Aspects of JT gravity in General Spacetimes

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Based on 2501.02614 with Norihiro Iizuka
and ongoing work with Onkar Parrikar, 2501.xxxx

Recent Developments in Black Holes and Quantum Gravity

January 24, 2025

Introduction

- Jackiw-Teitelboim (JT) gravity is a simple two dimensional model of dilaton gravity. It arises in the near-horizon limit of a large class of near-extremal black holes.
- It is almost a trivial theory with no local propagating degrees of freedom in the bulk and with all the dynamics encoded entirely in the boundary.
- The most important advantage is the analytical tractability leading to exact results.
- It has proven to be a very useful toy model and led to some novel insights in problems such as black hole information paradox, understanding of AdS_2/CFT_1 dictionary, connections to random matrix theory, questions related to gravity path-integrals, those tied to cosmology etc.

Dimensional reduction

- Start with the 4D theory described by the Euclidean action,

$$I = -\frac{1}{16\pi G_4}\left(\int d^4x\,\sqrt{g}\,(R - 2\Lambda_4 + F_{\mu\nu}F^{\mu\nu}) + 2\int d^3x\,\sqrt{\gamma}\,K\right).$$

- Solution corresponding to the Electrically charged RN black hole is

$$ds^2 = f(r)\,dt^2 + \frac{dr^2}{f(r)} + \Phi^2(r)d\Omega_2^2.$$

Here,

$$f(r) = \left(1 - \frac{2G_4M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{L^2}\right),\ \ \Phi(r) = r,\ \ F_{rt} = \frac{Q}{r^2}.$$

- f(r) has a double zero at extremality. The near horizon geometry for an extremal RN black hole is $AdS_2 \times S_2$.

- Taking a spherically symmetric ansatz of the form

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta + \Phi^2 d\Omega_2^2,$$

dimensionally reducing over the S^2 and expanding about the attractor value for $\Phi = \Phi_0(1+\phi)$, leads to the action for JT gravity, given by

$$\begin{aligned} I_{JT} = &-\frac{\Phi_0^2}{4G_4}\left(\int d^2x\,\sqrt{g}\,R + 2\int dx\,\sqrt{\gamma}\,K\right) \\ &-\frac{\Phi_0^2}{2G_4}\left(\int d^2x\,\sqrt{g}\,\phi\Big(R+\frac{2}{L_2^2}\Big) + 2\int dx\,\sqrt{\gamma}\,\phi K\right) + \cdots \end{aligned}$$

Outline

- Motivation
- Non-smooth boundary conditions in AdS
- Wavefunctions for mixed boundaries
- Centaur Geometries
- Conclusions

JT GRAVITY

JT gravity, is a simple model of 2D gravity, involving metric and dilaton.

$$I_{\text{JT}} = \frac{1}{16\pi G}\left(\int d^2x\,\sqrt{-g}\,\phi(R-2\Lambda) - 2\int_{\partial}\sqrt{-\gamma}\phi K\right)$$

Λ - Cosmological constant, ϕ - Dilaton
K - Extrinsic curvature of the boundary

Equations of motion are

$$R = 2\Lambda,$$

$$\nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\nabla^2\phi - \frac{\Lambda}{2}g_{\mu\nu}\phi = 0$$

Spacetime is locally AdS everywhere for $\Lambda = -1$.

Lorentzian AdS

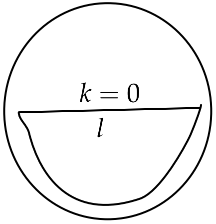
The global AdS solution is

$$
\begin{aligned}
ds^2 &= -(r^2-1)dt^2 + \frac{dr^2}{r^2-1} \\
\phi &= r
\end{aligned}
$$

- Slices of constant extrinsic curvature are spatial slices. So, extrinsic curvature is a potential choice for time in Lorentzian theory.
- Slices of constant extrinsic curvature are referred to as York slices.

- Wavefunctions in Lorentzian theory can be constructed by Euclidean path integral with appropriate boundary conditions.
- Specifically, wavefunction of fixed extrinsic curvature and fixed boundary length can be obtained by doing a Euclidean path integral where part of the boundary is asymptotic and part is extrinsic curvature.

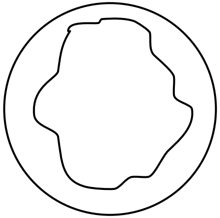
- An interesting observable that is studied is the HH wavefunction [Zhenbin '20]



- However, the matrix model interpretation of this is not yet fully well understood [Iliesiu-Levine-Lin-Maxfield-Mezei, Levine's talk].
- Another class of interesting observables that can be studied are the ones slice in the bulk is a non-zero constant extrinsic curvature slice.
- Requires studying JT gravity with non-smooth boundary conditions.

- Euclidean path integral with asymptotic boundary conditions with dilaton and length of the boundary fixed gives the partition function at finite temperature [Witten-Stanford, Saad-Shenker-Stanford, Moitra-SSK-Trivedi,...]

$Z(\beta) =$



$$= \frac{1}{\beta^{3/2}} e^{\frac{2\pi^2}{\beta}}$$
$$= \int dE \ \sinh\left(2\pi\sqrt{E}\right) e^{-\beta E}$$

$$\phi,\ l \to \infty, \beta = \frac{l}{\phi} = fixed$$

This satisfies the WDW equation

$$\left(\partial_l \partial_\phi - \frac{1}{l}\partial_\phi - l\phi\right)\left(e^{l\phi} Z(\beta)\right) = 0$$

VARIATIONAL PRINCIPLE

- Variation of the Euclidean AdS action for non-smooth boundaries gives,

$$S_{\text{bulk}} = -\frac{1}{2}\int d^2x\sqrt{g}\,\phi\,(R+2)$$
$$\delta S_{\text{bulk},\partial} = \frac{1}{2}\int_\partial \sqrt{h}\,\left[(\phi K - n\cdot\nabla\phi)h_{ab}\delta h^{ab} - 2\phi\,\delta K\right] + \sum_{j\in\text{Corners}} \phi_j\delta\theta_j$$

- Extrinsic curvature is the conjugate momentum to the dilaton. So fixing K corresponds to imposing Neumann boundary condition for the dilaton [Akash-Iliesiu-Kruthoff-Yang' 20.]

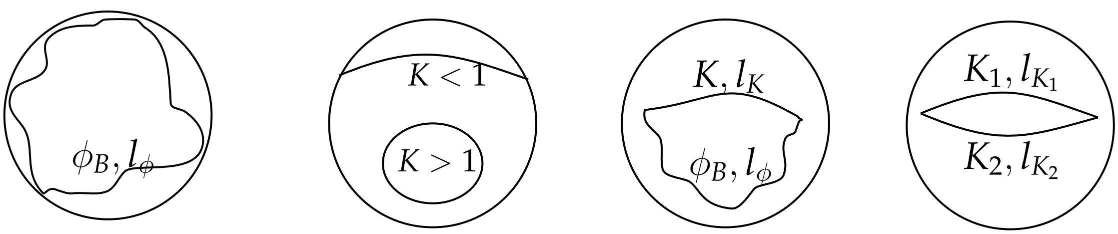
Boundary conditions

$$(a) \qquad S_{\phi,h} = -\frac{1}{2}\int d^2x\sqrt{g}\,\phi\,(R+2) - \int_{\partial}\sqrt{h}\phi K$$

$$(b) \qquad S_{K,h} = -\frac{1}{2}\int d^2x\sqrt{g}\,\phi\,(R+2)$$

$$(c) \quad S_{(\phi,h_\phi),(K,h_K)} = -\frac{1}{2}\int d^2x\sqrt{g}\,\phi\,(R+2) - \int_{\partial_\phi}\sqrt{h}\phi K + \sum_{j\in \text{Corners}} \phi_j\theta_j$$

$$(d) \quad S_{(K_1,h_1),(K_2,h_2)} = -\frac{1}{2}\int d^2\sqrt{g}\,\phi(R+2) + \sum_{j\in \text{Corners}} \phi_j\theta_j$$



- In Poincare coordinates,

$$ds^2 = \frac{dx^2 + dy^2}{y^2}$$

the curves of constant extrinsic curvature look like

$$x(\lambda) = x_0 + \frac{R \sinh(\lambda\sqrt{1-k^2})}{k + \cosh(\lambda\sqrt{1-k^2})}$$

$$y(\lambda) = \frac{R\sqrt{1-k^2}}{k + \cosh(\lambda\sqrt{1-k^2})}$$

- The trajectory is a circle in the UHP,

$$(x - x_0)^2 + \left(y + \frac{Rk}{\sqrt{1-k^2}}\right)^2 = \frac{R^2}{1-k^2}$$

- The length of such a curve between two points $(x_1, y_1), (x_2, y_2)$ is

$$l_k = \frac{1}{\sqrt{1-k^2}} \cosh^{-1}\left[1 + (1-k^2)\left(\frac{(x_1-x_2)^2 + (y_1-y_2)^2}{2y_1y_2}\right)\right]$$

- The appropriate scaling of various quantities for mixed boundaries are

$$\phi_{\partial_A} \equiv \phi_B \sim \frac{\phi_b}{\epsilon}, l_{\partial_A} \sim \frac{\beta}{\epsilon}$$
$$K_{\partial_K} = k, l_{\partial_K} \sim \frac{2}{\sqrt{1-k^2}} \log\left(\frac{L\sqrt{1-k^2}}{\epsilon}\right)$$

On-shell wavefunction

- For mixed boundary, the contribution comes from the extrinsic curvature and the corner terms

$$S_\partial = -\int_{\partial_A} dx\,\sqrt{\gamma}\phi(K-1) + \sum_{j\in\text{corners}} \phi_j\,\theta_j$$

leading to

$$\int_{\partial_A} \phi(K-1)\sqrt{\gamma}du = \frac{1}{2}\phi_b r_s^2 \beta$$

$$\phi_B\theta \simeq \phi_b r_c \arccos(k) + 2\phi_b r_s \cot(\frac{\beta r_s}{2})$$

- The on-shell wavefunction is

$$\Psi_A = e^{-S_{\text{on-shell}}} = \exp(\frac{1}{2}\phi_b r_s^2 \beta - 2\phi_b r_c \arccos(k) + 2\phi_b r_s \cot(\frac{\beta r_s}{2}))$$

$$L = \frac{2}{r_s}\sin(\frac{\beta r_s}{2}), \qquad r_c \sim \epsilon^{-1}$$

- For the York boundaries, the contribution entirely comes from the corner angles,

$$S_{\partial} = \sum_{j \in \text{Corners}} \phi_j \theta_j$$

with the value of corner angle being,

$$\theta = \arctan\left(\frac{k_1}{\sqrt{(1-k_1^2)}} \tanh\left(\frac{1}{2} l_1 \sqrt{1-k_1^2}\right)\right) + (k_1, l_1 \to k_2, l_2)$$

The wavefunction is

$$\Psi_Y = e^{-2\phi_B \theta}$$

Quantum wavefunction

- Easier to compute the wavefunction for various boundary conditions using the boundary particle formalism[Kitaev-Suh,Zhenbin].
- The dilaton integral sets $R = -2$ everywhere.
- Use the Gauss-Bonnet theorem which relates the extrinsic curvature and Ricci scalar to Euler character

$$\int_{\partial_A} K + \int_{\partial_K} K + \sum_i (\pi - \theta_i) = 2\pi\chi - \frac{1}{2}\int \sqrt{g}\, R$$

- The next step is to write the bulk term, integral of a top-form, as a boundary term

$$\int \sqrt{g} = \int_{\partial} a$$

- The boundary action, say for the case of Dirichlet boundary, becomes

$$S_\partial = \phi_B \left(l_A - \int_{\partial_A} a \right)$$

- Can be interpreted as the worldline action for a particle in an electric field with gauge field a.
- The partition function can be obtained by computing the propagator of the particle with given length.

$$Z(\beta) = G(\beta, \mathbf{x}, \mathbf{x})$$

- The exact propagator is given by [Comtet,Houston]

$$G(u, \mathbf{x_1}, \mathbf{x_2}) = e^{i\varphi(\mathbf{x_1},\mathbf{x_2})}\tilde{K}(u, \mathbf{x_1}, \mathbf{x_2})$$

$$\tilde{K}(u, \mathbf{x_1}, \mathbf{x_2}) = \int ds s \frac{\sinh(2\pi s)}{2\pi(\cosh(2\pi s) + \cosh(2\pi q))} \frac{e^{-\frac{us^2}{2}}}{d^{1+2is}} \; {}_2F_1\left(\frac{1}{2} - iq + is, \frac{1}{2} + iq + is, 1, 1 - \frac{1}{d^2}\right)$$

$$d = \sqrt{\frac{(x_1 - x_2)^2 + (y_1 + y_2)^2}{4y_1y_2}}$$

$$e^{i\varphi(\mathbf{x_1},\mathbf{x_2})} = e^{2q\arctan\left(\frac{x_2 - x_1}{y_1 + y_2}\right)}$$

$$q = \phi_B,$$

- In the limit of large $q = \phi_B$, the propagator gives the partition function

$$G(u, \mathbf{x}, \mathbf{x}) = \int ds\, s\, \sinh(2\pi s) e^{-\frac{us^2}{2}}$$

MIXED BOUNDARY

- The particle action in this case becomes

$$S_\partial = \phi_B \left(l_A - \int_{\partial_A} a \right) + \phi_B \left(k l_K - \int_{\partial_K} a \right)$$

- The full wavefunction is then a product of two propagators

$$\Psi_A = \psi_A \psi_K$$

- The asymptotic boundary contribution is

$$\psi_A = G(u, \mathbf{x_1}, \mathbf{x_2})$$

- In the large q limit, of the hypergeometric function becomes

$$\frac{1}{d^{1+2is}} {}_2F_1\left(\frac{1}{2} - iq + is, \frac{1}{2} + iq + is, 1, 1 - \frac{1}{d^2}\right) \xrightarrow{q\to\infty} \frac{e^{\pi q}}{\pi d} K_{2is}\left(\frac{2}{d_\infty}\right)$$

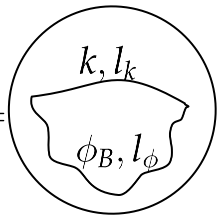
leading to

$$\psi_A \simeq \frac{1}{d} e^{i\varphi(\mathbf{x_1},\mathbf{x_2}) - \pi\phi_B} \int ds\, s \sinh(2\pi s) e^{-\frac{us^2}{2}}\, K_{2is}\left(\frac{2}{d_\infty}\right)$$

- For the extrinsic curvature boundary

$$\psi_K = e^{-\phi_B k l_k + \phi_B \int_{\partial_K} a}$$

Putting together

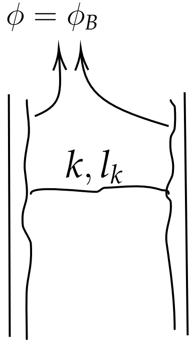
$$\Psi_A(\phi_B, l_\phi, l_k, k) =$$ 

$$= \quad e^{-2\phi_B \sin^{-1}(k)} \int dE\ \sinh\left(2\pi\sqrt{E}\right) e^{-\beta E} K_{2i\sqrt{E}}\left(\frac{2}{d_\infty}\right)$$

$$d_\infty = \frac{1}{\phi_B}\frac{1}{\sqrt{1-k^2}} e^{\frac{1}{2}l_k\sqrt{1-k^2}} = \frac{L}{\phi_b}$$

- The Bessel function is related to the energy eigenstates of the Lorentzian theory.

LORENTZIAN SETUP



- The ADM Hamiltonian of the Lorentzian theory is

$$H = H_L + H_R = \frac{P_k^2}{(1+k^2)} + \frac{4}{1+k^2} e^{-\tilde{l}_k \sqrt{1+k^2}}$$

- Eigenstates of fixed energy

$$\hat{H}\Psi(L,k) = E\Psi(L,k) \Rightarrow \Psi_E(L,k) = K_{2i\sqrt{E}}\left(\frac{2\phi_b}{L(ik)}\right)$$

- Leads to a nice interpretation for the euclidean results.

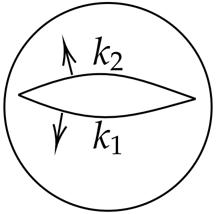
$$\Psi_A = \quad e^{-2\phi_B \sin^{-1}(k)} \int dE \sinh\left(2\pi\sqrt{E}\right) e^{-\beta E} K_{2i\sqrt{E}}\left(\frac{2\phi_b}{L}\right)$$

$$\underbrace{\sinh\left(2\pi\sqrt{E}\right)}_{\rho(E)} \qquad \underbrace{K_{2i\sqrt{E}}\left(\tfrac{2\phi_b}{L}\right)}_{\langle l_k|E\rangle}$$

Two Extrinsic curvature boundaries

In terms of the particle picture, the action is given by

$$S_\partial = \phi_B \left(k_1 l_1 - \int_{\partial_1} a \right) + \phi_B \left(k_2 l_2 - \int_{\partial_2} a \right)$$



$$\Psi_Y = \delta(d_{\infty,1}^2 - d_{\infty,2}^2) \exp[2\phi_B \tan^{-1}\left(\frac{k_1}{\sqrt{1-k_1^2}} \tanh\left(\frac{l_1}{2}\sqrt{1-k_1^2} \right) \right) + 2\phi_B \tan^{-1}\left(\frac{k_2}{\sqrt{1-k_2^2}} \tanh\left(\frac{l_2}{2}\sqrt{1-k_2^2} \right) \right)]$$

$$d_{\infty,i} = \frac{1}{\phi_B}\sqrt{1 + \frac{1}{1-k_i^2}\sinh^2\left(\frac{1}{2} l_i \sqrt{1-k_i^2} \right)}$$

CHECKS

- Reproduces the classical wavefunction in the classical limit. Need to use the integral representation of the Bessel function.

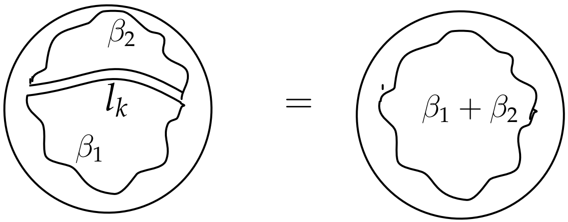
$$K_\alpha(x) = \int_{-\infty}^{\infty} d\xi e^{-x\cosh\xi}\cosh(\alpha\xi)$$

- Simple manipulations lead to

$$\frac{e^{-2\phi_B \arccos(k)}}{\pi^2 d}\int ds\, s\sinh(2\pi s)\, e^{-\frac{us^2}{2}}\; K_{2is}\left(\frac{2}{d_\infty}\right) = \frac{e^{-2\phi_B \arccos(k)}}{\pi^2 d}\frac{\sqrt{2\pi}}{u^{3/2}}\int_{-\infty}^{\infty} d\xi\,(\pi + i\xi)e^{-\frac{2}{d_\infty}\cosh\xi}e^{\frac{2(\pi+i\xi)^2}{u}}$$
$$= -\frac{1}{2}\frac{r_s\beta}{\pi^2 d}\frac{1}{u^{3/2}}\exp\left(\frac{1}{2}\phi_b r_s^2\beta + 2\phi_b r_s\cot(\frac{\beta r_s}{2}) - 2\phi_b r_c \arccos(k)\right)$$

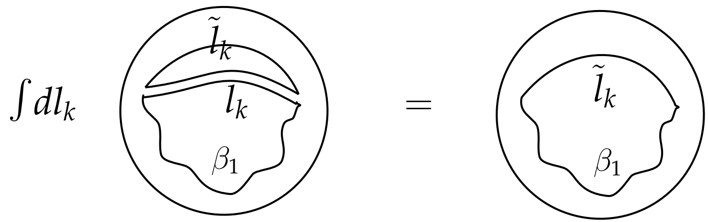
Convolution - Two mixed boundaries

$\int dl_k$



$$
\begin{aligned}
\int d(d_\infty^2)\Psi_A(\phi, l_\phi, k, l_k)\Psi_A(\phi, l_\phi, -k, l_k) &\propto \int ds_1 ds_2 \rho(s_1)\rho(s_2)\frac{\delta(s_1 - s_2)}{\rho(s_1)} e^{-u_1 s_1^2 - u_2 s_2^2} \\
&= \int ds \rho(s) e^{-(u_1+u_2)s^2} \\
&= Z_{\text{AdS}}(l_1 + l_2, \phi)
\end{aligned}
$$

Mixed boundary+ York boundary



$$\int d(d_\infty^2)\Psi_A(\phi, l_\phi, k, l_k)\Psi_Y(k, l_k, \tilde{k}, \tilde{l}_k) = \Psi_A(\phi, l_\phi, \tilde{k}, \tilde{l}_k)$$

Constraints

- Can check if the states constructed earlier satisfies constraint equations.
- The WDW equation can be obtained by working in the ADM-like gauge

$$ds^2 = N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

- The local WDW equation is

$$\frac{\delta S}{\delta N} \equiv \mathcal{H} = \sqrt{h}(-\phi - D^2\phi + Kn^\alpha \nabla_\alpha \phi)$$

Asymptotic boundary

- Conjugate momenta for the dilaton and boundary metric are

$$\pi_\phi = -\sqrt{h}K$$
$$\pi_h^{ij} = -\frac{\sqrt{h}}{2}h^{ij}n^\alpha\nabla_\alpha\phi$$

- The WDW equation becomes

$$\mathcal{H} = \frac{2}{\sqrt{h}}h_{ij}\pi_h^{ij}\pi_\phi - \sqrt{h}D^2\phi - \sqrt{h}\phi$$

- For non-smooth boundaries, the zero mode part of the WDW constraint get an extra contribution from the corners

$$\mathcal{H}_0^{(\phi)} = \pi_l\pi_\phi - l\phi - \sum_{j\in \text{corners}} r_j \cdot \nabla\phi$$
$$= \pi_l\pi_\phi - l\phi$$

York Boundary

- Conjugate momenta for the extrinsic curvature slice

$$\pi_K = \sqrt{h}\phi$$
$$\pi_h^{ij} = -\frac{\sqrt{h}h^{ij}}{2}(n^\alpha \nabla_\alpha \phi - \phi K)$$

- For the extrinsic curvature boundary, the corner contribution is important.

$$\mathcal{H}_0 = -(1-k^2)\pi_k - kl\pi_l - \sum_{j \in \text{corners}} r_j \cdot \nabla\phi$$
$$= -(1-k^2)\pi_k - kl\pi_l - \sum_{j \in \text{corners}} r_j \cdot \nabla\left(\frac{\pi_K}{\sqrt{h}}\right)$$

- The wavefunctions discussed above satisfy this modified WDW constraint, with the corner terms included.

Bulk Physical time

- The length states $|l,k\rangle$ form a Hilbert space at the instant of time k. Can be thought of as obtained by an evolution of the states at $k=0$ by the physical Hamiltonian H_{phy},

$$|l,k\rangle = e^{-i\int_0^k H_{\text{phy}}}|l,k=0\rangle$$

- In the Lorentzian theory

$$\langle E|l_k\rangle = \frac{1}{\tilde{d}_\infty}e^{2iq\sinh^{-1}k}K_{2i\sqrt{E}}\left(\frac{2}{\tilde{d}_\infty}\right), \tilde{d}_\infty = \frac{1}{\phi_B\sqrt{1+k^2}}e^{\frac{1}{2}l_k\sqrt{1+k^2}}$$

- Time evolution with respect to physical Hamiltonian can be obtained by

$$\langle E|l, k+\delta k\rangle = (1 - i\,\delta k\,H_{\text{phys}})\langle E|l, k\rangle$$

  which gives

$$H_{\text{phy}} = i\partial_k = \frac{lk}{1+k^2} i\partial_l - \frac{2q}{\sqrt{1+k^2}}$$

- It is just a rewriting of the WDW equation

$$\left(\pi_k - \frac{lk}{1+k^2}\pi_l - \frac{2q}{\sqrt{1+k^2}}\right)|l, k\rangle = 0$$

General potential

- The action for a more general potential is

$$I_{\mathrm{JT}} = \frac{1}{2}\left\{\int d^2x\,\sqrt{-g}\,(\phi R - U(\phi)) - 2\int_{bdy}\sqrt{-\gamma}\phi K\right\}$$

which give rise to the equation of motion

$$R - U'(\phi) = 0$$

$$\nabla_\mu \nabla_\nu \phi - g_{\mu\nu}\nabla^2\phi - \frac{1}{2}g_{\mu\nu}U(\phi) = 0$$

- Equations of motion guarantee the existence of a conserved quantity and a Killing vector.
- Conserved quantity is

$$M = \nabla_\mu \phi \nabla^\mu \phi + W(\phi), \quad W(\phi) = \int_0^\phi U(x) dx$$

- Killing vector

$$\xi^\mu = \epsilon^{\mu\nu} \nabla_\nu \phi$$

- The norm of the Killing vector is

$$\xi^\mu \xi_\mu = W(\phi) - M$$

- The most general metric can be written as

$$ds^2 = -\frac{dr^2}{W(r) - M} + (W(r) - M) dx^2, \qquad \phi = r$$

- We will be especially interested in our discussion of the quantum theory in what happens for potentials which asymptote, for large $-\phi$, to the AdS form, i.e. where

$$W(\phi) \to -\phi^2 \quad \text{as} \quad \phi \to \infty$$

- Of particular interest will be the geometries which have AdS in the UV and interpolate to dS spacetime in the IR
- The case of AdS and dS correspond to the potential [Anninos,Galante, Hofman'18, Anninos, Harris'22]

$$U_{\text{AdSdS}}(\phi) = \begin{cases} 2\phi & \text{AdS} \\ -2\phi & \text{dS} \end{cases}$$

- For a geometry with a horizon at $r = r_h$, the thermodynamic quantities are given by

$$T = \frac{U(r_h)}{2\pi}, \qquad S = 2\pi r_h$$

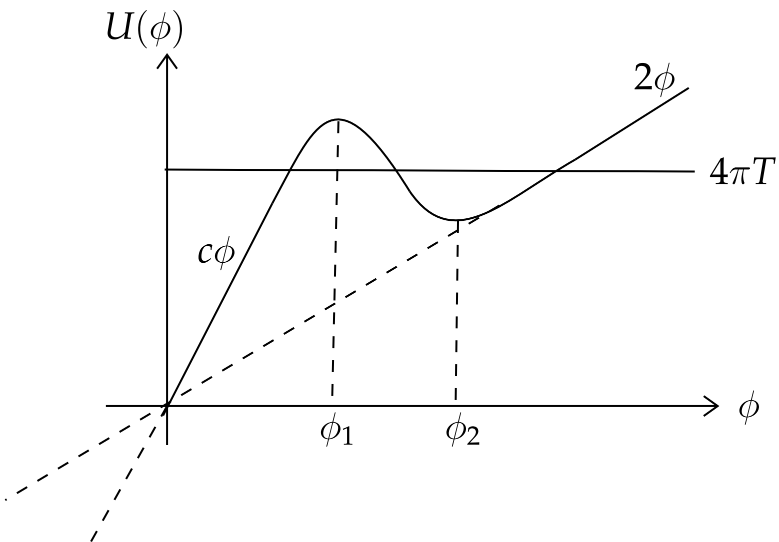
- Thermodynamic stability related to specific heat

$$C \equiv \frac{dE}{dT} \propto \frac{dE}{dr_h} \Big/ \frac{dT}{dr_h} \propto \frac{U(r_h)}{U'(r_h)} > 0\,.$$

Thus, the horizon in dS has negative specific heat.

- Thus, we shall consider the potential of the qualitative form as Fig. 1 and it can be modeled crudely as below [Anninos,Harris'22]

$$U(\phi) = \begin{cases} c\,\phi & \phi < \phi_1, \qquad (c > 2) \\ U_0 - \alpha\,\phi & \phi_1 < \phi < \phi_2 \\ 2\,\phi & \phi_2 < \phi \end{cases}$$



CANONICAL QUANTIZATION

- Working in the ADM gauge with radial coordinate r playing the role of time,

$$ds^2 = N^2 dr^2 + g_1 (dx + N_\perp dr)^2$$

- Hamiltonian and Momentum constraints become

$$0 = \mathcal{H} \equiv \frac{\delta I_{\text{JT}}}{\delta N} = 2\pi_\phi \pi_{g_1} \sqrt{g_1} + \left(\frac{\phi'}{\sqrt{g_1}} \right)' - \frac{1}{2}\sqrt{g_1} U$$

$$0 = \mathcal{P} \equiv \frac{\delta I_{\text{JT}}}{\delta N_\perp} = 2 g_1 \pi'_{g_1} + \pi_{g_1} g'_1 - \pi_\phi \phi'$$

[cf. Sandip's talk]

Solutions

- On a slice of constant dilaton, $\phi' = 0$, general solutions can be written as

$$\Psi(\phi) = \int dM \left(\rho_+(M) e^{\ell\sqrt{W(\phi)-M}} + \rho_-(M) e^{-\ell\sqrt{W(\phi)-M}} \right) .$$

- For Euclidean AdS_2 the coefficients functions are

$$\rho_+(M) = \sinh(2\pi\sqrt{M}) , \quad \rho_-(M) = 0 .$$

On-shell action

- The on-shell value of the action for fixed length and value of the dilaton on the boundary in the asymptotic limit,

$$l \gg 1\,, \quad W(\phi_b) \gg 1\,, \quad \frac{l}{W(\phi_b)} = \text{fixed}\,,$$

  is given by

$$Z = e^{-I_{\text{on-shell}}} = e^{-\frac{\beta M_*}{2} + 2\pi\phi_h} \equiv e^{-\beta E_* + 2\pi\phi_h}$$

  where M_* should be thought of as a function of temperature, given by

$$M_* = \int_0^{r_h} U(r), \quad r_h = U^{-1}(4\pi T)$$

Centaur Wavefunction at fixed T

- The location of the horizon is obtained by solving for ϕ_h

$$\phi_h = \begin{cases} \frac{4\pi T}{c} & \phi_h < \phi_1 \\ 2\pi T & \phi_h > \phi_2 \end{cases}$$

- The partition function at finite temperature, obtained by evaluating the on-shell action, is given by

$$\begin{aligned} Z(\beta) &= \theta(\beta - \beta_c) e^{-\beta F_1} + \theta(\beta_c - \beta) e^{-\beta F_2} \\ F_1 &= -\frac{4\pi^2}{c\beta^2} \\ F_2 &= -\frac{2\pi^2}{\beta^2} + \frac{2\pi^2}{\beta_c^2}\left(1 - \frac{2}{c}\right) \\ T_c &= \frac{1}{2\pi}\sqrt{\frac{c\phi_1\phi_2}{2}} \end{aligned}$$

- The entropy and the density of states are given by

$$S = \begin{cases} 2\pi\sqrt{\frac{2M}{c}} & T < T_c \\ 2\pi\sqrt{M - 4\pi^2\left(1 - \frac{2}{c}\right) T_c^2} & T > T_c \end{cases}$$

and so

$$\rho \approx \begin{cases} e^{2\pi\sqrt{\frac{2M}{c}}} & \text{for small } M \\ e^{2\pi\sqrt{M - \tilde{M}_0}} & \text{for large } M \end{cases}$$

- The density of states show that the number of degrees of freedom are decreased due to the presence of IR dS bubble.

Summary

- Natural to use York time in AdS. Wavefunctions specified by constant extrinsic curvature can be constructed from Euclidean path integrals.
- Constructed wavefunction corresponding to a general class of observables
- Carried out some basic checks of the wavefunctions, matching with the on-shell calculation in the classical limit, taking inner product to give expected results.
- The wavefunctions satisfy a modified WDW equation where the corner terms play an important role.
- Explored Centaur geometries with a dS bubble embedded inside AdS and computed density of states in WKB limit.

Some Open questions

- Understand how to compute the mixed boundary wavefunctions in the second order formalism directly.
- Compute the mixed boundary wavefunction away from the asymptotic region.
- Study the implications of the modified $T\bar{T}$ equation.
- Relation to Cauchy slice holography? [Onkar-Kruthoff-Pawel, Goncalo-Rifath-Wall,..]
- Compute the full path integral for Centaur geometry and obtain the full Density of States.
- Relation to deformed SYK models? [cf. Shira's talk]

THANKS