Replica Wormholes, Entanglement Wedges and the Black Hole Information Paradox

Geoff Penington, UC Berkeley

Talk based on:

Entanglement Wedge Reconstruction and the Information Paradox. GP. *arXiv:1905.08255*.

Replica Wormholes and the Black Hole Interior. GP, S. Shenker, D. Stanford, Z. Yang. *arXiv:1911.11977*.

See also:

The Entropy of Bulk Quantum Fields and the Entanglement Wedge of an Evaporating Black Hole. A. Almheiri, N. Engelhardt, D. Marolf, H. Maxfield. *arXiv:1905.08762*.

The Page Curve of Hawking Radiation From Semiclassical Geometry. A. Almheiri, R. Mahajan, J. Maldacena, Y. Zhao. *arXiv:1908.10996.*

Replica wormholes and the entropy of hawking radiation. A. Almheiri, T. Hartman, J. Maldacena, E. Shaghoulian, A. Tajdini. *arXiv:1911.12333*.

Other important work by: Akers, Harlow, Bousso, Tomasevic, Chen, Fisher, Hernandez, Myers, Ruan, Rozali, Van Raamsdonk, Sully, Waddell, Wakeham

The claim:

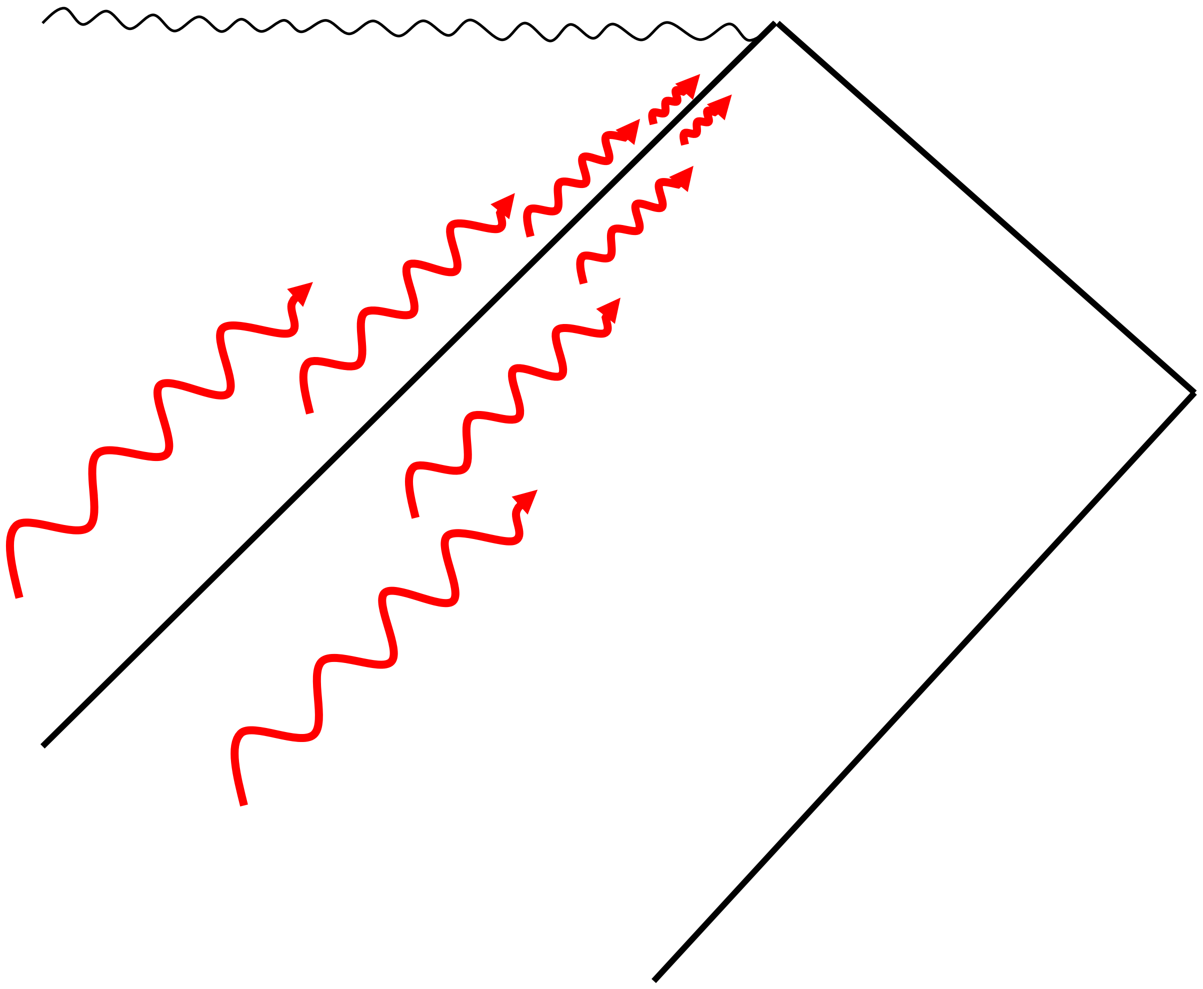
- In **1975**, Hawking famously argued that information that falls into a black hole is lost forever, in contradiction with the **unitarity** of quantum mechanics.
- Since then, many arguments that this **must** be wrong, but, until last year, no actual **gravity calculation** that showed the information escaping.
- We now know exactly the **mistake** Hawking made in his calculation. Using the tools of **quantum information**, we can see exactly how and when the information comes out. Agrees exactly with **predictions** from (non-gravitational) **toy models**.
- Original context: AdS/CFT. **However**, don’t need a CFT, string theory, anti-de Sitter space, etc.
- Hawking simply **missed** the existence of certain **saddle points** in the **gravitational path integral**, which contain ‘**spacetime wormholes**’. Including these saddles in calculations shows that information escapes.

The plan:

- **Part 0:** review the relevant features of the information paradox
- **Part I**: abstract everything away into a very simple but **unrealistic** toy model, where we can calculate everything completely **explicitly**.
- **Part II**: move to more realistic models, including **evaporating four-dimensional black holes**, at the cost of being somewhat less explicit.

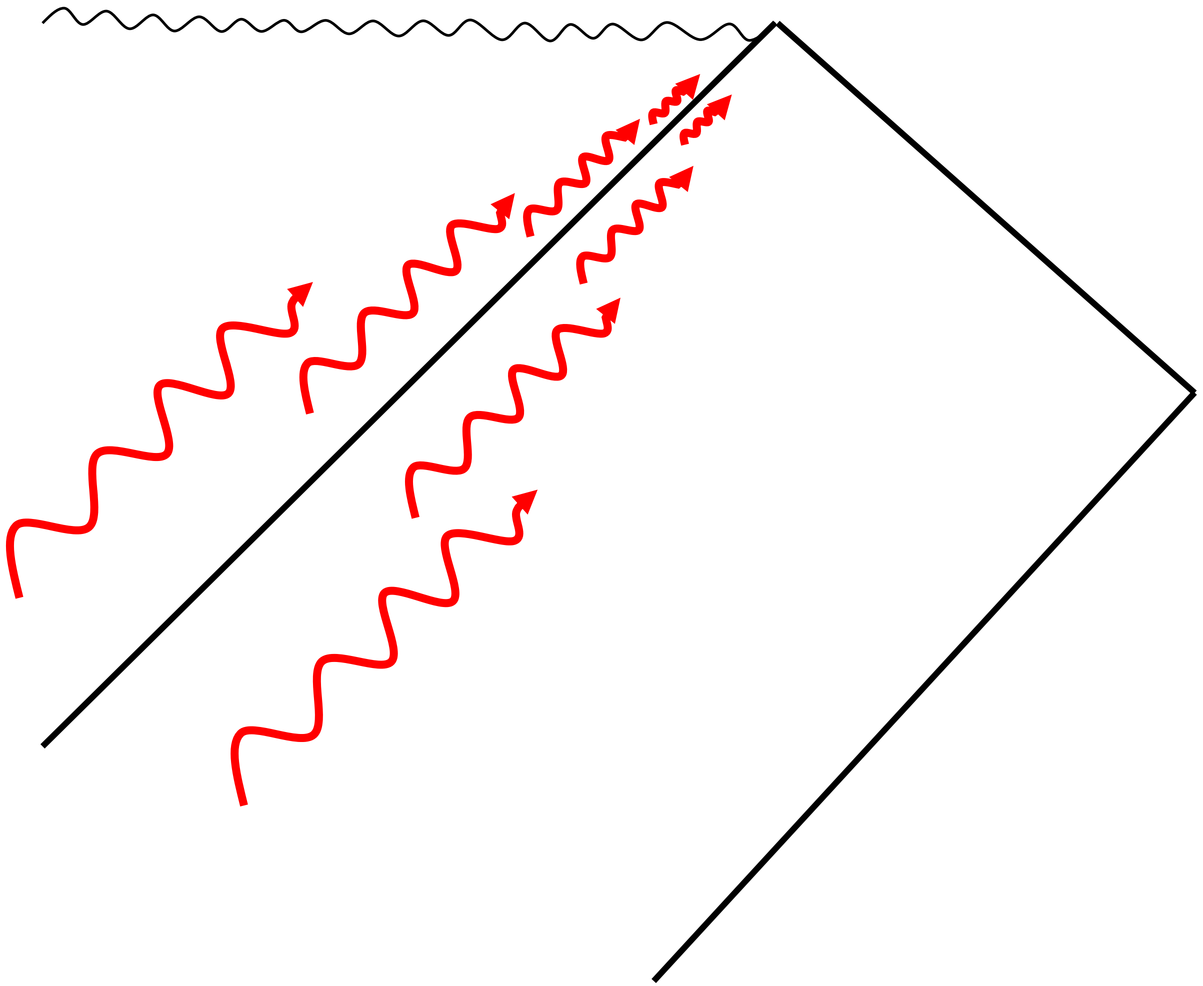
Part 0: The Information Paradox

Evaporating black holes



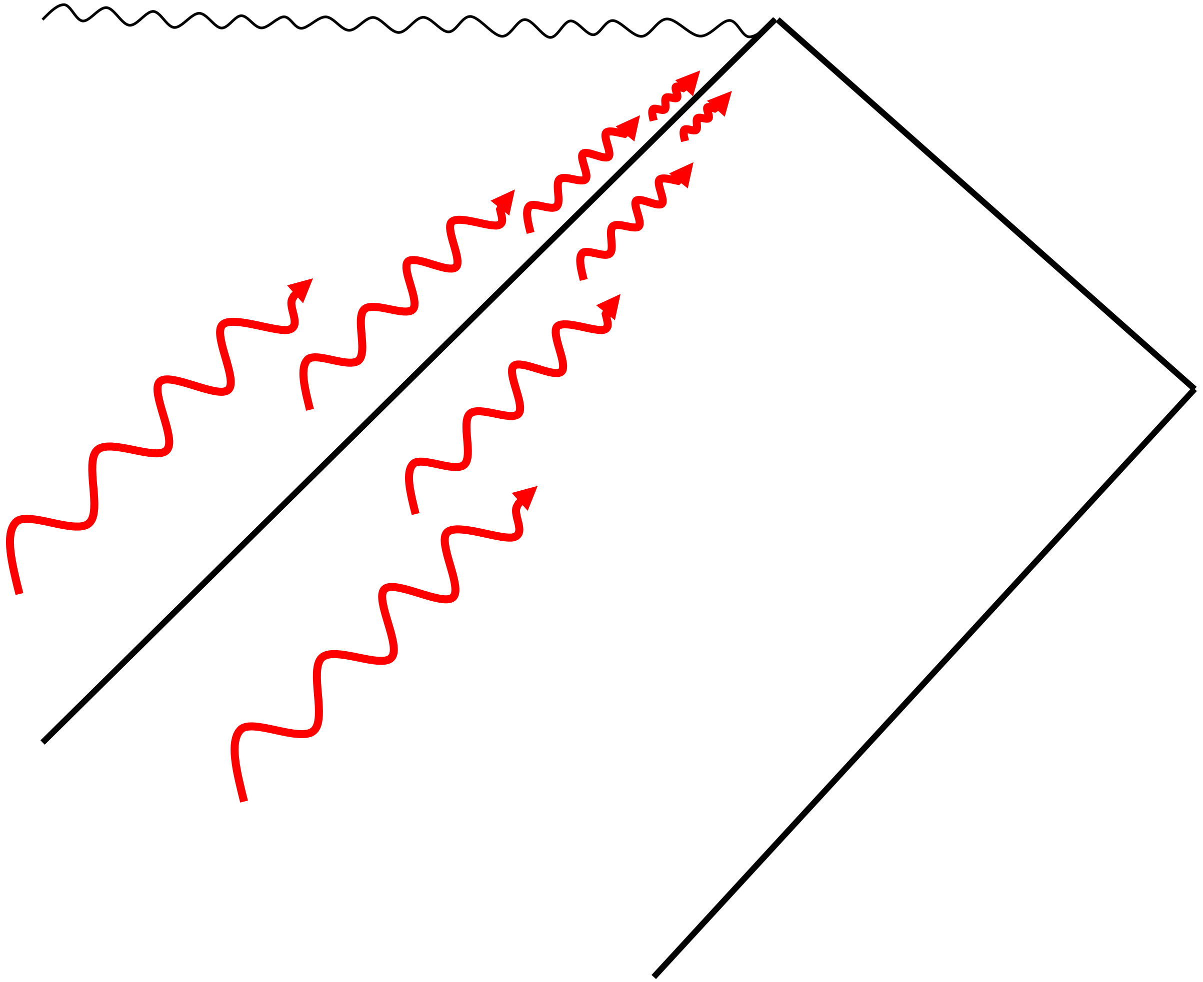
- **Quantum field theory:** short-range entanglement between outgoing modes slightly inside/outside the BH horizon
- BH dynamics **redshifts** these modes, creating **Hawking radiation** that is entangled with interior modes
- As the black holes evaporates, **more and more modes** escape. The entanglement (apparently) increases indefinitely

The Information Paradox



- Eventually the entanglement entropy becomes larger than the **Bekenstein-Hawking** entropy of the black hole (the **Page time**)
- If the BH entropy is truly the **statistical entropy** of black hole microstates (true in string theory, AdS/CFT), this is a **paradox**: not enough BH states to be able to purify the Hawking radiation
- Possible resolutions: a) **information loss** or b) entanglement entropy starts **decreasing** at/before Page time (**Page curve**)

The Information Paradox

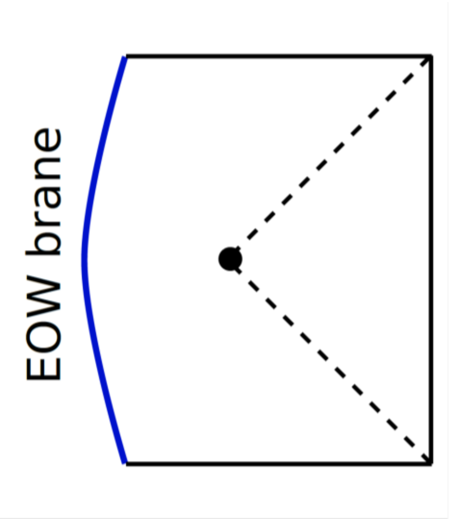


- If the evolution is **unitary** and the black hole is **maximally entangled** after the Page time, it cannot carry any information about stuff that had **fallen in**.
- Quantum information can't be **erased**. Hence the Hawking radiation must already carry some sort of information about the state.
- But, in the semiclassical description, there is **no interaction** between the infalling matter and the Hawking modes.

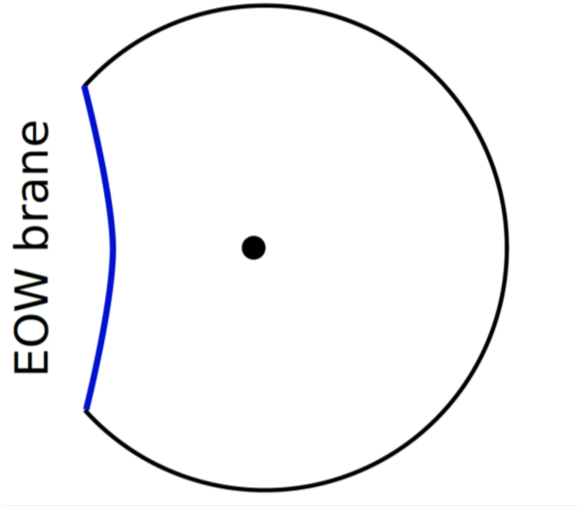
Part 1: A Very Simple Model

(**GP**, Shenker, Stanford, Yang *arXiv:1911.11977*)

A Very Simple Model: Pure JT gravity plus EOW Branes



Lorentzian



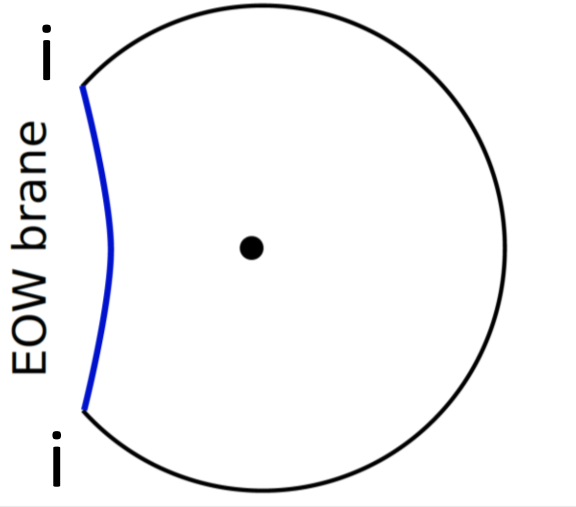
Euclidean

1+1-dimensional, one-sided eternal black hole, where the spacetime ends on an **‘end-of-the-world’ brane** in the BH interior

Analogue for Hawking radiation: add internal degrees of freedom to the EOW brane **(interior modes)** that are maximally entangled with a reference system

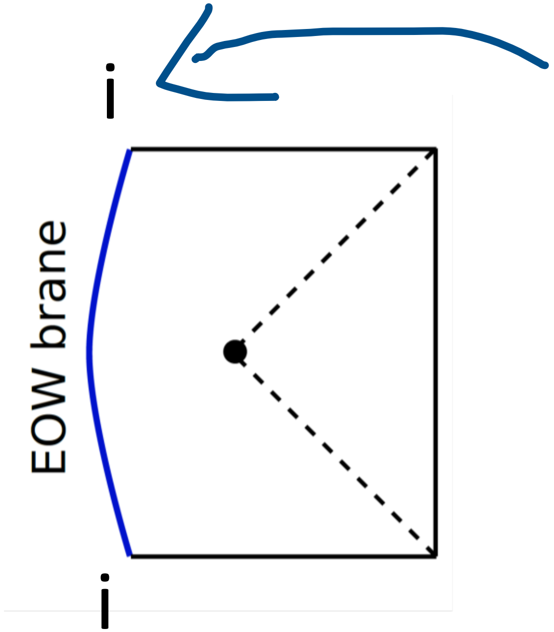
A Very Simple Information Paradox

$|\psi_i\rangle_{\mathrm{B}} =$



Euclidean

$=$



Lorentzian

Orthogonal basis for **internal state** of EOW brane

$$|\Psi\rangle = \frac{1}{\sqrt{k}} \sum_{i=1}^{k} |\psi_i\rangle_{\mathrm{B}} |i\rangle_{\mathrm{R}}.$$

(Simple) **information paradox** when $k \gg e^{S_{BH}}$. Entanglement entropy seemingly becomes **larger** than the **Bekenstein-Hawking** entropy

The Replica Trick

- How do you calculate **von Neumann entropies** using a path integral?
- **Answer**: the **integer n Renyi entropies**

$$\frac{1}{1-n}\log \mathrm{Tr}\rho_R^n$$

'Replica trick'

are proportional to the logarithm of an observable on **n copies of the system**.

- We can calculate the von Neumann entropy by **analytically continuing the Renyi entropies** to **n=1**.
- **The key idea**: the gravitational path integral includes topologies that connect the different replicas via **spacetime wormholes**.

Calculating the Purity

Einstein-Hilbert term
∼ Euler characteristic

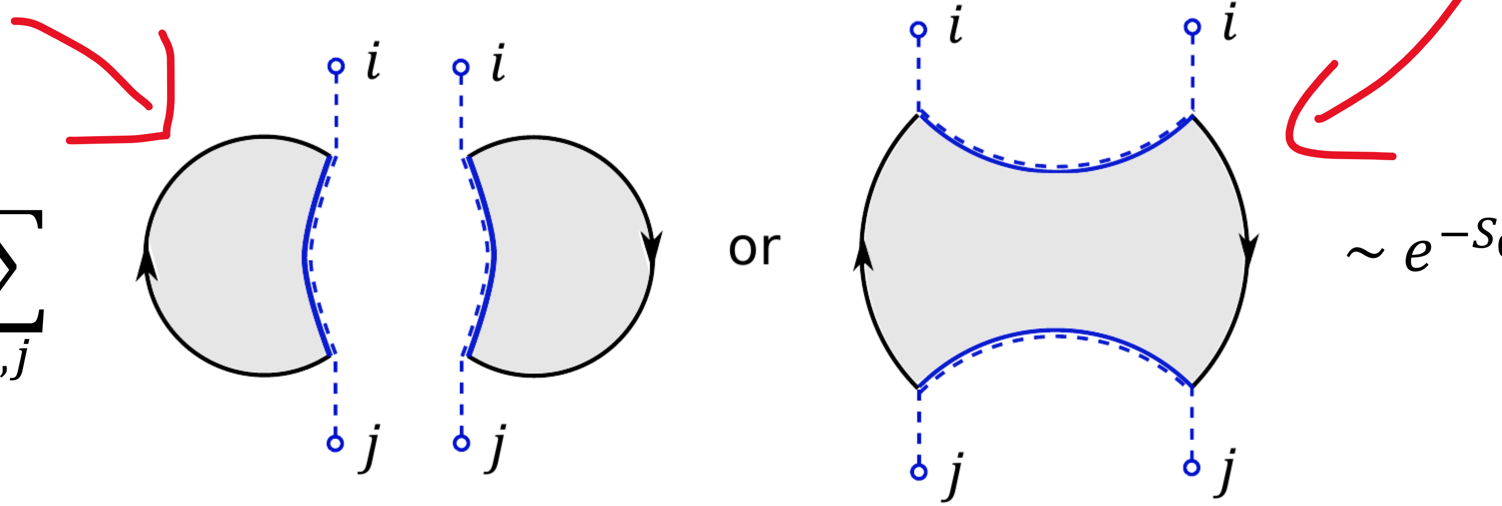
Calculate the purity $Tr(\rho_R^2)$ using a **Euclidean path integral**, where we sum over **all topologies** with the correct boundary conditions:

$$\mathrm{Tr}(\rho_{\mathsf{R}}^2) = \frac{1}{k^2} \sum_{i,j=1}^{k} |\langle \psi_i | \psi_j \rangle|^2.$$

Very small but doesn't decay at large k

$= \frac{1}{k}$

$\sum_{i,j}$



$\sim e^{-S_0}$

Two copies of the original black hole

Two black holes connected by a wormhole

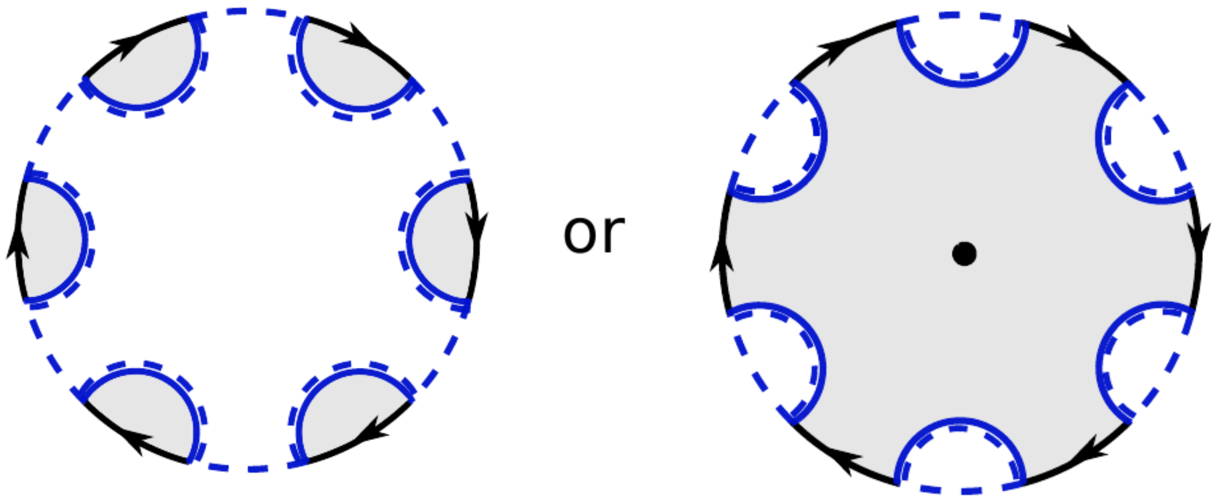
Calculating the von Neumann Entropy

In general, there are a lot of topologies that can contribute to $\mathrm{Tr}(\rho_R^n)$. However, in the limit where k is **very large/small** one of **two families of topologies** dominates

Small k

Large k

$$\frac{1}{k^{n-1}} =$$



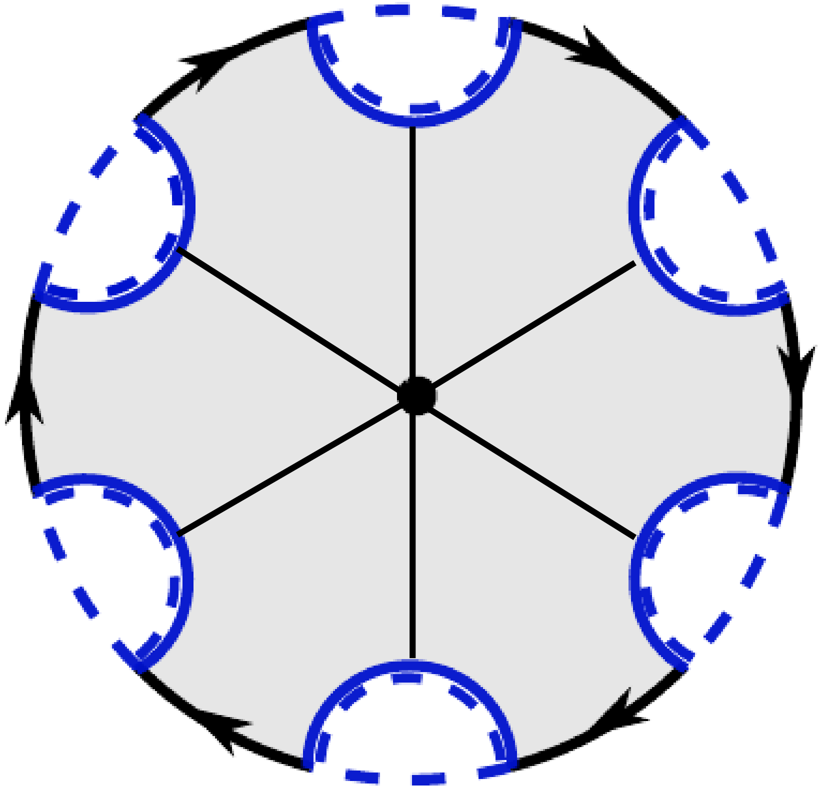
or

$$\sim e^{-(n-1)S_0}$$

Disconnected topology
Gives $S = \log k$

Connected topology
Gives $S = S_{BH}$

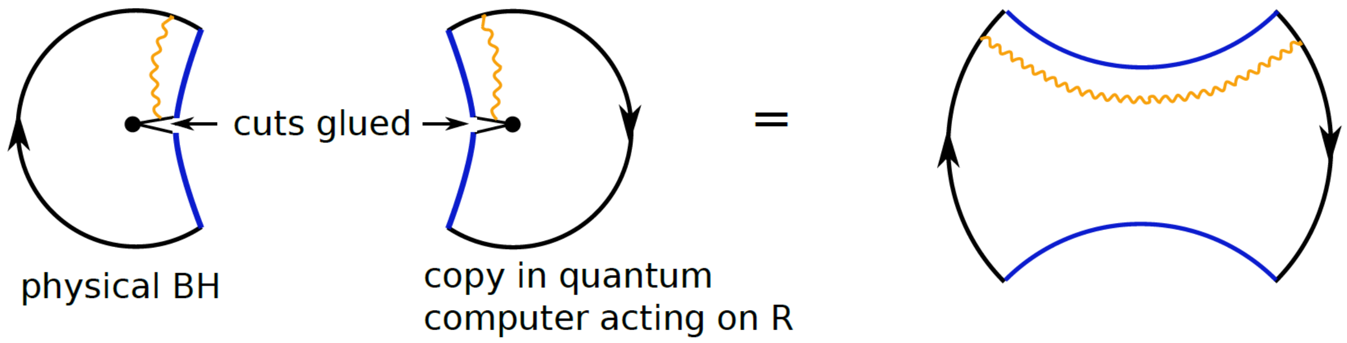
Calculating the von Neumann Entropy



- Connected topology has a Z_n replica symmetry.
- After **quotienting** by this symmetry, we get *roughly* the original black hole geometry, except that there is a **conical singularity** at the fixed point of the replica symmetry
- In the limit $n \to 1$, the singularity **vanishes** (get original unbackreacted geometry)
- Von Neumann entropy given by the “area” of replica fixed point (in this case the **bifurcation surface**)

Extracting Information from the Interior

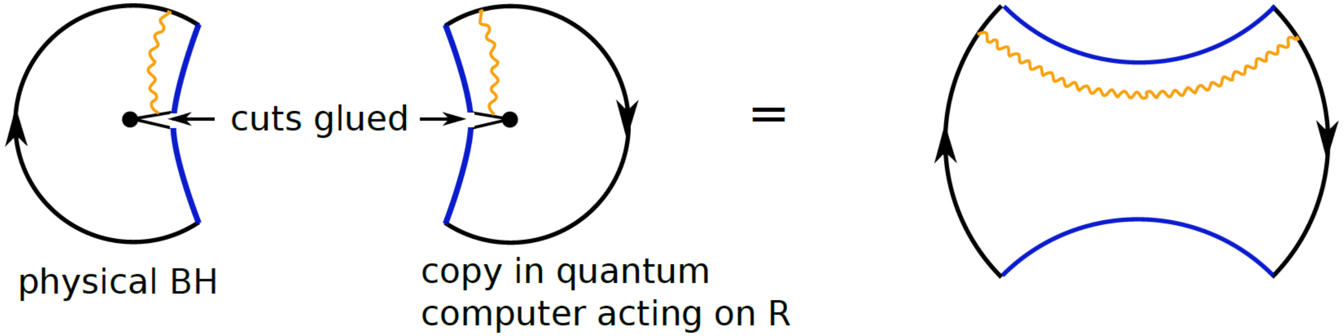
- Can extract information from the black hole interior using a **general purpose** QEC **recovery map** called the **Petz map**
- In order to do so, the **quantum computer** decoding the information effectively has to **simulate** the black hole evaporation process
- The information escapes because of **spacetime wormholes** connecting the real and simulated black holes
- **After the Page time** (and only after), the wormhole configuration **dominates**, and so information can be extracted



(Cotler, Hayden, **GP**, Salton, Swingle, Walter *arXiv:1704.05839*)
(Chen, **GP**, Salton *arXiv:1902.02844*)

Extracting Information from the Interior

- Should note: last slide somewhat **oversimplified**
- Just like the **von Neumann entropy**, the Petz map calculation involves an **analytic continuation** where the number of ‘simulated’ black holes goes to **zero**
- Obviously you can’t analytically continue a **quantum circuit**: this is just a mathematical trick for doing the calculation
- The actual way to implement the recovery is to do **Grover search**. This involves simulating the black hole **exponentially many times**.
- Hawking was **information-theoretically** wrong, but **computationally** right



Very, very hard

(Brown, Gharibyan, **GP**, Susskind *arXiv:1912.00288*)

Much more to say in this model!

- Simple enough that we can do the **full path integral**, rather than just looking at classical saddle points
- Can use tools from **free probability theory** to find the corrections to the von Neumann entropy, and even the **full entanglement spectrum,** near the Page transition, when the Renyi entropies are not dominated by a single topology.
- Transition is **complicated**, with **seven distinct phases**. However the main qualitative features agree with previous expectations.
- (In particular, there are $O(1/\sqrt{G_N})$ corrections near the transition from **energy fluctuations**.)

Part 2: Actual Evaporating Black Holes

(**GP** *arXiv:1905.08255*)

What about actual evaporating black holes?

- **Bad news:** no one has found analytic solution for the **replica wormhole** geometry at integer $n \geq 2$ in more realistic models.
- **Numerical results** for the **SYK model** suggest that the physics is inherently **messy**, with complicated backreaction related to the **fast scrambling** behaviour.
- **Good news**: we saw in the simple model that the **von Neumann entropy** was controlled by the area of the **replica fixed point** in the **original** geometry.

Generally true

(Also true of the Petz map calculations)

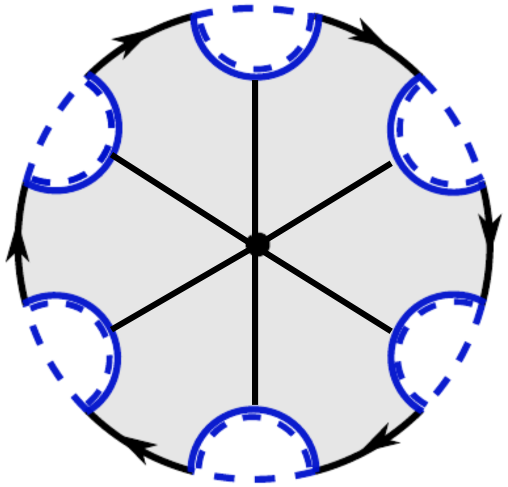
The Quantum Extremal Surface Prescription

1. In the limit $n \to 1$, the equations of motion imply that the replica fixed point needs to be a **quantum extremal surface**

$$\text{ext} \left[\frac{A(\chi)}{4G_N} + S_{\text{bulk}}(\chi) \right]$$

Generalised entropy

Entanglement between bulk fields on opposite sides of χ



2. **Von Neumann entropy** (from given family of saddles) is the generalised entropy of the corresponding QES.

3. The **dominant family of saddles** comes from the replica fixed point with **smallest generalised entropy** (the **minimal QES**)

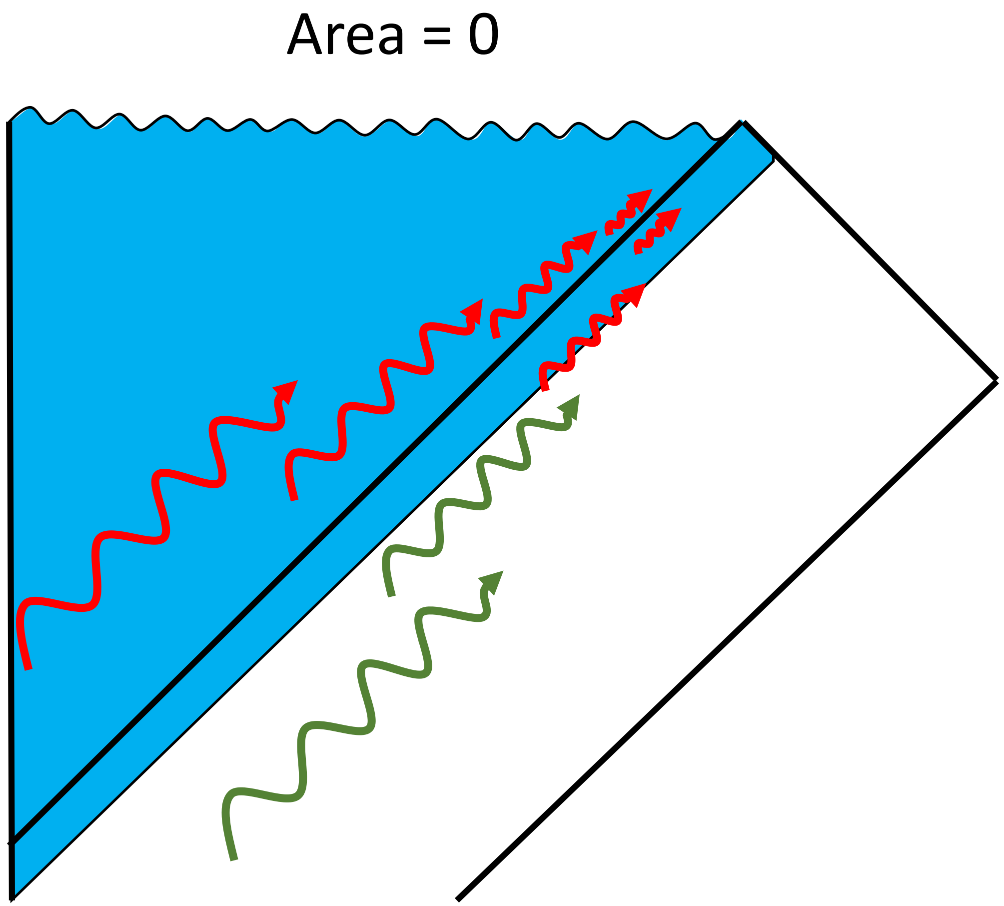
(Quantum-corrected) **Ryu-Takayanagi** formula

The Page Curve in Evaporating Black Holes

At **early times**, disconnected replica topologies dominate

No replica fixed point = **empty QES**

Generalised entropy = A/4G_N= $A/4G_N + S_{bulk}$ = **semiclassical entropy** of the Hawking radiation



Entanglement between **blue region** and escaped **Hawking radiation** grows linearly with time

The Page Curve in Evaporating Black Holes

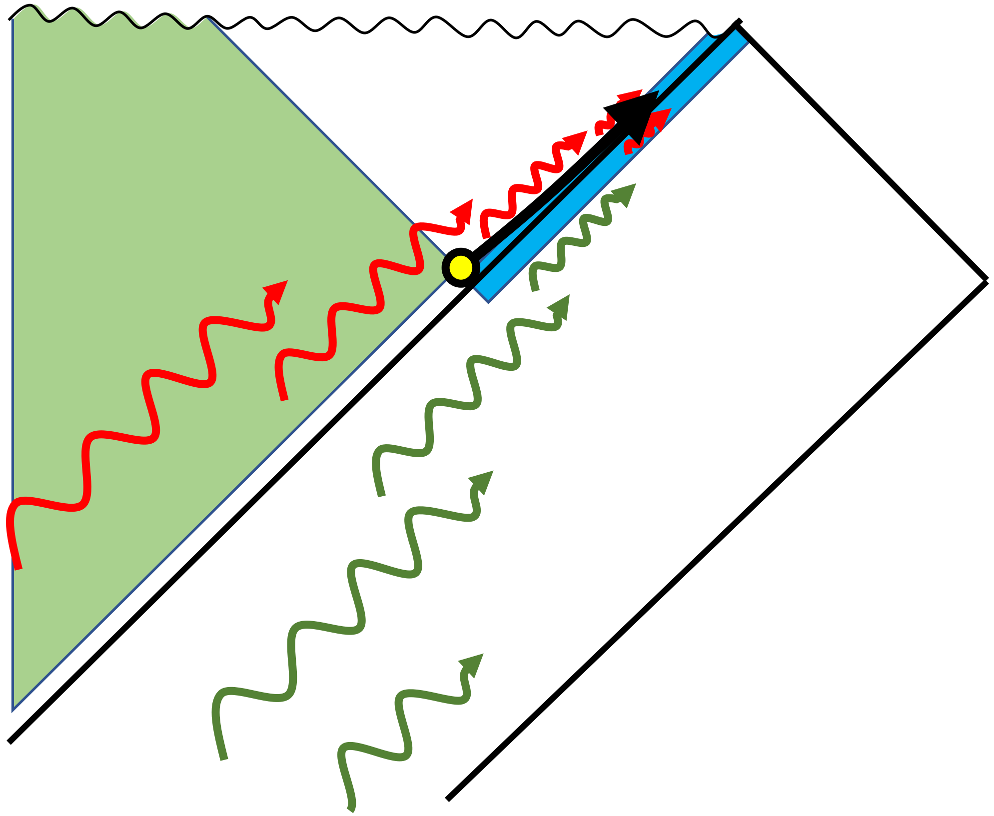
However, there also exists a **non-empty quantum extremal surface** that lies just inside the event horizon

Generalised entropy ≈ BH entropy

After the Page time, this becomes the **minimal QES** (this corresponds to the transition to a **fully connected replica wormhole topology**)

As the black hole continues to evaporate, the RT surface **tracks along the horizon**, travelling on a spacelike trajectory (generalised entropy **decreases** with time)

A ≈ horizon area

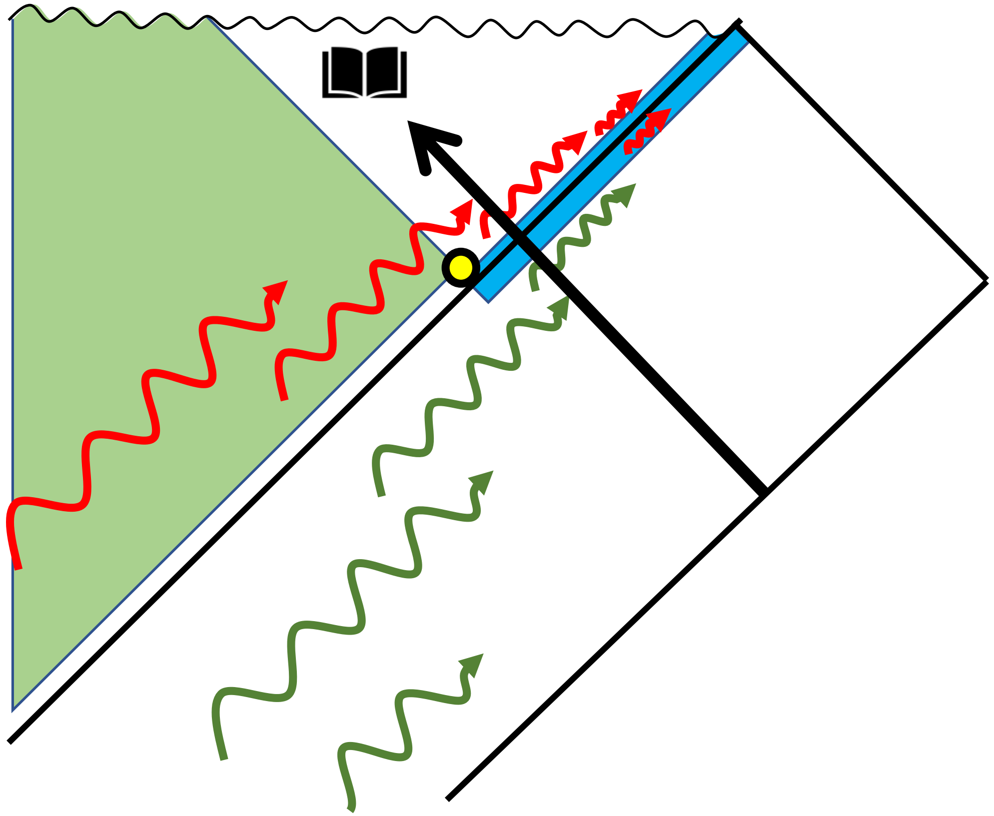


Entanglement between **blue region** and **green region + escaped Hawking radiation** is small (O(1))

Hayden-Preskill Decoding Criterion

Suppose we throw a **diary** into the black hole (after the Page time)

Initially, the worldline of the diary is in the **blue region**: this means that no information about the diary has escaped in the Hawking radiation



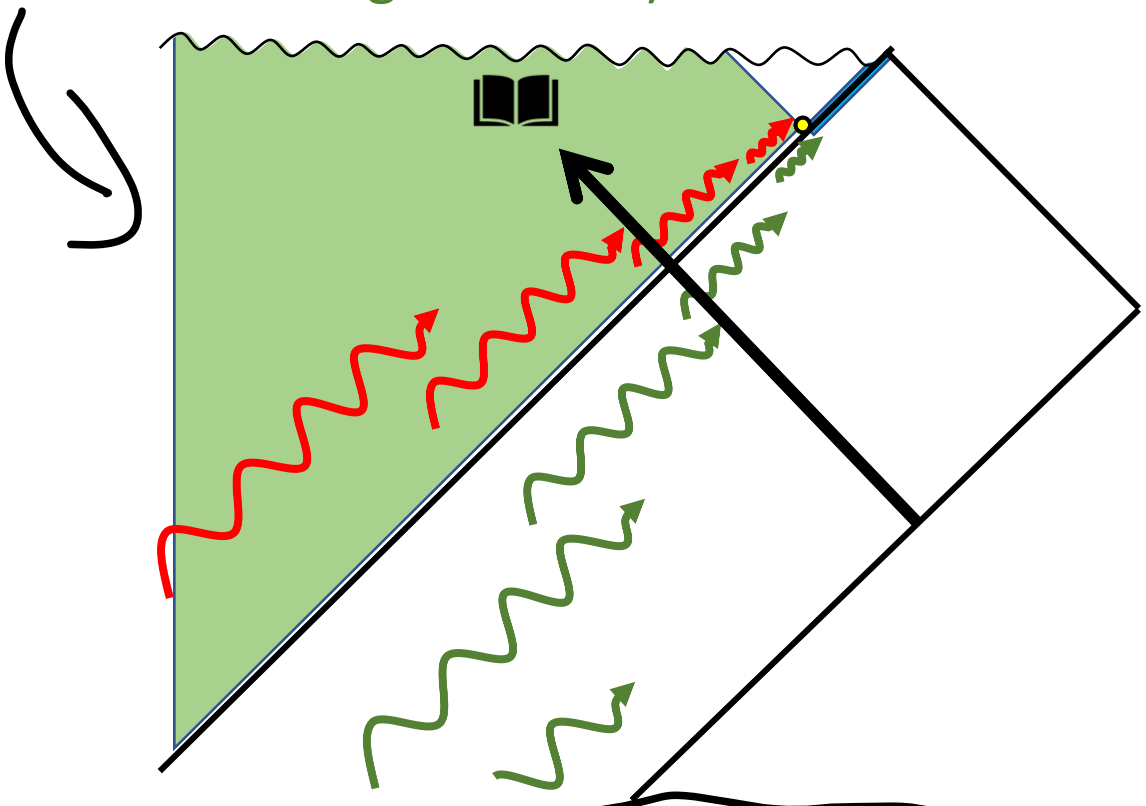
(**Petz map** won't learn anything, for example)

Hayden-Preskill Decoding Criterion

However as the black hole continues to evaporate, the RT surface continues to track along the horizon

After waiting for **more than the scrambling time**, the worldline of the diary will be in the **green region**

The information in the diary can now be **pulled out through a wormhole** by the Petz map: information has escaped!



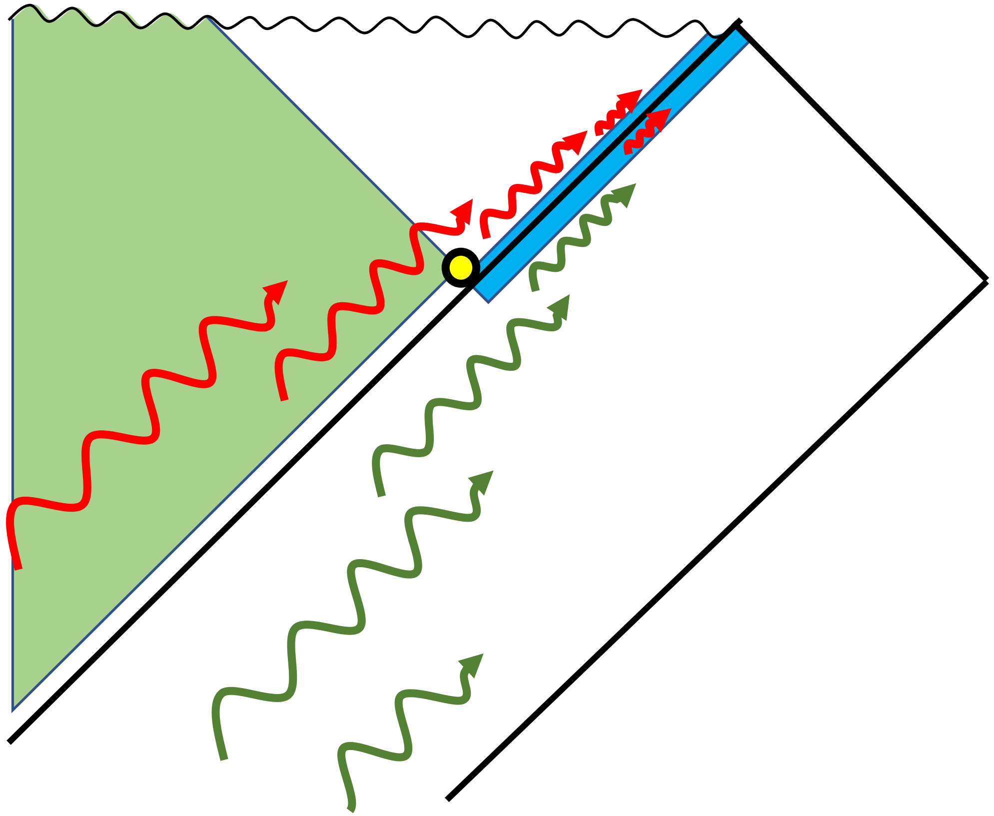
Famously predicted by **Hayden and Preskill**, based on simple toy models

The firewall paradox

AMPS Paradox: how can late time Hawking radiation be entangled with both **interior partner** and **early radiation** (given monogamy of entanglement)?

Answer: worldline of **interior partner** goes through **green region** so it is **encoded in** the early radiation (ER=EPR)

(Full story is more complicated with several important subtleties, but **quantum extremality** magically ensures that everything works out and you exactly avoid any firewall paradox.)

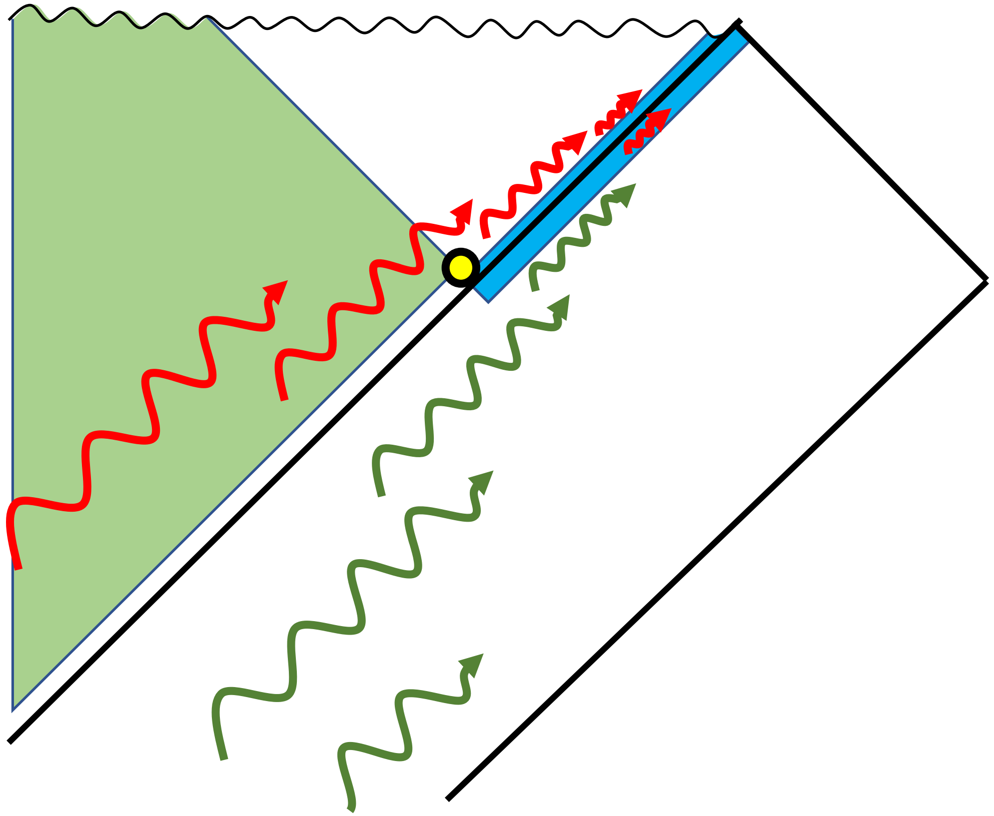


How does the information get out?

So far, I haven’t described a **mechanism** by which the state of matter that **fell into** the black hole can influence the state of the **Hawking radiation**

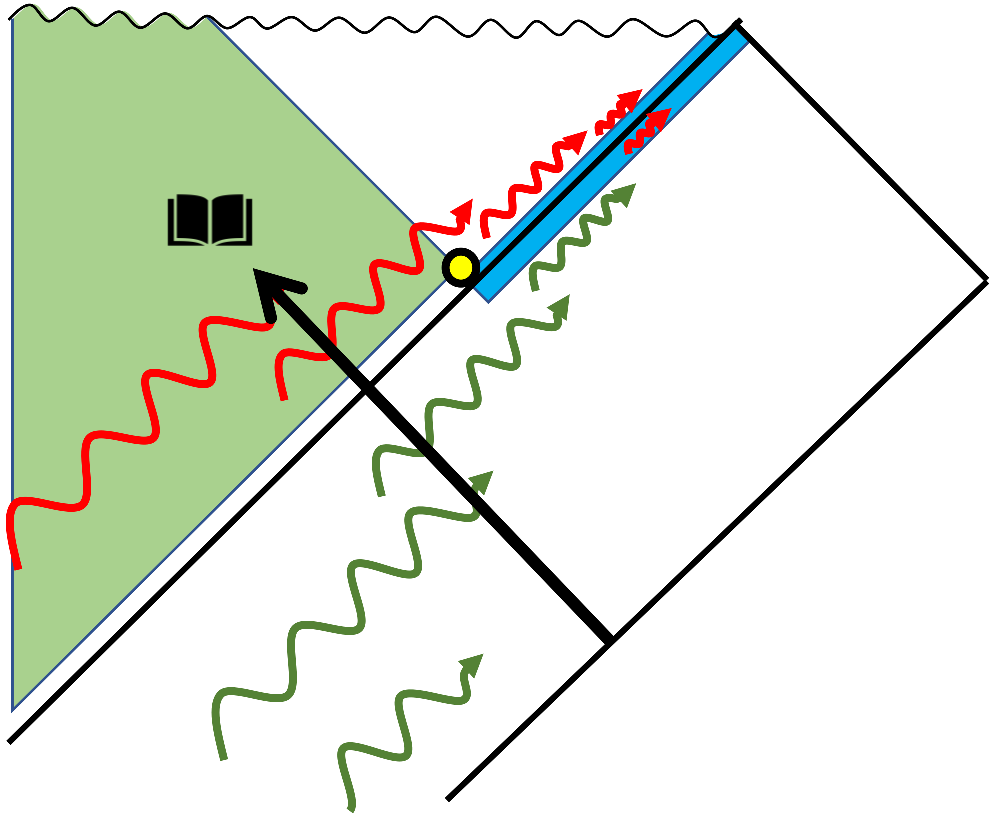
The state of any **Hawking mode** plus its interior partner is **fixed**, even if the interior partner is secretly encoded in the earlier radiation

Missing the last piece of the puzzle:
state dependence



What information comes out when?

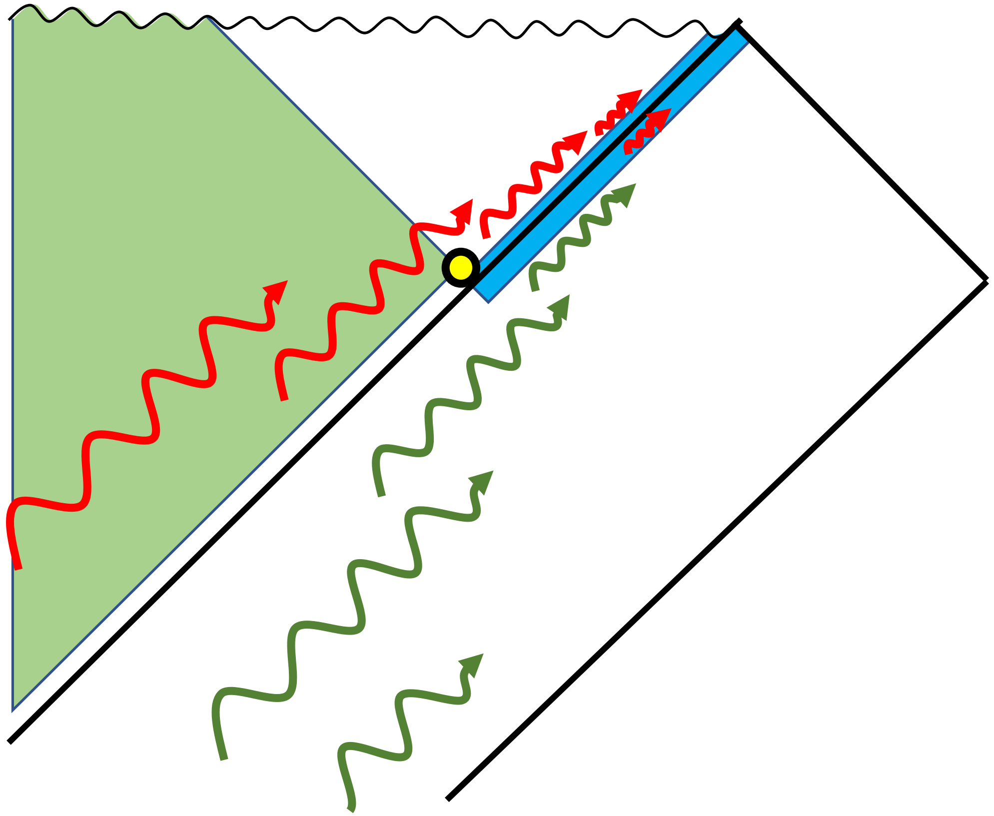
- Earlier claim: anything in the **green region** is encoded in the radiation after the Page time.
- Real claim: any **small subsystem** can be decoded **provided the state of everything else is known.**
- If you don't know that the state lies in a **sufficiently small code subspace**, you can't create a spacetime wormhole using a Petz map.
- The **encoding** of the interior in the Hawking radiation **depends on the state**
- As the black hole continues to evaporate, **larger and larger** subspaces can be decoded.
- Eventually, no **prior knowledge** of the state is required: all the information has escaped



(Hayden, **GP** *arXiv:1807.06041*)

How does this resolve the paradox?

- The **late time Hawking mode** is entangled with its **interior partner**, in a fixed state (as Hawking predicted).
- However, the **encoding** of the interior partner in the **early Hawking radiation**, depends on the state of stuff that fell into the black hole.
- Hence, the **combined state** of early and late radiation depends on the state of the matter that fell in.
- Information gets out!



Some final comments

- This whole story is only possible because the **error correction** is only **approximate** (approximate and exact QEC behave very differently when the Hilbert space dimension is large).
- Although the results I discussed are consistent with **both** a single unitary theory, and an average of an **ensemble** of unitary theories. In fact the **simple toy model** is calculating the average of an ensemble of unitary theories.
- We believe other more complicated theories of quantum gravity (N=4 SYM) are dual to a **single unitary theory**, but there are still questions about how this can be fully consistent with a fundamental theory containing **spacetime wormholes**.
- Lots of interesting questions left!

Thank you!