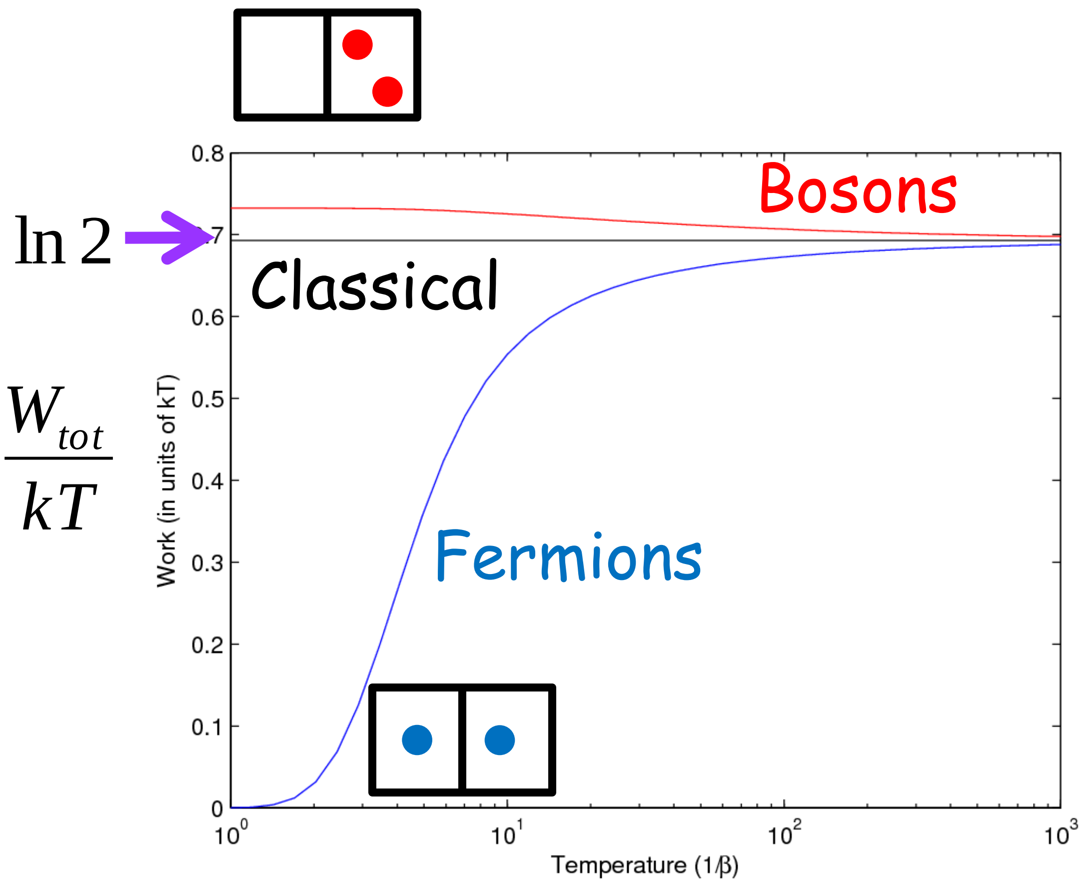
are prohibited due to Pauli exclusion principle in the low T.

(cf) classical work

$$W_{classical} = \frac{1}{2}\left(2kT\ln 2\right)$$



Two particle q-Szilard engine



SWK, Sagawa, De Liberato & Ueda, PRL (2011)

Viewpoint

• Statistical Mechanics • Quantum Information

Physics 4, 13 (2011)
DOI: 10.1103/Physics.4.13

Maxwell’s demon in the quantum world

Juan M. R. Parrondo and **Jordan M. Horowitz**
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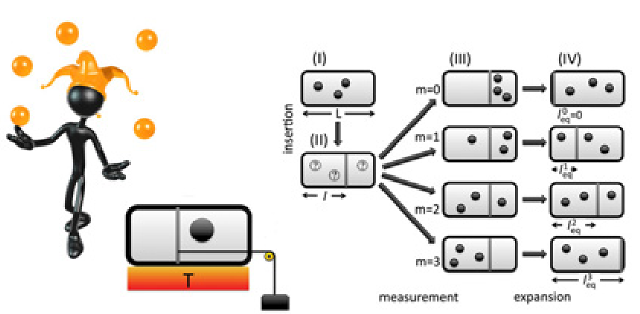
Published February 14, 2011

Extraction of work from a heat bath using the Szilard engine depends crucially on the statistics of indistinguishable particles.

A Viewpoint on:

Quantum Szilard Engine
Sang Wook Kim, Takahiro Sagawa, Simone De Liberato, and Masahito Ueda
Phys. Rev. Lett. **106**, 070401 (2011) – Published February 14, 2011
Download PDF (free)



(a) (b)

Information and thermodynamics are intimately connected. This idea was first illustrated by Maxwell with his celebrated demon: an intelligent being who uses his knowledge about the position and velocity of the molecules in a gas to transfer heat against a temperature gradient without expenditure of work, beating the second law of thermodynamics. The Szilard engine is a stylized version of the demon, where a yes/no measurement of a classical single-particle system allows one to extract a tiny amount of energy, $kT\ln 2$, k being the Boltzmann constant, from a thermal reservoir at temperature T. The engine has been around for almost a century now [1]. Along the way it has furnished insight into the foundations of statistical mechanics, become the canonical model for investigations of feedback-controlled systems, and even spurred the creation of a new field: the thermodynamics of

Physical Review Letters
moving physics forward

Phys. Rev. Lett. 111, 188901 (2013) [1 pages]

Comment on "Quantur

Abstract References

Martin Plesch[1,2], Oscar Dahlsten[3,4], John Gool
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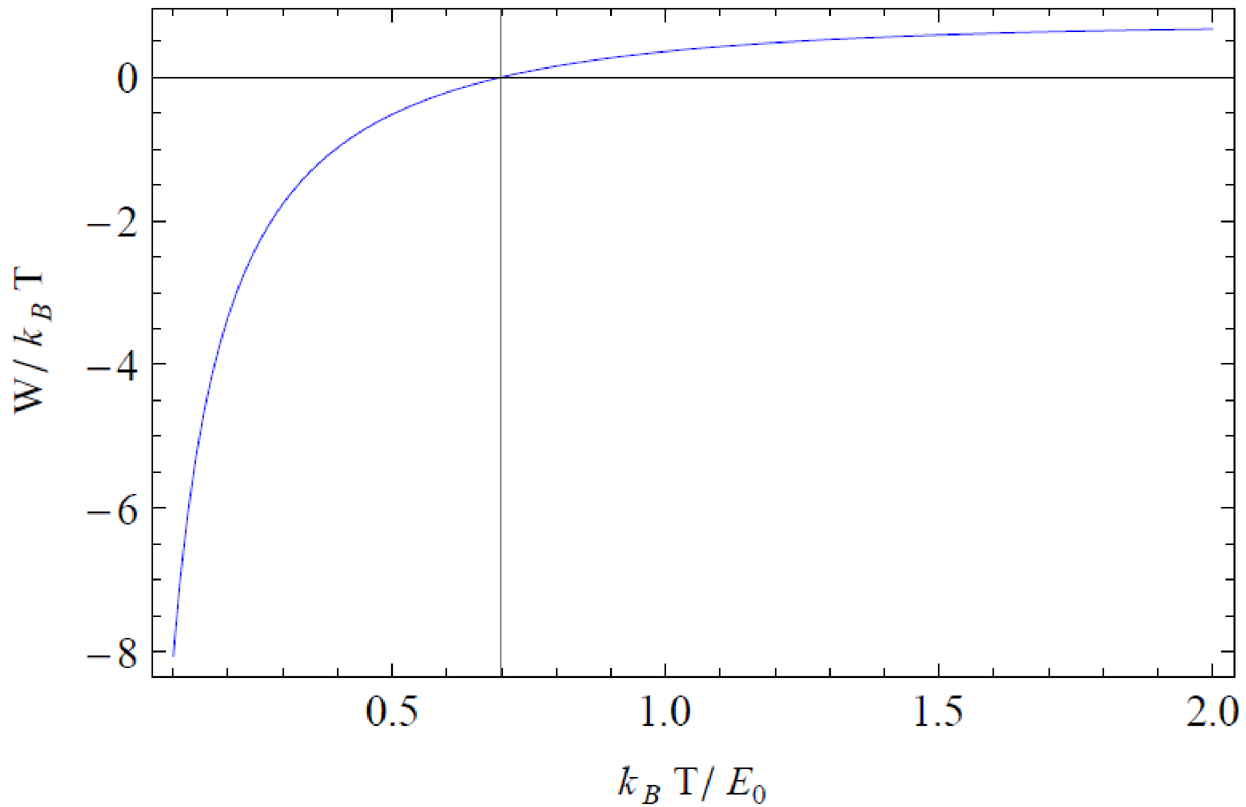
[3]Department of Physics, University of Oxford, C
[4]Center for Quantum Technology, National Uni

Received 22 November 2012; published 30 Oc

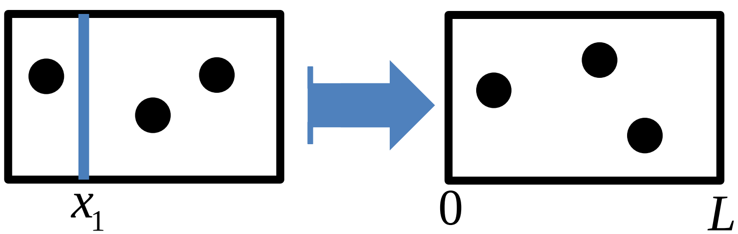
A Comment on the Letter by S. W. Kim *et al.*, Ph

URL: http://link.aps.org/doi/10.1103/PhysRevL
DOI: 10.1103/PhysRevLett.111.188901
PACS: 05.30.-d, 03.67.-a, 05.70.-a, 89.70.Cf

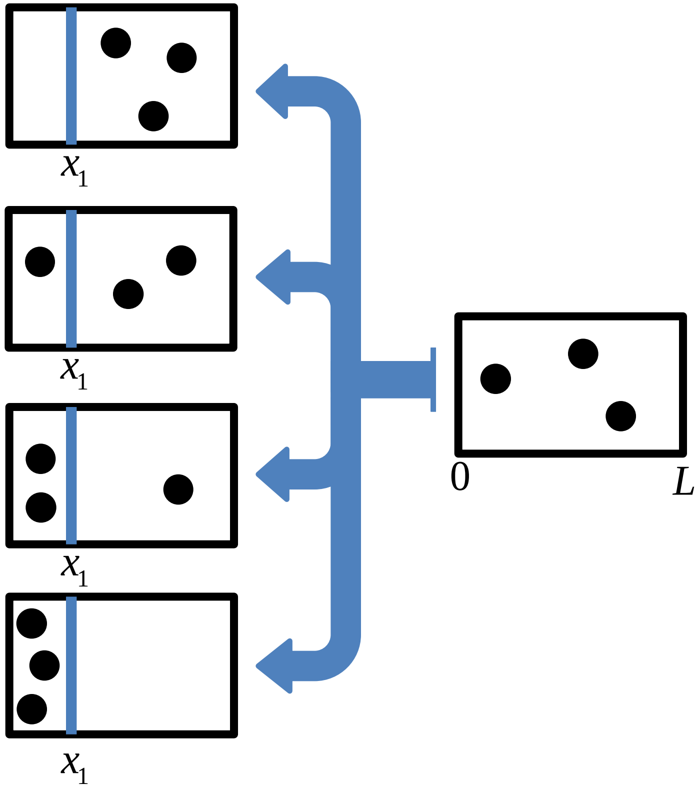


Irreversible process I

Time-forward



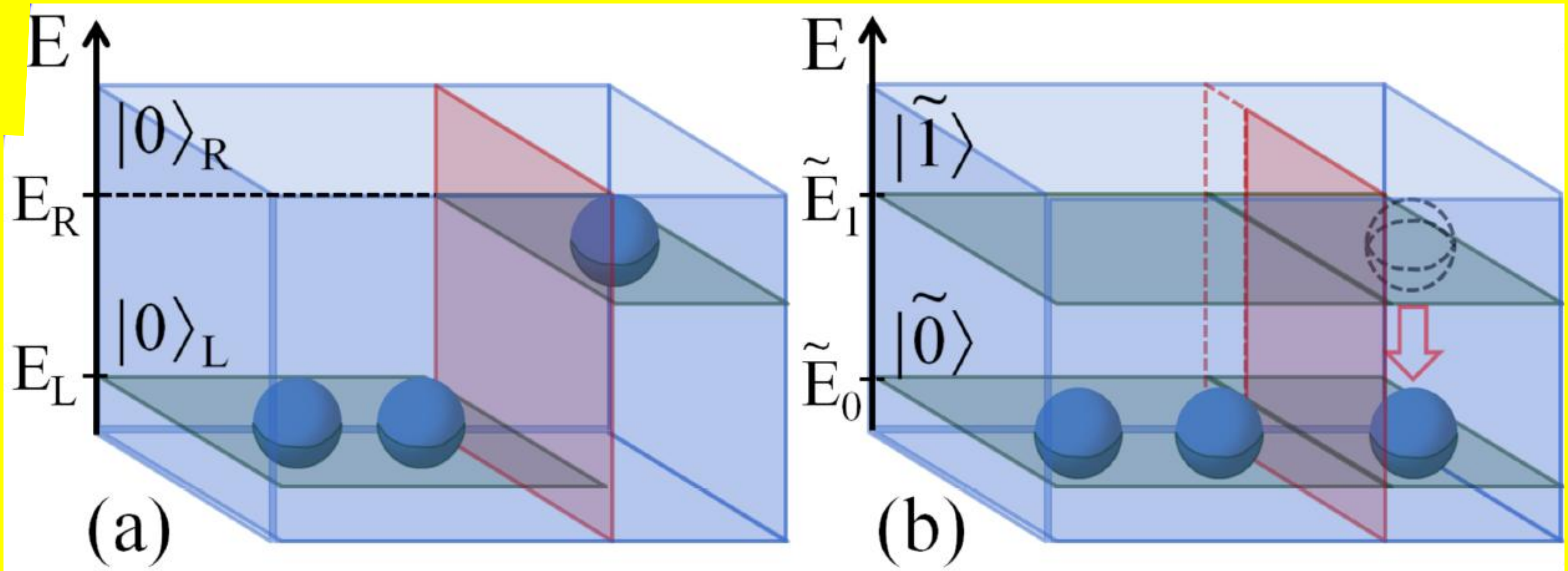
Time-backward



Inherently irreversible!

(cf) Murashita, Funo & Ueda, PRE (2014)
Ashida, Funo, Murashita & Ueda, PRE (2014)

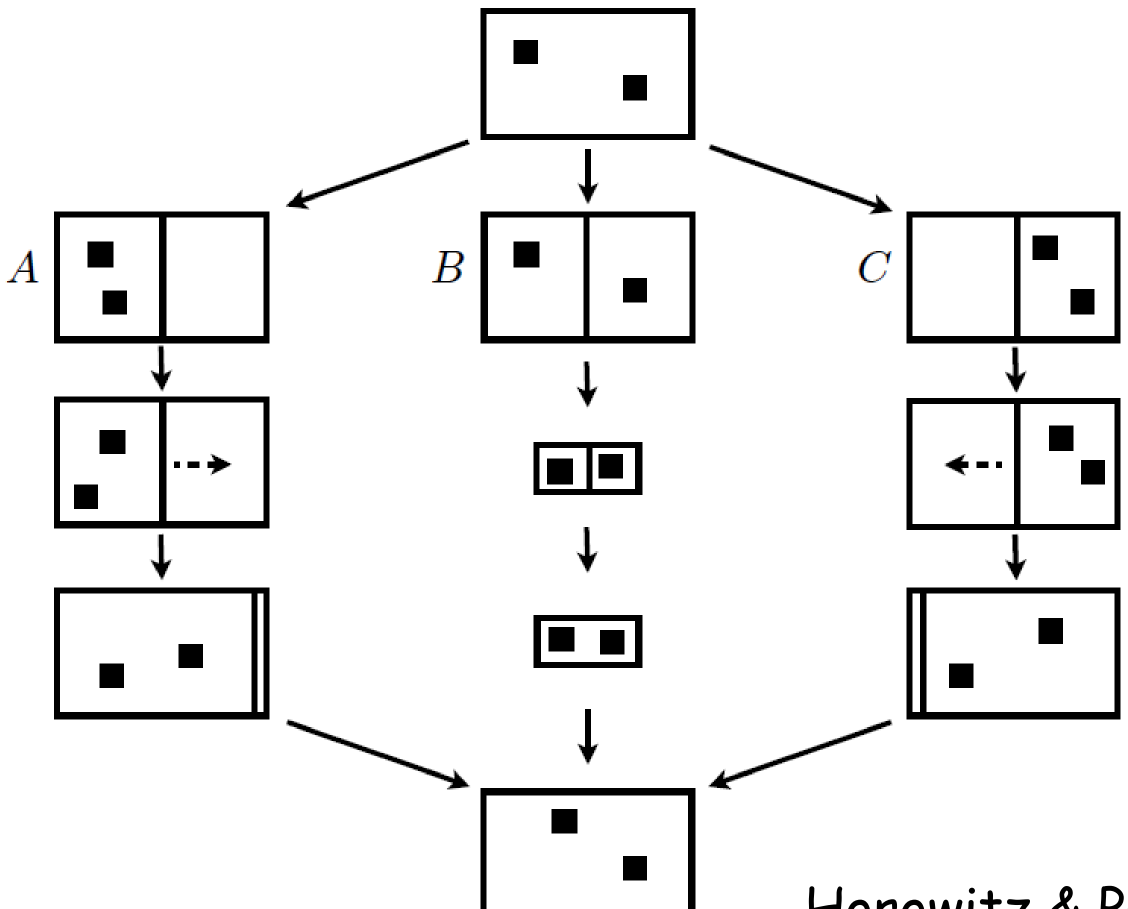
Irreversible process II



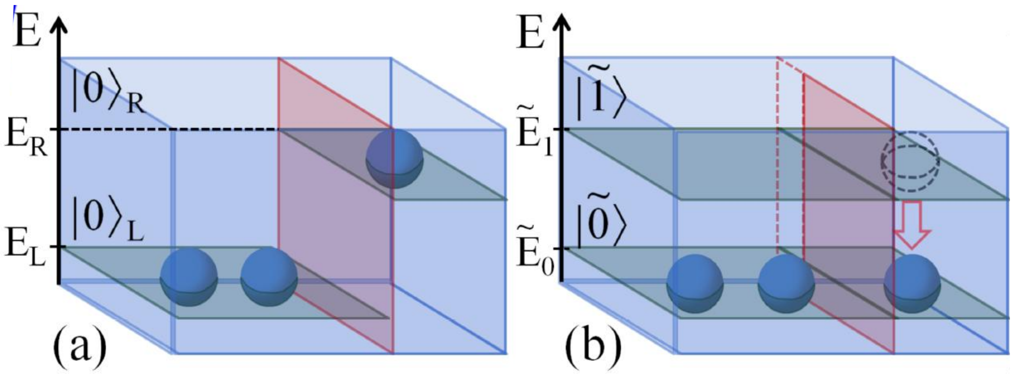
T

$$W_{tot} = kT\left\{-\sum_{m=0}^{N} f_m \ln f_m + \left(-\sum_{m=0}^{N} f_m \ln f_m^*\right)\right\}$$

(Option 1)
Make the protocol reversible



Horowitz & Parrond, NJP (2011)



(Option 2) Optimize work via math

$$W_{tot} = -kT\sum_{m=0}^{N} f_m \ln\left(\frac{f_m}{f_m^*}\right)$$

$$W_{tot}(l,\{x_m\})$$

Optimal condition:

$$\nabla_{l,x_m} W_{tot}(l,\{x_m\}) = 0$$

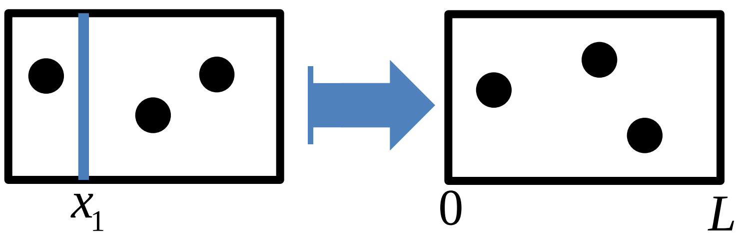
$$\frac{1}{Z_m}\frac{\partial Z_m}{\partial x_m} = \frac{1}{Z}\frac{\partial Z}{\partial x_m}$$

$$F_m(x_m) = \langle F_p(x_m)\rangle_p$$

$$W_{tot} \geq 0$$

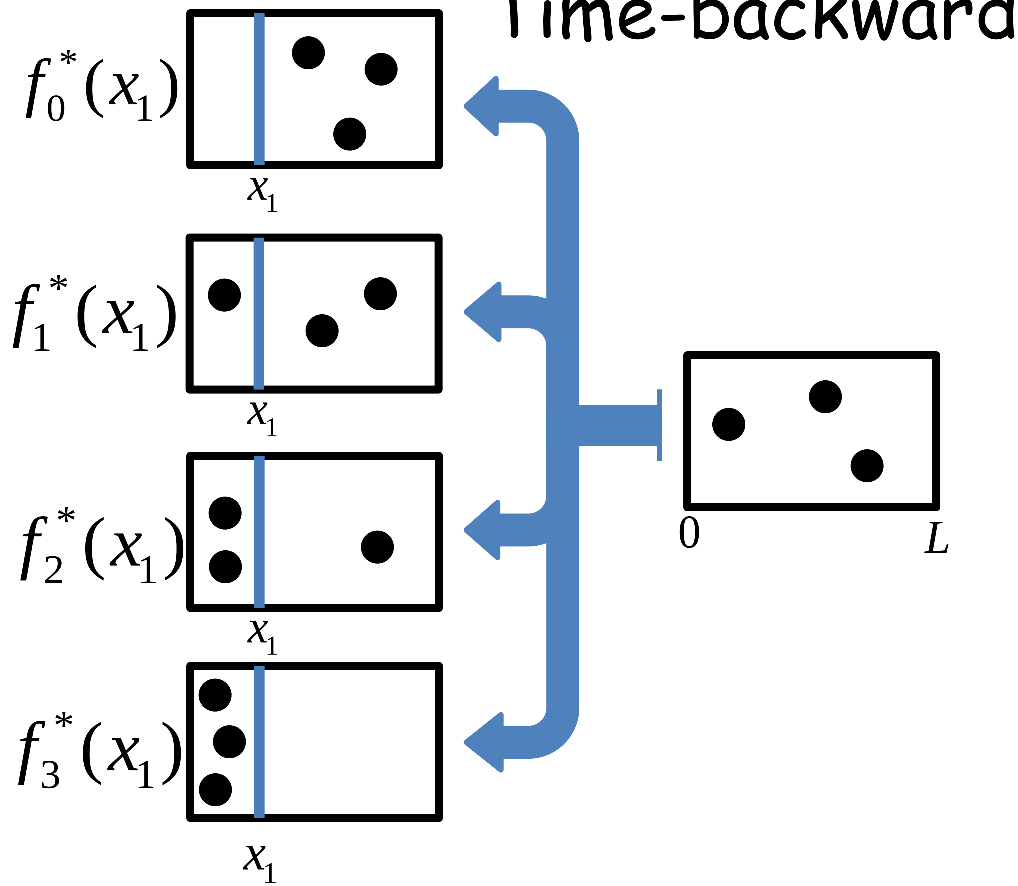
Physical meaning of optimal condition

Time-forward



(cf) $F(x) = F^{left} + F^{right} = 0$

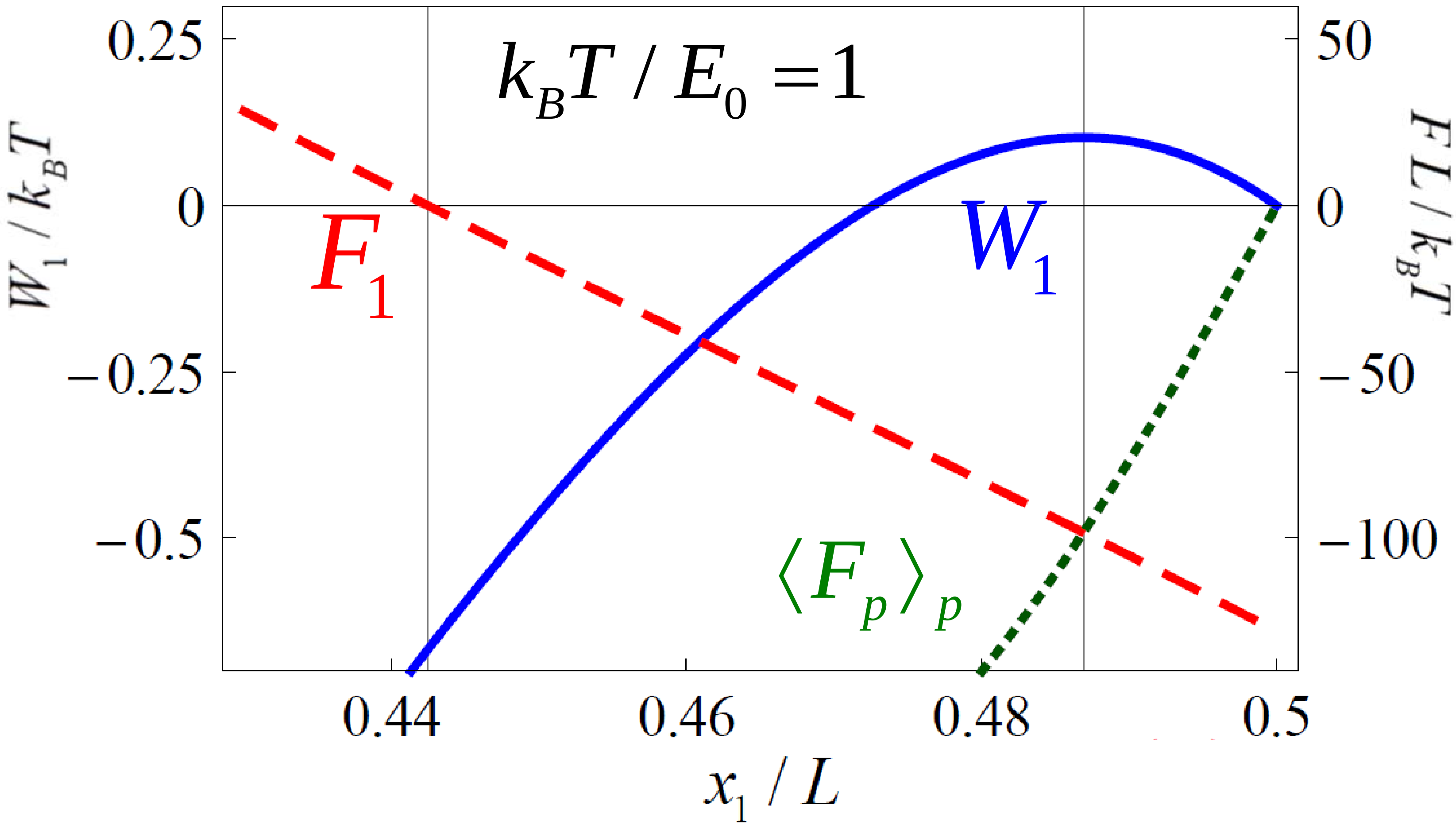
Time-backward



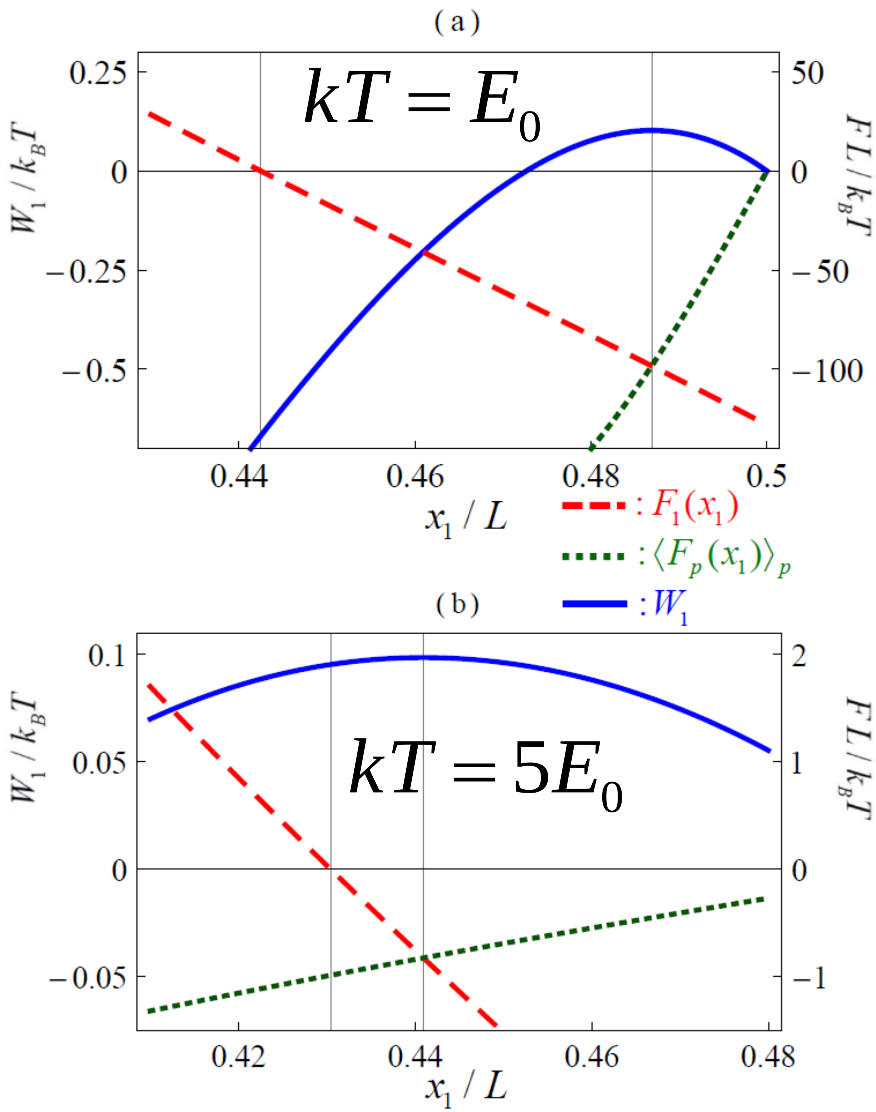
$$F_1(\boxed{x_1}) = \left\langle F_p(\boxed{x_1}) \right\rangle_p = \sum_{p=0}^{N} f_p^* F_m(x_1)$$

Numerical check I

$Boson \quad N = 3$



Numerical check II



$$\left\langle F_p(x_1) \right\rangle_p \to 0$$

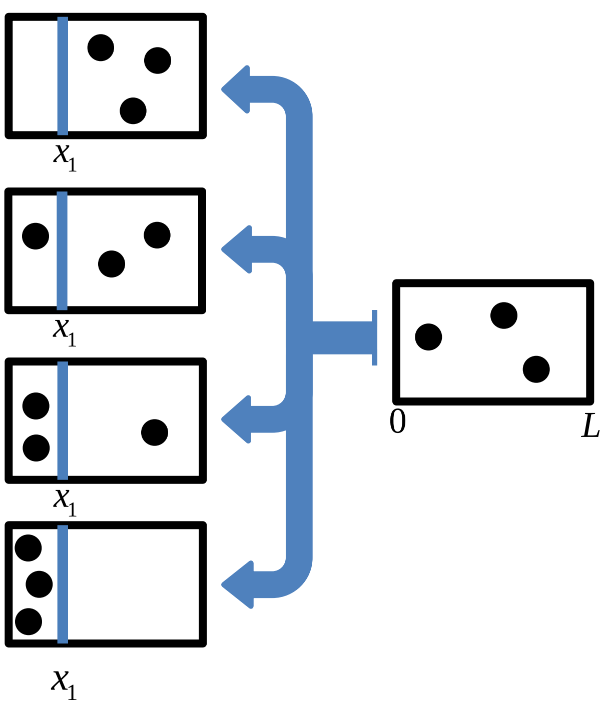
$$T \to \infty$$

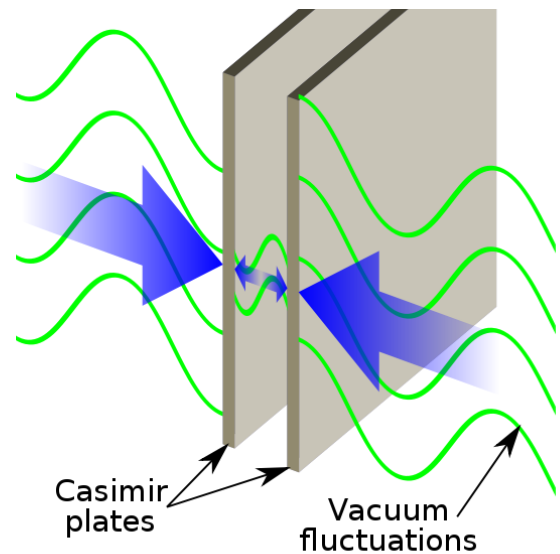
$$\left\langle F_p(x_1)\right\rangle_p = \sum_{p=0}^{N} f_p^* F_m(x_1) \neq 0$$

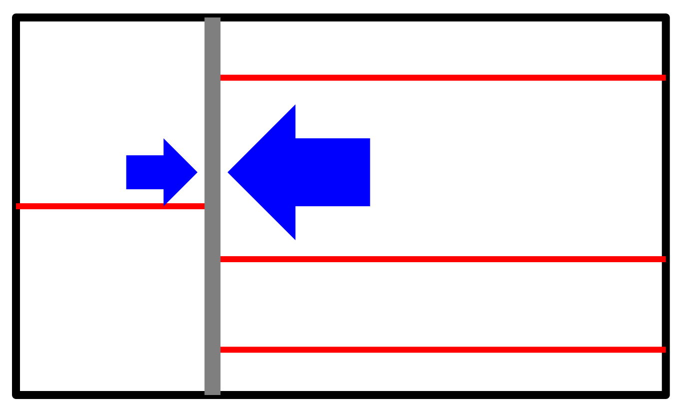
Why?

Casimir force

Time-backward

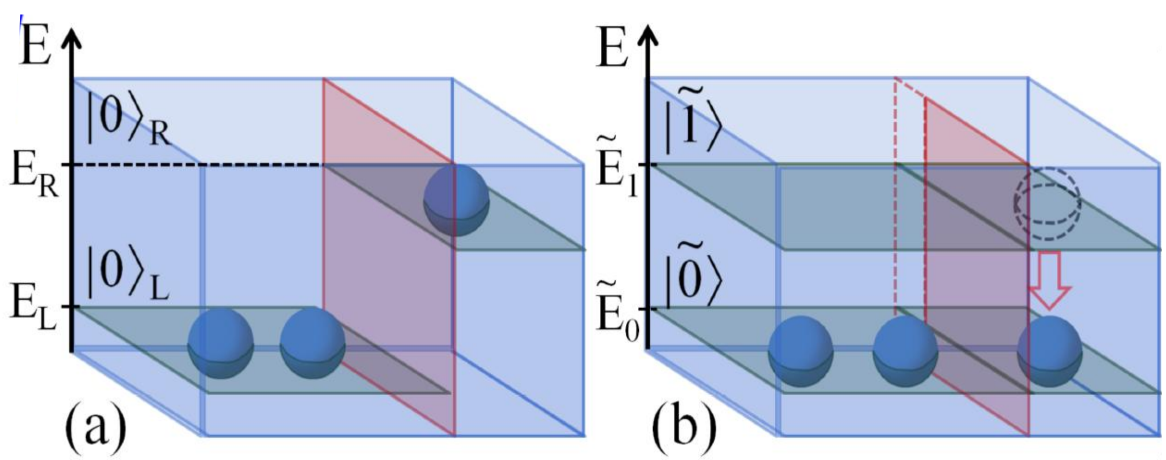






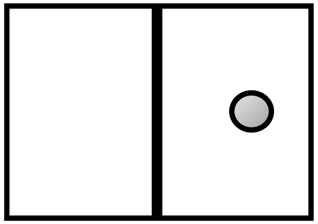
The optimal condition of the q-SZE with intrinsic irreversibility is achieved once the time-forward force is equivalent to the time back-ward force:

$$F_m(x_m) = \langle F_p(x_m) \rangle_p$$



Remark and a new question

$$W_{tot} = -kT\sum_{m=0}^{N} f_m \ln\left(\frac{f_m}{f_m^*}\right)$$



In fact, this equation can be derived from fully classical consideration. The point is that the above expression is mainly ascribed to multi-particle nature of SZE.

Is work extractable from a heat engine by using purely quantum mechanical information? If yes, what is its mathematical formula?

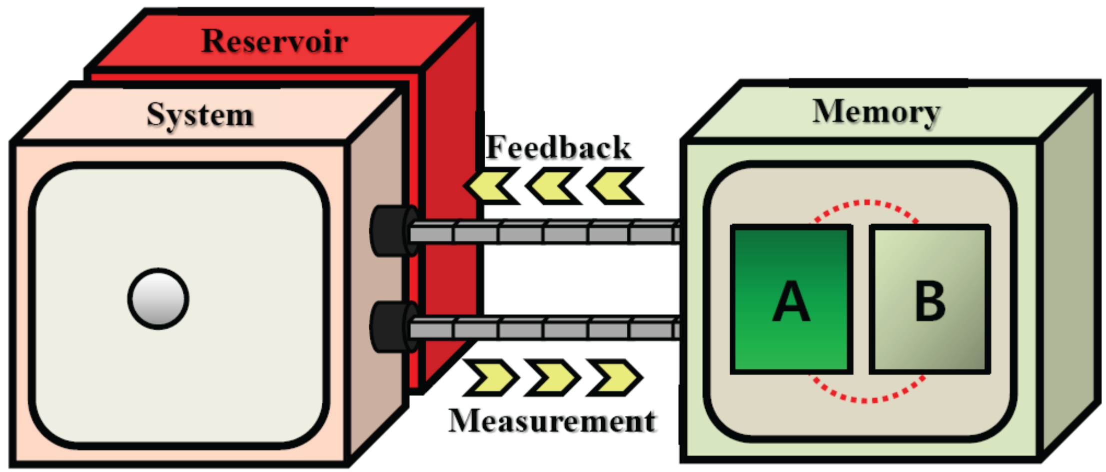
K.-H. Kim & SWK, PRE (2011)

Quantum information demon?

- Previous works

Oppenheim, Horodeki, Horodeki & Horodeki, PRL (2002)
Zurek, PRA (2003)
Rio, Aberg, Renner, Dahlsten & Vedral, Nature (2011)
Funo, Watanabe & Ueda, PRA (2013)

Our Set-up



$$\rho^{(i)} = \rho_{AB}^{(i)} \otimes \frac{\exp\left(-\beta H_S^{(i)}\right)}{Z_S^{(i)}} \otimes \frac{\exp\left(-\beta H_R^{(i)}\right)}{Z_R^{(i)}}$$

Park, K.-H. Kim, Sagawa & SWK, PRL (2013)

Origin of quantum-mechanical complementarity probed by a 'which-way' experiment in an atom interferometer

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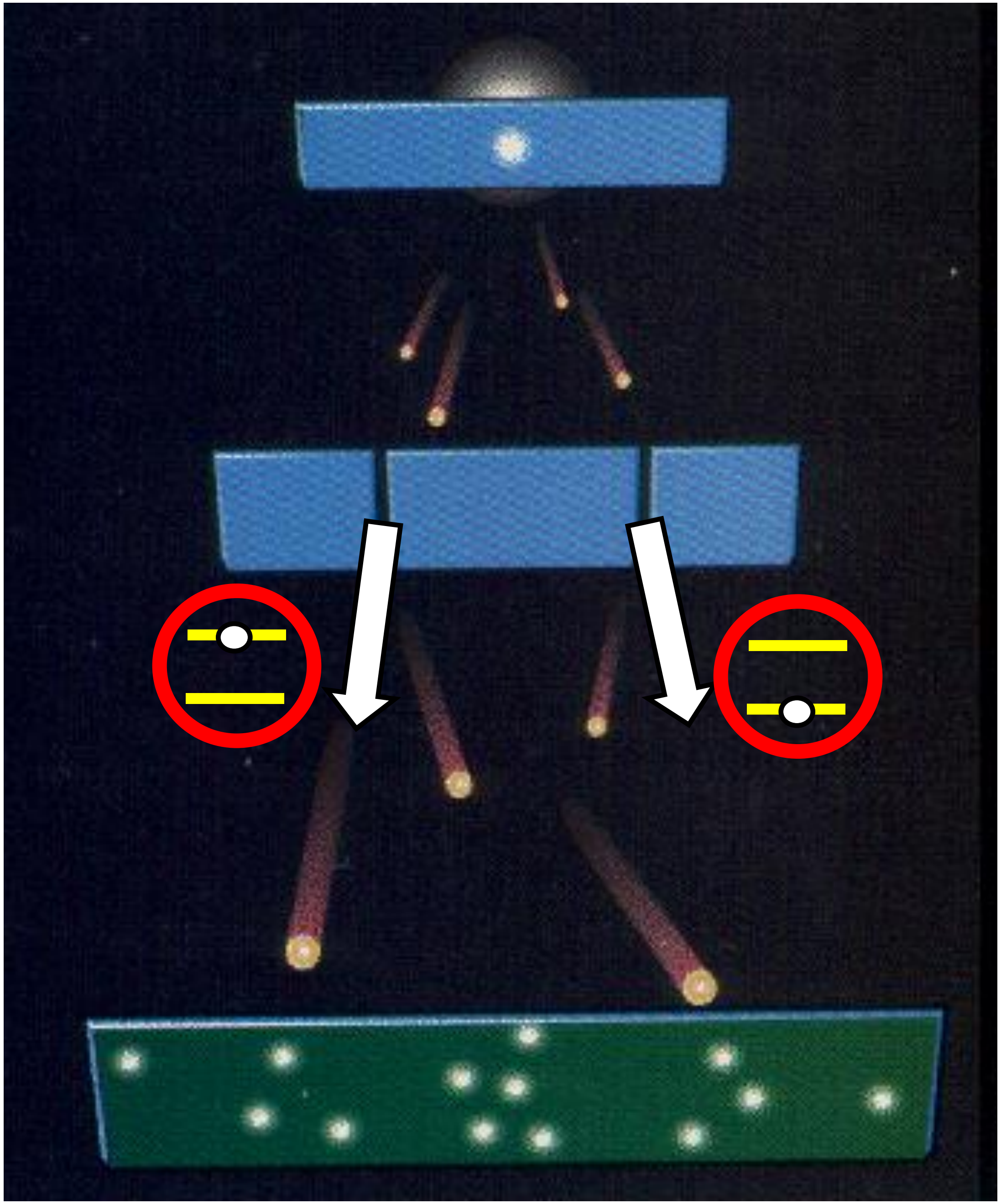
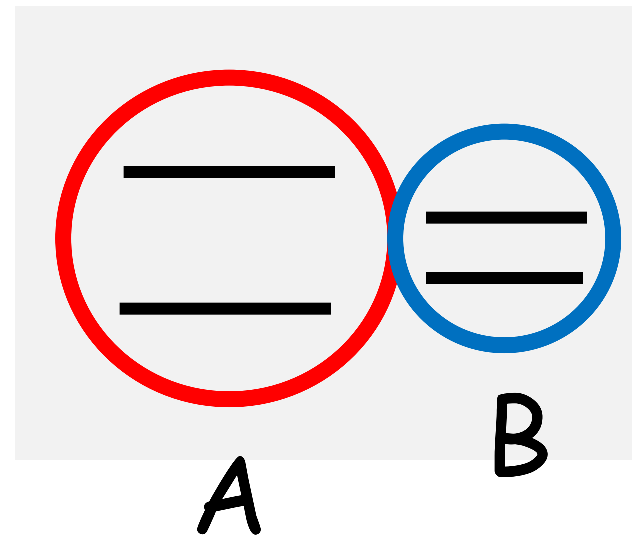
The principle of complementarity refers to the ability of quantum-mechanical entities to behave as particles or waves under different experimental conditions. For example, in the famous double-slit experiment, a single electron can apparently pass through both apertures simultaneously, forming an interference pattern. But if a 'which-way' detector is employed to determine the particle's path, the interference pattern is destroyed. This is usually explained in terms of Heisenberg's uncertainty principle, in which the acquisition of spatial information increases the uncertainty in the particle's momentum, thus destroying the interference. Here we report a which-way experiment in an atom interferometer in which the 'back action' of path detection on the atom's momentum is too small to explain the disappearance of the interference pattern. We attribute it instead to correlations between the which-way detector and the atomic motion, rather than to the uncertainty principle.




A
B

Thermodynamic process

Stage 1 (unitary evolution)

$$\rho^{(1)} = U^{(1)} \rho^{(i)} U^{(1)+} \qquad U^{(1)} = I_{AB} \otimes U_{SR}^{(1)}$$

This can also describe isothermal process.

Stage 2 (POVM)

$$\rho^{(2)} = \sum_k \Pi_A^k U^{(2)} \rho^{(1)} U^{(2)+} \Pi_A^k = \sum p_k \left| k \right\rangle_A \left\langle k \right| \otimes \rho_{BSR}^{(2)k}$$

$$p_k \equiv \mathrm{tr}\left[\Pi_A^k U^{(2)} \rho^{(1)} U^{(2)+} \Pi_A^k\right] \qquad \rho_{BSR}^{(2)k} \equiv \mathrm{tr}_\mathrm{A}\left[\Pi_A^k U^{(2)} \rho^{(1)} U^{(2)+} \Pi_A^k\right]$$

Stage 3 (feedback control)

$$\rho^{(f)} = U^{(3)} \rho^{(2)} U^{(3)+}$$

Entropy consideration

von Neumann entropy $S(\rho) \equiv -\mathrm{tr}(\rho \ln \rho)$

(1) concavity $\sum_k p_k S\left[\rho_{SR}^{(2)k}\right] \le S\left[\rho_{SR}^{(f)}\right]$

(2) POVM $S\left[\rho^{(i)}\right] \le S\left[\rho^{(2)}\right]$

(3) Klein inequality $\mathrm{tr}\left[\rho_{SR}^{(f)} \ln \rho_{SR}^{(f)\mathrm{can}}\right] \le S\left[\rho_{SR}^{(f)}\right]$

$$W \le -\Delta F_S + kT\left(\Delta S_A + \Delta S_B\right) - kT\Delta I$$

$$\Delta I \equiv I\left(A^{(2)} : B^{(2)}\right) - I\left(A^{(i)} : B^{(i)}\right)$$

mutual information $I(A:B) \equiv S(\rho_A) + S(\rho_B) - S(\rho_{AB}) = I(B:A)$

Park, K.-H. Kim, Sagawa & SWK, PRL (2013)

Mutual information and Discord

classical mutual information

$$J(B:A) \equiv H(B) + H(A) - H(B,A) = H(B) - H(B|A)$$

Shanon information $H(A) \equiv -\sum_{A} p(A) \ln p(A)$

conditional entropy $H(B|A) \equiv -\sum_{A,B} p(A,B) \ln p(B|A)$

quantum analogue

$$\tilde{J}(B:A) = S(B) - S(B|A)???$$

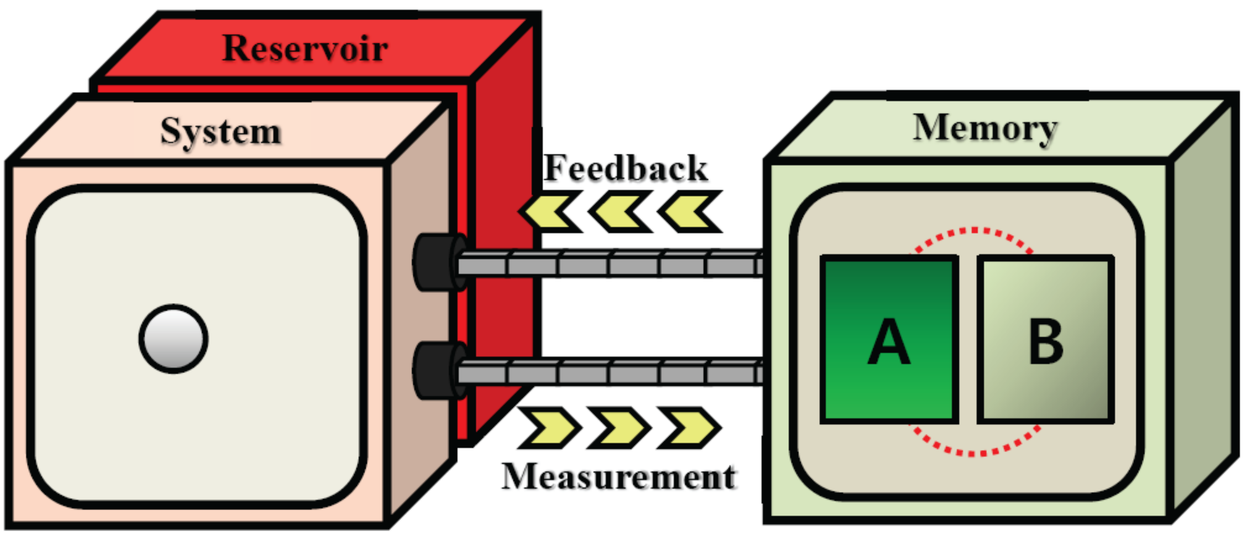
$$\tilde{J}_{\{\Pi_A^i\}}(B:A) = S(\rho_B) - S\left(\rho_B \middle| \{\Pi_A^i\}\right)$$

quantum discord $\delta(B|A) = \min\left[\mathrm{I(A:B)} - \tilde{J}_{\{\Pi_A^i\}}(B:A)\right]$

Ollivier & Zurek, PRL (2002)

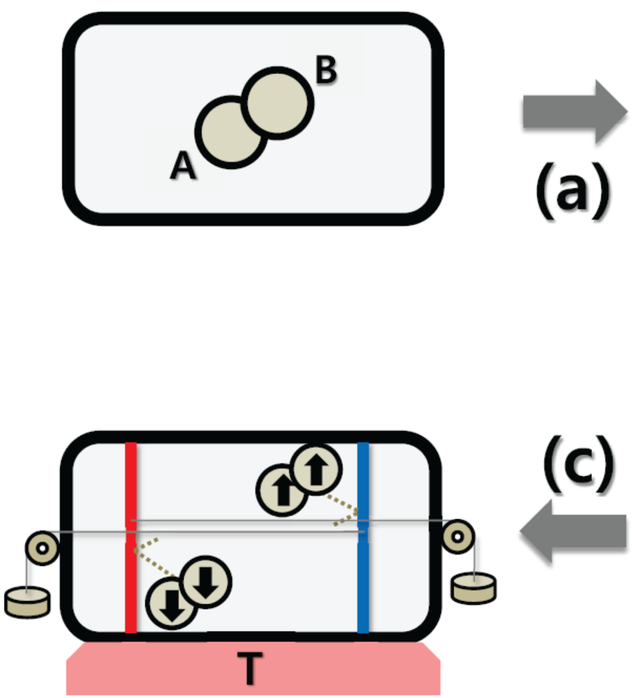
Final formula

$$W \le -\Delta F_S + kT\left(\Delta S_A + \Delta S_B\right) - kT\Delta J + kT\delta\left(B^{(i)} \middle| A^{(i)}\right)$$

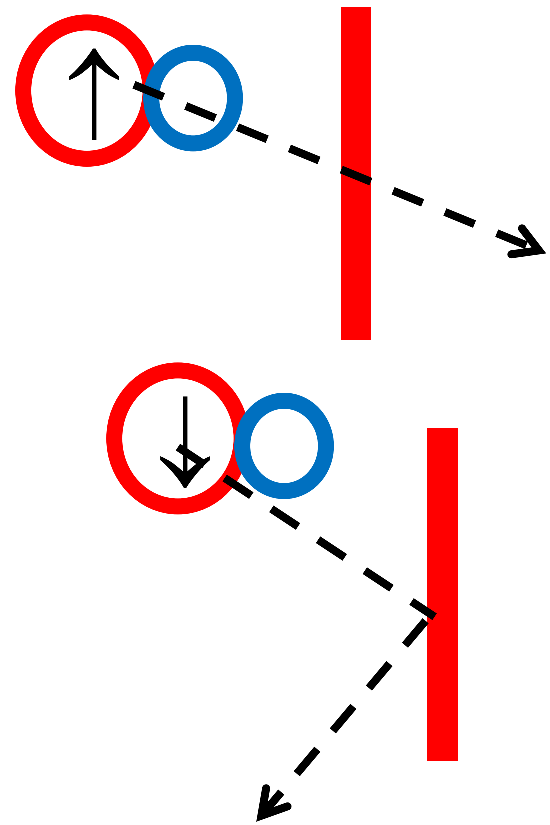


Park, K.-H. Kim, Sagawa & SWK, PRL (2013)

Szilard engine containing a heteronuclear diatomic molecule



semi-permeable wall



Von Neumann, A. Peres, V. Vedral, L. B. Levitin

$$\rho^{(1)} = \left|\Psi^+\right\rangle_{AB}\left\langle\Psi^+\right| \otimes \frac{1}{2}\left(\left|L\right\rangle_S\left\langle L\right| + \left|R\right\rangle_S\left\langle R\right|\right) \otimes \rho_R^{\mathrm{can}}$$

$$\left|\Psi^+\right\rangle_{AB} = \frac{1}{\sqrt{2}}\left(\left|\uparrow_A\uparrow_B\right\rangle + \left|\downarrow_A\downarrow_B\right\rangle\right)$$

$$U^{(2)} = \frac{1}{2}\left(\left|\uparrow\uparrow\right\rangle\left\langle\Psi^+\right| \otimes \left|L\right\rangle\left\langle L\right| + \left|\downarrow\downarrow\right\rangle\left\langle\Psi^+\right| \otimes \left|R\right\rangle\left\langle R\right|\right)$$

$$\Pi_A^k \in \left\{\left|\uparrow\right\rangle_A\left\langle\uparrow\right|, \left|\downarrow\right\rangle_A\left\langle\downarrow\right|\right\}$$

$$\rho^{(12)} = \frac{1}{2}\left(\left|\uparrow\uparrow\right\rangle\left\langle\uparrow\uparrow\right| \otimes \left|L\right\rangle_S\left\langle L\right| + \left|\downarrow\downarrow\right\rangle\left\langle\downarrow\downarrow\right| \otimes \left|R\right\rangle_S\left\langle R\right|\right) \otimes \rho_R^{\mathrm{can}}$$

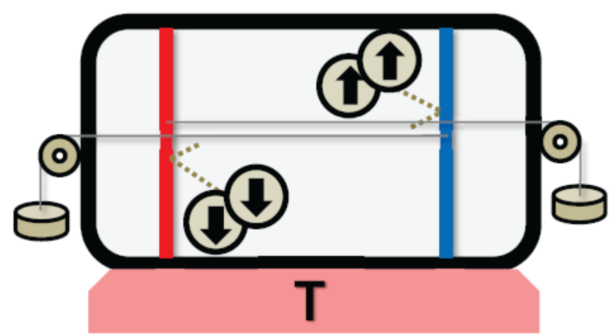
$$\rho^{(f)} = \frac{1}{2}\left(\left|\uparrow\uparrow\right\rangle\left\langle\uparrow\uparrow\right| + \left|\downarrow\downarrow\right\rangle\left\langle\downarrow\downarrow\right|\right) \otimes \frac{1}{2}\left(\left|L\right\rangle_S\left\langle L\right| + \left|R\right\rangle_S\left\langle R\right|\right) \otimes \rho_R^{\mathrm{can}}$$

$$W \le -\Delta F_S + kT\left(\Delta S_A + \Delta S_B\right) - kT\Delta J + kT\delta\left(B^{(i)}\middle|A^{(i)}\right)$$

$$\Delta F_S = 0 \qquad \Delta S_A = \Delta S_B = 0 \qquad \Delta J = 0$$

$$I(A^{(i)} : B^{(i)}) \equiv S(\rho_A^{(i)}) + S(\rho_B^{(i)}) - S(\rho_{AB}^{(i)})$$
$$= \ln 2 + \ln 2 - 0 = 2\ln 2$$

$$\delta(B^{(i)} \mid A^{(i)}) = \min\left[\mathrm{I(A:B)} - \tilde{J}_{\{\Pi_A^i\}}(B:A)\right]$$
$$= 2\ln 2 - \ln 2 = \ln 2$$



$$W = \frac{1}{2}kT\ln 2 + \frac{1}{2}kT\ln 2 = kT\ln 2$$

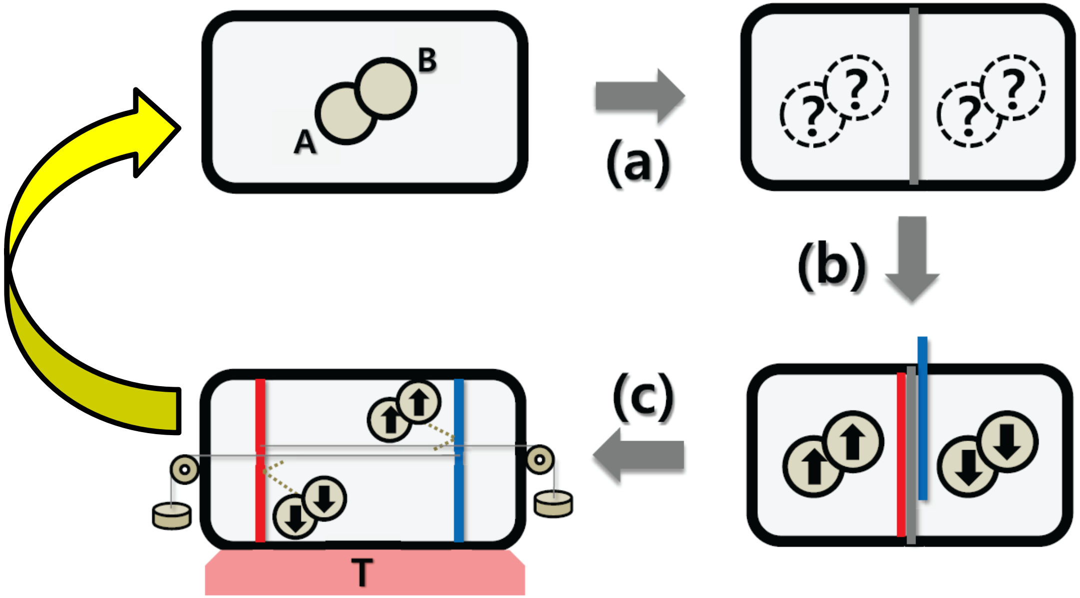
Thermodynamic 2nd law

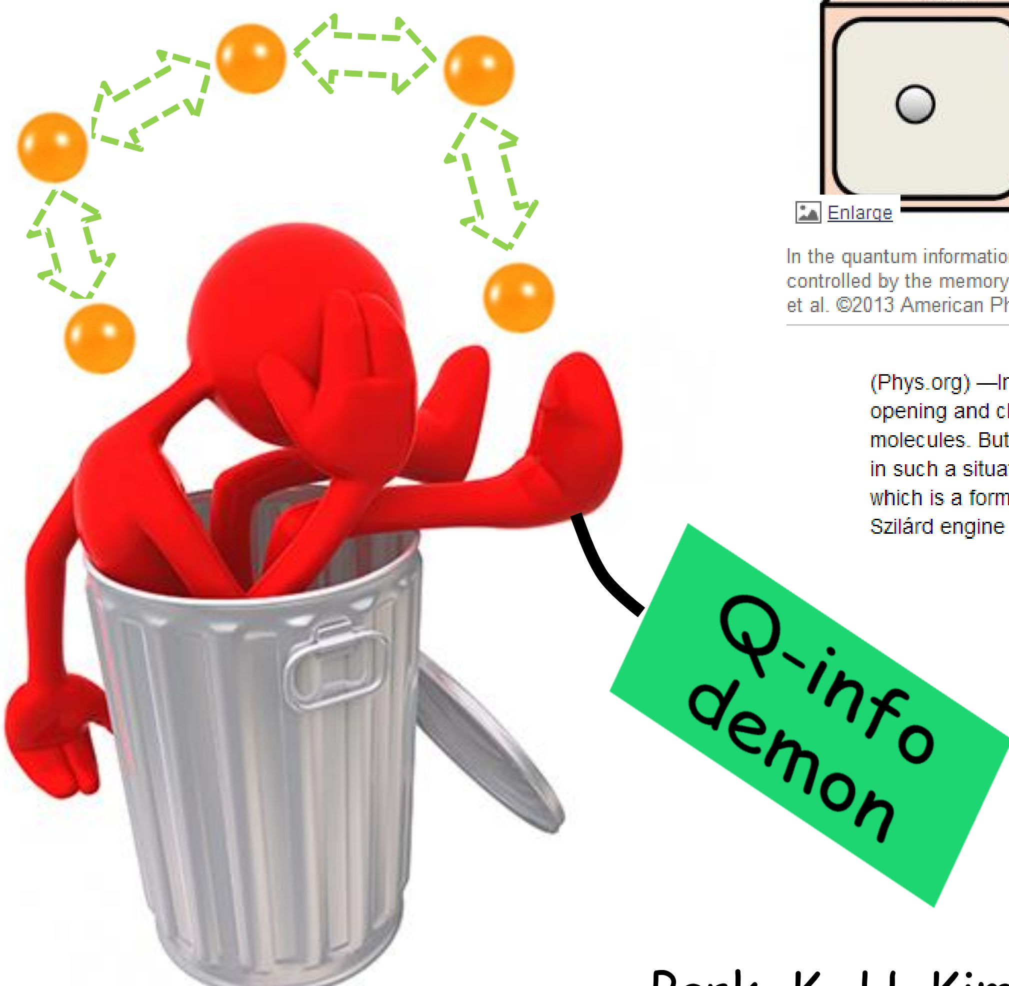
$$\rho^{(f)} = \frac{1}{2}\left(\left|\uparrow\uparrow\right\rangle\left\langle\uparrow\uparrow\right| + \left|\downarrow\downarrow\right\rangle\left\langle\downarrow\downarrow\right|\right) \otimes \frac{1}{2}\left(\left|L\right\rangle_S\left\langle L\right| + \left|R\right\rangle_S\left\langle R\right|\right) \otimes \rho_R^{\text{can}}$$

$$\rho^{(1)} = \left|\Psi^+\right\rangle_{AB}\left\langle\Psi^+\right| \otimes \frac{1}{2}\left(\left|L\right\rangle_S\left\langle L\right| + \left|R\right\rangle_S\left\langle R\right|\right) \otimes \rho_R^{\text{can}}$$

$$S\left[\rho^{(f)}\right] - S\left[\rho^{(1)}\right] = \ln 2$$

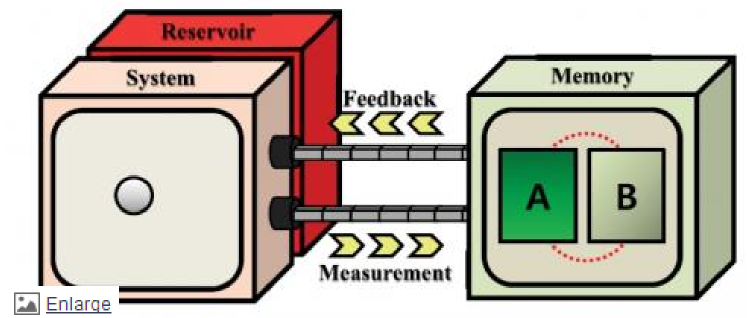
To prepare the initial state of memory, we need to pay kTln2.





Maxwell's demon can use quantum information

Dec 18, 2013 by Lisa Zyga feature



Enlarge

In the quantum information heat engine, the system is attached to the reservoir, and is measured and controlled by the memory consisting of a diatomic molecule with two entangled states. Credit: Park, et al. ©2013 American Physical Society

(Phys.org) —In theory, Maxwell's demon can decrease the entropy of a system by opening and closing a door at appropriate times to separate hot and cold gas molecules. But as physicist Leó Szilárd pointed out in 1929, entropy does not decrease in such a situation because the demon's measurement process requires information, which is a form of entropy. Szilárd's so-called information heat engine, now called the Szilárd engine (SZE), demonstrates how work can be generated by using information.

Park, K.-H. Kim, Sagawa & SWK, PRL (2013)

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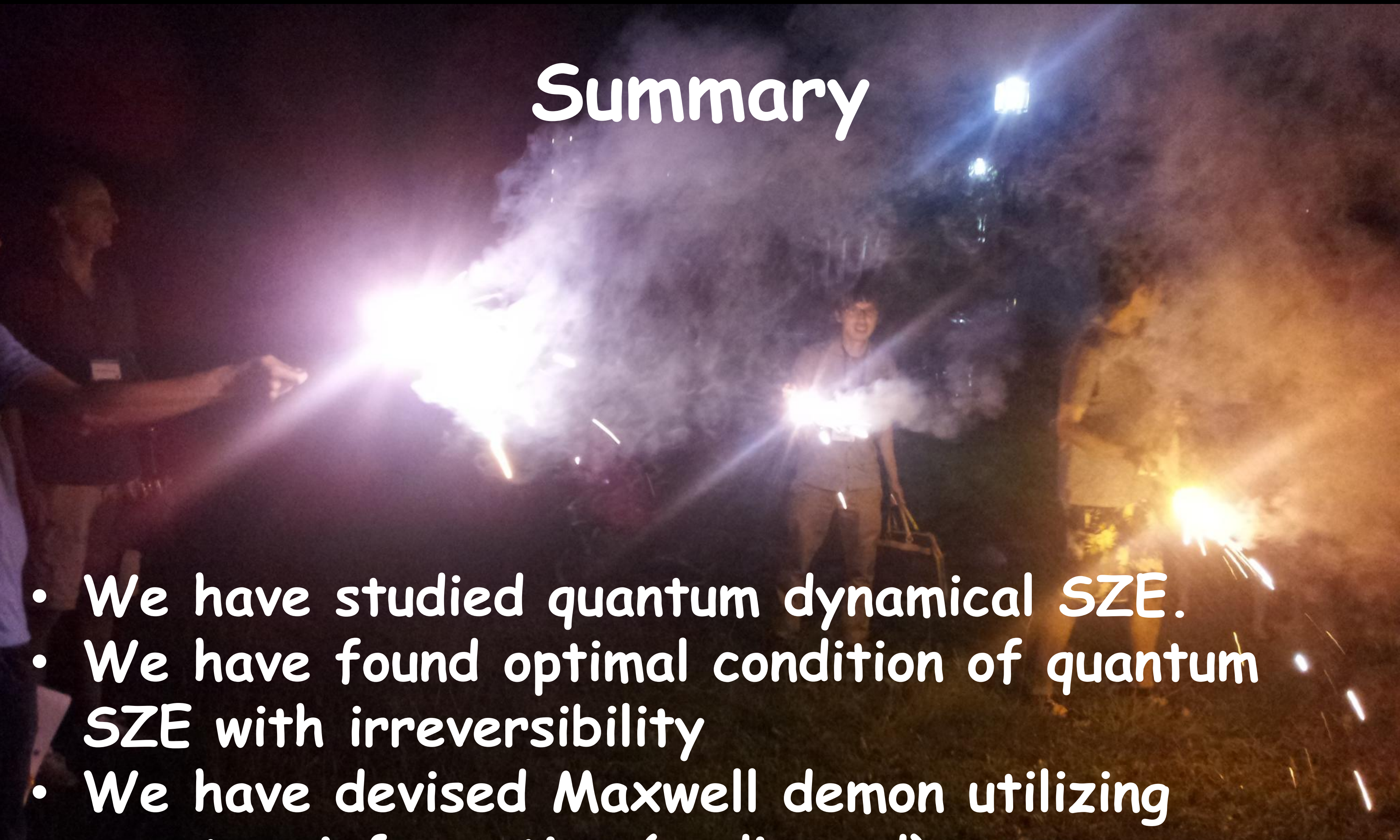
Phys.org, 18 Dec 2013 "Maxwell's demon can use quantum information to generate work"

H. J. Jeon and S. W. Kim
Optimal work of quantum Szilard engine with isothermal processes arXiv:1401.1685



Summary

- We have studied quantum dynamical SZE.
- We have found optimal condition of quantum SZE with irreversibility
- We have devised Maxwell demon utilizing quantum information (q-discord)

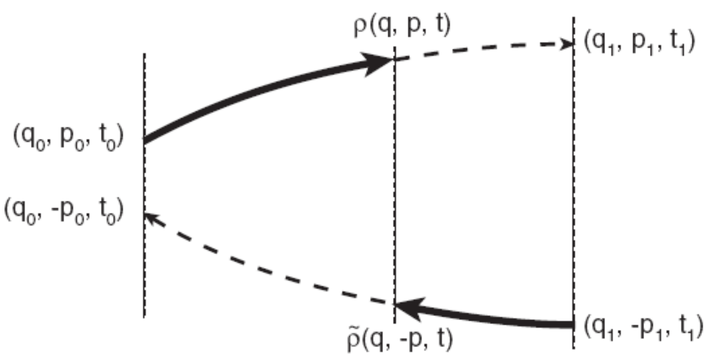
Non-equilibrium thermodynamics

- Jarzynski equality (1997)

$$\left\langle e^{-\beta W} \right\rangle_{\text{non-equilibrium}} = e^{-\beta \Delta F}$$

-The dissipative work for non-equilibrium process

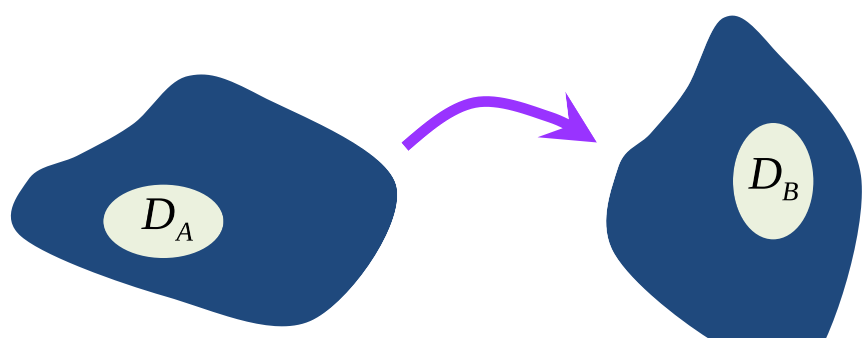
$$\langle W \rangle_{diss} = \langle W \rangle - \Delta F = kT \left\langle \ln \frac{\rho}{\tilde{\rho}} \right\rangle_{\rho}$$



Kawai, Parrando & Van den Broeck,

-The dissipative work for non-equilibrium process with filtering or feedback control

$$kT \left\langle \ln \frac{\rho}{\tilde{\rho}} \right\rangle_{\rho} + kT \ln \frac{p_{D_A}}{p_{D_B}} = \langle W \rangle - \Delta F$$



$t = t_0$ $\qquad$ $t = t_1$

Parrando, Van den Broeck & Kawai,

New J. Phys. (2009)

Physical meaning of $-\Sigma f\ln(f/f^*)$

$$\cancel{kT\left\langle \ln\frac{\rho}{\tilde{\rho}}\right\rangle_{\rho}} + kT\ln\frac{p_{D_A}}{p_{D_B}} = \langle W\rangle - \cancel{\Delta F}$$

quasi-static process

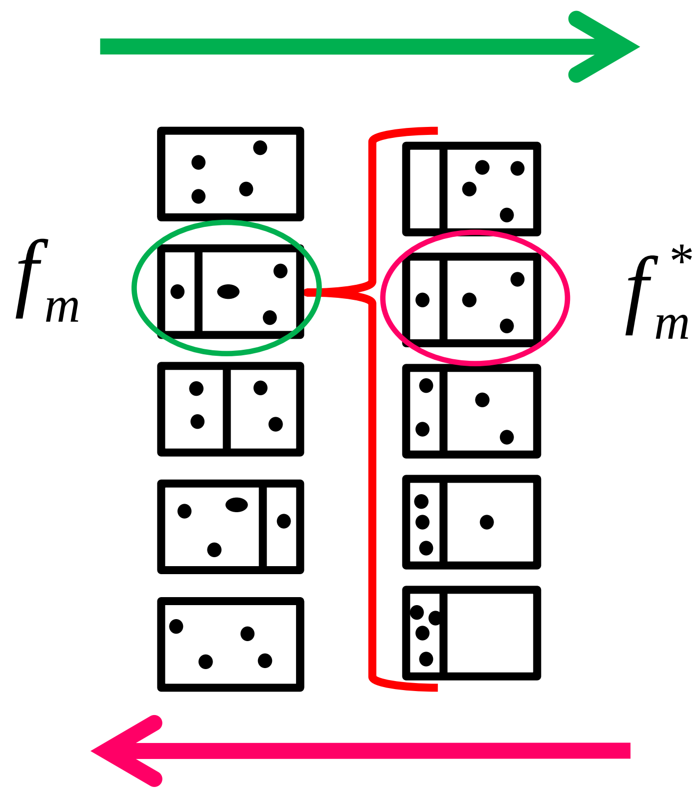
cyclic engine

forward filtering $p_{D_A} = f_m$

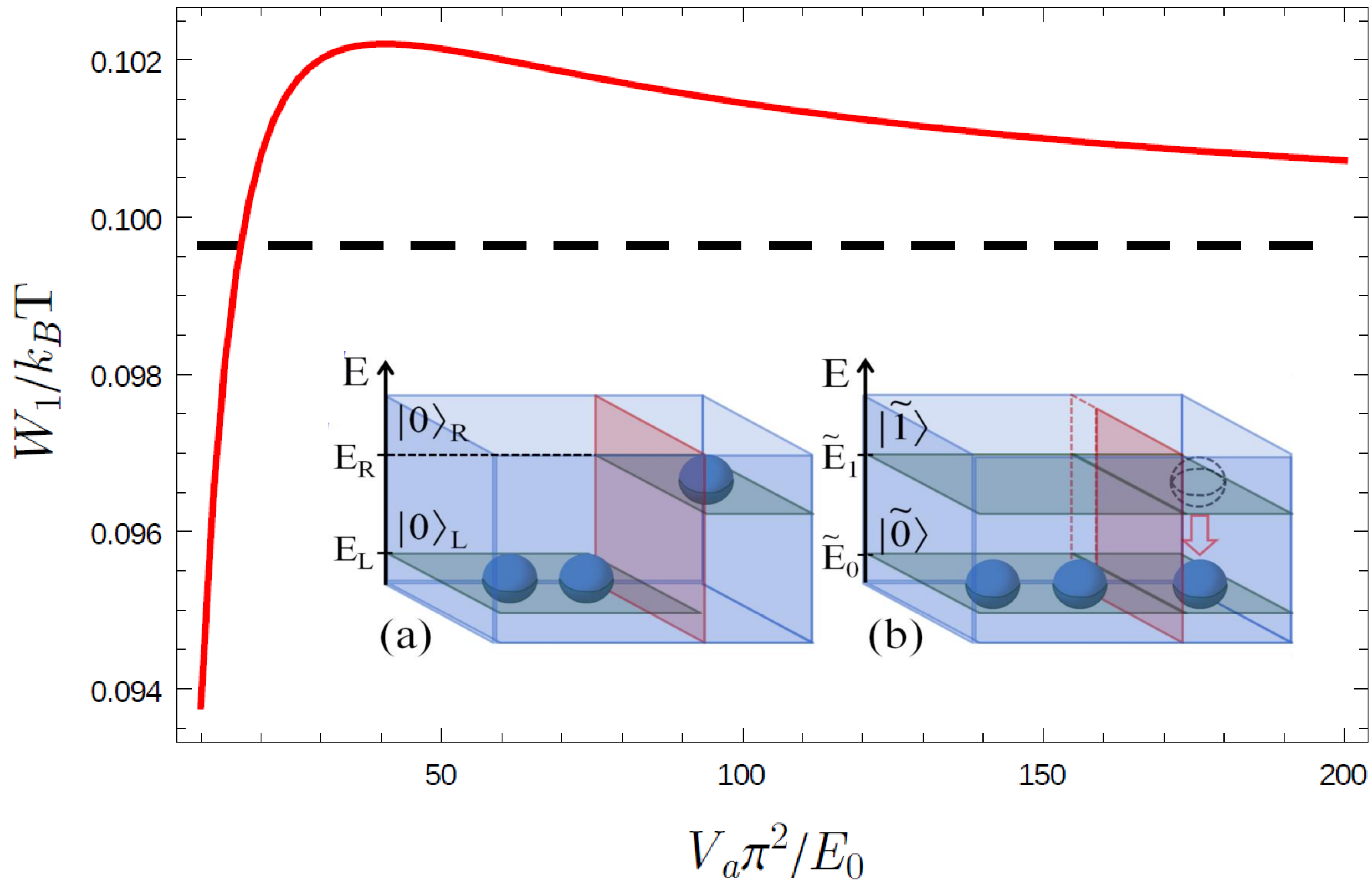
backward filtering $p_{D_B} = f_m^*$

$$\langle W\rangle_{tot} = \sum_m f_m \langle W\rangle_m$$

$$= kT\sum_m f_m \ln\left(\frac{f_m}{f_m^*}\right)$$



Remark I

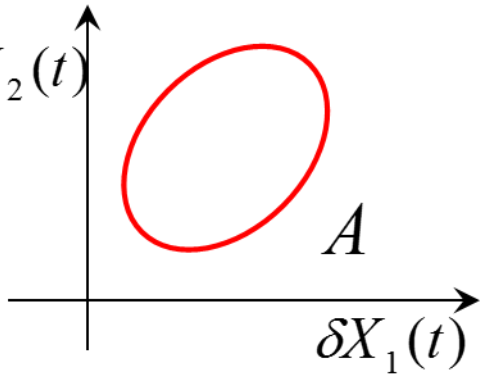


0.102
0.100
0.098
0.096
0.094
50
100
150
200
$W_1/k_B\mathrm{T}$
$V_a\pi^2/E_0$
E
$|0\rangle_R$
E_R
$|0\rangle_L$
E_L
(a)
E
$|\tilde{1}\rangle$
$\tilde{E}_1$
$|\tilde{0}\rangle$
$\tilde{E}_0$
(b)

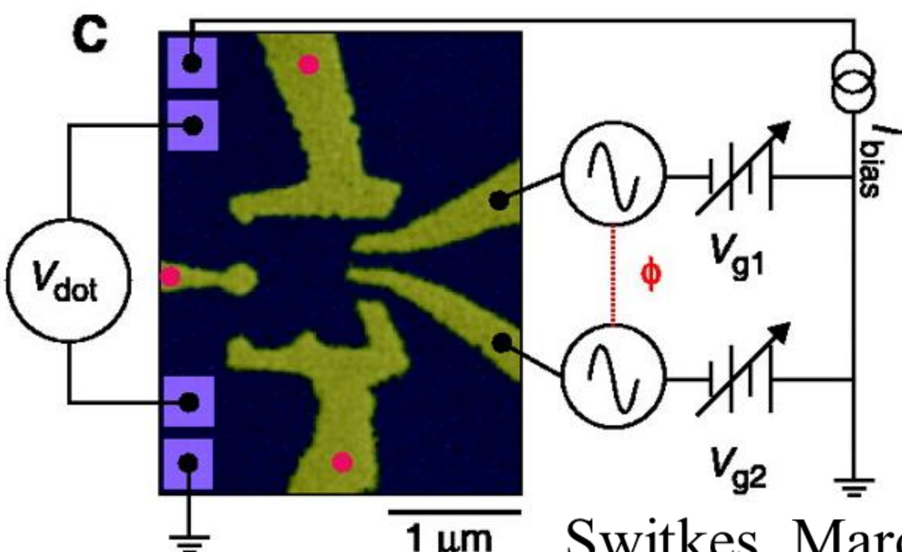
Adiabatic Q-pump

$$Q(m)=\frac{e}{\pi}\int_A dX_1 dX_2 \sum_{\beta,\alpha\in m} \mathrm{Im}\frac{\partial S_{\alpha\beta}}{\partial X_1}\frac{\partial S_{\alpha\beta}^{*}}{\partial X_2}$$

$$\delta X_1(t)=\delta X_1 \sin\omega t,\ \delta X_2(t)=\delta X_2 \sin(\omega t+\phi)$$

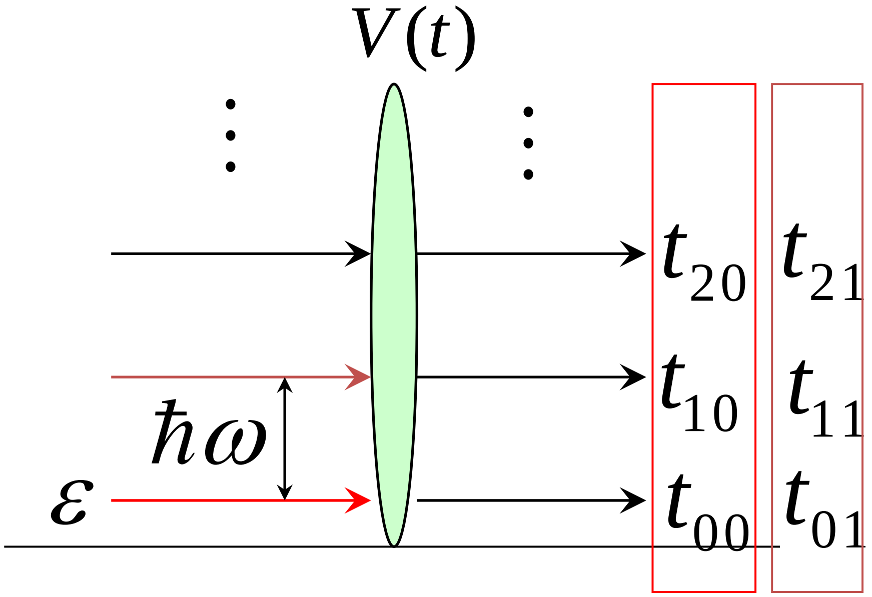


Brouwer (1998)



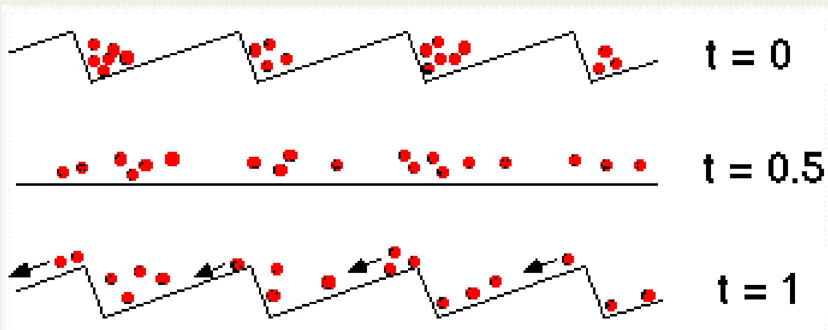
Switkes, Marcus, Campman, and Gossard (1999)

$$S = \begin{pmatrix} r_{00} & r_{01} & \cdots & t_{00}' & t_{01}' & \cdots \\ r_{10} & r_{11} & \cdots & t_{10}' & t_{11}' & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\ t_{00} & t_{01} & \cdots & r_{00}' & r_{01}' & \cdots \\ t_{10} & t_{11} & \cdots & r_{10}' & r_{11}' & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \end{pmatrix}$$

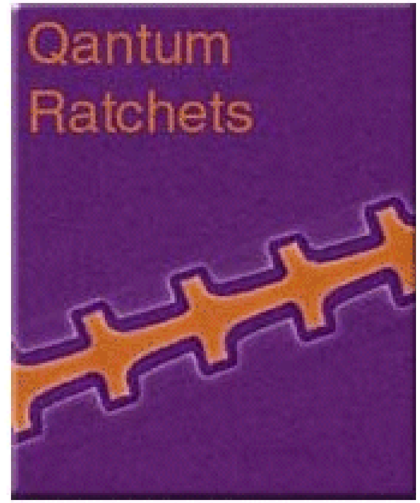


$$I = \frac{2e}{h}\int dEdE'\left[t_{\rightarrow}(E',E)\,f_L(E) - t_{\leftarrow}(E,E')\,f_R(E')\right]$$

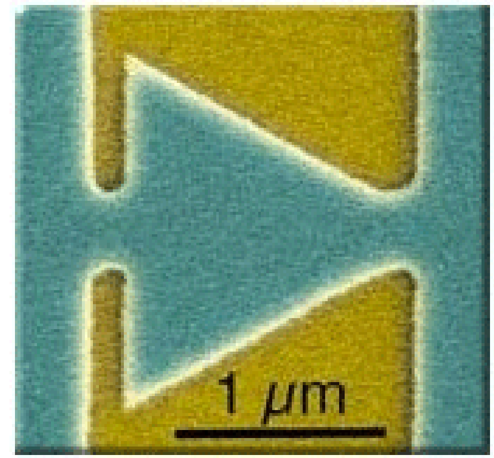
Classical/Quantum ratchet



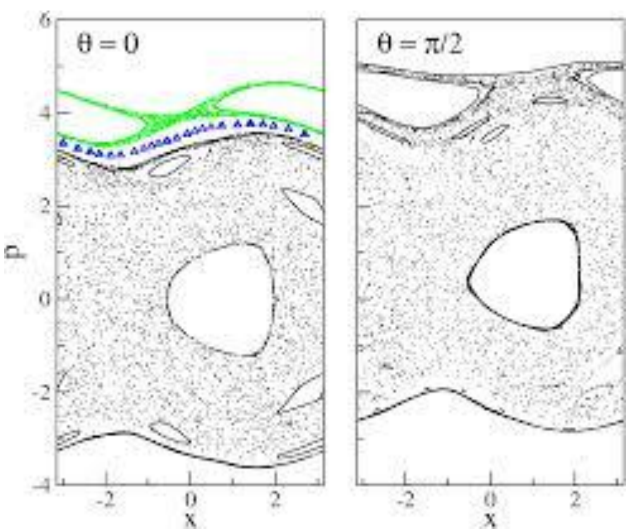
Classical ratchet



SEM image of a tunneling ratchet (highlighted in orange)



Electron microscopic image of a quantum dot ratchet



http://www.uoregon.edu/~linke/