Parametric decay instability of Alfvén wave in relativistically magnetized plasma

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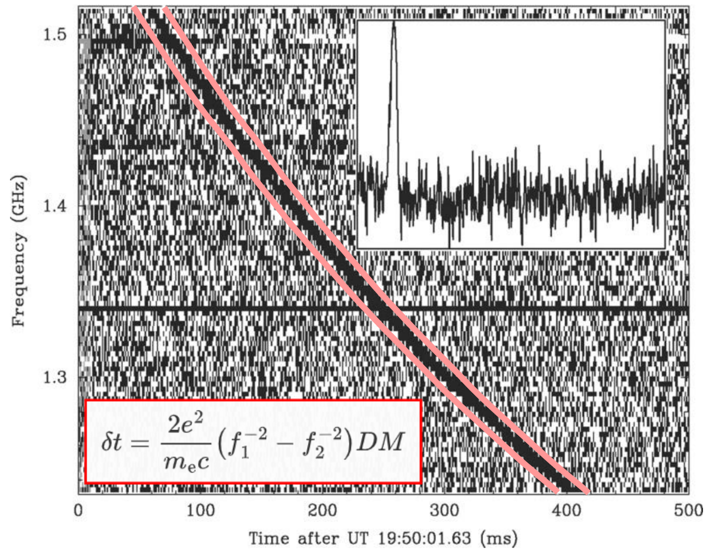
Fast Radio Burst

$$DM = \int_0^D n_e ds \sim \langle n_e \rangle D$$

- Transient phenomenon in the radio bands
  - Flux : S_ν~O(Jy) @ Ghz
  - Duration : δt~msec
  - Excess from the Dispersion Measure (DM) of Galaxy

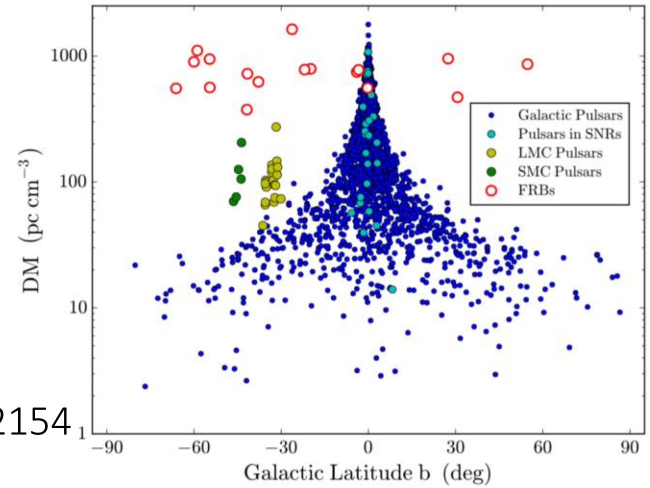
    DM~O(1000) pc cm^{-3} → Cosmological
  - Extremely high brightness temperature → coherent emission

$$T_b \simeq \frac{S_\nu D^2}{2\pi k_B (\nu \delta t)^2} = 1.2 \times 10^{36}\ \mathrm{K} \left(\frac{D}{10^{28}\ \mathrm{cm}}\right)^2 \frac{S_\nu}{\mathrm{Jy}} \left(\frac{\nu}{\mathrm{GHz}}\right)^{-2} \left(\frac{\delta t}{\mathrm{ms}}\right)^{-2}$$



(Lorimer et al., 2007)

- Source/Mechanism → Still Unknown
  - Short duration → compact star origin?
  - Repeating species (Repeater) → cataclysmic origin difficult?
  - Association with X-ray flare of galactic magnetar SGR1935+2154

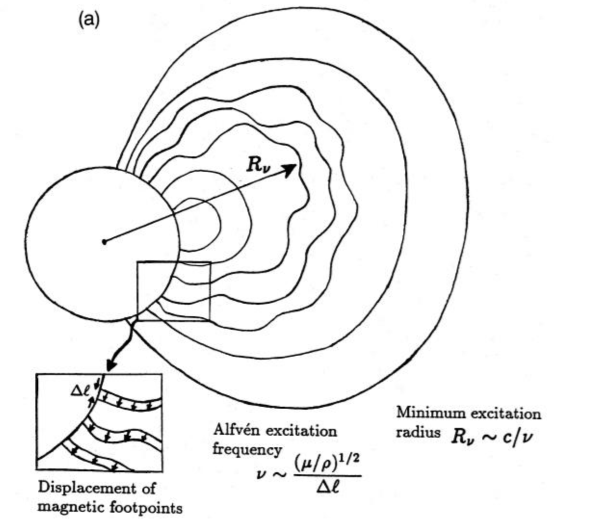


⇒Promising candidate: **Magnetosphere of magnetars**

(Cordes et al., 2016)

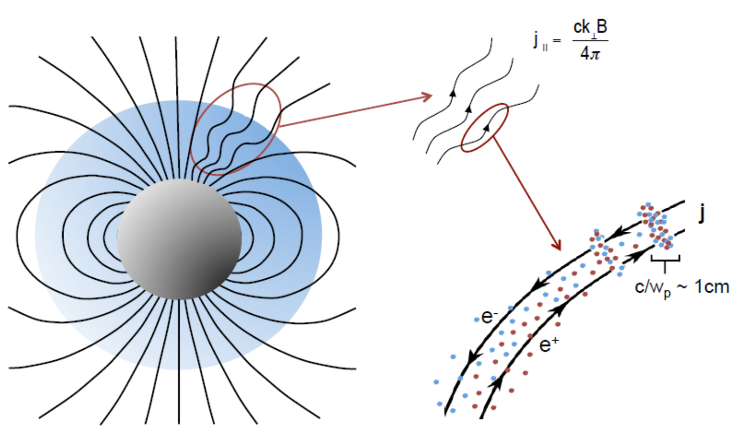
Alfven wave in a NS magnetosphere

- Alfvén wave in a magnetosphere
  - Generation: starquakes / reconnections (?)
  - Wave dissipation → Plasma heating → Photon emission (?)



(Thompson & Duncan, 1995)

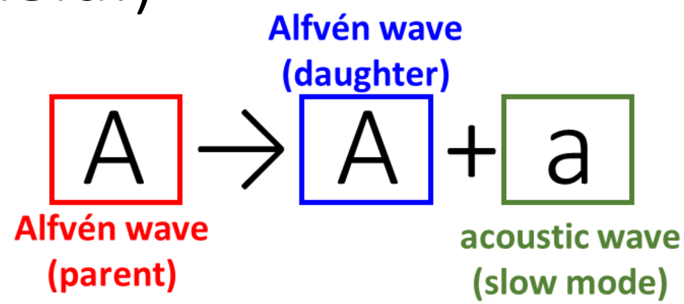
- Alfvén wave in FRB model: Examples
  - Alfvén wave becomes "charge starved", and then the particle acceleration process will turn on (e.g., Kumar & Bosnjak 2020)
  - Alfvén waves create large shear in magnetic fields during propagation and generate current sheets (Yuan+2020)



(Kumar & Bosnjak 2020)

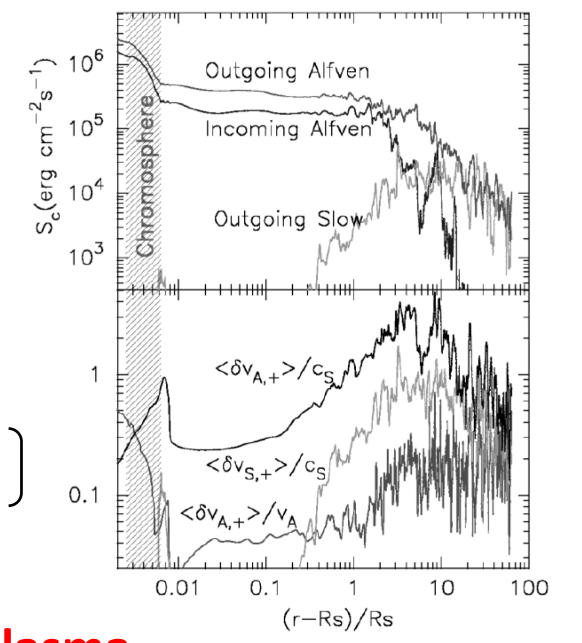
Propagation of Alfvén wave is a key issue

Stability of Alfvén wave (Non-Rela.)



- Decay instability of Alfvén wave
  - Instability due to resonant 3-wave interaction
  - Major process is **Alfven waves decay into slow and Alfven waves** for β<1 (non-rela. regime)

- Example : Corona heating
  - Alfven waves generated near the chromosphere
  - Converted to slow mode (compressible mode)
  - Slow mode creates shocks → plasma heats up
  - Growth rate: $\gamma/\omega_0 = \frac{1}{2}\eta\beta^{-1/4}$ $\left(\eta = \delta B/B_0\,,\ \beta = c_s^2/v_A^2 \right)$



(Suzuki & Inutsuka (2005))

Consider this process in a "relativistically magnetized" plasma

What is “Relativistically magnetized” plasma?

- “Relativistic magnetized”
  - The energy density of the electromagnetic field exceeds the rest mass energy density
  - Magnetization parameter σ :

$$\sigma = \frac{\text{(energy density of B-field)}}{\text{(enthalpy density of gas)}} = \frac{B_0^2/4\pi}{\rho c^2 + \epsilon_{\rm int} + p_{\rm gas}}$$

e.g., NS magnetosphere $\sigma \sim 10^{3\text{-}5}$(?)>>1

- Force-free approximation
  - Neglecting the contribution of matter field
  - The normal modes are Alfvén wave and fast wave; **there is no slow wave**.
  - In force-free regime, Alfvén wave is stable against 3-wave interaction

    (cannot be decayed by satisfying the resonance condition)

Without force-free approximation, we investigate the stability of Alfvén waves in a plasma with relativistic magnetization

Governing Equations

- Basic equation : Special relativistic ideal MHD equations

$$\frac{\partial}{\partial t}\left[(\epsilon+p)\gamma^2 - p + \frac{1}{8\pi}\left(E^2+B^2\right)\right] + \frac{\partial}{\partial z}\left[(\epsilon+p)\gamma^2 v_z + \frac{c}{4\pi}(\boldsymbol{E}\times\boldsymbol{B})_z\right] = 0$$

$$\frac{\partial}{\partial t}\left[(\epsilon+p)\gamma^2 v_x + \frac{c}{4\pi}(\boldsymbol{E}\times\boldsymbol{B})_x\right] + \frac{\partial}{\partial z}\left[(\epsilon+p)\gamma^2 v_x v_z - \frac{c^2}{4\pi}(E_x E_z + B_x B_z)\right] = 0$$

$$\frac{\partial}{\partial t}\left[(\epsilon+p)\gamma^2 v_y + \frac{c}{4\pi}(\boldsymbol{E}\times\boldsymbol{B})_y\right] + \frac{\partial}{\partial z}\left[(\epsilon+p)\gamma^2 v_y v_z - \frac{c^2}{4\pi}(E_y E_z + B_y B_z)\right] = 0$$

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$$\frac{\partial B_x}{\partial t} = -\frac{\partial}{\partial z}\left(v_z B_x - v_x B_z\right)$$

$$\frac{\partial B_y}{\partial t} = \frac{\partial}{\partial z}\left(v_y B_z - v_z B_y\right)$$

- Key Assumptions
  - **Consider only parallel propagation**

    ⇒ slow mode becomes pure acoustic wave, fast mode is degenerate with Alfven wave

  - For simplicity, the parent wave is set to the circularly polarized wave

Setting

$$\eta \equiv \delta B / B_0$$

- Perturbations are set as:

		Background	Parent wave	Daughter waves	
$\boldsymbol{B}$	$=$	$\boldsymbol{B_0}$	$+\ \boldsymbol{\delta B}$	$+\ \boldsymbol{b}_\perp$	
$\boldsymbol{v}$	$=$		$\boldsymbol{\delta v}$	$+\ \boldsymbol{v}_\perp$	$+\ \boldsymbol{v}_\parallel$
ϵ	$=$	ϵ_0			$+\ \epsilon_\parallel$
			Alfvén wave	Alfvén wave	Sound wave
		$O(1)$	$O(\eta)$	$O(\varepsilon)$	

Uniform plasma with B_0

Alfven wave propagating “clean” plasma

Setting

$$\eta \equiv \delta B / B_0$$

- Perturbations are set as:

		Background	Parent wave	Daughter waves	
B	$=$	B_0	$+\ \delta B$	$+\ b_\perp$	
v	$=$		δv	$+\ v_\perp$	$+\ v_\parallel$
ϵ	$=$	ϵ_0			$+\ \epsilon_\parallel$
			Alfvén wave	Alfvén wave	Sound wave
		$O(1)$	$O(\eta)$	$O(\varepsilon)$	

Uniform plasma with B_0

Alfven wave propagating "clean" plasma

An exact solution for finite amplitude waves exists!

Setting

$$\eta \equiv \delta B / B_0$$

- Perturbations are set as:

	Background	Parent wave	Daughter waves	
$\boldsymbol{B} =$	$\boldsymbol{B_0}$	$+\ \boldsymbol{\delta B}$	$+\ \boldsymbol{b}_\perp$	
$\boldsymbol{v} =$		$\boldsymbol{\delta v}$	$+\ \boldsymbol{v}_\perp$	$+\ \boldsymbol{v}_\parallel$
$\epsilon =$	ϵ_0			$+\ \epsilon_\parallel$
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$\boldsymbol{v} =$		$\delta\boldsymbol{v}$	$+ \,\boldsymbol{v}_\perp$	$+ \,\boldsymbol{v}_\parallel$
$\epsilon =$	ϵ_0			$+ \,\epsilon_\parallel$
		Alfvén wave	Alfvén wave	Sound wave
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Uniform plasma with B_0

Alfven wave propagating “clean” plasma

\+ perturbations

Do the perturbations grow?

Setting

MHD equations

$$\frac{\partial}{\partial t}\left[(\epsilon+p)\gamma^2 - p + \frac{1}{8\pi}\left(E^2+B^2\right)\right] + \frac{\partial}{\partial z}\left[(\epsilon+p)\gamma^2 v_z + \frac{c}{4\pi}(\boldsymbol{E}\times\boldsymbol{B})_z\right] = 0$$

$$\frac{\partial}{\partial t}\left[(\epsilon+p)\gamma^2 v_x + \frac{c}{4\pi}(\boldsymbol{E}\times\boldsymbol{B})_x\right] + \frac{\partial}{\partial z}\left[(\epsilon+p)\gamma^2 v_x v_z - \frac{c^2}{4\pi}(E_x E_z + B_x B_z)\right] = 0$$

$$\frac{\partial}{\partial t}\left[(\epsilon+p)\gamma^2 v_y + \frac{c}{4\pi}(\boldsymbol{E}\times\boldsymbol{B})_y\right] + \frac{\partial}{\partial z}\left[(\epsilon+p)\gamma^2 v_y v_z - \frac{c^2}{4\pi}(E_y E_z + B_y B_z)\right] = 0$$

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$$\frac{\partial B_x}{\partial t} = -\frac{\partial}{\partial z}(v_z B_x - v_x B_z)$$

$$\frac{\partial B_y}{\partial t} = \frac{\partial}{\partial z}(v_y B_z - v_z B_y)$$

+

Perturbations

	Background	Parent wave	Daughter waves	
		Alfvén wave	Alfvén wave	Sound wave
$\boldsymbol{B} =$	$\boldsymbol{B}_0$	$+\ \delta\boldsymbol{B}$	$+\ \boldsymbol{b}_\perp$	
$\boldsymbol{v} =$		$\delta\boldsymbol{v}$	$+\ \boldsymbol{v}_\perp$	$+\ \boldsymbol{v}_\parallel$
$\epsilon =$	ϵ_0			$+\ \epsilon_\parallel$
	$O(1)$	$O(\eta)$	$O(\varepsilon)$	

Perturbed equations

	Background	Parent wave	Daughter waves	
				Sound wave
$\boldsymbol{B} =$	$\boldsymbol{B_0}$	$+\,\boldsymbol{\delta B}$	$+\,\boldsymbol{b}_\perp$	
$\boldsymbol{v} =$		$\boldsymbol{\delta v}$	$+\,\boldsymbol{v}_\perp$	$+\,\boldsymbol{v}_\parallel$
$\epsilon =$	ϵ_0			$+\,\epsilon_\parallel$
		Alfvén wave	Alfvén wave	
	$O(1)$	$O(\eta)$	$O(\varepsilon)$	

- Acoustic wave
  - Energy conservation

$$\frac{1+\beta_s^2\delta\beta^2}{1-\delta\beta^2}\frac{1}{c}\frac{\partial\epsilon_\parallel}{\partial t}+\left(\delta\gamma^2\left(\epsilon_0+p_0\right)\right)\frac{\partial\beta_\parallel}{\partial z}=$$
$$-\frac{1}{c}\frac{\partial}{\partial t}\left[\left(2\delta\gamma^4\left(\epsilon_0+p_0\right)+\frac{B_0^2}{4\pi}\right)\left(\delta\boldsymbol{\beta}\cdot\boldsymbol{\beta}_\perp\right)-\frac{\delta\beta\delta BB_0\beta_\parallel}{4\pi}\right]+\frac{B_0}{4\pi}\left[\boldsymbol{\beta}_\perp\cdot\frac{\partial}{\partial z}\left(\delta\boldsymbol{B}\right)+\delta\boldsymbol{\beta}\cdot\frac{\partial\boldsymbol{b}_\perp}{\partial z}\right]$$

  - Momentum equation (z-component)

$$\left[\delta\gamma^2\left(\epsilon_0+p_0\right)+\frac{\delta B^2}{4\pi}\right]\frac{1}{c}\frac{\partial\beta_\parallel}{\partial t}+\beta_s^2\frac{\partial\epsilon_\parallel}{\partial z}=-\frac{\partial}{\partial z}\left(\frac{\delta\boldsymbol{B}\cdot\boldsymbol{b}_\perp}{4\pi}\right)+\frac{B_0}{4\pi}\left(\delta\boldsymbol{B}\cdot\frac{1}{c}\frac{\partial\boldsymbol{\beta}_\perp}{\partial t}+\boldsymbol{b}_\perp\cdot\frac{1}{c}\frac{\partial}{\partial t}\left(\delta\boldsymbol{\beta}\right)\right)$$

- Alfven wave
  - Momentum equations (in xy-plain)

$$\left[\delta\gamma^2\left(\epsilon_0+p_0\right)+\frac{B_0^2+\delta B^2}{4\pi}\right]\frac{1}{c}\frac{\partial\boldsymbol{\beta}_\perp}{\partial t}-\frac{B_0}{4\pi}\frac{\partial\boldsymbol{b}_\perp}{\partial z}=-\delta\gamma^2\left(\epsilon_0+p_0\right)\beta_\parallel\frac{\partial}{\partial z}\left(\delta\boldsymbol{\beta}\right)$$
$$-\delta\gamma^2\left(1+\beta_s^2\right)\epsilon_\parallel\frac{1}{c}\frac{\partial}{\partial t}\left(\delta\boldsymbol{\beta}\right)-\delta\boldsymbol{\beta}\beta_s^2\frac{1}{c}\frac{\partial\epsilon_\parallel}{\partial t}+\frac{B_0}{4\pi}\frac{1}{c}\frac{\partial}{\partial t}\left(\beta_\parallel\delta\boldsymbol{B}\right)$$
$$-2\delta\gamma^4\left(\epsilon_0+p_0\right)\left(\delta\boldsymbol{\beta}\cdot\boldsymbol{\beta}_\perp\right)\frac{1}{c}\frac{\partial}{\partial t}\left(\delta\boldsymbol{\beta}\right)+\frac{1}{4\pi}\frac{\partial}{\partial z}\left[\hat{z}\cdot\left\{\left(\delta\boldsymbol{\beta}\times\boldsymbol{b}_\perp\right)+\left(\boldsymbol{\beta}_\perp\times\delta\boldsymbol{B}\right)\right\}\right]\left(\delta\boldsymbol{\beta}\times\boldsymbol{B_0}\right)$$
$$-\frac{\delta\boldsymbol{B}\cdot\boldsymbol{b}_\perp}{4\pi}\frac{1}{c}\frac{\partial}{\partial t}\left(\delta\boldsymbol{\beta}\right)-\left(\delta\boldsymbol{B}\cdot\frac{1}{c}\frac{\partial\boldsymbol{b}_\perp}{\partial t}\right)\delta\boldsymbol{\beta}+\frac{\delta B\delta\beta}{4\pi}\frac{1}{c}\frac{\partial\boldsymbol{b}_\perp}{\partial t}+\frac{\delta\boldsymbol{B}}{4\pi}\left(\frac{1}{c}\frac{\partial\boldsymbol{\beta}_\perp}{\partial t}\cdot\delta\boldsymbol{B}\right)+\frac{\delta\boldsymbol{B}\cdot\boldsymbol{\beta}_\perp}{4\pi}\frac{1}{c}\frac{\partial}{\partial t}\left(\delta\boldsymbol{B}\right)$$

  - Induction euations (in xy-plain)

$$\frac{1}{c}\frac{\partial\boldsymbol{b}_\perp}{\partial t}-B_0\frac{\partial\boldsymbol{\beta}_\perp}{\partial z}=-\frac{\partial}{\partial z}\left(\beta_\parallel\delta\boldsymbol{B}\right)$$

too complicated... → First we ignore $O(\varepsilon\eta^2)$

Perturbed equations

	Background	Parent wave	Daughter waves	
$\boldsymbol{B} =$	$\boldsymbol{B}_0$	$+\,\delta\boldsymbol{B}$	$+\,\boldsymbol{b}_\perp$	
$\boldsymbol{v} =$		$\delta\boldsymbol{v}$	$+\,\boldsymbol{v}_\perp$	$+\,\boldsymbol{v}_\parallel$
$\epsilon =$	ϵ_0			$+\,\epsilon_\parallel$
		Alfvén wave	Alfvén wave	Sound wave
	$O(1)$	$O(\eta)$	$O(\varepsilon)$	

- Acoustic wave
  - Energy conservation

$$\frac{1}{c}\frac{\partial \epsilon_\parallel}{\partial t} + (\epsilon_0 + p_0)\frac{\partial \beta_\parallel}{\partial z} = -\frac{1}{c}\frac{\partial}{\partial t}\left[\left(2(\epsilon_0+p_0)+\frac{B_0^2}{4\pi}\right)(\delta\boldsymbol{\beta}\cdot\boldsymbol{\beta}_\perp)\right] + \frac{B_0}{4\pi}\left[\boldsymbol{\beta}_\perp\cdot\frac{\partial}{\partial z}(\delta\boldsymbol{B}) + \delta\boldsymbol{\beta}\cdot\frac{\partial \boldsymbol{b}_\perp}{\partial z}\right]$$

  - Momentum equation (z-component)

$$(\epsilon_0+p_0)\frac{1}{c}\frac{\partial \beta_\parallel}{\partial t} + \beta_s^2\frac{\partial \epsilon_\parallel}{\partial z} = -\frac{\partial}{\partial z}\left(\frac{\delta\boldsymbol{B}\cdot\boldsymbol{b}_\perp}{4\pi}\right) + \frac{B_0}{4\pi}\left(\delta\boldsymbol{B}\cdot\frac{1}{c}\frac{\partial \boldsymbol{\beta}_\perp}{\partial t} + \boldsymbol{b}_\perp\cdot\frac{1}{c}\frac{\partial}{\partial t}(\delta\boldsymbol{\beta})\right)$$

- Alfven wave
  - Momentum equations (in xy-plain)

$$\left(\epsilon_0+p_0+\frac{B_0^2}{4\pi}\right)\frac{1}{c}\frac{\partial \boldsymbol{\beta}_\perp}{\partial t} - \frac{B_0}{4\pi}\frac{\partial \boldsymbol{b}_\perp}{\partial z} =$$

$$-(\epsilon_0+p_0)\beta_\parallel\frac{\partial}{\partial z}(\delta\boldsymbol{\beta}) - (1+\beta_s^2)\frac{B_0\epsilon_\parallel}{4\pi\mathcal{E}}\frac{\partial}{\partial z}(\delta\boldsymbol{B}) - \delta\boldsymbol{\beta}\beta_s^2\frac{1}{c}\frac{\partial \epsilon_\parallel}{\partial t} + \frac{B_0}{4\pi}\frac{1}{c}\frac{\partial}{\partial t}(\beta_\parallel\delta\boldsymbol{B})$$

  - Induction euations (in xy-plain)

$$\frac{1}{c}\frac{\partial \boldsymbol{b}_\perp}{\partial t} - B_0\frac{\partial \boldsymbol{\beta}_\perp}{\partial z} = -\frac{\partial}{\partial z}(\beta_\parallel\delta\boldsymbol{B})$$

Neglecting $O(\varepsilon\eta^2)$, $O(\varepsilon^2)$

$$\left(\mathcal{E} \equiv \epsilon_0 + p_0 + \frac{B_0^2}{4\pi}\right)$$

Assumption: the amplitude of the parent wave is not so large

Perturbed equations

	Background	Parent wave	Daughter waves	
		Alfvén wave	Alfvén wave	Sound wave
$\boldsymbol{B} =$	$\boldsymbol{B}_0$	$+\ \boldsymbol{\delta B}$	$+\ \boldsymbol{b}_\perp$	
$\boldsymbol{v} =$		$\boldsymbol{\delta v}$	$+\ \boldsymbol{v}_\perp$	$+\ \boldsymbol{v}_\parallel$
$\epsilon =$	ϵ_0			$+\ \epsilon_\parallel$

- Acoustic wave

Equations are linear with respect to the perturbed quantities

⇩

Finding a solution is equivalent to finding a dispersion relation

⇩

Let us check if there is an unstable mode in the dispersion relation!

Dispersion relation

$$S_0 = k^2\left(\omega^3 + k\omega^2 - 3\omega + k\right)$$

$$S_3 = S_0 - \omega^2(\omega - k) - \eta^2 k\omega\left[\omega - k + \theta^2(\omega + k)(k\omega - 1)\right]$$
$$+ \theta^2\left[\omega^5 + 3k\omega^4 + \left(2k^2 - 3\right)\omega^3 - k\left(4k^2 + 7\right)\omega^2 - k^2\left(4k^2 - 11\right)\omega + k^3\right]$$

$$S_4 = \theta^2 k\omega(\omega - k)\left\{(\omega + k)^2 - 4\right\}$$

$\theta \equiv V_S/V_A$

- Dispersion relation with k and ω as independent variables sound wave

  (unit system ω_0=1, k_0=1)

$$(\omega - k)^2\left(\omega^2 - \theta^2 k^2\right)\left\{(\omega + k)^2 - 4\right\} = \frac{1}{(1+\sigma)^4}\eta^2(\omega - k)\left(S_0 + S_1\sigma + S_2\sigma^2 + S_3\sigma^3 + S_4\sigma^4\right)$$

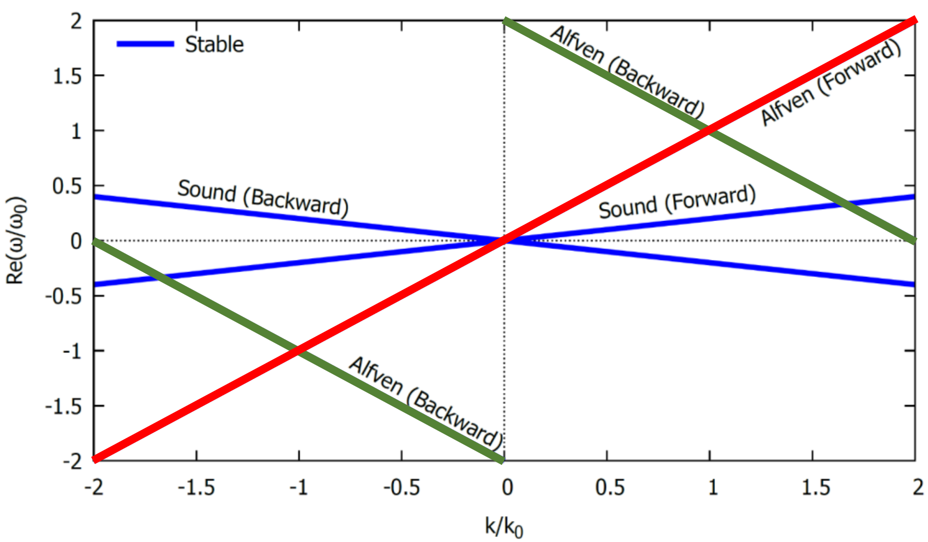
  - η=0 : reproduces the dispersion relation in a uniform plasma

$\eta \equiv \delta B/B_0$

$k_\pm = k \pm k_0, \quad \omega_\pm = \omega \pm \omega_0$

Parent wave **Parent wave**

$\eta = 0$ (no parent wave)



Forward Alfven wave (satisfying resonance)

Backward Alfven wave (satisfying resonance)

Acoustic wave

Dispersion relation

$$S_0 = k^2\left(\omega^3 + k\omega^2 - 3\omega + k\right)$$
$$S_3 = S_0 - \omega^2(\omega - k) - \eta^2 k\omega\left[\omega - k + \theta^2(\omega + k)(k\omega - 1)\right] + \theta^2\left[\omega^5 + 3k\omega^4 + (2k^2 - 3)\omega^3 - k(4k^2 + 7)\omega^2 - k^2(4k^2 - 11)\omega + k^3\right]$$
$$S_4 = \theta^2 k\omega(\omega - k)\left\{(\omega + k)^2 - 4\right\}$$

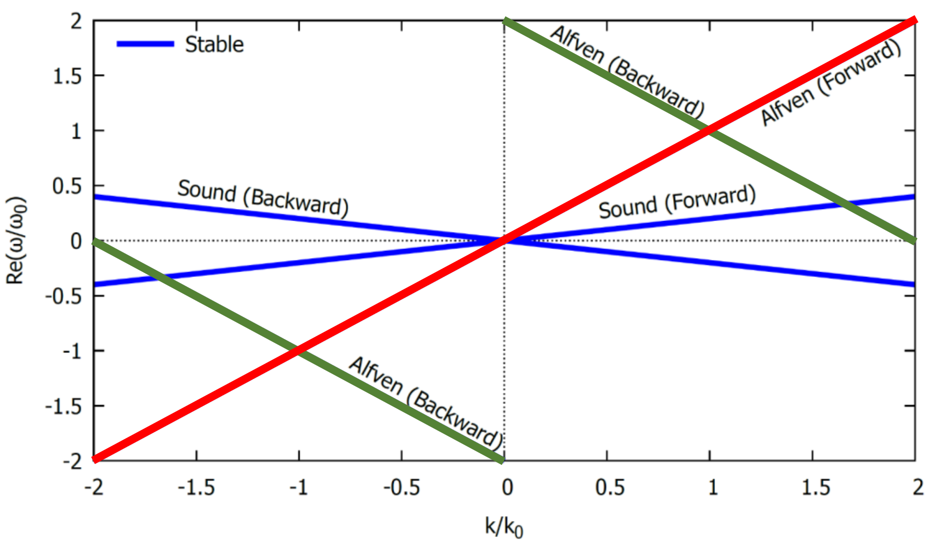
$\theta \equiv V_S/V_A$

- Dispersion relation with k and ω as independent variables sound wave

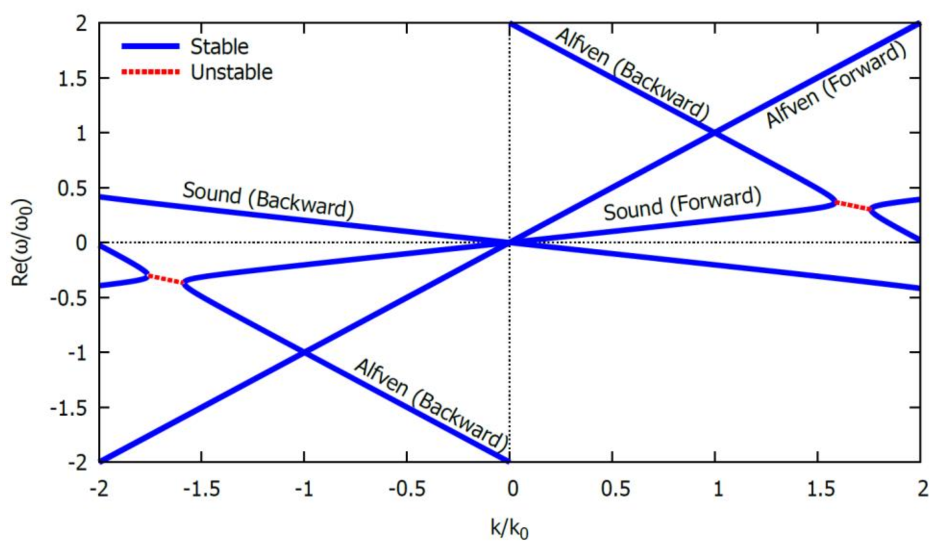
$$(\omega - k)^2\left(\omega^2 - \theta^2 k^2\right)\left\{(\omega + k)^2 - 4\right\} = \frac{1}{(1+\sigma)^4}\eta^2(\omega - k)\left(S_0 + S_1\sigma + S_2\sigma^2 + S_3\sigma^3 + S_4\sigma^4\right)$$

  - η=0 : reproduces the dispersion relation in a uniform plasma
  - η>0 : unstable solution exists for waves where backward propagating Alfvén waves and forward propagating sound waves satisfy the resonance condition

$\eta = 0$ (no parent wave)



$\eta = 0.1$ (parent wave exists)



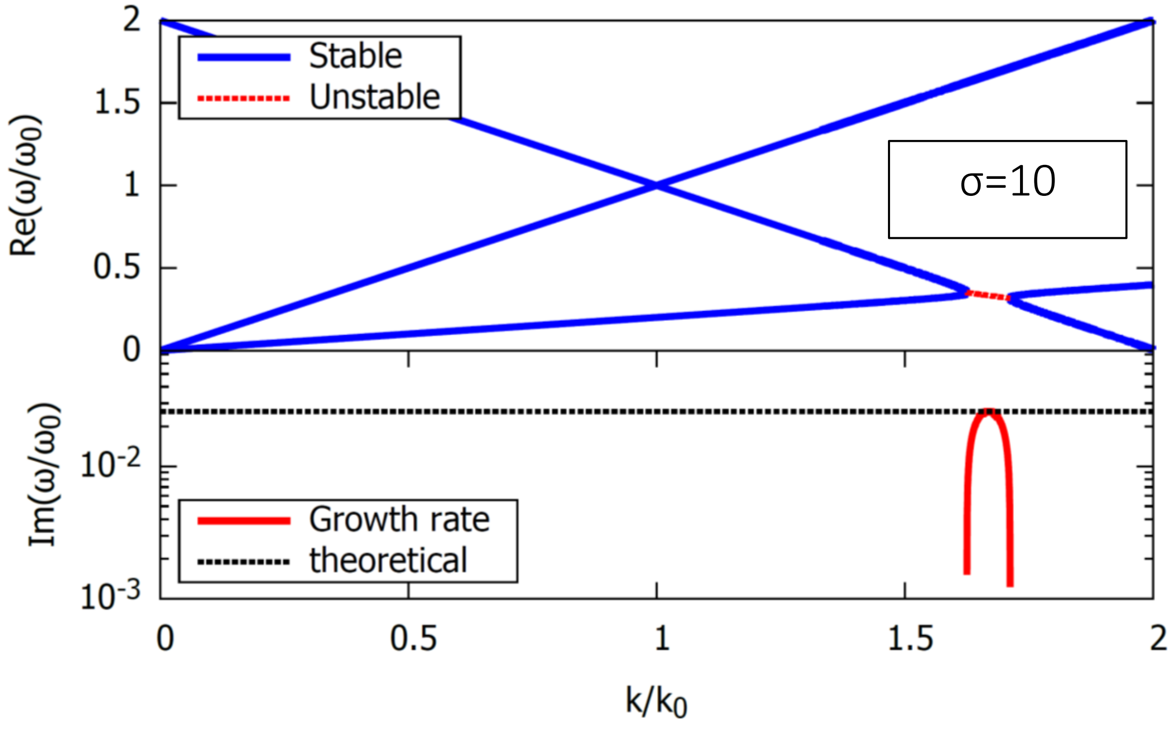
Growth rate of the instability

Alfvén sound Alfvén sound

$$k_0 = k_- + k, \quad \omega_0 = \omega_- + \omega$$

Parent wave Parent wave

- Growth rate (per 1 wave period)
  - Expand around a point that satisfies the resonance condition
  - Growth rate: $\Gamma = \frac{1}{2}\eta\theta^{-1/2}\frac{\sqrt{1-\theta}}{1+\theta}\frac{1+\sigma\left(1+\theta^2\right)}{\left(1+\sigma\right)^{3/2}} \sim \frac{1}{2}\eta\theta^{-1/2}\left(1+\sigma\right)^{-1/2}$



Key parameters

$$\eta \equiv \delta B/B_0$$

$$\theta \equiv V_S/V_A$$

$$\sigma \equiv \frac{B_0^2}{4\pi(\rho c^2 + \epsilon_{\rm int} + p)}$$

Growth rate of the instability

e.g., NS magnetosphere

~10^{-2} (for σ~10^{3-4})

- Growth rate (per 1 wave period)

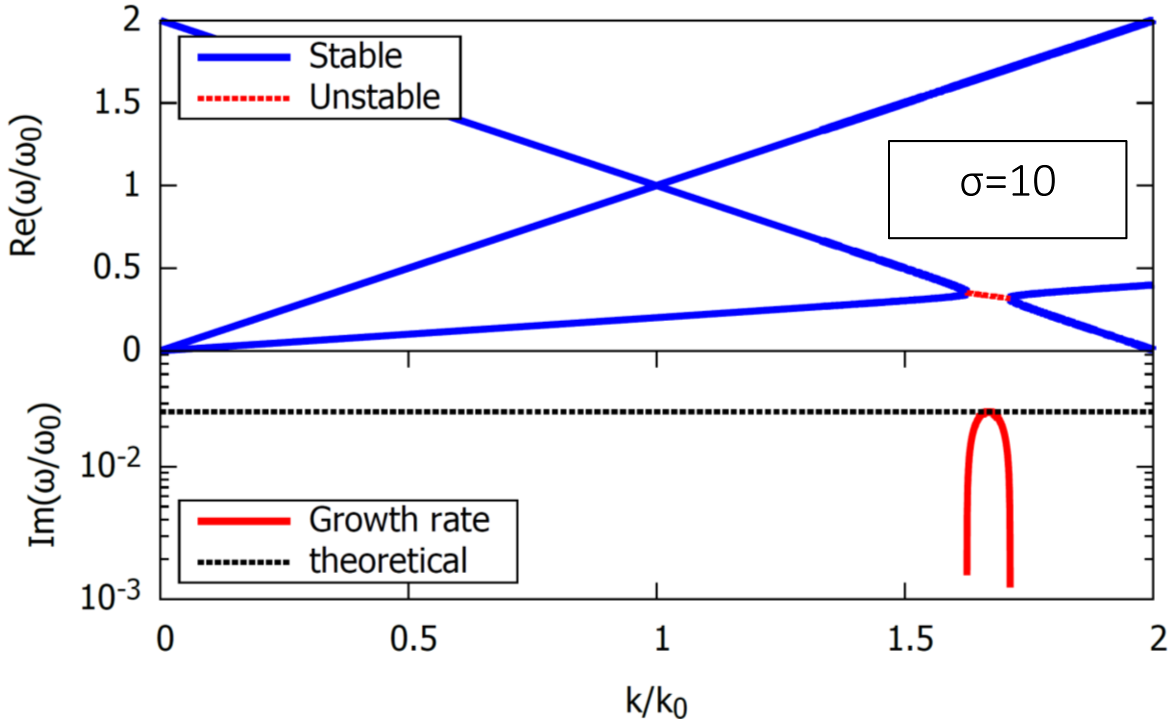
$$\Gamma = \frac{1}{2}\eta\theta^{-1/2}\frac{\sqrt{1-\theta}}{1+\theta}\frac{1+\sigma\left(1+\theta^2\right)}{(1+\sigma)^{3/2}} \sim \frac{1}{2}\eta\theta^{-1/2}(1+\sigma)^{-1/2}$$

Same as non-rela case

Rela. correction

Although larger σ suppresses the instability,

Alfven waves are still unstable with respect to the decay instability.



Key parameters

$$\eta \equiv \delta B / B_0$$

$$\theta \equiv V_S / V_A$$

$$\sigma \equiv \frac{B_0^2}{4\pi(\rho c^2 + \epsilon_{\rm int} + p)}$$

Summary

- What do we want to know?
  - In order to understand the mechanism of the FRB, we study the propagation of Alfven wave in the relativistically magnetized plasma without force-free approximation
- What did we did?
  - We derived the equations of sound and Alfven-like perturbations on a background with finite amplitude Alfven waves and obtained the dispersion relation for perturbations.
- What did we find?
  - As in the non-relativistic case, even for relativistically magnetized plasma, there is a decay instability in which Alfven waves excite forward propagating sound waves and backward propagating Alfven waves.
  - Obtained an analytical expression for the growth rate of the instability, finding that growth rate of the decay instability becomes smaller for large σ.

Backup

Sigma dependence

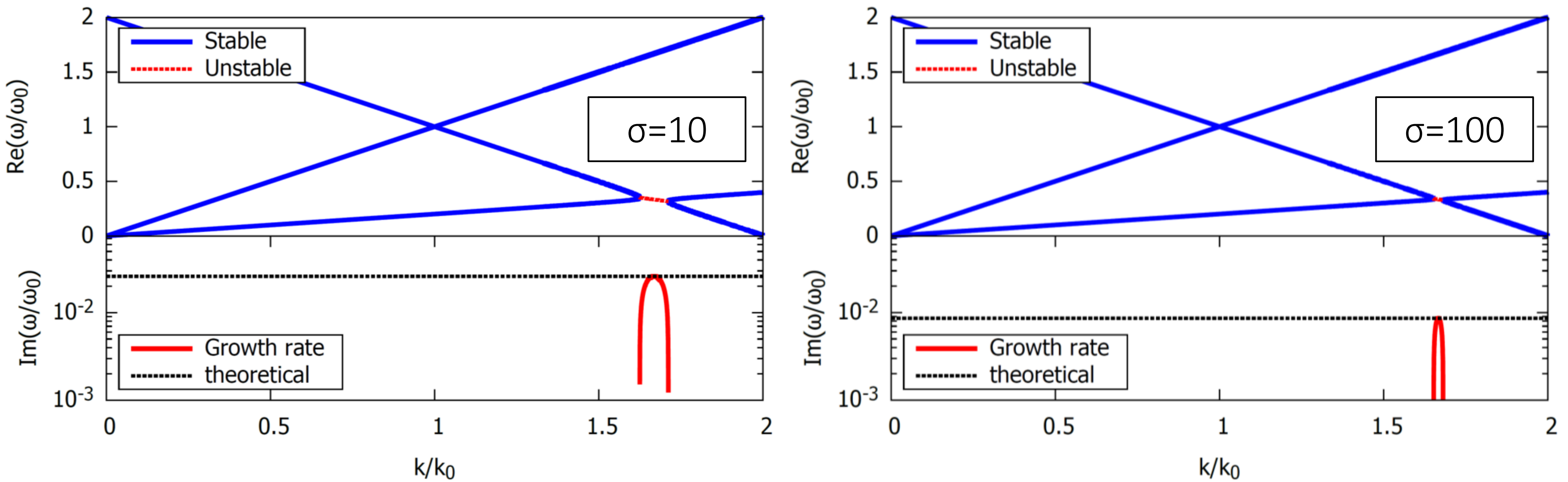
$$\theta \equiv V_S/V_A$$

$$\sigma \equiv \frac{B_0^2}{4\pi(\rho c^2 + \epsilon_{\rm int} + p)}$$

- Growth rate of the decay instability

$$\Gamma = \frac{1}{2}\eta\theta^{-1/2}\frac{\sqrt{1-\theta}}{1+\theta}\frac{1+\sigma\left(1+\theta^2\right)}{(1+\sigma)^{3/2}} \sim \frac{1}{2}\eta\theta^{-1/2}(1+\sigma)^{-1/2}$$

The larger σ, the narrower the range of wavenumbers that become unstable, and the instability disappears at σ → ∞.



Sigma dependence

$$\theta \equiv V_S/V_A$$

$$\sigma \equiv \frac{B_0^2}{4\pi(\rho c^2 + \epsilon_{\mathrm{int}} + p)}$$

- Growth rate of the decay instability

$$\Gamma = \frac{1}{2}\eta\theta^{-1/2}\frac{\sqrt{1-\theta}}{1+\theta}\frac{1+\sigma\left(1+\theta^2\right)}{\left(1+\sigma\right)^{3/2}} \sim \frac{1}{2}\eta\theta^{-1/2}\left(1+\sigma\right)^{-1/2}$$

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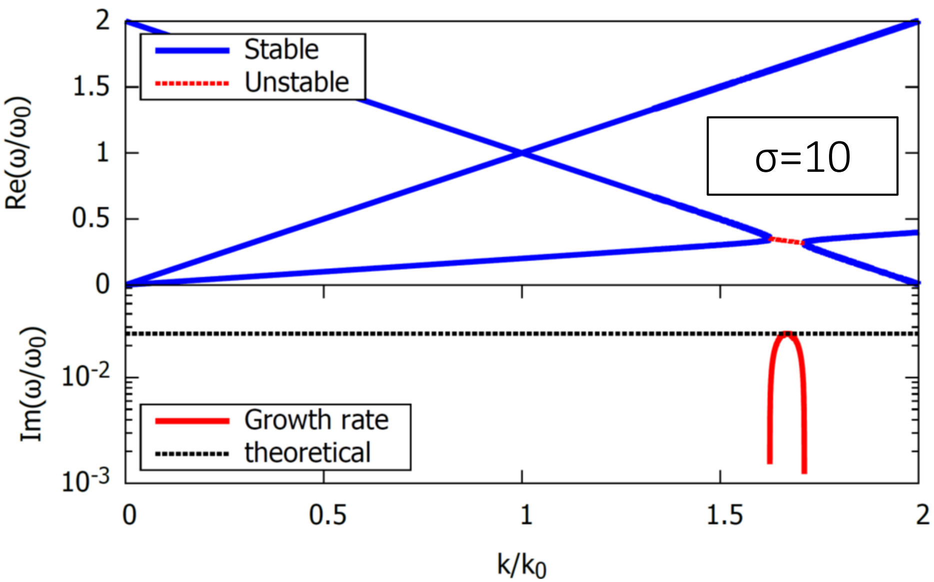
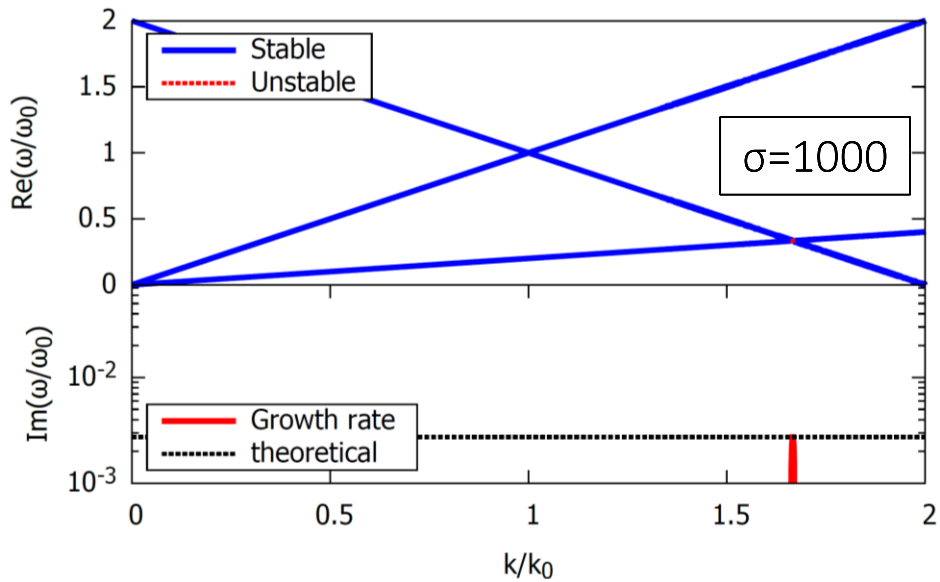


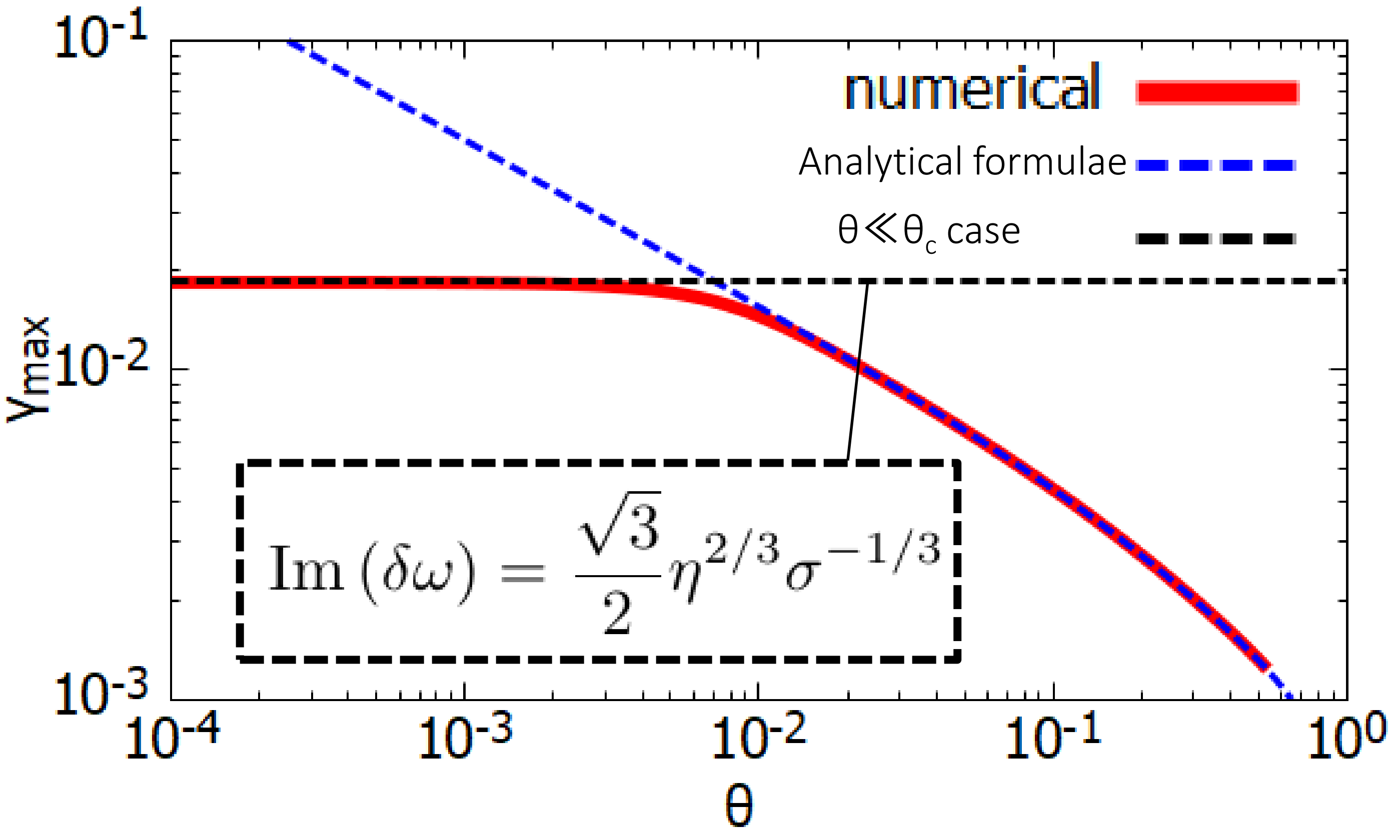


θ dependence

$\theta \equiv V_S/V_A$

$$\sigma \equiv \frac{B_0^2}{4\pi(\rho c^2 + \epsilon_{\mathrm{int}} + p)}$$

- Analytical formulae (for general sigma)
  - Formulae : $\Gamma = \frac{1}{2}\eta\theta^{-1/2}\frac{\sqrt{1-\theta}}{1+\theta}\frac{1+\sigma\left(1+\theta^2\right)}{(1+\sigma)^{3/2}} \sim \frac{1}{2}\eta\theta^{-1/2}(1+\sigma)^{-1/2}$
  - For θ<<1, a different expansion is needed, growth rate will no longer depend on θ



Discussion : Why instability vanishes?

- Why does the instability vanish for σ → ∞?
  - In the non-rela case, why the wave becomes unstable?

    Alfvén + Alfvén : Fluctuation of the magnetic pressure → generates sound wave

    Alfvén + Sound : Fluctuation of the inertia

    →Magnetic tension per mass is fluctuated→generates Alfven wave

  - In the relativistic case, the force perpendicular to the background magnetic field is dominated by that originating from the displacement current.
  - Alfven waves become like free photons and are no longer affected by the plasma.

$$\mathcal{E}\frac{1}{c}\frac{\partial \boldsymbol{\beta}_\perp}{\partial t} - \frac{B_0}{4\pi}\frac{\partial \boldsymbol{b}_\perp}{\partial z} = \underbrace{-(\epsilon_0+p_0)\,\beta_\parallel \frac{\partial}{\partial z}(\delta\boldsymbol{\beta})}_{\text{Convection}} \underbrace{- (1+\beta_s^2)\,\frac{B_0\epsilon_\parallel}{4\pi\mathcal{E}}\frac{\partial}{\partial z}(\delta \boldsymbol{B})}_{\text{Inertia}} \underbrace{- \delta\boldsymbol{\beta}\beta_s^2\frac{1}{c}\frac{\partial\epsilon_\parallel}{\partial t}}_{\text{Inertia (Internal energy)}} \underbrace{+ \frac{B_0}{4\pi}\frac{1}{c}\frac{\partial}{\partial t}(\beta_\parallel\delta\boldsymbol{B})}_{\text{Displacement current}}$$