Universal transitions to turbulence

from simple fluid to liquid crystal & quantum fluid

Kazumasa A. TAKEUCHI

(Tokyo Institute of Technology)

Collaboration teams

Liquid crystal: K. A. Takeuchi, M. Kuroda, H. Chaté, and M. Sano (2006-09)

Quantum fluid: M. Takahashi, M. Kobayashi, and K. A. Takeuchi (2014-)

Simple fluid: M. Sano and K. Tamai (2013-)

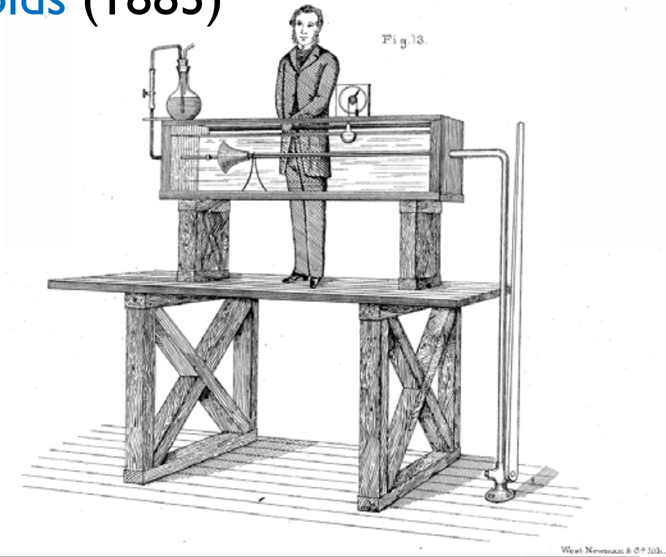
NB) unpublished data are omitted in this version posted on the website.

Turbulence

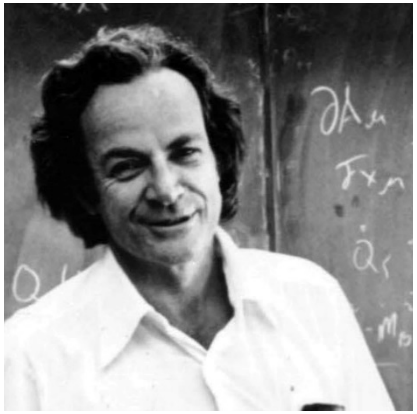
Leonardo da Vinci (around 1510)

O. Reynolds (1883)

[Phil. Trans. R. Soc. London A 174, 935 (1883)]



R. Feynman (1963)



Finally, there is a physical problem that is common to many fields, that is very old, and that has not been solved. It is not the problem of finding new fundamental particles, but something left over from a long time ago over a hundred years. Nobody in physics has really been able to analyze it mathematically satisfactorily in spite of its importance to the sister sciences. It is the analysis of circulating or turbulent fluids.

In a sense, turbulence is an ultimate open problem in nonlinear & nonequilibrium physics!

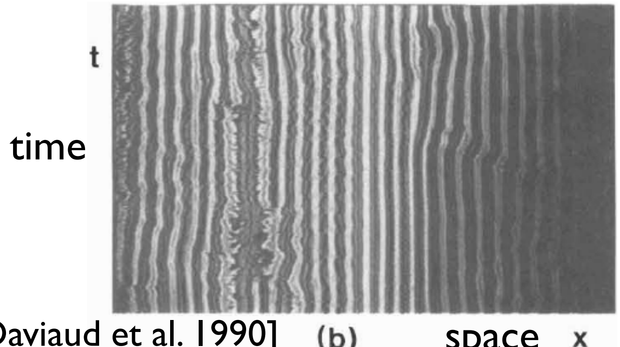
Onset of Turbulence

Some routes to turbulence (70-80's)

- Ruelle-Takens-Newhouse (RTN) route:
  periodic → quasi-periodic → chaos
- Period-doubling cascade:
  periodic (period 1) → period 2 → period 4 → … → chaos
- Intermittency:
  periodic flow interrupted by random bursts
  (life time diverges at Re_c)

Well understood in terms of bifurcations, despite complicated dependence on experimental conditions (e.g., aspect ratio)

- Abrupt transitions to turbulence, bypassing periodic state:
  typically occur in shear flow (pipe, Couette flow, channel flow)
- Spatio-temporal intermittency: laminar & turbulent regions coexist.



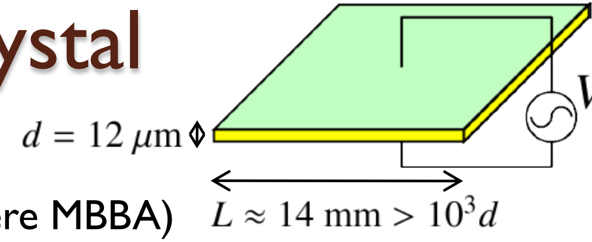
[Daviaud et al. 1990]

Pomeau's conjecture (1986) [Pomeau 1986]
"Transitions to spatiotemporal intermittency may belong to the directed percolation class."

negative results from experiments.

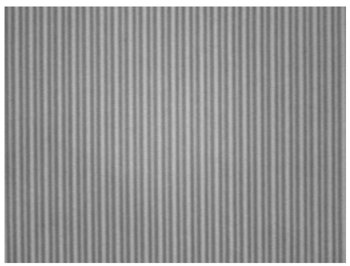
[Ciliberto & Bigazzi 1988, Daviaud et al. 1989 & many works afterward]

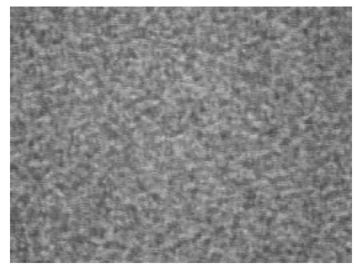
Electroconvection of Liquid Crystal

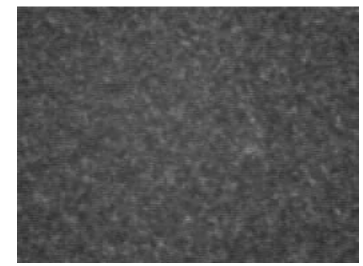


- Apply an ac electric field to nematic liquid crystal (here MBBA)
- Convection driven by Carr-Helfrich instability (due to nematic anisotropy)
- Quasi-2d system $(16\,\mathrm{mm} \times 16\,\mathrm{mm} \times 12\,\mu\mathrm{m})$ ⇨ large system size

$100\,\mu\mathrm{m}$

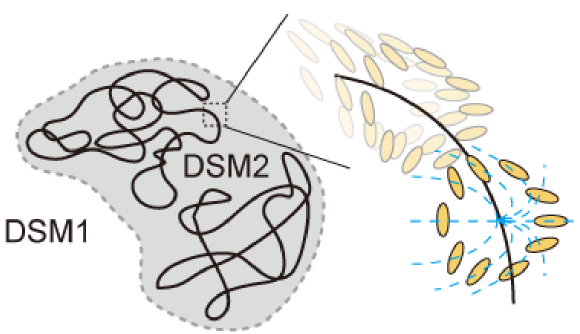






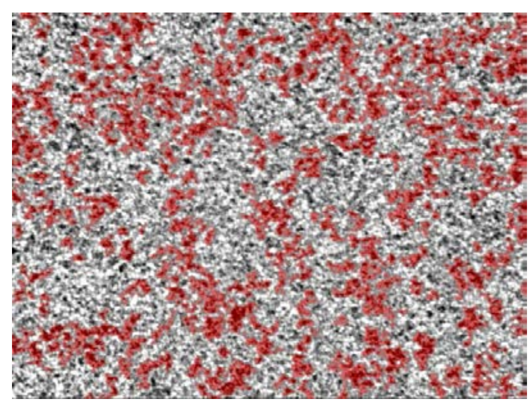
no convection | roll convection | … | dynamic scattering mode 1 (DSM1) | ≈ 35 V | dynamic scattering mode 2 (DSM2) | V

0 | ≈ 10 V | ≈ 25 V



DSM2 = topological-defect turbulence

DSM1-DSM2
spatio-temporal intermittency



red = DSM2

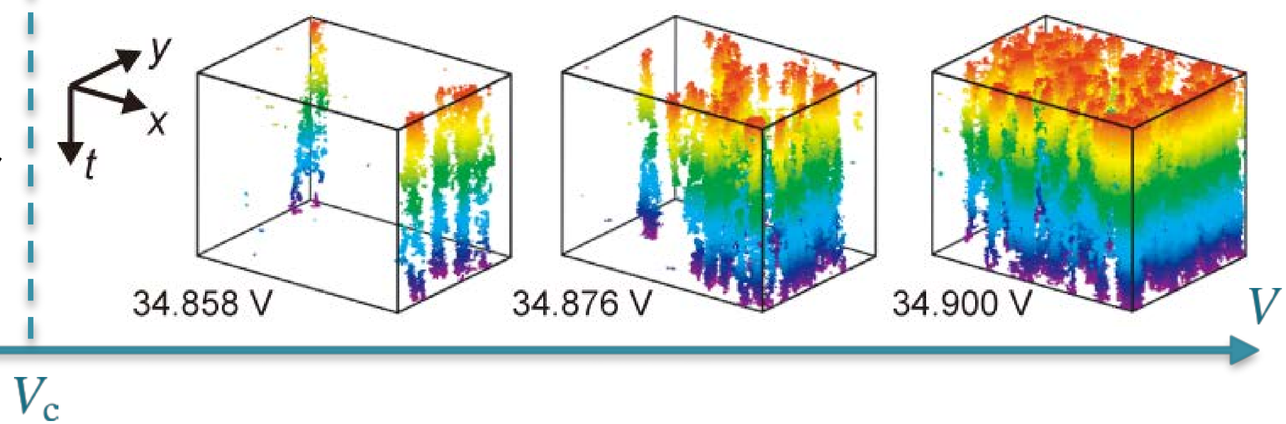
DSM1-DSM2 Transition

[KaT *et al.* PRL 99, 234503 (2007);
PRE 80, 051116 (2009)]

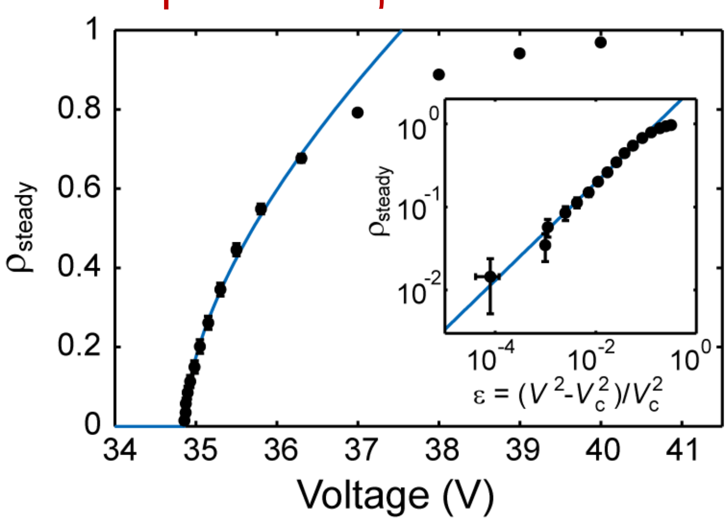
Near the DSM1-DSM2 transition

Space-time evolution (colored zone = DSM2)

All DSM2 patches
eventually disappear



Order parameter ρ = DSM2 area fraction



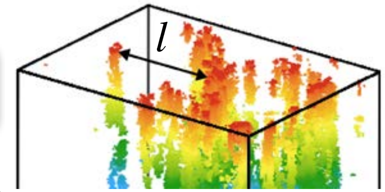
$$\rho_{\mathrm{steady}} \sim (V^2 - V_c^2)^{\beta}$$

$\beta = 0.59(4)$ | $\beta_{\mathrm{DP}} = 0.583(3)$

Good agreement with
(2+1)d directed percolation (DP) class

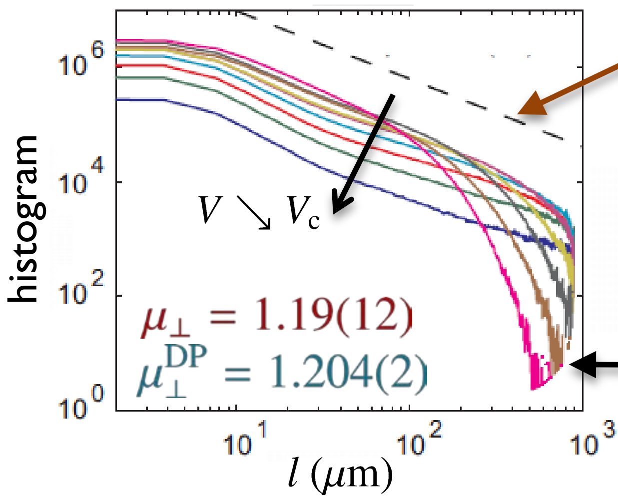
DSM1-DSM2 Transition

[KaT *et al.* PRL 99, 234503 (2007); PRE 80, 051116 (2009)]



We measured 12 exponents and all agreed with DP class

- Spatial correlation (measuring gap l between DSM2 patches)



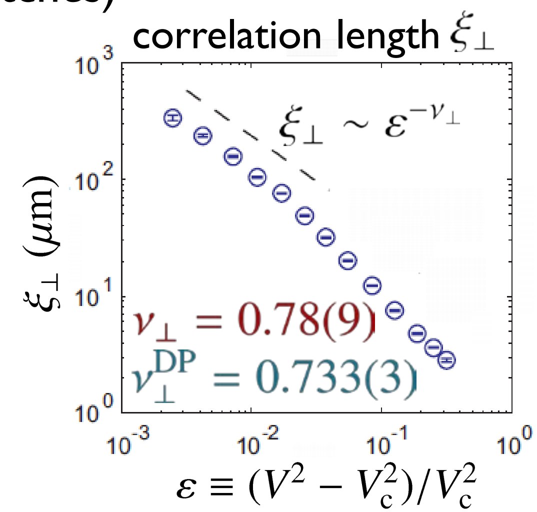
$l^{-\mu_\perp}$

scaling relation

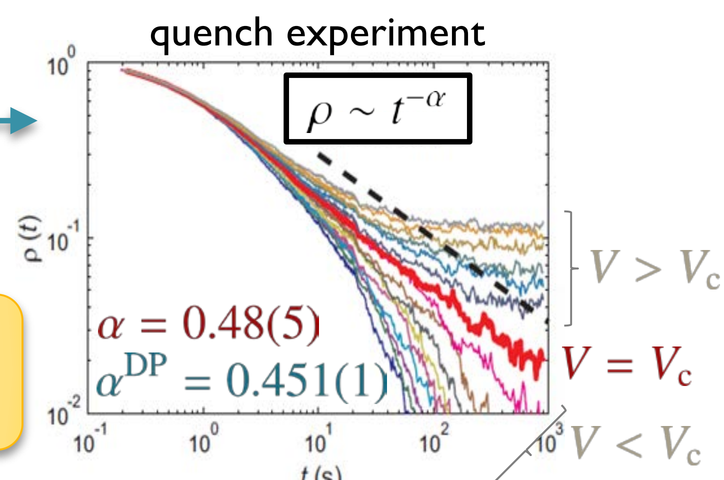
$$\mu_\perp = d_f + 1 = 2 - \beta/\nu_\perp$$

DP fractal dimension

$e^{-l/\xi_\perp}$ $(l \gg \xi_\perp)$



- Relaxation of order parameter
(after quench from $V \gg V_c$ to $\approx V_c$)



3 independent DP exponents are confirmed
⇨ DSM1-DSM2 transition is in the DP class

Directed Percolation Class? [review: Hinrichsen, Adv. Phys. 49, 815 (2000)]

DP class = basic universality class for transitions into an "absorbing state"
without extra symmetry or conservation law

under usual conditions, such as the absence of long-range interactions,
absence of quenched disorder, effectively stochastic dynamics, etc.

Absorbing state = system can enter, but can never escape once it enters.

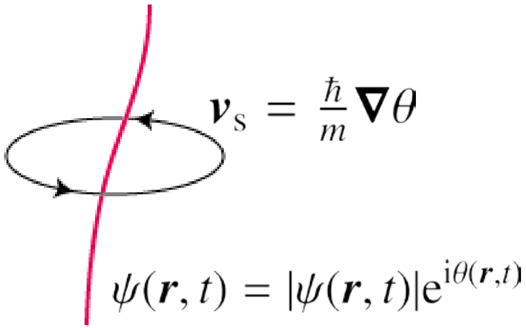
In our liquid-crystal system,
practically no spontaneous nucleation of DSM2 (made of topological defects)
⇨ state without any DSM2 patch = absorbing state

- Various models belong to DP class, so it's very robust.
  (epidemics, catalytic reactions, Ca waves in cells, population dynamics, galaxy…)

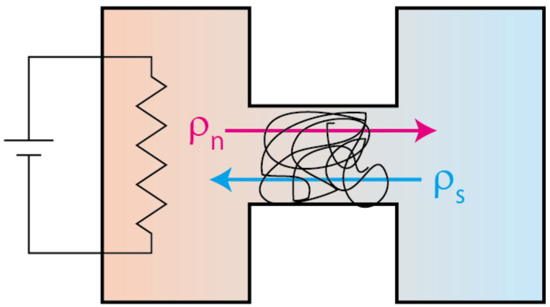
- Nevertheless, DP was found experimentally for the first time here.
  This gap between theory & experiments remains to be understood.

Another Topological-Defect Turbulence: Quantum Turbulence

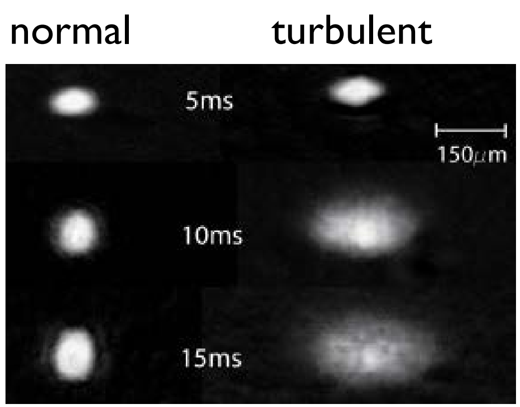
- In quantum fluids
  such as superfluid helium and cold atom gas (BEC),
  vortices are quantized (hence topological defects).



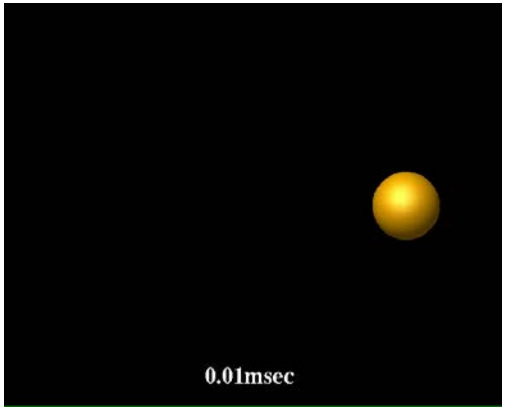
- Quantum turbulence (made of turbulent vortices)



thermal counterflow
of superfluid He
[review: Tsubota *et al.* Phys Rep. 2013]



experimental realization of
turbulence in cold atom BEC
[Henn *et al.* PRL 2009]



generating turbulence
by obstacle oscillation
[Goto *et al.* PRL 2009]

Quantum turbulence has been realized in various situations
and has attracted great theoretical & experimental interests

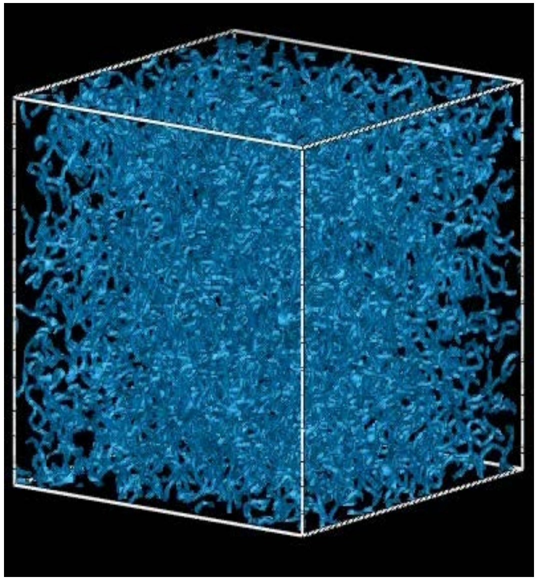
Kolmogorov Law in Quantum Turbulence

Simulation of developed quantum turbulence [Kobayashi & Tsubota, PRL 2005, JLTP 2006]

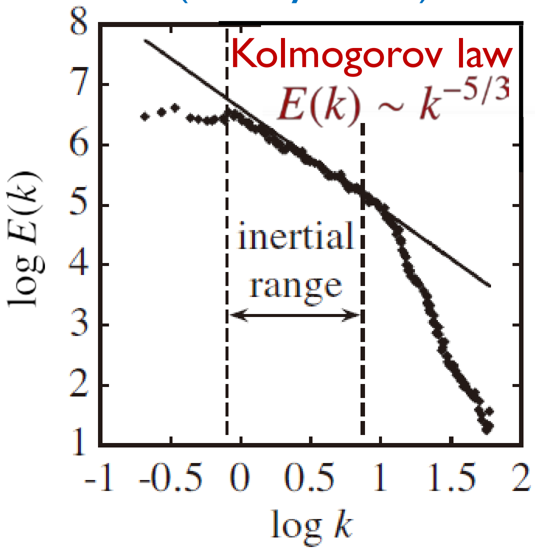
Model: Gross-Pitaevskii (GP) equation with dissipation term

$$(i\hbar - \gamma)\frac{\partial}{\partial t}\psi(\boldsymbol{r},t) = -\frac{\nabla^2}{2m}\psi + [V(\boldsymbol{r},t) - \mu]\psi + g|\psi|^2\psi$$

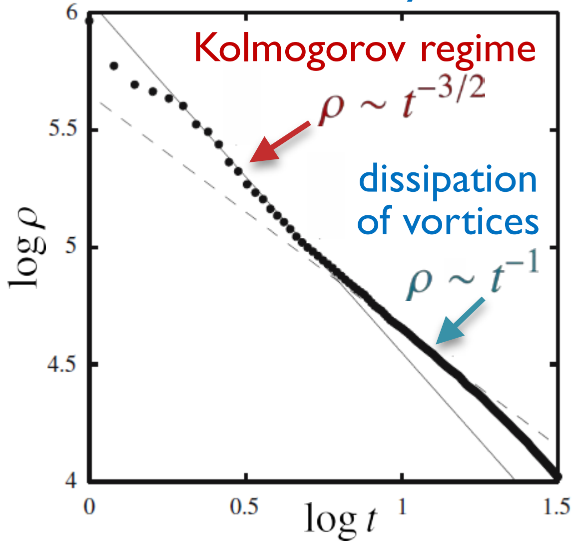
$V(\boldsymbol{r},t)$: random potential (amplitude V_0, correlation length l_v and time τ_v)



energy spectrum (steady state)



relaxation of vortex density ρ



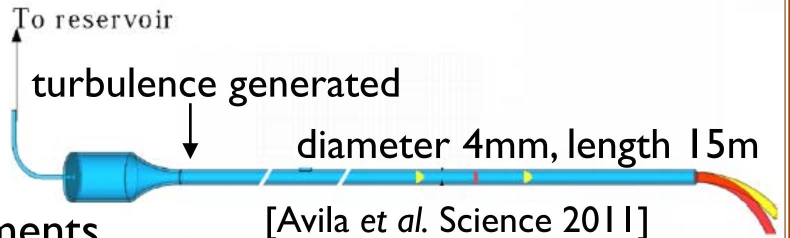
In contrast, less is known about phase transitions to quantum turbulence.

So… What about Simple Fluids?

Routes to turbulence

RTN route (via quasi-periodicity), period doubling, intermittency, abrupt transitions, spatio-temporal intermittency, …

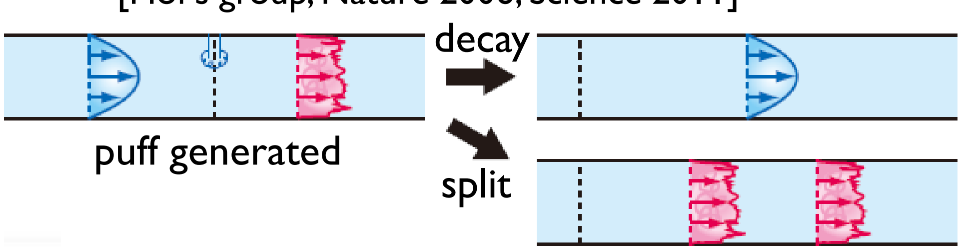
↳ important recent progress ↳ no example of DP yet (in simple fluids)

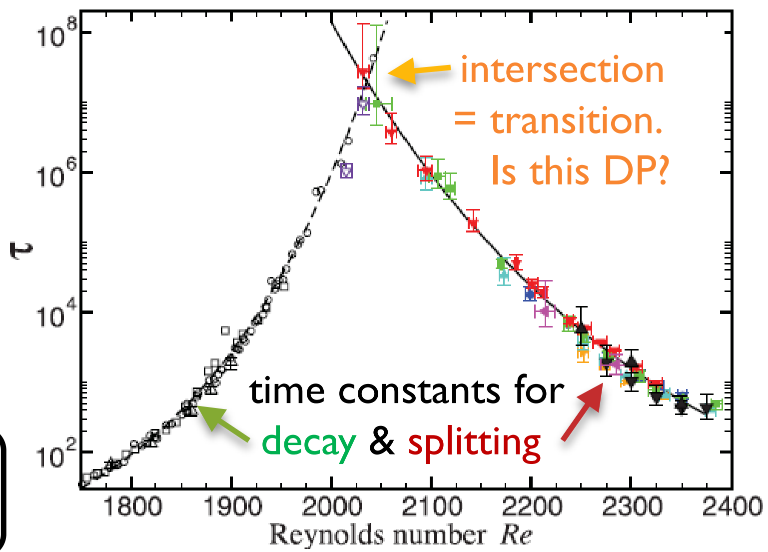


[Avila *et al.* Science 2011]

Abrupt transition in pipe flow

- Laminar flow linearly stable up to $\mathrm{Re} = \infty$
  Becomes turbulent at $\mathrm{Re} \approx 2000$ in experiments
- Localized turbulent objects (puffs) near Re_c
- Puffs' evolution: stochastic decay & splitting
  [Hof's group, Nature 2006; Science 2011]

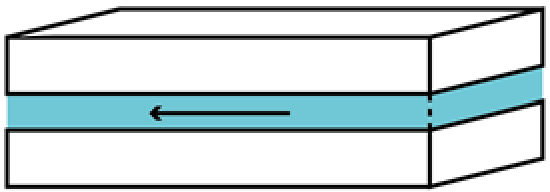




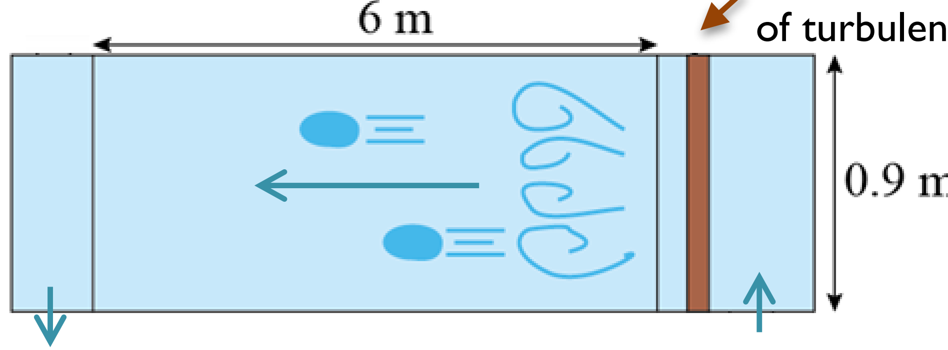
However, direct measurement of critical behavior is unrealistic

Channel Flow Experiment [Sano & Tamai, to be published]

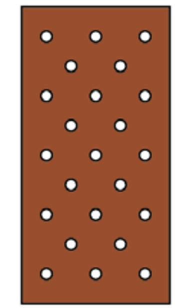
Laminar-turbulent transition in a plane channel, instead of a pipe



gap 5mm



Continuous generation of turbulence by a grid



Turbulent spots are visualized by flake particles.

$\mathrm{Re} = 788$

$\mathrm{Re} = 809$

$\mathrm{Re} = 954$

Re →

Linear stability analysis gives $\mathrm{Re}_c \approx 5772$ [Orszag 1971]

Summary

Directed percolation (DP) class tends to arise at transitions to turbulence in simple fluids, liquid crystal, and quantum fluids

DP class = basic universality class for transitions into an absorbing state

Topological-defect turbulence in liquid crystal (expt.) [KaT *et al.* PRL 99, 234503 (2007); PRE 80, 051116 (2009)]

- First experimental evidence of DP, found at the DSM1-DSM2 transition.
- No spontaneous creation of DSM2 (made of topological defects) = absorbing state

Quantum-vortex turbulence in quantum fluid (numerics) [Takahashi, Kobayashi, KaT, to be published]

- DP found in the (well-founded) GP equation ⇨ future experimental test?
- 2-step relaxation from Kolmogorov to DP

Abrupt transition in channel flow of simple fluid (expt.) [Sano & Tamai, to be published]

- DP found experimentally at laminar-turbulent transition in channel flow
- Laminar flow is linearly stable, even for $\mathrm{Re} > \mathrm{Re}_c$. = absorbing state
  DP arises universally at abrupt transitions? [cf. numerics on plane Couette by Shi *et al.* 2015]

⇨ Also toward better understanding of DP itself (noise vs chaos, UV divergence, …)