# BV double copy from homotopy algebras (Part 0)

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Homotopy Algebra of Quantum Field Theory and Its Application
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Joint work 2007.13803 and 2102.11390 with Branislav Jurčo,
Hyungrok Kim (Part 1), Tommaso Macrelli (Part 2), Christian Saemann,
Martin Wolf

# Gravity and gauge theory

- Gravity as a gauge theory:
  - Gauge theory of Lorentz, (super) Poincaré or de Sitter symmetries
    [Utiyama '56; Kibble '61; MacDowell-Mansouri '77; Chamseddine-West '77; Stelle-West 79]
  - Holographic principle - AdS/CFT correspondence
    ['t Hooft '93; Susskind '94; Maldacena '97]

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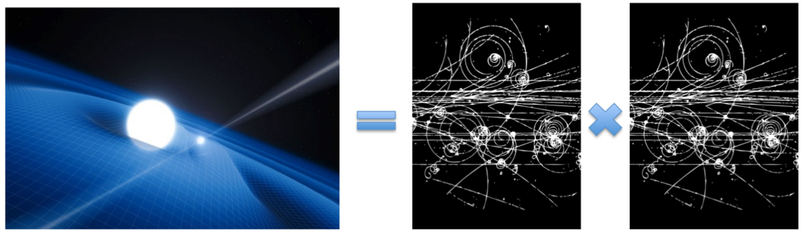
- Here, we appeal to a third and (superficially) independent perspective:

$$\text{Gravity} = \text{Gauge} \times \text{Gauge}$$

- The theme of gravity as the "square" of Yang-Mills has appeared in a variety of guises going back to the KLT relations of string theory [Kawai-Lewellen-Tye '85] Cf. Field theory [Feynman–Morinigo–Wagner; Papini '65]

- Bern–Carrasco–Johansson colour–kinematics (CK) duality and double-copy of (super) Yang-Mills (plus matter) scattering amplitudes
  [Bern-Carrasco-Johansson '08, '10; Bern-Dennen-Huang-Kiermaier '10]

# Gravity $=$ Gauge $\times$ Gauge



- BV/BRST quantised Yang-Mills $\longrightarrow$ $L_\infty$-algebra that factorises:

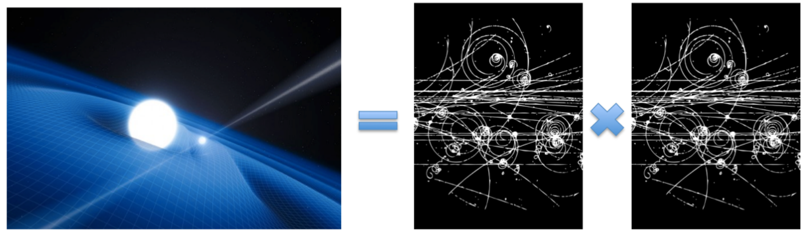
$$\mathfrak{L}_{\mathrm{YM}} = \mathfrak{g} \otimes \mathfrak{V} \otimes_\tau \mathfrak{S}$$

- BRST-Lagrangian (or homotopy) double-copy:

| Bi-adjoint $\phi^3$ theory | | YM theory | | $\mathcal{N} = 0$ supergravity |
|---|---|---|---|---|
| $\mathfrak{g} \otimes \tilde{\mathfrak{g}} \otimes \mathfrak{S}$ | $\longleftarrow$ | $\mathfrak{g} \otimes \mathfrak{V} \otimes_\tau \mathfrak{S}$ | $\longrightarrow$ | $\tilde{\mathfrak{V}} \otimes_{\tilde{\tau}} \otimes \mathfrak{V} \otimes_\tau \mathfrak{S}$ |

- Yang-Mills (integrands of) amplitudes double-copy to $\mathcal{N} = 0$ supergravity

# Gravity = Gauge × Gauge



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$$\begin{array}{ccccc} \text{Bi-adjoint } \phi^3 \text{ theory} & & \text{YM theory} & & \mathcal{N}=0 \text{ supergravity} \\ \mathfrak{g} \otimes \tilde{\mathfrak{g}} \otimes \mathfrak{S} & \longleftarrow & \mathfrak{g} \otimes \mathfrak{V} \otimes_\tau \mathfrak{S} & \longrightarrow & \tilde{\mathfrak{V}} \otimes_{\tilde{\tau}} \otimes \mathfrak{V} \otimes_\tau \mathfrak{S} \end{array}$$

- Yang-Mills (integrands of) amplitudes double-copy to $\mathcal{N} = 0$ supergravity

- Quantum gravity *is* the square of Yang-Mills (well, perturbatively and coupled to a Kalb-Ramond 2-form and dilaton) [2007.13803, 2102.11390]

# Order of Events

1. BCJ Colour–Kinematics Duality and Double-Copy: Review
2. The BRST Lagrangian Double Copy: A Heuristic Summary
3. Colour–Kinematics Duality Redux (Hyungrok Kim: Part 1)
4. Homotopy Double Copy (Tommaso Macrelli: Part 2)

BCJ Colour-Kinematic Duality and Double-Copy

# Amplitudology

- Consider pure Yang-Mills theory:

$$S_{\mathrm{YM}} = \frac{1}{2g^2} \int \mathrm{tr} F \wedge \star F$$

$F = dA + A \wedge A$

$A \in \Omega^1(M) \otimes \mathfrak{g}$

- Interested in $n$-point and $L$-loop amplitudes on Minkowski spacetime

- Conceptually clear and most direct route to reality

# Amplitudology

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$$S_{\mathsf{YM}} = \frac{1}{2g^2} \int \mathrm{tr} F \wedge \star F$$

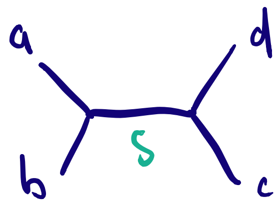
- Interested in $n$-point and $L$-loop amplitudes on Minkowski spacetime
- Conceptually clear and most direct route to reality
- Feynman diagram expansion quickly becomes unwieldy
- 'Going on-shell' reveals hidden structure in the madness

# Amplitudes as sums over cubic diagrams

- Can write $n$-point $L$-loop gluon amplitude in terms of only cubic diagrams:


$$A_{\text{YM}}^{n,L} = \int_L \sum_{i \in \text{cubic diag}} \frac{c_i n_i}{S_i d_i}$$

s-channel :




$c_s = f_{abx} f_{xcd}$

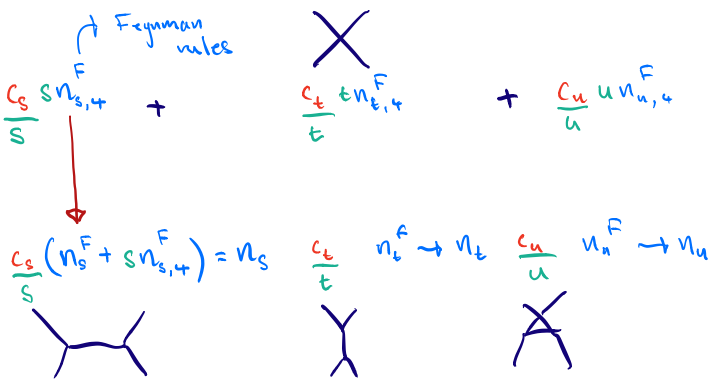
$d_s = (p_1 + p_2)^2 = s$

$n_s = 4\, \varepsilon_1 \cdot p_2\, \varepsilon_2 \cdot \varepsilon_4\, \varepsilon_3 \cdot p_4 + \cdots$

- $c_i$: colour numerator, built from $f^{abc}$, read off diagram $i$
- $n_i$: kinematic numerator, built from $p, \varepsilon$ ← Not unique!
- $d_i$: propagator, $\prod_{\text{int. lines}} p^2$, read off diagram $i$

# Amplitudes and cubic diagrams

- Consider tree-level 4-point example as given by standard Feynman diagrams:




$$A^{4,0}_{YM} = \frac{c_s n_s}{s} + \frac{c_t n_t}{t} + \frac{c_u n_u}{u}$$

$$s n_s = \alpha s$$
$$s n_t = \alpha t$$
$$s n_u = \alpha u$$

leaves $A^{4,0}$ invariant

$$c_s + c_t + c_u = 0$$

# Amplitudes and cubic diagrams

- Can be realised in the Lagrangian through auxiliary fields:

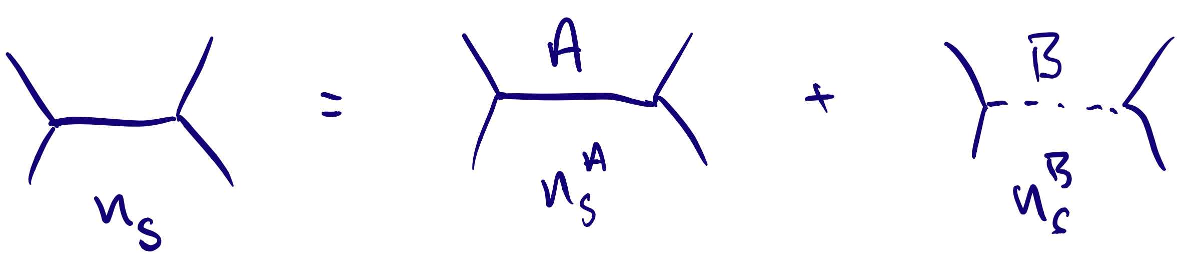
$$\mathcal{L}_{\text{YM}} = \cdots + g^2 [A_\mu, A_\nu][A^\mu, A^\nu] \rightarrow \tfrac{1}{2} B^{\mu\nu\kappa} \Box B_{\mu\nu\kappa} - g(\partial_\mu A_\nu + \frac{1}{\sqrt{2}} \partial^\kappa B_{\kappa\mu\nu})[A^\mu, A^\nu]$$

- Feynman diagrams give 'cubic' amplitudes directly:

$n_i = \sum_\varphi n_i^\varphi$

$$A^{n,L}_{\text{YM}} = \int_L \sum_{\alpha \in \text{Feynman diag}} \frac{c_\alpha n_\alpha}{S_\alpha d_\alpha} = \int_L \sum_{i \in \text{cubic diag}} \frac{c_i n_i}{S_i d_i}$$

- Example: 4-point $s$-channel diagram



# BCJ colour-kinematics duality

- There is an organisation of the $n$-point $L$-loop gluon amplitude:

$$A^{n,L}_{\text{YM}} = \int_L \sum_{i \in \text{cubic diag}} \frac{c_i n_i}{S_i d_i}$$

↑ BCJ numerators

such that

$$\begin{array}{ccc} c_i + c_j + c_k = 0 & \Rightarrow & n_i + n_j + n_k = 0 \\ c_i \longrightarrow -c_i & \Rightarrow & n_i \longrightarrow -n_i \end{array}$$

[Bern-Carrasco-Johansson '08]

4-points : $n_s + n_t + n_u = 0$ ✓

$n > 4$ : Not automatic

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[Bern-Carrasco-Johansson '08]

- CK duality established at tree-level

  [Stieberger 0907.2211, Bjerrum-Bohr–Damgaard–Vanhove 0907.1425]

- Significant evidence up to 4 loops in various (super)YM theories

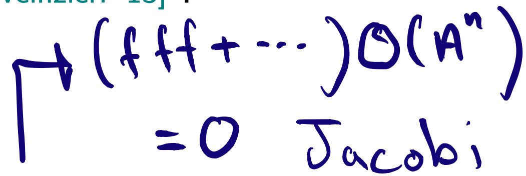
  [Carrasco–Johansson '11; Bern–Davies–Dennen–Huang–Nohle '13; Bern-Davies-Dennen '14. . . ]

- Quickly becomes difficult to check, even with on-shell methods

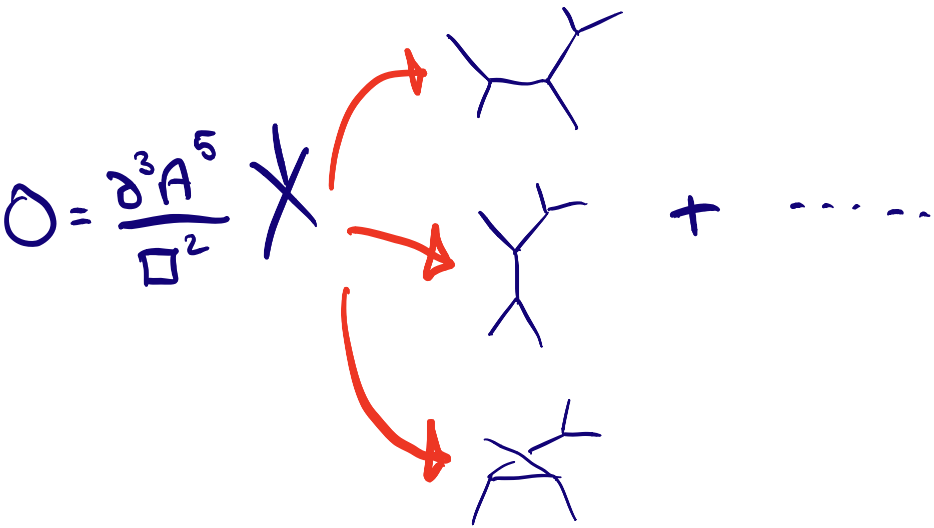
  [Bern–Carrasco–Chen–Edison–Johansson–Parra-Martinez–Roiban–Zeng '18]

# Colour-Kinematics via Feynman Diagrams

- Feynman diagrams yield amplitudes manifesting CK duality for tree-level amplitudes [Bern–Dennen–Huang–Kiermaier '10; Tolotti–Weinzierl '13] :




$$\mathcal{L}^{(2)}_{\mathsf{YM}} + \mathcal{L}^{(3)}_{\mathsf{YM}} + \frac{\square}{\square}\mathcal{L}^{(4)}_{\mathsf{YM}} + \sum_{n=5}^{\infty}\mathcal{L}^{(n)}_{\mathsf{YM}}$$

- Can make cubic through auxiliary field [2007.13803; 2102.11390]
- Nice homotopy interpretation, cf. Hyungrok Kim's talk

# BCJ double-copy prescription

- Given CK dual amplitude of pure Yang-Mills

$$A_{\mathrm{YM}}^{n,L} = \int_L \sum_{i \in \text{cubic diag}} \frac{c_i n_i}{S_i d_i}$$

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$$c_i \quad \longrightarrow \quad n_i$$

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- Double-copy:

$$c_i \quad \longrightarrow \quad n_i$$

- Gives an amplitude of $\mathcal{N} = 0$ supergravity

$$A^{n,L}_{\mathcal{N}=0} = \int_L \sum_{i \in \text{cubic diag}} \frac{n_i n_i}{S_i d_i}$$

$$S_{\mathcal{N}=0} = \frac{1}{2\kappa^2} \int \star R - \frac{1}{d-2} d\varphi \wedge \star d\varphi - \frac{1}{2} \mathrm{e}^{-\frac{4}{d-2}\varphi} dB \wedge \star dB$$

where $B$ is the Kalb-Ramond 2-form, $\varphi$ is the dilaton
[Bern-Carrasco-Johansson '08, '10; Bern-Dennen-Huang-Kiermaier '10]

# Implications

- Conceptually compelling: is gravity the square of gauge theory?
- Computationally powerful: $\mathcal{N} = 8$ supergravity four-point to 5 loops! (finite) [Bern–Carrasco–Chen–Edison–Johansson–Parra-Martinez–Roiban–Zeng '18]

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- Perhaps surprising, but can be explained by supersymmetry and $E_{7(7)}$ U-duality [Bjornsson–Green '10, Bossard–Howe–Stelle '11; Elvang–Freedman–Kiermaier '11; Bossard–Howe–Stelle–Vanhove '11]
- But at 7 loops any cancellations cannot be "consequences of supersymmetry in any conventional sense" [Bjornsson–Green '10]

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- But at 7 loops any cancellations cannot be "consequences of supersymmetry in any conventional sense" [Bjornsson–Green '10]
- $D = 4, \mathcal{N} = 5$ supergravity finite to 4 loops, contrary to expectations:

  "Enhanced" cancellations

  [Bern-Davies-Dennen '14]
- Such cancellations not seen for $\mathcal{N} = 8$ at 5 loops: implications unclear

# Origin, validity, generality, implications and applications

- Classical double-copy of solutions
  - Non-perturbative classical double-copy $\rightarrow$ black holes from gauge theory
    [Monteiro–O'Connell–White '14...]
  - Amplitudes and the double-copy $\rightarrow$ applications to gravity wave astronomy
    [Kosower–Maybee–O'Connell '18; Bern–Cheung–Roiban–Shen–Solon–Zeng '19; Bern–Luna–Roiban–Shen–Zeng '20...]
- Geometric/world-sheet picture
  - String theory monodromy $\rightarrow$ tree-level CK duality
    [Bjerrum-Bohr–Damgaard–Vanhove '09]
  - Ambitwistor string theories theories and scattering equation formalism
    [Cachazo–He–Yuan '13 '14; Mason–Skinner '13; Adamo–Casali–Skinner '13; Adamo–Casali–Mason–Nekovar '17 '18; Geyer–Monteiro '18; Geyer–Mason '19...]

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- Central question: does CK duality and/or the double copy hold to all orders?
  - Today's talks Part 1-2: *yes* for the double copy
  - Homotopy algebras abound!

# BRST-Lagrangian Double-Copy: A Heuristic Summary

# Off-shell BRST-Lagrangian double-copy

- CK duality and the double copy exposed by ‘on-shell’ lens
- Can we go back 'off-shell’ to establish the validity of the double-copy to all orders in perturbations theory?

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(on-shell, transverse gluons) $(h, \varphi, B)$

tree CK duality $\Rightarrow$ tree double-copy

BRST complex off-shell

loop CK duality $\Rightarrow$ loop double-copy

? ?

- $\rightarrow$ off-shell BRST-Lagrangian double-copy

# Some ingredients

- Field theory product of BRST gauge theories and Lagrangian double-copy
  [Bern–Dennen–Huang–Kiermaier '10; Anastasiou–LB-Duff–Hughes–Nagy '14; LB '17; Anastasiou–LB–Duff-Nagy–Zoccali '18; LB–Jubb–Makwana–Nagy '20; LB-Nagy '20]

- CK duality manifesting actions and kinematic algebras
  [Bern–Dennen–Huang–Kiermaier '10; Tolotti–Weinzierl '13; Cheung–Shen '16; Luna–Monteiro–Nicholson–Ochirov–O'Connell–Westerberg–White '16] [Monteiro–O'Connell '11, '13; Bjerrum–Bohr–Damgaard–Monteiro–O'Connell '12; Chen–Johansson–Teng–Wang '19; Reiterer '19]

- Left/right factorised form of $\mathcal{N} = 0$ supergravity action
  [Bern–Grant '99; Hohm 11; Cheung–Remmen '17]

- Also cf. pure spinor BRST cohomology approach to loop CK duality
  [Mafra–Schlotterer '14]

# BRST-Lagrangian double copy: lighting overview

Step 1. Cubic tree-level CK duality manifesting Yang-Mills BRST-action (cf. Hyungrok Kim's talk):

DeWitt index

tower

$$S_{\text{BRST-CK YM}} = c_{ab} C_{ij} A^{ai} \Box A^{aj} + f_{abc} F_{ijk} A^{ai} A^{bj} A^{ck}$$

$$A^i = (A, c, \bar{c}, b, \text{Aux})$$

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Step 2. BRST-action double-copy (cf. Tommaso Macrelli's talk):

$$S_{\text{BRST-CK}} \times \tilde{S}_{\text{BRST-CK}} = S_{\text{DC}} = C_{ij} C_{\tilde{\imath}\tilde{\jmath}} A^{i\tilde{\imath}} \Box A^{j\tilde{\jmath}} + F_{ijk} F_{\tilde{\imath}\tilde{\jmath}\tilde{k}} A^{i\tilde{\imath}} A^{j\tilde{\jmath}} A^{k\tilde{k}}$$

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Step 3. Double-copy BRST operator (cf. Tommaso Macrelli's talk):

$$(Q_{\text{YM}}, \tilde{Q}_{\text{YM}}) = Q_{\text{DC}} = Q^{\text{lin}}_{\text{diffeo}} + Q^{\text{lin}}_{\text{2-form}} + \cdots$$

Perfect CK duality $\Rightarrow$ $Q_{\text{DC}}$ is (up to quasi-isomorphisms) $Q_{\text{diffeo}} + Q_{\text{2-form}}$

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Step 4. Perturbative quantum equivalence (cf. Tommaso Macrelli's talk):

on-shell tree-level CK + BRST Ward identities $\Rightarrow S_{\text{DC}} \cong S_{\text{BRST}\,\mathcal{N}=0}$

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Corollary: $S_{\text{BRST-CK YM}} \rightarrow$ 'almost BCJ numerators' that correctly double-copy:

$$A^{n,L}_{\text{YM}} = \int_L \sum_{i \in \text{cubic diag}} \frac{c_i n_i}{S_i d_i} \quad \longrightarrow \quad \int_L \sum_{i \in \text{cubic diag}} \frac{n_i n_i}{S_i d_i} = A^{n,L}_{\mathcal{N}=0}$$

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- 'Almost': construction doesn't imply $n_i$ satisfy perfect loop CK duality, but close enough for double-copy, cf. generalised CK duality [Bern–Carrasco–Chen–Johansson–Roiban '17]
- Only tree-level CK duality required to construct loop almost BCJ $n_i$ - complicated, but purely algebraic
- Is there a precise weakened notion of on-mass-shell loop CK duality?

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  - Is there a precise weakened notion of on-mass-shell loop CK duality?

- Incorporating ideas from [Reiterer '19] we can possibly do better $\rightarrow$ perfect CK duality, cf. Hyungrok Kim's talk

# Things to come (past and future work)

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- $\rightarrow$ homotopy realisation of CK duality for extended Hilbert space of BRST complex

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- $\to$ homotopy realisation of CK duality for extended Hilbert space of BRST complex
- Part 2 (Tommaso Macrelli): Factorisation redux
- $\to$ homotopy factorisation of Yang-Mills and $\mathcal{N}=0$ supergravity

$$\begin{aligned}
\mathfrak{L}_{\mathsf{YM}} &= \overbrace{\underbrace{}_{}\overset{L_\infty}{\overbrace{\mathfrak{colour}}} \otimes \overset{C_\infty}{\overbrace{\mathfrak{kinematics} \otimes_\tau \overset{A_\infty}{\overbrace{\mathfrak{scalar}}}}}}^{L_\infty} \\
& \qquad\qquad\Downarrow \\
\mathfrak{L}_{\mathsf{DC}} &= \mathfrak{kinematics} \otimes_\tau \mathfrak{kinematics} \otimes_\tau \mathfrak{scalar}
\end{aligned}$$

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- Part 1 (Hyungrok Kim): CK duality redux

- $\to$ homotopy realisation of CK duality for extended Hilbert space of BRST complex

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- Part 1 + Part 2 $\Rightarrow$ pertubative equivalence 'gravity = gauge $\times$ gauge'