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Magnetars and the Chiral Plasma Instabilities

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We propose a possible new mechanism for a strong and stable magnetic field of compact stars due to an instability in the presence of a chirality imbalance of electrons—the chiral plasma instability. A large chirality imbalance of electrons inevitably occurs associated with the parity-violating weak process during core collapse of supernovae. We estimate the magnetic field due to this instability to be of order 10^{16} G at the core. This mechanism naturally generates a large magnetic helicity from the chiral asymmetry, which ensures the stability of the large magnetic field.

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Introduction.—The origin of compact stars with the most powerful magnetic field ($\sim 10^{15}$ G on the surface) in the Universe, called magnetars [1], is a mystery in astrophysics. Examples of the possible mechanisms include the fossil field or dynamo hypothesis among others [2, 3]. However, these mechanisms have an important problem that a strong magnetic field produced cannot be sustained for a long time scale [2, 3]. Indeed, a purely poloidal magnetic field [depicted in Fig. 1(a)] typically considered is known to be unstable [4], as confirmed with numerical magnetohydrodynamic (MHD) simulations [5]. For other issues in these mechanisms, see, e.g., Ref. [3].

It is suggested that *if* nonzero magnetic helicity

$$\mathcal{H} = \int d\boldsymbol{x}\, \boldsymbol{A} \cdot \boldsymbol{B} \tag{1}$$

(where $\boldsymbol{B}$ and $\boldsymbol{A}$ are the magnetic field and vector potential) is produced at the initial configuration for some reason, it can make the magnetic field stable [3]. This is because $\mathcal{H}$ is proportional to the Gauss linking number of the magnetic flux tubes and serves as an approximate conserved quantity [6].[1] For example, linking of poloidal and toroidal magnetic fields [depicted in Fig. 1(c)] has a nonzero magnetic helicity and can exist stably [7]. However, the origin of the magnetic helicity itself remains to be understood as well (see also below).

In this paper, we propose a new mechanism for a strong *and* stable magnetic field in magnetars due to a novel instability in the presence of an imbalance between right- and left-handed electrons—the chiral plasma instability

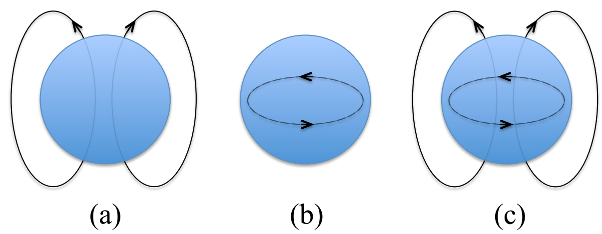

FIG. 1. Configurations of magnetic fields in magnetars: (a) poloidal, (b) toroidal, and (c) linked poloidal-toroidal magnetic fields.

(CPI). The CPI was recently found in the context of electromagnetic and quark-gluon plasmas [8] based on chiral kinetic theory [9]. A related instability had been previously argued for the electroweak theory [10, 11] and for the primordial magnetic field in the early Universe [12]. (See also Refs. [13] for recent works.) This instability appears somewhat similar to the Rayleigh-Taylor instability that occurs in the presence of a density imbalance of two fluids at an interface. However, the former is remarkable in that it is a consequence of relativistic and quantum effects related to quantum anomalies [14] unlike the latter.

Our mechanism for a strong and stable magnetic field is based only on the chirality asymmetry of electrons that is inevitably produced in the parity-violating weak process (electron capture) during core collapse of supernovae. The energy of the chirality imbalance is then converted to a large magnetic field by the CPI. Furthermore, we show that it naturally generates a large magnetic helicity at the same time. Possible generation of magnetic helicity by the parity-violating (modified) Urca process *after* the birth of a neutron star was previously suggested in Ref. [15] though the CPI was not considered. To the

[1] Magnetic helicity is a conserved quantity for a perfect conductor and is conserved approximately with finite conductivity; see the discussion below.

best of our knowledge, the CPI is the first and only microscopic mechanism to create a magnetic helicity in magnetars at any stage during its evolution. Note that our mechanism does not require any exotic hadron or quark phases inside compact stars under discussion, such as ferromagnetic nuclear matter [16], ferromagnetic quark matter [17], pion domain walls [18], and so on, which are essential in some of previous suggestions for the origin of magnetars.

In the following, we ignore the electron mass m_e, as electrons may be regarded as ultrarelativistic at high density, $\mu_e \gg m_e$. We will discuss the possible effects of m_e later on.

Chiral Plasma Instabilities.—Let us briefly review the physical argument of the CPI (see Ref. [19] for the detail). For simplicity, we here ignore the effect of dissipation which is not essential to understand the instability itself.

Suppose there is a homogeneous chiral asymmetry between right- and left-handed electrons in the core of compact stars, which we parametrize by a chiral chemical potential $\mu_5 \equiv (\mu_R - \mu_L)/2$. (We shall give an estimate of μ_5 just after the onset of core collapse of supernovae later.) Let us consider a perturbation of a small magnetic field B_z with wavelength λ in a cylindrical coordinate (r, θ, z) (see Fig. 2). In the presence of μ_5, this magnetic field leads to an electric current in the z direction (called the chiral magnetic effect) [20]:

$$j_z = \frac{2\alpha}{\pi}\mu_5 B_z, \tag{2}$$

where α is the fine structure constant. Intuitively, this current can be understood as follows: to minimize the energy of the system, the magnetic moment of an electron is aligned in the direction of B_z. Remembering the definition of chirality, momentum of a right- (left-)handed fermions is in the same (opposite) direction as the spin; hence, a net electric current flows in the direction of B_z at finite μ_5.

Now Ampère's law states that the current (2) leads to a magnetic field in the θ direction at a distance $R \sim \lambda$ as $B_\theta = \pi\lambda^2 j_z/(2\pi R)$. This in turn induces the current due to the chiral magnetic effect: $j_\theta = (2\alpha/\pi)\mu_5 B_\theta$. According to Ampère's law again, this current gives rise to a magnetic field in the z direction as

$$B_z' = \int dR\, j_\theta(R) \sim \left(\frac{2\alpha\mu_5\lambda}{\pi}\right)^2 B_z. \tag{3}$$

So if $\lambda \gtrsim (\alpha\mu_5)^{-1}$, it follows that $B_z' > B_z$: the original magnetic field gives a positive feedback to itself, and it grows exponentially. This is the chiral plasma instability. This unstable mode then reduces μ_5 so that the instability is attenuated [8, 19].

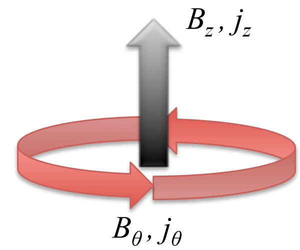

FIG. 2. Physical picture of the chiral plasma instability.

One also finds that this instability generates not only a poloidal magnetic field in the z direction but also a toroidal magnetic field in the θ direction; the resulting configuration has a finite magnetic helicity. Later, we will estimate the magnitude of magnetic helicity from the helicity conservation.

Estimate of the chirality imbalance of electrons.—How large can a magnetic field be due to this mechanism inside compact stars? From now on, we shall provide its estimate based on the neutron density generated during core collapse of supernovae. The core of a compact star is almost "neutronized" via the parity-violating weak process at this stage, where the largest chiral asymmetry of electrons is created [21].

We note that our estimate for the magnetic field below assumes a number of simplifications, so it should be regarded as schematic. Nonetheless, it turns out that the magnetic field due to the CPI can be of order 10^{16} G at the core (with a possible small deviation due to the uncertainty of the prefactor), and it could potentially explain the large magnetic field $\sim 10^{15}$ G on the surface. Our estimate can, in principle, be made more realistic by including various complications that we will discuss later.

The chirality imbalance of electrons is produced via electron capture inside a core,

$$p + e_L^- \to n + \nu_L^e, \tag{4}$$

where the subscript L stands for left-handedness. Here only left-handed fermions are involved, as it is described by the V$-$A type weak interaction. Note that neutrons and the chirality imbalance produced by the process (4) are not washed out by its inverse process during core collapse (and thus forming a *neutron* star eventually). This is because the time scale of (4) is much shorter than the neutrino diffusion time $\tau_{\rm NS} \sim 10$ s and their production occurs in a nonequilibrium manner. Note also that the other possible processes (thermal neutrino emission) [22] do not change the neutron number nor the chirality imbalance on average.

That huge neutrons are produced in these processes means that huge left-handed electrons are "eaten" by protons, leading to the Fermi surface imbalance, $\mu_R > \mu_L$, or a nonzero chiral chemical potential for electrons, $\mu_5 \equiv (\mu_R - \mu_L)/2 > 0$. Because the number of neutrons produced in this process is equal to the number difference between right- and left-handed electrons, N_5, we have

$$n_5 \approx \Delta n_{\rm n}, \tag{5}$$

where n_5 is the chiral density of electrons and $\Delta n_{\rm n}$ is the increased neutron density by electron capture. Considering the neutron density inside a neutron star, it is reasonable to take $\Delta n_{\rm n} \sim (0.1\text{–}1)\ {\rm fm}^{-3}$ at the core. In the natural units $\hbar = c = 1$, $\Delta n_{\rm n} \sim 0.1\text{–}1\Lambda^3$, where we introduced the mass scale $\Lambda = 200$ MeV for later convenience. In the following, we use the natural units unless otherwise stated.

The chiral number density is expressed by the chemical potentials as

$$n_5 = \frac{\mu_5}{3\pi^2}(\mu_5^2 + 3\mu^2) \tag{6}$$

at sufficiently low temperature, where $\mu \equiv (\mu_R + \mu_L)/2$ is the chemical potential associated with U(1) (vector-like) particle number. We here ignored the contribution of the temperature T to the density. Recalling that the typical electron chemical potential at the core is $\mu \lesssim \Lambda$, one finds

$$\mu_5^{\rm total} \sim \Lambda. \tag{7}$$

It should be remarked that $\mu_5^{\rm total}$ we obtained is the *total* (or time-integrated) chiral chemical potential *produced* during core collapse. In reality, the production of μ_5 by the process (6) occurs simultaneously with the reduction of μ_5 by the CPI (and with the reduction by the electron mass m_e, which we shall argue later). In general, the production of μ_5 varies as functions of t and $\boldsymbol{x}$, and so does the reduction of μ_5 due to the CPI; tracing the evolution of $\mu_5(t, \boldsymbol{x})$ requires the knowledge of the neutrino production rate depending on the evolution of a neutron star and MHD simulations, which is beyond the scope of the present paper. Below we shall ignore the inhomogeneity of μ_5 (and inhomogeneity of electromagnetic fields) to give a simple estimate of the magnetic field produced by our mechanism with taking into account the reduction of μ_5 by the CPI itself. The assumption of the homogeneous μ_5 was also made in Refs. [12] in a different context.

Evolution of the chirality imbalance.—Without the effects of the CPI, n_5 would gradually increase toward the value (5). Let us consider, for simplicity, a specific case where n and n_5 grow homogeneously at a constant speed at the core: converting it in terms of μ and μ_5, it means they would grow as $\mu \propto t^{1/3}$ and $\mu_5 \propto t^{1/3}$, so one can set $\mu = c_1(\Lambda^4 t)^{1/3}$ and $(d\mu_5/dt)_+ = c_2(\Lambda^4/t^2)^{1/3}$ (where "+" stands for the production). Considering that the time scale for the formation of a neutron star is $\tau_{\rm NS} \sim 10\ {\rm s} \sim 10^{24}\Lambda^{-1}$, we take $c_1 \sim 10^{-8}$ and $c_2 \sim 10^{-8}$ as typical values.

In practice, however, n_5 decreases due to the CPI, and its evolution is dictated by the anomaly relation [14]

$$\frac{dn_5}{dt} = \frac{2\alpha}{\pi}\boldsymbol{E}\cdot\boldsymbol{B}. \tag{8}$$

It is natural to take the typical scale characterizing the magnitude of $\boldsymbol{E}\cdot\boldsymbol{B}$ to be $\mu_5^4(t)$. Assuming $\mu(t) \gg \mu_5(t)$, which is justified a posteriori, the reduction rate is given by $(d\mu_5/dt)_- = -c_3\mu_5^4/\mu^2$ with some constant c_3.

When combined altogether, one arrives at the following evolution equation for μ_5:

$$\frac{d\mu_5}{dt} = c_2\left(\frac{\Lambda^4}{t^2}\right)^{\frac{1}{3}} - c_3\frac{\mu_5^4}{\mu^2} = c_2\left(\frac{\Lambda^4}{t^2}\right)^{\frac{1}{3}}\left[1 - C\left(\frac{\mu_5}{\Lambda}\right)^4\right], \tag{9}$$

where $C = c_3/(c_1^2 c_2)$. Hence, μ_5 saturates with the magnitude $\mu_5^{\rm sat} = C^{-1/4}\Lambda$ after the time $t \ll \tau_{\rm NS}$. This state will continue until the end of the neutronization of the star when the production of μ_5 becomes negligible. We note that this result should be taken with care, because, as we mentioned above, electromagnetic fields and μ_5 vary depending on $(t, \boldsymbol{x})$ in reality, where such saturation may not occur. Rather, the estimate here is to be understood as a simple model to incorporate the effects of the CPI that reduces μ_5 in Eq. (7).

Estimate of magnetic fields and magnetic helicity.—As explained above, the state with nonzero n_5 is unstable and decays rapidly by converting it to a magnetic field due to the CPI. Assuming this state will decay into a state with $n_5 \sim 0$ at the end, one can estimate the magnitudes of a resulting magnetic field and magnetic helicity from the energy and helicity conservations [8]. Note that the CPI will end before μ_5 is reduced to $\mu_5^{\rm end} \sim 10^{-16}\Lambda$ where the wavelength $\lambda \sim (\alpha\mu_5)^{-1}$ becomes macroscopic (~ 1 km). As $\mu_5^{\rm end} \ll \mu_5^{\rm sat}$, it occurs after the production of μ_5 stops.

The decrease of the energy density as a consequence of the reduction of μ_5 by the CPI is

$$\Delta\epsilon = \int dt\, \frac{dn_5}{dt}\mu_5 \sim c_3(\mu_5^{\rm sat})^5\tau_{\rm NS}. \tag{10}$$

Here we used the anomaly relation (8) and the fact that $\mu_5(t) = \mu_5^{\rm sat}$ through most of the time during $\tau_{\rm NS}$. The energy conservation requires that it is equal to the energy of the magnetic field $(1/2)\Delta B_{\rm inst}^2$. One can thus estimate the magnetic field generated by this instability as

$$B_{\rm inst} \sim (c_1^2 c_2)^{\frac{5}{8}} c_3^{-\frac{1}{8}} (\Lambda^5 \tau_{\rm NS})^{\frac{1}{2}}, \qquad (11)$$

assuming no dissipation of energy and perfect conversion efficiency (see the discussion below). The dependence of $B_{\rm inst}$ on c_3 is weak, so one may take $c_3^{-1/8} \sim 1$. One then finds that typical magnetic fields produced by the CPI are of order 10^{15} G – 10^{16} G in this model. Assuming the resulting poloidal magnetic field is dipole-like and is dominant, we can translate $B_{\rm inst}$ into the magnetic field on the surface, $B_{\rm surface} \approx (R_{\rm core}/R_{\rm star})^3 B_{\rm core}$ [23], where $R_{\rm core}$ and $R_{\rm star}$ are the radii of the core and the neutron star itself, respectively (here the "core" is meant by the region which has the density comparable to or larger than the nuclear density); e.g., when $R_{\rm core}/R_{\rm star} \simeq 0.5$, $B_{\rm surface} \sim 10^{14}$ G – 10^{15} G.

On the other hand, the helicity conservation reads

$$\frac{d}{dt}\left(N_5 + \frac{\alpha}{\pi}\mathcal{H}\right) = 0, \quad N_5 = \int d\boldsymbol{x}\, n_5, \qquad (12)$$

where N_5 is the global chiral charge of electrons and $\mathcal{H}$ is the magnetic helicity (also called Chern-Simons number in particle physics and mathematics) defined in Eq. (1). Note that this is the global version of the anomaly relation in Eq. (8). From the helicity conservation, one obtains the magnetic helicity at the end as

$$\Delta\mathcal{H} = -\frac{\pi}{\alpha}\int dt d\boldsymbol{x}\, \frac{dn_5}{dt} \sim \frac{1}{\alpha} V c_1^2 c_2 \Lambda^4 \tau_{\rm NS} \qquad (13)$$

with $V \approx 4\pi R_{\rm core}^3/3$; so a large magnetic helicity is naturally produced as a consequence of the CPI, which then ensures the stability of the strong magnetic field. The detailed configuration of the magnetic field with a large magnetic helicity is under study [24].

We note that any microscopic process concerning the electromagnetic and strong interactions respects parity and cannot generate a parity-odd magnetic helicity; microscopically, a parity-odd quantity can originate from the parity-violating weak interaction alone. However, the weak interaction violates parity in the fermionic sector (leptons), so it cannot generate magnetic helicity directly. It is this CPI that converts the parity-odd chiral asymmetry in the fermionic sector to the parity-odd magnetic helicity in the gauge sector.

Note also that the interior of a star could acquire the magnetic helicity macroscopically by accident, by losing helicity through the surface in its evolution. However, no such a evidence was observed in MHD simulations for a specific initial configuration with $\mathcal{H} = 0$ [5]. At least, our mechanism seems the only to ensure the inevitable (rather than accidental) production of magnetic helicity in magnetars from the microscopic point of view.

Discussion.—Let us discuss several possible effects we have ignored above, which can modify our simple estimate (11). One potentially important effect is the electron mass m_e which also reduces the chiral asymmetry. When $\mu_5(t) \ll \mu(t)$, the chirality flipping rate due to m_e is $\Gamma_{\rm mass} \sim \alpha^2 m_e^2/\mu$ [12]. For $\mu_5(t) = \mu_5^{\rm sat}$ in our model, the time scale $\sim \Gamma_{\rm mass}^{-1}$ can become comparable to that of the CPI, $\tau_{\rm inst} \sim (\alpha^2\mu_5)^{-1}$ [8]. Another important effect is the conversion efficiency of the chirality imbalance into the magnetic energy, which could be less than 100%. For example, a finite conductivity σ dissipates the energy and makes the magnetic field in Eq. (11) smaller. Also, heavier degrees of freedom (ions) inside a star may interfere with the electron dynamics and could reduce the magnetic field. Although evaluation of these effects is not easy, it is not entirely unreasonable to expect that the magnetic field induced by the CPI can occupy a non-negligible fraction of the gigantic magnetic field of magnetars.

What is the final configuration of the magnetic field? As μ_5 and the chiral magnetic effect are expected to disappear after the CPI, one might wonder how a magnetic field is kept without currents at the end. As the CPI has the quantum origin, this magnetic field may be accounted for by a specific configuration of the spins of electrons and/or that of the wave function of hadrons. In the context related to the former, recall that magnets, which originate from parallel alignment of spins, generate magnetic fields without macroscopic currents. In the context related to the latter, a specific configuration of pion fields is known to generate a magnetic field at finite density [18]. At this moment we do not have an intuitive picture for the final state after the CPI, but we stress that the resulting strong magnetic field is a consequence of the conservation laws and should be robust. It would be an interesting future question to explore in detail [24].

Finally, we comment on the possible evolution of the large magnetic field after the birth of magnetars by our mechanism. Remember that the magnetic helicity $\mathcal{H}$ is a strict conserved quantity without dissipation. In reality, the medium has a conductivity σ so that $\mathcal{H}$ is conserved approximately; it is the finiteness of σ that allows magnetic flux tubes to reconnect, which results in the decrease of $\mathcal{H}$. Therefore, one expects that magnetic fields decay slowly by dissipation and the reconnection which could manifest themselves in the form of giant outbursts.

Conclusion.—In conclusion, we proposed a possible new mechanism for a strong magnetic field with magnetic helicity in magnetars due to the chiral plasma instability. Our mechanism is based only on the chirality imbalance of electrons that is necessarily produced in the process of electron capture during core collapse. The magnetic field is estimated as $\sim 10^{16}$ G at the core (in our simple model) which may potentially explain the magnetic field $\sim 10^{15}$ G on the surface. This large magnetic field is a macroscopic consequence of the quantum effects tied to quantum anomalies. More realistic calculations, e.g., using magnetohydrodynamics for chiral plasmas, are necessary to reach a definite conclusion. We defer these calculations to future work.

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