October 25-27 YITP workshop

Scattering of Fast Radio Bursts in strongly magnetized electron-positron plasma

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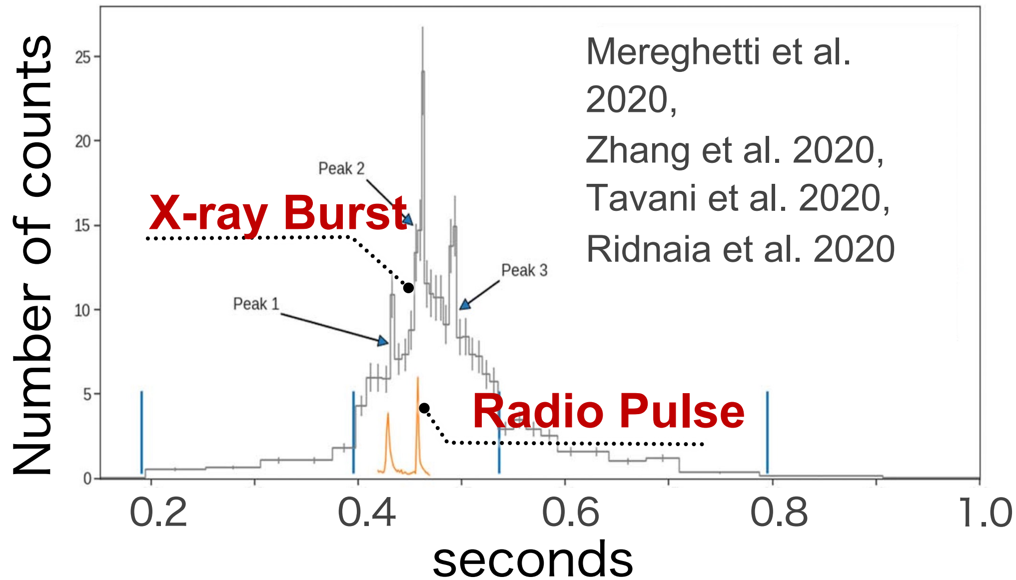
What is a Fast Radio Burst (FRB) ?

FRBs are the brightest radio transients, first discovered in 2007.

Lorimer et al. 2007

- FRBs origin is not fully understood.
- FRB 20200428 from the Galactic magnetar marked a step towards understanding the phenomena.

CHIME/FRB et al. 2020,
Bochenek et al. 2020



I would like to clarify the FRB emission in magnetar.

FRB Emission Models in Magnetars

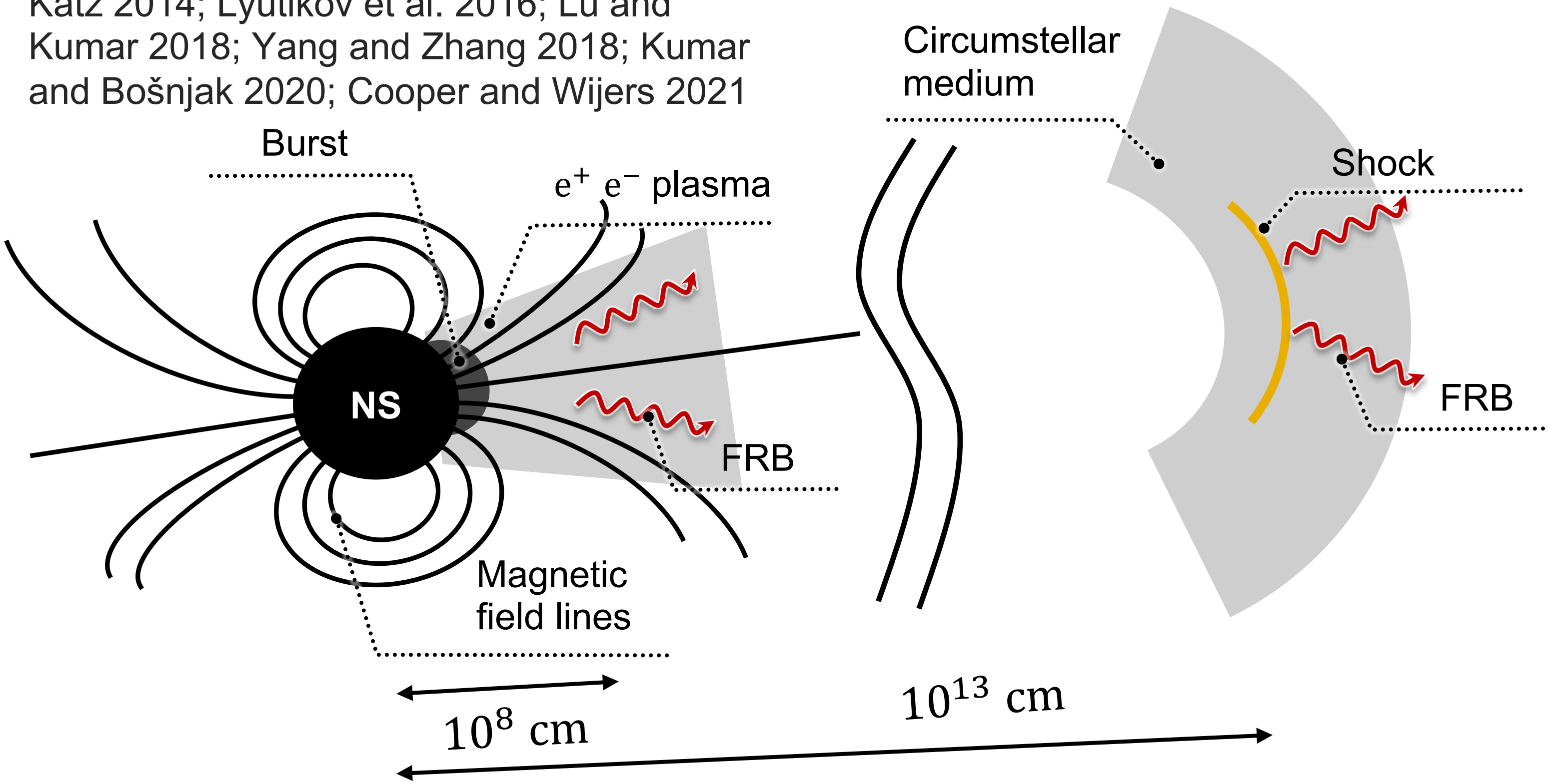
Two major locations for FRB generation in the magnetar are considered.

Magnetosphere models

Katz 2014; Lyutikov et al. 2016; Lu and Kumar 2018; Yang and Zhang 2018; Kumar and Bošnjak 2020; Cooper and Wijers 2021

Far-away models

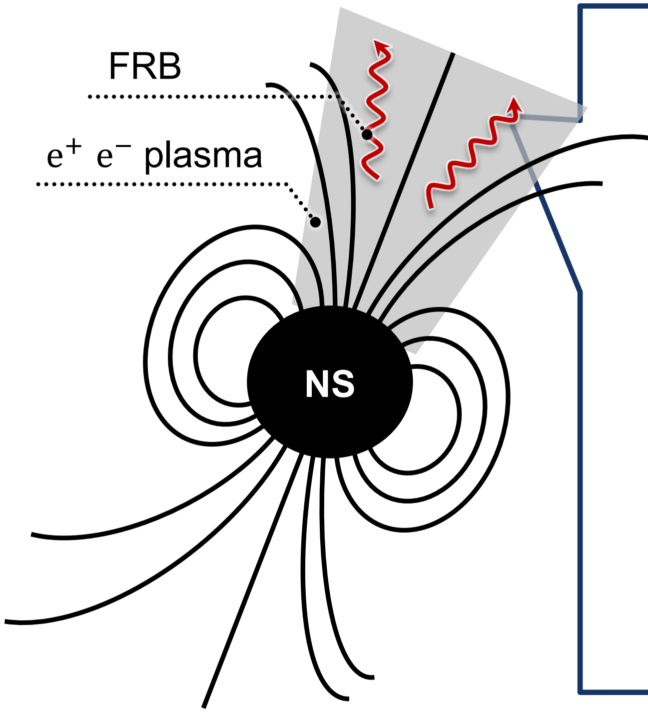
Lyubarsky 2014; Murase et al. 2016; Waxman 2017; Margalit et al. 2020; Beloborodov 2020



Discussion on Scattering in the Magnetosphere Model

Coherent waves (FRBs) can be significantly attenuated by induced Compton scattering in a magnetar's magnetosphere.

Blandford and Scharlemann 1975; Wilson and Rees 1978;
Wilson 1982; Lyubarskii and Petrova 1996; Lyubarsky 2008



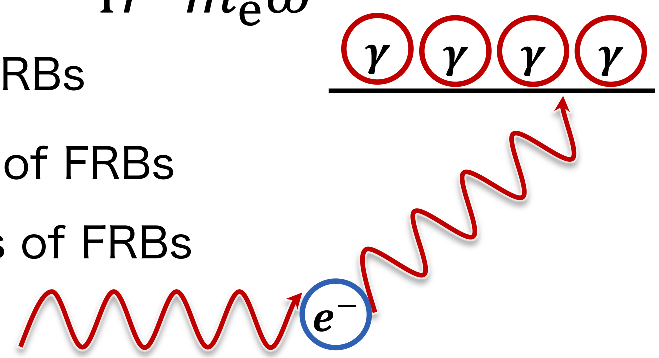
$$\tau_{\mathrm{T}} \sim n_{\mathrm{e}} \sigma_{\mathrm{T}} l \ll 1$$: Thomson optical depth

$$\tau_{\mathrm{ind}} \sim n_{\mathrm{e}} \sigma_{\mathrm{T}} c \Delta t_{\mathrm{FRB}} \frac{3\pi L_{\mathrm{FRB}}}{4 r^2 m_{\mathrm{e}} \omega^3} \gg 1$$

L_γ : Luminosity of FRBs

r : Scattering point of FRBs

Δt_{FRB} : Pulse widths of FRBs



Focus on the cross-section affecting the reaction rate.

Suppression Effects in Scattering Cross-Section

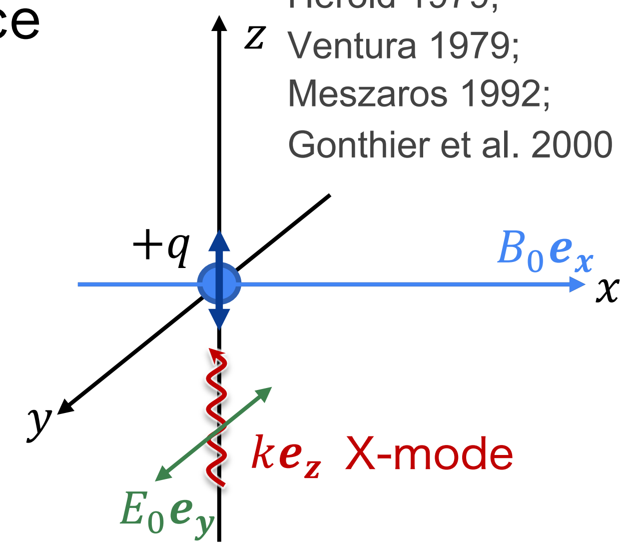
In a strongly magnetized $e^{\pm}$ plasma, scattering can be suppressed by various effects.

- X-mode waves in pair plasma experience significant suppression of scattering.

$$\sigma_{\mathrm{X}} = \frac{1}{2}\sigma_{\mathrm{T}}\left[\left(\frac{\omega}{\omega+\omega_{\mathrm{c}}}\right)^2 + \left(\frac{\omega}{\omega-\omega_{\mathrm{c}}}\right)^2\right]$$

$$\sim \sigma_{\mathrm{T}}\left(\frac{\omega}{\omega_{\mathrm{c}}}\right)^2 \qquad , \omega_{\mathrm{c}} \equiv \frac{eB_0}{m_{\mathrm{e}}c}$$

Canuto et al. 1971; Herold 1979; Ventura 1979; Meszaros 1992; Gonthier et al. 2000



- It is argued that the drift motion of electrons and positrons significantly cancels out the scattering.

Lyubarsky 2020; Golbraikh and Lyubarsky 2023

I want to cohesively understand the scattering in magnetized $e^{\pm}$ plasma.

Scattering of Electron-Positron ($e^{\pm}$) Pairs

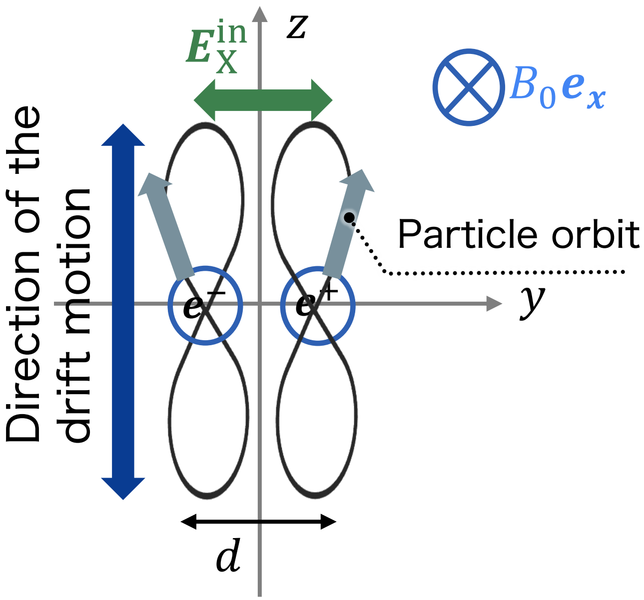
At first glance, when $e^{\pm}$ pairs scatter with X-mode waves,
the drift motion seems to cancel out the scattering.

- Assuming the electron and positron are stationary before scattering.

$$\sigma_{\mathrm{X}}^{(1)} \sim \sigma_{\mathrm{T}} \left(\frac{\omega}{\omega_{\mathrm{c}}}\right)^2$$

$$\Downarrow$$

$$\sigma_{\mathrm{X}}^{(1)} \sim \sigma_{\mathrm{T}} \left(\frac{\omega_0}{\omega_{\mathrm{c}}}\right)^2 \{1 - \cos(k_x d)\} \sim 0$$



Is the same scattering cancellation effect realized in multi-particle scattering ?

Collective Thomson Scattering

Collective Thomson scattering considers the interactions among many charged particles in plasma.

Fejer 1960; Dougherty and Farley 1960; Salpeter 1960; Hutchinson 2002; Froula et al. 2012

$$\frac{d\sigma^{(1)}}{d\Omega d\omega_1} = \frac{3\sigma_T}{8\pi} S(\boldsymbol{k}_1 - \boldsymbol{k}_0, \omega_1 - \omega_0) \sin^2\theta$$

in un-magnetized ion-electron plasma

$$\underbrace{S(\boldsymbol{k}, \omega)} \equiv \lim_{V,T\to\infty} \frac{\left\langle \left| \widetilde{\delta n_-}(\boldsymbol{k}, \omega) \right|^2 \right\rangle_{\text{ensemble}}}{V T n_e}$$

Spectral density function

ω_1: scattering frequency

ω_0: incident frequency

V: scattering region

T: scattering time

n_e: uniform density of plasma

δn_-: density fluctuations of electrons

- We must consider a strong magnetic field.
- We must replace ions with positrons in plasma.

→ **We must also consider scattering from positrons.**

Evaluation of Plasma Density Fluctuations

Plasma density fluctuations are determined by expanding the Vlasov equations for electric field variations.

$$\frac{\partial F_\pm}{\partial t} + \boldsymbol{v}_{0\pm} \cdot \frac{\partial F_\pm}{\partial \boldsymbol{r}} \pm \frac{e}{m_\mathrm{e}} \left(\delta \boldsymbol{E}(\boldsymbol{r}, t) + \boldsymbol{v}_{0\pm} \times \boldsymbol{B}_0 \right) \cdot \frac{\partial F_\pm}{\partial \boldsymbol{v}_{0\pm}} = 0$$

Electric field produced by density fluctuations

Cyclotron motions

$$F_\pm(\boldsymbol{r}, \boldsymbol{v}, t) = F_{0\pm}(\boldsymbol{v}) + \delta F_\pm(\boldsymbol{r}, \boldsymbol{v}, t) = \sum_{j=1}^{N_\pm} \delta^3\big(\boldsymbol{r} - \boldsymbol{r}_j(t)\big) \delta^3\big(\boldsymbol{v} - \boldsymbol{v}_j(t)\big)$$

Plasma distribution function

$$\Rightarrow \quad \widetilde{\delta n}_\pm(k, \omega - i\gamma) = \int \mathrm{d}^3\boldsymbol{v} \int \mathrm{d}^3\boldsymbol{r} \int_0^\infty \mathrm{d}t \, e^{-(i\omega+\gamma)t + i\boldsymbol{k}\cdot\boldsymbol{r}} \, \widetilde{\delta F}_\pm(\boldsymbol{k}, \omega, \boldsymbol{v})$$

$\delta n_\pm$: density fluctuations for positrons/electrons

Cross-Section in Magnetized $e^{\pm}$ Plasma

The scattering cross-section is described by
four types of spectral density functions.

$$\frac{4\pi\omega_0}{\sigma_\mathrm{T}}\frac{\mathrm{d}\sigma^{(1)}}{\mathrm{d}\Omega\mathrm{d}\omega_1}=\frac{3\omega_0}{8\pi}\left(\frac{\omega_0^2}{\omega_0^2-\omega_\mathrm{c}^2}\right)^2\left[\underbrace{(S_{++}+S_{+-}+S_{-+}+S_{--})(1-\sin^2\theta\sin^2\varphi)}_{\text{Direction of incident electric field: }\hat{\sigma}_{\mathrm{Electric}}}\right.$$

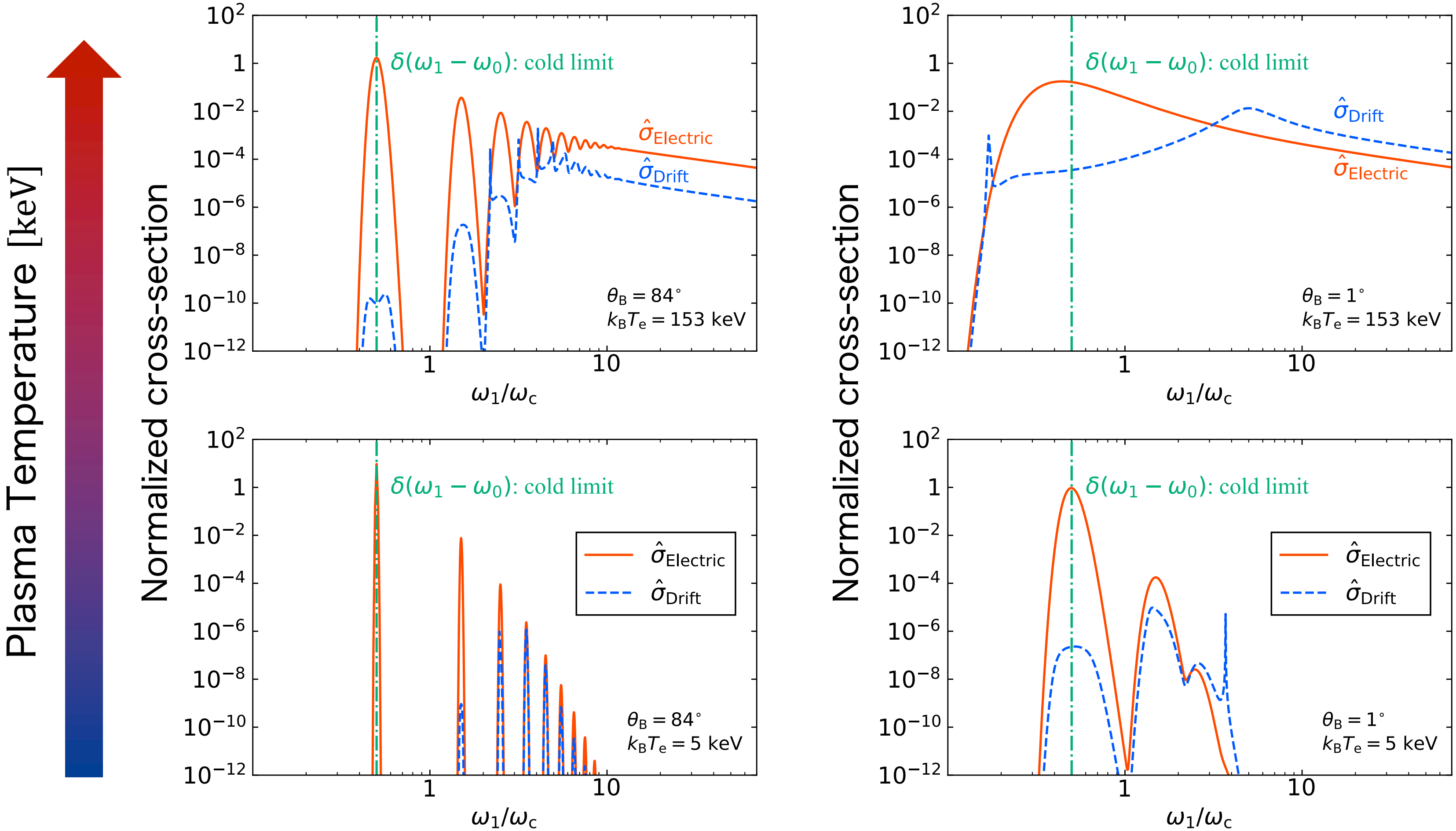
$$\left.\underbrace{+\left(\frac{\omega_\mathrm{c}}{\omega_0}\right)^2(S_{++}-S_{+-}-S_{-+}+S_{--})\sin^2\theta}_{\text{Direction of drift motion: }\hat{\sigma}_{\mathrm{Drift}}}\right]$$

$$S_{\pm\pm}(\boldsymbol{k},\omega)\equiv\lim_{V,T\to\infty}\frac{\left\langle\left|\widetilde{\delta n_\pm}(\boldsymbol{k},\omega)\right|^2\right\rangle_{\mathrm{ensemble}}}{VTn_e}$$

$$S_{\pm\mp}(\boldsymbol{k},\omega)\equiv\lim_{V,T\to\infty}\frac{\left\langle\widetilde{\delta n_\pm}(\boldsymbol{k},\omega)\widetilde{\delta n_\mp}^{\,*}(\boldsymbol{k},\omega)\right\rangle_{\mathrm{ensemble}}}{VTn_e}$$

$\delta n_\pm$: density fluctuations for positrons/electrons

Scattering Spectrum in Maxwellian Distribution



- The spectrums of waves propagating perpendicular to the magnetic field peaks at cyclotron intervals.

Scattering Cross-Section in Cold Plasma

The cross-section is consistent with single-particle scattering in the cold plasma limit.

$$\frac{\mathrm{d}\sigma_{\mathrm{cold}}^{(1)}}{\mathrm{d}\Omega\mathrm{d}\omega_1} = \frac{3\sigma_{\mathrm{T}}}{8\pi}\left(\frac{\omega_0^2}{\omega_0^2-\omega_{\mathrm{c}}^2}\right)^2\left[(1-\sin^2\theta\sin^2\varphi)+\left(\frac{\omega_{\mathrm{c}}}{\omega_0}\right)^2\sin^2\theta\right]\delta(\omega_1-\omega_0)$$

↓

$$\sigma_{\mathrm{cold}}^{(1)} = \frac{1}{2}\sigma_{\mathrm{T}}\left[\left(\frac{\omega_0}{\omega_0+\omega_{\mathrm{c}}}\right)^2+\left(\frac{\omega_0}{\omega_0-\omega_{\mathrm{c}}}\right)^2\right] = \sigma_{\mathrm{X}}$$

- The magnetic field still suppresses X-mode wave scattering.
- Drift motion of electrons and positrons does **NOT** cancel scattering.

$$\sigma_{\mathrm{T}} \longrightarrow \sigma_{\mathrm{X}}^{(1)} \sim \sigma_{\mathrm{T}}\left(\frac{\omega_0}{\omega_{\mathrm{c}}}\right)^2$$

Why is the Scattering NOT Canceled out ?

It is necessary to incorporate the particle statistics of plasma just before scattering.

- For $e^{\pm}$ pair scattering: The cross-terms remain.

$$\frac{\mathrm{d}\sigma_{\mathrm{drift}}^{(2)}}{\mathrm{d}\Omega} \propto \left|\widetilde{\boldsymbol{E}_{\mathrm{rad}}}\right|^2 \propto \left(e^{i\boldsymbol{k}\cdot\boldsymbol{r}_+} - e^{i\boldsymbol{k}\cdot\boldsymbol{r}_-}\right)\left(e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_+} - e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_-}\right)$$

$$\propto 2\{1 - \cos(k_x d)\}\frac{\mathrm{d}\sigma_{\mathrm{drift}}^{(1)}}{\mathrm{d}\Omega}$$

- For collective scattering: The cross-terms averages to zero.

e.g. Froula et al. 2012

$$\frac{\mathrm{d}\sigma_{\mathrm{drift}}^{(N_+ + N_-)}}{\mathrm{d}\Omega} \propto \left|\widetilde{\boldsymbol{E}_{\mathrm{rad}}}\right|^2 \propto \left\langle\left|\widetilde{\delta n_+} - \widetilde{\delta n_-}\right|^2\right\rangle_{\mathrm{ensemble}}$$

$$\propto \left\langle\left\{\sum_{j=1}^{N_-} e^{i\boldsymbol{k}\cdot\boldsymbol{r}_j(t=0)}(\cdots) + \sum_{h=1}^{N_+} e^{i\boldsymbol{k}\cdot\boldsymbol{r}_h(t=0)}(\cdots)\right\}\left\{\sum_{s=1}^{N_-} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_s(t=0)}(\cdots) + \sum_{g=1}^{N_+} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_g(t=0)}(\cdots)\right\}\right\rangle_{\mathrm{ensemble}}$$

$$\propto (N_+ + N_-)\frac{\mathrm{d}\sigma_{\mathrm{drift}}^{(1)}}{\mathrm{d}\Omega} + 0$$

Physical Interpretation (Without Magnetic Filed)

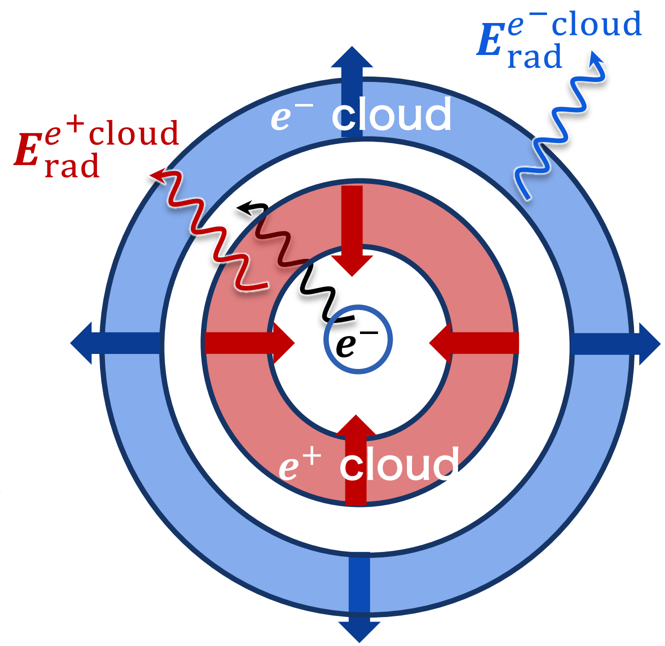
Without a magnetic field, the collective effect does NOT appear at all in $e^{\pm}$ plasma scattering.

Sincell and Krolik 1992

- In plasma, the response of particle groups to a single particle can be viewed as an averaged cloud (Hartree-Fock-like description).

- The radiative electric field, being charge-sign independent, results in mutually canceling radiation from the clouds.

$$\boldsymbol{E}^{\pm}_{\rm rad} = \pm\frac{e}{cR}\{\boldsymbol{n}\times(\boldsymbol{n}\times\dot{\boldsymbol{\beta}}_{\pm})\}_{\rm ret} = \boldsymbol{E}^{+}_{\rm rad} = \boldsymbol{E}^{-}_{\rm rad}$$

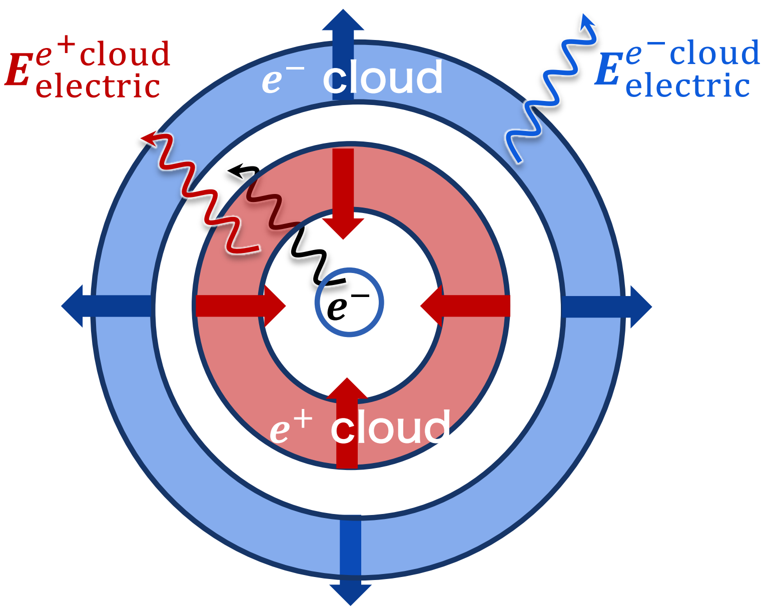


Physical Interpretation (With Magnetic Field)

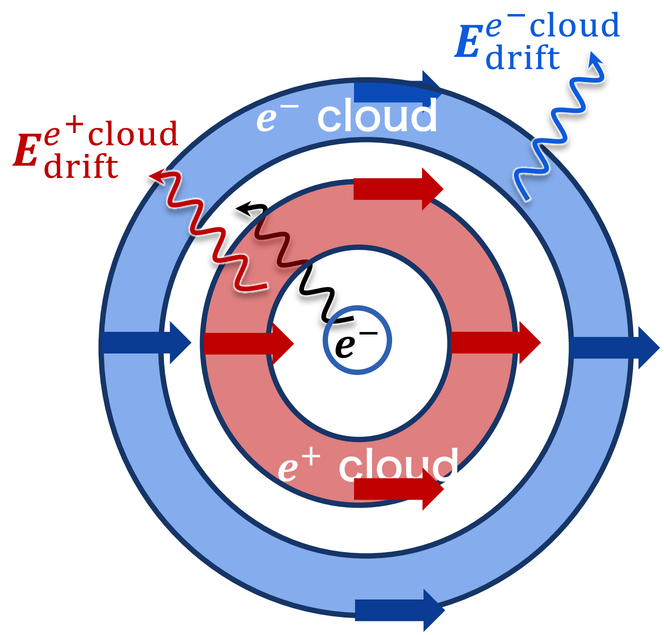
With a magnetic field, we have newly found that **the cross-section retains a collective effect.**

arXiv: 2310.02306

Direction of incident electric field: $\hat{\sigma}_{\mathrm{Electric}}$



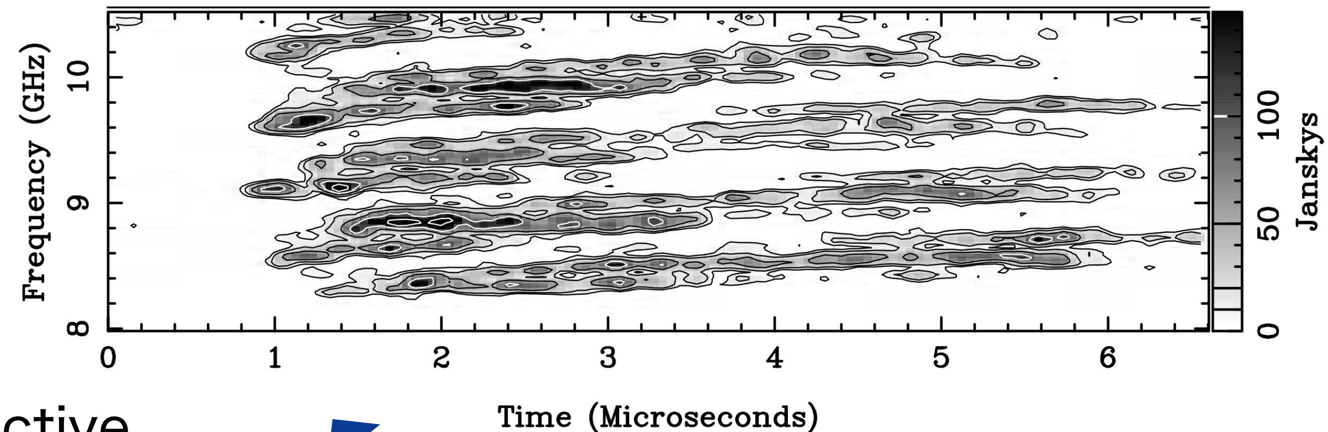
Direction of drift motion: $\hat{\sigma}_{\mathrm{Drift}}$



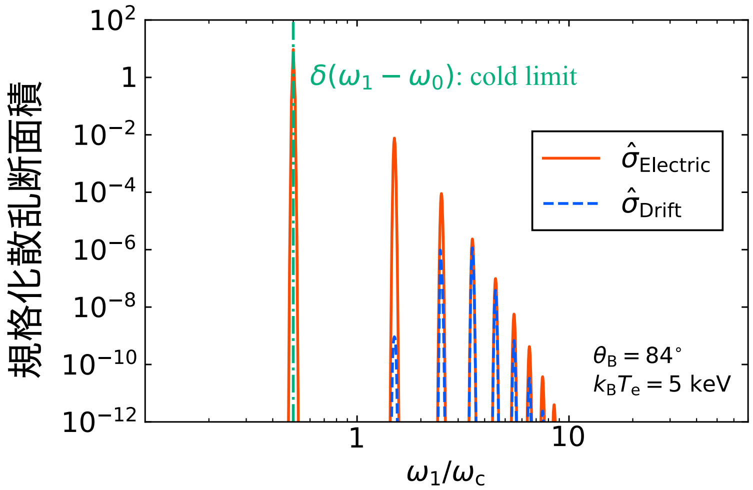
Observational Implications of Scattering Spectrum

Multiple radiation bands have been observed in the Crab Pulsar.

Hankins and Eilek 2007



- Traces of collective scattering in the pulsar magnetosphere ?



We may extract information about magnetic field and plasma in scattering regions.

Conclusion

Collective Thomson scattering in magnetized $e^{\pm}$ plasma has revealed the following:

- Considering particle interactions, the scattering wave cancellation effect still does **NOT** occur.

- X-mode FRBs can extend their propagation range due to magnetic field effects but not significantly due to particle interactions.

$$\sigma_{\mathrm{X}}^{(1)} \sim \sigma_{\mathrm{T}} \left(\frac{\omega_0}{\omega_{\mathrm{c}}}\right)^2$$

$$\Rightarrow \quad \tau_{\mathrm{ind}}^{\mathrm{X}} \sim n_{\mathrm{e}} \sigma_{\mathrm{T}} c \Delta t_{\mathrm{FRB}} \left(\frac{\omega_0}{\omega_{\mathrm{c}}}\right)^2 \frac{3\pi L_{\mathrm{FRB}}}{4 r^2 m_{\mathrm{e}} \omega_0^3}$$

議論　密度揺らぎの物理的解釈

散乱断面積を特徴づける密度揺らぎは3つの項に分類される

Froula et al. 2012

$$\widetilde{\delta n_-}(\boldsymbol{k},\omega) = \left(1 - \frac{H}{\varepsilon_\mathrm{L}}\right)\sum_{j=1}^{N_-} e^{i\boldsymbol{k}\cdot\boldsymbol{r}_j(t=0)} \times (\text{電子サイクロトロン運動の情報})$$

$$+\frac{H}{\varepsilon_L}\sum_{h=1}^{N_+} e^{i\boldsymbol{k}\cdot\boldsymbol{r}_h(t=0)} \times (\text{陽電子サイクロトロン運動の情報})$$

H: 電気感受率

ε_L: 縦誘電率

Noncollective term

- 1. 各電子が磁場中でサイクロトロン運動していることを表す項

Collective term ➡ 誘電率 (プラズマ効果) 依存性あり

- 2. 各電子が他の電子集団の雲に与える効果の項
- 3. 各陽電子が他の電子集団の雲に与える効果の項

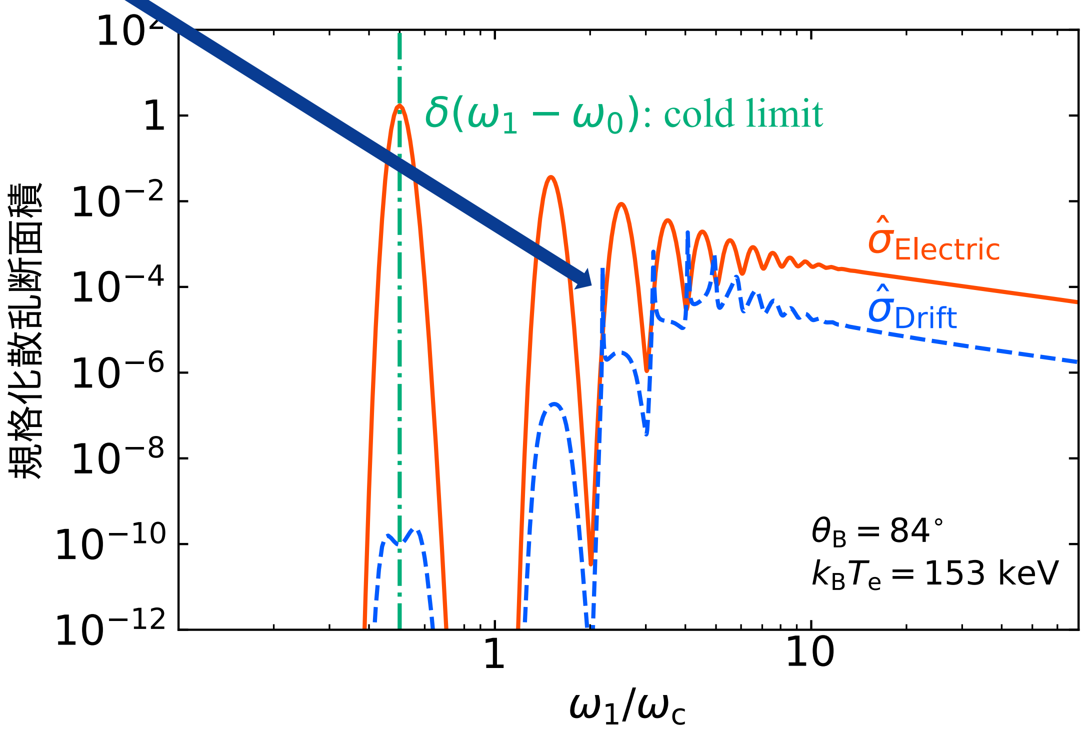
議論　散乱スペクトルの奇妙なピーク

Collective effectありの散乱スペクトルに
鋭いピークがある

- 鋭いピークは
  縦誘電率がゼロになる
  周波数に現れる

↓

プラズマ固有モードの場所



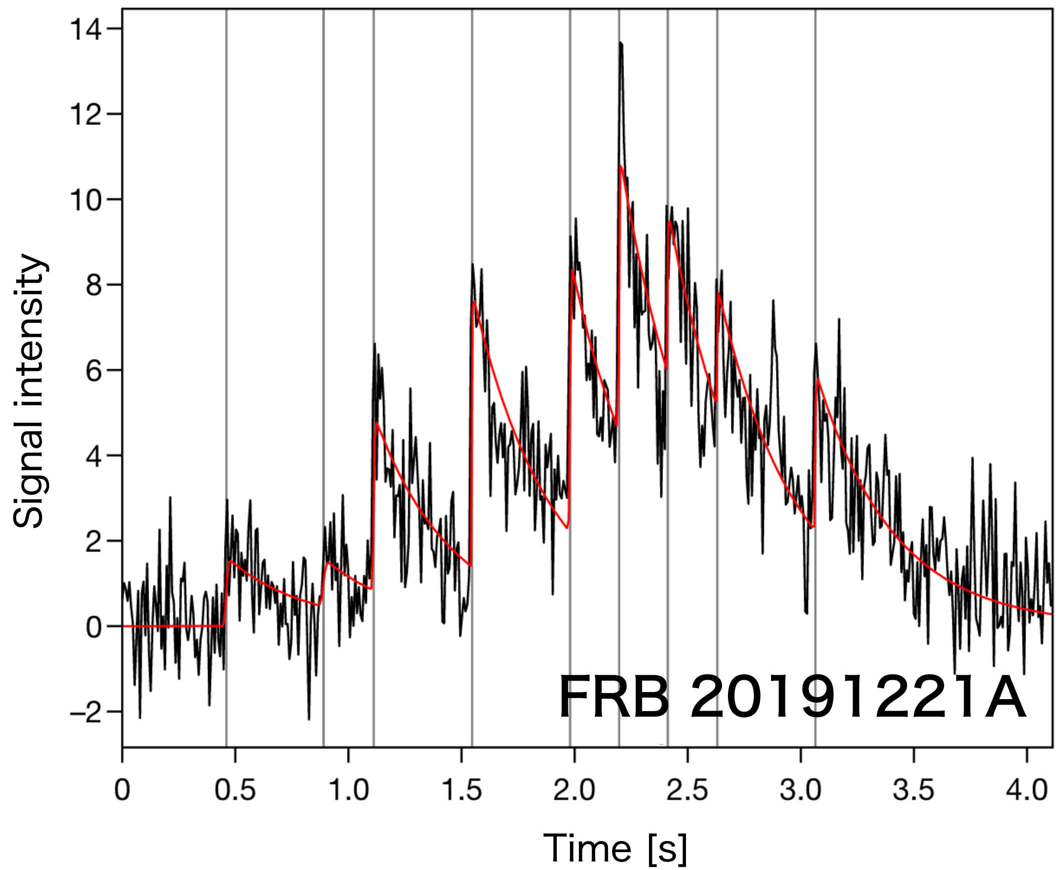
鋭いピークが何の固有モード由来か明らかにしたい
(おそらくプラズマ振動かBernstein波)

予備スライド　磁気圏から離れたモデルの問題点 (1)

パルサー放射のような**周期的なパルスを放つ**FRBが観測された

CHIME/FRB Collaboration et al. 2022

- 周期：数100ミリ秒
- 磁気圏から離れたモデルでこのような放射を作れるかは不明



パルサー放射と似た機構でマグネター磁気圏で作られる?

予備スライド　磁気圏から離れたモデルの問題点 (2)

Giant pulseのような**ナノ秒スケール**のFRBが観測された

Nimmo et al. 2022

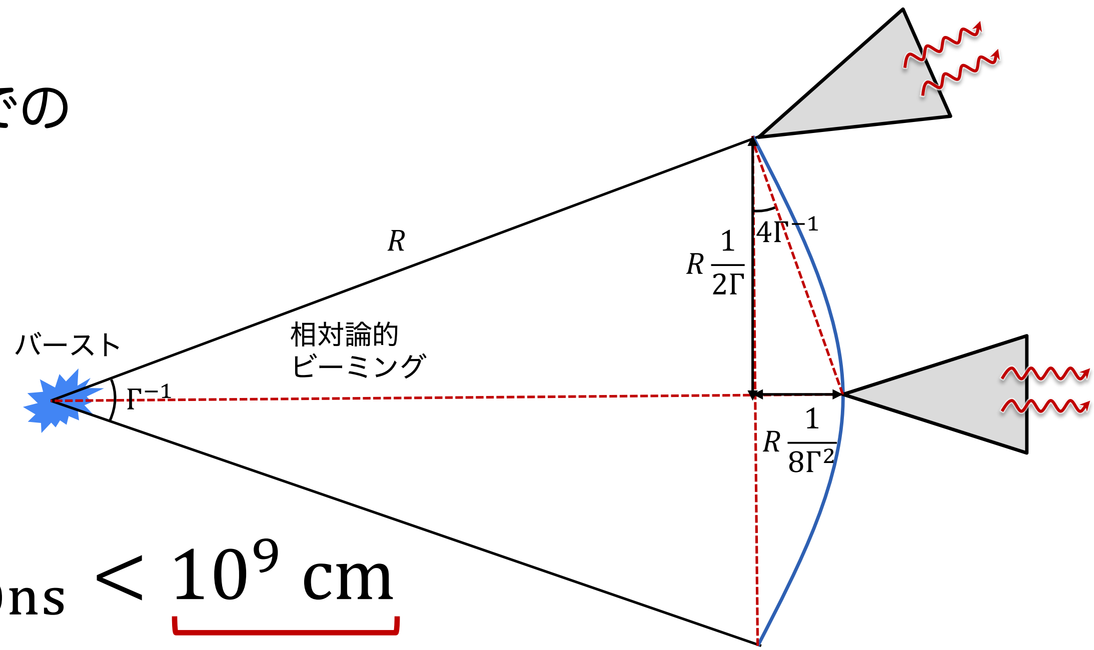
- FRB 20200120Eの持続時間：60 ns

- 磁気圏から離れたモデルでの
  FRB発生場所の推定

$$\Delta t_{\mathrm{FRB}} \sim \frac{R}{8c\Gamma^2}$$

$$R \sim 10^7\ \mathrm{cm}\ \Gamma_2^2 \Delta t_{10\mathrm{ns}} < 10^9\ \mathrm{cm}$$

磁気圏の大きさ



磁気圏から離れたモデルでは発生場所が
マグネター表面から近すぎる

予備スライド　FRBの輝度温度

黒体放射を仮定するとFRBの輝度温度が高すぎる

$$T_{\mathrm{b}} \sim \frac{L_\gamma}{8\pi\nu^3\Delta t_{\mathrm{FRB}}^2 k_{\mathrm{B}}} \sim 10^{36}\ \mathrm{K}\ \frac{L_{\gamma,42}}{\omega_{\mathrm{GHz}}^3\Delta t_{\mathrm{ms}}^2}$$

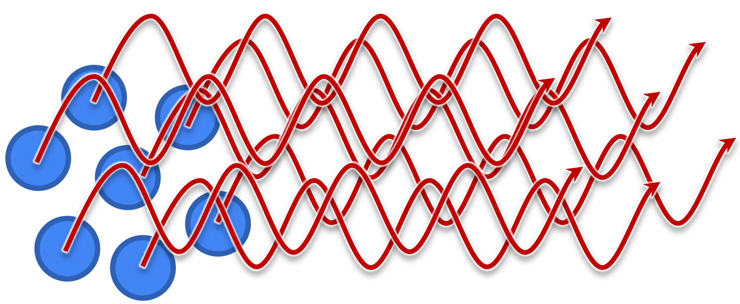
L_γ : FRBの光度

Δt_{FRB} : FRBのパルス幅

ω : FRBの周波数

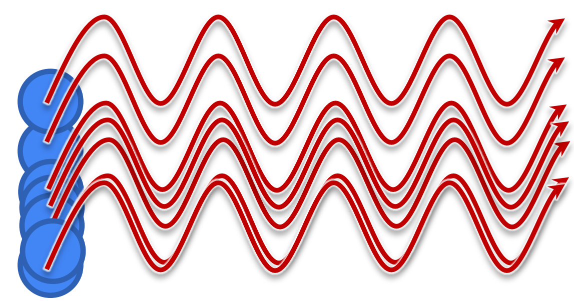
- 理論で予言される限界輝度温度 (10^{12} K) をはるかに上回る

  Kellermann and Pauliny-Toth 1969



(放射エネルギー) ∝ (粒子数)

コヒーレント放射



(放射エネルギー) ∝ (粒子数)^2

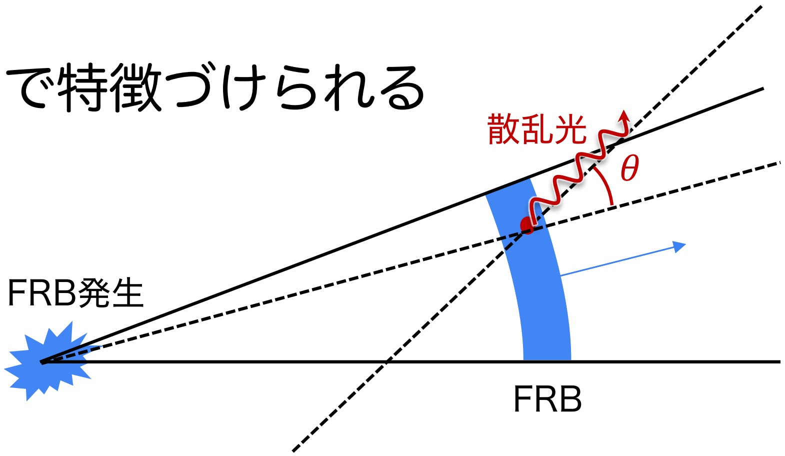
FRBは粒子集団からのコヒーレント放射である

散乱反応率が入射光の占有数に比例して増幅される

- FRBの減衰度合いは**散乱光の増幅因子**で特徴づけられる
  (“有効的”光学的厚さ：$\tau_{\rm ind}$)

$$n(\omega, \mathbf{\Omega}) \equiv n_0 e^{\tau_{\rm ind}}$$

散乱光の占有数　散乱光の種



$$\tau_{\rm ind} = \frac{3\pi\sigma_{\rm T} L_\gamma n_{\rm e} c\Delta t_{\rm FRB}}{4 r_0^2 m_{\rm e} \omega^3}$$

Lyubarsky 2008

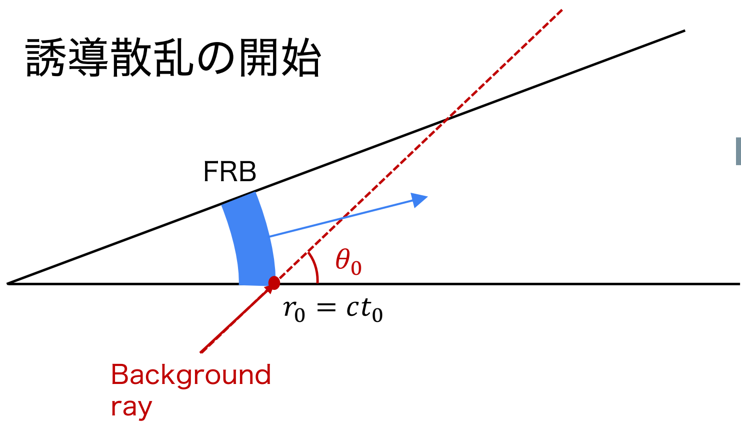
$$\sim 2\times10^{22} \frac{L_{\gamma,42} n_{\rm e,14} c\Delta t_{,-3}}{r_8^2 \omega_{\rm GHz}^3} \gg 1$$

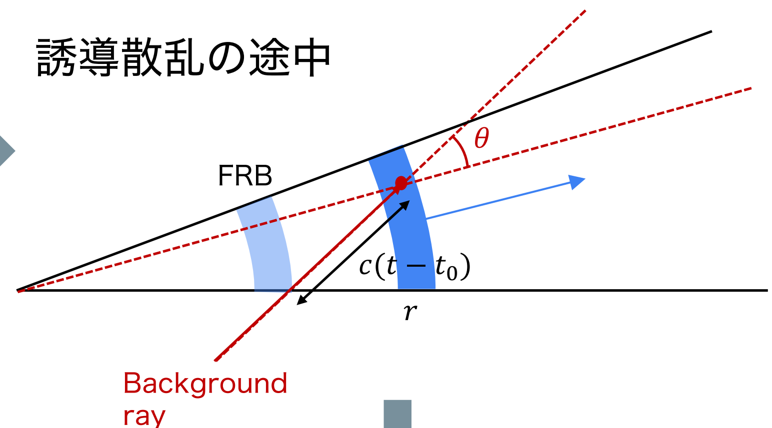
$\sigma_{\rm T}$：トムソン断面積
L_γ：FRBの光度
r_0：FRBの発生場所
ω：FRBの周波数
$\Delta t_{\rm FRB}$：FRBのパルス幅

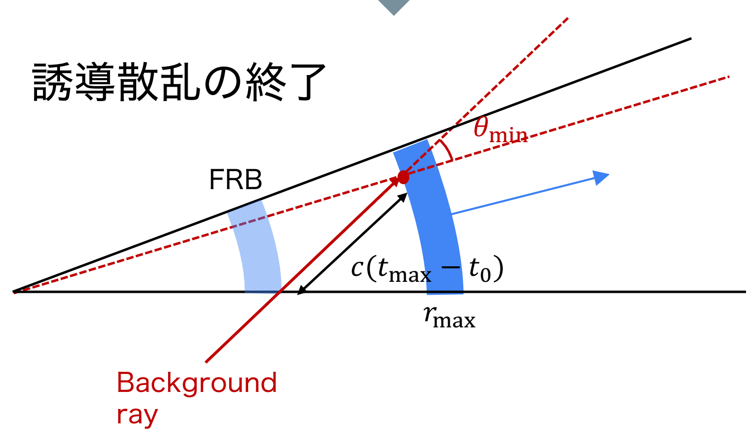
幅広いパラメーター範囲でFRBが大きく減衰する

予備スライド　誘導コンプトン散乱の描像

散乱光はFRBのパルスの中にいる間だけ成長する







$$\underbrace{n(\omega, \mathbf{\Omega})}_{\text{散乱光の占有数}} \equiv \underbrace{n_0}_{\text{散乱光の種}} e^{\tau_{\rm ind}}$$

FRBは散乱光から

占有数を奪われて減衰

予備スライド　電子・陽電子の$E \times B$ ドリフトによる影響

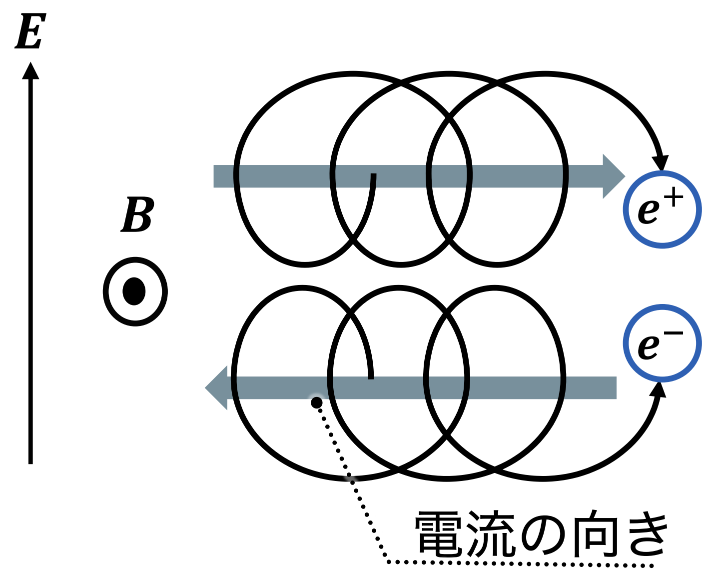
一様磁場中に電場が加わると電荷に依らない運動が生じる

$$\boldsymbol{v}_\perp = c\frac{\boldsymbol{E}\times\boldsymbol{B}}{B^2}$$

- 電子と陽電子のドリフト方向は同じ

➡ **ドリフトで生じる電流は打ち消す**

$$j_+^{\mathrm{drift}} + j_-^{\mathrm{drift}} \sim 0$$



$E\times B$ドリフトの描像

散乱においてこの効果を考えたら散乱が抑制されるのではないか?

Lyubarsky 2020

強磁場中の散乱粒子はドリフト運動が支配的

- 入射電磁波：X-mode波

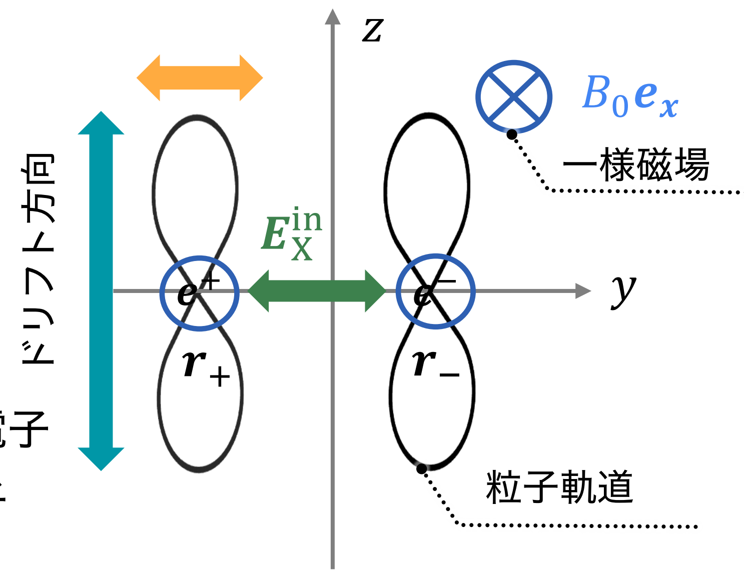
$$\boldsymbol{E}_{\mathrm{X}}^{\mathrm{in}}(t) = E_0 e^{-i\omega_0 t}\begin{pmatrix}0\\1\\0\end{pmatrix}$$

- 散乱粒子のEOM

$$m_{\mathrm{e}}\dot{\boldsymbol{v}}_{\pm} = \pm e\boldsymbol{E}_{\mathrm{X}}^{\mathrm{in}} \pm \frac{e}{c}\boldsymbol{v}_{\pm}\times\boldsymbol{B}_0$$

+: 陽電子

−: 電子



⬇

$$j_{\pm y}^{\mathrm{particle}}(\boldsymbol{r},t) \simeq i\frac{e^2E_0}{m_{\mathrm{e}}}\frac{\omega_0}{\omega_0^2-\omega_{\mathrm{c}}^2}e^{-i\omega_0 t}\delta^3(\boldsymbol{r}-\boldsymbol{r}_{\pm}) = \mathcal{O}(\omega_{\mathrm{c}}^{-2})$$

$$\wedge\!\!\wedge$$

$$j_{\pm z}^{\mathrm{particle}}(\boldsymbol{r},t) \simeq \mp\frac{e^2E_0}{m_{\mathrm{e}}}\frac{\omega_{\mathrm{c}}}{\omega_0^2-\omega_{\mathrm{c}}^2}e^{-i\omega_0 t}\delta^3(\boldsymbol{r}-\boldsymbol{r}_{\pm}) = \mathcal{O}(\omega_{\mathrm{c}}^{-1})$$

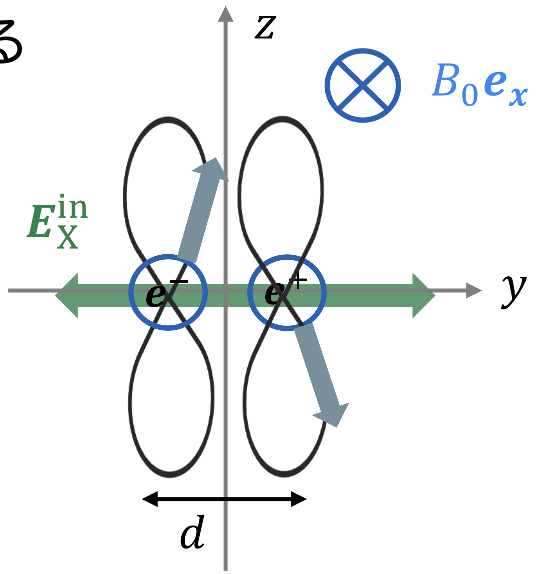
ドリフト方向の電流

電子・陽電子対が作る電流・電場

- 電子・陽電子各々が作る電流密度を足し合わせる

$$\boldsymbol{j}_{\mathrm{p}}(\boldsymbol{r},t)=\left[e\boldsymbol{v}_{+}\delta\left(x-\frac{d}{2}\right)-e\boldsymbol{v}_{-}\delta\left(x+\frac{d}{2}\right)\right]\delta(y)\delta(z)$$

陽電子の寄与　電子の寄与



- 放射電場は電磁場ポテンシャルの波動方程式から求める

$$\begin{cases}\left(k^2-\dfrac{\omega^2}{c^2}\right)\tilde{\phi}(\boldsymbol{k},\omega)=4\pi\tilde{\rho}_{\mathrm{p}}\\ \left(k^2-\dfrac{\omega^2}{c^2}\right)\widetilde{\boldsymbol{A}}(\boldsymbol{k},\omega)=\dfrac{4\pi}{c}\tilde{\boldsymbol{j}}_{\mathrm{p}}\end{cases}$$

$$\widetilde{\boldsymbol{E}}(\boldsymbol{k},\omega)=-i\boldsymbol{k}\tilde{\phi}(\boldsymbol{k},\omega)+i\frac{\omega}{c}\widetilde{\boldsymbol{A}}(\boldsymbol{k},\omega)$$

予備スライド　電子・陽電子対の散乱

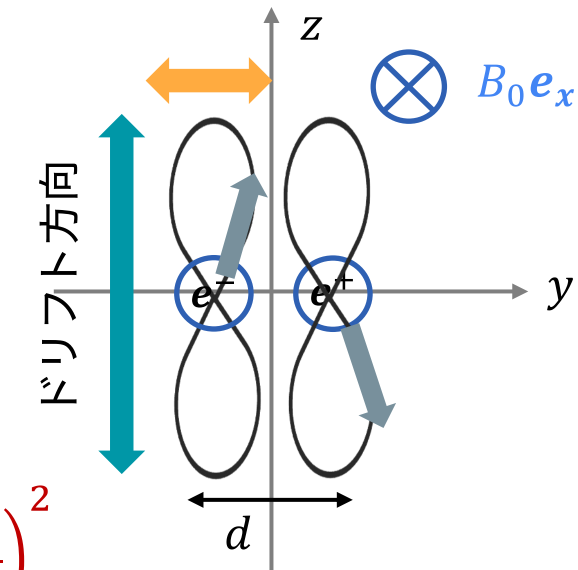
電子・陽電子対が出す散乱エネルギー

- 単位時間当たりの散乱エネルギーの時間平均

$$\left\langle P^{(2)}\right\rangle = \left\langle P_y^{(2)}\right\rangle + \left\langle P_z^{(2)}\right\rangle$$

8の字運動の小さい方向

8の字運動の大きい方向(ドリフト方向)



$$= \frac{4e^4E_0^2}{3m_e^2c^3}\frac{\omega_0^4}{(\omega_0^2-\omega_c^2)^2} + \frac{2e^4E_0^2}{15m_e^2c^3}\frac{\omega_0^2\omega_c^2}{(\omega_0^2-\omega_c^2)^2}\left(\frac{\omega_0 d}{c}\right)^2$$

散乱波の打ち消し効果

$$= \mathcal{O}\left(\left(\frac{\omega_0}{\omega_c}\right)^4\right) + \mathcal{O}\left(\left(\frac{\omega_0}{\omega_c}\right)^2\left(\frac{\omega_0 d}{c}\right)^2\right)$$

$$\sim 10^{-12}\,\frac{\omega_9^2}{d_{-5}^2}$$

ドリフト運動に起因する放射が大きく抑制される

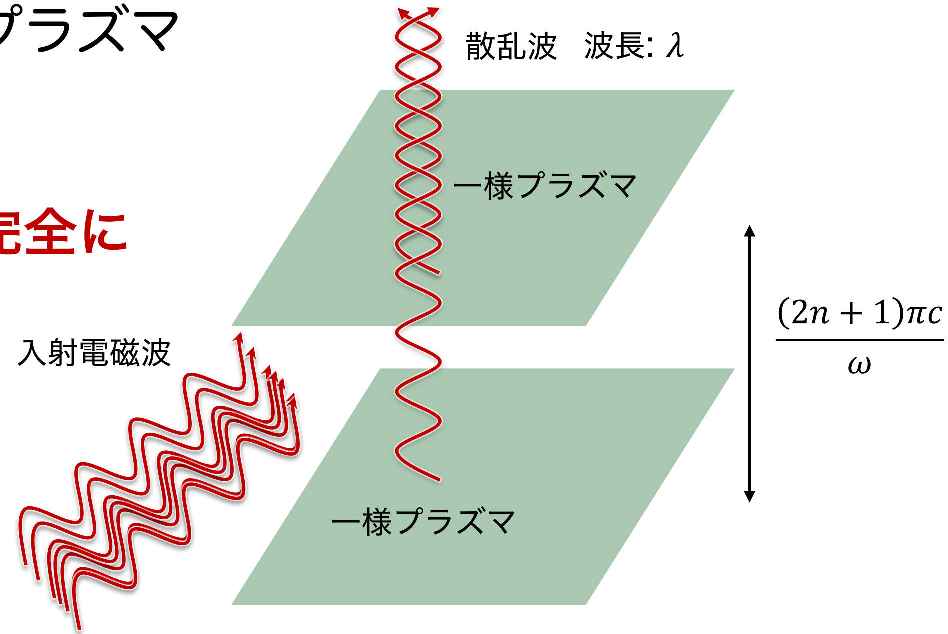
予備スライド　電子・陽電子プラズマの散乱

プラズマの一様密度成分からの散乱は無視できる

Bekefi 1966

$$\boldsymbol{j}_{\mathrm{plasma}}(\boldsymbol{r},t) = \underbrace{e\boldsymbol{v}_{+}(\boldsymbol{r},t)[\cancel{n_{\mathrm{e}}} + \delta n_{+}(\boldsymbol{r},t)]}_{\text{陽電子の寄与}} - \underbrace{e\boldsymbol{v}_{-}(\boldsymbol{r},t)[\cancel{n_{\mathrm{e}}} + \delta n_{-}(\boldsymbol{r},t)]}_{\text{電子の寄与}}$$

- 散乱波の半波長分離れた一様プラズマの薄い板のペアを考える
- 薄い板からの散乱波は互いに**完全にキャンセルする**
- 散乱領域全体に渡って同様の議論が成り立つ



電磁波の散乱は**プラズマの密度揺らぎにより生じる**

プラズマ密度揺らぎの評価

- プラズマ密度揺らぎは散乱前の粒子集団に対する
  無衝突ボルツマン方程式から求まる

$$\frac{\partial F_\pm}{\partial t} + \boldsymbol{v}_{0\pm} \cdot \frac{\partial F_\pm}{\partial \boldsymbol{r}} \pm \frac{e}{m_\mathrm{e}} \left(\underbrace{\delta \boldsymbol{E}(\boldsymbol{r}, t)} + \underbrace{\boldsymbol{v}_{0\pm} \times \boldsymbol{B}_0} \right) \cdot \frac{\partial F_\pm}{\partial \boldsymbol{v}_{0\pm}} = 0$$

散乱前に密度揺らぎが作る電場　サイクロトロン運動

$$\underbrace{F_\pm(\boldsymbol{r}, \boldsymbol{v}, t)} = F_{0\pm}(\boldsymbol{v}) + \delta F_\pm(\boldsymbol{r}, \boldsymbol{v}, t) = \sum_{j=1}^{N_\pm} \delta^3\left(\boldsymbol{r} - \boldsymbol{r}_j(t)\right) \delta^3\left(\boldsymbol{v} - \boldsymbol{v}_j(t)\right)$$

プラズマ分布関数

$$\widetilde{\delta n}_\pm(k, \omega - \underbrace{i\gamma}) = \int \mathrm{d}^3\boldsymbol{v} \int \mathrm{d}^3\boldsymbol{r} \int_0^\infty \mathrm{d}t \, e^{-(i\omega+\gamma)t + i\boldsymbol{k}\cdot\boldsymbol{r}} \, \widetilde{\delta F}_\pm(\boldsymbol{k}, \omega, \boldsymbol{v})$$

無限小の減衰因子

プラズマ密度揺らぎの評価

- プラズマ密度揺らぎは散乱前の粒子集団に対する
無衝突ボルツマン方程式から求まる

$$\widetilde{\delta n}_{\pm}(k,\omega-\underbrace{i\gamma}_{\text{無限小の減衰因子}})=\int \mathrm{d}^3\boldsymbol{v}\int \mathrm{d}^3\boldsymbol{r}\int_0^{\infty}\mathrm{d}t\, e^{-(i\omega+\gamma)t+i\boldsymbol{k}\cdot\boldsymbol{r}}\underbrace{\widetilde{\delta F}_{\pm}(\boldsymbol{k},\omega,\boldsymbol{v})}_{\text{分布関数の摂動}}$$

- 磁化プラズマの密度揺らぎはHughes et al. (1988)
により導出されている

$$\widetilde{\delta n}_{-}(\boldsymbol{k},\omega)=\alpha_{-}(\boldsymbol{k},\omega)\sum_{j=1}^{N_-}e^{i\boldsymbol{k}\cdot\boldsymbol{r}_j(t=0)}\times(\text{電子サイクロトロン運動の情報})$$

$$+\beta_{-}(\boldsymbol{k},\omega)\sum_{h=1}^{N_+}e^{i\boldsymbol{k}\cdot\boldsymbol{r}_h(t=0)}\times(\text{陽電子サイクロトロン運動の情報})$$

散乱直前の各々の粒子位相

Spectral density functionの評価

$$S_{--}(\boldsymbol{k},\omega) \equiv \lim_{V,T\to\infty} \frac{\left\langle \left| \widetilde{\delta n_-}(\boldsymbol{k},\omega) \right|^2 \right\rangle_{\text{ensemble}}}{VTn_e}$$

$$\sim \left\langle \left\{ \sum_{j=1}^{N_-} e^{i\boldsymbol{k}\cdot\boldsymbol{r}_j(t=0)} \times (\cdots) + \sum_{h=1}^{N_+} e^{i\boldsymbol{k}\cdot\boldsymbol{r}_h(t=0)} \times (\cdots) \right\} \times \left\{ \sum_{s=1}^{N_-} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_s(t=0)} \times (\cdots) + \sum_{g=1}^{N_+} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_g(t=0)} \times (\cdots) \right\} \right\rangle_{\text{ensemble}}$$

赤：電子運動の寄与

緑：陽電子運動の寄与

- **同じ粒子同士の積の項**のみ初期位相が打ち消して残る ($j = s, h = g$の項)
- 他の項は粒子運動のランダム性により**平均してゼロになる**

予備スライド　散乱における磁場の有無による違い

磁場無しの場合電子・陽電子プラズマ中の散乱は
collective effect (誘電率依存性) が完全に打ち消す

Sincell and Krolik 1992

- 磁場ありの場合、粒子の運動方向によってcollective effect が異なる

$$\underbrace{S_{++}+S_{+-}+S_{-+}+S_{--}}_{\text{入射電場方向: } \hat{\sigma}_{\mathrm{Electric}}} = 4\sqrt{\pi}\sum_{l=-\infty}^{+\infty}\exp\left\{-\frac{1}{2}\left(\frac{v_{\mathrm{th}}k_\perp}{\omega_{\mathrm{c}}}\right)^2\right\} I_l\left[\frac{1}{2}\left(\frac{v_{\mathrm{th}}k_\perp}{\omega_{\mathrm{c}}}\right)^2\right]\frac{\exp\left[-\left(\frac{\omega-l\omega_{\mathrm{c}}}{k_x v_{\mathrm{th}}}\right)^2\right]}{k_x v_{\mathrm{th}}}$$

ε_{L} : 縦誘電率　　H: 電気感受率

$$\underbrace{S_{++}-S_{+-}-S_{-+}+S_{--}}_{\text{ドリフト運動方向: } \hat{\sigma}_{\mathrm{Drift}}}$$

$$= 4\sqrt{\pi}\left\{1-4\mathrm{Re}\left(\frac{H}{\varepsilon_{\mathrm{L}}}\right)+4\left|\frac{H}{\varepsilon_{\mathrm{L}}}\right|^2\right\}\sum_{l=-\infty}^{+\infty}\exp\left\{-\frac{1}{2}\left(\frac{v_{\mathrm{th}}k_\perp}{\omega_{\mathrm{c}}}\right)^2\right\} I_l\left[\frac{1}{2}\left(\frac{v_{\mathrm{th}}k_\perp}{\omega_{\mathrm{c}}}\right)^2\right]\frac{\exp\left[-\left(\frac{\omega-l\omega_{\mathrm{c}}}{k_x v_{\mathrm{th}}}\right)^2\right]}{k_x v_{\mathrm{th}}}$$

磁場ありの場合散乱断面積にcollective effectが残る
ことが新たに分かった

議論　Collective effectの物理的解釈

磁場無しの場合電子・陽電子プラズマ中の散乱は
collective effect (誘電率依存性) が完全に打ち消す

Sincell and Krolik 1992

- 彼らの主張：「1つのテスト電子に応答する陽電子集団が電子分布の「hole」とちょうど反対方向に動くからである。」

→ あんま分からん...
もっと良い物理的解釈？

- 電子・陽電子はイオン・電子と違って質量と電荷の大きさが等しいのでcollective effect が完全に打ち消すのは不思議ではない

議論　磁場あり電子・陽電子プラズマからの散乱

磁場ありの場合散乱断面積にcollective effectが残ることが新たに分かった

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ε_{L} : 縦誘電率　　H: 電気感受率

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$$= 4\sqrt{\pi}\left\{1-4\mathrm{Re}\left(\frac{H}{\varepsilon_{\mathrm{L}}}\right)+4\left|\frac{H}{\varepsilon_{\mathrm{L}}}\right|^2\right\}\sum_{l=-\infty}^{+\infty}\exp\left\{-\frac{1}{2}\left(\frac{v_{\mathrm{th}}k_{\perp}}{\omega_{\mathrm{c}}}\right)^2\right\} I_l\left[\frac{1}{2}\left(\frac{v_{\mathrm{th}}k_{\perp}}{\omega_{\mathrm{c}}}\right)^2\right]\frac{\exp\left[-\left(\frac{\omega-l\omega_{\mathrm{c}}}{k_x v_{\mathrm{th}}}\right)^2\right]}{k_x v_{\mathrm{th}}}$$