$T\bar{T}$ and the Black Hole Interior

(with Shadi Ali Ahmad and Ahmed Almheiri — coming soon)

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YITP workshop:
Recent Developments in BHs and QG
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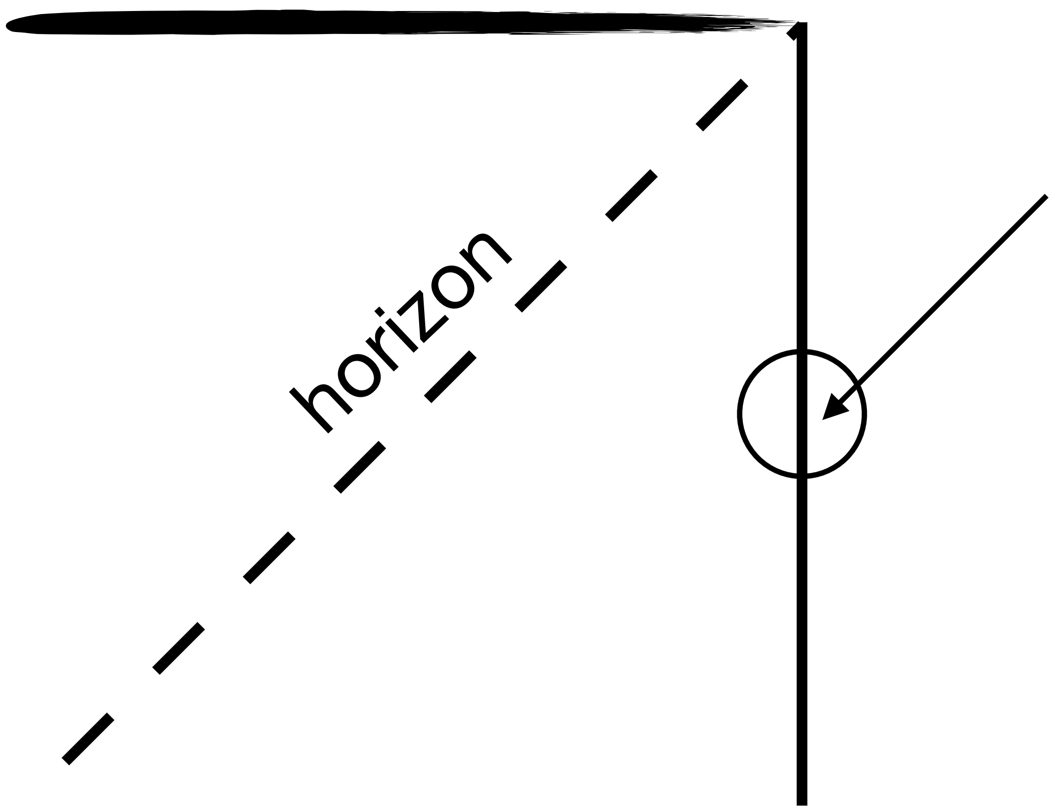
Problem:

no non-perturbative boundary description of interior operators



$$\phi_{\text{bdy}}(x) = \lim_{z\to\infty} z^{\Delta}\phi_{\text{blk}}(z,x)$$

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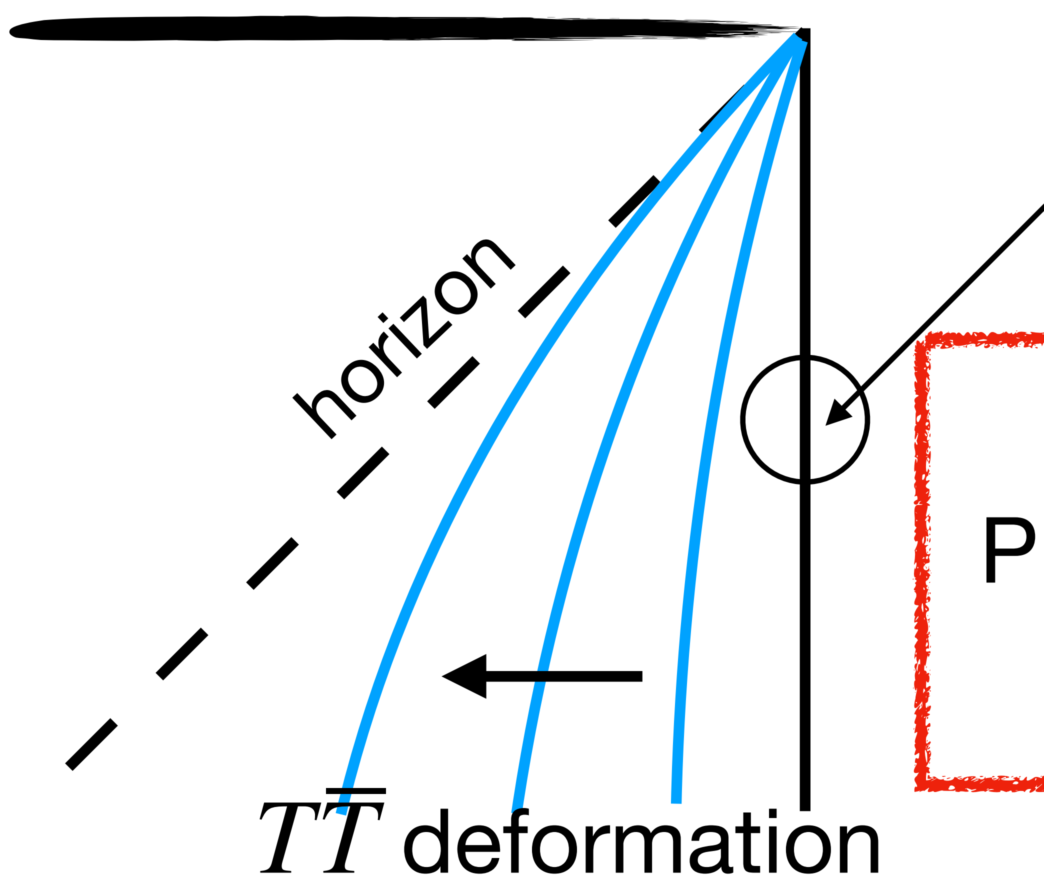


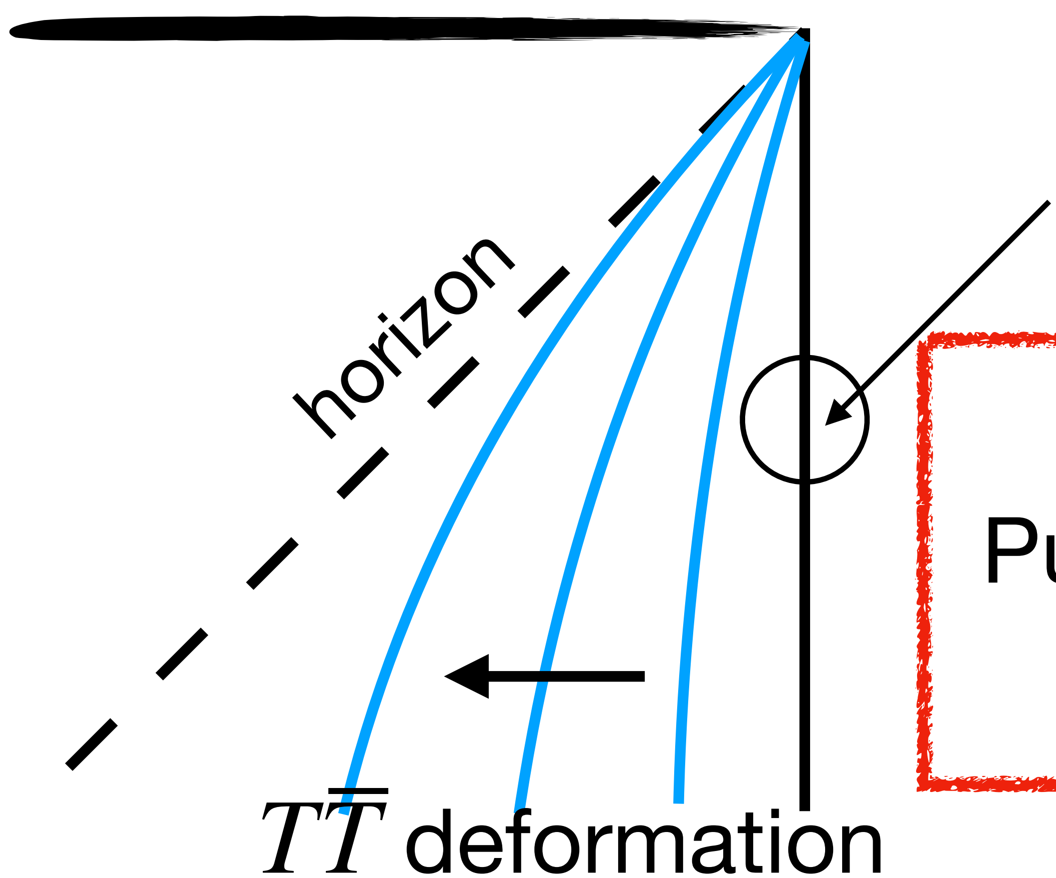
$$\phi_{\rm bdy}(x) = \lim_{z\to\infty} z^{\Delta}\phi_{\rm blk}(z,x)$$

Solution:

Put the holographic screen at finite z

Problem #2:

TT pushes the boundary *until* the horizon…



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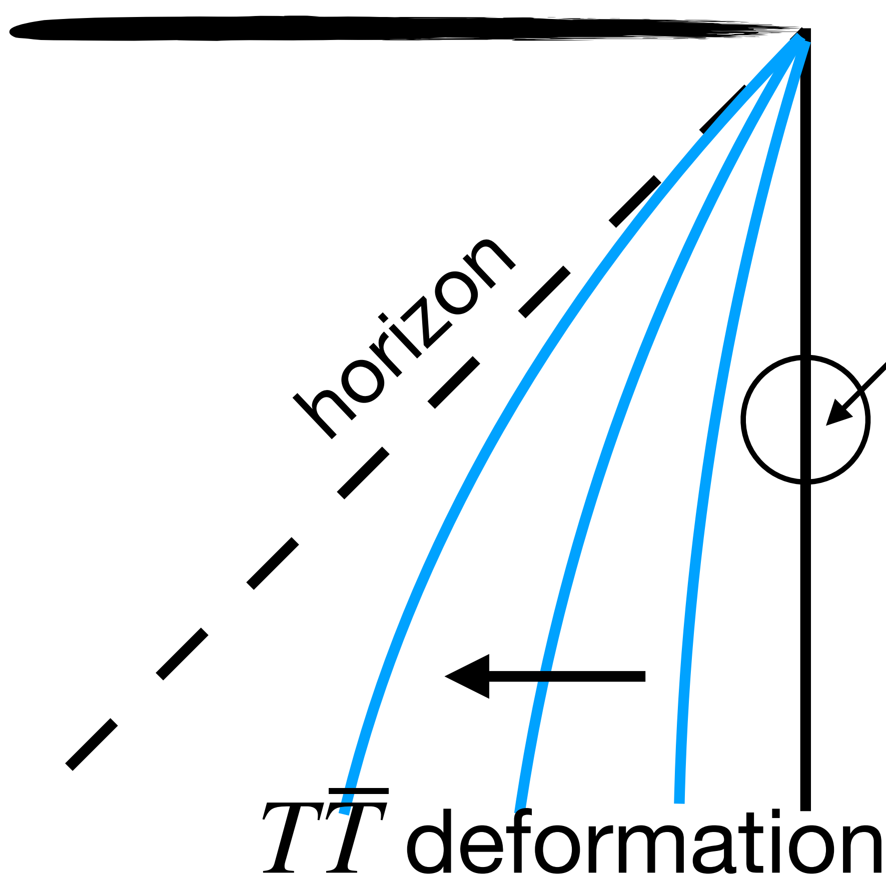
This Talk:

Extend further and put the screen at the *interior* of the black hole



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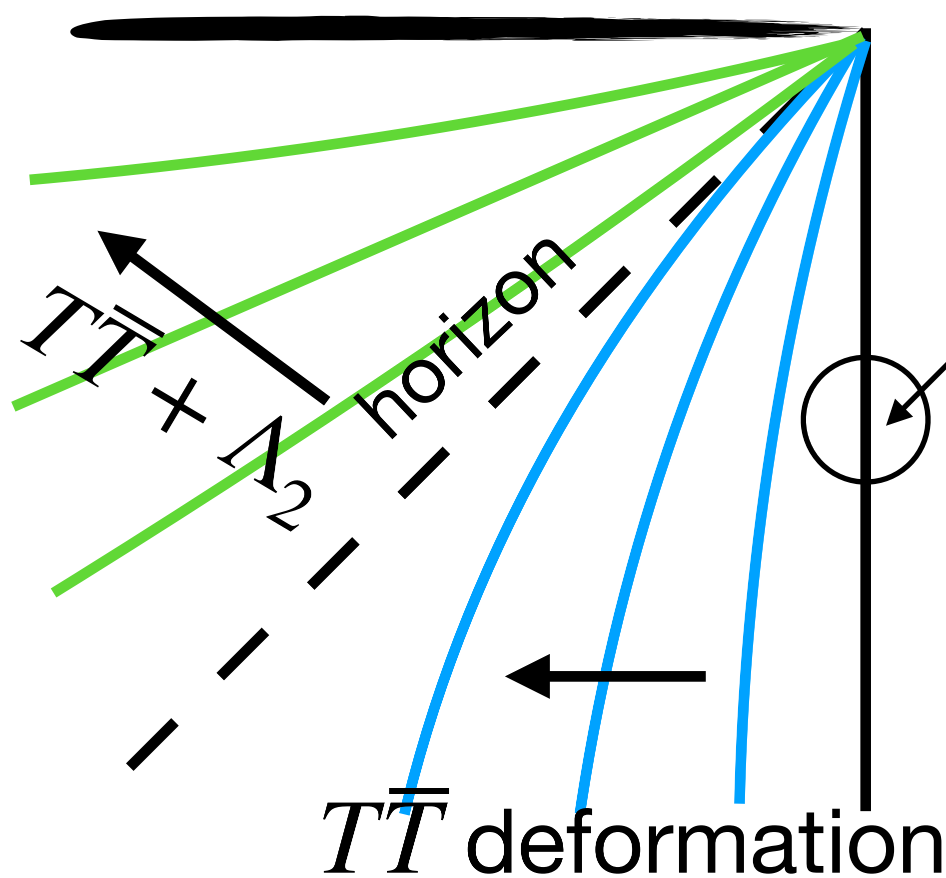


$$\phi_{\text{bdy}}(x) = \lim_{z\to\infty} z^{\Delta} \phi_{\text{blk}}(z,x)$$

Prescription:

Generalized deformation

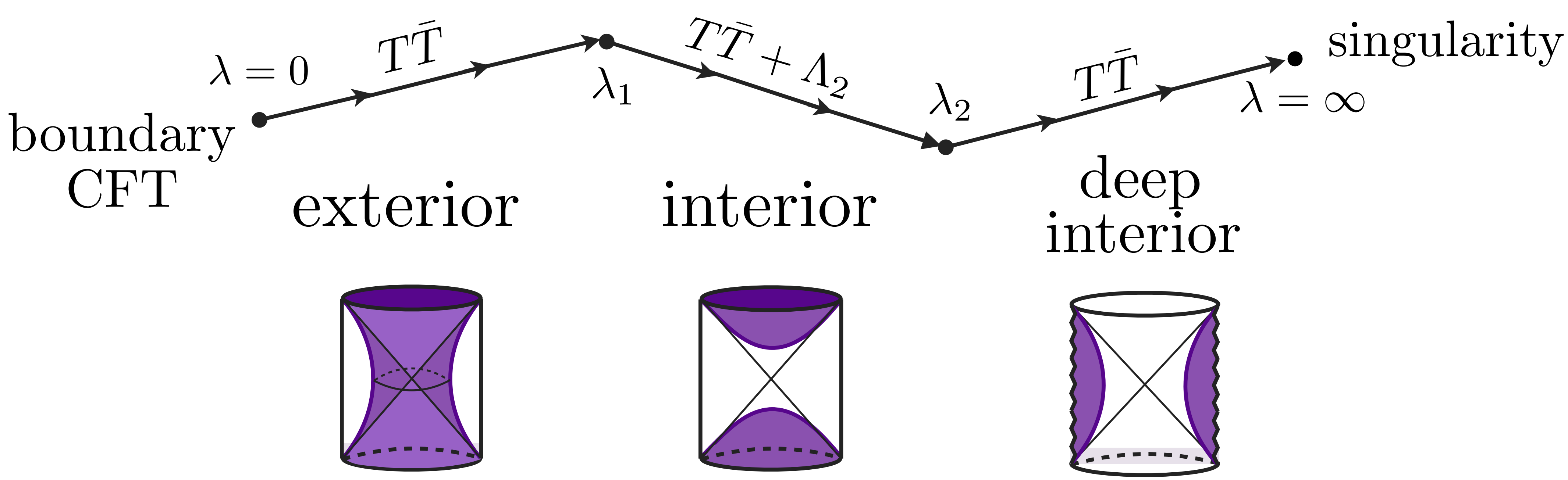
$$T\overline{T} + \Lambda_2$$

But how exactly?

- Generalized $T\overline{T}$ deformation (2d bdy/3d bulk)

$$\partial_\lambda S^\lambda_{\mathrm{QFT}} = \int dx^2 \sqrt{|h|}\,(T\overline{T} + b\Lambda_2)$$

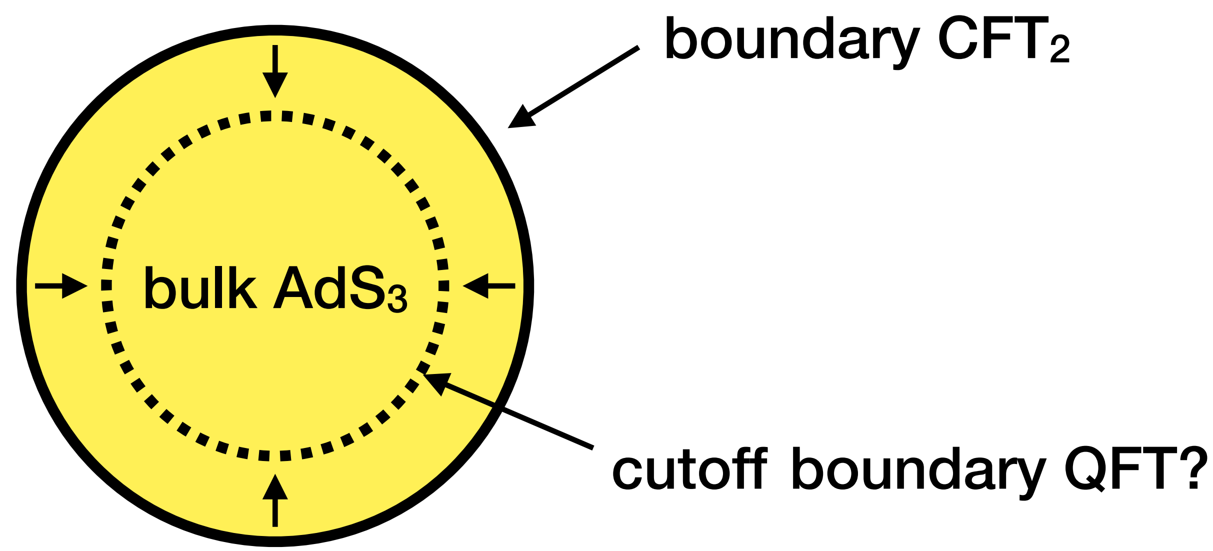
- Metric signature change when crossing the BH horizon



What I will (actually) talk about

1. Review of $T\overline{T}$
    - derivation from bulk [Hartman-Kruthoff-Shaghoulian-Tajdini]
    - "operator formalism" (energy flow) [McGough-Mezei-Verlinde]
    - "variational formalism" (metric flow) [Guica-Monten]
2. $T\overline{T} + \Lambda_2$
    - derivation from bulk (motivation of Λ_2 term)
    - operator formalism (energy matching)
    - variational formalism (metric signature change)
3. Gravitational path integral for the interior
    - what boundary condition to put at finite cutoff?

$T\bar{T}$ and finite cutoff holography [Hartman-Kruthoff-Shaghoulian-Tajdini]



- How to setup a finite cutoff?

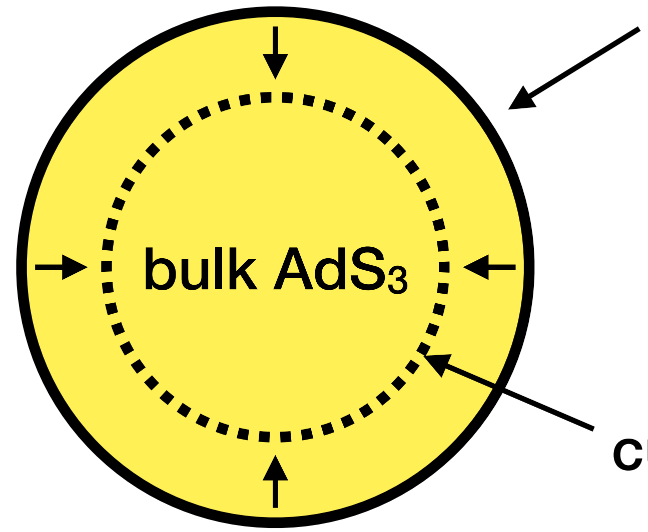
- (radial) Hamiltonian constraint

$$G_{ab}n^a n^b = \frac{1}{2}\left(R - K^2 - K_{ab}K^{ab} - 2n^a n_a\right) = 0$$

- K can be related to the Brown-York (BY) stress tensor

$$T_{ab} = \frac{2}{\sqrt{-h}}\frac{\delta S}{\delta h^{ab}} = -K_{ab} + h_{ab}(K-1)$$

$T\bar{T}$ and finite cutoff holography [Hartman-Kruthoff-Shaghoulian-Tajdini]



- Assuming radial decomp

$$ds^2 = \frac{d\rho^2}{4\rho^2} + \frac{\gamma_{ab}}{\rho} dx^a dx^b$$

- Need to renormalize the metric

$$h_{ab} = \gamma_{ab}/\rho$$

- Hamiltonian constraint $\longrightarrow$ "trace flow equation"

$$T = -\frac{\rho}{2}\left(\cancel{R} - T_{ab}T^{ab} + T^2\right)$$

- Relate this to the deformation of the CFT action

λ is the only scale $\Rightarrow$ $$\partial_\lambda S = \frac{1}{2\lambda}\int \sqrt{-\gamma}T$$

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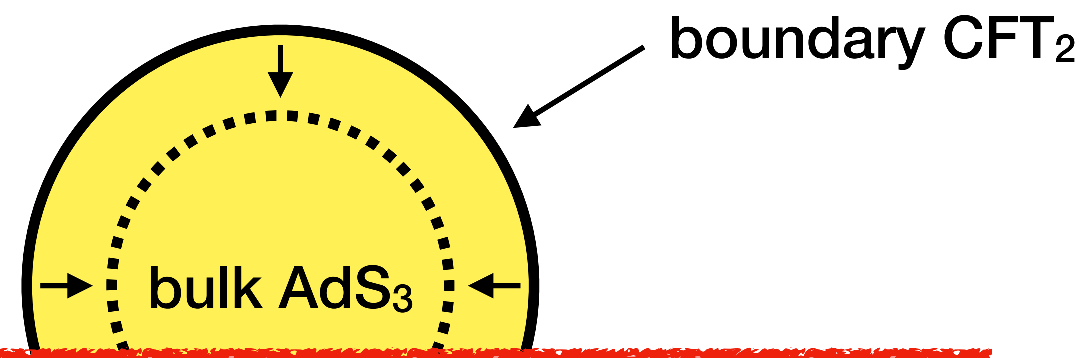

ary QFT?

$$\rho = 4\lambda$$

$$\partial_\lambda S = \frac{\rho}{4\lambda} \int dx^2 \sqrt{-\gamma} \left(T_{ab} T^{ab} - T^2 \right)$$

$$= 8 \int dx^2 \sqrt{-\gamma}\, T\bar{T}$$

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$T\overline{T}$ deformation [Zamolochikov-Smirnov]

- 1-parameter family of 2d QFT defined by

$$\partial_\lambda S_{QFT} = 8 \int dx^2 \sqrt{\gamma}\, T\overline{T}, \quad S^0_{QFT} = S_{CFT}$$

- $T\overline{T}$ operator $\quad T\bar{T} = \frac{1}{4} \det T_{\mu\nu} = \frac{1}{8} \left(T^{\mu\nu} T_{\mu\nu} - (T^\mu_\mu)^2 \right)$

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    - Well-defined as limit of a composite operator

$$T\bar{T}(z) \equiv \frac{1}{8} \lim_{x \to y} \epsilon^{\alpha\beta} \epsilon^{\gamma\delta} T_{\alpha\delta}(x) T_{\beta\delta}(y) + \text{total derivatives}$$

    - Irrelevant deformation
    - Factorization property $\quad \langle T\bar{T} \rangle = \langle T \rangle \langle \bar{T} \rangle - \langle \Theta \rangle^2$

$$(T = T_{zz},\ \bar{T} = T_{\bar{z}\bar{z}},\ \Theta = T_{z\bar{z}})$$

Operator flow formalism

[Zamolochikov-Smirnov]
[McGough-Mezei-Verlinde]

- Despite being an irrelevant factorization, the energy levels of the deformed theory is *exactly solvable*

- To see this:

  - From the definition of the deformation

$$\partial_\lambda S_\lambda = 8 \int dx^2 \sqrt{-\gamma}\, T\overline{T} = 2 \int dt d\theta \left(T_{tt} T_{\theta\theta} - T_{t\theta}^2\right)$$

  - Factorization property

$$\begin{aligned}\partial_\lambda E &= L \left\langle E, J \,\middle|\, T\overline{T} \,\middle|\, E, J\right\rangle \\ &= 2L \left(\langle T_{tt}\rangle \langle T_{\theta\theta}\rangle - \langle T_{t\theta}\rangle^2\right)\end{aligned}$$

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$$\frac{E}{L} \qquad -\frac{\partial E}{\partial L} \qquad \frac{J}{L}$$

Operator flow formalism

[Zamolochikov-Smirnov]
[McGough-Mezei-Verlinde]

- The "flow equation":

$$\frac{1}{2}\partial_\lambda E + E\frac{\partial E}{\partial L} + \frac{J^2}{L} = 0$$

- Solutions: $$E(\lambda) = \frac{1}{4\lambda}\left(1 \pm \sqrt{1 - 8E_0\lambda + 16J^2\lambda^2}\right)$$

- Match to the quasi-local energy in the bulk (BTZ BHs)

$$E_{\rm BY} = T_{\mu\nu}u^\mu u^\nu = \frac{r_c^2}{4}\left(1 - \sqrt{f(r_c)}\right)$$
$$= \frac{r_c^2}{4}\left(1 - \sqrt{1 - \frac{8M}{r_c^2} + \frac{16J^2}{r_c^4}}\right)$$

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$$\boxed{r_c^2 = \lambda^{-1}}$$

Variational (metric flow) formalism [Guica-Monten]

- The operator method tells us how ***energy*** changes along the flow
  - It however masks how the metric gets modified
- Variational method [Guica-Monten] keeps track of ***deformation of metric*** during the flow

Variational (metric flow) formalism [Guica-Monten]

- The operator method tells us how ***energy*** changes along the flow
  - It however masks how the metric gets modified
- Variational method [Guica-Monten] keeps track of ***deformation of metric*** during the flow
- Starting from the defining relation $\partial_\lambda S^\lambda = 8 \int dx^2 \sqrt{-\gamma} T\overline{T}$
- Vary both sides w.r.t. γ_{ab} we get

$$\partial_\lambda \left(\sqrt{-\gamma} T_{ab} \delta\gamma^{ab} \right) = 8\delta(\sqrt{-\gamma} T\overline{T})$$

- Note that we now assume γ_{ab} changes along the flow (c.f. the operator formalism where it is held fixed $\gamma_{ab} = \eta_{ab}$)

Variational (metric flow) formalism [Guica-Monten]

- Evaluating the variation gives rise to “metric flow equations”

$$\partial_\lambda \gamma_{ab} = 4\hat{T}_{ab}, \quad \partial_\lambda \hat{T}_{ab} = 2\hat{T}_{ac}\hat{T}^c_b$$

$$\hat{T}_{ab} \equiv T_{ab} - \gamma_{ab} T$$

- Generic solutions

$$\gamma_{ab} = \gamma^0_{ab} + 4\lambda \hat{T}^0_{ab} + 4\lambda^2 \hat{T}^0_{ac} (\gamma^0)^{cd} \hat{T}^0_{db}$$

$$\hat{T}^\lambda_{ab} = \hat{T}^0_{ab} + 2\lambda \hat{T}^0_{ac} (\gamma^0)^{cd} \hat{T}^0_{db}$$

  - quadratic in λ
  - $\gamma^0_{ab}, \hat{T}^0_{ab}$: initial conditions. Chosen to match asym CFT data

$$\gamma^0_{ab} = \eta_{ab}, \quad T^0_{ab} = \begin{pmatrix} -E & J \\ J & E \end{pmatrix}$$

Variational (metric flow) formalism [Guica-Monten]

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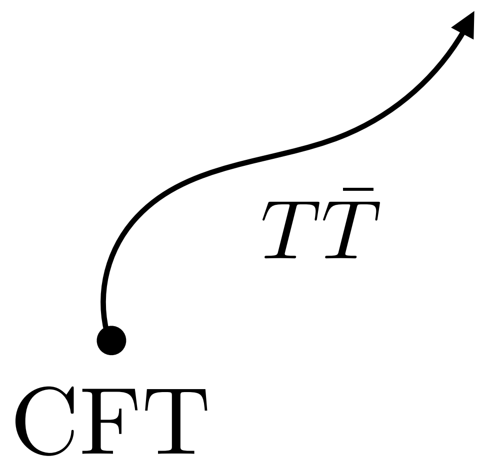
- match the BTZ solution in Fefferman-Graham gauge

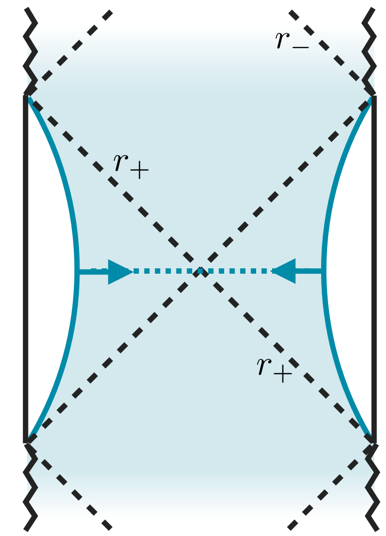
$$ds^2 = \frac{d\rho^2}{4\rho^2} + \frac{\gamma^{\rm FG}_{ab}}{\rho} dx^a dx^b, \qquad \gamma^{\rm FG}_{ab} = \gamma^{(0)}_{ab} + \rho \gamma^{(2)}_{ab} + \rho^2 \gamma^{(4)}_{ab}$$

- Einstein eq fix $\quad \gamma^{(4)}_{ab} = \frac{1}{4} \gamma^{(2)}_{ac} (\gamma^{(0)})^{cd} \gamma^{(2)}_{db}$

- Identification $\quad \rho = 4\lambda$

Story so far…





TT deformed CFT ⟷ AdS_3 at finite cutoff

deformation parameter λ ⟷ cutoff radius $r_c^2 = \lambda^{-1}$ [MMV]

$\rho = 4\lambda$ [Guica-Monten]

- Additional evidences:
  - Entanglement entropies [Donnelly-Shyam, Chen-Chen-Hao, Kraus-Liu-Marolf, …]
  - Correlation functions [He-Song-Yin, Li-Zhou, Cardy, Aharony-Barel, …]
  - Other approaches (as WdW wave functions, as coupling to JT gravity, as random geometry, equivalent gravitational path integral, …) [Cardy, Dubovsky-Gorbenko-(Hernandez-Chifflet), Iliesiu-Kruthoff-Turiaci-Verlinde, …]

Limitations

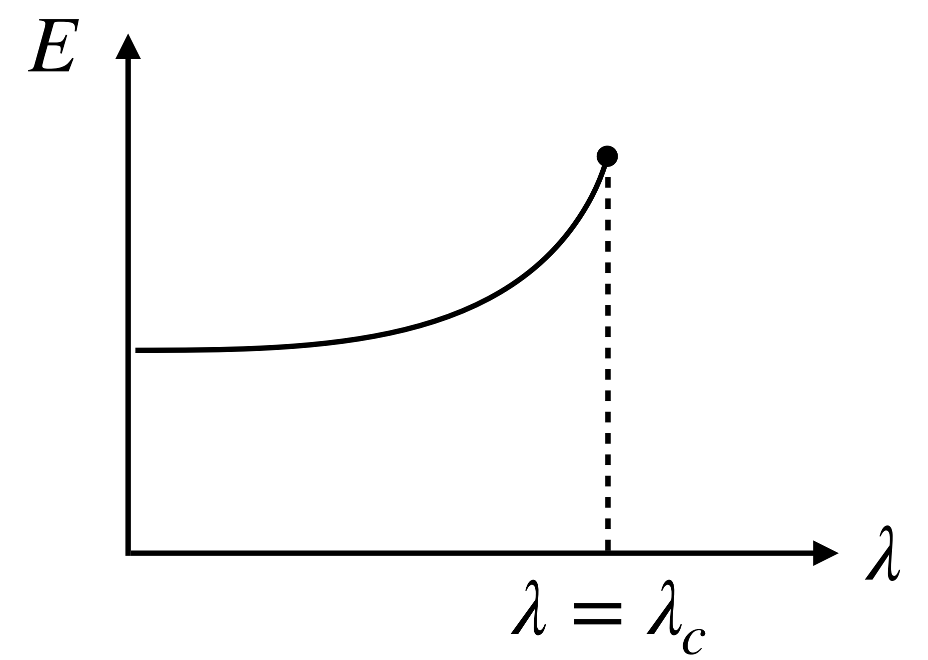
- Funny behaviors when we try to push the deformation through BH horizon

- complexification of energy:

$$f_\lambda = 1 - 8E_0\lambda + 16J^2\lambda^2$$

"emblackening factor"

$$E(\lambda) = \frac{1}{4\lambda}\left(1 - \sqrt{f_\lambda}\right)$$



- Not sure how to deal with them…
  - throw complex energies away?
  - BH interior non-Hermitian?
  - interior constant r surfaces are space-like

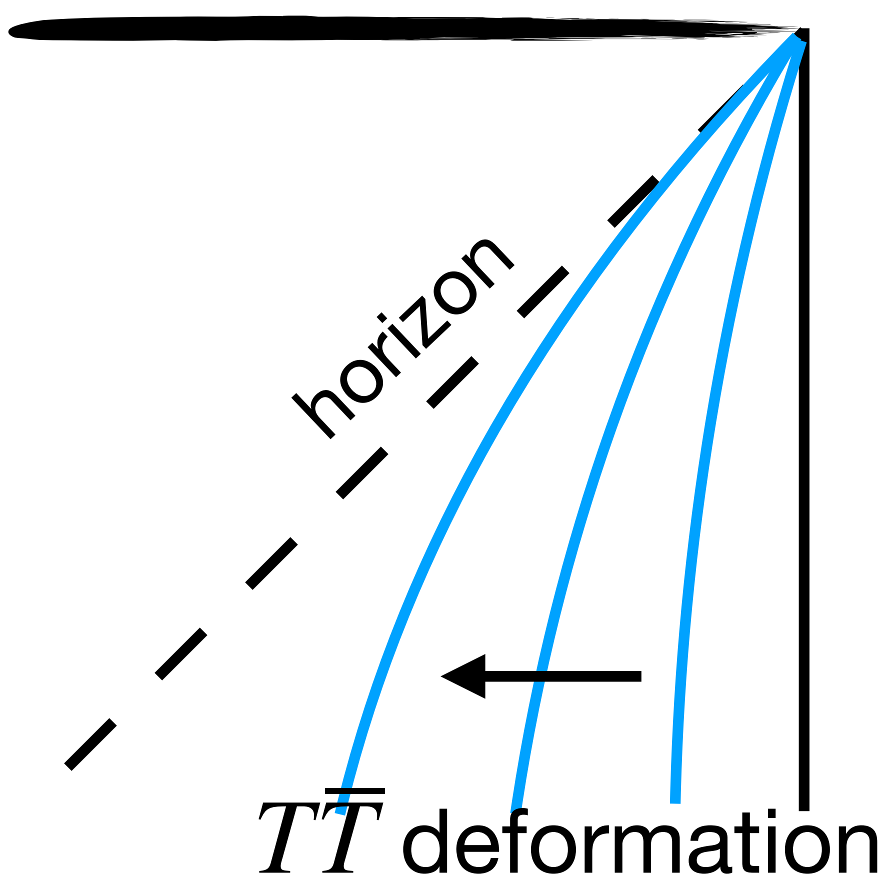
(c.f. Kotaro's talk)

Limitations

- Similarly, from the variational formalism:

$$\det \gamma_{ab}^{\lambda} \sim -\left(\frac{\sqrt{f_\lambda}}{\# + \sqrt{f_\lambda}}\right)^2$$

- The metric **degenerates** at horizon and is **complex** in the interior!



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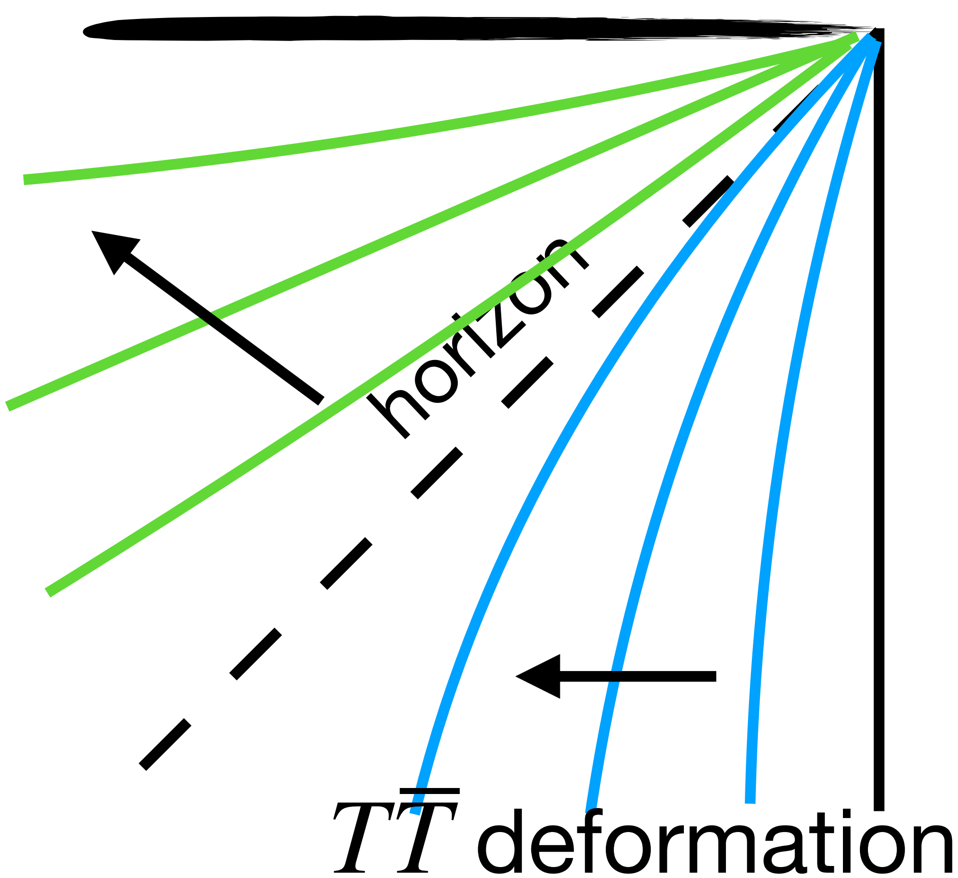
- From the bulk: γ_{ab} simply **changes signature**, not become complex

- Can we really trust that the same $T\bar{T}$ deformation will continue to push us further in?



$T\overline{T} + \Lambda_2$

- Fortunately we already knew how to ***derive*** the deformation from the bulk

- Hamiltonian constraint

bdy normal is now *timelike*: $n^a n_a = -1$

$$G_{ab} n^a n^b = \frac{1}{2}\left(R - K^2 - K_{ab}K^{ab} - 2n^a n_a\right) = 0$$

- Also need to be careful about defining T_{ab}

$$T_{ab} = \frac{2}{\sqrt{h}} \frac{\delta(iS)}{\delta h^{ab}} = i\left(K_{ab} - h_{ab}(K-1)\right)$$

- $n^a n_a$ sources an extra term in the constraint: we get

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- Hamiltonian constraint bdy normal is now *timelike*: $n^a n_a = -1$

$\rho = 4\lambda$

$$\partial_\lambda S = \frac{\rho}{4\lambda} \int dx^2 \sqrt{\gamma} \left(T_{ab}T^{ab} - T^2 + \frac{4}{\rho^2} \right)$$

$$= 8 \int dx^2 \sqrt{\gamma} \left(T\overline{T} + \frac{1}{32\lambda^2} \right)$$

0

-

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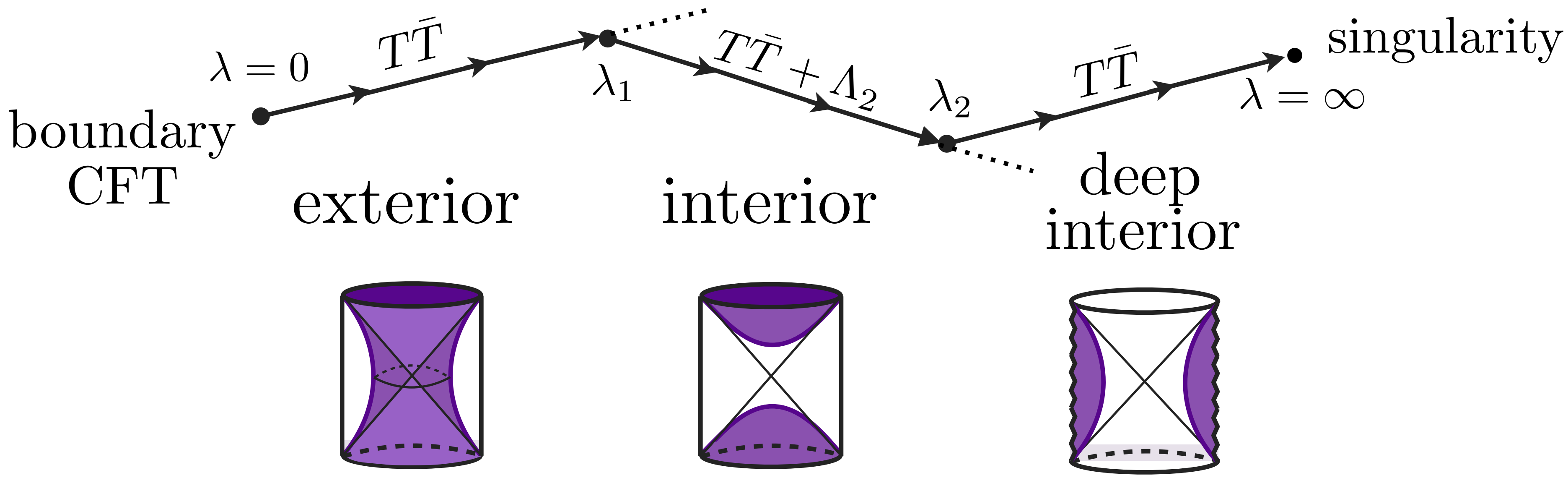
The CC term

- $n^a n_a$ sources an extra term in the constraint: we get

The recipe



- Claim: This sequence of flow matches the correct
  - bulk quasi-local energy (via operator formalism)
  - bulk induced metric in FG gauge (via variational formalism)
- Due to time constraint I will focus on the intermediate flow in this talk

$T\overline{T} + \Lambda_2$: operator formalism

$$\partial_\lambda S_{\mathrm{QFT}} = 8\int dx^2\sqrt{\gamma}\left(T\overline{T} + \frac{b}{32\lambda}\right)$$
$$= -2\int dt d\theta\left(T_{tt}T_{\theta\theta} - T_{t\theta}^2 - \frac{1}{8\lambda^2}\right)$$

- In spite of the newly added term the deformed energy levels still remain exactly solvable

- Using the factorization property:

$$\partial_\lambda E = 2L\left(\langle T_{tt}\rangle\langle T_{\theta\theta}\rangle - \langle T_{t\theta}\rangle^2 - \frac{1}{8\lambda^2}\right)$$

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$\frac{iE}{L}$ ↗ $\langle T_{tt} \rangle$, $\frac{i\partial E}{\partial L}$ ↗ $\langle T_{\theta\theta} \rangle$, $\frac{iJ}{L}$ ↗ $\langle T_{t\theta} \rangle$

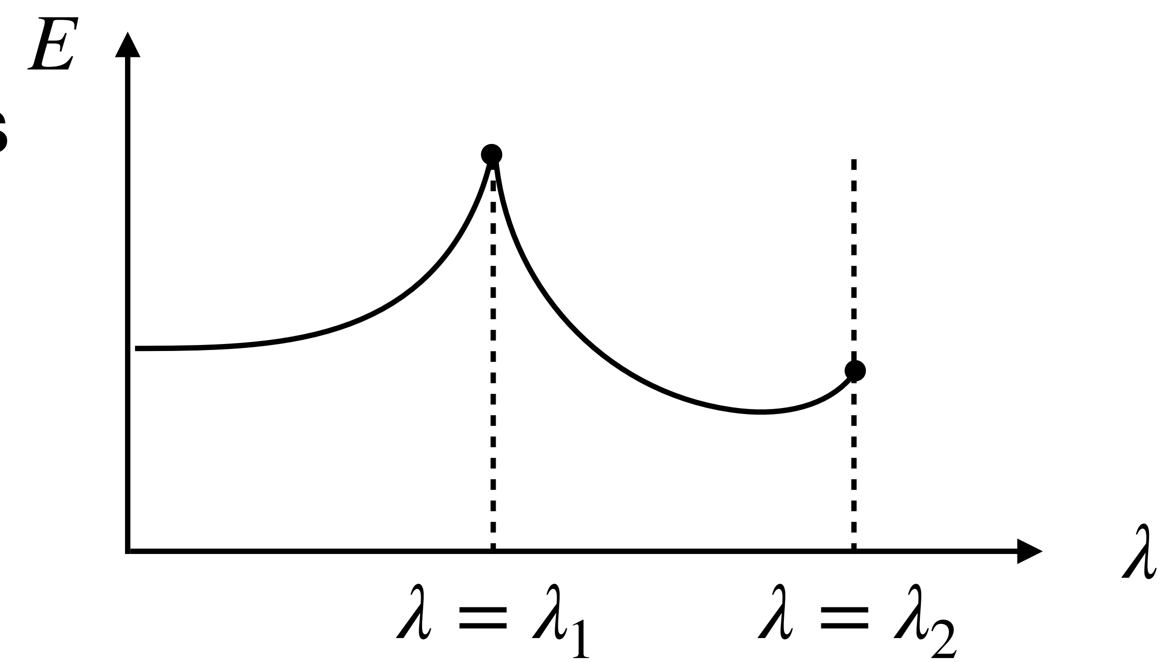
$T\overline{T} + \Lambda_2$: operator formalism

- Energy flow equation for the interior:

$$\frac{1}{2}\partial_\lambda E = -E\frac{\partial E}{\partial L} + \frac{J^2}{L} - \frac{L}{8\lambda^2}$$

- Solutions: $$E(\lambda) = \frac{1}{4\lambda}\left(1 - \sqrt{-1 + 8E_0\lambda - 16J^2\lambda^2}\right)$$

- Agree with bulk calculations



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$= -f_\lambda$, positive in the interior

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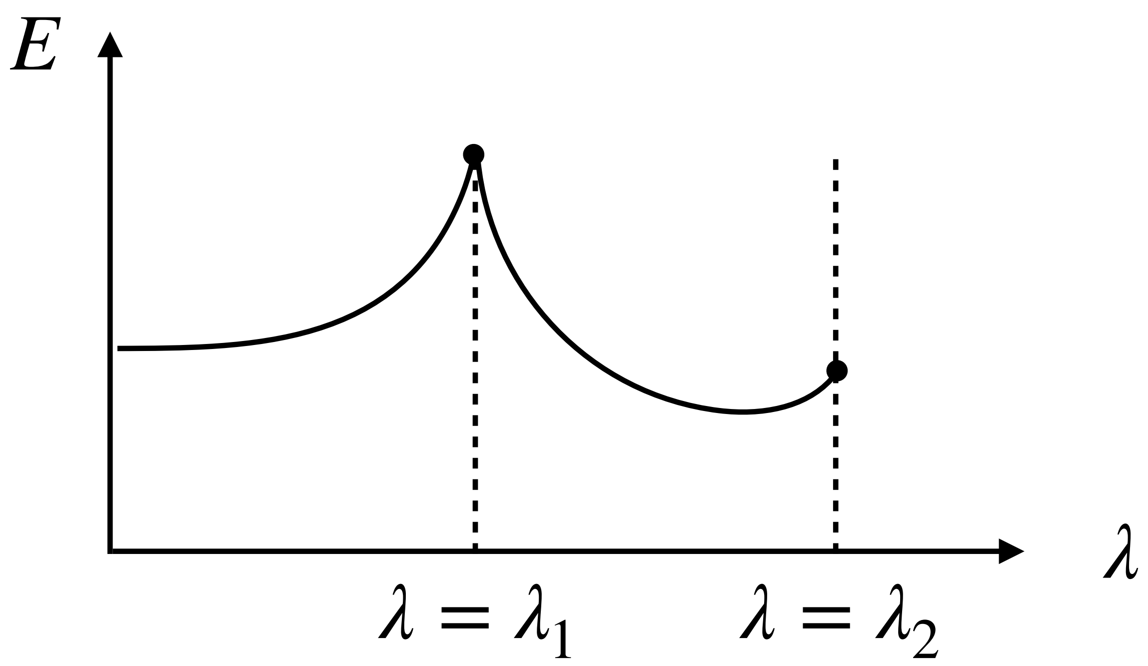

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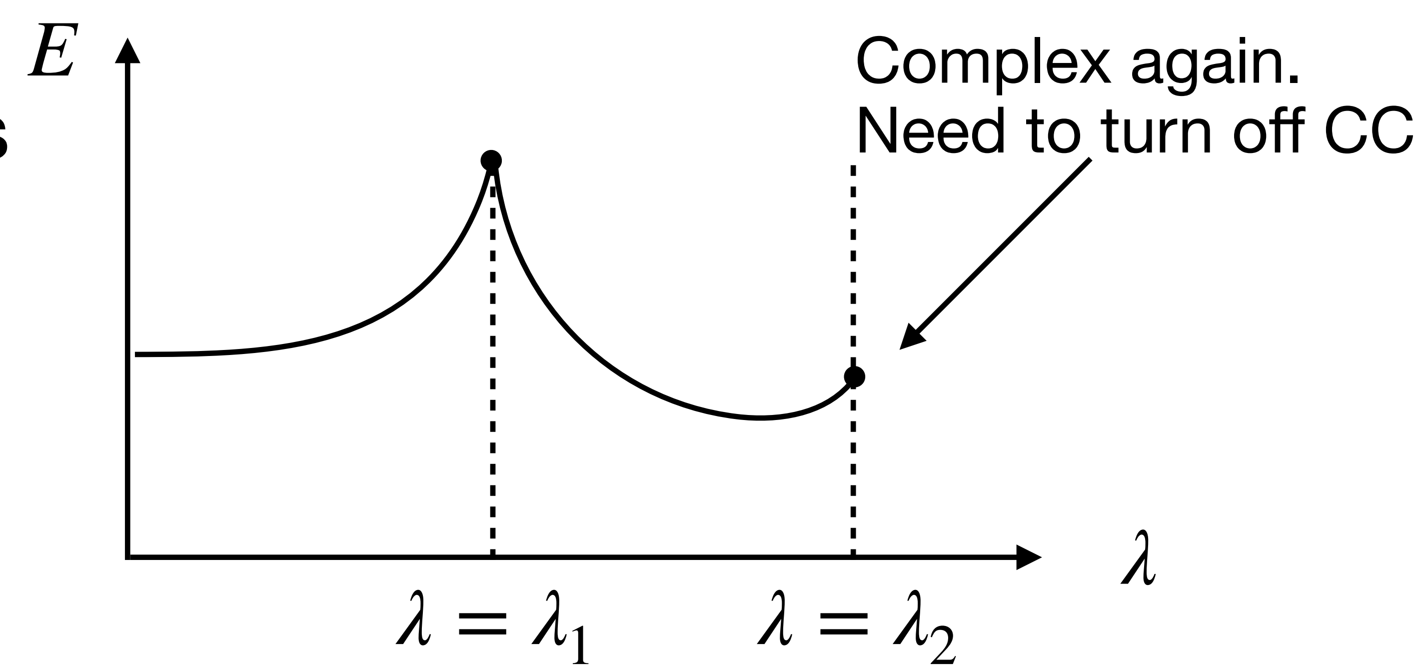


$T\overline{T} + \Lambda_2$: variational formalism

- Variation of the action gives

$$\partial_\lambda(\sqrt{\gamma}\delta\gamma^{ab}T_{ab}) = -8i\delta\left(\sqrt{\gamma}(T\overline{T} + 1/32\lambda^2)\right)$$

- Metric flow equations

$$\partial_\lambda \gamma_{ab} = 4i\hat{T}_{ab}, \quad \partial_\lambda \hat{T}_{ab} = 2i\hat{T}_{ab}\hat{T}^c_b + i\frac{\gamma_{ab}}{4\lambda^2}$$

- Boundary condition:

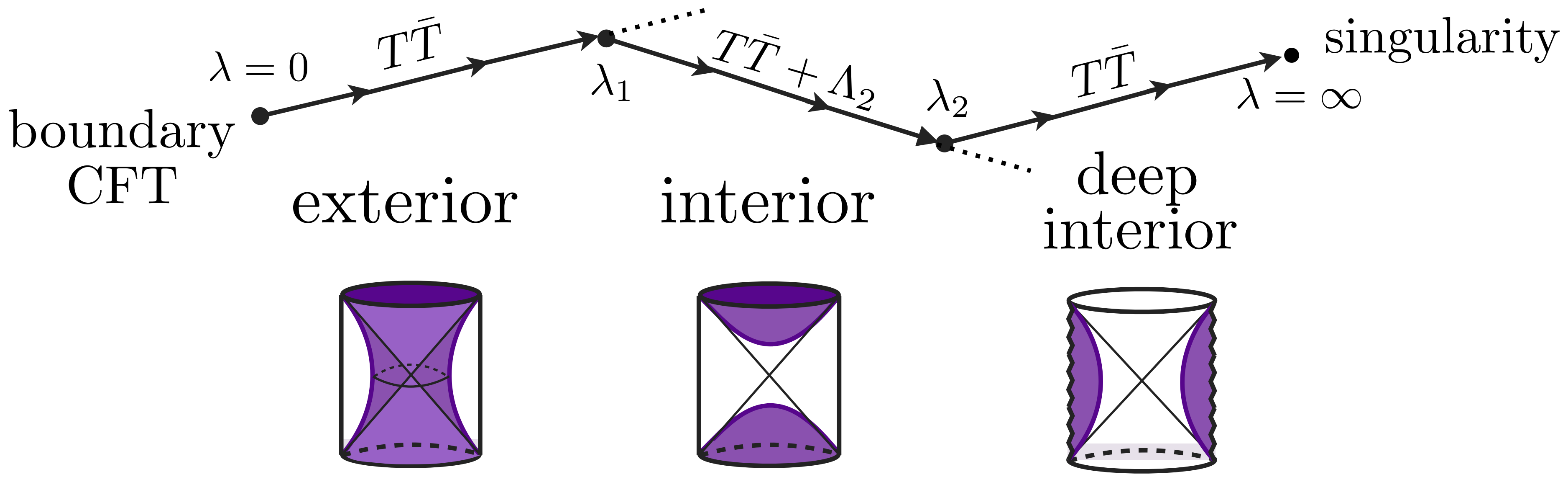
Define $\tilde{T}_{ab} = \begin{cases} T_{ab}, & \lambda < \lambda_c \\ iT_{ab}, & \lambda > \lambda_c \end{cases}$

Then γ_{ab}, $\tilde{T}_{ab}$ continuous, $\partial\tilde{T}_{ab}$ anti-continuous

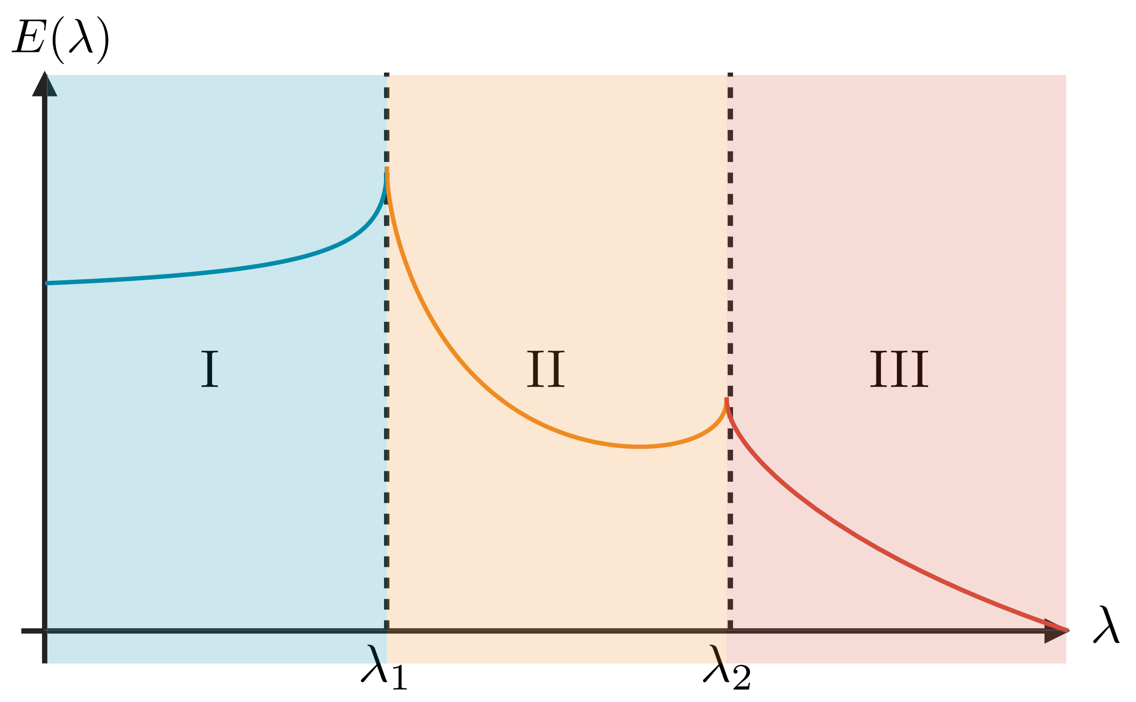
Can check that the solution matches the interior BTZ metric in BY gauge!

← not sure why… gives correct answer

Recap



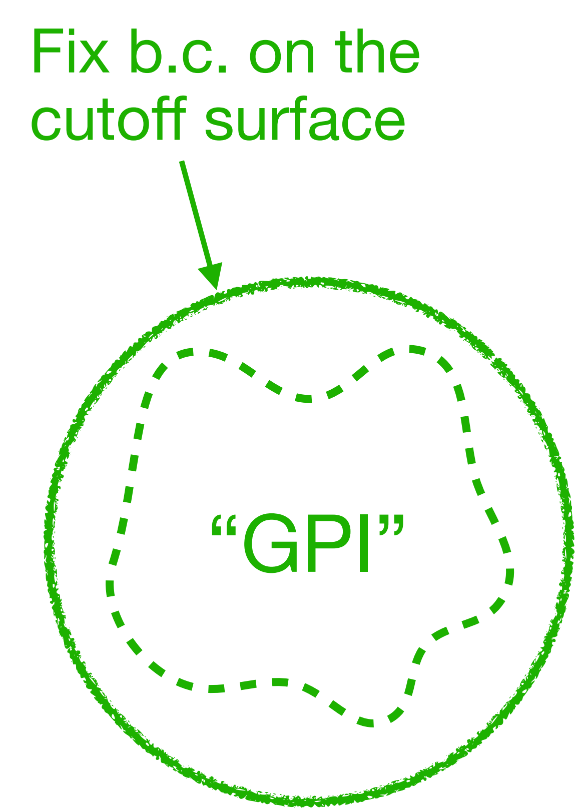
$\lambda = 0$
$T\bar{T}$
λ_1
$T\bar{T} + \Lambda_2$
λ_2
$T\bar{T}$
singularity
$\lambda = \infty$
boundary CFT
exterior
interior
deep interior



$E(\lambda)$
I
II
III
λ
λ_1
λ_2

A bulk gravitation path integral

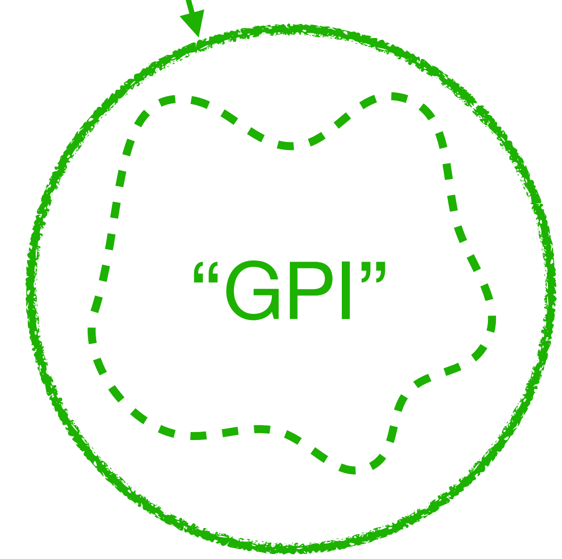
- The deformed boundary theory is defined by the initial conditions $T^0_{\tau\tau} = M, T^0_{\tau\theta} = iJ$ and flow parameter λ
- Q: What are we fixing in the bulk?
- A (naive): ADM mass M, angular momentum J, radial cutoff r_c



A bulk gravitation path integral

- The deformed boundary theory is defined by the initial conditions $T^0_{\tau\tau} = M, T^0_{\tau\theta} = iJ$ and flow parameter λ
- Q: What are we fixing in the bulk?
- A (naive): ADM mass M, angular momentum J, radial cutoff r_c
- To truly state a holographic dictionary we need equivalence of partition functions $Z_{T\bar{T}}(T^0_{ab}, \lambda) = Z_{\text{grav}}(?)$
- Need to define a (Euclidean) **gravitational path integral** (GPI) with boundary conditions defined solely on the finite cutoff surface!
- None of of (M, J, r_c) are defined on the bdy naively

Fix b.c. on the cutoff surface



A bulk gravitation path integral

- However, there are boundary-local quantities whose values are the same as the triple $(T^0_{\tau\tau}, T^0_{\tau\theta}, \lambda)$:

  - Hawking mass (spherical symm) [Soni-Wall]

$$M = \frac{1}{8G}\frac{K^2_{\theta t} - K^2_{\theta\theta}}{h_{\theta\theta}}, \qquad J = \frac{1}{8G}K_{\theta t}$$

  - 1d induced metric $h_{\theta\theta}$

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(Equivalent to fixing the BY energy)

$$E_{\rm BY} = \int d\theta \sqrt{h_{\theta\theta}}\, u^a u^b T^{\rm BY}_{ab}$$

Think of microcanonical ensemble!

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(Equivalent to fixing the BY energy)

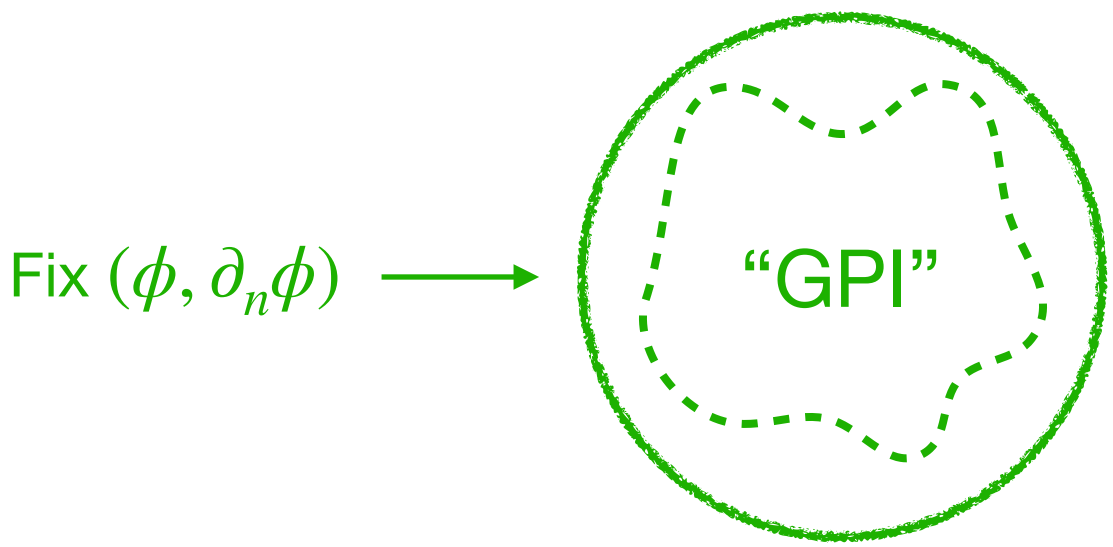
$$E_{\mathrm{BY}} = \int d\theta \sqrt{h_{\theta\theta}}\, u^a u^b T^{\mathrm{BY}}_{ab}$$

Think of microcanonical ensemble!

- In JT case ($J = 0$) we have
  - $M_H = \phi^2 - (\partial_n \phi)^2$
  - $E_{BY} = \phi(\phi - \partial_n \phi)$

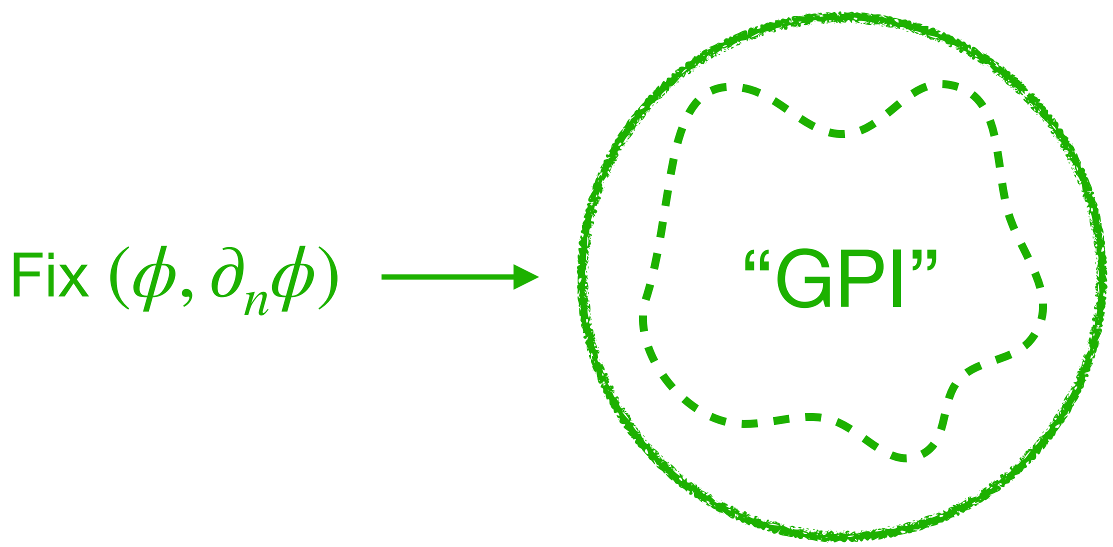
← We are actually fixing $(\phi, \partial_n \phi)$ on $r = r_c$ [Iliesiu-Kruthoff-Turiaci-Verlinde]

GPI for BH interior?



- Seem that there exists $\phi_{\text{min}} = \sqrt{M}$ where $\partial_n \phi$ becomes imaginary!
- Since $\phi \sim r$, The GPI does not have classical saddles for the BH interior.

GPI for BH interior?



- Seem that there exists $\phi_{\text{min}} = \sqrt{M}$ where $\partial_n \phi$ becomes imaginary!
- Since $\phi \sim r$, The GPI does not have classical saddles for the BH interior.

What gives?

GPI for BH interior!

- We implicitly assumed that the normal to the boundary is space like $n^a n_a = +1$, which is not true for the interior

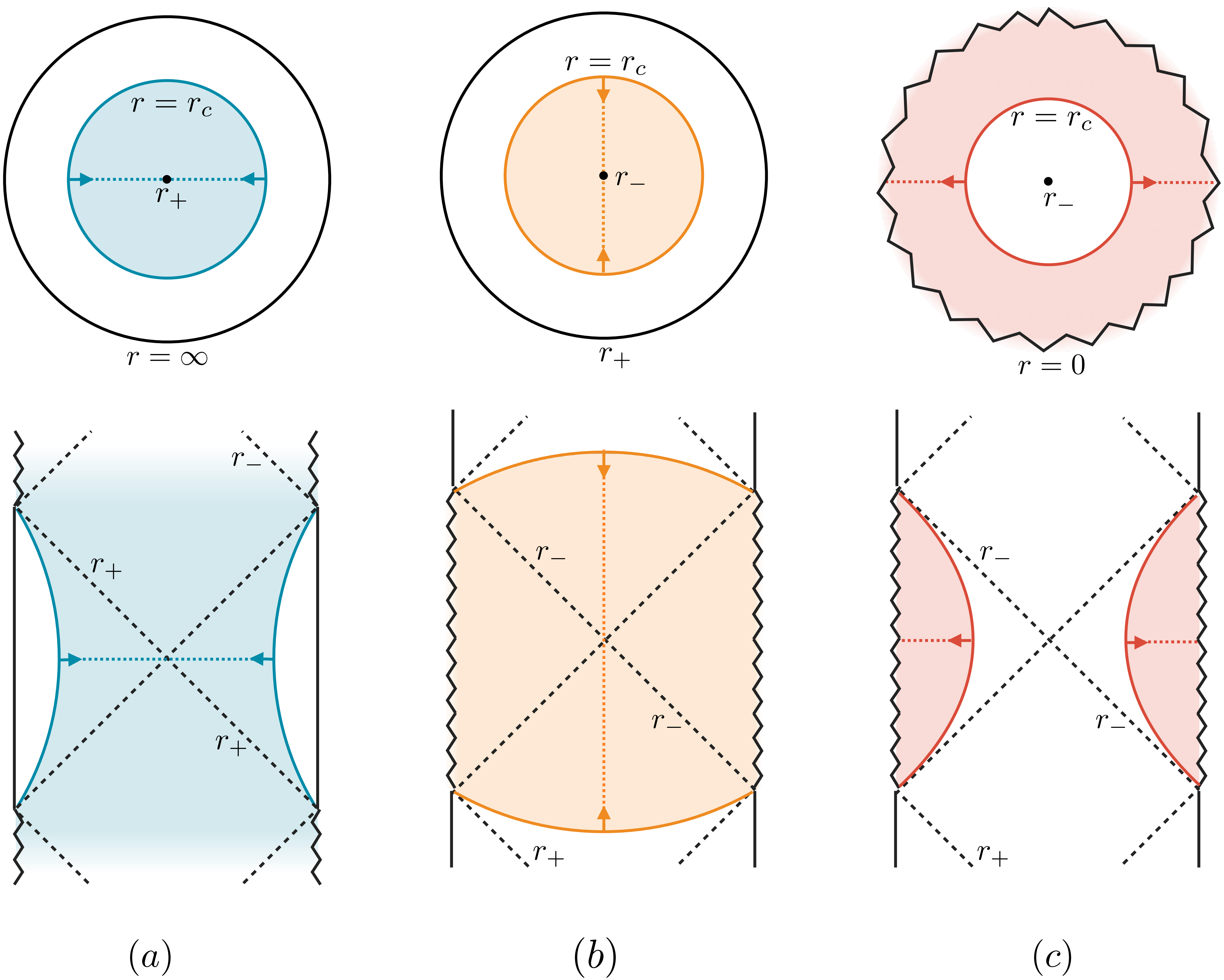
- Relaxing this by allowing metrics with $n^a n_a = +1$ modifies the formula for the boundary energies

$$M_H = \phi^2 \pm (\partial_n \phi)^2$$

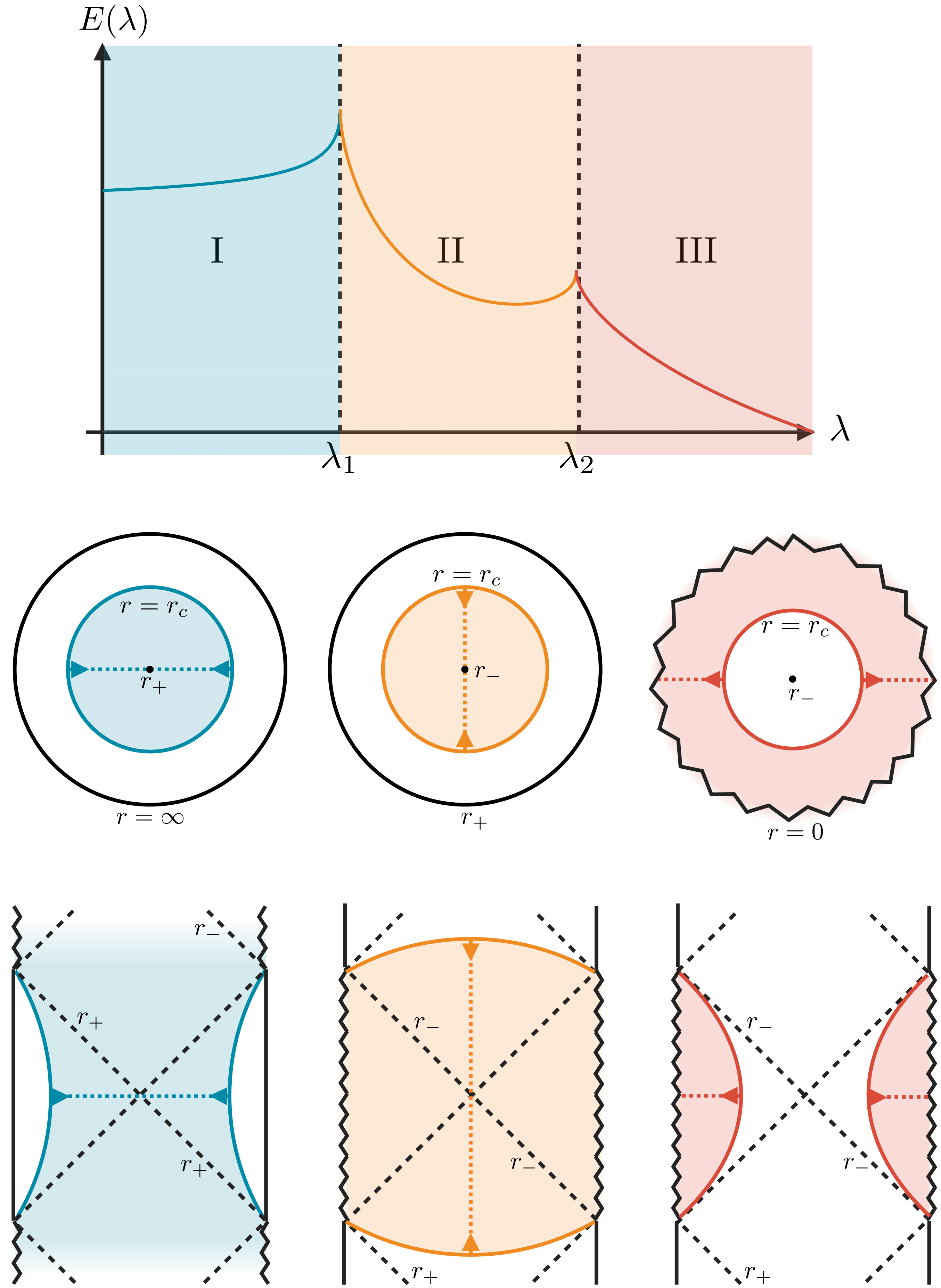
$$E_{BY} = \phi(\phi - \partial_n \phi) = \phi^2 - \phi\sqrt{|M_H - \phi^2|}$$

- Now our GPI **does** have saddle solutions past $\phi_{\min}$!

- Matches to the BH interior with the correct energy



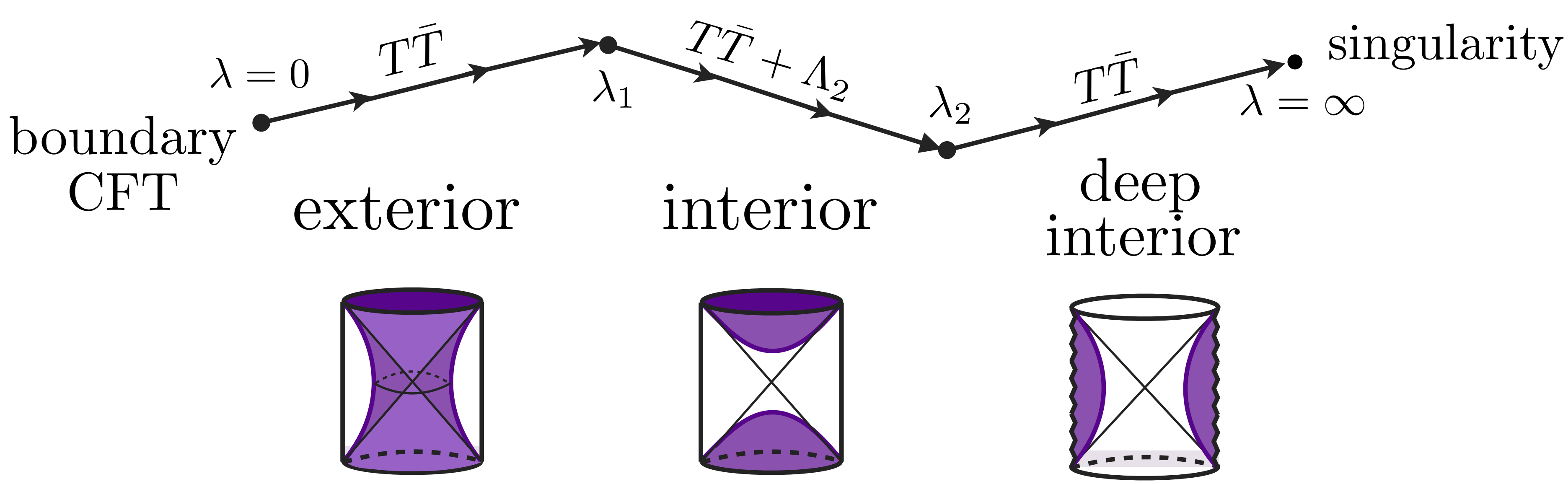
$r = r_c$
r_+
$r = \infty$
$r = r_c$
r_-
r_+
$r = r_c$
r_-
$r = 0$
r_-
r_+
r_+
r_-
r_-
r_+
r_-
r_-
r_+
(a)
(b)
(c)



$E(\lambda)$
I
II
III
λ
λ_1
λ_2
$r = r_c$
r_+
$r = \infty$
$r = r_c$
r_-
r_+
$r = r_c$
r_-
$r = 0$
r_-
r_+
r_+
r_-
r_-
r_+
r_-
r_-
r_+

Summary

- TT deformation of 2d CFT corresponds to 3d AdS spacetimes with finite boundary cutoff outside the BH horizon
- I described a new sequence of generalized TT deformation that appears to push this cutoff inside the BH horizon
- I also provided a bulk gravitational path integral whose saddles reproduces these geometries with finite cutoff

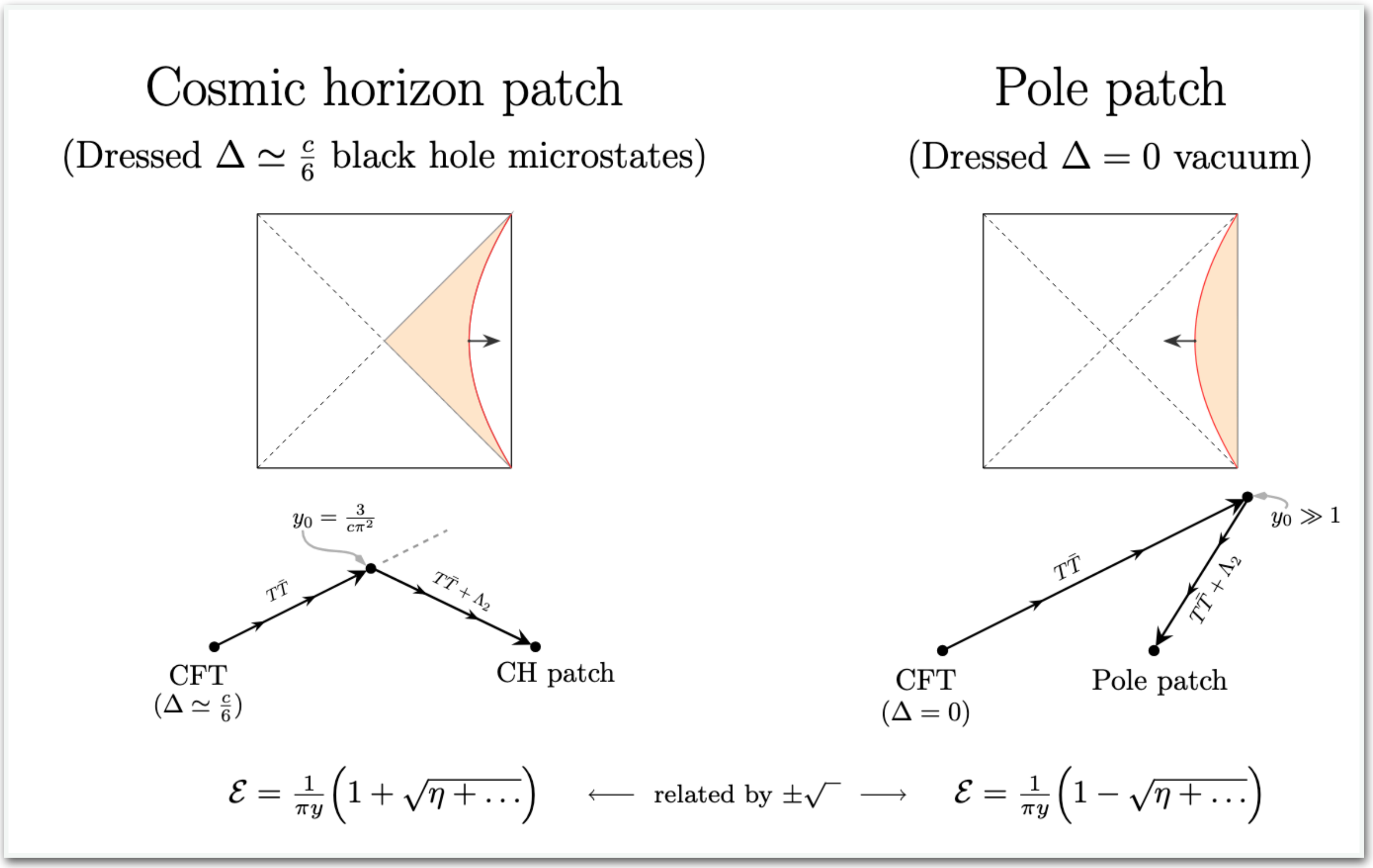


$T\bar{T}$ is surely very interesting…

Comparison with works by Silverstein et. al.

(Backup slides)

- Their claim: $T\overline{T} + \Lambda_2$ flows from AdS -> dS



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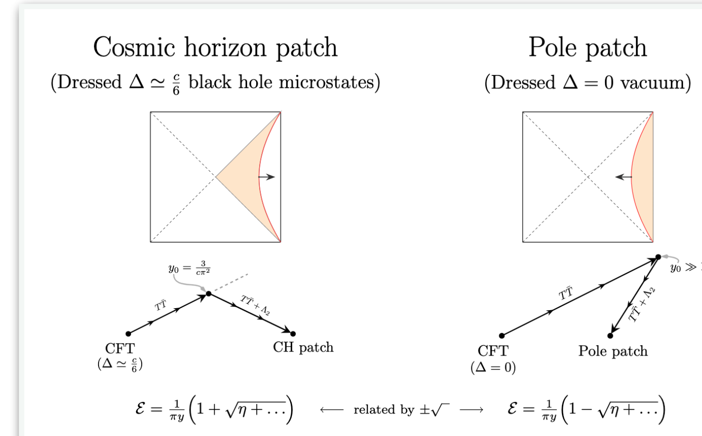
Comparison with works by Silverstein et. al.

(Backup slides)

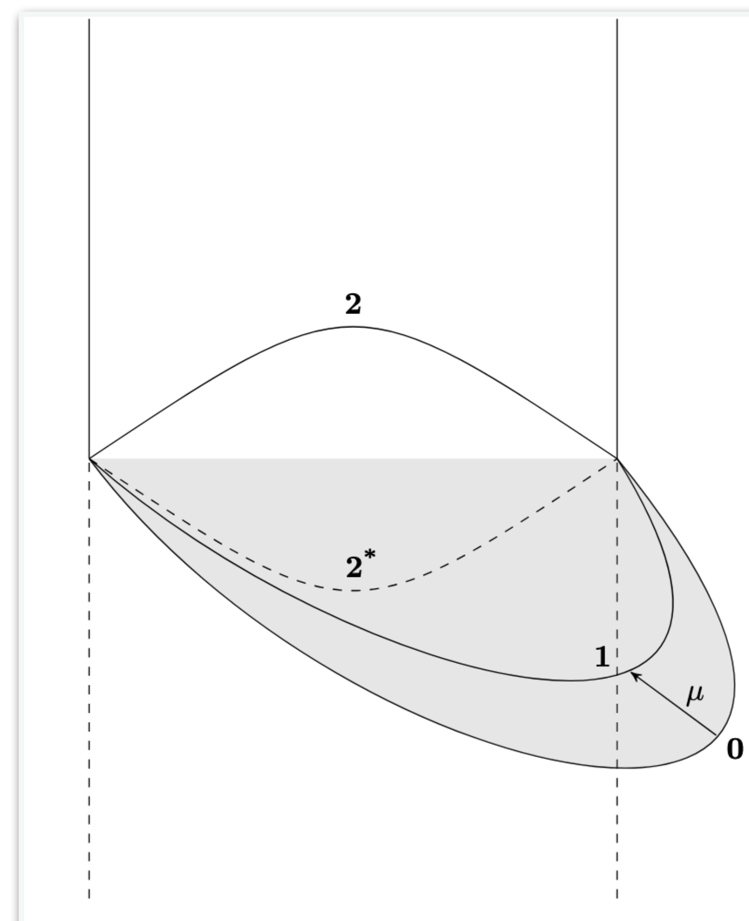


- Differences:
  - Their relative sign between $T\bar{T}$ and Λ_2 is different from ours
  - They did not consider the signature change of γ_{ab}
- Our prescription seems to be more natural, in the sense that
  - They have to (?) match to dS microstates at the horizon
  - Their generalization to higher dims require “uplifting”, which seems very complicated
  - While our proposal only make use of the data from AdS solution

Comparison with Cauchy slice holography [(Araujo-Regado)-Khan-Wall]

(Backup slides)

- Their claim: A $T\overline{T}$ deformation from Euclidean bulk can be "analytically continued" to a Cauchy slice in Lorentzian bulk
  - The construction is more technically different from ours
  - e.g. For BH setup the Cauchy slice contains both the exterior **and** interior
  - We do not have much to say about CSH at this moment



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