Exotic clustering in light nuclear systems
N. Itagaki
Yukawa Institute for Theoretical Physics, Kyoto University
single-particle motion of protons and neutrons

weakly interacting state of (strongly bound) clusters

decay threshold to clusters

Excitation energy

Nuclear structure

Energy (MeV)
Synthesis of $^{12}$C from three alpha particles

The necessity of 3$\alpha$-cluster state has been pointed out from astrophysical side, and experimentally confirmed afterwards.
Microscopic Study of the Triple-α Reaction

A. S. Umar, J. A. Maruhn, N. Itagaki, and V. E. Oberacker

1Department of Physics and Astronomy, Vanderbilt University, Nashville, Tennessee 37235, USA
2Institut für Theoretische Physik, Goethe-Universität, D-60438 Frankfurt am Main, Germany
3Department of Physics, University of Tokyo, Hongo, Tokyo 113-0033, Japan

(Received 24 March 2010; published 27 May 2010)

FIG. 1: (Color online) Selected density profiles from TDHF time-evolution of the $^4\text{He}+^8\text{Be}$ head-on collision for initial Be orientation angle $\beta = 0^\circ$ (see small graphs in Fig. 2) using the SLy4 interaction. The initial energy is $E_{\text{c.m.}} = 2$ MeV. For $T < 2500$ fm/c the system vibrates about the linear chain configuration shown in the top pane, subsequently the system changes its mode to a bending configuration shown in the middle pane, and finally relaxes into a more compact configuration as shown in the bottom pane. Note that the region shown is only a part of the computational mesh.

FIG. 2: (Color online) Potential energy curves for the collision of the $^4\text{He}+^8\text{Be}$ system as a function of $R$ for three initial alignments of the Be nucleus and at $E_{\text{c.m.}} = 2$ MeV.
Lifetime of linear chain as a function of impact parameter

\[ ^4\text{He} + ^8\text{Be} \]
\[ E_{\text{c.m.}} = 2 \text{ MeV} \]

**FIG. 4:** Time spent in the linear chain configuration as a function of the impact parameter \( b \) for the \(^4\text{He}+^8\text{Be}\) system at \( E_{\text{c.m.}} = 2 \text{ MeV} \) and \( \beta = 0^\circ \) alignment.
single-particle motion of protons and neutrons

weakly interacting state of clusters

decay threshold to clusters

cluster structure with geometric shapes

Excitation energy

cluster structure with geometric shapes

single-particle motion of protons and neutrons

Excitation energy

weakly interacting state of clusters
\( ^{10}\text{Be} \)

RIビームファクトリーで得られる重イオン（1次ビーム）エネルギーは、あらゆる核種についてRIビーム発生に必要なエネルギー（核子あたり百メガ電子ボルト以上）を大幅に上回ります。その結果、現在世界トップクラスの性能を誇る理研リングサイクロトロンのビームもしても数十個程度しか発見できなかった新RIの種類が飛躍的に増大し、その数は、千個にも及ぶと予想されます。これらの新RIの性質を系統的かつ詳細に調べることが、宇宙の元素合成のメカニズムの謎を解明する手掛かりとなります。更には、実験に利用可能な強度が得られるRIの種類も大幅に増加し、原子核物理学の分野のみならず基礎物理学の問題から生物・医学の分野にわたりて新たなプローブを提供することもできます。
How about in neutron-rich nuclei?

It becomes stable due to the glue effect of the neutrons?
Interactions (Skyme) and model space are ones for mean-field models

<table>
<thead>
<tr>
<th>Force</th>
<th>$E_B$</th>
<th>$\pi^2 \delta^2$</th>
<th>$\pi^2 \pi'^2$</th>
<th>$\pi^2 \delta \pi'$</th>
<th>$\pi^2 \sigma \pi'$</th>
<th>$\pi^2 \delta \pi''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SkI3</td>
<td>101.5</td>
<td>19.5</td>
<td>14.5</td>
<td>17.0</td>
<td>19.1</td>
<td>17.5</td>
</tr>
<tr>
<td>SkI4</td>
<td>100.8</td>
<td>19.9</td>
<td>15.7*</td>
<td>17.6</td>
<td>19.7</td>
<td>18.0</td>
</tr>
<tr>
<td>Sly6</td>
<td>100.6</td>
<td>18.9</td>
<td>15.4*</td>
<td>17.0</td>
<td>19.0</td>
<td>17.3</td>
</tr>
<tr>
<td>SkM*</td>
<td>115.0</td>
<td>17.5</td>
<td>16.4*</td>
<td>16.9</td>
<td>19.7</td>
<td>17.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Force</th>
<th>$\beta_{g.s.}$</th>
<th>$\pi^2 \delta^2$</th>
<th>$\pi^2 \pi'^2$</th>
<th>$\pi^2 \delta \pi'$</th>
<th>$\pi^2 \sigma \pi'$</th>
<th>$\pi^2 \delta \pi''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SkI3</td>
<td>0.34</td>
<td>0.82</td>
<td>0.69</td>
<td>0.76</td>
<td>0.88</td>
<td>0.76</td>
</tr>
<tr>
<td>SkI4</td>
<td>0.33</td>
<td>0.80</td>
<td>0.68*</td>
<td>0.75</td>
<td>0.86</td>
<td>0.74</td>
</tr>
<tr>
<td>Sly6</td>
<td>0.32</td>
<td>0.81</td>
<td>0.68*</td>
<td>0.75</td>
<td>0.87</td>
<td>0.75</td>
</tr>
<tr>
<td>SkM*</td>
<td>0.28</td>
<td>0.79</td>
<td>0.66*</td>
<td>0.73</td>
<td>0.85</td>
<td>0.73</td>
</tr>
</tbody>
</table>
Stability against bending motion

Solid $^{20}\text{C}$ ($\pi^4 \sigma^2 \delta^2$)
Dotted $^{16}\text{C}$ ($\pi^4$)

\[ R(Q_{31}) \text{ [fm}^3\text{]} \]

\[ t \text{ [fm/c]} \]
Linear Chain Structure of Four-\(\alpha\) Clusters in \(^{16}\)O

T. Ichikawa,\( ^1\) J. A. Maruhn,\( ^2\) N. Itagaki,\( ^1\) and S. Ohkubo\( ^3,^4\)

\( ^1\)Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
\( ^2\)Institut fuer Theoretische Physik, Universitaet Frankfurt, D-60438 Frankfurt, Germany
\( ^3\)Department of Applied Science and Environment, University of Kochi, Kochi 780-8515, Japan
\( ^4\)Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan
(Received 17 June 2011; published 9 September 2011)

We investigate the linear chain configurations of four-\(\alpha\) clusters in \(^{16}\)O using a Skyrme cranked Hartree-Fock method and discuss the relationship between the stability of such states and angular momentum. We show the existence of a region of angular momentum (13–18\(\hbar\)) where the linear chain configuration is stabilized. For the first time we demonstrate that stable exotic states with a large moment of inertia (\(\hbar^2/2\Theta \sim 0.06–0.08\) MeV) can exist.

DOI: 10.1103/PhysRevLett.107.112501 

PACS numbers: 21.60.Jz, 21.30.Fe, 21.60.Cs

Rod-Shaped Nucleus

We picture atomic nuclei as spherical globs of protons and neutrons, although they can also be egg-shaped. Now calculations published 9 September in Physical Review Letters show that an even more exotic shape is possible: a rapidly spinning nucleus can form into a linear chain of several small clusters of neutrons and protons. Such exotic nuclear states could play important intermediary roles in the evolution of stars.
FIG. 5: Angular momentum as a function of rotational frequency $\omega$ for the Skyrme forces. The lines with solid symbols denote the calculated results for the rigid-body moment of inertia, while the open symbols denote the results for the cranking method.

FIG. 6. Calculated excitation energies of the four-$\alpha$ linear chain states with the SkI3 force versus the angular momentum. The dotted lines denote the corresponding cluster-decomposition threshold energies.
Threshold rule: gas like structure

Cluster structure with geometric shape

Excitation energy

Cluster-threshold

Competition between shell and cluster

Single particle motion of protons and neutrons
What is the key mechanism for the cluster-breaking?

Spin-orbit interaction is driving force of breaking $\alpha$ clusters and restoring the single particle motions of nucleons.

Is there some control parameter in the cluster wave function to take into account the spin-orbit contribution?


**20Ne case**

Cluster model – $^{16}$O+alpha model

Present model – $^{16}$O+quasi cluster

Four nucleons in the quasi cluster perform single particle motions around $^{16}$O

Simplified modeling of cluster-shell competition in $^{20}$Ne and $^{24}$Mg

G3RS interaction spin-orbit term

\[ V_{ls} = V_0 \left( e^{-d_1 r^2} - e^{-d_2 r^2} \right) P^3O \vec{L} \cdot \vec{S} \]

Total energy of $^{20}\text{Ne}$

- $V_0 = 0$ MeV
- $V_0 = 1000$ MeV
- $V_0 = 2000$ MeV

Solid, Dotted, Dashed lines $\rightarrow R = 0.5, 2.0, 4.0$ fm
• $^{12}\text{C}$ case

3alpha model $\Lambda = 0$

2alpha+quasi cluster $\Lambda = $ finite
Various configurations of $3\alpha$'s with $\Lambda=0$
Various configurations of $3\alpha$'s with $\Lambda=0$.
Λ is a good tool to prepare the α breaking configurations

However this is a control parameter introduced in the wave function and not an observable

After superposing Slater determinants with different Λ values, it is difficult to estimate the extent to which the α cluster is broken
We need to introduce an operator and calculate the expectation value of $\alpha$ breaking.

What is the operator related to the $\alpha$ breaking?

$$\sum_{i=\text{protons}} (l^*s)_i$$

one-body spin-orbit operator for the proton part
Various configurations of 3α’s with Λ=0

Energies of the $0^+$ states (MeV)

Number of the basis states

$\Lambda \neq 0$

One body $ls$
How about cluster-shell competition in neutron-rich nuclei?

We generate many Slater determinants with different configurations for the valence neutrons and superpose them
The graph shows the energy of the $0^+$ states (MeV) as a function of the number of basis states. The energy decreases as the number of states increases. There are two curves labeled $\Lambda = 0$ and $\Lambda \neq 0$. The values $0.51$, $0.51$, $0.37$, $0.99$, and $0.91$ are shown on the right side of the graph.
Lifetime Measurement of the $2_1^+$ State in $^{20}$C

M. Petri,1,* P. Fallon,1 A. O. Macchiavelli,1 S. Paschalis,1 K. Starosta,1,2,3,4 T. Baugher,3,4 D. Bazin,3 L. Cartegni,5 R. M. Clark,1 H. L. Crawford,1,5 M. Cromaz,1 A. Dewald,7 A. Gade,3,4 G. F. Grinyer,3 S. Gros,1 M. Hackstein,7 H. B. Jeppesen,1 I. Y. Lee,1 S. McDaniel,3,4 D. Miller,3,4 M. M. Rajabali,5 A. Ratkiewicz,3,4 W. Rother,7 P. Voss,3,4 K. A. Walsh,3,4 D. Weisshaar,3 M. Wiedeking,8 and B. A. Brown4

1Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA
2Department of Chemistry, Simon Fraser University, Burnaby, British Columbia, V5A 1S6, Canada
3National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824, USA
4Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA
5Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA
6Department of Chemistry, Michigan State University, East Lansing, Michigan 48824, USA
7Institut für Kernphysik der Universität zu Köln, D-50937 Köln, Germany
8Lawrence Livermore National Laboratory, Livermore, California 94551, USA

(Received 21 January 2011; published 30 August 2011)

![Chart showing B(E2) trend](chart.png)

**FIG. 3** (color online). $B(E2; 2_1^+ \rightarrow 0^+_{\text{g.s.}})$ trend in even mass carbon isotopes for $A = 16–20$ including only statistical errors.
Fig. 15. Family of kaonic few-body states.

Fig. 16. Schematic structure of the $\bar{K}KN$ quasi-bound state with the inter-hadron distances.
Summary

• Nuclear structure changes as a function of excitation energy
• Cluster structure appears around the decay threshold, and geometric configurations are stabilized in neutron-rich nuclei
• We can simultaneously discuss the cluster-shell competition in the ground state and appearance of cluster states in the excited states
Summary

• Nuclear structure changes as a function of excitation energy

• Cluster structure appears around the decay threshold, and geometric configurations are stabilized in neutron-rich nuclei

• We can simultaneously discuss the cluster-shell competition in the ground state and appearance of cluster states in the excited states
Summary

• Nuclear structure changes as a function of excitation energy
• Cluster structure appears around the decay threshold, and geometric configurations are stabilized in neutron-rich nuclei
• We can simultaneously discuss the cluster-shell competition in the ground state and appearance of cluster states in the excited states
The optimum values of the parameters $R_1$ and $\Lambda$ for the carbon isotopes.

<table>
<thead>
<tr>
<th></th>
<th>$^{12}$C</th>
<th>$^{14}$C</th>
<th>$^{16}$C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$ (fm)</td>
<td>1.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>0.4</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>$E$ (MeV)</td>
<td>$-88.6$</td>
<td>$-106.4$</td>
<td>$-108.5$</td>
</tr>
</tbody>
</table>

3α cluster state is important in the excited states

0^+ states of $^{16}$C

Shell model

Cluster model

independent particle state

weak coupling state

SU(2)

U(3)

SO(3)

increasing Λ

increasing R

small Λ

small R

jj-coupling state

rigid rotor state

\( ^{18}\text{C} \)

Graph showing the energy of the \( 0^+ \) states in MeV as a function of the number of basis states. The graph includes lines with different slopes, indicating varying numbers of one body states.
\[ V_{ls} = V_0 \left( e^{-d_1 r^2} - e^{-d_2 r^2} \right) P^{(3)}O \cdot \vec{L} \cdot \vec{S} \]

\[ \begin{array}{cccc}
0 \text{ MeV} & 1000 \text{ MeV} & 2000 \text{ MeV} & \text{Exp.} \\
8^+ & 8^+ & 8^+ & \\
6^+ & 6^+ & 8^+ & \\
4^+ & 4^+ & 6^+ & 6^+ \\
2^+ & 2^+ & 4^+ & 4^+ \\
0^+ & 0^+ & 2^+ & 0^+ \\
\end{array} \]

\[ ^{20}\text{Ne} \]

After GCM

FIG. 1: The GCM calculations of yrast levels in \(^{20}\text{Ne}\) for three different values of the strength of the spin-orbit interaction: \(V_0 = 0, 1000, 2000\) MeV, are compared with the experimental data (Exp.). For more details, see the description in the text.
Give wave functions an initial boost factor with $r=3\text{fm}$, $\alpha=1\text{fm}$.

Then do time-dependent calculation.

Result shows long-term return to ground state, but some oscillations before.

\[
A \frac{\exp[iQ(\vec{r})]}{1 + e^{(r-r_0)/\alpha}}
\]

Figure 9: Time-dependent reaction of the chain state in $^{20}\text{C}$ to a slight bending-type excitation. Plotted is the $\Re(Q_{31})$ expectation value described in the text for an excitation of 0.04 MeV relative to the unperturbed chain-state energy.
FIG. 2: (Color online) Total nucleon density distribution calculated using the cranking method for (a) the initial wave function, (b) the ground state, (c) the quasi-stable state, and (d) the four-\(\alpha\) linear chain state. The isolines correspond to multiples of 0.02 fm\(^{-3}\). We normalize the color to the density distribution at the maximum of each plot.

FIG. 3: Coefficient of the rotational energy \(\hbar^2/2\Theta\), calculated using the cranking method versus the HF iterations with various rotational frequencies \(\omega\). The symbols (b), (c), and (d) correspond to the density distributions given in Fig. 2.

FIG. 4: Coefficient of the rotational energy \(\hbar^2/2\Theta\) as a function of rotational frequency \(\omega\). The lines correspond to the different Skyrme forces as indicated.
Single particle wave function of nucleons in quasi cluster (spin-up):

$$\psi_i = \left(\frac{2\nu}{\pi}\right)^{3/4} \exp[-\nu(\vec{r} - \vec{\zeta}/\sqrt{\nu})^2]$$

$$\vec{\zeta}/\sqrt{\nu} = R(\vec{e}_x + i\Lambda\vec{e}_y)$$

$$\psi_i = \left(\frac{2\nu}{\pi}\right)^{3/4} \exp[-\nu\vec{r}^2 - \vec{\zeta}^2 + 2\nu \vec{r} \cdot \vec{\zeta}/\sqrt{\nu}]$$

the cross term can be Taylor expanded as:

$$\exp[2\nu \vec{r} \cdot \vec{\zeta}/\sqrt{\nu}] = 1 + \sum_{k=1}^{\infty} \frac{1}{k!} (2\nu R(x + i\Lambda y))^k$$

For $\Lambda = 1$, one finds:

$$\exp[2\nu \vec{r} \cdot \vec{\zeta}/\sqrt{\nu}] = 1 + \sum_{k=1}^{\infty} \frac{1}{k! s_k} \frac{1}{2\nu r R} (2\nu r R)^k Y_{k,k}(\Omega)$$
For \( \Lambda = 1 \)

the single particle wave function in the quasi cluster becomes

\[
\psi_i = \left( \frac{2\nu}{\pi} \right)^{3/4} \left\{ 1 + s^{-1}_1 2\nu r_i R Y_{11}(\Omega_i) \right.
\]
\[
+ \frac{1}{2!} s^{-1}_2 (2\nu r_i R)^2 Y_{22}(\Omega_i)
\]
\[
+ \frac{1}{3!} s^{-1}_3 (2\nu r_i R)^3 Y_{33}(\Omega_i)
\]
\[
+ \cdots + \frac{1}{n!} s^{-1}_n (2\nu r_i R)^n Y_{nn}(\Omega_i)
\]
\[
+ \cdots \} \exp[-\nu r_i^2].
\]

for the spin-up nucleon (complex conjugate for spin-down).
Figure 1: Convergence behavior in a static HF iteration showing an intermediate quasistable state of $^{16}\text{C}$. Plotted are the relative change in total energy from one iteration to the next, $\Delta E = \left| \frac{E_{n+1} - E_n}{E_n} \right|$ and the average fluctuation in the single-particle energies as defined in Eq.(eq:hfclcut).

Figure 2: Convergence behavior in a static HF iteration showing an intermediate quasistable state of $^{16}\text{C}$. Plotted are the quadrupole deformation parameter $\beta$ (full curve) and the $\Re(Q_{31})$ expectation value described in the text (dashed curve).
Rod-Shaped Nucleus

We picture atomic nuclei as spherical globs of protons and neutrons, although they can also be egg-shaped. Now calculations published 9 September in *Physical Review Letters* show that an even more exotic shape is possible: a rapidly spinning nucleus can form into a linear chain of several small clusters of neutrons and protons. Such exotic nuclear states could play important intermediary roles in the formation of carbon-12 and oxygen-16--elements essential for life--in the interiors of stars. The authors' new technique for calculating such structures also allows for the study of even more exotic arrangements.

The shape of a nucleus has important effects on nuclear reactions, such as those in the interior of stars or in particle accelerators, where understanding nuclear structure is key to designing efficient reactions. Calculating the new, exotic shape allows physicists to better understand these processes and their implications for understanding the natural world.

*All in a row.* A spinning oxygen-16 nucleus can spread out into a linear chain of four clusters, according to calculations. This is the first clear evidence for such a "linear-chain" state.
How we can stabilize such states?

- Adding valence neutrons
- Orthogonalizing to low-lying states
- Rotating the system