



# **Thermodynamics in $(2+1)$ -flavor QCD by the gradient flow method**



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in collaboration with  
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(WHOT-QCD Collaboration)

# QCD Thermodynamics with Gradient Flow

## Gradient flow

Lüscher(2009–), Narayanan-Neuberger(2006)

Imaginary evolution of the system into a fictitious "time"  $t$  preserving gauge sym. etc.:

(ex) pure gauge theory  $\dot{B}_\mu = D_\nu G_{\nu\mu}$ ,  $B_\mu|_{t=0} = A_\mu$  — original gauge field

We may view the flowed field  $B_\mu$  as a smeared  $A_\mu$  over a physical range of  $\sqrt{(8t)}$ .

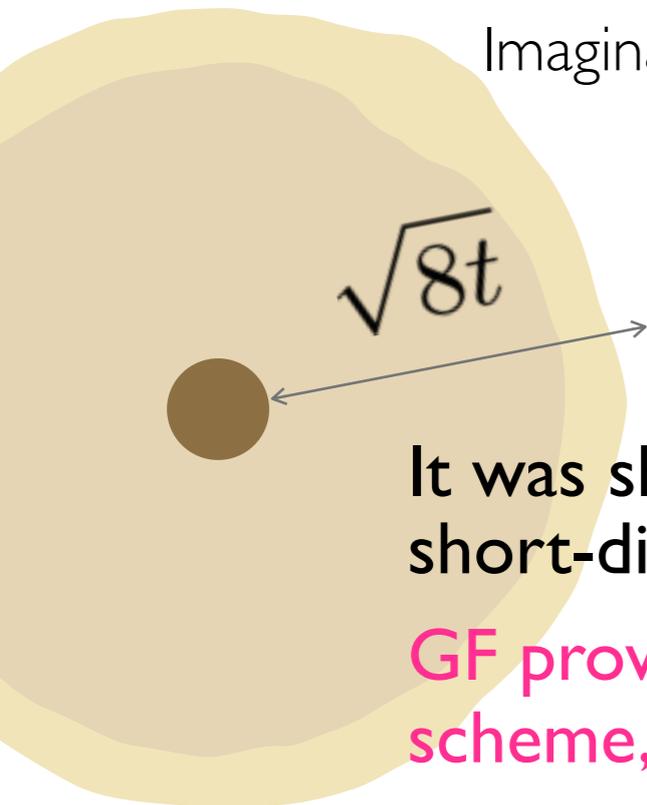
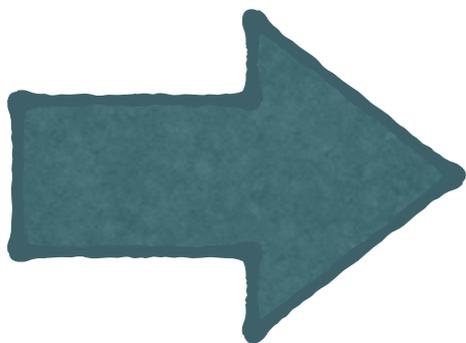
It was shown that operators of flowed fields have no UV divergences nor short-dist. singularities at  $t > 0$ . Lüscher-Weisz(2011)

GF provides us with a new physical (i.e. non-perturbative) renormalization scheme, which is directly calculable on the lattice in the  $a \rightarrow 0$  limit.

This opened many possibilities to drastically simplify lattice evaluation of physical observables.

## Our project: Application of GF to thermodyn. of (2+1)-flavor QCD

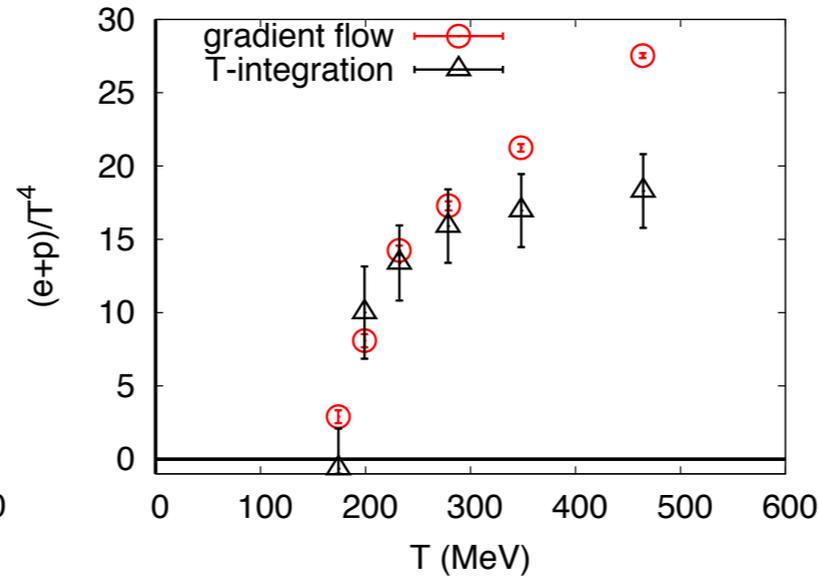
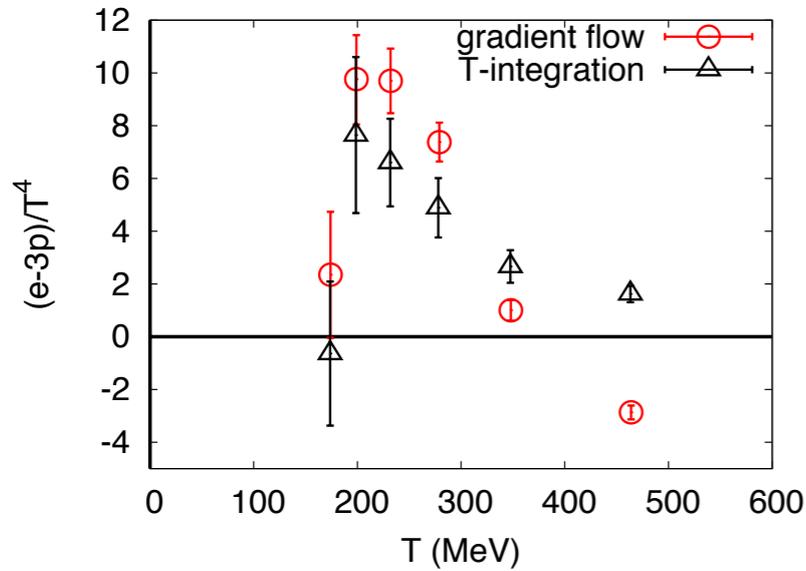
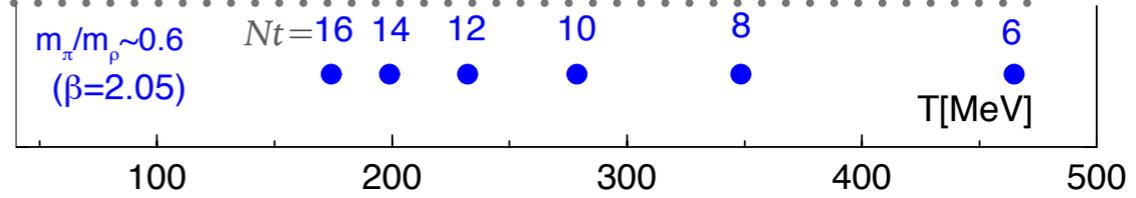
- EMT / EOS H. Suzuki (2013), Makino-Suzuki (2014)
- Chiral condensate Hieda-Suzuki (2016)
- Topological charge / susceptibility, ...



# Main Results

Iwasaki gauge + NP clover at  $a \approx 0.07\text{fm}$ ,  $m_{PS}/m_V \approx 0.63$   
 $T/T_{pc} \approx 0.92 - 2.44$  with the fixed-scale approach.

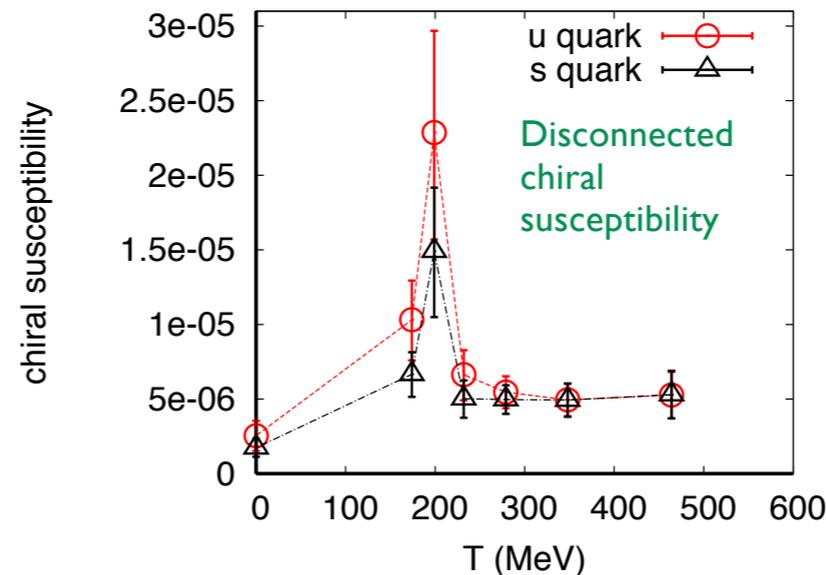
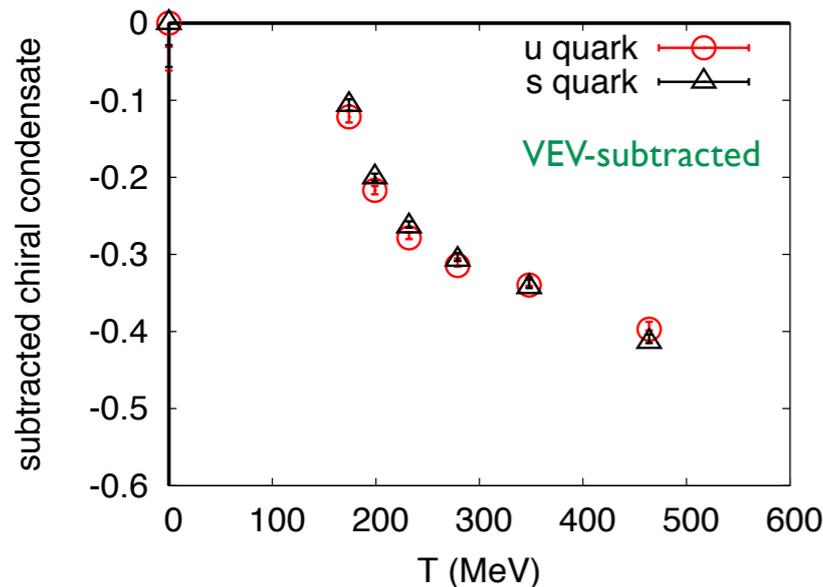
## Energy-momentum tensor and EOS



$T_{pc}$

★ Good agreement with the conventional method at  $T \leq 300$  MeV ( $Nt \geq 10$ ), suggesting small- $Nt$  artifacts at  $Nt < 10$ .

## Chiral condensate / disconnected susceptibility



★ Clear signal of  $T_{pc}$  at  $\approx 190$  MeV, with Wilson-type quarks! Peak higher with lighter quark.

# Main Results

Iwasaki gauge + NP clover at  $a \approx 0.07\text{fm}$ ,  $m_{PS}/m_V \approx 0.63$   
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## Topological susceptibility

We test two alternative definitions:

(a) gauge definition

$$q(t, x) = \frac{1}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} G_{\mu\nu}^a(t, x) G_{\rho\sigma}^a(t, x)$$

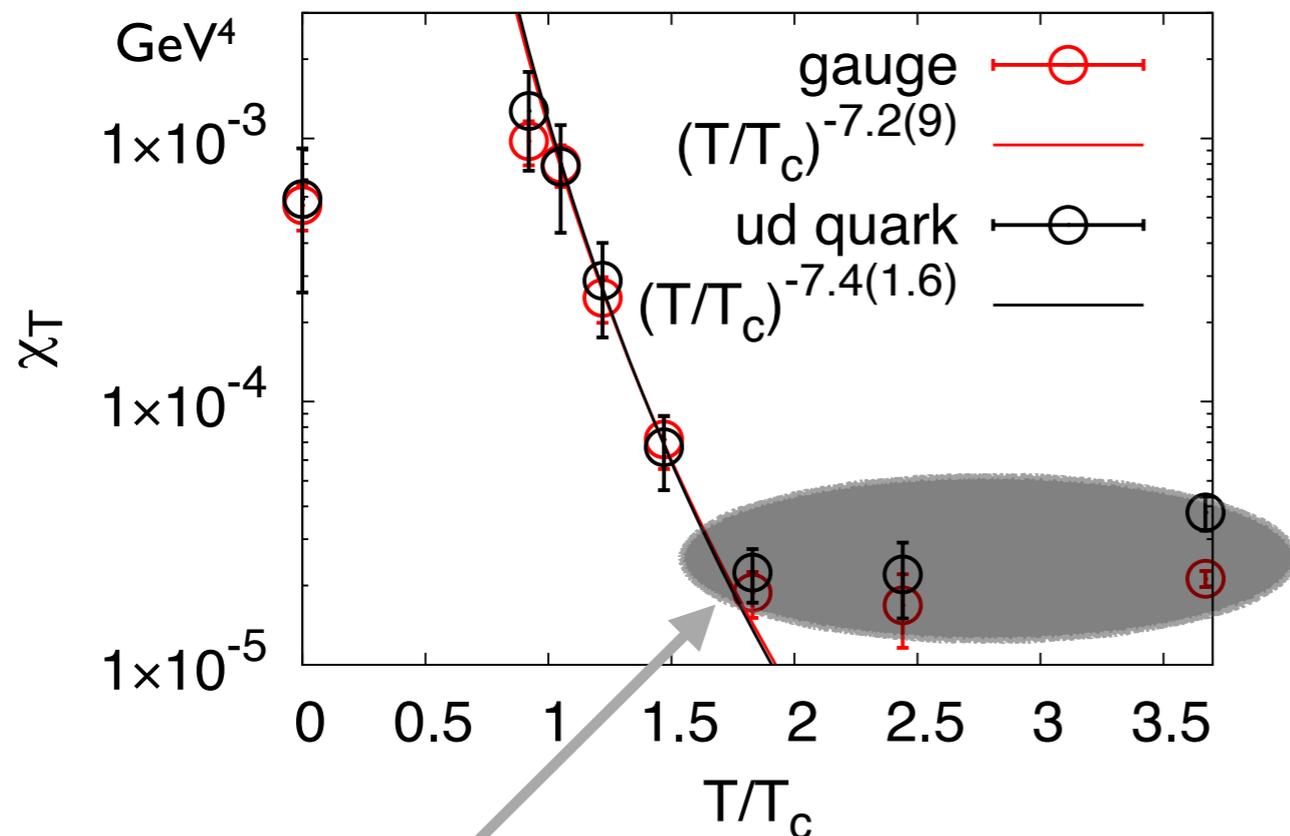
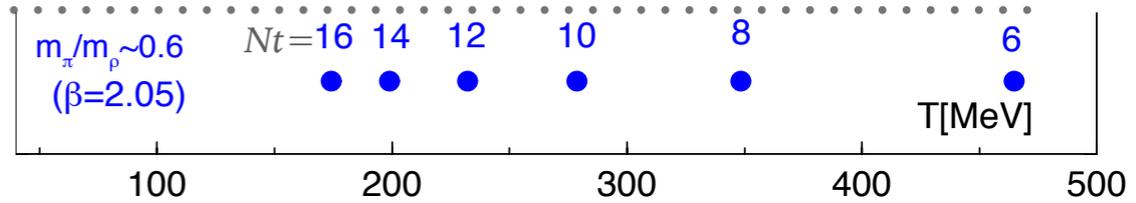
(b) fermion definition

$$N_f^2 \langle Q^2 \rangle = m^2 (\langle P^0 P^0 \rangle - N_f \langle P^a P^a \rangle)$$

Lüscher('10), Consonni-Engel-Giusti('15)  $T_{pc}$

$$Q(t) = \int d^4x q(t, x)$$

Giusti-Rossi-Testa('04)  
 using chiral W-T identities



- ★ Gauge definition and fermion definition agree well with each other at  $T \leq 1.5$ , even with Wilson-type quarks!
- ★ Power-low behavior of gauge and fermion definitions consistent with the dilute instant gas approximation (DIGA) which predicts the exponent to be  $-8$ .

Flow for  $t < t_{1/2}$  insufficient for gauge definition, giving upper bounds.  
 Small- $Nt$  error severe for  $Nt \leq 8$  (from our study of EMT).  
 At  $T/T_{pc} > 2$ ,  $Q$  is frozen to 0. Need a different method, e.g. Frison et al, arXiv:1606.07175.

# Thermodynamics in (2+1)-flavor QCD by the gradient flow method

WHOT-QCD Collaboration: Kazuyuki Kanaya\*, Shinji Ejiri<sup>1</sup>, Ryo Iwami<sup>1</sup>, Masakiyo Kitazawa<sup>2</sup>, Hiroshi Suzuki<sup>3</sup>, Yusuke Taniguchi\*, Takashi Umeda<sup>4</sup>, Naoki Wakabayashi<sup>1</sup>  
 \*: U. Tsukuba, <sup>1</sup>: Niigata U., <sup>2</sup>: Osaka U., <sup>3</sup>: Kyushu U., <sup>4</sup>: Hiroshima U.

We study the energy momentum tensor (EMT), chiral condensate, and topological susceptibility in (2+1)-flavor QCD with improved Wilson quarks. These quantities have been not straightforward to be studied on the lattice due to the explicit violation of

the relevant symmetry on the lattice. To calculate them, we adopt the strategy of H. Suzuki using the gradient (GF) flow method.

As our first study with dynamical quarks, we reuse configurations generated in our previous study of EOS in (2+1)-flavor QCD on

a fine lattice with approximately physical s quark and heavy u and d quarks ( $m_{PS}/m_V \approx 0.63$ ) in the range  $T \approx 174-697$  MeV.

We found that GF with Suzuki's strategy works quite well for all of these observables.

## 1. Gradient flow Lüscher('09-), Narayanan-Neuberger('06)

Imaginary evolution of the system into a fictitious "time"  $t$ .

$$\hat{B}_\mu = D_\nu G_{\nu\mu}, \quad B_\mu|_{t=0} = A_\mu$$

We may view the flowed field  $B_\mu$  as a smeared  $A_\mu$  over a physical range  $\sqrt{8t}$ .

It was shown that operators in terms of  $B_\mu$  have no UV divergences nor short-dist. singularities at  $t > 0$ . Lüscher-Weisz('11)

=> GF defines a physical (non-pert.) renormalization scheme, which can be calculated directly on the lattice.

This opened many possibilities to drastically simplify lattice evaluation of renormalized observables.

## 2. QCD thermodynamics with GF Suzuki ('12-)

The strategy of H. Suzuki to compute observables with GF:

- 1) Define observable in a continuum regularization.
- 2) Relate it with lattice operator thru finite quantities at  $t > 0$ .
- 3) Remove unwanted contributions at  $t > 0$  by taking  $t \rightarrow 0$ .

This avoids difficulties in LQCD to define observable related to a symmetry which is violated on the lattice.

We apply the strategy to 2+1 flavor QCD at finite temperature:

\* Energy-momentum tensor and EOS  $\epsilon = -(T_{00}), p = \frac{1}{3} \sum_i T_{ii}$

Makino-Suzuki, PTEP 2014, 063B02 (2014)

\* Chiral condensate and its susceptibility

\* Topological susceptibility

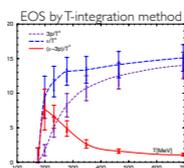
Hieda-Suzuki, arXiv:1606.04193 (2016)

which are otherwise difficult to calculate on the lattice due to explicit violation of relevant symmetry (continuous coordinate transformation, chiral symmetry, etc.).

## 3. Simulation parameters

- ☑  $N_f=2+1$  QCD, Iwasaki gauge + NP-clover // fine lattice, physical s & heavy ud
- ☑ CP-PACS+JLQCD's  $T=0$  config. ( $\beta = 2.05, 28^3 \times 56, a \approx 0.07 \text{ fm}, m_{PS}/m_V \approx 0.63$ ) available on ILDG/JLDG
- ☑ WHOT-QCD's  $T > 0$  fixed-scale config.set ( $32^3 \times N_t, N_t = 4, 6, 8, 10, 12, 14, 16$ ) WHOT-QCD, Phys.Rev.D85, 094508 ('12)
- ☑ gauge measurements at every config..
- ☑ quark measurements every 5(10) config's
- ☑ continuum extrapolation => next step study

$T$ (MeV)	$T/T_{pc}$	$N_t$	$t_{1/2}$	gauge confs.
0	0	56	24.5	650
174	0.92	16	8	1440
199	1.05	14	6.125	1270
232	1.22	12	4.5	1290
279	1.47	10	3.125	780
348	1.83	8	2	510
464	2.44	6	1.125	500
697	3.67	4	0.5	700



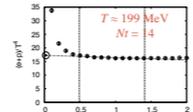
To avoid oversmearing wrapping around the lattice,  
 $\sqrt{8t/a^2} \leq \min(N_s/2, N_t/2)$   
 i.e.,  $t/a^2 \leq t_{1/2} = [\min(N_s/2, N_t/2)]^2 / 8$

$T_{pc} \approx 190$  MeV assumed

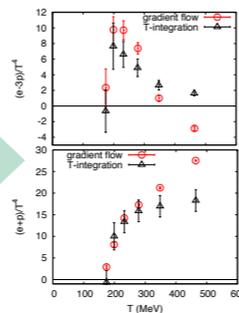
## 4. Energy-momentum tensor and EOS

GF enables us to determine EMT and EOS without additional inputs such as the beta-function.

Clear linear windows found for  $t \rightarrow 0$  at  $N_t \geq 6$ .

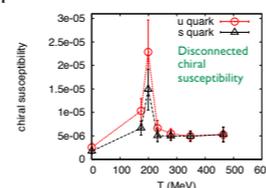
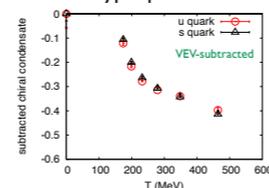


★ Good agreement with the conventional method at  $T \leq 300$  MeV ( $N_t \geq 10$ ), suggesting small- $N_t$  artifacts at  $N_t < 10$ .



## 5. Chiral condensate

GF works well also for the chiral cond., which has been difficult with Wilson-type quarks due to explicit chiral violation.



★ Clear signal of  $T_{pc}$  at  $\approx 190$  MeV. Peak higher with lighter quark.

## 6. Topological susceptibility

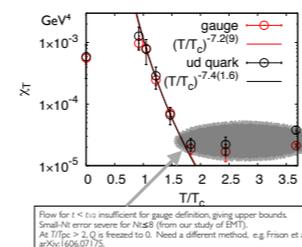
We test two alternative definitions:

(a) gauge definition Lüscher('10), Consonni-Engel-Giusti('15)

$$q(t, x) = \frac{1}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} G_{\mu\nu}^a(t, x) G_{\rho\sigma}^a(t, x) \quad Q(t) = \int d^4x q(t, x)$$

(b) fermion definition Guisti-Rossi-Testa('04)

$$N_f^2 \langle Q^2 \rangle = m^2 (\langle P^0 P^0 \rangle - N_f \langle P^a P^a \rangle) \quad \text{using chiral W-T identities}$$



★ Gauge definition and fermion definition agree well with each other at  $T \leq 1.5$ , even with Wilson-type quarks!

★ Power-low behavior of gauge and fermion definitions consistent with the dilute instant gas approximation (DIGA) which predicts the exponent to be  $-8$ .

## 7. Discussions

- ★ GF opens us a big possibility to accelerate the study of QCD thermodynamics.
- ★ Though a definite conclusion possible only after cont. extrapolation, our nice results at  $N_t \geq 10$  suggests that our lattices are sufficiently fine, while small- $N_t$  artifact visible at  $N_t < 10$ .
- ★ Further studies called.

*please visit our poster for details*

# SUMMARY



- We apply gradient flow ideas to investigate thermodynamics of (2+1)-flavor QCD. As the first test, we choose heavy  $ud$  quarks with physical  $s$  quark, on a fine lattice ( $a \approx 0.07\text{fm}$ ,  $m_{PS}/m_V \approx 0.63$ ), and adopt the fixed-scale approach.
- EOS agrees with conventional  $T$ -integration method at  $T \leq 300$  MeV ( $Nt \geq 10$ ).
- Chiral condensate and its disconnected susceptibility also calculated. Even with the explicit chiral violation of Wilson-type quarks, we obtain reasonable results,
- Topological susceptibility by gauge definition and by fermion definition beautifully agree with each other, and reproduce the  $T$ -dep. of DIGA.
- But, note that our  $m_\pi \sim 400$  MeV and finite  $a$ . A definite conclusion possible only after cont. extrapolation.
- Our good results at  $Nt \geq 10$ , however, suggests that our lattices are sufficiently fine, while small- $Nt$  artifacts visible at  $Nt < 10$ .
- Further study needed to complete the cont. extrapolation, and at lighter  $m_{ud}$ .