Time asymmetric driving and entropy production

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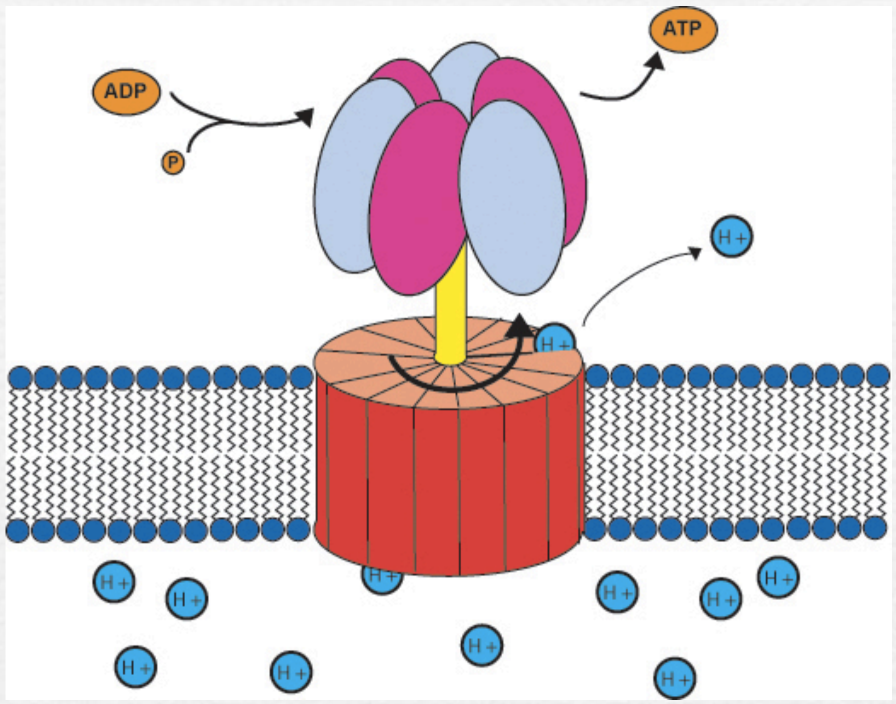
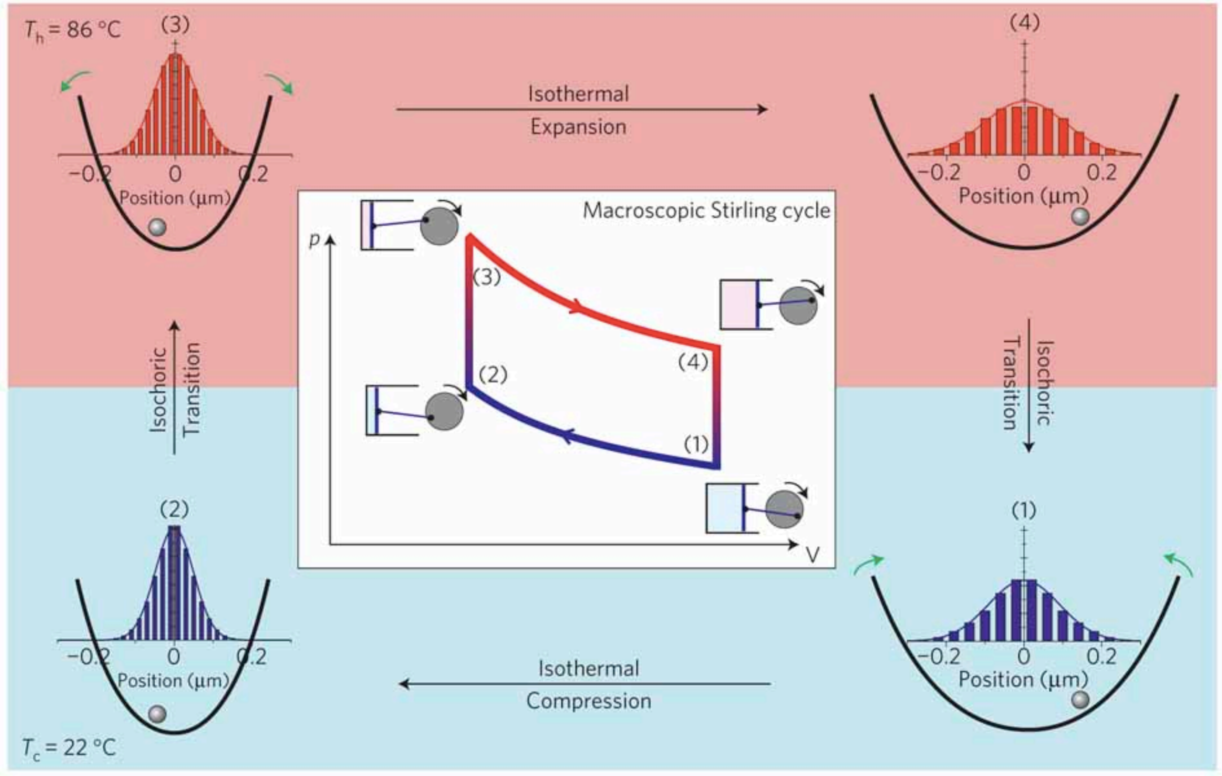
- Micromachines
- Reversible transport (1998)
- Hidden pumps (M. Esposito, 2014)
- Carnot cycles (Leo Granger and Johannes Hoppenau, 2015)

Kyoto, July 28th 2015

Micromachines





Autonomous

Driven

Non-feedback

Feedback

Symmetric

Non-symmetric

Reversible transport

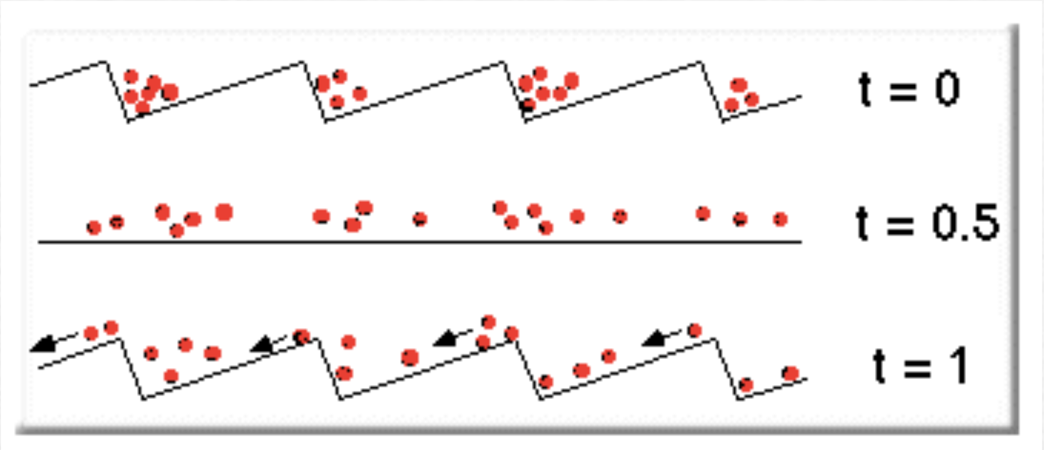
JMRP, PRE (1998). Reversible ratchets as Brownian particles in an adiabatically changing periodic potential.

An overdamped Brownian particle in a driven periodic potential:

$$\dot{x}(t) = -V'(x;\lambda(t)) + \xi(t)$$

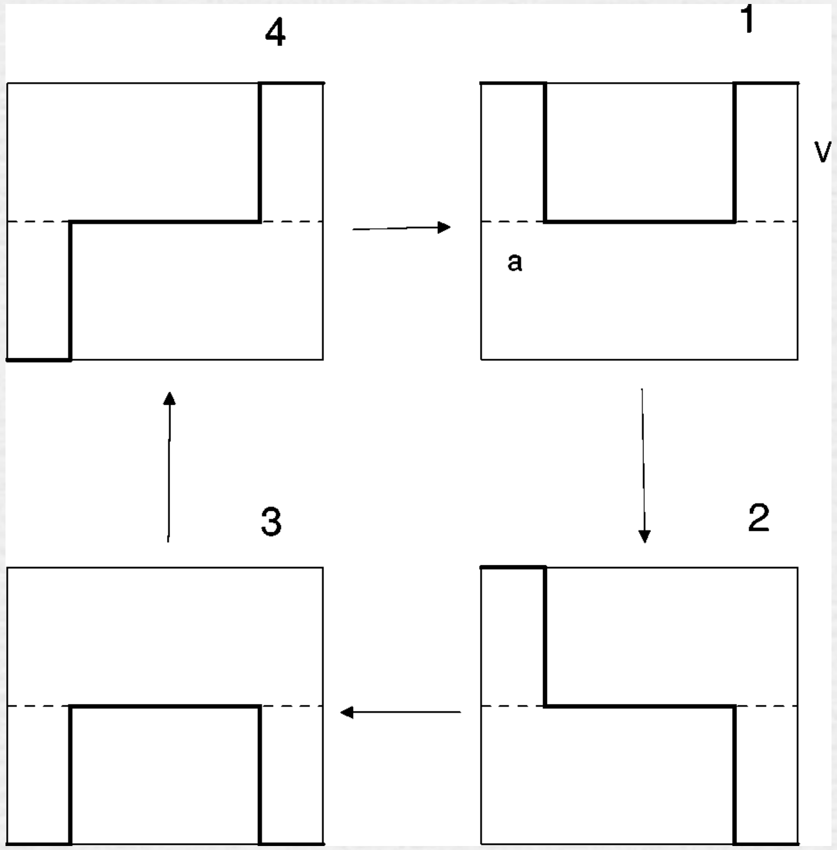
$$V(0;\lambda) = V(L;\lambda)$$

$$\lambda(t),\ 0 \le t \le \tau$$



Quasistatic limit:

$$\dot{\lambda}(t) \to 0 \Rightarrow \rho(x,t) = \frac{1}{Z(\lambda(t))} e^{-\beta V(x;\lambda(t))}$$



Zero current, BUT...

Reversible transport

Integrated current:

$$J = \int_0^\tau \delta J(t)$$

$$\delta J(t) = \int_0^L dx \int_0^x dx' \rho_+(x;\lambda(t)) \left[\vec{\nabla}_\lambda \rho_-(x';\lambda(t))\right] \cdot \dot{\lambda}(t) dt$$

$$\rho_\pm(x;\lambda) = \frac{e^{\pm\beta V(x;\lambda)}}{Z_\pm}$$

Not an exact differential

Total work:

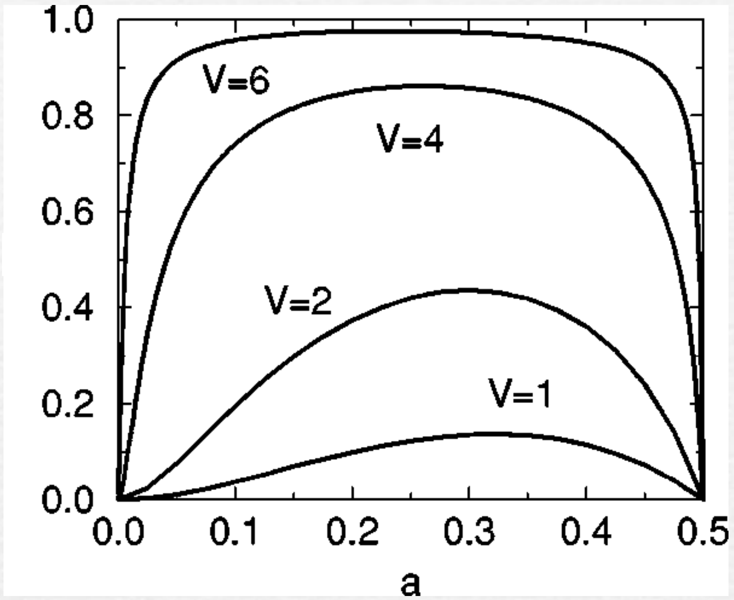
$$W = \int_0^\tau \delta W(t)$$

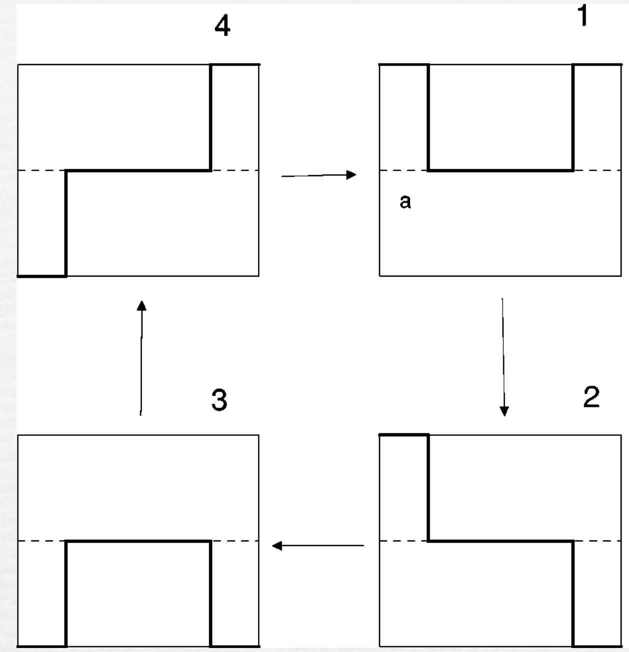
$$\delta W(t) = -kT \left[\vec{\nabla}_\lambda \ln Z_-(\lambda(t))\right] \cdot \dot{\lambda}(t) dt$$

Exact differential

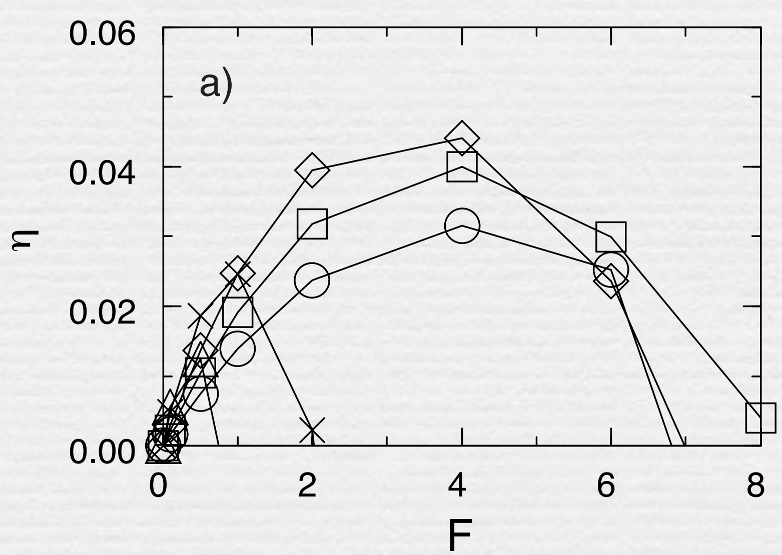
Reversible transport

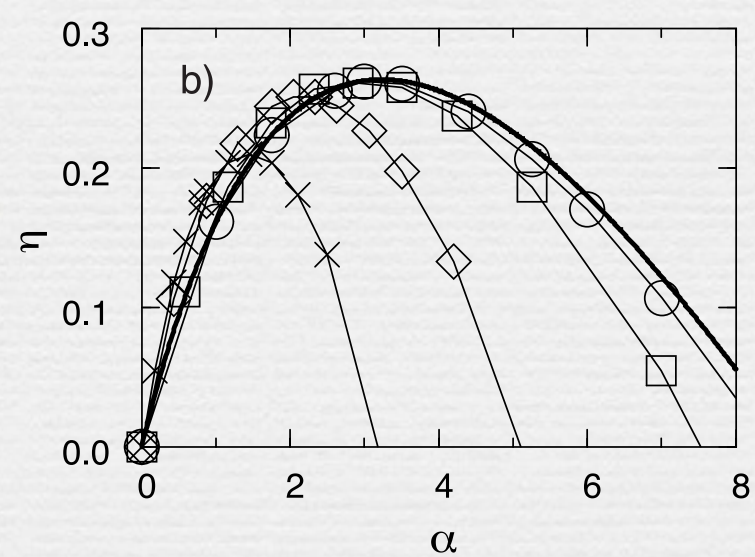
$$J = \int_0^\tau \delta J(t)$$

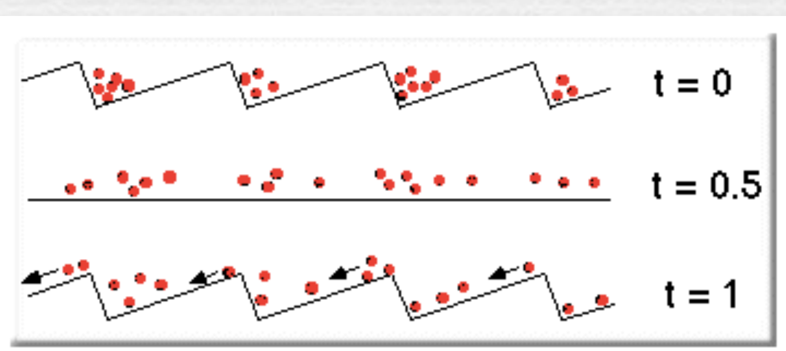




Efficiency:

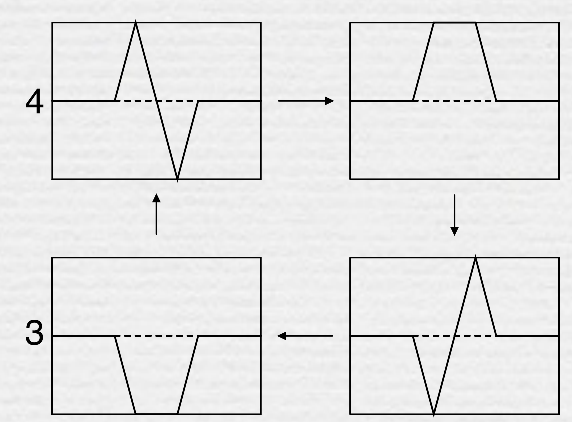






Irreversible ratchet

Reversible ratchet



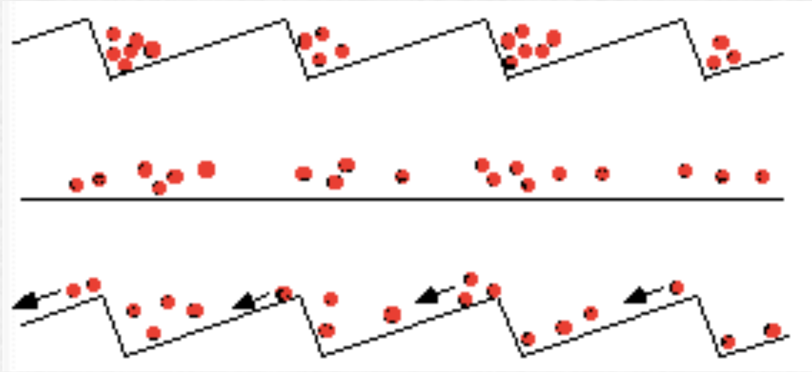
Reversible transport

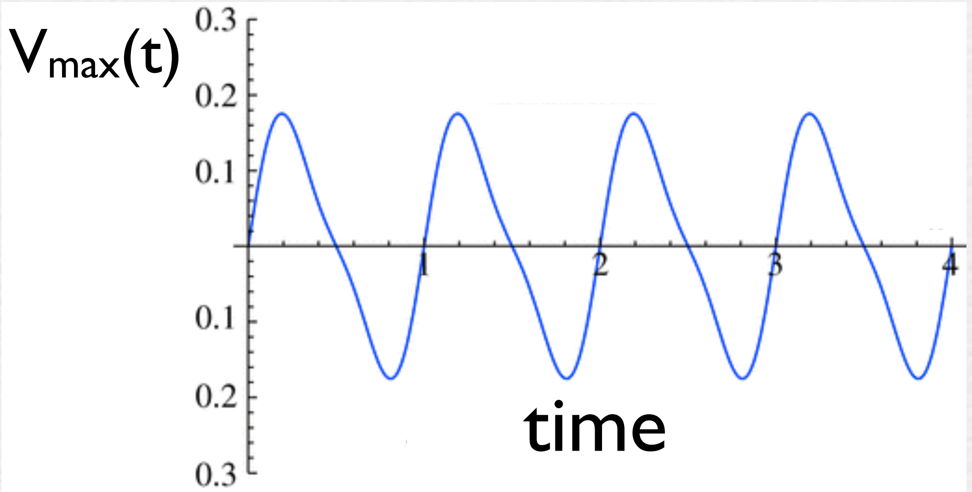
$$J = \int_0^\tau \delta J(t)$$

Line integral (it only depends on the path in the parameter space)

$$\delta J(t) = \int_0^L dx \int_0^x dx' \rho_+(x;\lambda(t)) \left[\vec{\nabla}_\lambda \rho_-(x';\lambda(t))\right] \cdot \dot{\lambda}(t)dt$$

$$= d\lambda(t)$$



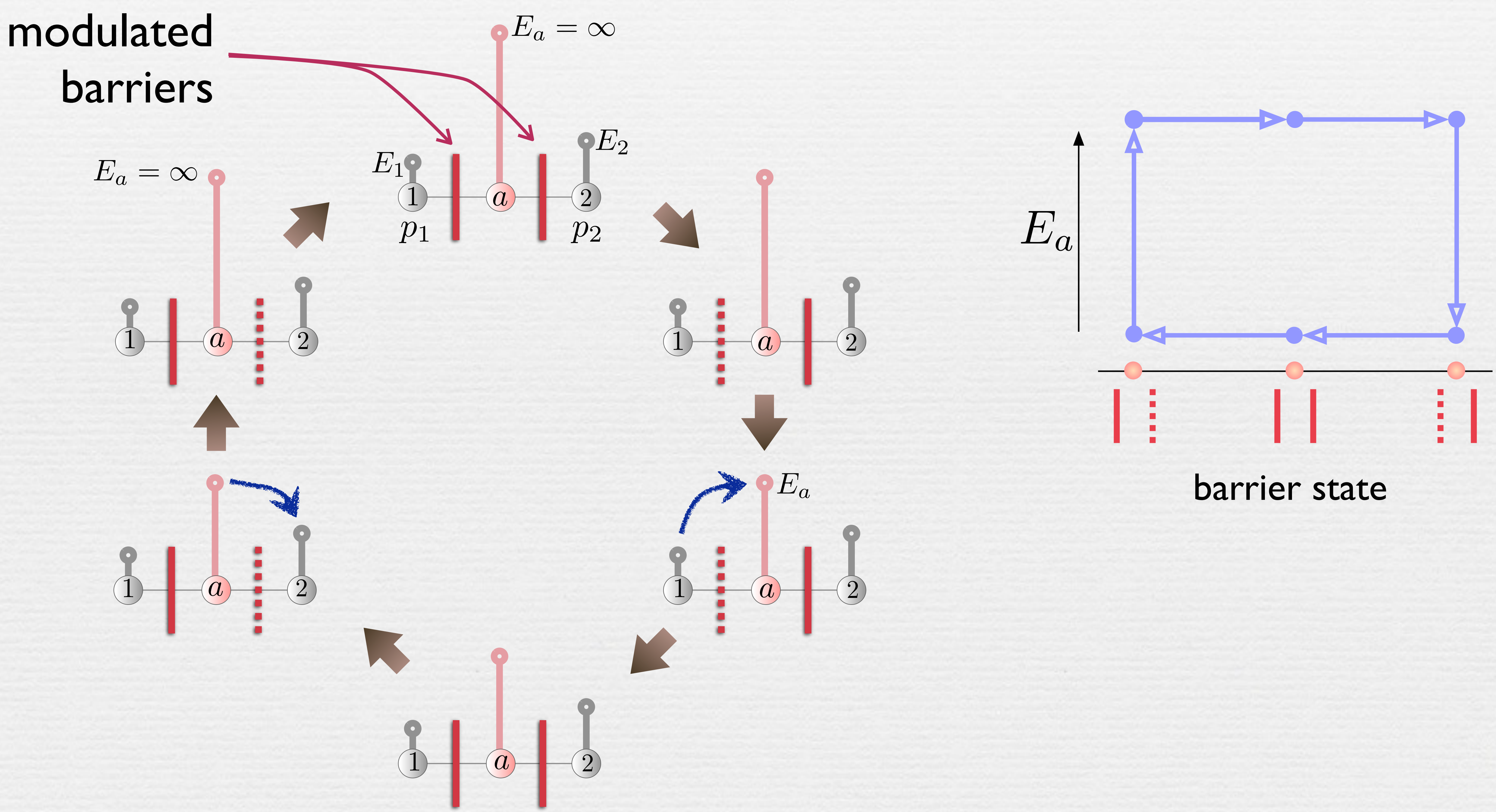


In a flashing ratchet with an asymmetric potential J=0

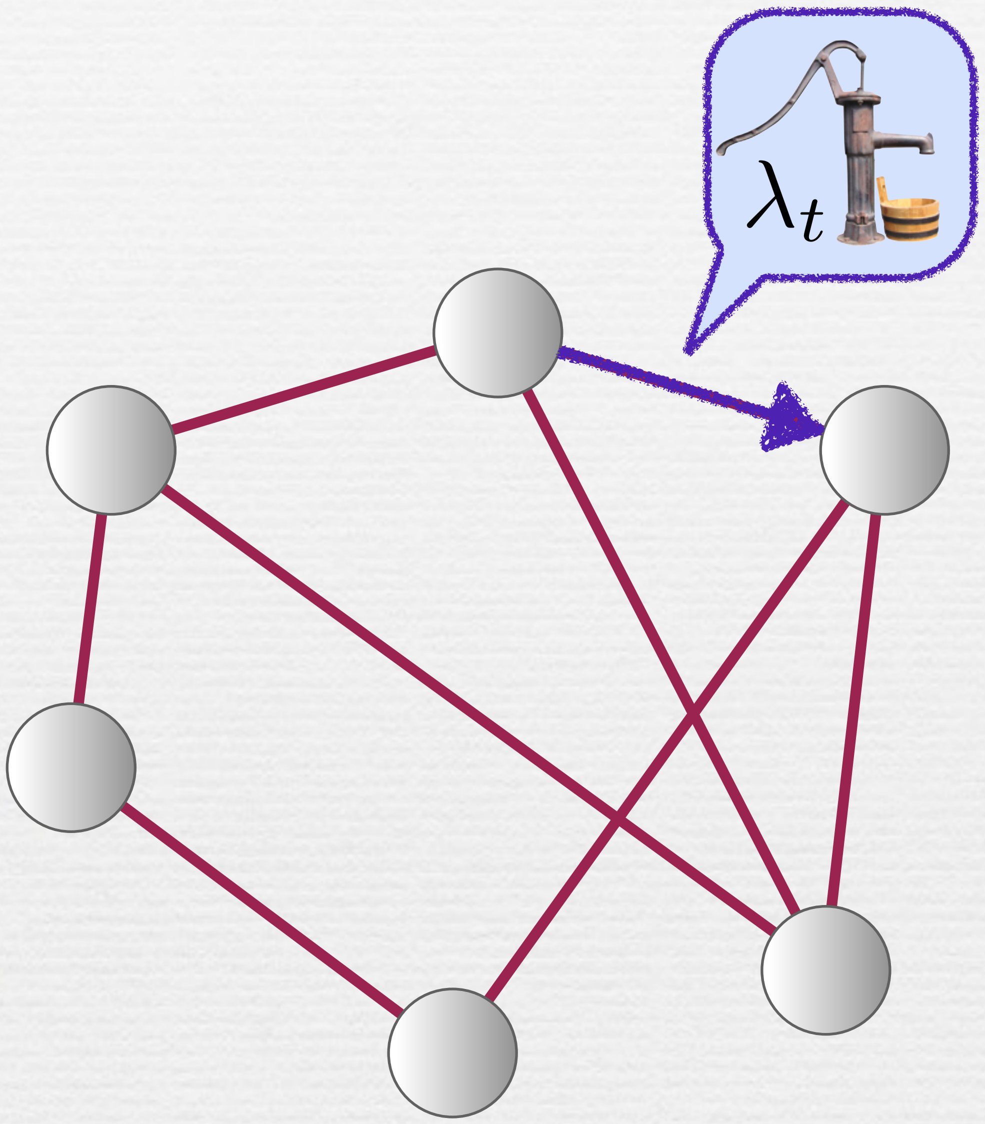
Time asymmetry is not enough to induce reversible transport

Adiabatic pumps (Astumian, PRL 2003)

An auxiliary state a

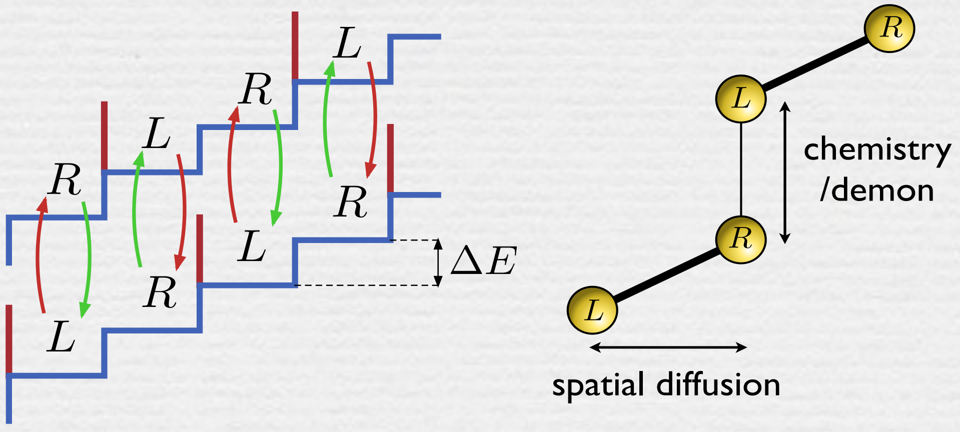


Hidden pumps (Esposito & JMRP. PRE 2015)

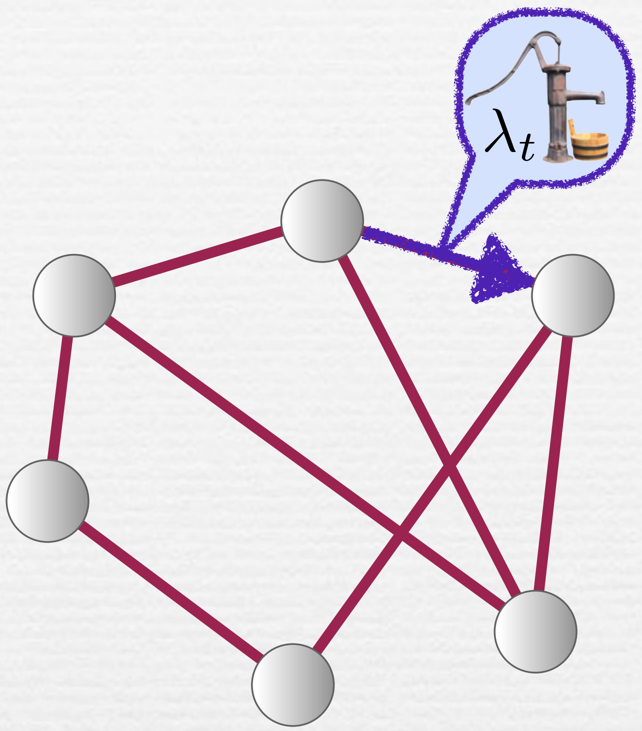


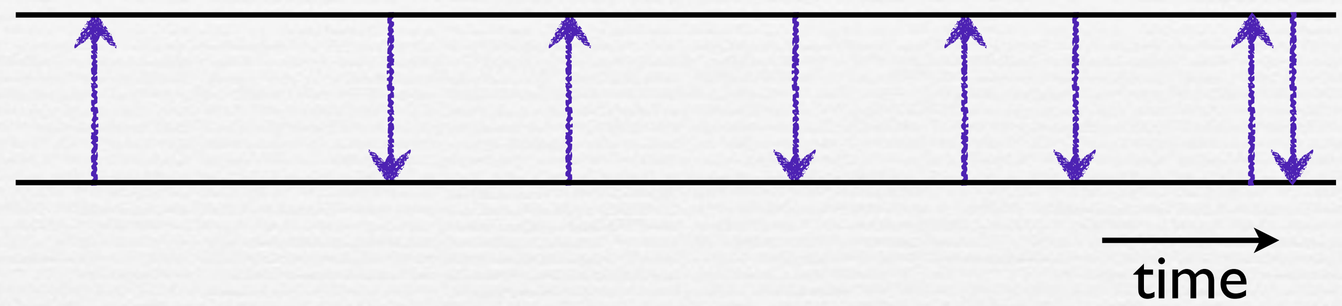
- A pump biases a transition, i.e., creates an effective force.
- The idea: design a protocol such that, at some coarse-grain level (hidden pump), the dynamics of the network is identical to that of an autonomous Markovian system.

The original motivation: To compare chemical motors and Maxwell demons (Horowitz, Sagawa, JMRP. PRL 2013).

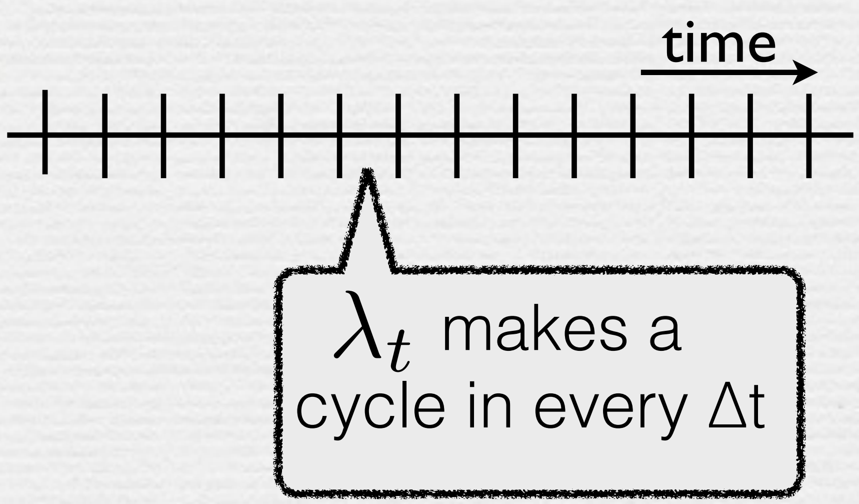


Hidden pumps





$$p_i\,(t+\Delta t) = p_i(t) + \sum_{j\neq i} (w_{j\to i} - w_{i\to j})\, p_j(t)\Delta t$$



Markovianity at the coarse grained level:

- A cyclic protocol with period $\Delta t \ll 1/w_{i\to j}$
- In each cycle a small amount of probability is transferred.

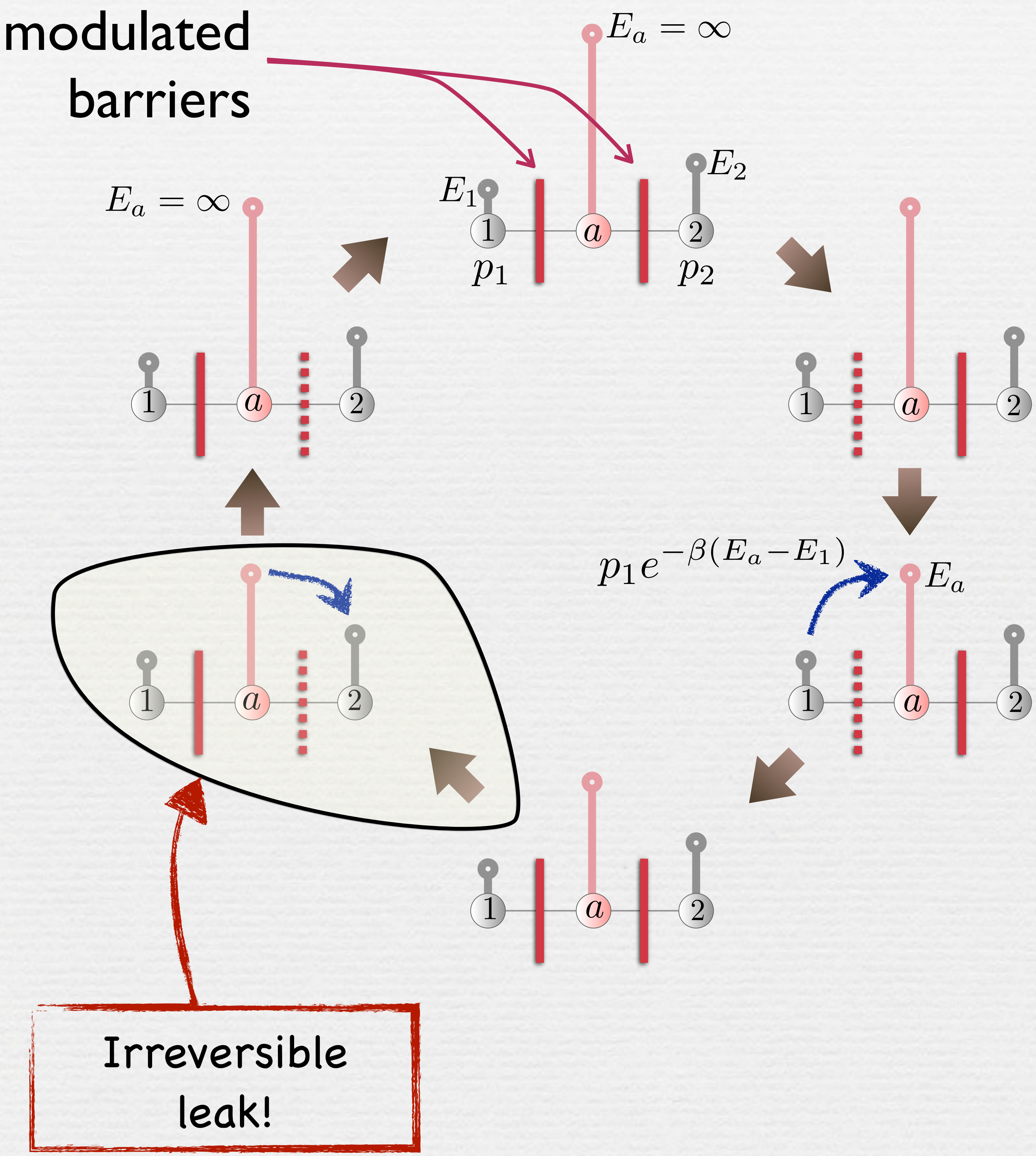
Low entropy production:

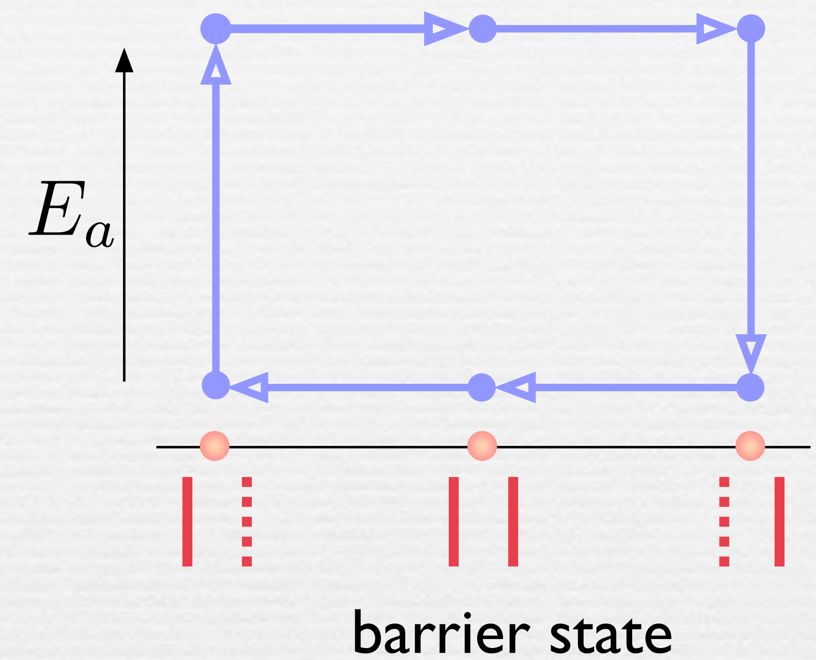
- Protocol must be slow compared to the kinetics of the hidden states:

$$\Delta t \gg (\text{hidden rates})^{-1}$$

Hidden pumps

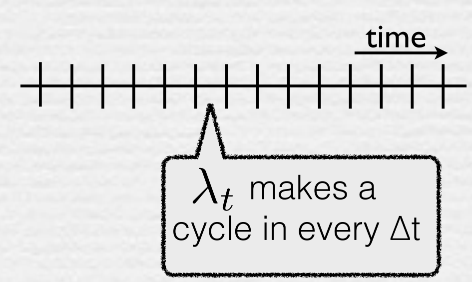
An auxiliary state a





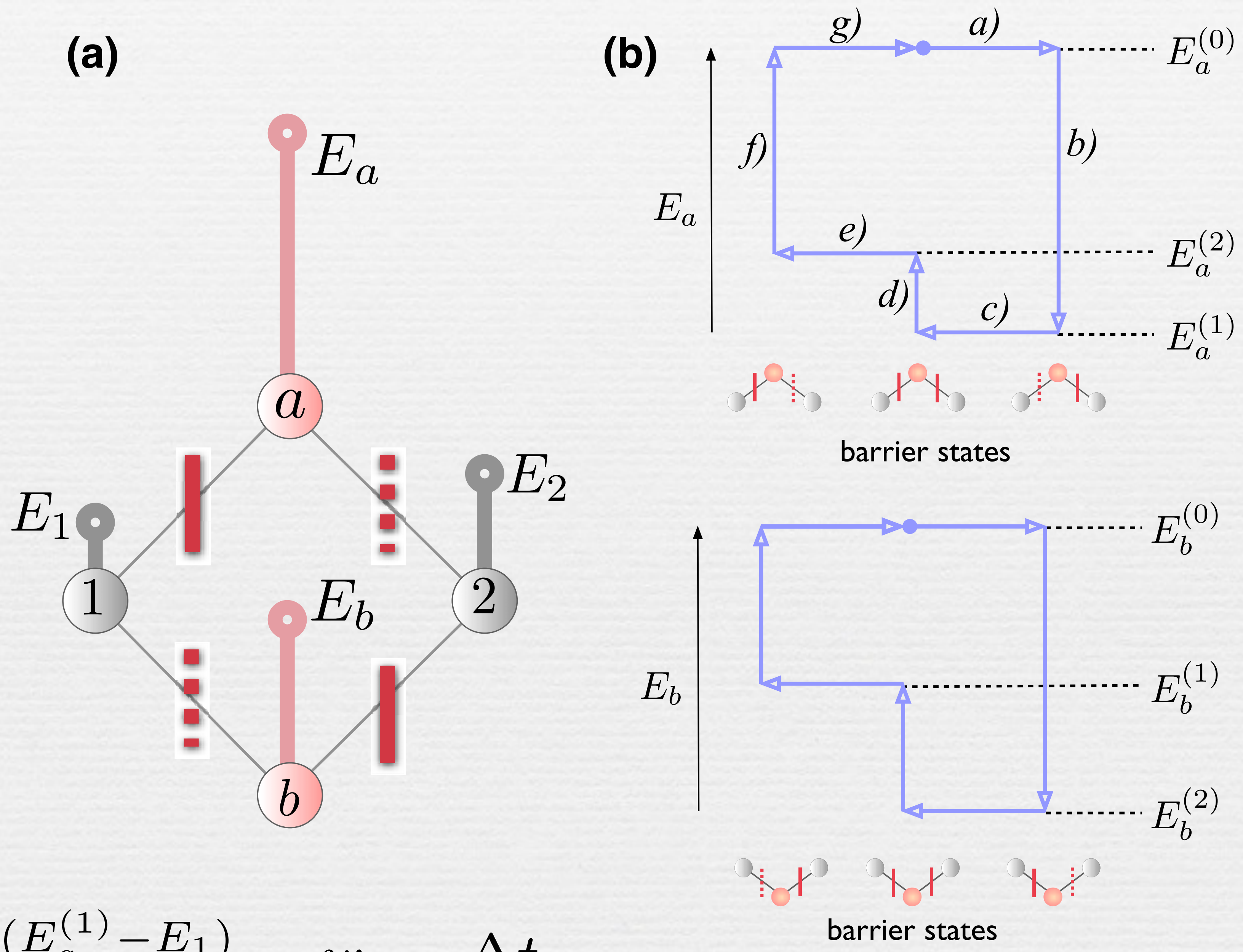
We transfer an amount of probability from 1 to 2:

$$p_1 e^{-\beta(E_a - E_1)} \simeq p_1 w_{1\to 2} \Delta t$$



Poissonian
rate

Hidden pumps



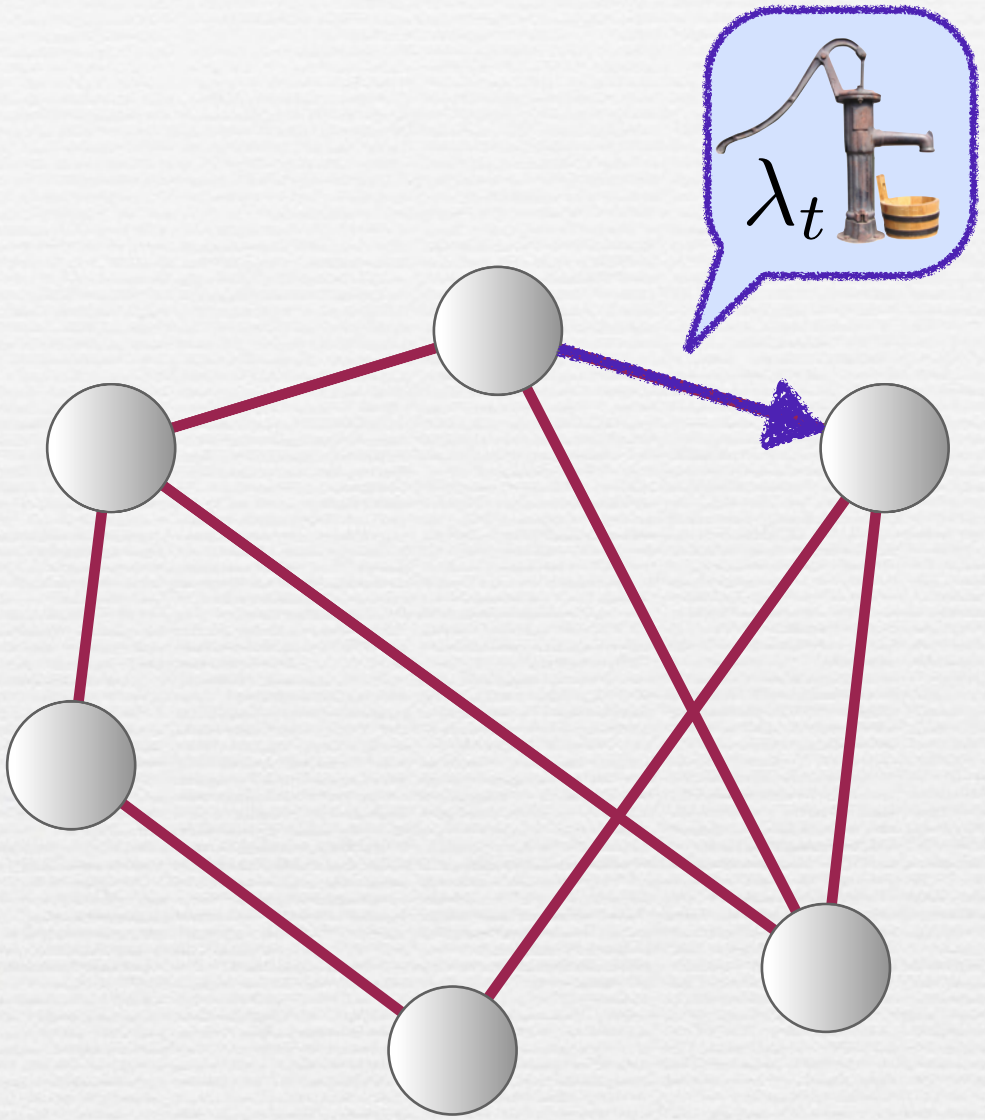
$$e^{-\beta(E_a^{(1)} - E_1)} = w_{1\to 2}\Delta t$$

Effective force induced by the pump

$$\frac{w_{1\to 2}}{w_{2\to 1}} = e^{-\beta(E_2 - E_1 - F_{21}^{\mathrm{eff}})}$$

$$\boxed{F_{21}^{\mathrm{eff}} \equiv E_b^{(2)} - E_a^{(1)}}$$

Hidden pumps



Entropy production of the coarse-grained description:

$$T\dot{S}_{\text{tot}}^{(\text{cg})} = J_{12}F_{12}^{(\text{eff})} - \sum_{\text{all links}} \delta\dot{\mathcal{F}}_{ij} \geq 0$$

Real entropy production:

$$T\dot{S}_{\text{tot}} = - \sum_{\text{all links but 12}} \delta\dot{\mathcal{F}}_{ij} \geq 0$$

$$\boxed{T\dot{S}_{\text{tot}}^{(\text{cg})} \geq T\dot{S}_{\text{tot}}}$$

It is even possible to have zero entropy production with finite current!

Hidden variables

Entropy production:

$$S_{\text{tot}} = k \sum_{\substack{(x,y)\\(x',y')}} p(x,y;x',y';\{\lambda_t\}) \ln \frac{p(x,y;x',y';\{\lambda_t\})}{p(\tilde{x}',\tilde{y}';\tilde{x},\tilde{y};\{\tilde{\lambda}_t\})}$$

~ = time reversal

Coarse grained entropy production:

$$S_{\text{tot}}^{(\text{cg})} = k \sum_{x,x'} p(x,x';\{\lambda_t\}) \ln \frac{p(x,x';\{\lambda_t\})}{p(\tilde{x}',\tilde{x};\{\tilde{\lambda}_t\})}$$

Marginal probability distribution

$$p(x,x';\{\lambda_t\}) = \sum_{y,y'} p(x,y;x',y';\{\lambda_t\})$$

One can prove:

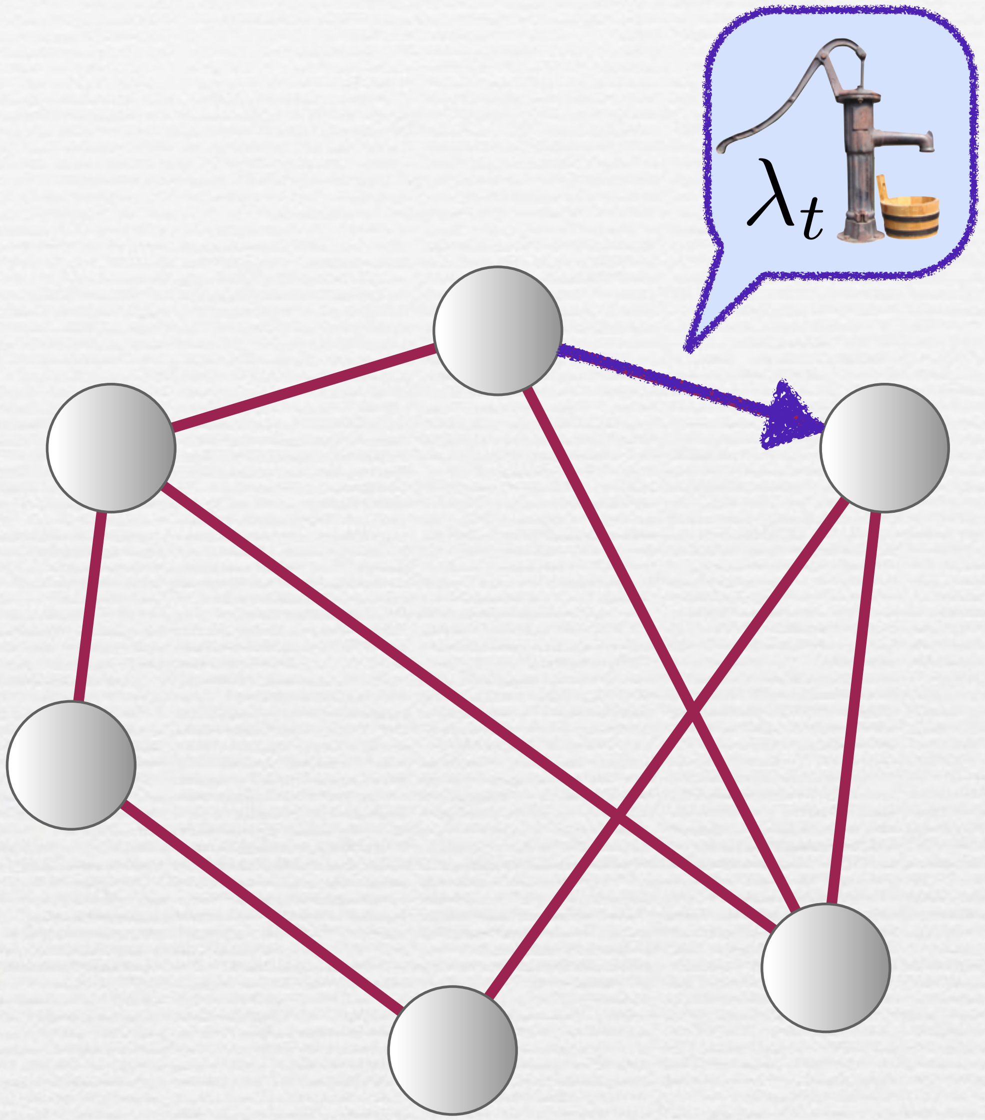
$$\boxed{S_{\text{tot}}^{(\text{cg})} \leq S_{\text{tot}}}$$

Two assumptions!

$\tilde{y} = y$ (overdamped systems + no magnetic fields)

No hidden driving.

Hidden pumps



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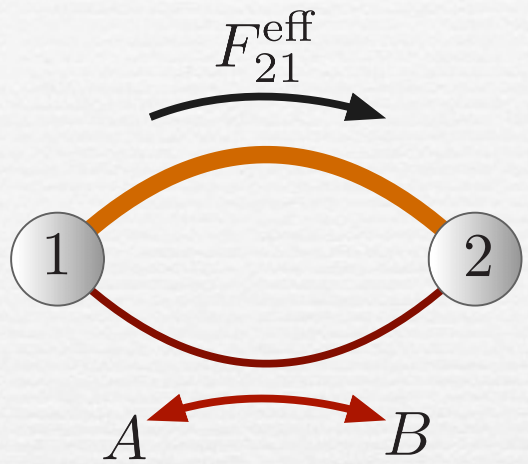
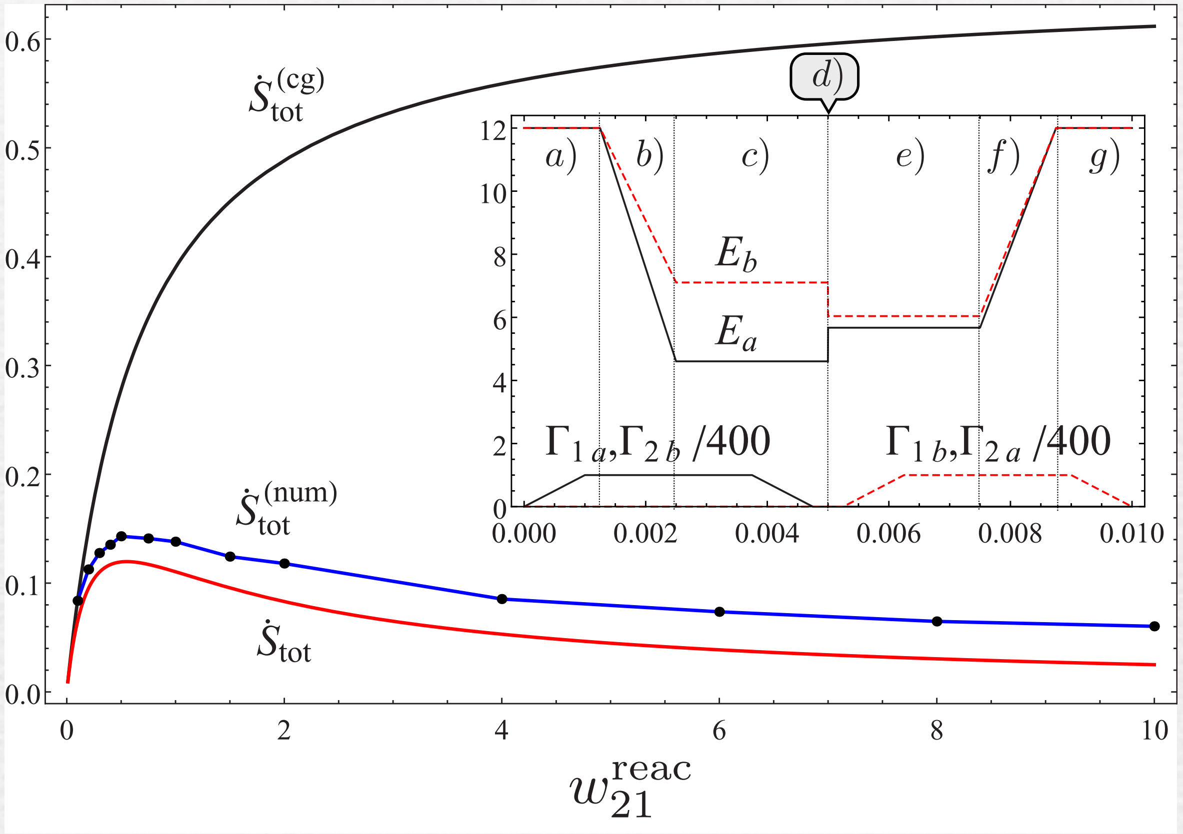
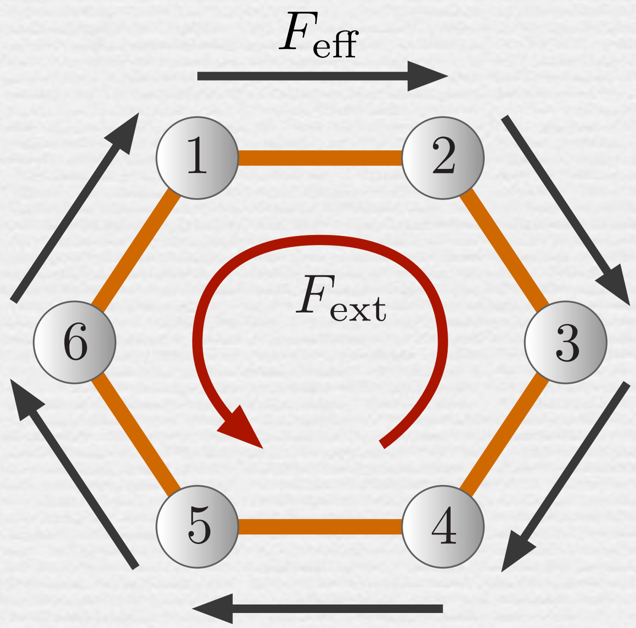
Hidden pumps



$\boxed{\mu_A - \mu_B > 0}$

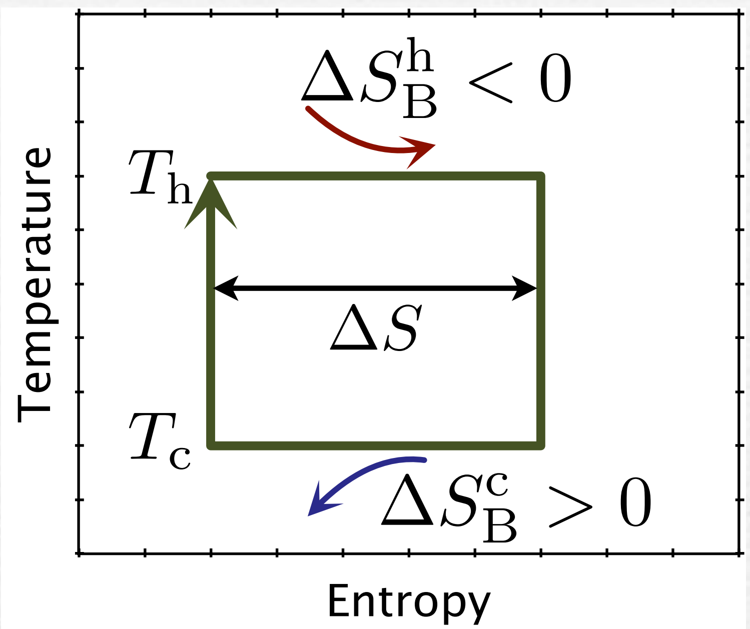
The pump induces the production of A.

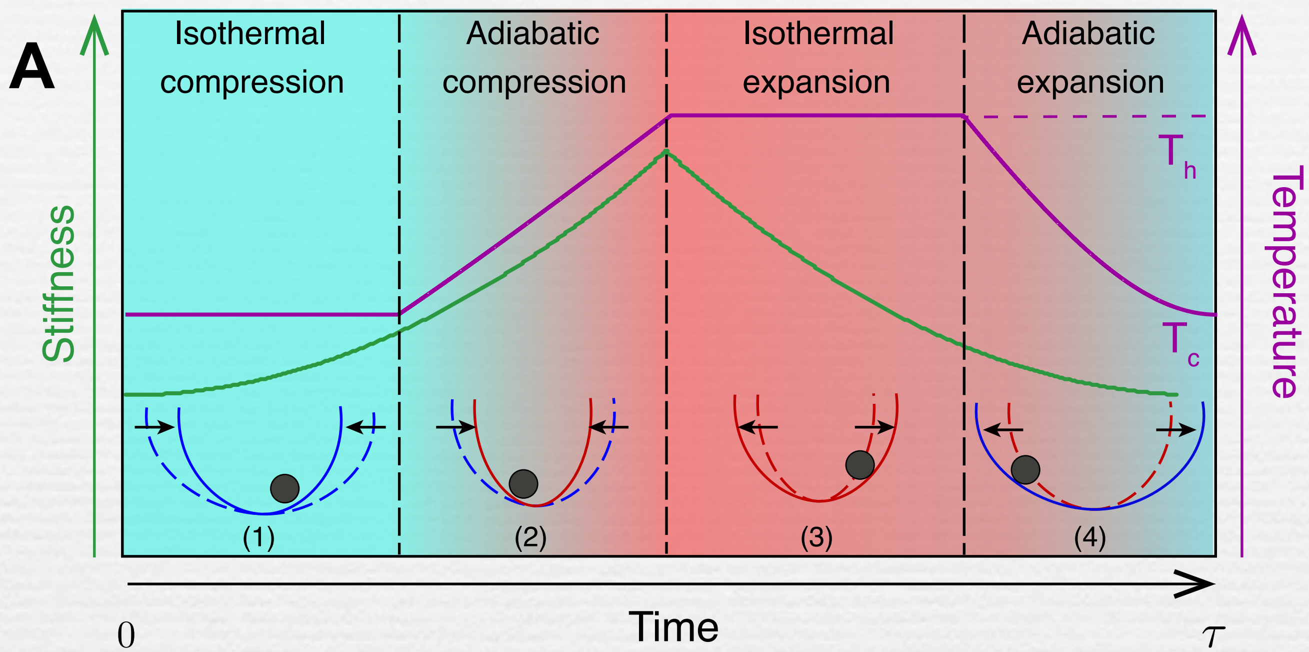


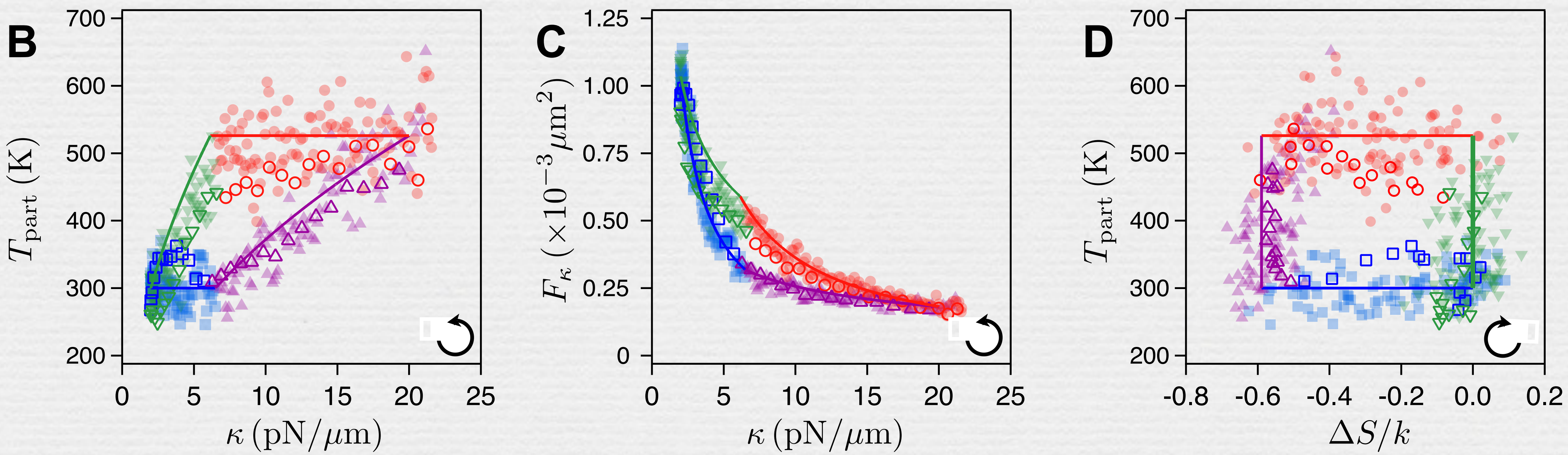


Finite current with zero entropy production

Carnot cycles



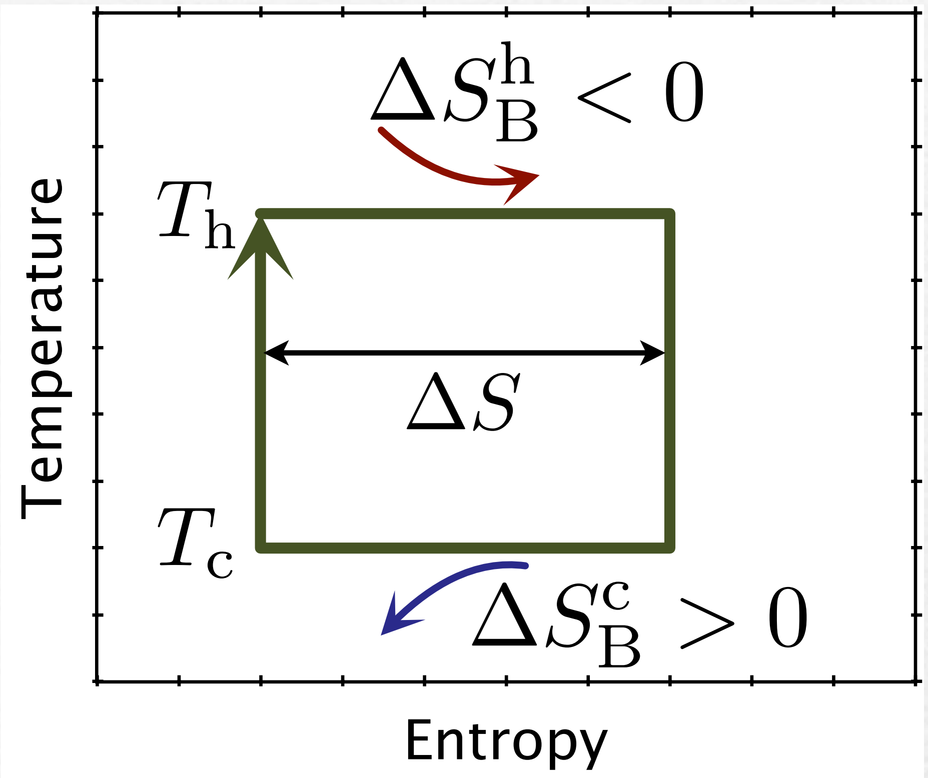


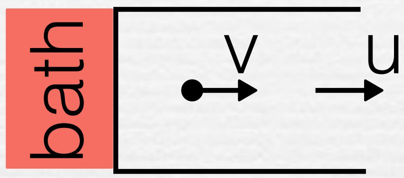


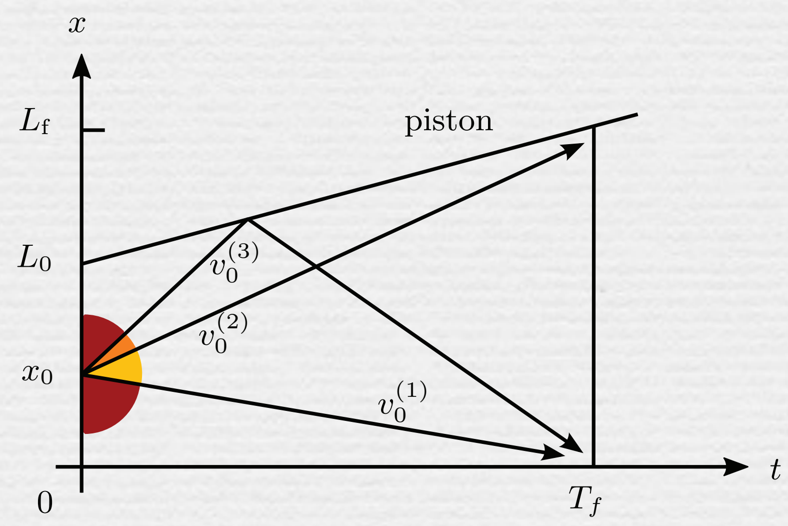
- Martínez, Roldán, Dinis, Petrov, JMRP, Rica. *Brownian Carnot engine*. arXiv:1412.1282v3 (2015).

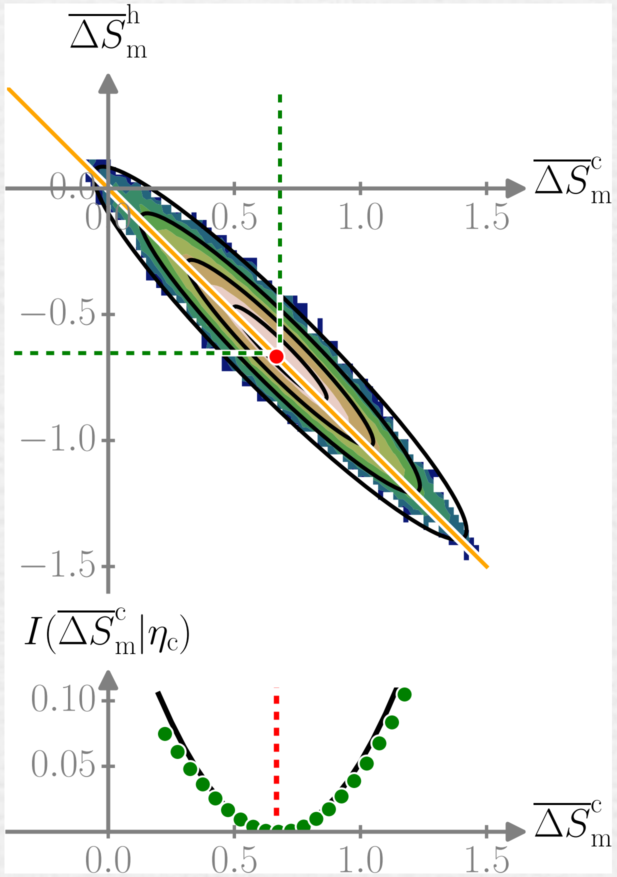
Carnot cycles

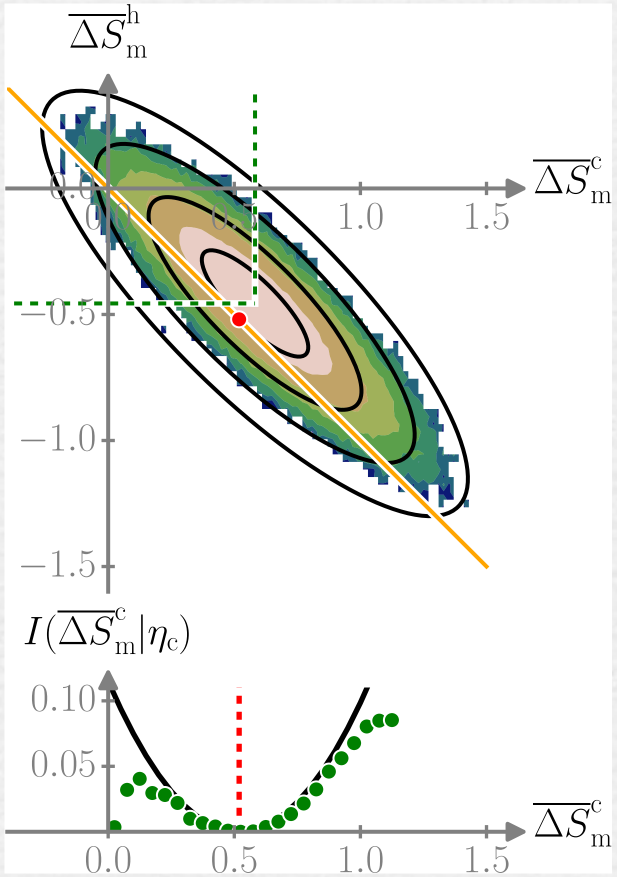
Hoppenau, Granger, Dinis, JMRP





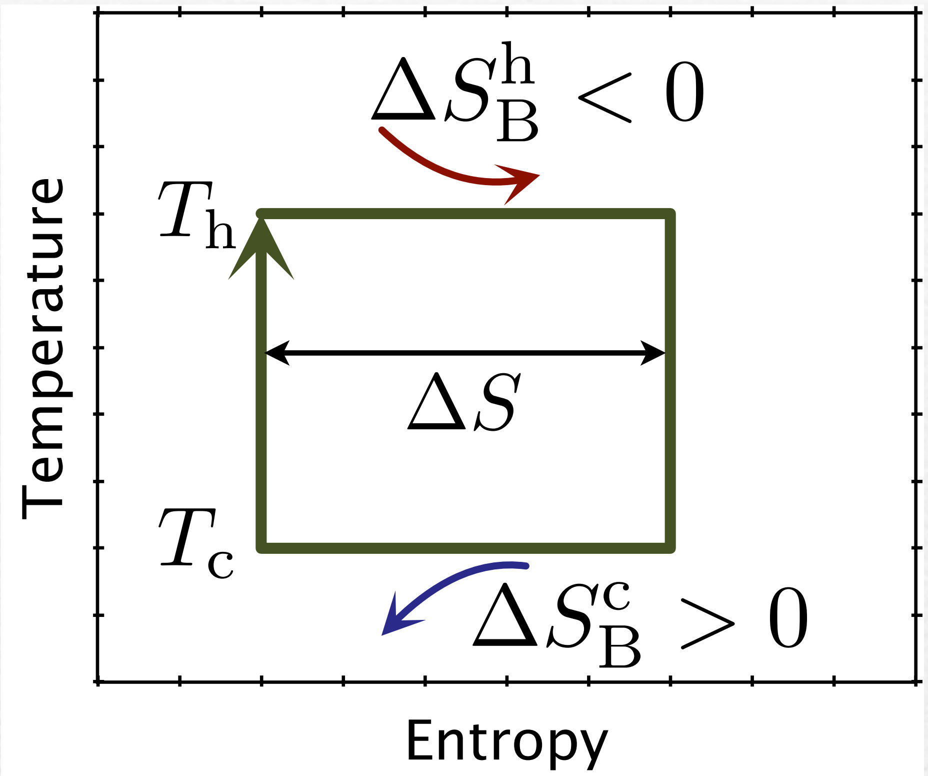




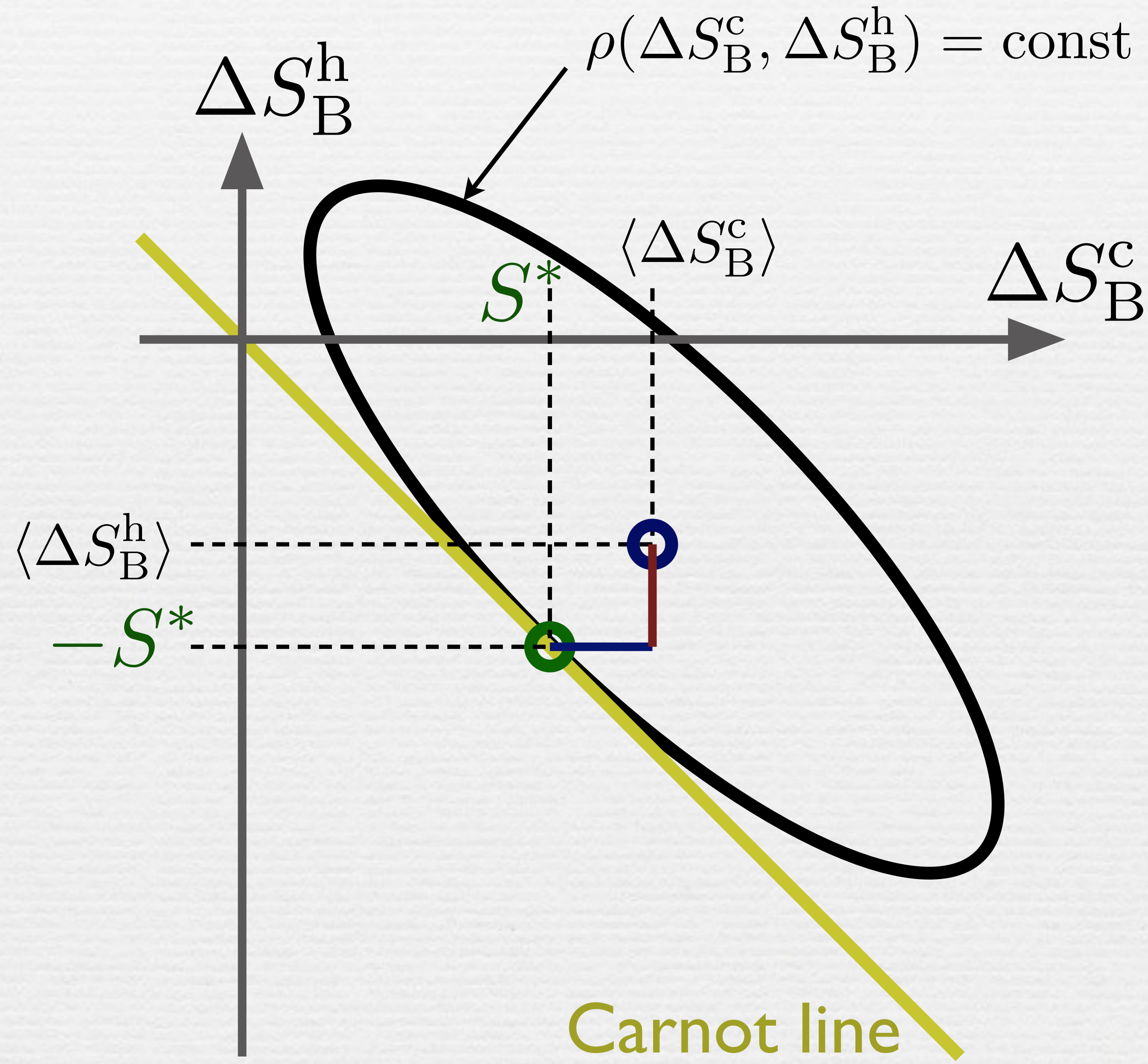


1. Hoppenau, J., Niemann, M. & Engel, A. Carnot process with a single particle. *Phys Rev E* **87,** 062127 (2013).

Carnot cycles



Hoppenau, Granger, Dinis, JMRP



Conclusions

- Driven systems apparently perform much better than autonomous systems.
- Time asymmetry is not enough to have reversible transport.
- Most likely reversible trajectories have finite power.

•JMRP. *Reversible ratchets as Brownian particles in an adiabatically changing periodic potential*. Phys Rev E 57, 7297 (1998).
•JMRP, Blanco, Cao, Brito. *Efficiency of Brownian motors*. Europhys Lett 43, 248 (1998).
•Horowitz, Sagawa, JMRP. *Imitating Chemical Motors with Optimal Information Motors*. Phys. Rev. Lett. 111, 010602 (2013).
•Esposito, JMRP. *Stochastic thermodynamics of hidden pumps*. Phys Rev E 91, 052114 (2015).
•Martínez, Roldán, Dinis, Petrov, JMRP, Rica. *Brownian Carnot engine*. arXiv:1412.1282v3 (2015).
•Hoppenau, Granger, Dinis, JMRP. *Efficiency of finite-time small Carnot engines: optimal designs and fluctuations*. In preparation.