BV double copy from homotopy algebras

Tommaso Macrelli
t.macrelli@surrey.ac.uk

University of Surrey

31 March 2021

Based on joint work with L Borsten, B Jurčo, H Kim, C Saemann, M Wolf
2007.13803, 2102.11390

Introduction: gravity = gauge × gauge

- Naive idea: "$A_\mu \otimes \bar{A}_\nu \; = \; g_{\mu\nu} \oplus B_{\mu\nu} \oplus \phi$"
- First concrete incarnation: KLT relations
- A purely field theoretic approach: BCJ colour–kinematic duality and double copy prescription
- We propose an homotopy algebra realisation of this principle

Flash review of BCJ duality and double copy

- Every Lagrangian field theory is equivalent to a theory with only cubic interactions
- n-points L-loops YM amplitude as sums of trivalent graphs

$$\mathcal{A}_{n,L}^{\text{YM}} = \sum_i \int \prod_{l=1}^{L} \mathrm{d}^d p_l \frac{1}{S_i} \frac{C_i N_i}{D_i}$$

- i ranges over all trivalent L-loops graphs
- C_i: colour factor, composed of gauge group structure constants
- N_i: kinematic factor, composed of Lorentz-invariant contractions of polarisations and momenta

Flash review of BCJ duality and double copy

BCJ colour–kinematic duality

There is a choice of kinematic factors such that N_is obey the same algebraic relations (e.g., Jacobi identity) of the correspondent C_i

- True at tree-level, conjectured for loop-level
- Gravity amplitudes can be represented as sum over trivalent graphs, too

Yang–Mills double copy

If BCJ duality holds true, replacing the colour factor with a copy of the kinematic factor in $\mathcal{A}_{n,L}^{\text{YM}}$ produces a $\mathcal{N} = 0$ supergravity amplitude

- All-loop statement, the problem is then to validate BCJ duality at loop level

BRST-Lagrangian perspective on double copy

- Until now, on-shell scattering amplitude approach
- An off-shell Lagrangian realisation of colour-kinematic duality and double copy could solve the all-loop conundrum

Our approach

Double-copy YM Lagrangian and BRST operator to obtain a theory equivalent to $\mathcal{N} = 0$ supergravity

Yang–Mills theory in the BV formalism

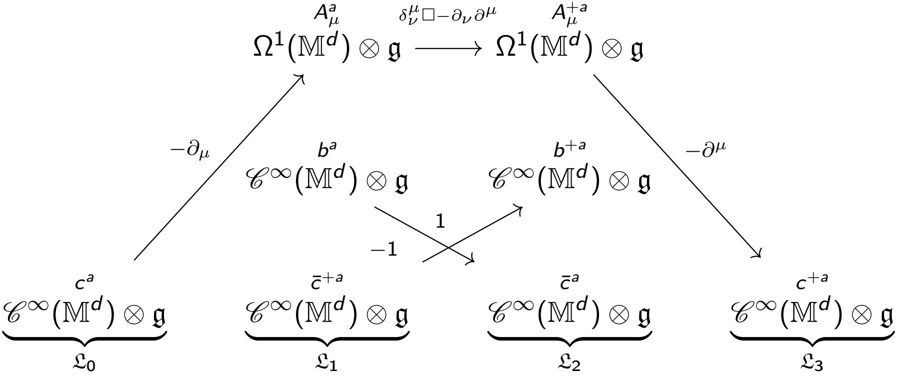
- Extend YM action with antifields (A^+, c^+) and trivial pairs (b, $\bar{c}^+$, b^+, $\bar{c}$)

$$S_{\mathrm{BV}}^{\mathrm{YM}} = \int_{\mathbb{M}^d} \mathrm{d}^d x \left\{ -\tfrac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + A^{+a}_{\mu} (\nabla^\mu c)^a + \tfrac{g}{2} f^a_{bc} c^{+a} c^b c^c + b^a \bar{c}^{+a} \right\}$$

- We can formulate YM theory as the Maurer–Cartan homotopy theory associated to a cyclic L_∞-algebra $(\mathfrak{L}, \{\mu_i\}, \langle -, - \rangle)$

$$S_{\mathrm{MC}}[a] = \sum_{i \geq 1} \frac{1}{(i+1)!} \langle a, \mu_i(a, \ldots, a) \rangle$$

Chain complex (μ_1)



- Other non-vanishing higher products

$$
\begin{aligned}
[\mu_2(A,c)]^a &= g f^a_{bc} c^b c^c \;, \quad [\mu_2(A,c)]^a_\mu = -g f^a_{bc} A^b_\mu c^c \\
[\mu_2(A^+,c)]^a_\mu &= -g f^a_{bc} A^{+b}_\mu c^c \;, \quad [\mu_2(c,c^+)]^a = g f^a_{bc} c^b c^{+c} \\
[\mu_2(A,A)]^a_\mu &= -3!\kappa f^a_{bc} \partial^\nu (A^b_\nu A^c_\mu) \\
[\mu_2(A,A^+)]^a &= 2g f_{bc}{}^a \Big(\partial^\nu (A^b_\nu A^c_\mu) + 2A^{b\nu} \partial_{[\nu} A^c_{\mu]} \Big) \\
[\mu_3(A,A,A)]^a_\mu &= 3! g^2 f_{ed}{}^b f_{bc}{}^a A^{\nu c} A^d_\nu A^e_\mu
\end{aligned}
$$

- Cyclic structure

$$
\begin{aligned}
\langle A, A^+ \rangle &= \int_{\mathbb{M}^d} \mathrm{d}^d x \, A^a_\mu A^{+a\mu} \;, \qquad \langle b, b^+ \rangle = \int_{\mathbb{M}^d} \mathrm{d}^d x \, b^a b^{+a} \;, \\
\langle c, c^+ \rangle &= \int_{\mathbb{M}^d} \mathrm{d}^d x \, c^a c^{+a} \;, \qquad \langle \bar{c}, \bar{c}^+ \rangle = -\int_{\mathbb{M}^d} \mathrm{d}^d x \, \bar{c}^a \bar{c}^{+a}
\end{aligned}
$$

- Gauge-fixing: gauge-fixing fermion

$$\Psi \;=\; -\int \mathrm{d}^d x\, \bar{c}_a \left(\partial^\mu A^a_\mu + \tfrac{\xi}{2} b^a\right) .$$

with ξ real parameter

- Gauge-fixed action

$$S^{\mathrm{gf}}_{\mathrm{YM}} \;=\; \int \mathrm{d}^d x \left\{ -\tfrac{1}{4} F_{a\mu\nu} F^{a\mu\nu} - \bar{c}_a \partial^\mu (\nabla_\mu c)^a + \tfrac{\xi}{2} b_a b^a + b_a \partial^\mu A^a_\mu \right\}$$

$$S^{\mathrm{gf}}_{\mathrm{YM}} \;=\; \int \mathrm{d}^d x \left\{ \tfrac{1}{2} A_{a\mu} \Box A^{a\mu} + \tfrac{1}{2} (\partial^\mu A^a_\mu)^2 - \bar{c}_a \Box c^a + \tfrac{\xi}{2} b_a b^a + b_a \partial^\mu A^a_\mu \right\} + S^{\mathrm{int}}_{\mathrm{YM}}$$

- Canonical field redefinition

$$\tilde{c}^a = c^a \qquad \tilde{c}^{+a} = c^{+a}$$

$$\tilde{A}^a_\mu = A^a_\mu \qquad \tilde{A}^{+a}_\mu = A^{+a}_\mu + \frac{1-\sqrt{1-\xi}}{\xi}\,\partial_\mu b^{+a}$$

$$\tilde{b}^a = \sqrt{\frac{\xi}{\Box}}\left(b^a + \frac{1-\sqrt{1-\xi}}{\xi}\,\partial^\mu A^a_\mu\right) \qquad \tilde{b}^{+a} = \sqrt{\frac{\Box}{\xi}}\,b^{+a}$$

$$\tilde{\bar{c}}^a = \bar{c}^a \qquad \tilde{\bar{c}}^{+a} = \bar{c}^{+a}$$

- New action

$$\tilde{S}_{\mathrm{YM}} = \int \mathrm{d}^d x \left\{\tfrac{1}{2}\tilde{A}_{a\mu}\Box\tilde{A}^{a\mu} - \tilde{\bar{c}}_a\Box\tilde{c}^a + \tfrac{1}{2}\tilde{b}_a\Box\tilde{b}^a + \tilde{\xi}\,\tilde{b}_a\sqrt{\Box}\,\partial^\mu\tilde{A}^a_\mu\right\} + \tilde{S}^{\mathrm{int}}_{\mathrm{YM}}$$

with $\tilde{\xi} = \sqrt{\frac{1-\xi}{\xi}}$

- New chain complex

$$
\begin{array}{ccc}
\overset{\tilde{A}^a_\mu}{\Omega^1(\mathbb{M}^d)\otimes\mathfrak{g}} & \xrightarrow{\ \square\ } & \overset{\tilde{A}^{+a}_\mu}{\Omega^1(\mathbb{M}^d)\otimes\mathfrak{g}} \\
 & {\scriptstyle -\tilde{\xi}\sqrt{\square}\,\partial^\mu}\ \searrow \quad \nearrow\ {\scriptstyle \tilde{\xi}\sqrt{\square}\,\partial_\mu} & \\
\overset{\tilde{b}^a}{\mathscr{C}^\infty(\mathbb{M}^d)\otimes\mathfrak{g}} & \xrightarrow[\ \square\ ]{} & \overset{\tilde{b}^{+a}}{\mathscr{C}^\infty(\mathbb{M}^d)\otimes\mathfrak{g}}
\end{array}
$$

$$
\underbrace{\overset{\tilde{c}^a}{\mathscr{C}^\infty(\mathbb{M}^d)\otimes\mathfrak{g}}}_{=\tilde{\mathfrak{L}}^{\mathrm{YM}}_0} \xrightarrow{\ -\square\ } \underbrace{\overset{\tilde{\tilde{c}}^{+a}}{\mathscr{C}^\infty(\mathbb{M}^d)\otimes\mathfrak{g}}}_{=\tilde{\mathfrak{L}}^{\mathrm{YM}}_1} \qquad \underbrace{\overset{\tilde{\tilde{c}}^a}{\mathscr{C}^\infty(\mathbb{M}^d)\otimes\mathfrak{g}}}_{=\tilde{\mathfrak{L}}^{\mathrm{YM}}_2} \xrightarrow{\ -\square\ } \underbrace{\overset{\tilde{c}^{+a}}{\mathscr{C}^\infty(\mathbb{M}^d)\otimes\mathfrak{g}}}_{=\tilde{\mathfrak{L}}^{\mathrm{YM}}_3}
$$

- This chain complex is readily interpreted as a (twisted) tensor product

Tensor products

- Tensor product between associative, commutative and Lie algebras

$\otimes$	Ass	Com	Lie
Ass	Ass	Ass	–
Com	Ass	Com	Lie
Lie	–	Lie	–

For two compatible algebras $\mathfrak{A}, \mathfrak{B}$

$$m_2^{\mathfrak{A}\otimes\mathfrak{B}}(a_1 \otimes b_1, a_2 \otimes b_2) \;=\; m_2^{\mathfrak{A}}(a_1, a_2) \otimes m_2^{\mathfrak{B}}(b_1, b_2)$$

- Tensor product between chain complexes $\mathfrak{A}, \mathfrak{B}$

$$\mathfrak{A} \otimes \mathfrak{B} \;=\; \bigoplus_i (\mathfrak{A} \otimes \mathfrak{B})_i, \quad (\mathfrak{A} \otimes \mathfrak{B})_i \;=\; \bigoplus_{k+l=i} \mathfrak{A}_k \otimes \mathfrak{B}_l$$

$$m_1^{\mathfrak{A}\otimes\mathfrak{B}}(a \otimes b) \;=\; m_1^{\mathfrak{A}}(a) \otimes b \pm a \otimes m_2^{\mathfrak{B}}(b)$$

- Strict homotopy algebras are nothing but differential graded algebras

Factorisation of YM theory

$$\mathfrak{L}_{\mathsf{YM}} \equiv \color{red}{\mathfrak{colour}} \otimes \color{blue}{\mathfrak{kinematic}} \otimes_\tau \mathfrak{scalar}$$

- $\mathfrak{scalar}$ is the A_∞-algebra of a cubic scalar theory, with basis s_x
- $\mathfrak{colour}$ is the gauge Lie algebra, with basis e_a
- $\mathfrak{kinematic}$ is the following graded vector space

$$\overset{\mathsf{g}}{\mathbb{R}}[1] \oplus (\mathbb{R}^{\overset{\mathsf{v}^\mu}{d}} \otimes \overset{\mathsf{n}}{\mathbb{R}}) \oplus \mathbb{R}[\overset{\mathsf{a}}{-}1] \oplus \ldots$$

where basis $(\mathsf{g}, \mathsf{v}^\mu, \mathsf{b}, \mathsf{a}, \ldots)$ correspond to fields $(c, A, b, \bar{c}, \ldots)$

$$c = \mathsf{e}_a \mathsf{g} \mathsf{s}_x c^a(x), \quad A = \mathsf{e}_a \mathsf{v}^\mu \mathsf{s}_x A^a_\mu(x), \quad \ldots$$

Factorisation of YM theory

- $\otimes_\tau$ is a *twisted* tensor product, where the twist is controlled by a twist datum τ

$$\tau : \mathfrak{kinematic}^{\otimes n} \to \mathfrak{kinematic} \otimes \mathrm{End}(\mathfrak{scalar})^{\otimes n}$$

$$(\mathsf{k}_1 \cdots \mathsf{k}_\mathsf{n}) \mapsto \tau(\mathsf{k}_1, \ldots, \mathsf{k}_\mathsf{n}) \otimes \bigotimes_{i=1}^{n} \tau^i(\mathsf{k}_1, \ldots, \mathsf{k}_\mathsf{n})$$

- Thanks to the twist, $\mathfrak{kinematic}$ becomes a kinematic operator algebra, acting on $\mathfrak{scalar}$ with $\tau^i(\mathsf{k}_1, \ldots, \mathsf{k}_\mathsf{n})$

$$\begin{aligned} \mathrm{m}_1(k \otimes a) &= \pm\tau(k) \otimes \mathrm{m}_1(\tau^1(k)(a)) \;, \\ \mathrm{m}_2(k_1 \otimes a_1, k_2 \otimes a_2) &= \\ &= \pm\tau(k_1, k_2) \otimes \mathrm{m}_2(\tau^2(k_1, k_2)(a_1), \tau^2(k_1, k_2)(a_2)) \;. \end{aligned}$$

- τ is dictated by the field theory we consider

Factorisation of YM theory: chain complex level

- Scalar theory chain complex

$$* \to \underbrace{\overset{\mathsf{s}_x}{\mathfrak{F}[-1]}}_{\mathfrak{scalar}_1} \xrightarrow{\ \Box\ } \underbrace{\overset{\mathsf{s}_x^+}{\mathfrak{F}[-2]}}_{\mathfrak{scalar}_2} \to *$$

- Twist datum

$$\tau_1(\mathsf{g}) = \mathsf{g} \otimes \mathsf{id} \ , \qquad \begin{aligned} \tau_1(\mathsf{v}^\mu) &= \mathsf{v}^\mu \otimes \mathsf{id} + \tilde{\xi}\mathsf{n} \otimes \frac{1}{\sqrt{\Box}}\partial^\mu \ , \\ \tau_1(\mathsf{n}) &= \mathsf{n} \otimes \mathsf{id} - \tilde{\xi}\mathsf{v}^\mu \otimes \frac{1}{\sqrt{\Box}}\partial_\mu \ , \end{aligned} \qquad \tau_1(\mathsf{a}) = \mathsf{a} \otimes \mathsf{id}$$

To factorise the full interacting theory as

$$\mathfrak{L}_{\mathsf{YM}} \equiv \mathfrak{colour} \otimes \mathfrak{kinematic} \otimes_{\tau} \mathfrak{scalar}$$

we need to strictify YM theory

Strictification of YM theory

- There exist a non-local YM Lagrangian with manifest tree-level BCJ duality for on-shell physical gluons (Tolotti, Weinzierl, '13)
- We can insert auxiliary fields to make it local, and strictify to a Lagrangian with only cubic interaction

$$\mathcal{L} = \frac{1}{2}\Phi^{\alpha i} g_{\alpha\beta} G_{ij} \Box \Phi^{\beta j} + \frac{1}{3!}\Phi^{\alpha i} f_{\alpha\beta\gamma} F_{ijk} \Phi^{\beta j}\Phi^{\gamma k}$$

$$(Q\Phi)^{\alpha i} = \delta^{\alpha}_{\beta} q^{i}_{j} \Phi^{\beta j} + \frac{1}{2} f^{\alpha}_{\beta\gamma} Q^{i}_{jk} \Phi^{\beta j}\Phi^{\gamma k} + \frac{1}{3!} f^{\alpha}_{\beta\gamma\delta} Q^{i}_{jkl} \Phi^{\beta j}\Phi^{\gamma k}\Phi^{\delta l}$$

- We want to double copy the BRST-extended field space, but BCJ duality is satisfied only on-shell for physical gluons: eventual BCJ violations due to unphysical gluons and ghosts!
- We compensate for these eventual BCJ violations with suitable field redefinitions

Double copy of YM theory

$$\mathcal{L}_{\mathrm{DC}} = \frac{1}{2}\Phi^{i'i}\mathsf{G}_{i'j'}\mathsf{G}_{ij}\Box\Phi^{j'j} + \frac{1}{3!}\Phi^{i'i}\mathsf{F}_{i'j'k'}\mathsf{F}_{ijk}\Phi^{j'j}\Phi^{k'k}$$

$$(Q_{\mathrm{DC}}\Phi)^{i'i} = \dots$$

- Is the new theory consistent? Do we obtain a new BRST operator $Q_{\mathrm{DC}}^2 = 0$, $Q_{\mathrm{DC}}S_{\mathrm{DC}} = 0$?
- If F_{ijk} satisfies the same algebraic relations of $\mathsf{f}_{\alpha\beta\gamma}$, then $Q_{\mathrm{DC}}S_{\mathrm{DC}} = 0$.
- This requires off-shell BCJ duality! And we have it only on-shell
- We have then $Q_{\mathrm{DC}}S_{\mathrm{DC}} = 0$ up to equations of motion
- This is actually enough for quantum equivalence, since it provides the right on-shell Ward identities

Double copy of YM theory

$\mathfrak{colour} \otimes \mathfrak{colour} \otimes \mathfrak{scalar}$
Biadjoint
scalar theory

$\longleftarrow$

$\mathfrak{colour} \otimes \mathfrak{kinematic} \otimes_{\tau} \mathfrak{scalar}$
Yang–Mills
theory

$\longrightarrow$

$\mathfrak{kinematic} \otimes_{\tau} \mathfrak{kinematic} \otimes_{\tau} \mathfrak{scalar}$
$\mathcal{N}=0$
supergravity

Biadjoint scalar theory

$$\mathfrak{L}_{\text{biadj}} \equiv \mathfrak{colour} \otimes \mathfrak{colour} \otimes \mathfrak{scalar}$$

- $\mathfrak{g}$ and $\bar{\mathfrak{g}}$ semi-simple compact matrix Lie algebras with basis e_a and $\bar{\mathsf{e}}_{\bar{a}}$
- Fields: scalar $\varphi = \varphi^{a\bar{a}} \mathsf{e}_a \otimes \bar{\mathsf{e}}_{\bar{a}}$ taking values in $\mathfrak{g} \otimes \bar{\mathfrak{g}}$
- Action

$$S = \int \mathrm{d}^d x \left\{ \tfrac{1}{2} \varphi_{a\bar{a}} \Box \varphi^{a\bar{a}} - \tfrac{\lambda}{3!} f_{abc} f_{\bar{a}\bar{b}\bar{c}} \varphi^{a\bar{a}} \varphi^{b\bar{b}} \varphi^{c\bar{c}} \right\}$$

- Chain complex and L_∞ algebra

$$* \longrightarrow \underbrace{\overset{\varphi^{a\bar{a}}}{\mathfrak{g} \otimes \bar{\mathfrak{g}} \otimes \mathfrak{F}}}_{\mathfrak{L}_{\text{biadj},1}} \xrightarrow{\ \Box\ } \underbrace{\overset{\varphi^{+a\bar{a}}}{\mathfrak{g} \otimes \bar{\mathfrak{g}} \otimes \mathfrak{F}}}_{\mathfrak{L}_{\text{biadj},2}} \longrightarrow *$$

$$[\mu_2(\varphi_1, \varphi_2)]^{a\bar{a}} = -\lambda f_{bc}{}^{a} f_{\bar{b}\bar{c}}{}^{\bar{a}} \varphi_1^{b\bar{b}} \varphi_2^{c\bar{c}}$$

Double copy: field content

$$\mathfrak{L}_{\text{DC}} \equiv \mathfrak{kinematic} \otimes_\tau \mathfrak{kinematic} \otimes_\tau \mathfrak{scalar}$$

Antisymmetric sector

fields				anti-fields		
factorisation	$\lvert - \rvert_{\text{gh}}$	$\lvert - \rvert_{\mathfrak{L}}$	dim	factorisation	$\lvert - \rvert_{\mathfrak{L}}$	dim
$\tilde{\lambda} = -[\mathsf{g},\mathsf{g}]\mathsf{s}_x \frac{1}{2}\tilde{\lambda}(x)$	2	-1	$\frac{d}{2}-3$	$\tilde{\lambda}^+ = -[\mathsf{a},\mathsf{a}]\mathsf{s}_x^+ \frac{1}{2}\tilde{\lambda}^+(x)$	4	$\frac{d}{2}+3$
$\tilde{\Lambda} = [\mathsf{g},\mathsf{v}^\mu]\mathsf{s}_x \frac{1}{\sqrt{2}}\tilde{\Lambda}_\mu(x)$	1	0	$\frac{d}{2}-2$	$\tilde{\Lambda}^+ = [\mathsf{a},\mathsf{v}^\mu]\mathsf{s}_x^+ \frac{1}{\sqrt{2}}\tilde{\Lambda}^+_\mu$	3	$\frac{d}{2}+2$
$\tilde{\gamma} = [\mathsf{g},\mathsf{n}]\mathsf{s}_x \frac{1}{\sqrt{2}}\tilde{\gamma}(x)$	1	0	$\frac{d}{2}-2$	$\tilde{\gamma}^+ = [\mathsf{a},\mathsf{n}]\mathsf{s}_x^+ \frac{1}{\sqrt{2}}\tilde{\gamma}^+(x)$	3	$\frac{d}{2}+2$
$\tilde{B} = [\mathsf{v}^\mu,\mathsf{v}^\nu]\mathsf{s}_x \frac{1}{2\sqrt{2}}\tilde{B}_{\mu\nu}(x)$	0	1	$\frac{d}{2}-1$	$\tilde{B}^+ = [\mathsf{v}^\mu,\mathsf{v}^\nu]\mathsf{s}_x^+ \frac{1}{2\sqrt{2}}\tilde{B}^+_{\mu\nu}(x)$	2	$\frac{d}{2}+1$
$\tilde{\alpha} = [\mathsf{n},\mathsf{v}^\mu]\mathsf{s}_x \frac{1}{\sqrt{2}}\tilde{\alpha}_\mu(x)$	0	1	$\frac{d}{2}-1$	$\tilde{\alpha}^+ = [\mathsf{n},\mathsf{v}^\mu]\mathsf{s}_x^+ \frac{1}{\sqrt{2}}\tilde{\alpha}^+_\mu(x)$	2	$\frac{d}{2}+1$
$\tilde{\epsilon} = -[\mathsf{g},\mathsf{a}]\mathsf{s}_x \frac{1}{\sqrt{2}}\tilde{\epsilon}(x)$	0	1	$\frac{d}{2}-1$	$\tilde{\epsilon}^+ = -[\mathsf{g},\mathsf{a}]\mathsf{s}_x^+ \frac{1}{\sqrt{2}}\tilde{\epsilon}^+(x)$	2	$\frac{d}{2}+1$
$\tilde{\tilde{\Lambda}} = [\mathsf{a},\mathsf{v}^\mu]\mathsf{s}_x \frac{1}{\sqrt{2}}\tilde{\tilde{\Lambda}}_\mu(x)$	-1	2	$\frac{d}{2}$	$\tilde{\tilde{\Lambda}}^+ = [\mathsf{g},\mathsf{v}^\mu]\mathsf{s}_x^+ \frac{1}{\sqrt{2}}\tilde{\tilde{\Lambda}}^+_\mu(x)$	1	$\frac{d}{2}$
$\tilde{\tilde{\gamma}} = [\mathsf{a},\mathsf{n}]\mathsf{s}_x \frac{1}{\sqrt{2}}\tilde{\tilde{\gamma}}(x)$	-1	2	$\frac{d}{2}$	$\tilde{\tilde{\gamma}}^+ = [\mathsf{g},\mathsf{n}]\mathsf{s}_x^+ \frac{1}{\sqrt{2}}\tilde{\tilde{\gamma}}^+(x)$	1	$\frac{d}{2}$
$\tilde{\tilde{\lambda}} = -[\mathsf{a},\mathsf{a}]\mathsf{s}_x \frac{1}{2}\tilde{\tilde{\lambda}}(x)$	-2	3	$\frac{d}{2}+1$	$\tilde{\tilde{\lambda}}^+ = -[\mathsf{g},\mathsf{g}]\mathsf{s}_x^+ \frac{1}{2}\tilde{\tilde{\lambda}}^+(x)$	0	$\frac{d}{2}-1$

Double copy: field content

$$\mathfrak{L}_{\mathrm{DC}} \equiv \mathfrak{kinematic} \otimes_\tau \mathfrak{kinematic} \otimes_\tau \mathfrak{scalar}$$

Symmetric sector

fields				anti-fields		
factorisation	$\lvert - \rvert_{\mathrm{gh}}$	$\lvert - \rvert_{\mathfrak{L}}$	dim	factorisation	$\lvert - \rvert_{\mathfrak{L}}$	dim
$\tilde{X} = (\mathrm{g}, \mathrm{v}^\mu)\mathrm{s}_x \frac{1}{\sqrt{2}}\tilde{X}_\mu(x)$	1	0	$\frac{d}{2}-2$	$\tilde{X}^+ = (\mathrm{a}, \mathrm{v}^\mu)\mathrm{s}_x^+ \frac{1}{\sqrt{2}}\tilde{X}_\mu^+(x)$	3	$\frac{d}{2}+2$
$\tilde{\beta} = (\mathrm{g}, \mathrm{n})\mathrm{s}_x \frac{1}{\sqrt{2}}\tilde{\beta}(x)$	1	0	$\frac{d}{2}-2$	$\tilde{\beta}^+ = (\mathrm{a}, \mathrm{n})\mathrm{s}_x^+ \frac{1}{\sqrt{2}}\tilde{\beta}^+(x)$	3	$\frac{d}{2}+2$
$\tilde{h} = (\mathrm{v}^\mu, \mathrm{v}^\nu)\mathrm{s}_x \frac{1}{2\sqrt{2}}\tilde{h}_{\mu\nu}(x)$	0	1	$\frac{d}{2}-1$	$\tilde{h}^+ = (\mathrm{v}^\mu, \mathrm{v}^\nu)\mathrm{s}_x^+ \frac{1}{2\sqrt{2}}\tilde{h}_{\mu\nu}^+(x)$	2	$\frac{d}{2}+1$
$\tilde{\varpi} = -(\mathrm{n}, \mathrm{v}^\mu)\mathrm{s}_x \frac{1}{\sqrt{2}}\tilde{\varpi}_\mu(x)$	0	1	$\frac{d}{2}-1$	$\tilde{\varpi}^+ = -(\mathrm{n}, \mathrm{v}^\mu)\mathrm{s}_x^+ \frac{1}{\sqrt{2}}\tilde{\varpi}_\mu^+(x)$	2	$\frac{d}{2}+1$
$\tilde{\pi} = (\mathrm{n}, \mathrm{n})\mathrm{s}_x \frac{1}{2\sqrt{2}}\tilde{\pi}(x)$	0	1	$\frac{d}{2}-1$	$\tilde{\pi}^+ = (\mathrm{n}, \mathrm{n})\mathrm{s}_x^+ \frac{1}{2\sqrt{2}}\tilde{\pi}^+(x)$	2	$\frac{d}{2}+1$
$\tilde{\delta} = -(\mathrm{g}, \mathrm{a})\mathrm{s}_x \frac{1}{\sqrt{2}}\tilde{\delta}(x)$	0	1	$\frac{d}{2}-1$	$\tilde{\delta}^+ = -(\mathrm{g}, \mathrm{a})\mathrm{s}_x^+ \frac{1}{\sqrt{2}}\tilde{\delta}^+(x)$	2	$\frac{d}{2}+1$
$\tilde{\tilde{X}} = (\mathrm{a}, \mathrm{v}^\mu)\mathrm{s}_x \frac{1}{\sqrt{2}}\tilde{\tilde{X}}_\mu(x)$	-1	2	$\frac{d}{2}$	$\tilde{\tilde{X}}^+ = (\mathrm{g}, \mathrm{v}^\mu)\mathrm{s}_x^+ \frac{1}{\sqrt{2}}\tilde{\tilde{X}}_\mu(x)$	1	$\frac{d}{2}$
$\tilde{\tilde{\beta}} = (\mathrm{a}, \mathrm{n})\mathrm{s}_x \frac{1}{\sqrt{2}}\tilde{\tilde{\beta}}(x)$	-1	2	$\frac{d}{2}$	$\tilde{\tilde{\beta}}^+ = (\mathrm{g}, \mathrm{n})\mathrm{s}_x^+ \frac{1}{\sqrt{2}}\tilde{\tilde{\beta}}^+(x)$	1	$\frac{d}{2}$

...+ auxiliary fields

Double copy: action

$$
\begin{aligned}
S_{\mathrm{DC}} \;=\; \int \mathrm{d}^d x \Big\{ & \tfrac{1}{4}\tilde{B}_{\mu\nu}\Box\tilde{B}^{\mu\nu} - \tilde{\tilde{\Lambda}}_\mu\Box\tilde{\Lambda}^\mu + \tfrac{1}{2}\tilde{\alpha}_\mu\Box\tilde{\alpha}^\mu - \\
& - \tfrac{\tilde{\xi}^2}{2}(\partial^\mu\tilde{\alpha}_\mu)^2 + \tfrac{1}{2}\tilde{\epsilon}\Box\tilde{\epsilon} - \tilde{\tilde{\lambda}}\Box\tilde{\lambda} - \tilde{\tilde{\gamma}}\Box\tilde{\gamma} + \tilde{\xi}\tilde{\alpha}^\nu\sqrt{\Box}\partial^\mu\tilde{B}_{\mu\nu} + \\
& + \tilde{\xi}\tilde{\gamma}\sqrt{\Box}\partial_\mu\tilde{\tilde{\Lambda}}^\mu - \tilde{\xi}\tilde{\tilde{\gamma}}\sqrt{\Box}\partial_\mu\tilde{\Lambda}^\mu + \\
& + \tfrac{1}{4}\tilde{h}_{\mu\nu}\Box\tilde{h}^{\mu\nu} - \tilde{\tilde{X}}_\mu\Box\tilde{X}^\mu + \tfrac{1}{2}\tilde{\varpi}_\mu\Box\tilde{\varpi}^\mu + \\
& + \tfrac{\tilde{\xi}^2}{2}(\partial^\mu\tilde{\varpi}_\mu)^2 - \tfrac{1}{2}\tilde{\delta}\Box\tilde{\delta} + \tfrac{1}{4}\tilde{\pi}\Box\tilde{\pi} - \tilde{\tilde{\beta}}\Box\tilde{\beta} + \\
& + \tilde{\xi}\tilde{\varpi}^\nu\sqrt{\Box}\partial^\mu\tilde{h}_{\mu\nu} + \tilde{\xi}\tilde{\pi}\sqrt{\Box}\partial_\mu\tilde{\varpi}^\mu + \tfrac{1}{2}\tilde{\xi}^2\tilde{\pi}\partial_\mu\partial_\nu\tilde{h}^{\mu\nu} + \\
& + \tilde{\xi}\tilde{\beta}\sqrt{\Box}\partial_\mu\tilde{\tilde{X}}^\mu - \tilde{\xi}\tilde{\tilde{\beta}}\sqrt{\Box}\partial_\mu\tilde{X}^\mu \Big\} + \ldots
\end{aligned}
$$

Double copy: action

$$
\begin{aligned}
S_{\mathrm{DC}} \; = \; \int \mathrm{d}^d x \Big\{ & \tfrac{1}{4}\tilde{B}_{\mu\nu}\Box\tilde{B}^{\mu\nu} - \tilde{\tilde{\Lambda}}_{\mu}\Box\tilde{\Lambda}^{\mu} + \tfrac{1}{2}\tilde{\alpha}_{\mu}\Box\tilde{\alpha}^{\mu} - \\
& - \tfrac{\tilde{\xi}^2}{2}(\partial^{\mu}\tilde{\alpha}_{\mu})^2 + \tfrac{1}{2}\tilde{\epsilon}\Box\tilde{\epsilon} - \tilde{\tilde{\lambda}}\Box\tilde{\lambda} - \tilde{\tilde{\gamma}}\Box\tilde{\gamma} + \tilde{\xi}\tilde{\alpha}^{\nu}\sqrt{\Box}\partial^{\mu}\tilde{B}_{\mu\nu} + \\
& + \tilde{\xi}\tilde{\gamma}\sqrt{\Box}\partial_{\mu}\tilde{\tilde{\Lambda}}^{\mu} - \tilde{\xi}\tilde{\tilde{\gamma}}\sqrt{\Box}\partial_{\mu}\tilde{\Lambda}^{\mu} + \\
& + \tfrac{1}{4}\tilde{h}_{\mu\nu}\Box\tilde{h}^{\mu\nu} - \tilde{\tilde{X}}_{\mu}\Box\tilde{X}^{\mu} + \tfrac{1}{2}\tilde{\varpi}_{\mu}\Box\tilde{\varpi}^{\mu} + \\
& + \tfrac{\tilde{\xi}^2}{2}(\partial^{\mu}\tilde{\varpi}_{\mu})^2 - \tfrac{1}{2}\tilde{\delta}\Box\tilde{\delta} + \tfrac{1}{4}\tilde{\pi}\Box\tilde{\pi} - \tilde{\tilde{\beta}}\Box\tilde{\beta} + \\
& + \tilde{\xi}\tilde{\varpi}^{\nu}\sqrt{\Box}\partial^{\mu}\tilde{h}_{\mu\nu} + \tilde{\xi}\tilde{\pi}\sqrt{\Box}\partial_{\mu}\tilde{\varpi}^{\mu} + \tfrac{1}{2}\tilde{\xi}^2\tilde{\pi}\partial_{\mu}\partial_{\nu}\tilde{h}^{\mu\nu} + \\
& + \tilde{\xi}\tilde{\beta}\sqrt{\Box}\partial_{\mu}\tilde{\tilde{X}}^{\mu} - \tilde{\xi}\tilde{\tilde{\beta}}\sqrt{\Box}\partial_{\mu}\tilde{X}^{\mu} \Big\} + \ldots
\end{aligned}
$$

Antisymmetric sector

Double copy: action

$$
\begin{aligned}
S_{\mathrm{DC}} \; = \; \int \mathrm{d}^d x \Big\{ & \tfrac{1}{4}\tilde{B}_{\mu\nu}\Box\tilde{B}^{\mu\nu} - \tilde{\tilde{\Lambda}}_\mu\Box\tilde{\Lambda}^\mu + \tfrac{1}{2}\tilde{\alpha}_\mu\Box\tilde{\alpha}^\mu - \\
& - \tfrac{\tilde{\xi}^2}{2}(\partial^\mu\tilde{\alpha}_\mu)^2 + \tfrac{1}{2}\tilde{\epsilon}\Box\tilde{\epsilon} - \tilde{\tilde{\lambda}}\Box\tilde{\lambda} - \tilde{\tilde{\gamma}}\Box\tilde{\gamma} + \tilde{\xi}\tilde{\alpha}^\nu\sqrt{\Box}\partial^\mu\tilde{B}_{\mu\nu} + \\
& + \tilde{\xi}\tilde{\gamma}\sqrt{\Box}\partial_\mu\tilde{\tilde{\Lambda}}^\mu - \tilde{\xi}\tilde{\tilde{\gamma}}\sqrt{\Box}\partial_\mu\tilde{\Lambda}^\mu + \\
& + \tfrac{1}{4}\tilde{h}_{\mu\nu}\Box\tilde{h}^{\mu\nu} - \tilde{\tilde{X}}_\mu\Box\tilde{X}^\mu + \tfrac{1}{2}\tilde{\varpi}_\mu\Box\tilde{\varpi}^\mu + \\
& + \tfrac{\tilde{\xi}^2}{2}(\partial^\mu\tilde{\varpi}_\mu)^2 - \tfrac{1}{2}\tilde{\delta}\Box\tilde{\delta} + \tfrac{1}{4}\tilde{\pi}\Box\tilde{\pi} - \tilde{\tilde{\beta}}\Box\tilde{\beta} + \\
& + \tilde{\xi}\tilde{\varpi}^\nu\sqrt{\Box}\partial^\mu\tilde{h}_{\mu\nu} + \tilde{\xi}\tilde{\pi}\sqrt{\Box}\partial_\mu\tilde{\varpi}^\mu + \tfrac{1}{2}\tilde{\xi}^2\tilde{\pi}\partial_\mu\partial_\nu\tilde{h}^{\mu\nu} + \\
& + \tilde{\xi}\tilde{\beta}\sqrt{\Box}\partial_\mu\tilde{\tilde{X}}^\mu - \tilde{\xi}\tilde{\tilde{\beta}}\sqrt{\Box}\partial_\mu\tilde{X}^\mu \Big\} + \dots
\end{aligned}
$$

Symmetric sector

Perturbative equivalence: $\mathcal{N} = 0$ supergravity is quantum equivalent to Yang–Mills double copy

Classical equivalence

Antisymmetric sector is related to Kalb–Ramond theory by the following field redefinition

$$\tilde{\lambda} = \lambda\,, \qquad \tilde{\Lambda}_\mu = \Lambda_\mu\,,$$

$$\tilde{\gamma} = \sqrt{\frac{\xi}{\Box}}\left(\gamma + \frac{1-\sqrt{1-\xi}}{\xi}\partial^\mu \Lambda_\mu\right), \qquad \tilde{B}_{\mu\nu} = B_{\mu\nu}\,,$$

$$\tilde{\epsilon} = \epsilon + \frac{1-\xi}{2\Box}\partial^\mu \alpha_\mu\,, \qquad \tilde{\bar{\Lambda}}_\mu = \bar{\Lambda}_\mu\,,$$

$$\tilde{\bar{\gamma}} = \sqrt{\frac{\xi}{\Box}}\left(\bar{\gamma} + \frac{1-\sqrt{1-\xi}}{\xi}\partial^\mu \bar{\Lambda}_\mu\right), \qquad \tilde{\bar{\lambda}} = \bar{\lambda}\,,$$

$$\tilde{\alpha}_\mu = \sqrt{\frac{\xi}{\Box}}\left(\alpha_\mu - \partial_\mu \epsilon - \frac{1-\xi}{2\Box}\partial_\mu \partial^\nu \alpha_\nu + \frac{1-\sqrt{1-\xi}}{\xi}\partial^\nu B_{\nu\mu}\right)$$

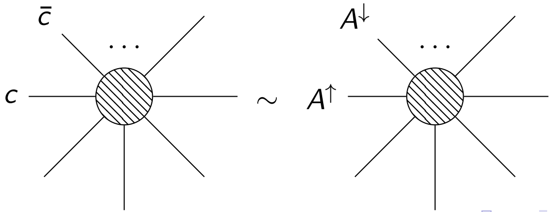
Analogously, symmetric sector can be related to Einstein–Hilbert gravity+dilaton

Classical equivalence

- Integrating out the auxiliary fields: DC and $\mathcal{N} = 0$ supergravity have the same field content and kinematic terms up to field redefinition
- The same field redefinition relates linearised BRST operators of DC and $\mathcal{N} = 0$ supergravity
- The tree-level double copy holds: setting the unphysical fields to zero, DC and $\mathcal{N} = 0$ supergravity are classically equivalent

Quantum equivalence

- Auxiliary fields of ghost number 0: tree-level amplitudes with physical and ghost number 0 auxiliary fields coincide for DC and $\mathcal{N} = 0$ supergravity, since they are determined by collinear limits of physical tree-level amplitudes
- Nakanishi–Lautrup fields: modify the gauge-fixing fermion and redefine NL fields to match
- Ghosts: linearised BRST operators of DC and $\mathcal{N} = 0$ supergravity coincide, and on-shell Ward identity links the tree-level scattering amplitudes containing ghosts to physical tree-level scattering amplitudes



Quantum equivalence

Conclusion: both theories are local and the tree-level scattering amplitudes on the BRST extended field space coincide. Yang–Mills double copy and $\mathcal{N} = 0$ supergravity are perturbatively quantum equivalent

Future

- Extend our approach to supersymmetric theories
- Establish a web of dualities between a zoology of QFT
- Renormalisation?
- Homotopic description of open–closed string duality?

Thank you for listening!