Multi-center solutions and effective superstrata

Masaki Shigemori
(Nagoya U / YITP Kyoto)

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Work in progress with
Iosif Bena, Raphaël Dulac, Emil Martinec, David Turton, and Nick Warner

Introduction

Black hole microstates

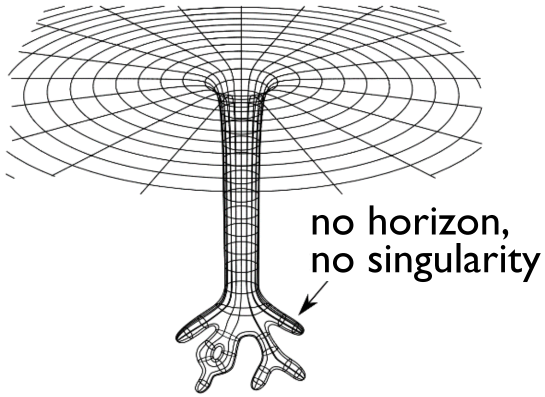
- BHs have entropy: $S_{BH} = \frac{A}{4G}$
- Where are those microstates?

General microstates

- Have the same asymptotic M, Q, J as the BH.
- Some states of string theory/quantum gravity.
  Cf. "central dogma"
- No horizon or singularity in the sense that the scattering matrix is unitary.
  Cf. "bags of gold", such as geometries with shells of dust inside a classical horizon

Microstate geometries

- *Classical* solutions of (super)gravity with the same asymptotic M, Q, J.
- No horizon or singularity.
- Many MGs have been constructed.
- Some "look like" a BH.



(schematic)

Status of MGs

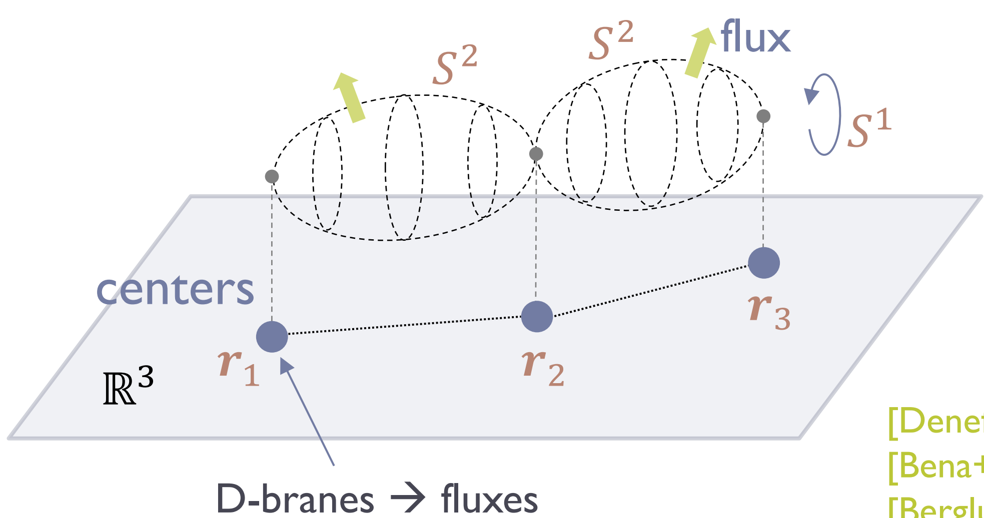
* We're restricting ourselves to BPS states

Known MGs are roughly made of two ingredients:

1. Multi-center solutions
2. Superstrata

Multi-center solutions

- Solutions of 5D sugra
- Bound states of branes
- Generally, centers are BHs
- If centers are "primitive", represent smooth geometries



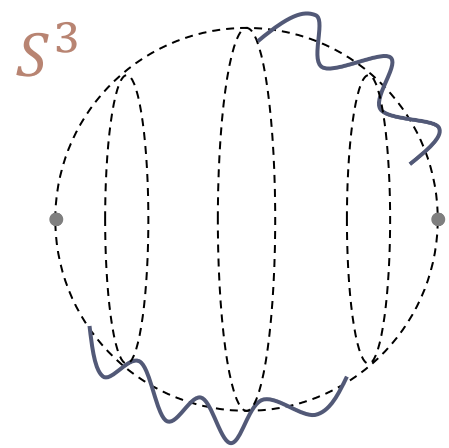
"Bubble equations"

[Denef+Bates 2003]
[Bena+Warner 2006]
[Berglund+Gimon+Levi 2006]

[Bena, Giusto, Russo, MS, Warner 2015]
[Bena, Giusto, Martinec, Russo, MS, Turton, Warner 2016-17]

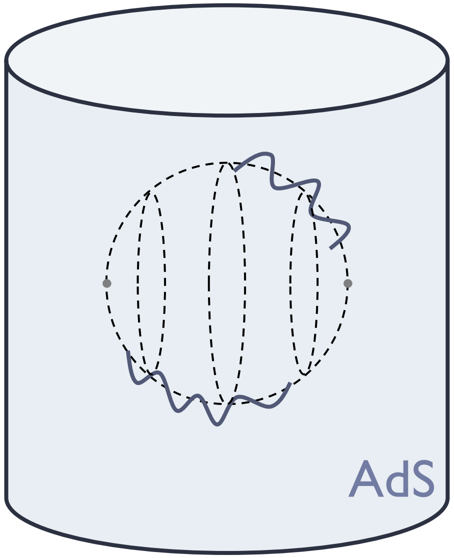
Superstrata

- 6D sugra
- A "graviton gas" on top of multi-center solutions
  - Coherent excitations of many gravitons
  - Backreacted geometry
  - Smooth & horizonless
- Can have multiple waves with various quantum numbers
  - Large entropy



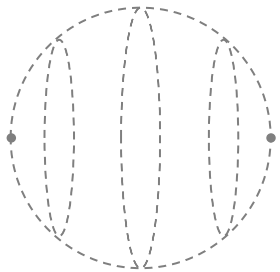
Entropy & AdS/CFT dictionary

- We focus on $\mathrm{AdS}_3/\mathrm{CFT}_2$ ("D1-D5 CFT")
- Can put MGs in the AdS/CFT context
  The holographic dictionary for basic (2-center) superstrata is known



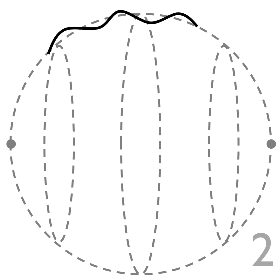
Bulk	Boundary

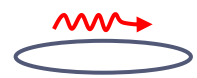
empty
$AdS_3 \times S^3$



$|\mathrm{vac}\rangle$

a graviton

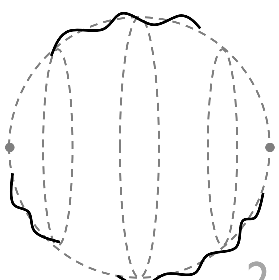




$(J_0^-)^m (L_{-1})^n \, |\phi\rangle_k$

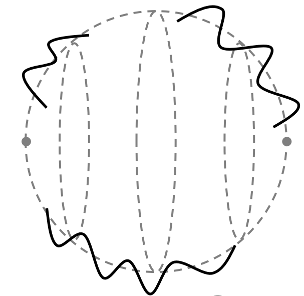
“single-trace”
“graviton state”

multiple
gravitons



“multi-trace”

Entropy: large but not enough



$$S_{\text{strata}} \sim N^{5/4} \ll S_{\text{BH}} \sim N^{3/2}$$

- D1-D5-P [MS, 2020] [Mayerson, MS, 2020]
- D0-D4-D4-D4
  [de Boer, El-Showk, Messamah, Van den Bleeken 2009]

"monotone"
[Chang, Lin, Zhang '25]

Generalizations of 2-center MGs:

$(\text{AdS}_3 \times S^3)/\mathbb{Z}_p$, GLMT, JMaRT

[Giusto, Lunin, Mathur, Turton 2012]
[Jejjala, Madden, Ross, Titchener 2005]

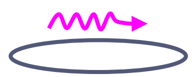
Going beyond?

- What are "BH states" and their holographic duals?

Cf. "fortuity" phenomenon

CFT side

- Graviton states: very special
- More general states involve:



$$L_{-\frac{n}{k}},\ \alpha_{-\frac{n}{k}},\ \psi_{-\frac{n}{k}},\ \ldots$$ "fractional modes"

Many such states lift.
(Cf. fortuity)

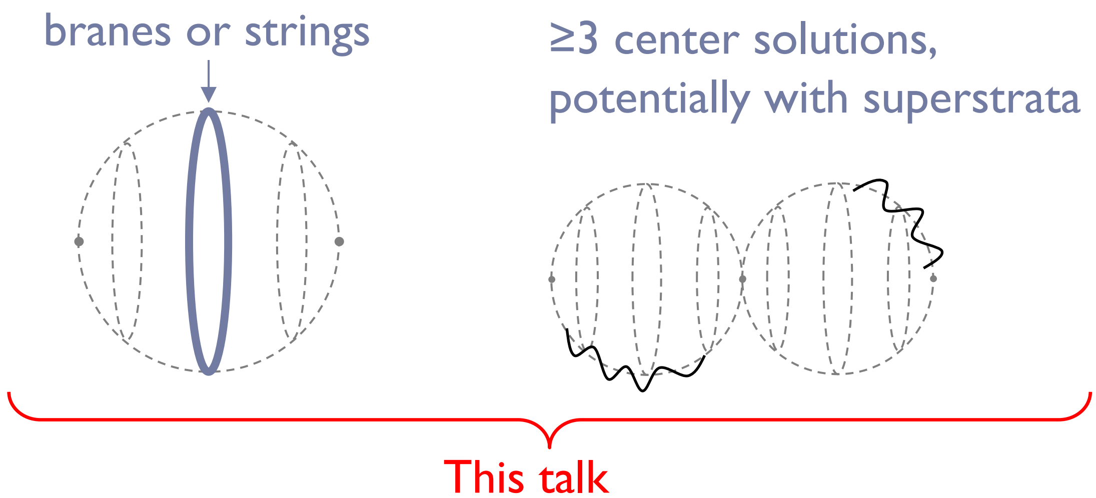
[BMN 02]
[Lunin-Mathur 02] [Gomis-Motl-Strominger 02] [Gava-Narain 02]
…

- What is their bulk dual in general?
- Relation to BHs?

: Long-standing questions

Modest goals:

There are relatively simple bulk configurations whose holographic duals are not understood.   Let us study those.



Other direction: internal space

- “Themelia”, “super-maze”, “mohawk” [Bena, Warner, …]
- Cf. MSW [Maldacena-Strominger-Witten ’97]

(Modest) results:

A "stringy" multi-center solution:

- Constructed a 3-center solution involving D1-P waves
- Matching with worldsheet CFT (F1-P side)
  – Not holographic CFT, but the microscopic control should help us study the HCFT dual!

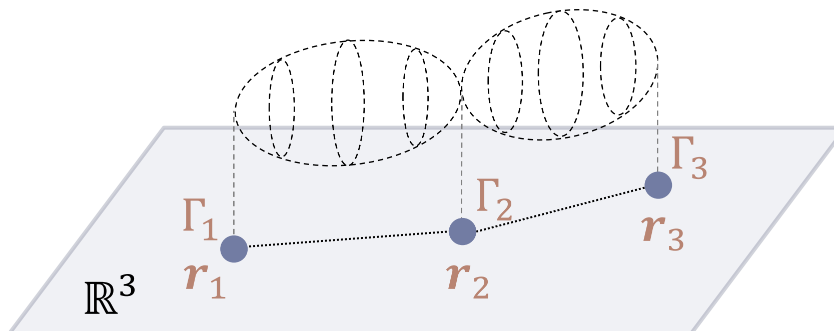
"Effective superstrata"

- Effective description of MGs in terms of multi-center solutions
- Reversing the process of resolving a singular solution into smooth MGs. Avoid having to deal with fine detail

Multi-center solutions & D1 waves

Multi-center solutions

- Solutions of 5D sugra
- S^1 fibered over $\mathbb{R}^3$
- Bound states of charge centers



Charge vector: $\Gamma = (p^0, p^I, q_I, q_0)$

M on $CY_3 \times S^1$ → KKM M5 M2 P

IIA on CY_3 → D6 D4 D2 D0

In this talk
$CY_3 = T^2 \times T^2 \times T^2$,
$I = 1,2,3$

$$q_1 = -\frac{p^1 p^2}{p^0}, \ \dots, \ q_0 = \frac{p^1 p^2 p^3}{2(p^0)^2}$$

- When Γ are "primitive", the geometry is smooth and represents a microstate
- Positions $\boldsymbol{r}_i$ must satisfy so-called "bubble equations"

$$\sum_{j(\neq i)} \frac{\langle \Gamma_i, \Gamma_j \rangle}{a_{ij}} = \langle h, \Gamma_i \rangle$$

Example

A single center with

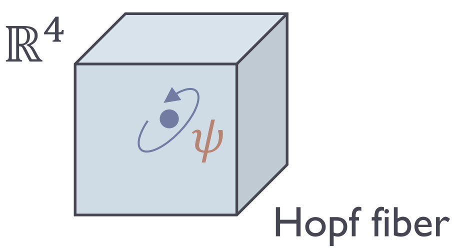
$$\Gamma = (0, (p^1, p^2, p^3), (0,0,0), q_0)$$

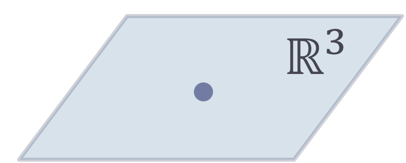
→ 4-charge D0-D4^3 BH with $S_{BH} = 2\pi\sqrt{p^1 p^2 p^3 q_0}$

D1-D5 sys in terms of multi-ctr solns

- We can relate D1-D5 sys ($AdS_3 \times S^3 \times T^4$) to MCS

$\mathrm{IIB}/\mathbb{R}_t \times \mathbb{R}^4 \times S^1_y \times T^4_{6789}$						$\mathrm{IIA}/\mathbb{R}_t \times \mathbb{R}^3 \times T^6_{456789}$
D1(y)		D2		M2		D2(45)
D5$(y6789)$	$\xrightarrow{\mathrm{T}_{y67}}$	D2	$\xrightarrow{\text{lift}}$	M2	$\xrightarrow{\text{cptfy}_\psi}$	D2(67)
P(y)		F1		M2		D2(89)
$J(\psi)$		$J(\psi)$		$J(\psi)$		D0
KKM$(y6789, \psi)$		KKM		KKM		D6(456789)





$\mathbf{AdS_3 \times S^3}$ (vacuum)

- D1 & D5 puff out into a "KKM supertube"; branes disappear into geometry with flux

$$\mathrm{KKM}(\psi 6789, y) \xrightarrow{T_{y67}} \mathrm{NS5} \xrightarrow{\text{lift}} \mathrm{M5} \xrightarrow{\text{cptfy}_\psi} \mathrm{D4}(6789)$$

➡ Two centers

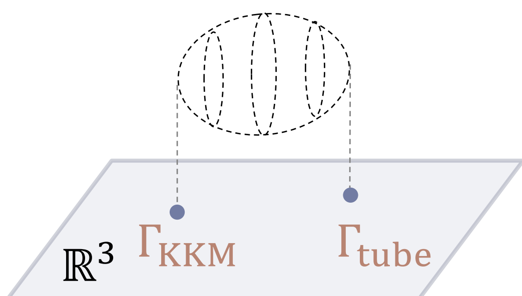
$$\Gamma_{\rm tube} = (0,(0,0,k_3),(l_1,l_2,0),m), \qquad m = \frac{l_1 l_2}{2k^3}$$

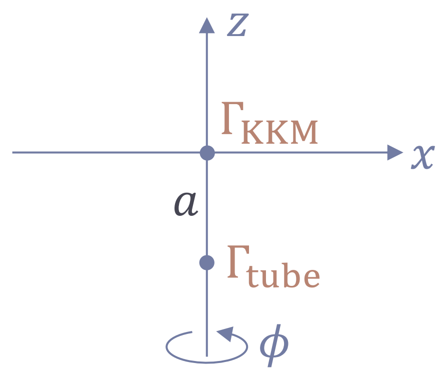
$$\Gamma_{\rm KKM} = (1,(0,0,-k_3),(0,0,0),0),$$

- Locally ½ BPS
- Pure flux

- $-k_3$ needed for well-behaved 5D geom at ∞
- Or conservation of D4 Page charge

➡ In MCS language, $AdS_3 \times S^3 \longleftrightarrow$ 2-center (More precisely, $(AdS_3 \times S^3)/\mathbb{Z}_{k_3}$)





bubble eq

$\to a = \dfrac{2m}{k^3} \equiv a_0$

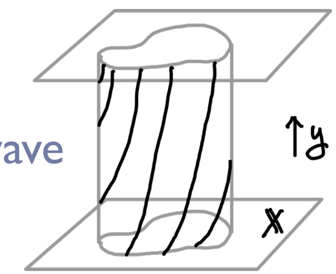
Putting a 3rd center

"1/8-BPS giant gravitons" [Mandal-Raju-Smedback 07] [Raju 07]

⇨ Any D1 waves are susy in IIB; $\boldsymbol{X} = \boldsymbol{X}(v), \;\; v = \frac{1}{\sqrt{2}}(t+y)$

Any $AdS_3 \times S^3$ coordinate (except t, y)

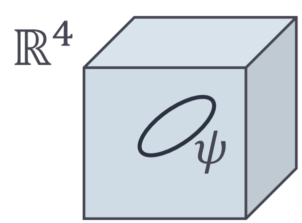
D1 wave



"barber pole"

Wikipedia commons

Take a ψ-circle as the profile



helix

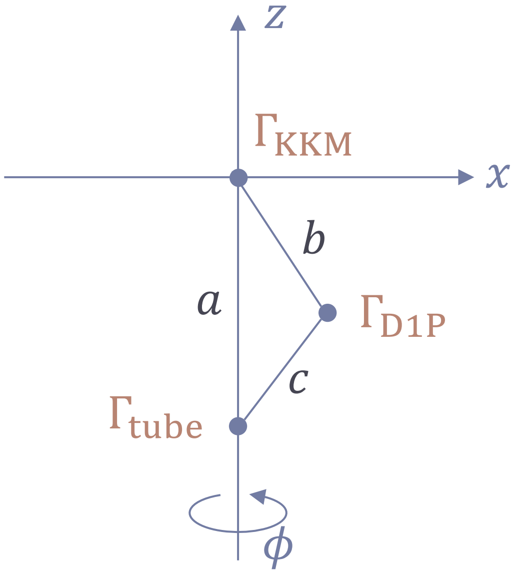
Charges: $\mathrm{D1}(\psi), \mathrm{D1}(y), J(\psi), P(y)$

$\mathrm{D1}(\psi), \mathrm{D1}(y), J(\psi), P(y)$

Dualize to IIA

Wave structure (v dep) gets smeared

$\mathrm{D2}(4589), \mathrm{D2}(45), \mathrm{D0}, D2(89)$



3-center solution

$$\Gamma_{\mathrm{D1}P} = (0, (0, k_2', 0), (l_1', 0, l_3'), m'), \quad m' = \frac{l_1' l_3'}{2k_2'}$$

- Wavy profile along y, ψ averaged away
- a, b, c determined by bubble eqs.
- Can sit anywhere along ϕ

The 3-ctr solution

Background:

$$\Gamma_{\text{tube}} = (0,(0,0,k_3),(l_1,l_2,0),m)$$
$$\Gamma_{\text{KKM}} = (1,(0,0,-k_3),(0,0,0),0)$$

Now we are adding a probe center

$$\Gamma_{\text{D1}P} = (0,(0,k_2',0),(l_1',0,l_3'),m')$$

(primed $\ll$ unprimed)

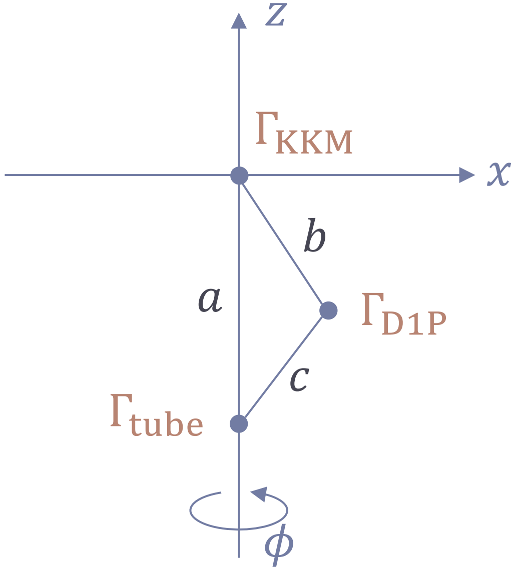
Backreaction on the background charges:

- D4 Page charge should not change
- Fluxes through the orig S^2 should not change

⇨

$$\Gamma_{\text{tube}} = (0,(0,0,k_3),(l_1+k_2'k_3,l_2,0),m)$$
$$\Gamma_{\text{KKM}} = (1,(0,-k_2',-k_3),(-k_2'k_3,0,0),0)$$
$$\Gamma_{\text{D1}P} = (0,(0,k_2',0),(l_1',0,l_3'),m')$$

$$\Gamma_{\text{tube}} = (0, (0,0,k_3), (l_1 + k_2' k_3, l_2, 0), m)$$
$$\Gamma_{\text{KKM}} = (1, (0, -k_2', -k_3), (-k_2' k_3, 0,0), 0)$$
$$\Gamma_{\text{D}1P} = (0, (0, k_2', 0), (l_1', 0, l_3'), m')$$



Bubble eqs →
$$\frac{c}{b} = \frac{l_2 k_2' - k_3 l_3'}{2m' - k_3 l_3'}$$
$$a = a_0 + \Delta a,$$
$$\Delta a = \frac{4m}{k_3^2} \frac{m' - \frac{1}{2} k_3 l_3'}{b} - \frac{l_2 k_2'}{k_3}$$

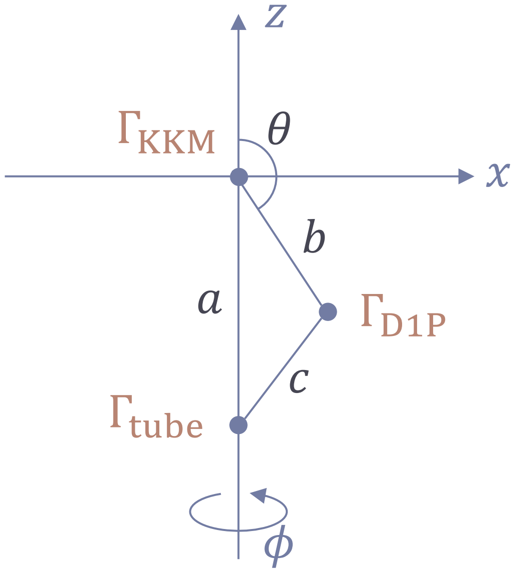
Charges:

$$Q_1 = l_1 + l_1', \qquad Q_2 = l_2.$$

$$J_L = J_{L,0} + J_L' \qquad J_L' = \frac{Q_2}{2}\left(-\frac{l_1'}{k_3} + k_2'\right) + m'$$

$$J_R = J_{R,0} + J_R' \qquad J_R' = \frac{m(Q_2 k_2' - k_3 l_3')}{k_3 c} - \frac{Q_2 l_1'}{2}$$

Given the probe charges $(k_2', l_1', l_3', m', J_L', J_R')$, its location is fixed.



It's more convenient to go to coordinates $(\tilde{r}, \tilde{\theta})$ often used in 6D:

$$(\tilde{r}^2 + \tilde{a}^2)\sin^2\tilde{\theta} = 4r\,\sin^2\frac{\theta}{2}$$

$$\tilde{r}^2\cos^2\tilde{\theta} = 4r\,\cos^2\frac{\theta}{2}$$

"Position formula"

$$\cos^2\tilde{\theta} = \frac{1}{2}\left(1 - \frac{J_L' - Q_2(k_2' - \frac{l_1'}{2k_3})}{J_R' + \frac{Q_2 l_1'}{2}}\right)$$

$$\tilde{r}^2 = \frac{\tilde{a}_0^2}{2}\left(-1 + \frac{J_L' + \frac{Q_2 l_1'}{2k_3} - k_3 l_3'}{J_R' + \frac{Q_2 l_1'}{2}}\right)$$

This can be reproduced from worldsheet CFT

Worldsheet CFT

D1-D5 F_3^{RR} $\xleftrightarrow{S}$ F1-NS5 H_3^{NSNS}

D1 waves F1 waves

Worldsheet CFT as a WZW model

[Maldacena-Ooguri '00-01]

$$AdS_3 = SL(2,\mathbb{R})$$
$$S^3 = SU(2)$$

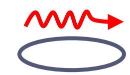
[Martinec, Massai, Turton '17–]:

- Extended this to $(AdS_3 \times S^3)/\mathbb{Z}_p$, GLMT, JMaRT using gauged WZW

$$\frac{SL(2,\mathbb{R}) \times SU(2) \times \mathbb{R}_t \times S_y^1}{U(1)_L \times U(1)_R}$$

[Martinec, Massai, Turton '17–]:

- Classified BPS spectrum of strings

			Holog. CFT dual	
1/4-BPS strings	=	1/4-BPS gravitons	known	
1/8-BPS strings	=	1/8-BPS gravitons	known	
		BPS stringy excitations	unknown	?

- S-dual to D1 waves
- Given quantum numbers, position fixed [Martinec, Turton, to appear]
  → **Exactly reproduces the D1 position formula!**
- More generally can be along ψ and ϕ

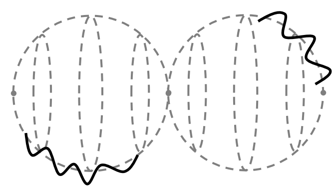
Comments

- Backreaction on background charges is important for matching with worldsheet CFT
- The microscopic control in worldsheet CFT must help us identify the holographic CFT dual (work in progress…) (Caveat: orbifold CFT and sugra are far away in moduli space. Lifting. But see also "moulting" [Bena, Chowdhury, de Boer, El-Showk, MS, 2011])
- Matching can be generalized to GLMT

Effective superstrata

Motivational words

- Singular BH sol'ns can (often/sometimes) be resolved into MGs
- Explicitly constructing MG solutions is quite demanding
  - Need to solve PDEs that depend on the base
  - Nontrivial requirement of regularity (cf. coiffuring)
  - Explicit solutions known only for specific cases (such as the "(1,0,n)" superstrata)



- Reverse the philosophy
  - Replace smooth MGs by "effective geometries"
  - Avoid having to handle the details of the exact geometry
  - Accurate for physics at scales much larger than detailed microstructure

- Averaging
  - Typical situation: brane intersection + P
  - Branes dissolve into fluxes through topological cycles. Added P localizes away from brane sources, at high freq.

→ Average out P waves & replace them with a singular source

Let's see how this works.

6D equations

- The superstratum in IIB on $S^1 \times T^4$ have fields

$$ds_6^2 = -\frac{2}{\sqrt{\mathcal{P}}}(dv+\beta)\left[du+\omega+\frac{\mathcal{F}}{2}(dv+\beta)\right]+\sqrt{\mathcal{P}}\,ds^2(\mathcal{B}) \qquad \mathcal{P} \equiv Z_1\,Z_2 - Z_4^2$$

$\ldots$

- The quantities Z_I, Θ_I, ω are functions $x_m, v = \frac{1}{\sqrt{2}}(t+y)$ and satisfy:

1st-layer eqs:

$$\begin{aligned} \mathcal{D} *_4 \mathcal{D} Z_1 &= -\Theta_2 \wedge d\beta, & \Theta_2 &= *_4 \Theta_2, \\ \mathcal{D} *_4 \mathcal{D} Z_2 &= -\Theta_1 \wedge d\beta, & \Theta_1 &= *_4 \Theta_1, \\ \mathcal{D} *_4 \mathcal{D} Z_4 &= -\Theta_4 \wedge d\beta, & \Theta_4 &= *_4 \Theta_4. \end{aligned}$$

$$\mathcal{D} \equiv d_4 - \beta \wedge \partial_v$$

2nd-layer eqs:

$$\begin{aligned} *_4\mathcal{D} *_4 \left(\dot{\omega} - \frac{1}{2}\mathcal{D}\mathcal{F}\right) &= (\dot{Z}_1\dot{Z}_2 - \dot{Z}_4^2) + (Z_1\ddot{Z}_2 + Z_2\ddot{Z}_1 - 2Z_4\ddot{Z}_4) \\ &\quad - \frac{1}{2} *_4 (\Theta_1 \wedge \Theta_2 - \Theta_4 \wedge \Theta_4), \\ (1+*_4)\mathcal{D}\omega + \mathcal{F}d\beta &= Z_1\Theta_1 + Z_2\Theta_2 - 2Z_4\Theta_4. \end{aligned}$$

Averaging 6D equations

Quantities include v-waves $(\cos\omega v, \sin\omega v)$.
Introduce averaging:

$$\langle \dots \rangle \equiv \frac{1}{2\pi R_v}\int dv \ \dots$$

⇩

assumed β is v indep

1st-layer eqs:

$$\begin{aligned} d *_4 d\langle Z_1\rangle &= -\langle\Theta_2\rangle \wedge d\beta, & \langle\Theta_2\rangle &= *_4\langle\Theta_2\rangle, \\ d *_4 d\langle Z_2\rangle &= -\langle\Theta_1\rangle \wedge d\beta, & \langle\Theta_1\rangle &= *_4\langle\Theta_1\rangle, \\ d *_4 d\langle Z_4\rangle &= -\langle\Theta_4\rangle \wedge d\beta, & \langle\Theta_4\rangle &= *_4\langle\Theta_4\rangle, \end{aligned}$$

Reduce to zero modes

2nd-layer eqs:

$$\begin{aligned} *_4 d *_4 d\langle\mathcal{F}\rangle &= 2\langle \dot{Z}_1\dot{Z}_2 - \dot{Z}_4^2\rangle + *_4\langle\Theta_1\wedge\Theta_2 - \Theta_4\wedge\Theta_4\rangle, \\ (1+*_4)d\langle\omega\rangle &= \langle Z_1\Theta_1 + Z_2\Theta_2 - 2Z_4\Theta_4\rangle - \langle\mathcal{F}\rangle d\beta. \end{aligned}$$

RMS on the RHS

In the standard superstratum,

- $\Theta_1 = \dot{Z}_2 = 0, \quad \langle Z_4 \rangle = \langle \Theta_{2,4} \rangle = 0$
- $Z_2, \Theta_3 \equiv d\beta$ have no v dependence

⇩

Non-trivial equations:

$$d *_4 d\langle Z_1 \rangle = d *_4 d\langle Z_2 \rangle = 0,$$

$$*_4 d *_4 d\langle Z_3 \rangle = \langle \dot{Z}_4^2 \rangle + *_4\langle \Theta_4 \wedge \Theta_4 \rangle,$$

$$(1 + *_4) d\langle k \rangle = \langle Z_3 \rangle \Theta_3 - \langle Z_4 \Theta_4 \rangle$$

$$Z_3 = 1 - \mathcal{F}/2, \; d\beta = \Theta_3, \; k = \frac{1}{\sqrt{2}}(\omega + \beta)$$

(We rescaled fields for better matching with 5D convention)

Comparison with 5D eqs

6D eqs, averaged

$$d *_4 d\langle Z_1\rangle = d *_4 d\langle Z_2\rangle = 0,$$
$$*_4 d *_4 d\langle Z_3\rangle = \boxed{\langle \dot{Z}_4^2\rangle + *_4\langle\Theta_4 \wedge \Theta_4\rangle},$$
$$(1 + *_4)d\langle k\rangle = \langle Z_3\rangle\Theta_3 \boxed{- \langle Z_4\Theta_4\rangle}$$

5D eqs that lead to multi-ctr solns:

$$d *_4 dZ_1 = d *_4 dZ_2 = 0,$$
$$*_4 d *_4 dZ_3 = 0,$$
$$(1 + *_4)dk = Z_3\Theta_3$$

- Same except that we have extra sources on the RHS
- For the superstratum, the extra sources are sharply peaked for high frequency and approximated by δ-function
- The superstratum can effectively be described by an extra center in MCS!
- The extra sources contribute to P, J (long distance physics)

Check

- Explicit superstratum:

$$Z_4 = b_4 R_y \frac{\Delta_{\mathsf{k,m,n}}}{\Sigma} \cos v_{\mathsf{k,m,n}},$$

$$\Delta_{\mathsf{k,m,n}} \equiv \left(\frac{a}{\sqrt{r^2+a^2}}\right)^{\mathsf{k}} \left(\frac{r}{\sqrt{r^2+a^2}}\right)^{\mathsf{n}} \cos^{\mathsf{m}}\theta \, \sin^{\mathsf{k-m}}\theta,$$

$$v_{\mathsf{k,m,n}} \equiv (\mathsf{m}+\mathsf{n}) \frac{\sqrt{2}\, v}{R_y} + (\mathsf{k}-\mathsf{m})\phi - \mathsf{m}\psi,$$

- Peak of Δ:

$$r^2 = a^2 \frac{\mathsf{n}}{\mathsf{k}}, \qquad \cos^2\theta = \frac{\mathsf{m}}{\mathsf{k}}.$$

This superstratum must effectively be described by a MCS with a center at this point

- In the first half, we did study a wavy probe center in $\mathrm{AdS}_3 \times S^3$. We did average out v dependence.
- Must serve as an effective description of the superstratum

Position formula:

$$\cos^2\tilde{\theta} = \frac{1}{2}\left(1 - \frac{J'_L - Q_2(k'_2 - \frac{l'_1}{2k_3})}{J'_R + \frac{Q_2 l'_1}{2}}\right), \qquad \tilde{r}^2 = \frac{\tilde{a}_0^2}{2}\left(-1 + \frac{J'_L + \frac{Q_2 l'_1}{2k_3} - k_3 l'_3}{J'_R + \frac{Q_2 l'_1}{2}}\right)$$

$$\Gamma_{\mathrm{D1P}} = (0, (0, k'_2, 0), (l'_1, 0, l'_3), m')$$

$$\cos^2\tilde\theta = \frac{1}{2}\left(1 - \frac{J'_L - Q_2(k'_2 - \frac{l'_1}{2k_3})}{J'_R + \frac{Q_2 l'_1}{2}}\right), \qquad \tilde r^2 = \frac{\tilde a_0^2}{2}\left(-1 + \frac{J'_L + \frac{Q_2 l'_1}{2k_3} - k_3 l'_3}{J'_R + \frac{Q_2 l'_1}{2}}\right)$$

$$\Gamma_{\mathrm{D1P}} = (0, (0, k'_2, 0), (l'_1, 0, l'_3), m')$$

- Superstratum:

Only $\mathrm{P}(y), J(\psi)$ ⇨ $k'_2 = l'_1 = 0$

Microstates $L^{\mathsf{n}}_{-1}(J^+_{-1})^{\mathsf{m}}|\psi\rangle_{\mathsf{k}}$ ⇨ $l'_3 = \mathsf{n} + \mathsf{m}. \quad m' = J'_L = \mathsf{m} - \frac{\mathsf{k}}{2}, \quad J'_R = -\frac{\mathsf{k}}{2}.$

⇨ $$\cos^2\tilde\theta = \frac{1}{2} + \frac{Q_2 \cdot 0 - 2(\mathsf{m} - \mathsf{k}/2)}{4(-\mathsf{k}/2 + 0)} = \frac{\mathsf{m}}{\mathsf{k}}$$

$$\tilde r^2 = 4m(-1 + \frac{m' - l'_3}{J'_R}) = \tilde a^2 \frac{\mathsf{n}}{\mathsf{k}}.$$

- Agree on the nose!
- Confirms validity of "effective superstratum"

$a = \frac{2m}{k_3} = 2m$ and $\tilde a = 2\sqrt{a}$

Comments

- In this case, the MCS is primitive (microstate ↔ microstate)
- Can also consider reduction to non-primitive MCS
  - Shockwaves
- Readily generalized to superstrata on ≥3 centers
- Applicable to v dependent base (zeroth layer)?

Conclusions

Concluding remarks

- BH microstates
  - Going beyond known microstate geometries?
  - Graviton states, string states, BH states, and what not
- 3-center solution with D1 waves
  - Interesting configuration from micro viewpoint
  - Matching with worldsheet results
  - Holographic dual?
- Effective superstrata
  - Reverse of resolving singular geom into smooth MG
  - Can avoid having to handle details of exact MG solution