Quasi-stationary states in periodically driven quantum systems

Takashi Mori

Univ. of Tokyo

with Tomotaka Kuwahara, Keiji Saito

Outline

- Introduction
- Result for local driving

  setup, theorem, outline of the proof
- Result for global driving

  setup, theorem, outline of the proof
- Discussion

Periodically driven systems

Typical nonequilibrium problem

Rich phenomena due to periodic driving

- Dynamical localization — Dunlap and Kenkre, PRB (1986)
- Coherent destruction of tunneling — Grossmann, et al. PRL (1991)
- Dynamical phase transition — Prosen and Ilievski, PRL (2011); Bastidas, et al. PRL (2012)

Quantum engineering ultracold atoms, trapped ions

- Control of quantum transport — Kitagawa, et. al. PRB (2011)
- Control of quantum topological phases — Lindner, et. al. Nat. Phys. (2011)

Floquet Theory

time evolution operator in one period

$$\mathcal{T} e^{-i\int_0^T dt H(t)} = e^{-iH_F T} \qquad \omega = \frac{2\pi}{T}$$

Floquet Hamiltonian

$$H_F|\phi_\alpha\rangle = \varepsilon_\alpha|\phi_\alpha\rangle \qquad \varepsilon_\alpha \in \left[-\frac{\omega}{2}, \frac{\omega}{2}\right)$$

"1st Brillouin zone"

Quantum state at time *t*

$$|\psi(t)\rangle = \sum_\alpha C_\alpha e^{-i\varepsilon_\alpha t}|\phi_\alpha(t)\rangle$$

$$|\phi_\alpha(t)\rangle = |\phi_\alpha(t+T)\rangle$$

$$|\phi_\alpha(t)\rangle = \mathcal{T} e^{-i\int_0^t ds[H(s)-\varepsilon_\alpha]}|\phi_\alpha\rangle$$

stroboscopic observation

$$|\psi(mT)\rangle = \sum_\alpha C_\alpha e^{-i\varepsilon_\alpha mT}|\phi_\alpha\rangle$$

High-frequency regime

n-th order truncation of Floquet-Magnus expansion

$$H_F^{(n)} = \sum_{m=0}^{n} T^m \Omega_m$$

effective static Hamiltonian

$$\Omega_0 = \frac{1}{T}\int_0^T dt H(t) \qquad \Omega_1 = \frac{1}{2T^2}\int_0^T dt_2 \int_0^{t_2} dt_1 [H(t_2), H(t_1)]$$

$$\Omega_n = \frac{1}{(n+1)^2 n!}\sum_\sigma (-1)^{n-\theta(\sigma)} \theta(\sigma)!(n-\theta(\sigma))!$$

$$\times \frac{1}{T^{n+1}}\int_0^T dt_{n+1}\int_0^{t_{n+1}} dt_n \ldots \int_0^{t_2} dt_1 [H(t_{\sigma(n+1)}), H(t_{\sigma(n)}), \ldots, [H(t_{\sigma(2)}), H(t_{\sigma(1)})] \ldots]]$$

σ permutation

$$\theta(\sigma) = \sum_{i=1}^{n} \theta(\sigma(i+1) - \sigma(i))$$

I. Bialynicki-Birula et al (1969)

$$\|\Omega_n\| \leq \frac{1}{(n+1)^2} \sup_{0\leq t_1, t_2, \ldots, t_{n+1} \leq T} \|[H(t_{n+1}), [H(t_n), \ldots, [H(t_2), H(t_1)] \ldots]]\|$$

Recent theoretical studies

Perspective from the Eigenstate Thermalization Hypothesis (ETH)

ETH: All the energy eigenstates with macroscopically same energy eigenvalues look the same

✓ Each energy eigenstate is indistinguishable from the microcanonical (or canonical) ensemble

Floquet ETH: All the Floquet eigenstates look the same

✓ Each Floquet eigenstate is indistinguishable from the infinite-temperature state (completely random state)

D'Alessio and Rigol, PRX (2014)
Lazarides, Das, and Moessner, PRE (2014)
Ponte, Chandran, Papic, and Abanin, Ann. Phys. (2015)

Long-time behavior

$$|\psi(t)\rangle = \sum_{\alpha} C_\alpha e^{-i\varepsilon_\alpha t} |\phi_\alpha(t)\rangle$$

Stroboscopic infinite-time average

$$\lim_{\mathcal{M}\to\infty} \frac{1}{\mathcal{M}} \sum_{m=1}^{\mathcal{M}} |\psi(mT)\rangle\langle\psi(mT)| = \sum_{\alpha} |C_\alpha|^2 |\phi_\alpha\rangle\langle\phi_\alpha|$$

Floquet ETH → The system heats up to infinite temperature

$$|\phi_\alpha\rangle\langle\phi_\alpha| \approx \lim_{\beta\to+0} \rho_\beta^{\mathrm{can}}$$

$$|\psi(t)\rangle\langle\psi(t)| \approx \lim_{\beta\to+0} \rho_\beta^{\mathrm{can}}$$

Truncated Floquet Hamiltonian → Energy is localized

$$e^{-iH_F^{(n)}t} |\psi(0)\rangle\langle\psi(0)| e^{iH_F^{(n)}t} \approx \frac{e^{-\beta H_F^{(n)}}}{Z}, \quad \beta \neq 0$$

Convergence radius of FM expansion

Convergence of FM expansion is ensured only when

$$\|H(t)\|T \leq \mathcal{O}(1)$$

For macroscopic systems or systems with unbounded Hamiltonian, the above condition is not satisfied.

Validity of FM expansion is not clear.

In some works on condensed matter, some nontrivial states of matter are predicted by the truncation of FM expansion.

Many-body systems

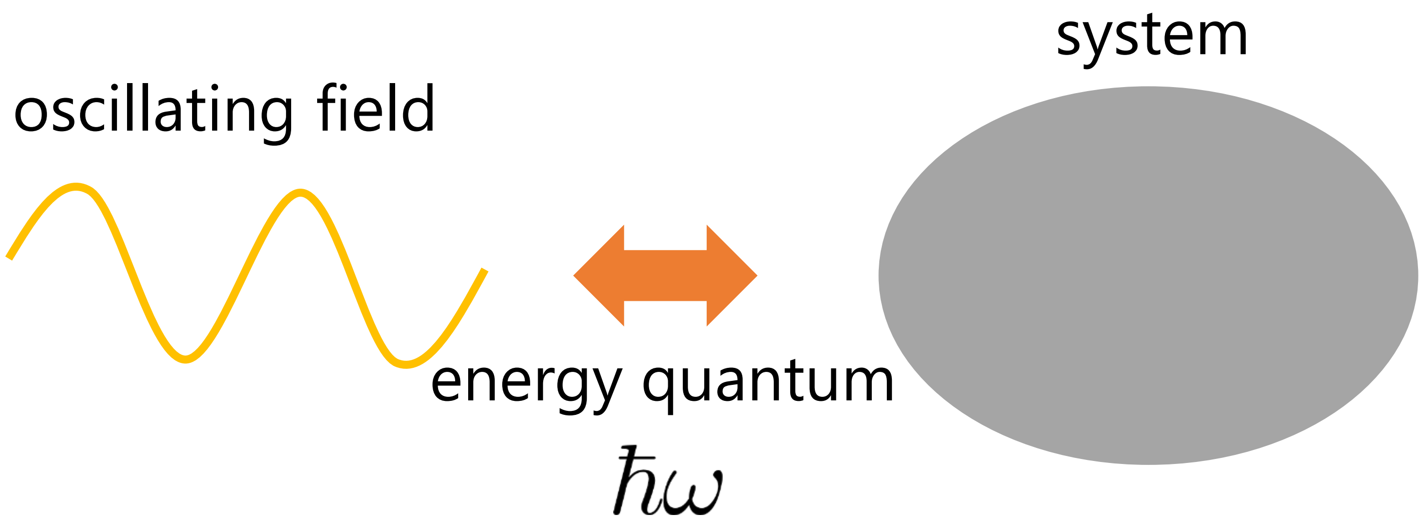
On the continuous space

→ Energy absorption is a single-particle process

(one-particle model will be enough to capture the physics)

On the lattice

→ Energy absorption as a **many-body phenomenon**



Motivation and Summary of the results

Validity of FM expansion and truncated Floquet Hamiltonian for high frequency regime

- Rigorous inequality for local driving

  For any bounded observable, its expectation value at time *t* is very close to that calculated by the truncated Floquet Hamiltonian up to exponentially long time.

- Rigorous inequality for global driving

  The energy absorption is exponentially slow.

Setup: Many-body lattice system

k-local and g-extensive Hamiltonian

\# of sites is N

$$H(t) = \sum_{|X| \le k} h_X(t) \qquad \forall x, \quad \sum_{X \ni x} \|h_X(t)\| \le g$$

- k-local: up to k-body interactions

  $H(t)$ may include any long-range interactions.

- g-extensive: single-site energy is bounded by g

$$H(t) = H_0 + V(t) \qquad \int_0^T V(t)dt = 0$$

periodicity $H(t) = H(t+T)$ $\qquad \omega = \frac{2\pi}{T}$

Fundamental inequalities

For k_A-local and g_A-extensive operator A and k_B-local operator B,

$$\|[A, B]\| \leq 2 g_A k_B \overline{B}$$

$$\|[A, B]\| \leq 2\|A\|\|B\|$$

✖ in the analysis of many-body systems

$$B = \sum_{|X| \leq k_B} b_X \qquad \overline{B} = \sum_{|X| \leq k_B} \|b_X\|$$

$$\boxed{\left\| [A_n, [A_{n-1}, \dots, [A_1, B] \dots]] \right\| \leq \left(\prod_{i=1}^{n} 2 g_{A_i} K_i \right) \overline{B}}$$

$$K_i = k_B + \sum_{j=1}^{i-1} k_{A_j}$$

Improved upper bound of each term of FM expansion

$$\|\Omega_n\| \leq \frac{1}{(n+1)^2} \sup_{0\leq t_1,t_2,\dots,t_{n+1}\leq T} \|[H(t_{n+1}),[H(t_n),\dots,[H(t_2),H(t_1)]\dots]]\|$$

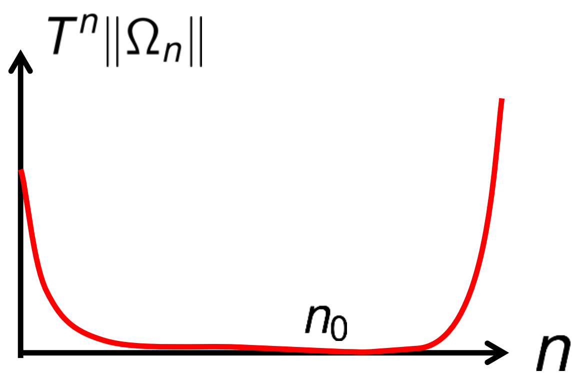
Naive upper bound: $\|[A,B]\| \leq 2\|A\|\|B\|$

$$\|\Omega_n\| \leq \frac{1}{(n+1)^2} \left(\sup_{0\leq t\leq T} \|H(t)\| \right)^{n+1}$$

Improved upper bound:

The driving field $V(t)$ is applied only to M sites

$$\|\Omega_n\| \leq \frac{2g(2gk)^n n!}{(n+1)^2} M$$



asymptotic expansion?

$$H_F^{(n_0)} = \sum_{n=0}^{n_0} T^n \Omega_n$$

$T^n\|\Omega_n\|$ is decreasing up to $n = n_0 = \mathcal{O}(\omega)$

Main result for local driving

Theorem (for more precise statement, see T. Kuwahara, TM, K. Saito, in preparation)

Consider k-local and g-extensive operators H_0, $V(t)$, and $H(t)$, and assume that the driving field $V(t)$ acts nontrivially only to M sites. If the period T satisfies $MT^2 \le \alpha$ with α some constant depending only on g and k, for any initial state $|\psi\rangle$,

$$\left\| \left(\mathcal{T} e^{-i\int_0^T dt H(t)} - e^{-iH_F^{(n_0)}T} \right) |\psi\rangle \right\| \le C \exp\left(-\frac{D}{T}\right) T$$

where C and D are also constants depending only on k and g, and n_0 is the maximum constant not exceeding $\frac{1}{8gkT}$.

$MT^2 \le \alpha$ implies $M \le \mathcal{O}(\omega^2)$. (local driving)

Truncated FM expansion is valid up to exponentially long time

one period

$$\left\| \left(\mathcal{T} e^{-i\int_0^T dt H(t)} - e^{-iH_F^{(n_0)} T} \right) |\psi\rangle \right\| \le C \exp\left(-\frac{D}{T} \right) T$$

m period $t = mT$

$$\left\| \left(\mathcal{T} e^{-i\int_0^t ds H(s)} - e^{-iH_F^{(n_0)} t} \right) |\psi\rangle \right\| \le C \exp\left(-\frac{D}{T} \right) t$$

Quasi-stationary states

$$\left\| \left(\mathcal{T} e^{-i\int_0^t ds H(s)} - e^{-iH_F^{(n_0)} t} \right) |\psi\rangle \right\| \leq C \exp\left(-\frac{D}{T} \right) t$$

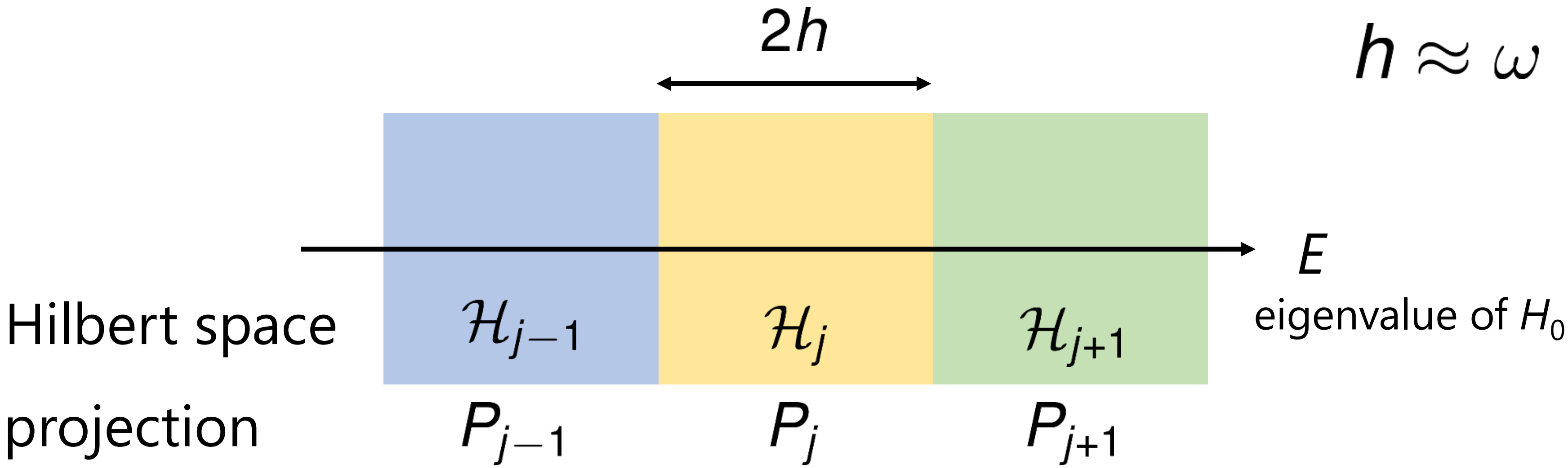
Time evolution is approximately governed by the truncated Floquet Hamiltonian $H_F^{(n_0)}$. If this is ergodic, the system will reach the quasi-stationary state described by the microcanonical ensemble concerned with $H_F^{(n_0)}$,

$$\mathrm{Tr} O |\psi(t)\rangle\langle\psi(t)| \approx \mathrm{Tr} O \rho_{\mathrm{mc}}^{(n_0)}$$

Here, for any $n<n_0$, we can show $H_F^{(n)} = H_F^{(n_0)} + \mathcal{O}(T^{n+1})$ and therefore we expect $\rho_{\mathrm{mc}}^{(n)} \approx \rho_{\mathrm{mc}}^{(n_0)}$ when T is small.

Quasi-stationary state described by $\rho_{\mathrm{mc}}^{(n)}$ whose lifetime is $\tau \sim e^{\mathcal{O}(\omega)}$

Outline of the proof 1: division of Hilbert space into energy blocks



$$\Delta U := \mathcal{T} e^{-i\int_0^T dt H(t)} - e^{-iH_F^{(n_0)} T}$$

$$\|\Delta U|\psi\rangle\| = \left\| \Delta U \sum_{j=-\infty}^{\infty} P_j |\psi\rangle \right\| = \sqrt{\sum_{j,j'=-\infty}^{\infty} \langle\psi|P_{j'}\Delta U^\dagger \Delta U P_j|\psi\rangle}$$

$$= \sqrt{\underbrace{\sum_{|j-j'|>1} \langle\psi|P_{j'}\Delta U^\dagger \Delta U P_j|\psi\rangle}_{\text{locality of the energy excitation}} + \underbrace{\sum_{|j-j'|\le 1} \langle\psi|P_{j'}\Delta U^\dagger \Delta U P_j|\psi\rangle}_{\text{we can avoid the large norm of } H_0}}$$

Outline of the proof 2: Locality of the energy excitation

$$\sum_{|j-j'|>1} \langle\psi|P_{j'}\Delta U^\dagger \Delta U P_j|\psi\rangle \leq 4 \sum_{j=-\infty}^{\infty} \|P_j|\psi\rangle\|^2 \sum_{j'=j+2}^{\infty} \|P_{j'} e^{iH_F^{(n_0)}T} e^{-iH_F T} P_j\|$$

$$e^{-iH_F T} := \mathcal{T} e^{-i\int_0^T dt H(t)}$$

$$\|P_{j'} e^{iH_F^{(n_0)}T} e^{-iH_F T} P_j\| = \|\underline{P_{j'} e^{-\tau H_0}} e^{\tau H_0} e^{iH_F^{(n_0)}T} e^{-iH_F T} e^{-\tau H_0} \underline{e^{\tau H_0} P_j}\|$$

$$\leq e^{-\tau(2j'-1)h} \|P_{j'} e^{\tau H_0} e^{iH_F^{(n_0)}T} e^{-iH_F T} e^{-\tau H_0} P_j\| e^{\tau(2j+1)h}$$

$$\tau = \frac{1}{8gk} \qquad \leq \underline{e^{-2(j'-j-1)\tau h}} \|e^{\tau H_0} e^{iH_F^{(n_0)}T} e^{-\tau H_0}\| \cdot \|e^{\tau H_0} e^{-iH_F T} e^{-\tau H_0}\|$$

energy change is exponentially suppressed!

Outline of the proof 3: effective bound of Hamiltonian for single energy block

$$\|\Delta U P_j\| \leq \epsilon \quad \Rightarrow \quad \sum_{|j-j'|\leq 1} \langle \psi | P_{j'} \Delta U^\dagger \Delta U P_j |\psi\rangle \leq 3\epsilon^2$$

$$\|\mathcal{P}(\underbrace{H_0, H_0, \ldots, H_0}_{p}, A_1, A_2, \ldots, A_q) P_j\| \leq (h + 6gkK_{p,q})^p \|A_1\| \cdot \|A_2\| \ldots \|A_q\|$$

Product with some permutation

$$K_{p,q} = pk + \sum_{i=1}^{q} k_i$$

$$\leq \|H_0\| \cdot \|H_0\| \ldots \|H_0\| \cdot \|A_1\| \cdot \|A_2\| \ldots \|A_q\|$$ (if there is no P_j)

$$\|H_0\| \to h + 6gkK_{p,q}$$

effectively we can bound the Hamiltonian for single energy block P_j

Summary of the result for local driving

Theorem (for more precise statement, see T. Kuwahara, TM, K. Saito, in preparation)

Consider k-local and g-extensive operators H_0, $V(t)$, and $H(t)$, and assume that the driving field $V(t)$ acts nontrivially only to M sites. If the period T satisfies $MT^2 \leq \alpha$ with α some constant depending only on g and k, for any initial state $|\psi\rangle$,

$$\left\| \left(\mathcal{T} e^{-i\int_0^T dt H(t)} - e^{-iH_F^{(n_0)}T} \right) |\psi\rangle \right\| \leq C \exp\left(-\frac{D}{T} \right) T$$

where C and D are also constants depending only on k and g, and n_0 is the maximum constant not exceeding $\frac{1}{8gkT}$.

$MT^2 \leq \alpha$ implies $M \leq \mathcal{O}(\omega^2)$. (local driving)

Motivation and Summary of the results

Validity of FM expansion and truncated Floquet Hamiltonian for high frequency regime

✓ Rigorous inequality for local driving

For any bounded observable, its expectation value at time t is very close to that calculated by the truncated Floquet Hamiltonian up to exponentially long time.

- Rigorous inequality for global driving

The energy absorption is exponentially slow.

Setup

$$\left\|\left(\mathcal{T}e^{-i\int_0^T dtH(t)} - e^{-iH_F^{(n_0)}T}\right)|\psi\rangle\right\| \leq C\exp\left(-\frac{D}{T}\right)T$$

➡ Only for $M \leq \mathcal{O}(\omega^2)$

✖ Global driving $M{\sim}N$

Focus on the dynamics of local operators!

O: $(l+1)k$-local operator with some integer l

$$O(t) = \overline{\mathcal{T}}e^{i\int_0^t dsH(s)}O\mathcal{T}e^{-i\int_0^t dsH(s)} \equiv \overline{\mathcal{T}}e^{i\int_0^t dsL(s)}O$$

$$\tilde{O}^{(n_0)}(t) = e^{iH_F^{(n_0)}t}Oe^{-iH_F^{(n_0)}t} \equiv e^{iL_F^{(n_0)}t}O$$

$$\boxed{\|O(T) - \tilde{O}^{(n_0)}(T)\|}$$

Main result for global driving

Theorem (TM, T. Kuwahara, K. Saito, in preparation)

Consider an arbitrary $(l+1)k$-local operator $O = \sum_{|X| \le (l+1)k} o_X$.

If H_0, $V(t)$, and $H(t)$ are k-local and g-extensive,

for $T < \frac{1}{8gk}$,

$$\|O(T) - \tilde{O}^{(n_0)}(T)\| \le 16gk2^{-(n_0-l)}\overline{O}T$$

where $\overline{O} := \sum_X \|o_X\|$ and

n_0 is the maximum integer not exceeding $\frac{1}{8gkT} - 1$.

In particular, for $O=H_0$, the following stronger bound exists:

$$\|H_0(T) - \tilde{H}_0^{(n_0)}(T)\| \le 8g^2k2^{-n_0}MT$$

where we assume that driving is applied only to M sites.

Implication of the theorem

$$\|O(T)-\tilde{O}^{(n_0)}(T)\| \le \overline{O}\exp[-\mathcal{O}(\omega)]T$$

Time evolution of any local operator in one period is well approximated by the Hamilton dynamics of $H_F^{(n_0)}$.

$t = mT$

$$\cancel{\|O(t)-\tilde{O}^{(n_0)}(t)\| \le \overline{O}\exp[-\mathcal{O}(\omega)]t}$$

O is $(l+1)k$-local

$$\|O(T)-\tilde{O}^{(n_0)}(T)\| \le 16gk2^{-(n_0-l)}\overline{O}T$$

$O(t)$ is highly nonlocal (l is not constant but grows with time)

Exponentially slow heating

$$O = H_F^{(n_0)} \qquad \tilde{H}_F^{(n_0)}(t) = e^{iH_F^{(n_0)}t} H_F^{(n_0)} e^{-iH_F^{(n_0)}t} = H_F^{(n_0)}$$

$$\boxed{H_F^{(n_0)} = \sum_{n=0}^{n_0} T^n \Omega_n}$$

$$\Rightarrow \quad \| H_F^{(n_0)}(mT) - H_F^{(n_0)} \| \leq m \| H_F^{(n_0)}(T) - H_F^{(n_0)} \|$$

$$\| H_F^{(n_0)}(T) - H_F^{(n_0)} \| \leq \| H_0 - \tilde{H}_0^{(n_0)}(T) \| + \sum_{n=1}^{n_0} T^n \| \Omega_n(T) - \tilde{\Omega}_n^{(n_0)}(T) \|$$

$$\boxed{\| H_0(T) - \tilde{H}_0^{(n_0)}(T) \| \leq 8 g^2 k 2^{-n_0} M T} \qquad \boxed{\| O(T) - \tilde{O}^{(n_0)}(T) \| \leq 16 g k 2^{-(n_0 - l)} \overline{O} T}$$

$$\| \Omega_n \| \leq \overline{\Omega_n} \leq \frac{2g(2gk)^n n!}{(n+1)^2} M \qquad n! \leq n_0^n$$

Quasi-conserved quantity

$$\boxed{\| H_F^{(n_0)}(t) - H_F^{(n_0)} \| \leq 16 g^2 k 2^{-n_0} M t}$$

$$\| H_F^{(n_0)} - H_0 \| \leq M \mathcal{O}(T)$$

$$\Rightarrow \quad \frac{\| H_0(t) - H_0 \|}{N} \leq \frac{M}{N} \left[16 g^2 k 2^{-n_0} t + \mathcal{O}(T) \right]$$

Quasi-stationary states

$$\|H_F^{(n_0)}(t) - H_F^{(n_0)}\| \leq 16g^2 k 2^{-n_0} Mt$$

Quasi conserved energy → Microcanonical ensemble

$$\mathrm{Tr} O|\psi(t)\rangle\langle\psi(t)| \approx \mathrm{Tr} O\rho_{\mathrm{mc}}^{(n_0)}$$

microcanonical density matrix concerned with $H_F^{(n_0)}$

For any $n<n_0$, we can show

$$H_F^{(n)} = H_F^{(n_0)} + M\mathcal{O}(T^{n+1})$$

and therefore we expect $\rho_{\mathrm{mc}}^{(n)} \approx \rho_{\mathrm{mc}}^{(n_0)}$ when T is small.

Quasi-stationary state described by $\rho_{\mathrm{mc}}^{(n)}$ whose lifetime is $\tau \sim e^{\mathcal{O}(\omega)}$

Outline of the proof 1: compare Dyson expansions

$$\overline{\mathcal{T}} e^{i\int_0^T dtL(t)} = \sum_{n=0}^{\infty} T^n \mathcal{A}_n \qquad e^{iL_F^{(n_0)}T} = \sum_{r=0}^{\infty} \frac{(iL_F^{(n_0)}T)^r}{r!} = \sum_{n=0}^{\infty} T^n \tilde{\mathcal{A}}_n^{(n_0)}$$

$$\boxed{L_F^{(n_0)} = \sum_{l=0}^{n_0} T^l [\Omega_l, \cdot] \equiv \sum_{l=0}^{n_0} T^l L_l}$$

$$\mathcal{A}_n = \frac{i^n}{T^n} \int_0^T dt_n \int_{t_n}^T dt_{n-1} \dots \int_{t_2}^T dt_1 L(t_n) L(t_{n-1}) \dots L(t_1)$$

$$\tilde{\mathcal{A}}_n^{(n_0)} = \sum_{r=0}^{n} \sum_{\substack{\{l_i\}_{i=1}^r \\ 0 \le l_i \le n_0}} \frac{i^r}{r!} \chi\left(\sum_{i=1}^{r} (l_i + 1) = n \right) L_{l_1} \dots L_{l_r}$$

$$\boxed{\mathcal{A}_n = \tilde{\mathcal{A}}_n^{(n_0)} \text{ for } n \le n_0}$$

$$\| O(T) - \tilde{O}^{(n_0)}(T) \| \le \sum_{n=n_0+1}^{\infty} T^n \left(\| \mathcal{A}_n O \| + \| \tilde{\mathcal{A}}_n^{(n_0)} O \| \right)$$

Outline of the proof 2: Inequality for multi-commutators

$$\|O(T) - \tilde{O}^{(n_0)}(T)\| \leq \sum_{n=n_0+1}^{\infty} T^n \left(\|\mathcal{A}_n O\| + \|\tilde{\mathcal{A}}_n^{(n_0)} O\|\right)$$

$$\boxed{\left\| [A_n, [A_{n-1}, \dots, [A_1, B] \dots]] \right\| \leq \left(\prod_{i=1}^{n} 2 g_{A_i} K_i \right) \overline{B}}$$

$$K_i = k_B + \sum_{j=1}^{i-1} k_{A_j}$$

$$\|\mathcal{A}_n O\| \leq \frac{1}{n!} \sup_{0 \leq t_1, \dots, t_n \leq T} \|L(t_n) L(t_{n-1}) \dots L(t_1) O\| \leq (2gk)^n \frac{(n+l)!}{n! l!} \overline{O}$$

$$\|\tilde{\mathcal{A}}_n^{(n_0)} O\| \leq \sum_{r=0}^{n} \sum_{\substack{\{l_i\}_{i=1}^r \\ 0 \leq l_i \leq n_0}} \frac{1}{r!} \chi\left(\sum_{i=1}^{r} (l_i + 1) = n \right) \boxed{\|L_{l_1} \dots L_{l_r} O\|}$$

$$\|L_{l_1} \dots L_{l_r} O\| \leq (2gk)^r (2gkn_0)^{n-r} \frac{(n+l)!}{(n+l-r)!} \overline{O}$$

Summary of the result for global driving

Theorem (TM, T. Kuwahara, K. Saito, in preparation)

Consider an arbitrary $(l+1)k$-local operator $O = \sum_{|X| \le (l+1)k} o_X$.

If H_0, $V(t)$, and $H(t)$ are k-local and g-extensive,

for $T < \dfrac{1}{8gk}$,

$$\|O(T) - \tilde{O}^{(n_0)}(T)\| \le 16gk2^{-(n_0-l)}\overline{O}T$$

where $\overline{O} := \sum_X \|o_X\|$ and

n_0 is the maximum integer not exceeding $\dfrac{1}{8gkT} - 1$.

In particular, for $O=H_0$, the following stronger bound exists:

$$\|H_0(T) - \tilde{H}_0^{(n_0)}(T)\| \le 8g^2k2^{-n_0}MT$$

Where we assume that driving is applied only to M sites.

Open question 1: stronger result??

Assumption: k-locality and g-extensivity of the Hamiltonian

up to k-body interactions

single site energy is bounded by g

No assumption on the range of interactions

short-range interacting systems ⟺ long-range interacting systems

Lieb-Robinson bound

Can we obtain stronger results by restricting ourselves into short-range interacting systems?

T. Kuwahara, TM, K. Saito, in progress…

Open question 2: Final state

Up to an exponentially long time scale in frequency, the time evolution is governed by the truncated Floquet Hamiltonian.

What is the eventual long-time asymptotic state?

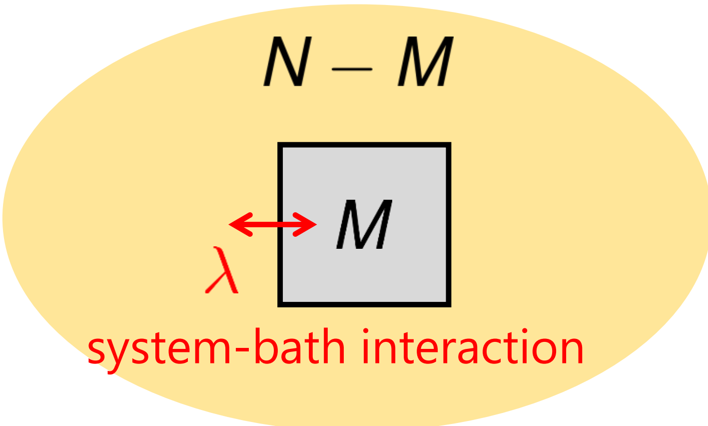
Floquet ETH: infinite temperature state (completely random state)

D'Alessio and Rigol, PRX (2014)
Lazarides, Das, and Moessner, PRE (2014)
Kim, Ikeda, and Huse, PRE (2014)
Ponte, Chandran, Papic, and Abanin, Ann. Phys. (2015)

Some reports: certain non-integrable systems do not heat up to infinite temperature

T. Prosen, PRL (1998)
D'Alessio and Polkovnikov, Ann. Phys. (2013)

Open question 3: open quantum systems



$N \to +\infty \qquad 1 \ll M \ll N$

open quantum system

system of interest

$\rho_S(t)$ reduced density matrix

thermal bath $\rho_B(0) = \dfrac{e^{-\beta H_B}}{\mathrm{Tr}e^{-\beta H_B}}$

$$\rho_S(t) \to \frac{e^{-\beta H_F^{(n)}}}{Z} \quad (t \lesssim \exp[\mathcal{O}(\omega)])$$

$$\boxed{\rho_{ss} = \lim_{t\to\infty} \lim_{N\to\infty} \rho_S(t)}$$

<u>In the van Hove limit</u> $\lambda \to 0, t \to \infty$ with $\lambda^2 t = \tau$ fixed

→ Floquet Born-Markov master equation

$$\rho_{ss} \neq \frac{e^{-\beta H_F^{(n)}}}{Z}$$

heating rate dissipation

$$\exp[-\mathcal{O}(\omega)] \gg \lambda^2$$

(in the van Hove limit)

↕

$$\exp[-\mathcal{O}(\omega)] \sim \lambda^2$$

T. Shirai, TM, S. Miyashita (2015)

Conclusion

Validity of FM expansion and truncated Floquet Hamiltonian for high frequency regime

✓ Rigorous inequality for local driving

For any bounded observable, its expectation value at time t is very close to that calculated by the truncated Floquet Hamiltonian up to exponentially long time.

✓ Rigorous inequality for global driving

The energy absorption is exponentially slow.