BRST in the ERG

BV-QFT 2021

(virtually) YITP, Kyoto 27/03/21

Tim Morris,

Physics & Astronomy,

University of Southampton, UK.

"BRST in the Exact RG", Y Igarashi, K Itoh & TRM, PTEP 2019 (2019) 10, 103B01 [1904.08231]

"Quantum Gravity, renormalizability and diffeomorphism invariance", TRM, SciPost Phys. 5 (2018) no.4, 040 [1806.02206]

Motivation:

Conceptual: Wilsonian renormalization group (RG) provides a deep understanding of what it means to form the continuum limit of a quantum field theory.

Practical: Wilsonian Exact RG (ERG) equations allow for very general approximations of continuum solutions.

However, need to combine with non-Abelian gauge invariance (gravity, Yang-Mills).

Quantum gravity has power law UV divergences. ERG makes these manifest. They could be crucial in defining a continuum limit. Also Euclidean signature Einstein-Hilbert action is unbounded from below (conformal factor instability): no partition function. ERG well defined but profoundly altered. This could also be crucial to defining continuum limit.

Batalin Vilkovisky Quantum Master Equation

$$\Sigma = 0 \qquad\qquad \Sigma = \frac{1}{2}(S,S) - \Delta S$$

Antibracket: $$(X,Y) = \frac{\partial_r X}{\partial \phi^A} K \frac{\partial_l Y}{\partial \phi^*_A} - \frac{\partial_r X}{\partial \phi^*_A} K \frac{\partial_l Y}{\partial \phi^A}$$

Measure operator: $$\Delta X = (-)^{A+1} \frac{\partial_r}{\partial \phi^A} K \frac{\partial_r}{\partial \phi^*_A} X$$

UV cutoff function $K(p^2/\Lambda^2)$

Regularises measure operator whilst preserving important identities, and compatible with ERG.

QME follows straightforwardly from:

$$\Delta\, \mathrm{e}^{-S} = 0$$

Wilsonian Exact RG

$$\partial_t \, \mathrm{e}^{-S} = \frac{\partial_r}{\partial \phi^A} \left(\hat{\Psi}^A \, \mathrm{e}^{-S} \right) \qquad \Longrightarrow \qquad \partial_t \int \mathcal{D}\phi \, \mathrm{e}^{-S} = 0$$

$t = \ln(\mu/\Lambda)$

field reparametrisation (blocking functional)

Polchinski Equation

$$\partial_t \, \mathrm{e}^{-S} = \frac{\partial_r}{\partial \phi^A} \left(\hat{\Psi}^A \, \mathrm{e}^{-S} \right) \qquad \Longrightarrow \qquad \partial_t \int \mathcal{D}\phi \, \mathrm{e}^{-S} = 0$$

$t = \ln(\mu/\Lambda)$ field reparametrisation (blocking functional)

$$\hat{\Psi}^A = -\frac{1}{2} \dot{\triangle}^{AB} \frac{\partial_r}{\partial \Phi^B} (S - 2S_0) \qquad\qquad S = S_0 + S_I$$

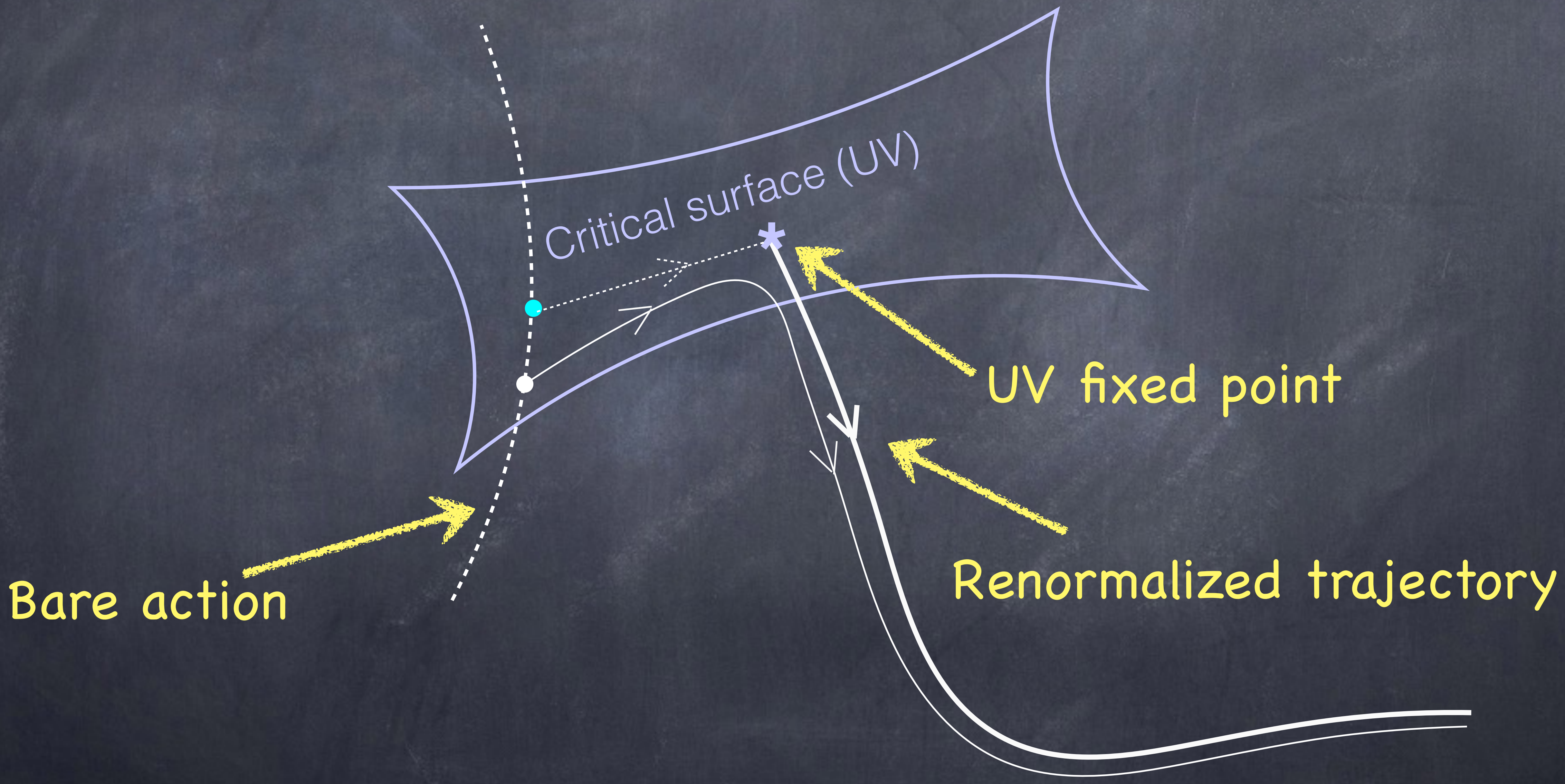
IR cutoff propagator $\bar{K} = 1 - K$

$$\dot{S}_I = -\tfrac{1}{2} \partial^r_A S_I \dot{\triangle}^{AB} \partial^l_B S_I + \tfrac{1}{2} (-)^A \dot{\triangle}^{AB} \partial^l_B \partial^r_A S_I = \tfrac{1}{2} a_0[S_I, S_I] - a_1[S_I]$$

Compatibility: $\dot{\Sigma} = a_0[S_I, \Sigma] - a_1[\Sigma]$

Continuum limit

`Theory space' of effective actions S

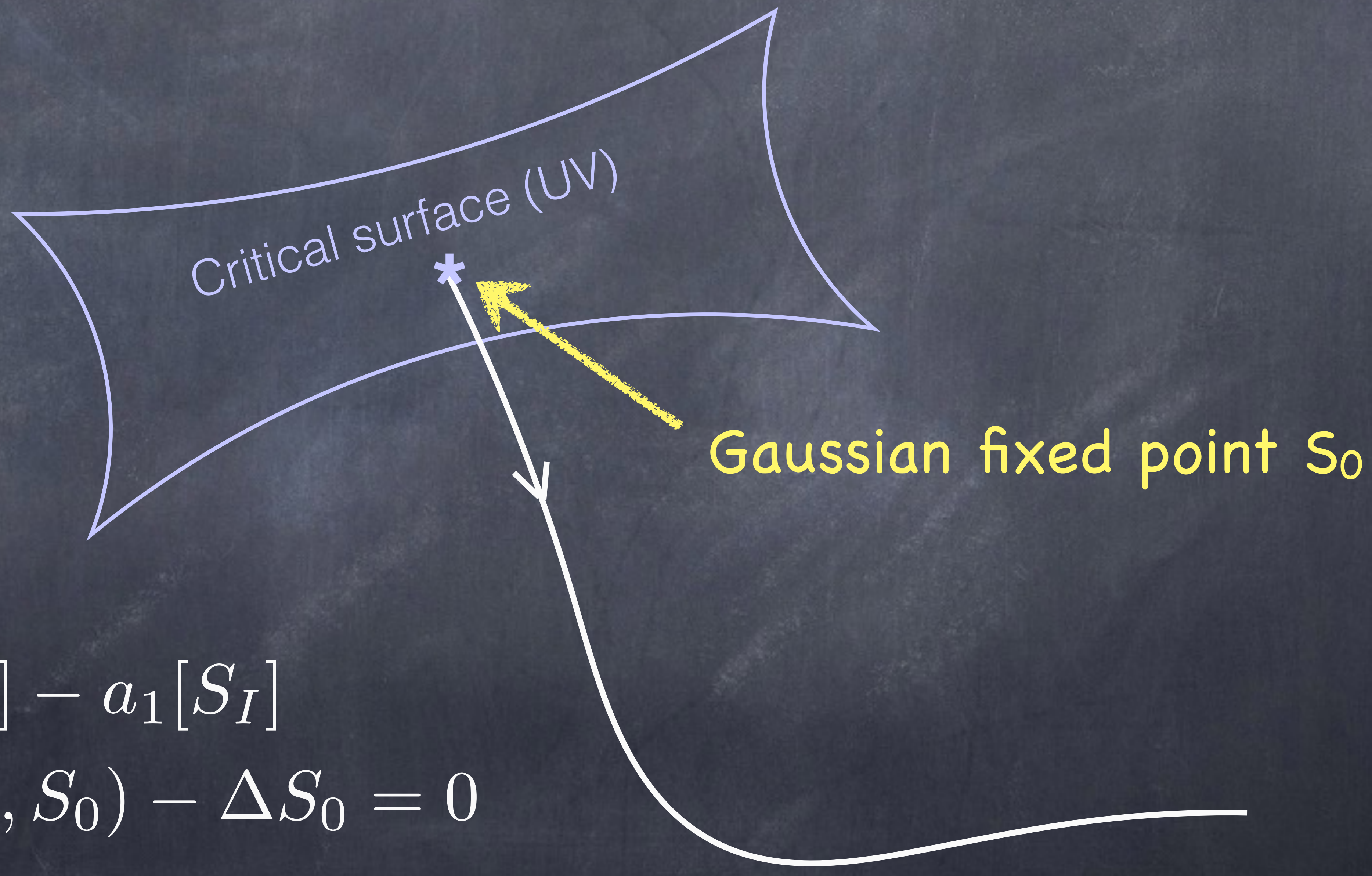


Example: perturbative continuum limit

$$S_0[\phi, \phi^*] = \tfrac{1}{2}\phi^A K^{-1} \triangle^{-1}_{AB}\, \phi^B + \phi^*_A K^{-1} R^A{}_B \phi^B$$

Free BRST transformations: $\quad Q_0\, \phi^A = R^A{}_B\, \phi^B$

$$S = S_0 + \cancel{S_I}$$



$$\dot{S}_I = \tfrac{1}{2} a_0 [S_I, S_I] - a_1 [S_I]$$

$$\Sigma = \Sigma_0 = \tfrac{1}{2}(S_0, S_0) - \Delta S_0 = 0$$

(QME)

Full BRST charge: $Q\phi^A = (\phi^A, S) = K\dfrac{\partial_l S}{\partial \phi^*_A}$

Koszul-Tate charge: $Q^-\phi^*_A = (\phi^*_A, S) = -K\dfrac{\partial_l S}{\partial \phi^A}$

If $S + \varepsilon\,\mathcal{O}$ satisfies $\Sigma = 0$: $\hat{s}\,\mathcal{O} = 0$

$\hat{s}^2 = 0$

$$\hat{s}\,\mathcal{O} = (\mathcal{O}, S) - \Delta\mathcal{O} = (Q + Q^- - \Delta)\,\mathcal{O}$$

Therefore: $\mathcal{O} = \hat{s}\,\mathcal{K} = (\mathcal{K}, S) - \Delta\mathcal{K}$ satisfies $\Sigma = 0$

But this just corresponds to field and source redefinitions:

$$\delta\phi^A = -\varepsilon K\frac{\partial_r \mathcal{K}}{\partial \phi^*_A}\,, \qquad \delta\phi^*_A = \varepsilon K\frac{\partial_r \mathcal{K}}{\partial \phi^A}$$

Therefore want: $\hat{s}\,\mathcal{O} = 0$ such that $\mathcal{O} \neq \hat{s}\,\mathcal{K}$

Quantum BRST cohomology

regularised Δ

derivative expansion

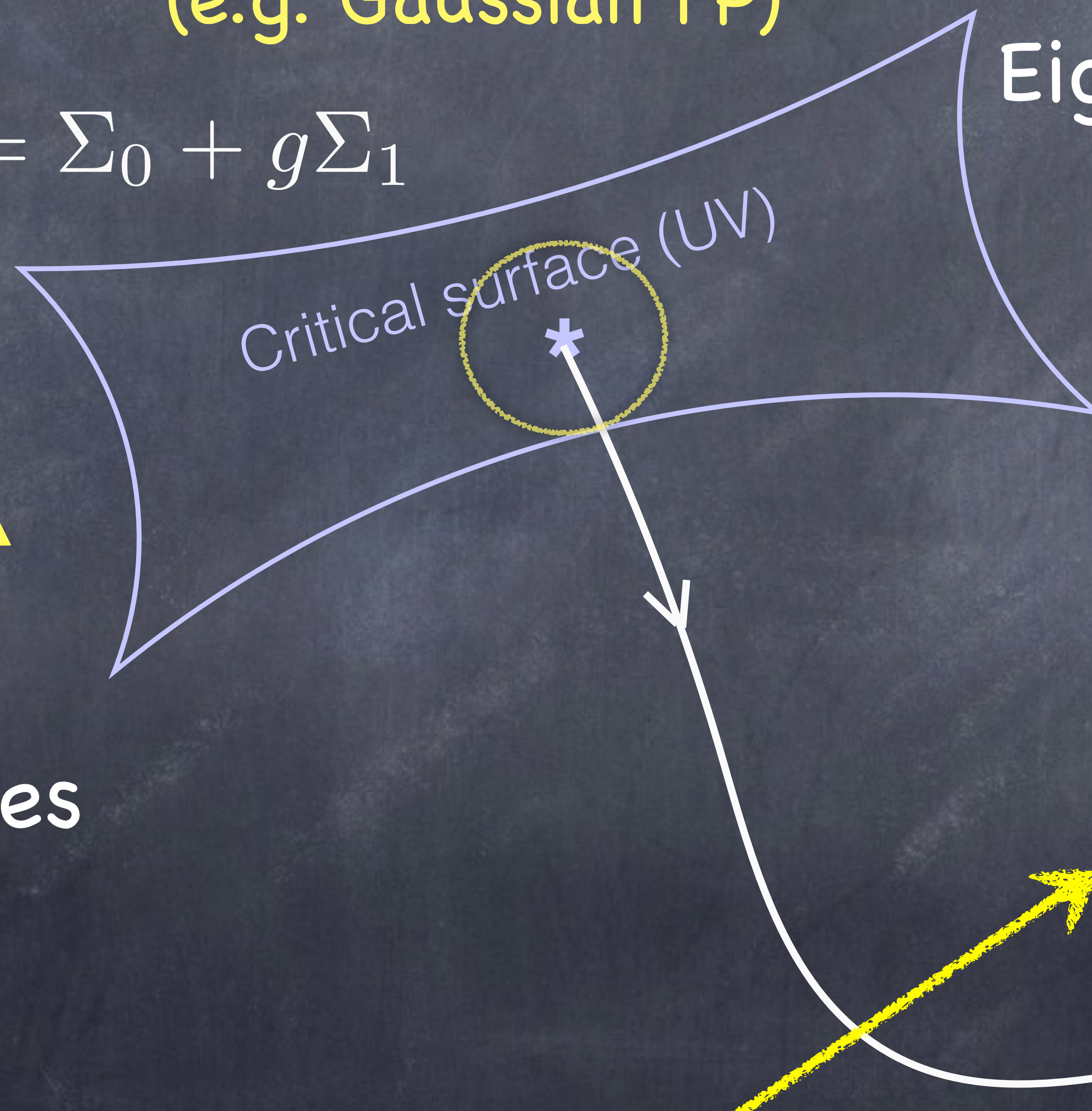
(close to fixed point)

Eigenoperators

$$\dot{\Sigma} = a_0[S_I, \Sigma] - a_1[\Sigma] \qquad \dot{S}_I = \tfrac{1}{2}a_0[S_I, S_I] - a_1[S_I]$$

(e.g. Gaussian FP)

$$S = S_0 + gS_1 \quad \Sigma = \Sigma_0 + g\Sigma_1$$



Eigenoperator eqn:

$$\dot{S}_1 = -a_1[S_1]$$

$$\hat{s}_0 = Q_0 + Q_0^- - \Delta$$

General soln is expansion over eigenoperators with constant coefficients g.

$\Sigma_1 = \hat{s}_0\, S_1$ satisfies

$$\dot{\Sigma}_1 = -a_1[\Sigma_1]$$

Quantum BRST cohomology in this space

(close to fixed point)

Eigenoperators

$$\dot{\Sigma} = a_0[S_I, \Sigma] - a_1[\Sigma] \qquad \dot{S}_I = \tfrac{1}{2}a_0[S_I, S_I] - a_1[S_I]$$

(e.g. Gaussian FP)

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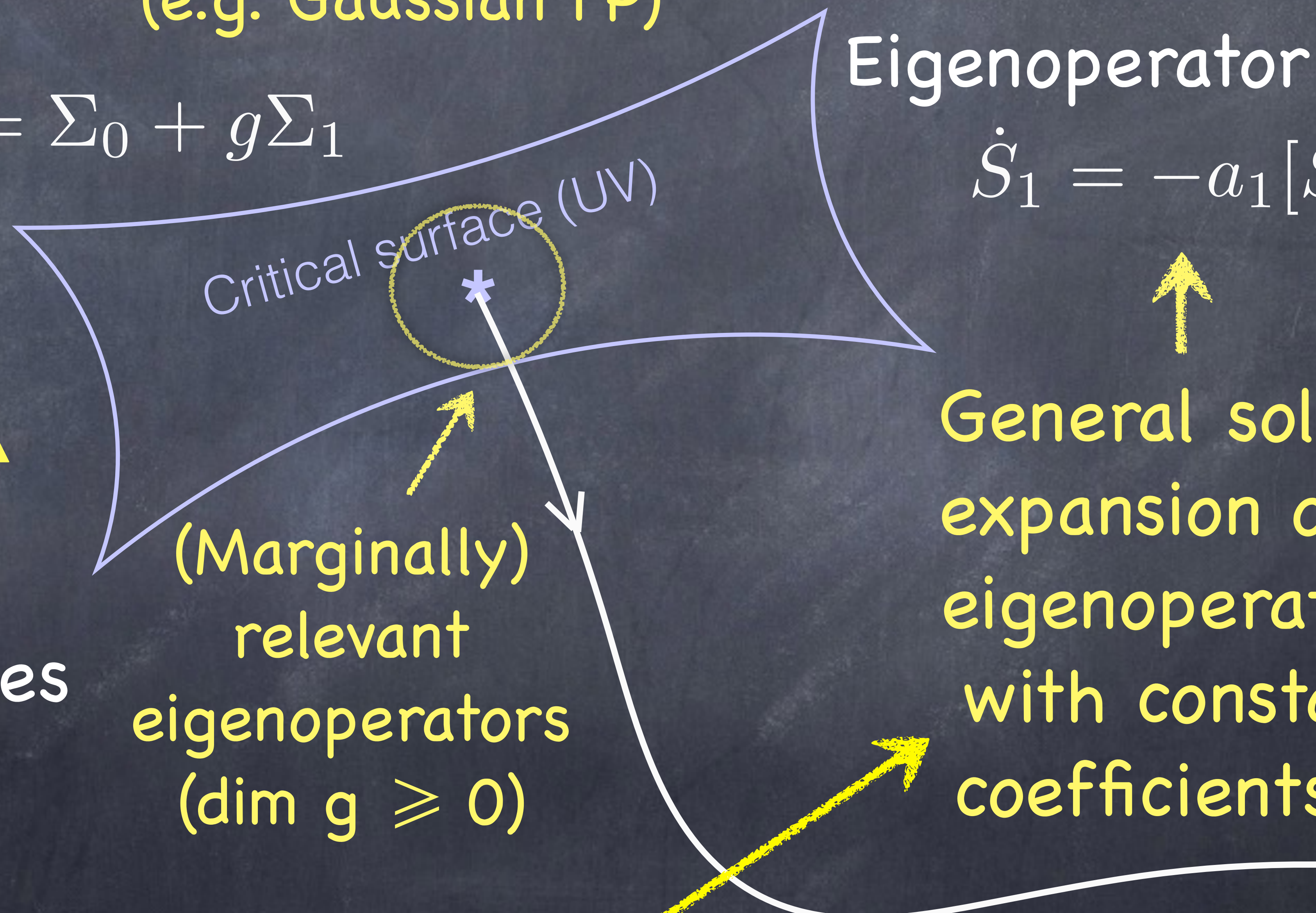

$$\hat{s}_0 = Q_0 + Q_0^- - \Delta$$

(Marginally) relevant eigenoperators (dim g $\geqslant$ 0)

General soln is expansion over eigenoperators with constant coefficients g.

$\Sigma_1 = \hat{s}_0\, S_1$ satisfies

$$\dot{\Sigma}_1 = -a_1[\Sigma_1]$$

Quantum BRST cohomology in this space

Renormalized trajectory close to FP

$$S = S_0 + gS_1 + g^2 S_2 + \cdots \qquad \dot{S}_n = \frac{1}{2}\sum_{m=1}^{n-1} a_0[S_{n-m}, S_m] - a_1[S_n]$$

$$\Sigma = \Sigma_0 + g\Sigma_1 + g^2\Sigma_2 + \cdots \qquad \dot{\Sigma}_n = \sum_{m=1}^{n-1} a_0[S_{n-m}, \Sigma_m] - a_1[\Sigma_n]$$

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$$\Sigma = \Sigma_0 + g\Sigma_1 + g^2\Sigma_2 + \cdots \qquad \dot{\Sigma}_n = \cancel{\sum_{m=1}^{n-1} a_0[S_{n-m}, \Sigma_m]} - a_1[\Sigma_n]$$

If QME already solved up to $\Sigma_{m<n} = 0$

then $\dot{\Sigma}_n = -a_1[\Sigma_n]$

$\Longrightarrow \Sigma_n$ can only be violated by a linear combination of eigenoperators

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$$\Sigma = \Sigma_0 + g\Sigma_1 + g^2\Sigma_2 + \cdots \quad \dot{\Sigma}_n = \sum_{m=1}^{n-1} a_0[S_{n-m}, \Sigma_m] - a_1[\Sigma_n]$$

If QME already solved up to $\Sigma_{m<n} = 0$

then $\dot{\Sigma}_n = -a_1[\Sigma_n]$

$\Longrightarrow \Sigma_n$ can only be violated by a

linear combination of eigenoperators

$$\Sigma_n = \hat{s}_0\, S_n + \cancel{\frac{1}{2}\sum_{m=1}^{n-1}(S_{n-m}, S_m)}$$

to be repaired by a

linear combination of eigenoperators

Renormalized trajectory ...

$$S = S_0 + gS_1 + g^2 S_2 + \cdots \qquad \dot{S}_n = \frac{1}{2}\sum_{m=1}^{n-1} a_0[S_{n-m}, S_m] - a_1[S_n]$$

$$\Sigma = \Sigma_0 + g\Sigma_1 + g^2\Sigma_2 + \cdots \qquad \dot{\Sigma}_n = \sum_{m=1}^{n-1} a_0[S_{n-m}, \Sigma_m] - a_1[\Sigma_n]$$

...follows from perturbative development of the BRST cohomology

$\Longrightarrow \Sigma_n$ can only be violated by a linear combination of eigenoperators

$$\Sigma_n = \hat{s}_0\, S_n + \frac{1}{2}\sum_{m=1}^{n-1}(S_{n-m}, S_m)$$

to be repaired by a linear combination of eigenoperators

Renormalized trajectory

Only freedom is to change coeffs in linear sum over eigenoperators

Soln of flow eqn (body of the physical amplitudes) guaranteed correct up to this linear combination of eigenoperators

Fix remaining freedom with renormalization conditions

Causes coupling constants to run.

Loop expansion

Eigenoperator eqn: $\dot{S}_1 = -\cancel{a_1[S_1]}$

(Marginally) relevant operators built from local terms with dimension $\leq$ 4

space-time dimension

Only freedom is in these Λ-independent local terms (counter-terms)

N.B. derivative expansion property at finite Λ, crucial.

$2\,\mathrm{tr}\int_x \{\cdots\}$

Example: Yang-Mills

$$S_0 = \tfrac{1}{2}\partial_\mu a_\nu K^{-1}\partial_\mu a_\nu - \tfrac{1}{2}\partial\cdot a K^{-1}\partial\cdot a + a^*_\mu K^{-1}\partial_\mu c$$

$$\Longrightarrow$$

$$Q_0\, a_\mu = \partial_\mu c\,, \qquad Q_0^-\, a^*_\mu = \Box\, a_\mu - \partial_\mu\partial\cdot a\,, \qquad Q_0^-\, c^* = -\partial\cdot a^*$$

$2\,\mathrm{tr}\int_x \{\cdots\}$

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$$S_0 = \tfrac{1}{2}\partial_\mu a_\nu K^{-1}\partial_\mu a_\nu - \tfrac{1}{2}\partial\cdot a K^{-1}\partial\cdot a + a^*_\mu K^{-1}\partial_\mu c$$

-1

$$\Longrightarrow$$

+1 +1

$$Q_0\, a_\mu = \partial_\mu c\,, \qquad Q_0^-\, a^*_\mu = \Box\, a_\mu - \partial_\mu\partial\cdot a\,, \qquad Q_0^-\, c^* = -\partial\cdot a^*$$

0 1 -2

Ghost numbers

$2\,\mathrm{tr}\int_x \{\cdots\}$

Example: Yang-Mills

$$S_0 = \tfrac{1}{2}\partial_\mu a_\nu K^{-1}\partial_\mu a_\nu - \tfrac{1}{2}\partial\cdot a K^{-1}\partial\cdot a + \overset{2}{\underset{-1}{a^*_\mu}} K^{-1}\partial_\mu c$$

$\Longrightarrow$

$$\overset{+1}{\overset{+1}{Q_0}}\,\underset{0}{a_\mu} = \partial_\mu \underset{1}{c}\,, \qquad \overset{+1}{\overset{+1}{Q_0^-}}\,a^*_\mu = \Box\, a_\mu - \partial_\mu\partial\cdot a\,, \qquad Q_0^-\,\overset{2}{\underset{-2}{c^*}} = -\partial\cdot a^*$$

Ghost numbers Dimension

$2\,\mathrm{tr}\int_x \{\cdots\}$

Example: Yang-Mills

2

$$S_0 = \tfrac{1}{2}\partial_\mu a_\nu K^{-1}\partial_\mu a_\nu - \tfrac{1}{2}\partial\cdot a K^{-1}\partial\cdot a + a^*_\mu K^{-1}\partial_\mu c$$

-1

+1   +1

+1   +1 $\Longrightarrow$ 2

$$Q_0\, a_\mu = \partial_\mu c\,, \qquad Q_0^-\, a^*_\mu = \Box\, a_\mu - \partial_\mu\partial\cdot a\,, \qquad Q_0^-\, c^* = -\partial\cdot a^*$$

+0   0   1   -1   1   -2

0   0   2

Ghost numbers   Dimension   Anti-field number: treats pieces differently!

$$2\,\mathrm{tr}\int_x \{\cdots\}$$

Example: Yang-Mills

2

$$S_0 = \tfrac{1}{2}\partial_\mu a_\nu K^{-1}\partial_\mu a_\nu - \tfrac{1}{2}\partial\cdot a K^{-1}\partial\cdot a + a^*_\mu K^{-1}\partial_\mu c$$

-1

+1 +1 $\Longrightarrow$

+1 +1 2

$$Q_0\, a_\mu = \partial_\mu c\,, \qquad Q_0^-\, a^*_\mu = \Box\, a_\mu - \partial_\mu\partial\cdot a\,, \qquad Q_0^-\, c^* = -\partial\cdot a^*$$

+0 0 1 -1 1 -2

0 0 2

Ghost numbers Dimension Anti-field number: treats pieces differently!

$$\hat{s}_0\, S_1 = 0$$

Want a solution up to $S_1 \mapsto S_1 + \hat{s}_0\mathcal{K}$

$2\,\mathrm{tr}\int_x \{\cdots\}$

Example: Yang-Mills

2

$$S_0 = \tfrac{1}{2}\partial_\mu a_\nu K^{-1}\partial_\mu a_\nu - \tfrac{1}{2}\partial\cdot a K^{-1}\partial\cdot a + a^*_\mu K^{-1}\partial_\mu c$$

-1

$\Longrightarrow$

+1   +1   2

+1   +1

$$Q_0\, a_\mu = \partial_\mu c\,, \qquad Q_0^-\, a^*_\mu = \Box\, a_\mu - \partial_\mu\partial\cdot a\,, \qquad Q_0^-\, c^* = -\partial\cdot a^*$$

+0   0   1   -1   1   -2

0   0   2

Ghost numbers   Dimension   Anti-field number: treats pieces differently!

$$\hat{s}_0\, S_1 = 0$$

Want a solution up to $S_1 \mapsto S_1 + \cancel{\hat{s}_0 \mathcal{K}}$

But $\mathcal{K}$ must be Λ independent, max dimension 3, ghost number -1, and have 3 fields!

Example: Yang-Mills

$2\,\mathrm{tr}\int_x \{\cdots\}$

2

$$S_0 = \tfrac{1}{2}\left(\partial_\mu a_\nu\right)^2 - \tfrac{1}{2}\left(\partial\cdot a\right)^2 + a^*_\mu \partial_\mu c$$

-1

+1 +1 $\Longrightarrow$

+1 +1

2

$$Q_0\, a_\mu = \partial_\mu c\,, \qquad Q_0^-\, a^*_\mu = \Box\, a_\mu - \partial_\mu \partial\cdot a\,, \qquad Q_0^-\, c^* = -\partial\cdot a^*$$

+0 0 1 -1 1 -2

0 0 2

Ghost numbers Dimension Anti-field number: not conserved by action!

$\hat{s}_0\, S_1 = 0$

For same reasons, S_1 has maximum anti-field number 2

where it is unique: $S_1^2 = -ic^*c^2$ [M Henneaux et al]

Thus: $S_1 = S_1^2 + S_1^1 + S_1^0$

is the unique extension of Maxwell theory

$$Q_0\, a_\mu = \partial_\mu c\,, \qquad Q_0^-\, a^*_\mu = \Box\, a_\mu - \partial_\mu \partial\!\cdot\! a\,, \qquad Q_0^-\, c^* = -\partial\!\cdot\! a^*$$

$$\hat{s}_0\, S_1 = (Q_0 + Q_0^-)\, S_1 = 0 \qquad\qquad S_1^2 = -ic^* c^2$$

Descendents:

$$Q_0 S_1^2 = 0 \qquad \checkmark$$

$$Q_0 S_1^1 = -Q_0^- S_1^2$$

$$i\partial\!\cdot\! a^* c^2 = -i a^*_\mu \{\partial_\mu c, c\} = -i a^*_\mu \{Q_0 a_\mu, c\} = iQ_0 \left(a^*_\mu [a_\mu, c]\right)$$

$\Longrightarrow \quad S_1^1 = -i a^*_\mu [a_\mu, c]$ unique `deformation' to $a^*_\mu D_\mu c$

$$Q_0 S_1^0 = -Q_0^- S_1^1$$

$\Longrightarrow \quad S_1^0 = -i \partial_\mu a_\nu [a_\mu, a_\nu]$ unique cubic interaction

is the unique deformation of Maxwell theory

Classical solution at O(g²)

$$s_0\, S_{2,\mathrm{cl}} = -\tfrac{1}{2}(S_1, S_1)$$

$$s_0\, S_{2,\mathrm{cl}} = \big([c^*, c] + [a^*_\mu, a_\mu]\big)\, K(c^2) \;-\; \{a^*_\mu, c\} K[a_\mu, c] \;-\; \Theta_\mu K[a_\mu, c]$$

$$\partial_\rho[a_\rho, a_\mu] + [\partial_\mu a_\rho - \partial_\rho a_\mu, a_\rho]$$

$$Q_0\, S^2_2 = [c^*, c] K(c^2)$$

$$K(-\partial^2/\Lambda^2) = 1 + \sum_{n=1}^{\infty} K_n(-\partial^2/\Lambda^2)^n$$

No canonical choice

(infinitely many solutions differing by s_0-exact piece)

Gauge fixed basis

Extended (non-minimal) gauge invariant basis:

$$S_0|_{\mathrm{gi}} = \tfrac{1}{2}\partial_\mu a_\nu K^{-1}\partial_\mu a_\nu - \tfrac{1}{2}\partial\cdot a K^{-1}\partial\cdot a + a^*_\mu K^{-1}\partial_\mu c + \frac{\xi}{2} b K^{-1} b + \bar{c}^* K^{-1} b$$

Canonical transformation:

$$\phi^*_A|_{\mathrm{gf}} = \phi^*_A|_{\mathrm{gi}} + \partial^r_A \Psi$$ e.g. $$\Psi = -i\bar{c}\,\partial\cdot a \quad \Longrightarrow$$

$$\bar{c}^*|_{\mathrm{gi}} = \bar{c}^*|_{\mathrm{gf}} + i\partial\cdot a\,,$$

$$a^*_\mu|_{\mathrm{gi}} = a^*_\mu|_{\mathrm{gf}} - i\partial_\mu\bar{c}$$

$$\langle c^a(p)\,\bar{c}^b(-p)\rangle = i\delta^{ab}\triangle(p)\,, \quad \text{where} \quad \triangle(p) := 1/p^2\,,$$

$$\langle a^a_\mu(p)\,a^b_\nu(-p)\rangle = \delta^{ab}\triangle_{\mu\nu}(p)\,, \quad \text{where} \quad \triangle_{\mu\nu}(p) := \frac{1}{p^2}\left(\delta_{\mu\nu} + (\xi-1)\frac{p_\mu p_\nu}{p^2}\right)\,,$$

$$\langle b^a(p)\,a^b_\mu(-p)\rangle = -\langle a^a_\mu(p)\,b^b(-p)\rangle = \delta^{ab}p_\mu\triangle(p)\,,$$

$$\langle b^a(p)\,b^b(-p)\rangle = 0$$

Classical solution at O(g^2)

$$\dot{S}_{2,\mathrm{cl}} = \tfrac{1}{2} a_0 [S_1, S_1] = -\tfrac{1}{2}\, \partial^r_A S_1 \dot{\bar{\triangle}}^{AB} \partial^l_B S_1$$

$$S_{2,\mathrm{cl}} = -\tfrac{1}{2}\, \partial^r_A S_1 \bar{\triangle}^{AB} \partial^l_B S_1 + \mathcal{O}_2$$

Unique solution with derivative expansion up to Λ independent piece

Classical solution at O(g²)

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$$\tfrac{1}{2}\left(\{a^*_\mu, c\} + \Theta_\mu\right) \bar{\triangle}_{\mu\nu} \left(\{a^*_\nu, c\} + \Theta_\nu\right) - \left([c^*, c] + [a^*_\mu, a_\mu]\right) \bar{\triangle}\, \partial_\nu [a_\nu, c]$$

$$s_0\, S_{2,\mathrm{cl}} = -\tfrac{1}{2}(S_1, S_1) \qquad \Longrightarrow \qquad \mathcal{O}_2 = -\tfrac{1}{4}[a_\mu, a_\nu]^2$$

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One-loop at O(g²)

$$\dot{S}_{2,\mathrm{q}} = -a_1[S_{2,\mathrm{cl}}] \quad \Longrightarrow \quad S_{2,\mathrm{q}} = \tfrac{1}{2}\, a_\mu\, \mathcal{A}_{\mu\nu}(\partial)\, a_\nu + a^*_\mu\, \mathcal{B}(-\partial^2)\, \partial_\mu c$$

$$s_0 S_{2,\mathrm{q}} = \Delta S_{2,\mathrm{cl}} = a_\mu\, \mathcal{F}(-\partial^2)\, \partial_\mu c$$

$$C_A \int_q \bar{\triangle}(q) K(p+q) \left\{ 4\frac{p\cdot q}{p^2} + 2 + \xi + (1-\xi)\frac{(p\cdot q)^2}{p^2 q^2} \right\}$$

$$p_\mu\, \mathcal{A}_{\mu\nu}(p) = \mathcal{F}(p^2)\, p_\nu$$

Λ dependent mass term + ...

Why?

$$\hat{s}\,\mathcal{O} = 0 \qquad\qquad \hat{s}\,\mathcal{O} = (\mathcal{O}, S) - \Delta\mathcal{O} = (Q + Q^{-} - \Delta)\,\mathcal{O}$$

Formally, BRST-invariant operators that are exact have vanishing correlators:

$$\langle \underline{\hat{s}\mathcal{K}\,\mathcal{O}_1 \cdots \mathcal{O}_n} \rangle = \langle \hat{s}\,(\mathcal{K}\mathcal{O}_1 \cdots \mathcal{O}_n) \rangle = -\frac{1}{\mathcal{Z}}\int \mathcal{D}\phi\,\Delta\left(\mathcal{K}\mathcal{O}_1 \cdots \mathcal{O}_n\,\mathrm{e}^{-S}\right) = 0$$

disjoint support

$\Longrightarrow$ decoupling of longitudinal modes, gauge parameter independence ...

Why?

$\hat{s}\,\mathcal{O} = 0$

$\hat{s}\,\mathcal{O} = (\mathcal{O}, S) - \Delta\mathcal{O} = (Q + Q^{-} - \Delta)\,\mathcal{O}$

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disjoint support

$\Longrightarrow$ decoupling of longitudinal modes, gauge parameter independence ...

But now this holds only in the limit $\Lambda \to \infty$ or $\Lambda \to 0$

$$\Delta(\mathcal{O}_1\mathcal{O}_2) = \mathcal{O}_1\Delta\mathcal{O}_2 + (-)^{\mathcal{O}_2}(\Delta\mathcal{O}_1)\mathcal{O}_2 + (-)^{\mathcal{O}_2}(\mathcal{O}_1, \mathcal{O}_2)$$

$$(\mathcal{O}_1, \mathcal{O}_2) = \frac{\partial_r\mathcal{O}_1}{\partial\phi^A}\,K\,\frac{\partial_l\mathcal{O}_2}{\partial\phi^*_A} - \frac{\partial_r\mathcal{O}_1}{\partial\phi^*_A}\,K\,\frac{\partial_l\mathcal{O}_2}{\partial\phi^A} \to 0$$

for $\Lambda \to 0$ or $\Lambda|x_1 - x_2| \to \infty$

Legendre effective action:

Legendre transformation:

$$\Gamma_I[\Phi, \Phi^*] = S_I[\phi, \phi^*] - \tfrac{1}{2}\,(\phi - \Phi)^A\,\bar{\Delta}^{-1}_{AB}\,(\phi - \Phi)^B$$

$\Phi^* = \phi^*$

$\Rightarrow$

$$\dot{\Gamma}_I = -\tfrac{1}{2}\,\mathrm{Str}\left(\dot{\bar{\Delta}}\bar{\Delta}^{-1}\left[1 + \bar{\Delta}\Gamma^{(2)}_I\right]^{-1}\right)$$

$$\Gamma^{(2)}_{I\;AB} = \frac{\overrightarrow{\partial}}{\partial\Phi^A}\Gamma_I\frac{\overleftarrow{\partial}}{\partial\Phi^B}$$

with IR cutoff Λ

Legendre effective action:

Effective average action:

$$\Gamma = \Gamma_0 + \Gamma_I\,, \qquad \Gamma_0 = \tfrac{1}{2}\,\Phi^A \triangle^{-1}_{AB}\Phi^B + \Phi^*_A R^A{}_B \Phi^B$$

Antibracket:

$$(\Xi, \Upsilon) = \frac{\partial_r \Xi}{\partial \Phi^A}\,\frac{\partial_l \Upsilon}{\partial \Phi^*_A} - \frac{\partial_r \Xi}{\partial \Phi^*_A}\,\frac{\partial_l \Upsilon}{\partial \Phi^A}$$

$$\left(\Gamma^{(2)}_{I*}\right)^A{}_B = \frac{\overrightarrow{\partial}}{\partial \Phi^*_A}\Gamma_I \frac{\overleftarrow{\partial}}{\partial \Phi^B}$$

$$\Rightarrow \qquad \Sigma = \tfrac{1}{2}(\Gamma,\Gamma) - \mathrm{Tr}\left(K\Gamma^{(2)}_{I*}\left[1 + \bar{\triangle}\Gamma^{(2)}_I\right]^{-1}\right)$$

These give modified Slavnov-Taylor identities

U Ellwanger, Phys Lett B335 (1994) 364

⇨ Zinn-Justin identities as $\Lambda \to 0$

Legendre effective action

Effective average action:

$$\Gamma = \Gamma_0 + \Gamma_I \,, \qquad \Gamma_0 = \tfrac{1}{2}\,\Phi^A \triangle^{-1}_{AB}\Phi^B + \Phi^*_A R^A{}_B \Phi^B$$

Antibracket:

$$(\Xi, \Upsilon) = \frac{\partial_r \Xi}{\partial \Phi^A}\,\frac{\partial_l \Upsilon}{\partial \Phi^*_A} - \frac{\partial_r \Xi}{\partial \Phi^*_A}\,\frac{\partial_l \Upsilon}{\partial \Phi^A}$$

$$\left(\Gamma^{(2)}_{I*}\right)^A{}_B = \frac{\overset{\rightarrow}{\partial}}{\partial \Phi^*_A}\Gamma_I \frac{\overset{\leftarrow}{\partial}}{\partial \Phi^B}$$

$$\Sigma = \tfrac{1}{2}(\Gamma, \Gamma) - \mathrm{Tr}\left(K\Gamma^{(2)}_{I*}\left[1 + \bar{\triangle}\Gamma^{(2)}_I\right]^{-1}\right)$$

$$\Gamma = \Gamma_0 + g\Gamma_1 + g^2\Gamma_2 + g^3\Gamma_3 + \cdots$$

$$\hat{s}_0\,\Gamma_1 = (Q_0 + Q_0^- - \Delta)\,\Gamma_1 = 0 \qquad\qquad \Delta\Gamma = \mathrm{Tr}\left(K\Gamma^{(2)}_*\right)$$

Legendre effective action:

Gives direct access to physics in limit $\Lambda \to 0$

Is one-particle irreducible

But BRST invariance obscured at $\Lambda > 0$:

$$\Sigma = \frac{1}{2}(\Gamma, \Gamma) - \mathrm{Tr}\left(K \Gamma^{(2)}_{I*} \left[1 + \bar{\triangle} \Gamma^{(2)}_{I} \right]^{-1} \right)$$

Wilsonian effective action:

Is entirely equivalent

UV regularisation makes Δ well defined

Unbroken quantum BRST invariance: $\Sigma = \frac{1}{2}(S, S) - \Delta S = 0$

But $\Delta S \neq 0$ leads to Λ-dependent mass terms etc..

In interacting case neither equation has local solutions:

UV cutoff function $K(p^2/\Lambda^2)$

$$\Sigma = \frac{1}{2}(S,S) - \Delta S = 0$$

$$\Sigma = \frac{1}{2}(\Gamma,\Gamma) - \mathrm{Tr}\left(K\Gamma^{(2)}_{I*}\left[1+\bar{\Delta}\Gamma^{(2)}_{I}\right]^{-1}\right)$$

so no local bare action that satisfies regularised QME or mST

But don't need a bare action.
Can solve these equations directly for the renormalized trajectory to all orders in derivative expansion.

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

$$\hat{s}_0\, \Gamma_1 = (Q_0 + Q_0^-)\, \Gamma_1 = 0 \qquad \Gamma_1^2 = -iC^* C^2$$

Descendents:

$$Q_0 \Gamma_1^2 = 0 \qquad \checkmark$$

$$Q_0 \Gamma_1^1 = -Q_0^- \Gamma_1^2$$

$$i\partial\cdot A^* C^2 = -iA^*_\mu \{\partial_\mu C, C\} = -iA^*_\mu \{Q_0 A_\mu, C\} = iQ_0 \left(A^*_\mu [A_\mu, C]\right)$$

$\Longrightarrow \quad \Gamma_1^1 = -iA^*_\mu [A_\mu, C]$ unique `deformation' to $A^*_\mu D_\mu C$

$$Q_0 \Gamma_1^0 = -Q_0^- \Gamma_1^1$$

$\Longrightarrow \quad \Gamma_1^0 = -i\partial_\mu A_\nu [A_\mu, A_\nu]$ unique cubic interaction

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

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$$\Longrightarrow \quad \Gamma_1^1 = -iA^*_\mu [A_\mu, C]$$ unique `deformation' to $A^*_\mu D_\mu C$

$$Q_0 \Gamma_1^0 = -Q_0^- \Gamma_1^1$$

$$\Longrightarrow \quad \Gamma_1^0 = -i\partial_\mu A_\nu [A_\mu, A_\nu]$$ unique cubic interaction

Classical solution at O(g^2)

$$s_0\, \Gamma_{2,\mathrm{cl}} = (Q_0 + Q_0^-)\, \Gamma_{2,\mathrm{cl}} = -\tfrac{1}{2}(\Gamma_1, \Gamma_1) = Q_0\left(-\tfrac{1}{4}[A_\mu, A_\nu]^2\right)$$

unique quartic interaction

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

Define coupling to be coefficient of unique s_0-closed:

$$g(\Lambda)\,\Gamma_1 = Z_g^{-1}(\Lambda)\, g\,\Gamma_1$$

Only other possibilities are s_0-closed two-point vertices.

Uniquely two options:

$$\tfrac{1}{2} z_A (Q_0 + Q_0^-)(A^*_\mu A_\mu) = -\tfrac{1}{2} z_A \left\{ (\partial_\mu A_\nu)^2 - (\partial\cdot A)^2 \right\} + \tfrac{1}{2} z_A A^*_\mu \partial_\mu C$$

$$\tfrac{1}{2} z_C (Q_0 + Q_0^-)(C^* C) = \tfrac{1}{2} z_C\, A^*_\mu \partial_\mu C$$

But these are s_0-exact, so just canonical reparametrisation:

$$\mathcal{K} = Z_E^{\frac{1}{2}} \Phi^*_E \Phi^E_{(r)}$$

$$\Phi^E = \frac{\partial_l}{\partial \Phi^*_E} \mathcal{K}[\Phi_{(r)}, \Phi^*]\,,\ \ \Phi^*_{(r)E} = \frac{\partial_r}{\partial \Phi^E_{(r)}} \mathcal{K}[\Phi_{(r)}, \Phi^*]$$

$$A_\mu = Z_A^{\frac{1}{2}} A_{(r)\mu}\,, \quad A^*_\mu = Z_A^{-\frac{1}{2}} A^*_{(r)\mu}\,, \quad C = Z_C^{\frac{1}{2}} C_{(r)}\,, \quad C^* = Z_C^{-\frac{1}{2}} C^*_{(r)}$$

Thus RG flow generates flow along consistent deformations of the QME:

$$\tfrac{1}{2} Z_A^{-1} A_\mu(-\Box\delta_{\mu\nu} + \partial_\mu\partial_\nu)A_\nu \;+\; Z_A^{\frac{1}{2}} Z_C^{-\frac{1}{2}} A^*_\mu \partial_\mu C$$

$$- igZ_g^{-1} Z_C^{-\frac{1}{2}} \left(C^* C^2 + A^*_\mu [A_\mu, C]\right) - igZ_g^{-1} Z_A^{-\frac{3}{2}} \partial_\mu A_\nu [A_\mu, A_\nu]$$

$$- \frac{1}{4} g^2 Z_g^{-2} Z_A^{-2} [A_\mu, A_\nu]^2$$

Standard parameterisation:

$$Z_3 = Z_A\,, \;\; \tilde{Z}_3 = Z_C^{\frac{1}{2}} Z_A^{-\frac{1}{2}}\,, \;\; Z_1 = Z_g Z_A^{\frac{3}{2}}\,, \;\; \tilde{Z}_1 = Z_g Z_C^{\frac{1}{2}}\,, \;\; Z_4 = Z_g^2 Z_A^2$$

$$\frac{Z_1}{Z_3} = \frac{\tilde{Z}_1}{\tilde{Z}_3} = \frac{Z_4}{Z_1} = Z_g Z_A^{\frac{1}{2}}$$

Slavnov-Taylor identities!

One-loop at O(g²)

$$\tfrac{1}{2} Z_A^{-1} A_\mu(-\Box\delta_{\mu\nu} + \partial_\mu\partial_\nu) A_\nu \; + \; Z_A^{\frac{1}{2}} Z_C^{-\frac{1}{2}} A^*_\mu \partial_\mu C$$

$$\Gamma_{2,\mathrm{q}} = \tfrac{1}{2}\, A_\mu\, \mathcal{A}_{\mu\nu}(\partial)\, A_\nu + A^*_\mu\, \mathcal{B}(-\partial^2)\, \partial_\mu C$$

$$p_\mu\, \mathcal{A}_{\mu\nu}(p) = \mathcal{F}(p^2)\, p_\nu$$

$$\dot{\mathcal{A}}^T_{\mu\nu}(p) = -\dot{z}_A(p^2\delta_{\mu\nu} - p_\mu p_\nu) + O(p^4/\Lambda^2)$$

$$\dot{z}_A = \frac{C_A}{(4\pi^2)^2}\left(\frac{13}{3} - \xi\right)\underbrace{\int_0^\infty du\, \frac{\partial}{\partial u}\bar{K}^2(u)}_{1}$$

$$\dot{\mathcal{B}}(0) = \tfrac{1}{2}(\dot{z}_A - \dot{z}_C) = \frac{\xi - 3}{2(4\pi)^2} C_A \int_0^\infty du\, \frac{\partial}{\partial u}\bar{K}^2(u)$$

One-loop at O(g²)

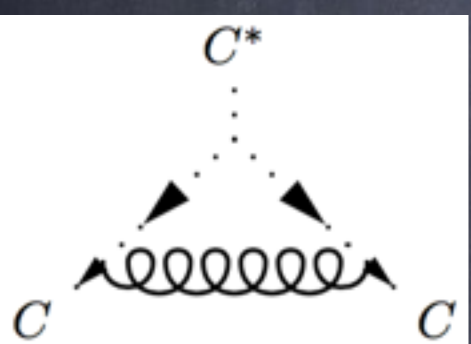
$$\tfrac{1}{2} Z_A^{-1} A_\mu(-\Box\delta_{\mu\nu} + \partial_\mu\partial_\nu) A_\nu \; + \; Z_A^{\frac{1}{2}} Z_C^{-\frac{1}{2}} A^*_\mu \partial_\mu C$$

$$\Gamma_{2,\mathrm{q}} = \tfrac{1}{2}\, A_\mu\, \mathcal{A}_{\mu\nu}(\partial)\, A_\nu + A^*_\mu\, \mathcal{B}(-\partial^2)\, \partial_\mu C \qquad p_\mu\, \mathcal{A}_{\mu\nu}(p) = \mathcal{F}(p^2)\, p_\nu$$

$$\dot{\mathcal{A}}^T_{\mu\nu}(p) = -\dot{z}_A(p^2\delta_{\mu\nu} - p_\mu p_\nu) + O(p^4/\Lambda^2) \qquad \dot{z}_A = \frac{C_A}{(4\pi^2)^2}\left(\frac{13}{3} - \xi\right)\int_0^\infty du\, \frac{\partial}{\partial u}\bar{K}^2(u)$$

$$\dot{\mathcal{B}}(0) = \tfrac{1}{2}(\dot{z}_A - \dot{z}_C) = \frac{\xi - 3}{2(4\pi)^2} C_A \int_0^\infty du\, \frac{\partial}{\partial u}\bar{K}^2(u)$$

1



One-loop at O(g³)

$$-\, igZ_g^{-1} Z_C^{-\frac{1}{2}}\left(C^*C^2 + A^*_\mu[A_\mu, C]\right) - igZ_g^{-1} Z_A^{-\frac{3}{2}} \partial_\mu A_\nu [A_\mu, A_\nu]$$

$$\dot{z}_g + \tfrac{1}{2}\dot{z}_C = -\frac{C_A}{2}\frac{\partial}{\partial t}\int_q \bar{\triangle}^2(q) q_\mu q_\nu \bar{\triangle}_{\mu\nu}(q) = -\frac{\xi C_A}{(4\pi)^2}\int_0^\infty du\, \frac{\partial}{\partial u}\bar{K}^3(u) = -\frac{\xi C_A}{(4\pi)^2}$$

$$\beta(g) = \Lambda\partial_\Lambda(gZ_g^{-1}) = g^3 \dot{z}_g$$

$$\left(\dot{z}_g + \tfrac{1}{2}\dot{z}_C\right) + \tfrac{1}{2}\left(\dot{z}_A - \dot{z}_C\right) - \tfrac{1}{2}\dot{z}_A = -\frac{11}{3}\frac{C_A}{(4\pi)^2}$$

Conclusions.

- Despite breaking by cutoff BRST invariance still very much present in flow equation.
- Regularisation of Δ plus existence of derivative expansion, allows to define quantum BRST cohomology.
- Together yields a formalism that is still elegant and not much harder than Dim Reg.