Dissipation in the $1/D$ expansion for planar matrix models

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collaboration with Norihiro Iizuka and Daniel Kabat

The main result

- We'll consider the QM of a large number D of $N\times N$ Hermitian matrices

$$S = \int d\tau \, \frac{1}{2}\mathrm{Tr}\left(\partial_\tau X^i \partial_\tau X^i\right) + \frac{1}{2}m_0^2 \,\mathrm{Tr}\left(X^i X^i\right) + \frac{1}{2}g_A^2 \,\mathrm{Tr}\left(X^i X^i X^j X^j\right) - \frac{1}{2}g_C^2 \,\mathrm{Tr}\left(X^i X^j X^i X^j\right)$$

(Euclidean action)

Matrices $X^i_{AB}(\tau)$ $\quad i = 1, \dots, D$

$A, B = 1, \dots, N$

This is usually hard to solve.

However, we will consider the following limit

- First, taking large N = planar limit.
- After that, taking a large D limit with a proper 't Hooft coupling fixed.

This leads to perturbation in $1/D$ and we can solve = compute a thermal correlator !

Matrix Model is important

As you know well, the matrix model has various intersections with gravity

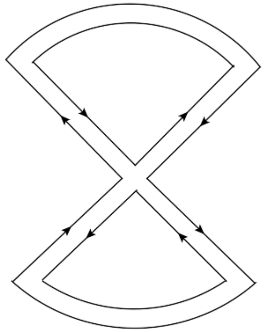
- **'t Hooft expansion** ['74 t'hooft]

  The clue for Gauge/Gravity. 't Hooft suggested taking a large number of colors $N \to \infty$ with 't Hooft coupling fixed and doing $1/N$ expansion. This expansion has the same structure as string theory.

- The essence of this discussion is that the field is a **matrix**.
- Let us consider the vacuum bubble of $\mathcal{L} = -\frac{1}{g^2}\mathrm{Tr}\left[\frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi + \frac{1}{4}\Phi^4\right]$
- If we define $\lambda = g^2 N$, N dependence are

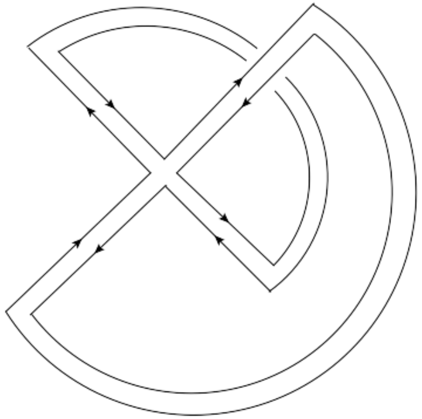
Φ_{ab}

Matrix element index



$= O(N^2)$

Planer (Leading)



$= O(N^0)$

Non-planer (subleading)

Double line notation

Matrix Model is important

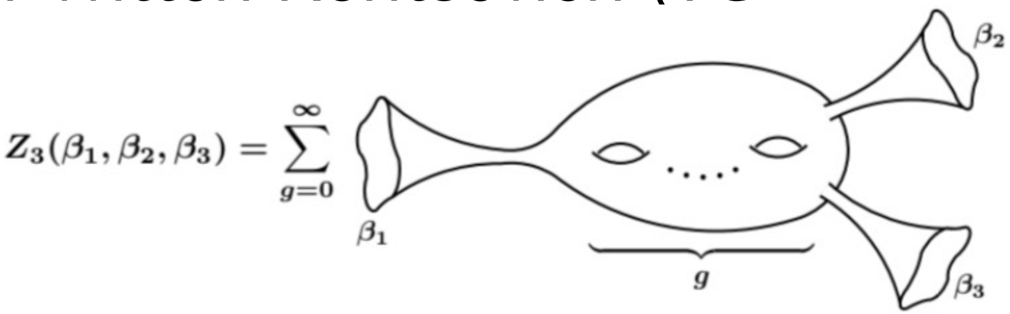
And many important works…

- **SSS** ['91 Witten, '92 Kontsevich] ['19 Saad, Shenker, Stanford]

JT gravity/GUE with proper potential correspondence. PF matches.

genus expansion in JT = topological expansion in RMT.

This can be regarded as a special case of Witten-Kontsevich (TG = DSRMT).



- **Chaotic system and RMT**

In chaotic systems, the energy level spacing is expected to obey a Wigner-Dyson distribution, which is a characteristic behavior in RMT.

Commutator squared potential

In particular, (commutator)^2 potential is important like

BFSS Matrix Model $L = \frac{1}{2g^2}\left[\mathrm{Tr}\left\{\frac{1}{2}(D_\tau X_i)^2 - \frac{1}{4}[X_i, X_j]^2\right\} + \text{fermion part}\right]$

$X^i_{AB}(\tau)$ $i = 1, \ldots, 9$, $A, B = 1, \ldots, N$

Dynamics for a collection of D0-brane

Dimensional reduction of $U(N)$ Yang-Mills theory from 10 to 1 dim

- Developing the solving method for this type of matrix model is important from the non-perturbative aspect of the string theory.
- However, the multi-matrix model is hard to solve in general. In fact, there are many references to attack this.
- e.g. Bootstrap (Numerically, @large N). The main focus is often ⋯

$$H = \mathrm{tr}\left(P_X^2 + P_Y^2 + m^2(X^2 + Y^2) - g^2[X, Y]^2\right)$$

['20 Han, Hartnoll, Kruthoff]
(with fermion part) ['23 H. W. Lin]

Our Model ['24 TA, Iizuka, Kabat]

$$S = \int d\tau \, \frac{1}{2}\mathrm{Tr}\left(\partial_\tau X^i \partial_\tau X^i\right) + \frac{1}{2}m_0^2 \,\mathrm{Tr}\left(X^i X^i\right) + \frac{1}{2}g_A^2 \,\mathrm{Tr}\left(X^i X^i X^j X^j\right) - \frac{1}{2}g_C^2 \,\mathrm{Tr}\left(X^i X^j X^i X^j\right)$$

(Euclidean action)

$(1+0)$ dim QM of a collection of $N \times N$ Hermitian matrices.

First term is Kinetic one, second term is mass one.

- Hidden index exists. X^i_{AB}

  i runs from 1 to D (the number of matrix).

  A, B runs from 1 to N (the size of matrix).

- m_0 is bare mass and there are two types of coupling g_A and g_C. These have a different role in this model.

Our Model - Motivation

$$S = \int d\tau \, \frac{1}{2}\mathrm{Tr}\left(\partial_\tau X^i \partial_\tau X^i\right) + \frac{1}{2} m_0^2 \,\mathrm{Tr}\left(X^i X^i\right) + \frac{1}{2} g_A^2 \,\mathrm{Tr}\left(X^i X^i X^j X^j\right) - \frac{1}{2} g_C^2 \,\mathrm{Tr}\left(X^i X^j X^i X^j\right)$$

- If we set $g_A = g_C = g_{YM}$, this reduces to

$$S = \int d\tau \, \frac{1}{2}\mathrm{Tr}\left(\partial_\tau X^i \partial_\tau X^i\right) - \frac{1}{4} g_{YM}^2 \,\mathrm{Tr}\left([X^i, X^j]^2\right)$$

- This model has a BFSS-like potential. In general, it's hopeless to solve this dynamics.

We consider the following two limits,

- First, we take large N limit = only planar diagrams contribute.
- Second, we take large D limit (with some fixed quantity) = Hopeful.

t' Hooft coupling

$$S = \int d\tau \, \frac{1}{2}\mathrm{Tr}\left(\partial_\tau X^i \partial_\tau X^i\right) + \frac{1}{2} m_0^2 \, \mathrm{Tr}\left(X^i X^i\right) + \frac{1}{2} g_A^2 \, \mathrm{Tr}\left(X^i X^i X^j X^j\right) - \frac{1}{2} g_C^2 \, \mathrm{Tr}\left(X^i X^j X^i X^j\right)$$

- More concretely, we consider the following limit.

$$\lambda_A = g_A^2 N \to 0 \quad \text{with} \quad \tilde{\lambda}_A = \lambda_A D \quad \text{fixed}$$

$$\lambda_C = g_C^2 N \to 0 \quad \text{with} \quad \tilde{\lambda}_C = \lambda_C D \quad \text{fixed}$$

- As we will see later, By setting $g_C = 0$ and taking our double-scaled limit, calculating the correlator comes down to a similar one of the $O(D)$ vector model.

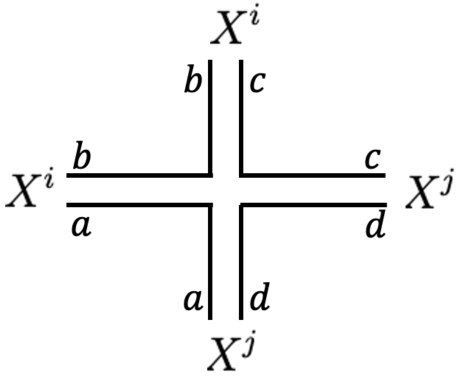
(Note that there is an essential difference in whether fields are commutable or not.)

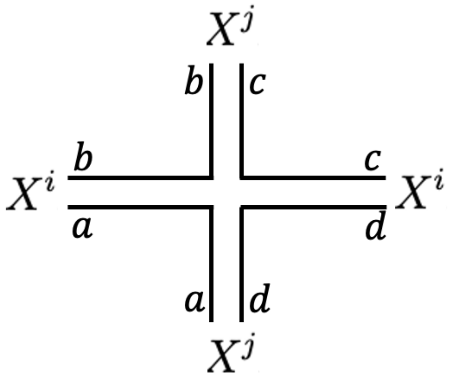
Leading contribution

$$S = \int d\tau \, \frac{1}{2} \mathrm{Tr}\left(\partial_\tau X^i \partial_\tau X^i\right) + \frac{1}{2} m_0^2 \, \mathrm{Tr}\left(X^i X^i\right) + \frac{1}{2} g_A^2 \, \mathrm{Tr}\left(X^i X^i X^j X^j\right) - \frac{1}{2} g_C^2 \, \mathrm{Tr}\left(X^i X^j X^i X^j\right)$$

- Now this model has the following two couplings.

$$+\frac{1}{2} g_A^2 \, \mathrm{Tr}\left(X^i X^i X^j X^j\right) \qquad -\frac{1}{2} g_C^2 \, \mathrm{Tr}\left(X^i X^j X^i X^j\right)$$

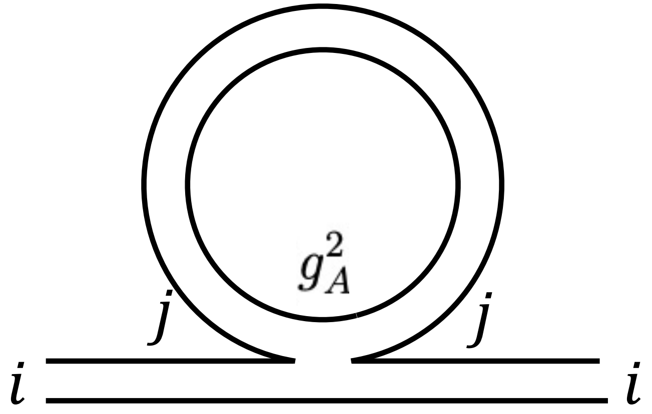




- Today, let us devote ourselves to computing the $1/D$ correction to the correlation.

Leading contribution

- First, let us consider the leading contribution.
- Only planar diagrams survive since we take large N.



$g_A^2 N D = \tilde{\lambda}_A$

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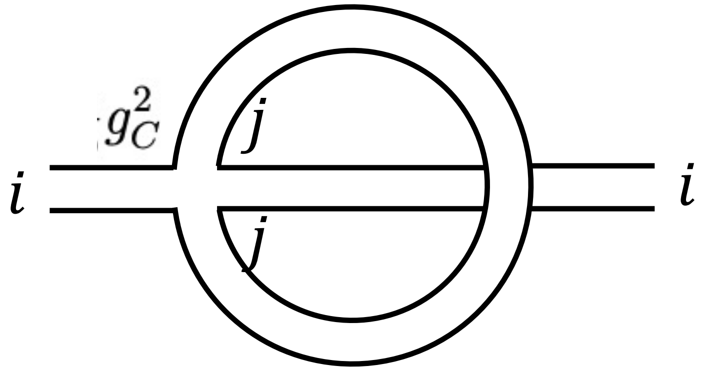
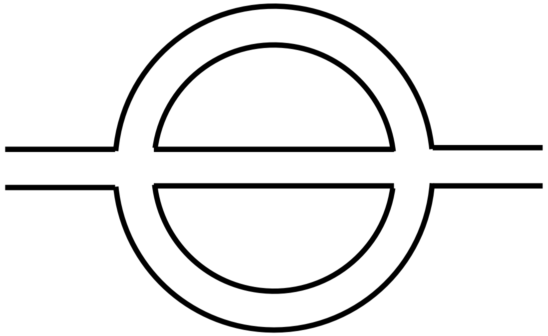


$g_C^2 D = \frac{1}{N}\tilde{\lambda}_C$



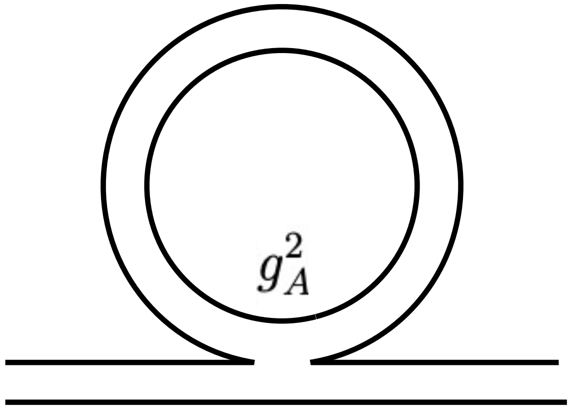
Planar but

$(g_C^2)^2 N^2 D = \frac{\tilde{\lambda}^2}{D}$

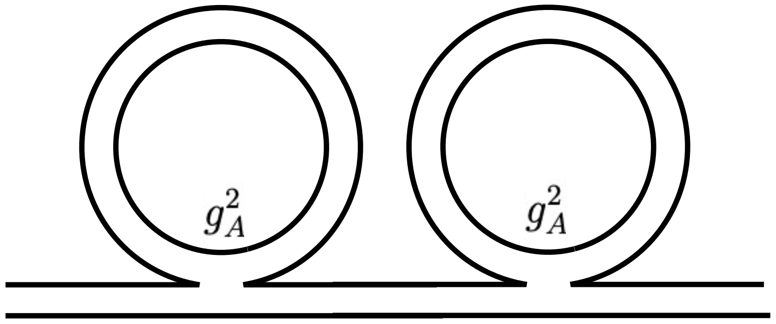
Single line loop = N (size of matrix) choices = $O(N)$
Double line loop = D(the # of matrix) choices = $O(D)$

Leading contribution

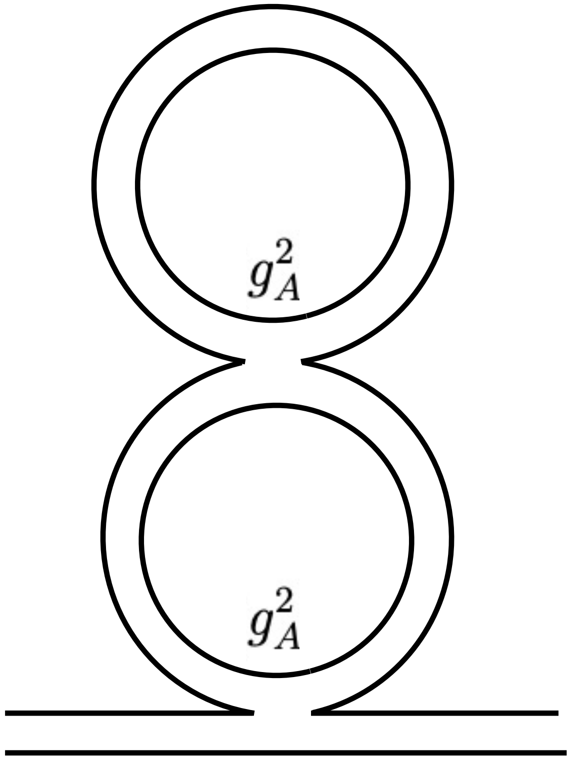
- After trial and error, the following types of diagrams contribute in leading order $O(N^0 D^0)$.



$g_A^2 ND = \tilde{\lambda}_A$



$(g_A^2 ND)^2 = \tilde{\lambda}_A^2$



$(g_A^2 ND)^2 = \tilde{\lambda}_A^2$

Only a series of snail diagrams contribute.

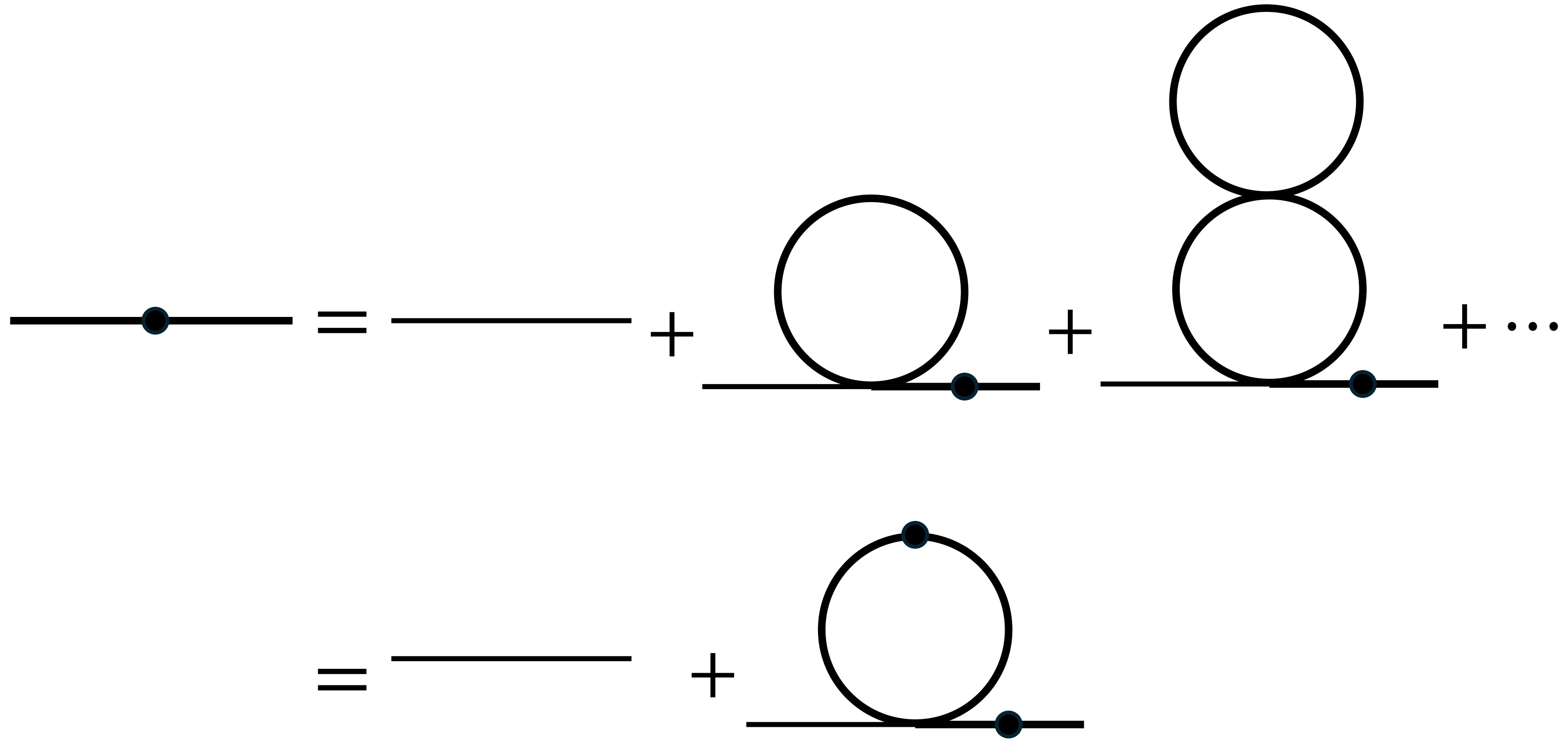
From now on, only planar diagrams will be considered at all times. Therefore, the double line notation can be eliminated.

Leading contribution

SD equation for leading order

$$G(\omega) = G_0(\omega) - c_L \tilde{\lambda}_A G_0(\omega) G(\omega) \int \frac{d\omega'}{2\pi} G(\omega')$$

$c_L = 2$
is a coefficient



Leading contribution

SD equation for leading order

$$G(\omega) = G_0(\omega) - c_L \tilde{\lambda}_A G_0(\omega) G(\omega) \int \frac{d\omega'}{2\pi} G(\omega')$$

- Let us assume $G_0 = \frac{1}{\omega^2 + m_0^2}, \quad G = \frac{1}{\omega^2 + m_1^2}$ $\qquad c_L = 2$
- And 1-loop integral can be performed

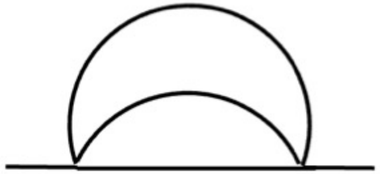
$$\int d\omega' G(\omega') = \int d\omega' \frac{1}{\omega'^2 + m_1^2} = \frac{\pi}{m_1}$$

- Therefore, the mass of leading two-point function becomes

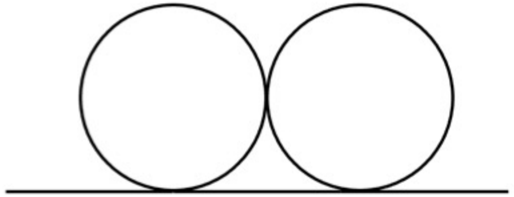
$$G_0(\omega)^{-1} = G(\omega)^{-1} - \frac{\tilde{\lambda}_A}{m_1} \quad \Rightarrow \quad m_1^2 = m_0^2 + \frac{\tilde{\lambda}_A}{m_1}$$

The correction from g_A

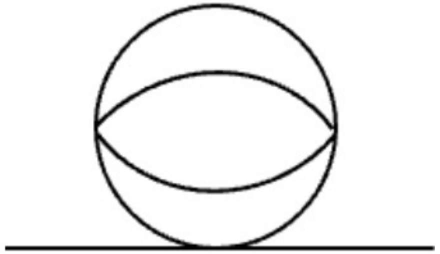
- Next, let us compute the $1/D$ correction.
- By using g_A, Many diagrams contribute to the correction.



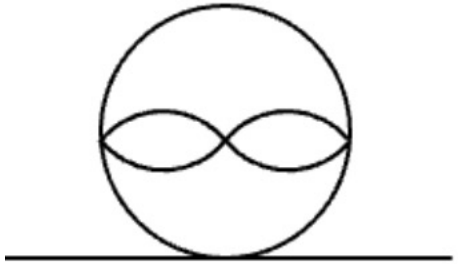
$O((g_A^2)^2 N^2 D) = \frac{\tilde{\lambda}_A^2}{D})$



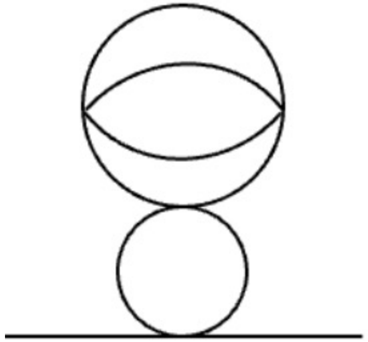
$O((g_A^2)^3 N^3 D^2) = \frac{\tilde{\lambda}_A^3}{D})$



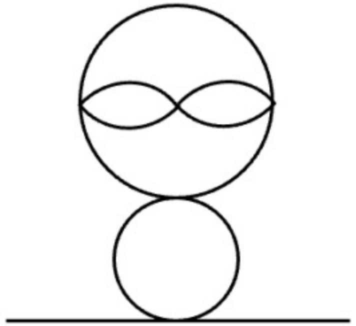
$O((g_A^2)^3 N^3 D^2) = \frac{\tilde{\lambda}_A^3}{D})$

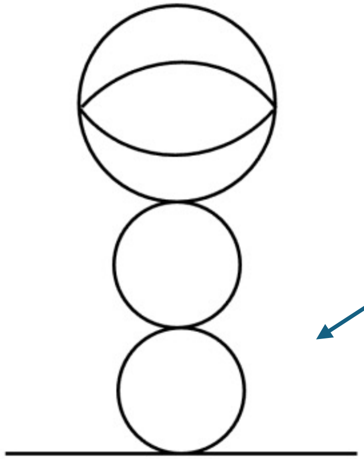


$O((g_A^2)^4 N^4 D^3) = \frac{\tilde{\lambda}_A^4}{D})$



$O((g_A^2)^4 N^4 D^3) = \frac{\tilde{\lambda}_A^4}{D})$

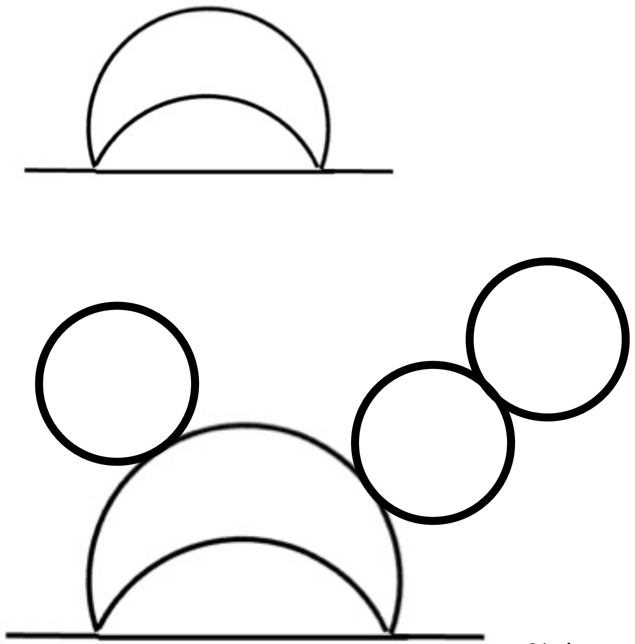




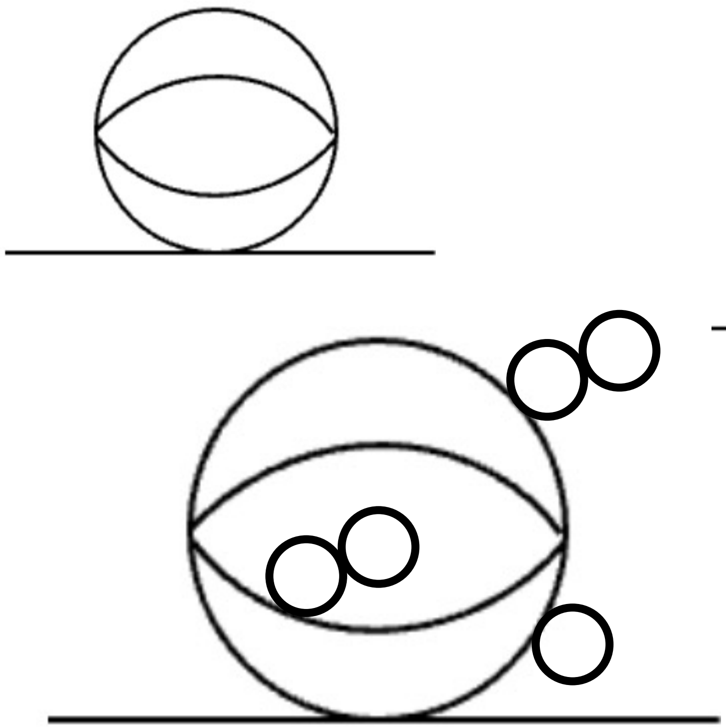
$O((g_A^2)^5 N^5 D^4) = \frac{\tilde{\lambda}_A^5}{D})$

The correction from g_A

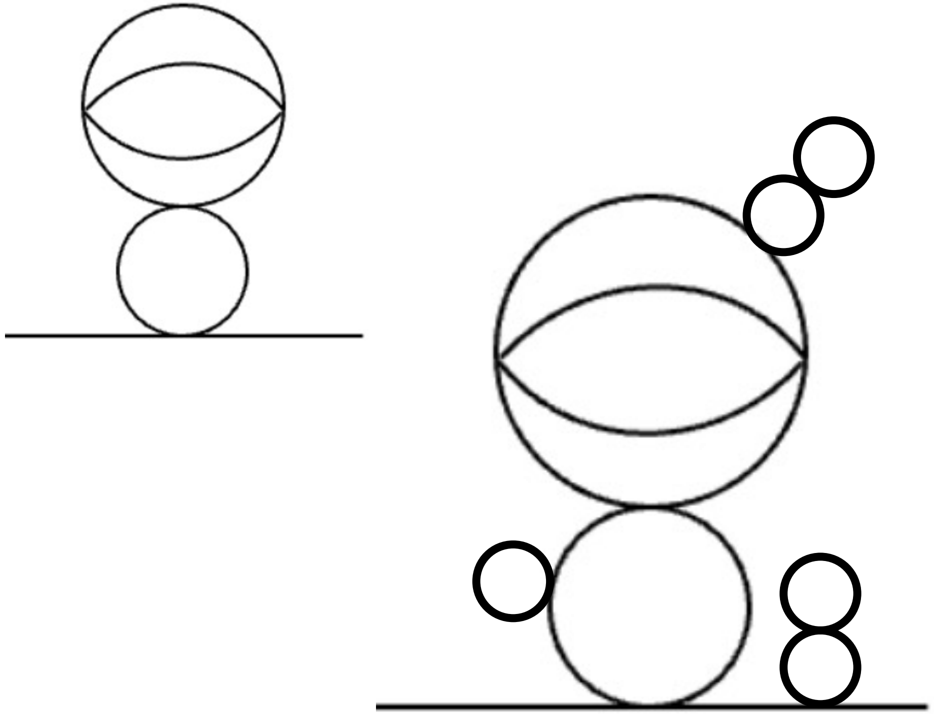
- Adding snails, which is an effect of leading, the diagrams still becomes the $1/D$ order. For example,



$O((g_A^2)^5 N^5 D^4) = \frac{\tilde{\lambda}_A^5}{D})$



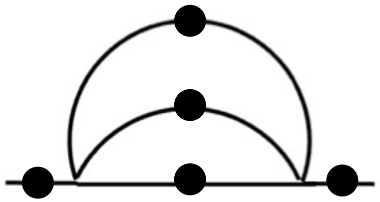
$O((g_A^2)^8 N^8 D^7) = \frac{\tilde{\lambda}_A^8}{D})$



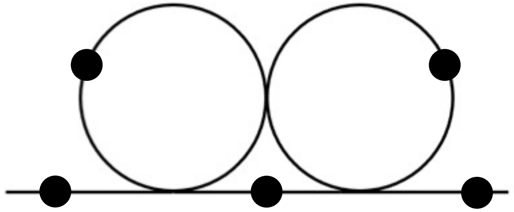
$O((g_A^2)^9 N^9 D^8) = \frac{\tilde{\lambda}_A^9}{D})$

The correction from g_A

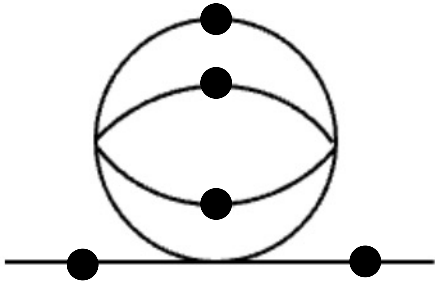
- We can replace all bare propagators to dressed one.
- All the following are the $1/D$ corrections (using only g_A).



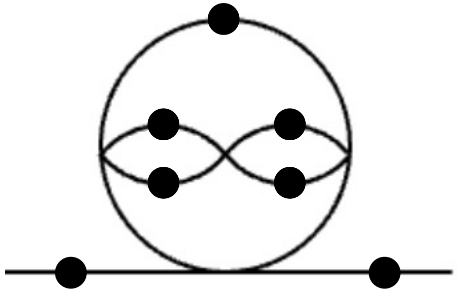
$O((g_A^2)^2 N^2 D) = \frac{\tilde{\lambda}_A^2}{D})$



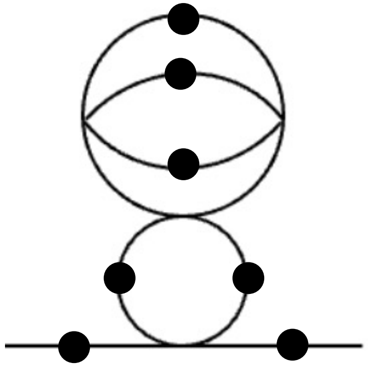
$O((g_A^2)^3 N^3 D^2) = \frac{\tilde{\lambda}_A^3}{D})$

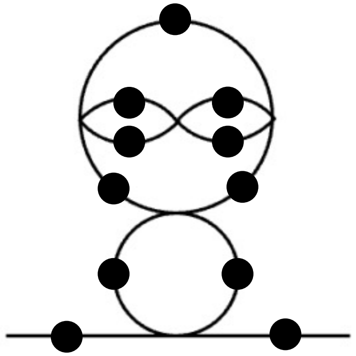


$O((g_A^2)^3 N^3 D^2) = \frac{\tilde{\lambda}_A^3}{D})$

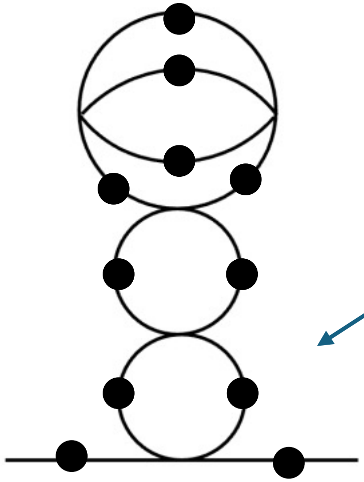


$O((g_A^2)^4 N^4 D^3) = \frac{\tilde{\lambda}_A^4}{D})$





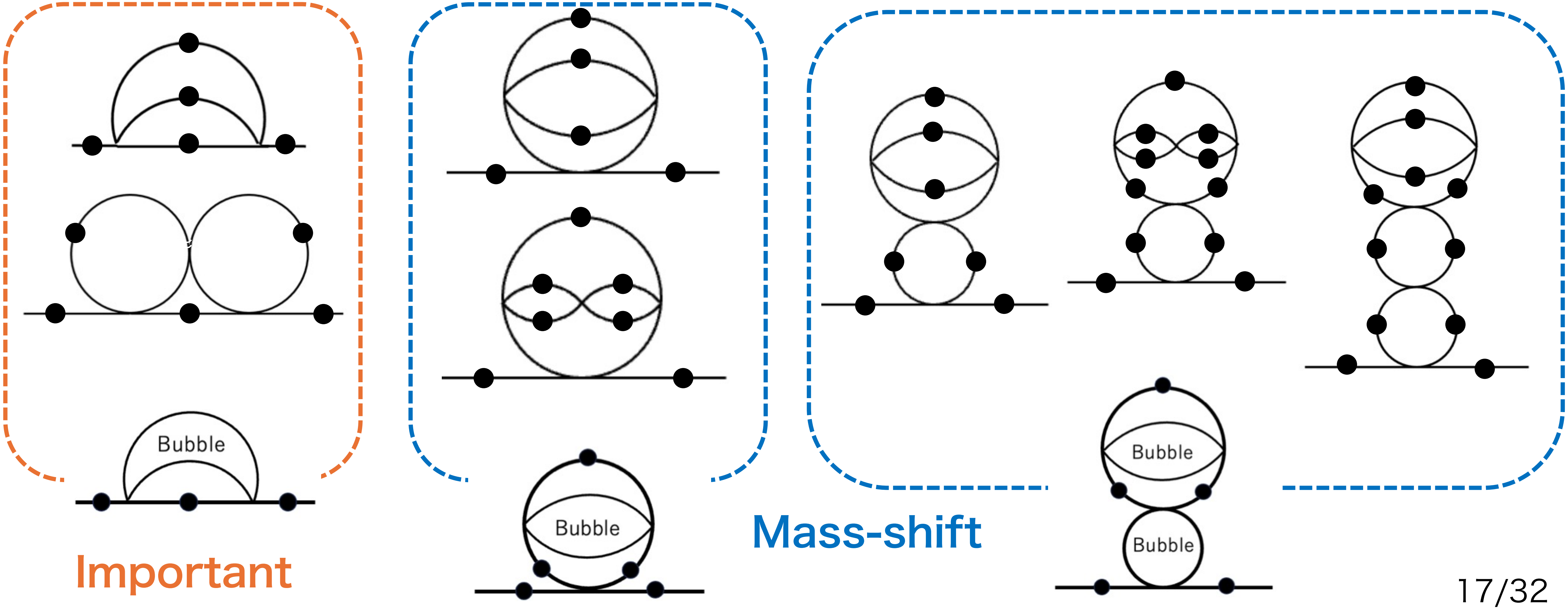
$O((g_A^2)^4 N^4 D^3) = \frac{\tilde{\lambda}_A^4}{D})$



$O((g_A^2)^5 N^5 D^4) = \frac{\tilde{\lambda}_A^5}{D})$

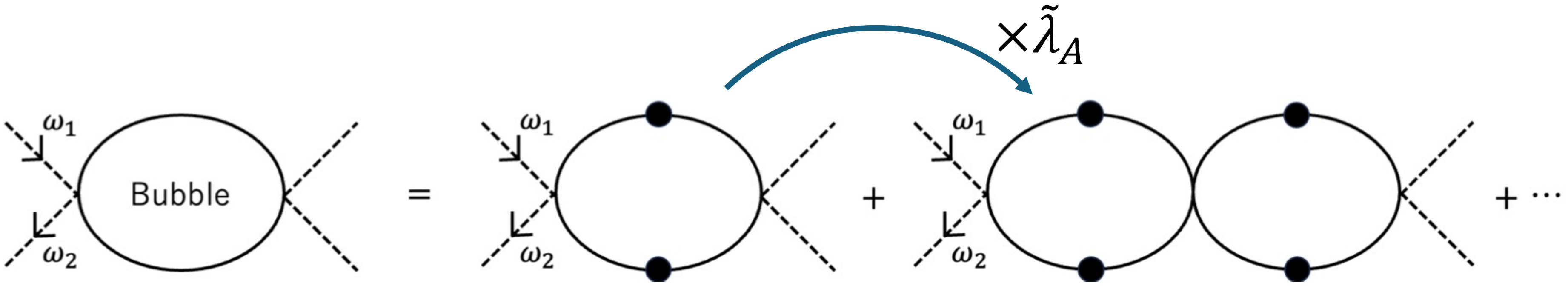
The correction from g_A

- There are three types of diagram which contributes correction
- This is all the corrections using only g_A.

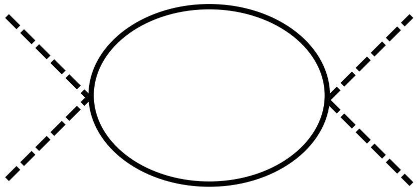


Bubble contribution

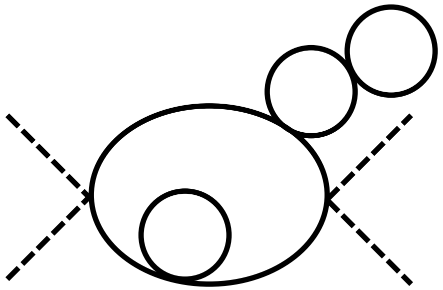
- We have to consider the contribution which contains all loop (we call this **Bubble**) to calculate the correction to the 2pt function.



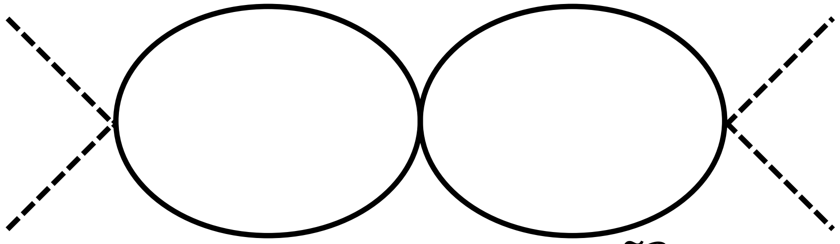
- Please note that only planar diagrams survive again. The following are all same order



$O((g_A^2)^2 ND = \frac{\tilde{\lambda}_A^2}{ND})$

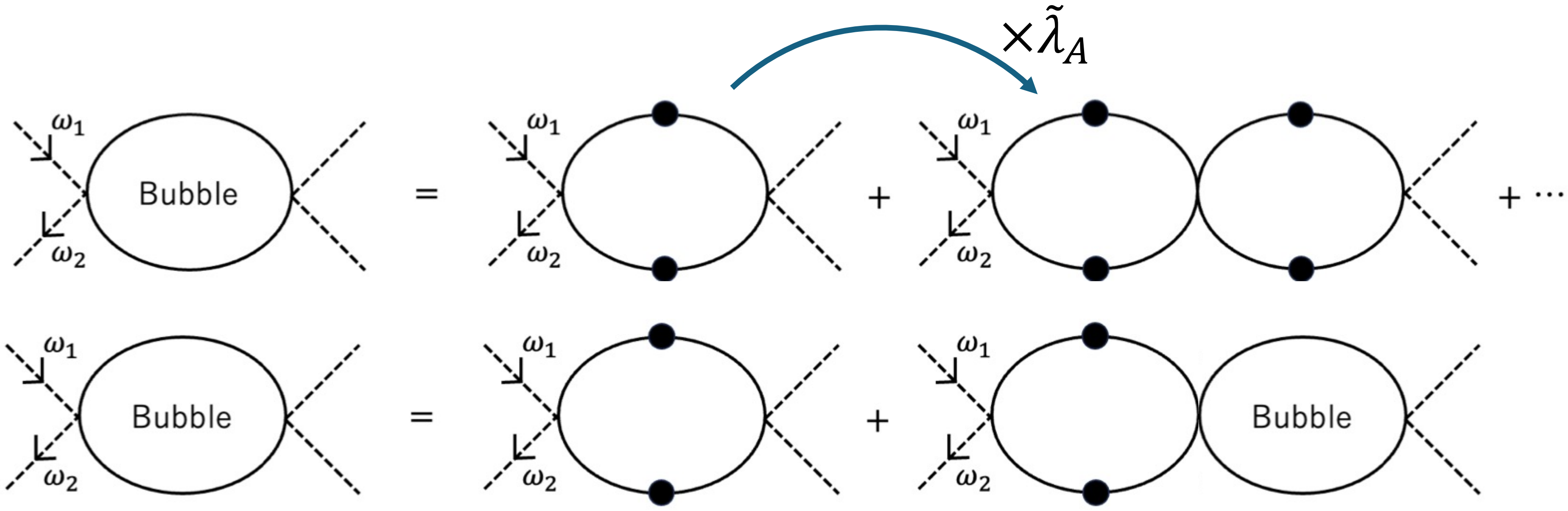


$O((g_A^2)^5 N^4 D^4 = \frac{\tilde{\lambda}_A^5}{ND})$



$O((g_A^2)^3 N^2 D^2 = \frac{\tilde{\lambda}_A^3}{ND})$

Bubble contribution

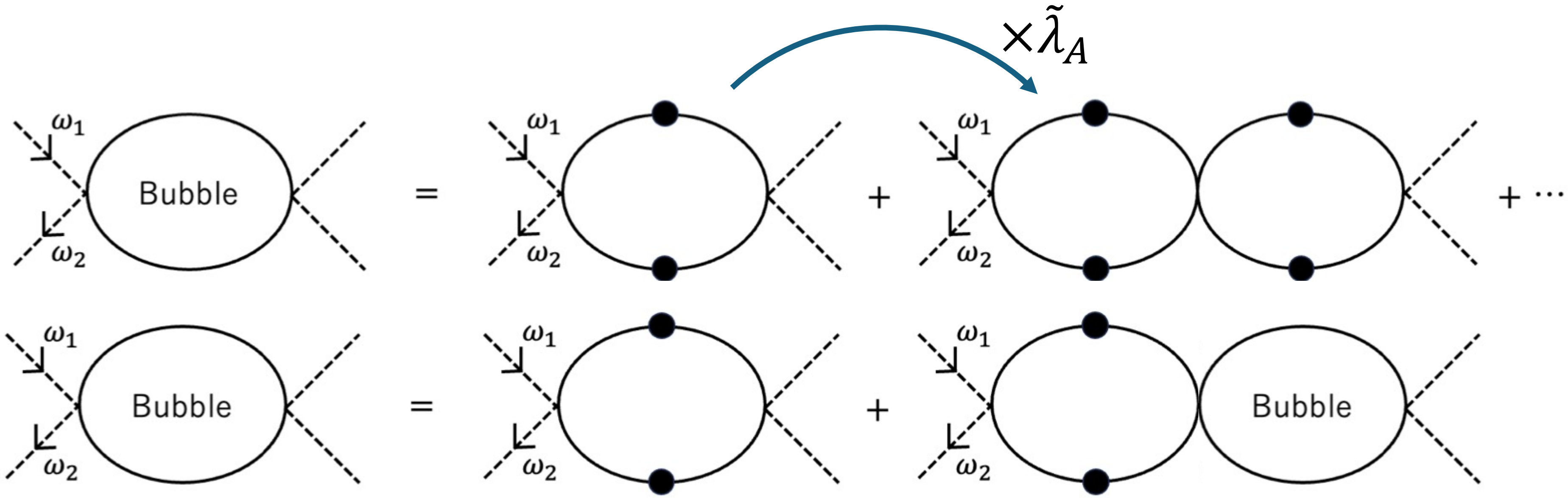


- Schwinger-Dyson equation can be written as

$$B(\omega_{1-2}) = \int \frac{d\omega}{2\pi} G(\omega) G(\omega_{1-2} - \omega) - c_B \lambda_A D \int \frac{d\omega}{2\pi} G(\omega) G(\omega_{1-2} - \omega) B(\omega_{1-2}) \qquad c_B = 1$$

- Of course, this has an overall factor, which will be re-counted later, but what is important is its coefficient ratio.

Bubble contribution



$$B(\omega_{1-2}) = \int \frac{d\omega}{2\pi} G(\omega)G(\omega_{1-2}-\omega) - c_B\lambda_A D \int \frac{d\omega}{2\pi} G(\omega)G(\omega_{1-2}-\omega)B(\omega_{1-2}) \qquad c_B = 1$$

- This has a geometric sum structure

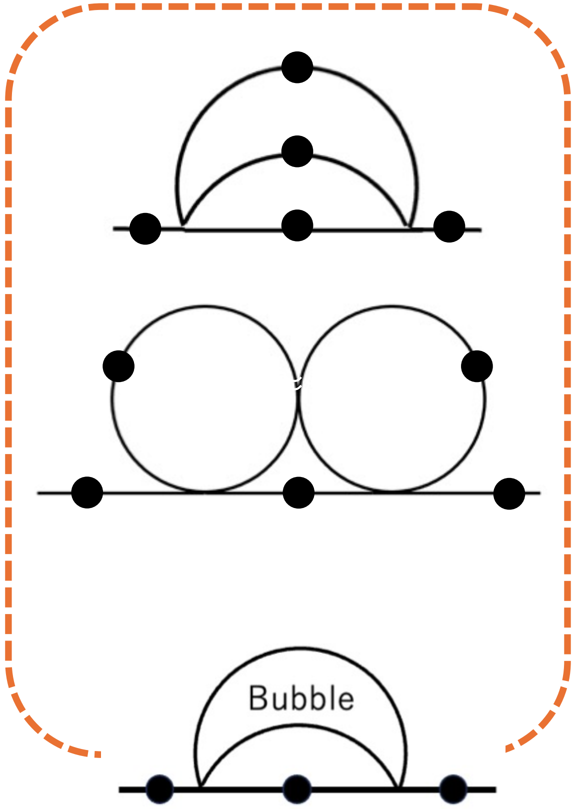
$$B(\omega_{1-2}) = \frac{\int \frac{d\omega}{2\pi} G(\omega)G(\omega_{1-2}-\omega)}{1 + c_B\tilde{\lambda}_A \int \frac{d\omega}{2\pi} G(\omega)G(\omega_{1-2}-\omega)}$$

- We can perform all integrals in r.h.s. where $G = \frac{1}{\omega^2 + m_1^2}$ $\qquad m_1^2 = m_0^2 + \frac{\tilde{\lambda}_A}{m_1}$

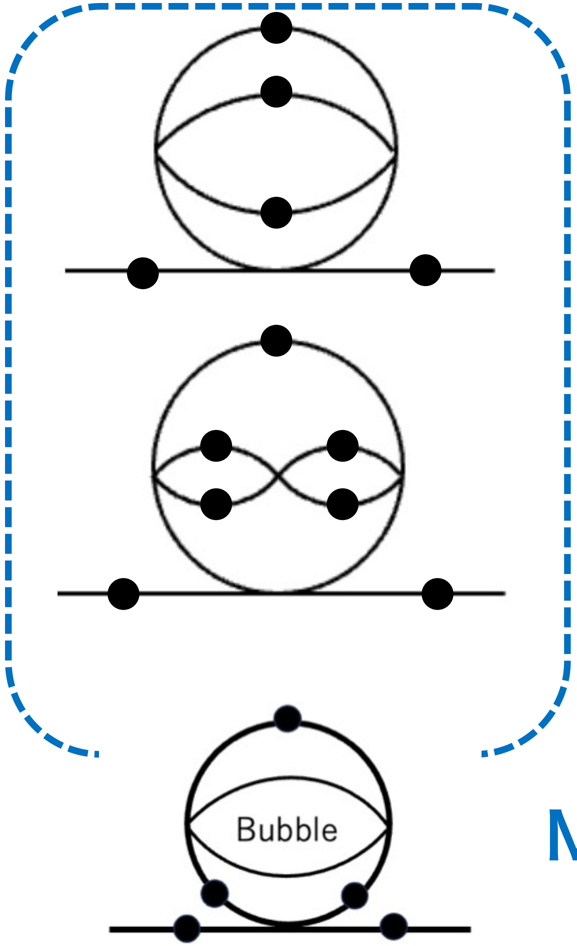
$$B(\omega) = \frac{1}{m_1}\frac{1}{\omega^2 + m_\sigma^2}, \quad \text{where} \quad m_\sigma^2 = 4m_1^2 + \frac{\tilde{\lambda}_A}{m_1}$$

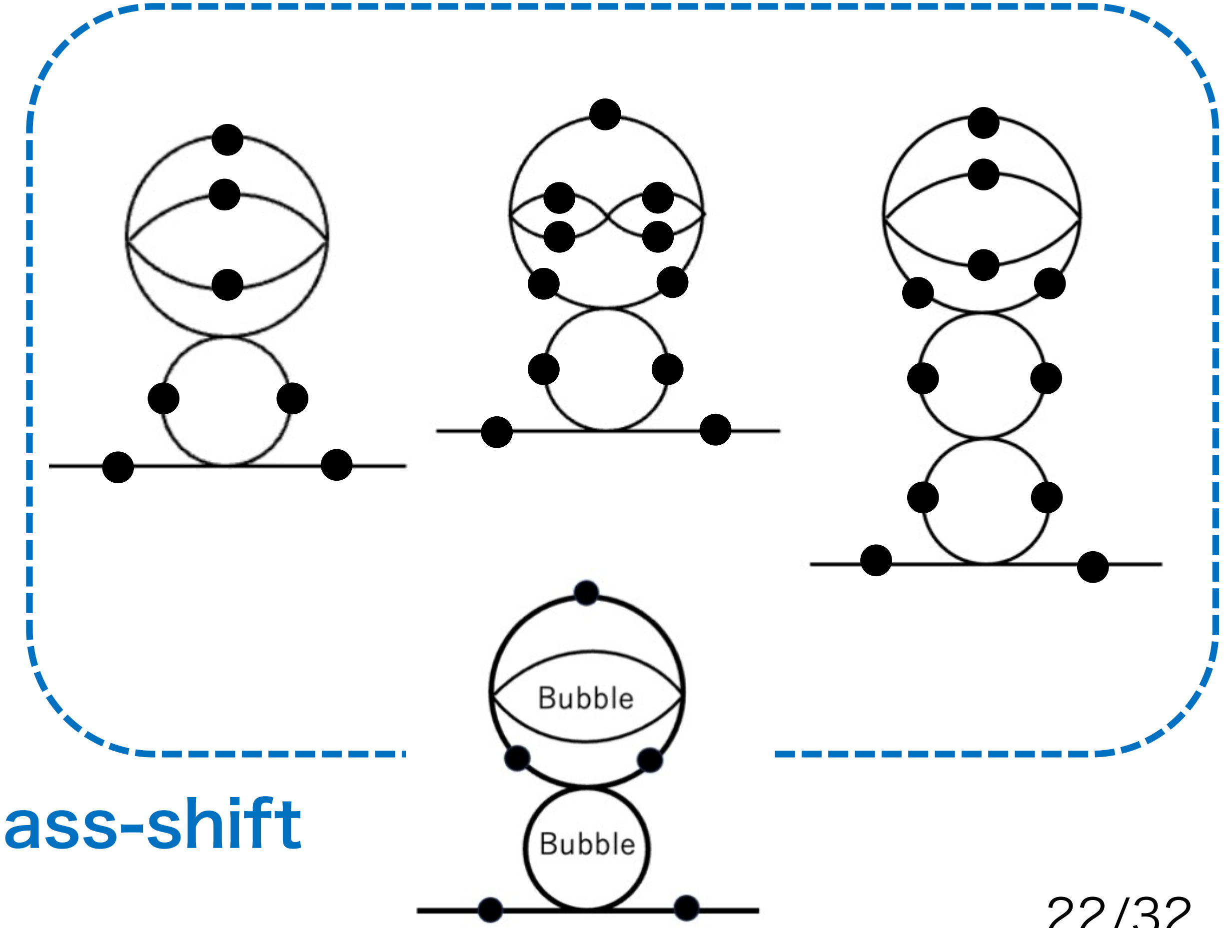
The correction from g_A

- There are three types of diagram which contributes correction
- This is all the corrections using only g_A.



Important

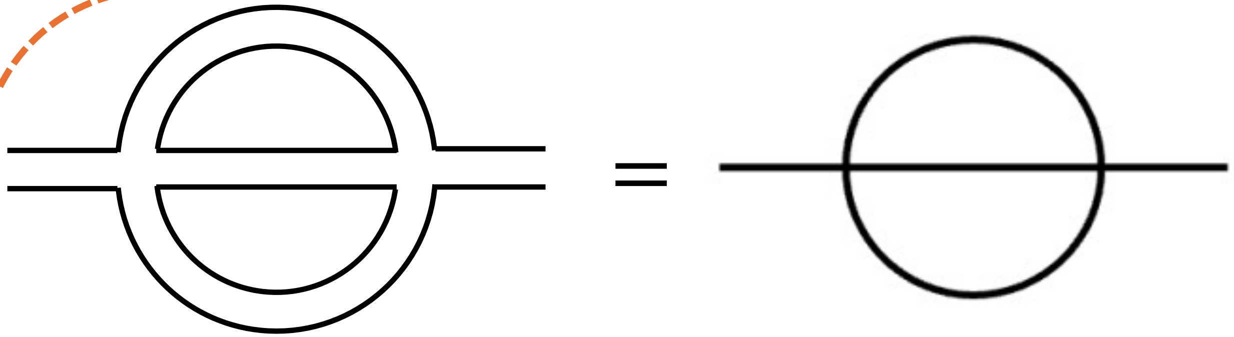




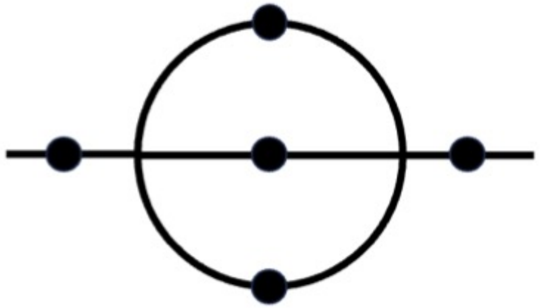
Mass-shift

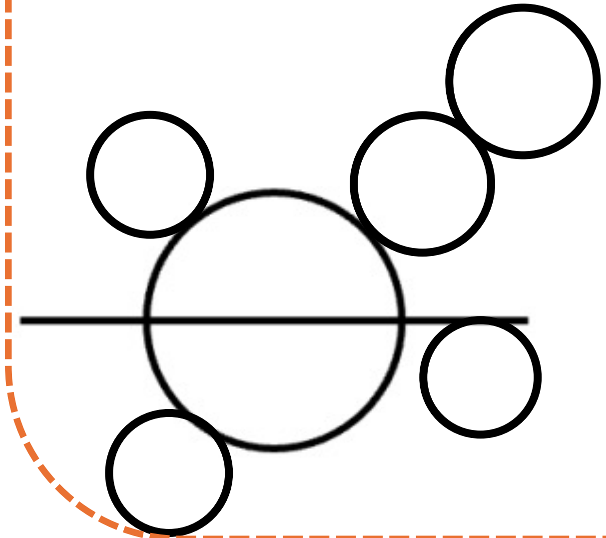
The correction from g_C

- By using g_C, there are the additional diagram which contributes.



Planar $O((g_C^2)^2 N^2 D) = \frac{\tilde{\lambda}_C^2}{D})$

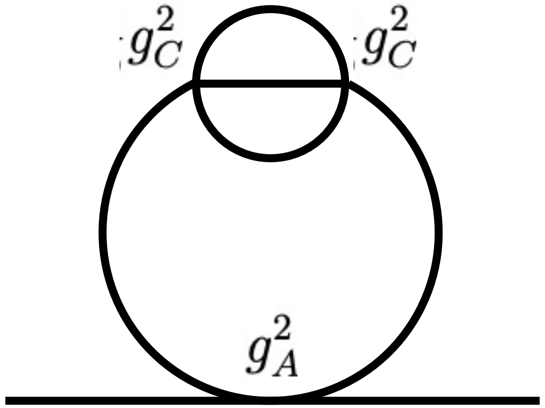




Adding snails does not change the order of D

$O((g_C^2)^2 N^2 D (g_A^2)^5 N^5 D^5) = \frac{\tilde{\lambda}_C^2 \tilde{\lambda}_A^5}{D})$

Important



$O(g_A^2 (g_C^2)^2 N^3 D^2) = \frac{\tilde{\lambda}_A \tilde{\lambda}_C^2}{D})$

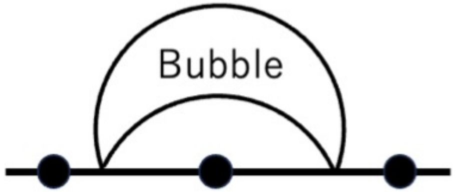
Mass-shift

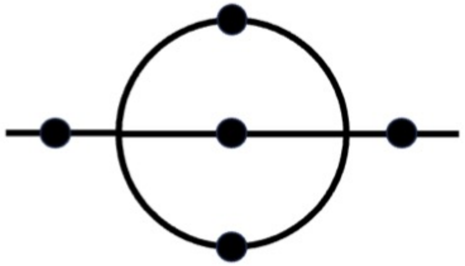
SD equation up to $O(1/D)$

Important   Mass-shift

$$\widetilde{G}(\omega)^{-1} = \omega^2 + m_1^2 + \frac{1}{D}\left(\Pi_A(\omega) + \Pi_C(\omega)\right) + \delta m_A + \delta m_C + \mathcal{O}\left(\frac{1}{D^2}\right)$$

SD equation up to $1/D$



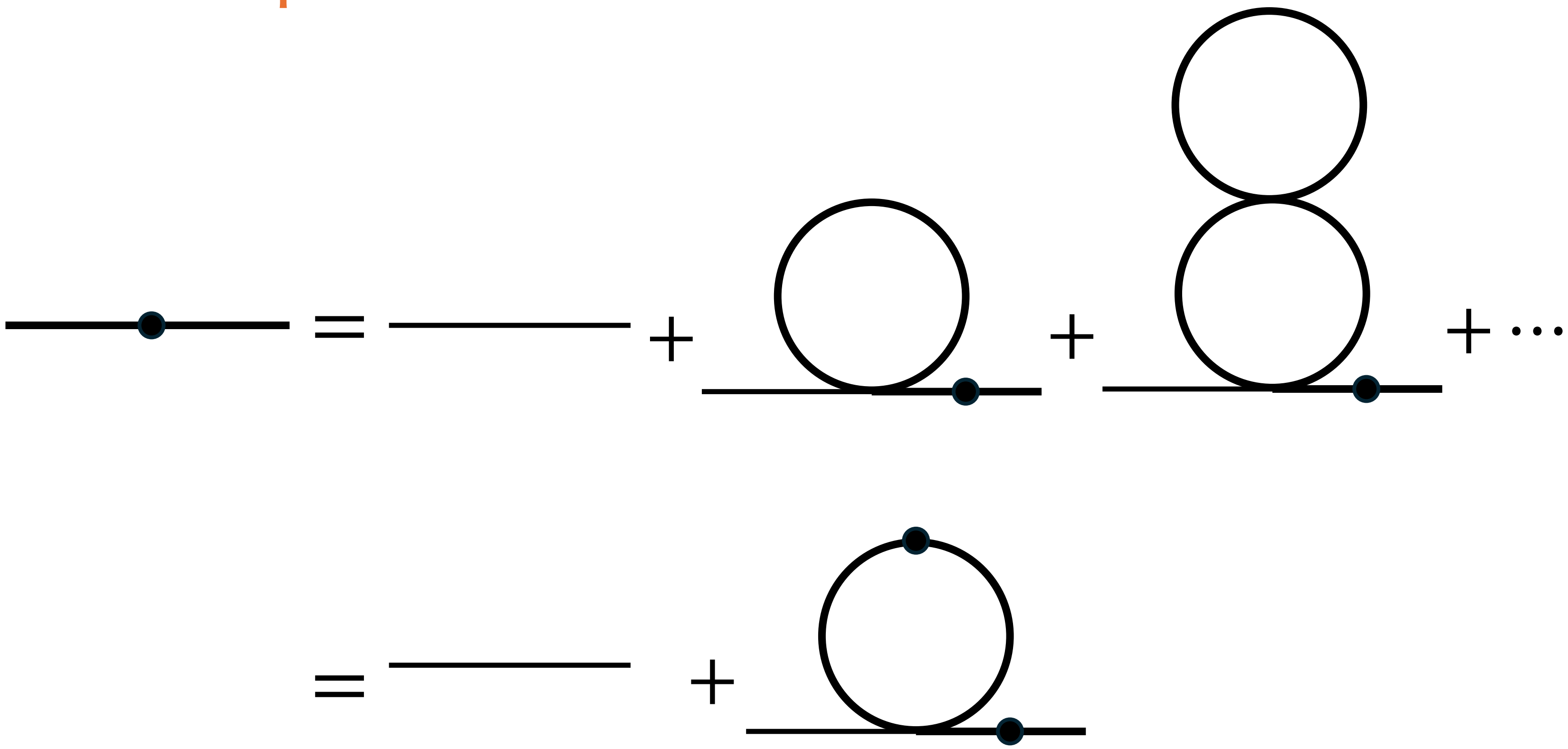


This is zero-temperature result.
We are interested in finite temperature case.

Finite temperature - leading

SD equation for leading order @Zero-temperature

$$G(\omega) = G_0(\omega) - c_L \tilde{\lambda}_A G_0(\omega) G(\omega) \int \frac{d\omega'}{2\pi} G(\omega') \qquad c_L = 2$$



Finite temperature - bubble

SD equation for leading order @Zero-temperature

$$G(\omega) = G_0(\omega) - c_L \tilde{\lambda}_A G_0(\omega) G(\omega) \int \frac{d\omega'}{2\pi} G(\omega') \qquad c_L = 2$$

@Finite temp (Matsubara formalism)

$$G(\omega_n) = G_0(\omega_n) - c_L \frac{\tilde{\lambda}_A}{\beta} G_0(\omega_n) G(\omega_n) \sum_k G(\omega_k)$$

- Let us assume $G_0 = \dfrac{1}{\omega^2 + m_0^2}, \quad G = \dfrac{1}{\omega^2 + m_1^2}$
- We can determine the mass of leading two-point function becomes

$$m_1^2 = m_0^2 + \frac{\tilde{\lambda}_A}{m_1} \coth \frac{\beta m_1}{2}$$

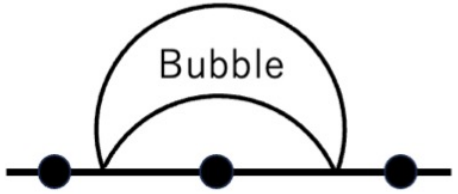
- Matsubara summation of the bubble diagram can be performed in the same way.

Finite temperature - correction

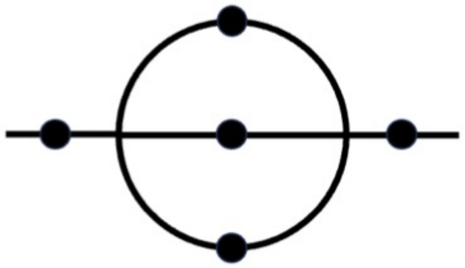
$$\widetilde{G}(\omega_n)^{-1} = G(\omega_n)^{-1} + \frac{1}{D}(\Pi_A(\omega_n) + \Pi_C(\omega_n)) + \delta m_A + \delta m_C + O\left(\frac{1}{D^2}\right)$$

SD equation
Up to 1/D

$$\frac{1}{D}\Pi_A(\omega_n) = -\frac{c_A}{D}\frac{\tilde{\lambda}_A^2}{\beta}\sum_k G(\omega_k)B(\omega_k - \omega_n)$$



$$\frac{1}{D}\Pi_C(\omega_n) = -\frac{c_C}{D}\frac{\tilde{\lambda}_C^2}{\beta^2}\sum_{k,k'} G(\omega_k)G(\omega_{k'})G(\omega_n - \omega_{k+k'})$$



- It's possible to calculate these contribution

$$\frac{1}{D}(\Pi_A(\omega_n) + \Pi_C(\omega_n)) = -\frac{B}{\omega_n^2 + m_1^2} + \cdots \quad \text{in the vicinity of } \omega^2 + m_1^2 = 0$$

$$B = B_1 + B_2 = \frac{1}{D}\left(\frac{2\tilde{\lambda}_A A}{\beta} + \frac{3\tilde{\lambda}_C^2}{m_1^2}\left(\frac{1}{e^{\beta m_1} - 1}\right)^2 e^{\beta m_1}\right)$$

A is a complicated β-dependent function.

Important point is this isn't just mass shift

Finite temperature - correction

$$\tilde{G}(\omega_n)^{-1} = G(\omega_n)^{-1} + \frac{1}{D}\left(\Pi_A(\omega_n) + \Pi_C(\omega_n)\right) + \delta m_A + \delta m_C + O\left(\frac{1}{D^2}\right)$$

$$\frac{1}{D}\left(\Pi_A(\omega_n) + \Pi_C(\omega_n)\right) = -\frac{B}{\omega_n^2 + m_1^2} + \cdots$$

$\delta m_A + \delta m_C$ B

- Appropriately redefining the mass, we obtain

determines

$$\tilde{B} \sim \frac{1}{D}$$

$$\tilde{G}(\omega) \propto \left(\frac{1}{\omega^2 + m^2 + \sqrt{\tilde{B}}} + \frac{1}{\omega^2 + m^2 - \sqrt{\tilde{B}}}\right)$$

- In the leading, the pole was single, but the correction at finite temperature decomposes the pole into a pair of poles.

$$m_\pm^2 = m^2 \pm \sqrt{\tilde{B}}$$

- There is no true dissipation, but there is destructive interference.

Finite temperature and dissipation

$$\tilde{G}(\omega) \propto \left(\frac{1}{\omega^2 + m^2 + \sqrt{\tilde{B}}} + \frac{1}{\omega^2 + m^2 - \sqrt{\tilde{B}}} \right) \qquad m_\pm^2 = m^2 \pm \sqrt{\tilde{B}}$$

- retarded Green's function which can be obtained from a Euclidean correlator by analytically continuing

$$\tilde{G}_R(\omega) = \tilde{G}(\omega \to -i(\omega + i\epsilon)) = \# \left(\frac{1}{-(\omega + i\epsilon)^2 + m_+^2} + \frac{1}{-(\omega + i\epsilon)^2 + m_-^2} \right)$$

$$\tilde{G}_R(t) = \int \frac{d\omega}{2\pi} e^{-i\omega t} \tilde{G}_R(\omega)$$

$$\propto \theta(t) \left(\frac{1}{m_+} \sin(m_+ t) + \frac{1}{m_-} \sin(m_- t) \right) \propto \theta(t) \sin(mt) \cos\left(\frac{\sqrt{\tilde{B}} t}{2m} \right)$$

- lifetime $\tau = \frac{\pi m}{\sqrt{\tilde{B}}}$ $\sim\sqrt{D}$, recurrence $t = \frac{4\pi m}{\sqrt{\tilde{B}}}$ $\sim\sqrt{D}$, width(rate) $\Gamma = \frac{1}{\tau} = \frac{\sqrt{\tilde{B}}}{\pi m}$ $\sim 1/\sqrt{D}$
- The dissipation is generically an $O(1/\sqrt{D})$ effect in many-matrix th?

The role of two couplings

$$\widetilde{G}(\omega_n)^{-1} = G(\omega_n)^{-1} + \frac{1}{D}\left(\Pi_A(\omega_n) + \Pi_C(\omega_n)\right) + \delta m_A + \delta m_C + O\left(\frac{1}{D^2}\right)$$

$$\frac{1}{D}\left(\Pi_A(\omega_n) + \Pi_C(\omega_n)\right) = -\frac{B}{\omega_n^2 + m_1^2} + \cdots$$

$$B = B_1 + B_2 = \frac{1}{D}\left(\frac{2\tilde{\lambda}_A A}{\beta} + \frac{3\tilde{\lambda}_C^2}{m_1^2}\left(\frac{1}{e^{\beta m_1} - 1}\right)^2 e^{\beta m_1}\right)$$

- In high temperature, second term in B dominates and all the effects of pole splits, comes from g_C. The effect of g_A is subleading.
- Why? The potential of our model can be rewritten as

$$V = \frac{1}{2}m_0^2 \operatorname{Tr}\left(X^i X^i\right) - \frac{1}{4}g_C^2 \operatorname{Tr}\left([X^i, X^j]^2\right) + \frac{1}{2}\left(g_A^2 - g_C^2\right) \operatorname{Tr}\left(X^i X^i X^j X^j\right)$$

Stable

If $g_A > g_C$, stable
If $g_C > g_A$, unstable

- **Is this instability related with? (Future direction)**

Summary

$$S = \int d\tau \, \frac{1}{2}\mathrm{Tr}\left(\partial_\tau X^i \partial_\tau X^i\right) + \frac{1}{2}m_0^2 \,\mathrm{Tr}\left(X^i X^i\right) + \frac{1}{2}g_A^2 \,\mathrm{Tr}\left(X^i X^i X^j X^j\right) - \frac{1}{2}g_C^2 \,\mathrm{Tr}\left(X^i X^j X^i X^j\right)$$

We'll consider the QM of a large number D of $N\times N$ Hermitian matrices

- First, we take large N = only planar diagrams survive.
- After that, we take large D and do perturbation in $1/D$.

We computed a thermal two-point correlator to $O(1/D)$

- In the leading, the pole was single, but the correction at finite temperature decomposes the pole into a pair of poles.
- This implies a timescale for thermal dissipation $\sim O(\sqrt{D})$
- At high temperatures dissipation is predominantly due to one of the two quartic couplings.

Future direction

- Higher order? And expansion convergent or not?

  The splitting continues and additional poles develop?
- 4pt (OTOC) calculation?

  In some particular limit, we can expect the order of Lyapunov exponent in comparison with usual (commutable) vector model. Can we perform the calculation explicitly?
- Application for Tensor models?

  The fundamental dof of our model $X^i_{AB}(\tau)$ can be regarded as tensor. What can we find by using the same double-scaled method?

Thank you for listening!

Four point, OTOC (Future direction)

$$S = \int d\tau \, \frac{1}{2}\mathrm{Tr}\left(\partial_\tau X^i \partial_\tau X^i\right) + \frac{1}{2} m_0^2 \,\mathrm{Tr}\left(X^i X^i\right) + \frac{1}{2} g_A^2 \,\mathrm{Tr}\left(X^i X^i X^j X^j\right) - \frac{1}{2} g_C^2 \,\mathrm{Tr}\left(X^i X^j X^i X^j\right)$$

When $g_C = 0$, our model has the similar structure with (non-commutable) $O(D)$ vector model. I'll introduce the simple thing which we can see soon.

- As a reference, what happens in **commutable case** ?

$$S = \int dt \left[\sum_{i=1}^{D} \left(\frac{1}{2}\dot{\phi}_i^2 - \frac{m^2}{2}\phi_i^2 \right) - \frac{\lambda}{4N} \sum_{i,j=1}^{D} \phi_i^2 \phi_j^2 \right]$$

This is integrable and OTOC oscillates = does not show chaos. However, if $O(D)$ symmetry is slightly broken, the Lyapunov exponent is non-zero = slightly chaotic. For example ['22 Kolganov, Trunin]

$$S = \int dt \left[\sum_{i=1}^{D} \left(\frac{1}{2}\dot{\phi}_i^2 - \frac{m^2}{2}\phi_i^2 \right) - \underbrace{\frac{\lambda}{4N} \sum_{i,j=1}^{D} \phi_i^2 \phi_j^2}_{\text{symmetric}} + \underbrace{\frac{\lambda}{4N} \sum_{i=1}^{D} \phi_i^4}_{\text{nonsymmetric}} \right]$$

Slightly chaotic!
Lyapunov $\sim 1/D$

Appendix

Four point, OTOC (Future direction)

$$S=\int d\tau\,\frac{1}{2}\mathrm{Tr}\left(\partial_\tau X^i\partial_\tau X^i\right)+\frac{1}{2}m_0^2\,\mathrm{Tr}\left(X^iX^i\right)+\frac{1}{2}g_A^2\,\mathrm{Tr}\left(X^iX^iX^jX^j\right)-\frac{1}{2}g_C^2\,\mathrm{Tr}\left(X^iX^jX^iX^j\right)$$

- Returning our model, if we set $g_A = g_C = g$, then part of the effect of g_C corresponds to this.

$$\underbrace{\sum_{i,j}\frac{1}{2}g^2\mathrm{Tr}(X^iX^iX^jX^j)}_{\text{Symmetric}}-\underbrace{\sum_{i}\frac{1}{2}g^2\mathrm{Tr}(X^iX^iX^iX^i)}_{\text{Nonsymmetric}}-\underbrace{\sum_{i\neq j}\frac{1}{2}g^2\mathrm{Tr}(X^iX^jX^iX^j)}_{\text{The rest}}$$

Maybe slightly chaotic! Lyapunov $\sim 1/D$

There are some differences since our model is noncommutative,

But the symmetry argument still seems valid, and we should be able to make a similar argument for our model.

- Remaining issues are the following.
  1. What is the contribution of the rest terms?
  2. Our model is nonconmmutable. 3. When $g_A \neq g_C$?

Appendix