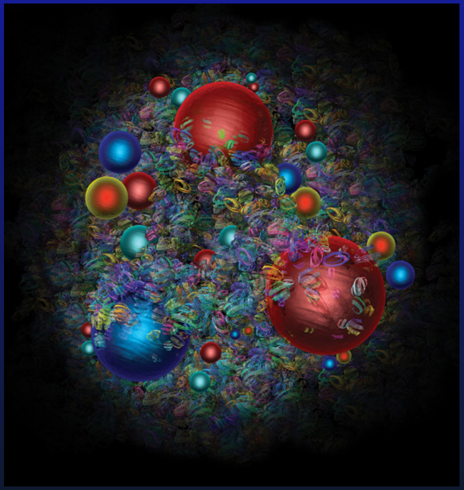
Matter under extreme conditions
-- from neutron stars to nucleons

Gordon Baym
University of Illinois
&
RIKEN iTHEMS

neutron star over Kyoto

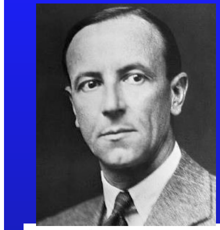
condensed
matter of QCD

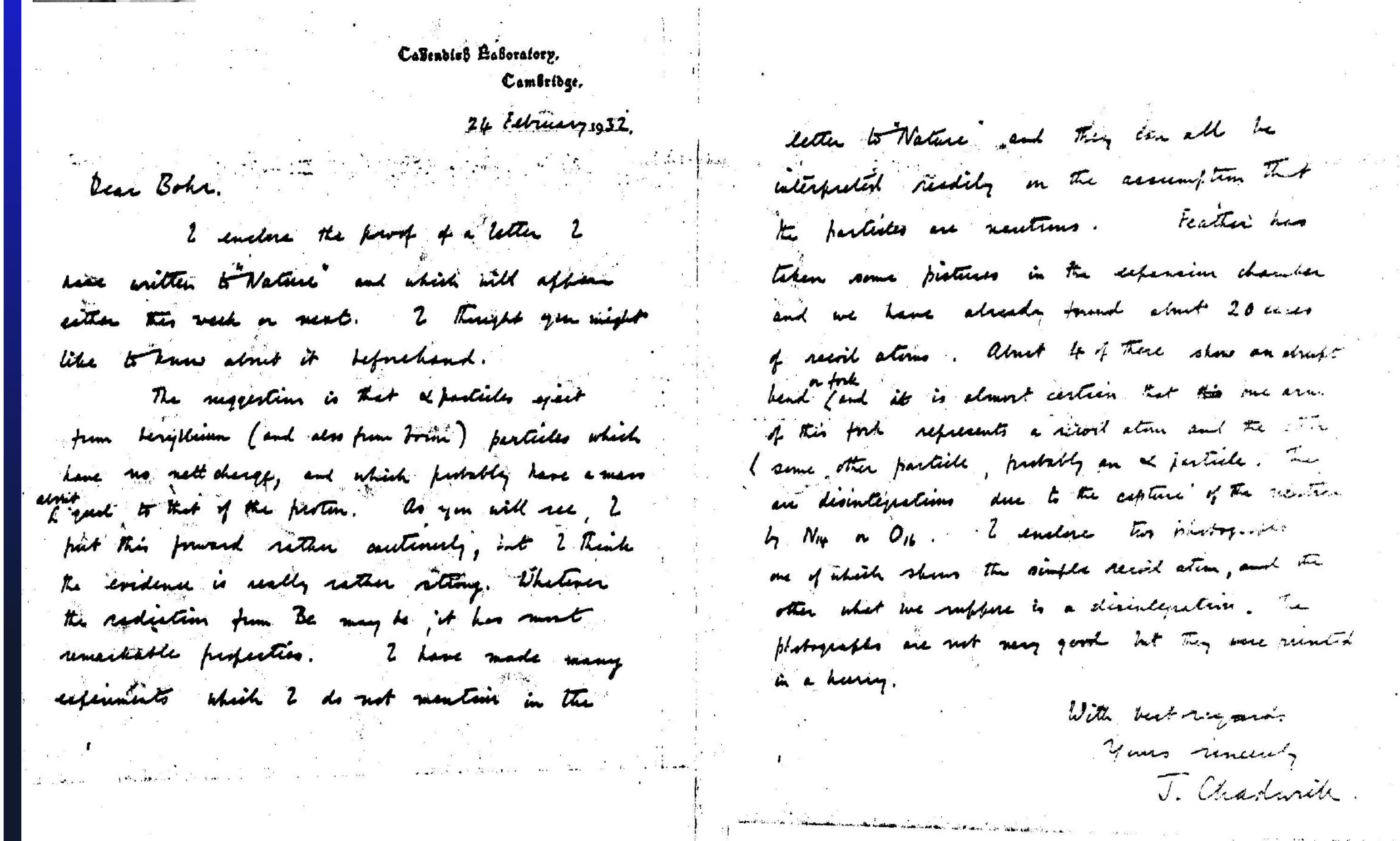


inside the proton

Condensed Matter Physics of QCD 2024
Yukawa Institute
京都大学　9 March 2024

Chadwick to Bohr, 24 Feb. 1932 announcing the discovery of the neutron





Cavendish Laboratory,
Cambridge.

24 February 1932.

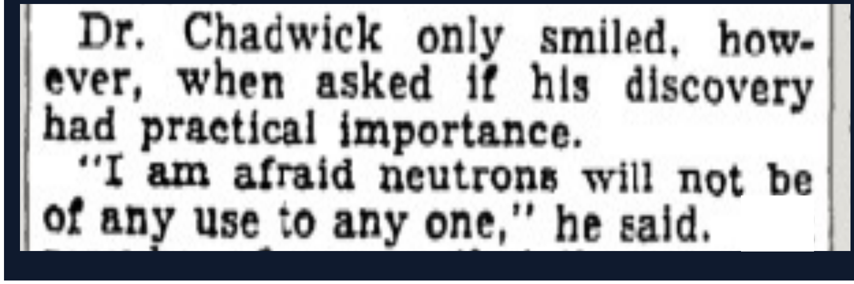
Dear Bohr.

I enclose the proof of a letter I have written to "Nature" and which will appear either this week or next. I thought you might like to know about it beforehand.

The suggestion is that α particles eject from beryllium (and also from boron) particles which have no nett charge, and which probably have a mass about equal to that of the proton. As you will see, I put this forward rather cautiously, but I think the evidence is really rather strong. Whatever the radiation from Be may be, it has most remarkable properties. I have made many experiments which I do not mention in the letter to "Nature" and they can all be interpreted readily on the assumption that the particles are neutrons. Feather has taken some pictures in the expansion chamber and we have already found about 20 cases of recoil atoms. About 4 of these show an abrupt bend or fork (and it is almost certain that ~~this~~ one arm of this fork represents a recoil atom and the other (some other particle, probably an α particle. These are disintegrations due to the capture of the neutron by N_{14} or O_{16}. I enclose two photographs one of which shows the simple recoil atom, and the other what we suppose is a disintegration. The photographs are not very good but they were mounted in a hurry.

With best regards
Yours sincerely
J. Chadwick.

"The suggestion is that α particles eject from beryllium (...) particles which have no nett charge, and which probably have a mass almost equal to that of the proton."

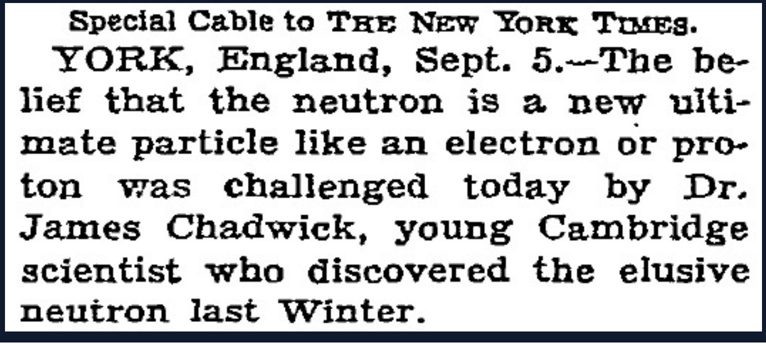


Dr. Chadwick only smiled, however, when asked if his discovery had practical importance.

"I am afraid neutrons will not be of any use to any one," he said.

NY Times
02/29/32

09/05/32



Special Cable to THE NEW YORK TIMES.

YORK, England, Sept. 5.—The belief that the neutron is a new ultimate particle like an electron or proton was challenged today by Dr. James Chadwick, young Cambridge scientist who discovered the elusive neutron last Winter.

Neutron star interior

Mass ~ 1-2+ M_{sun}

Radius ~ 12-13 km

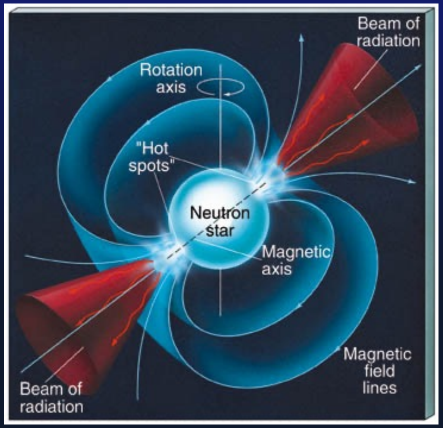
Temperature

~ 10^6-10^9 K

Baryon no. ~ 10^{57}

Magnetic fields

~ 10^6 – 10^{16} G

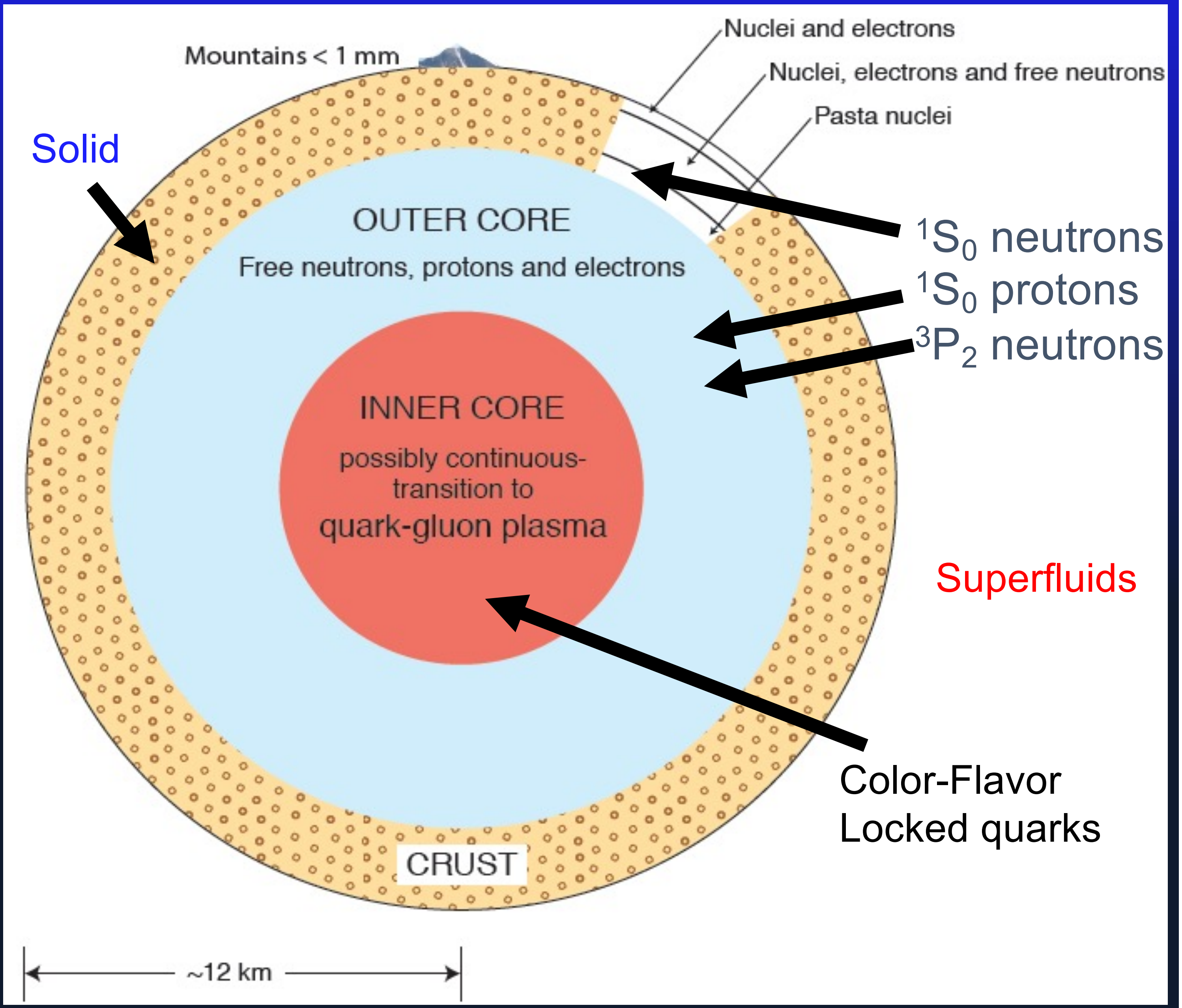


Surface gravity

~10^{11} that of Earth

Surface binding

~ 1/10 mc^2



Solid

$^1S_0$ neutrons

$^1S_0$ protons

$^3P_2$ neutrons

Superfluids

Color-Flavor Locked quarks

Made in gravitational collapse of massive stars (supernovae)
and can be a remnant of binary neutron star mergers

Matter in neutron stars is densest in universe:
baryon density n up to ~ 5-10 n_0

$n_0 = 0.16\ /\mathrm{fm}^3$ <-> 3×10^{14} g/cm^3 = density of matter in atomic nuclei
[cf. white dwarfs: $\rho \sim 10^5$-10^9 g/cm^3] 1 fm = 10^{-13} cm

Supported against gravitational collapse by nucleon
(and at higher densities quark) degeneracy pressure

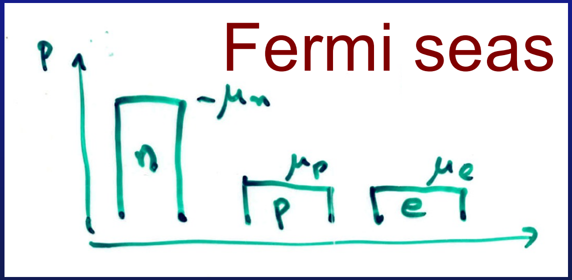
Central engines in variety of compact energetic systems:
pulsars, binary X-ray sources, Soft Gamma Repeaters,
magnetars, ultraluminous X-ray sources, Fast Radio Bursts (FRB),
binary neutron star (and ns-black hole) mergers

Astrophysical laboratory for study of cold high-density matter and QCD

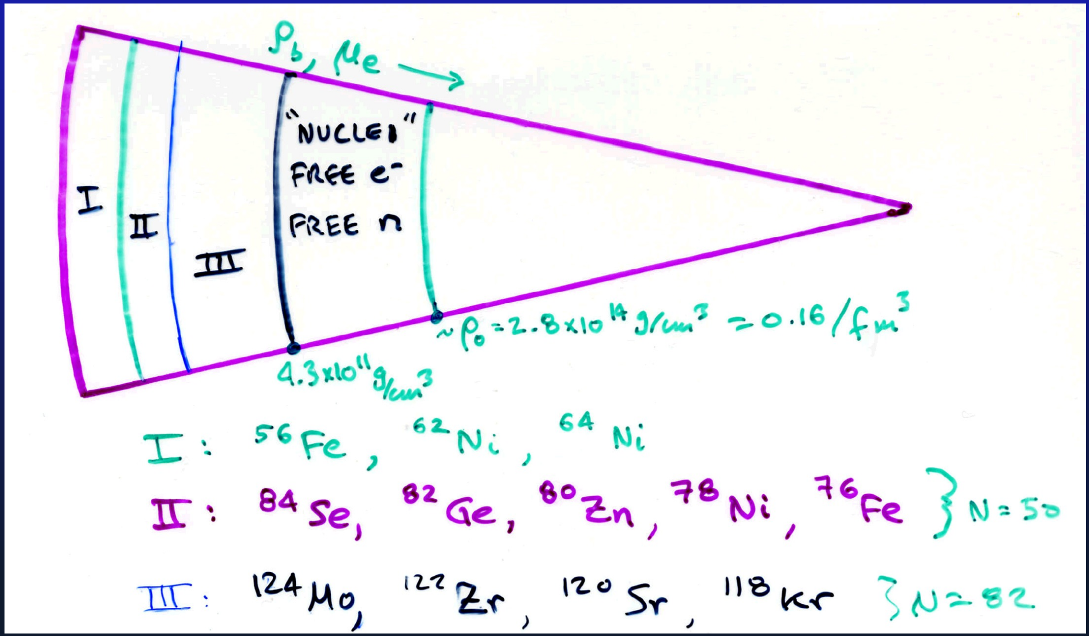
Nuclei before neutron drip (outer crust)

$e^- + p \longrightarrow n + \nu$: makes nuclei neutron rich as electron Fermi energy increases with depth

$n \longrightarrow p + e^- + \bar{\nu}$: not allowed if e^- state already occupied



Fermi seas



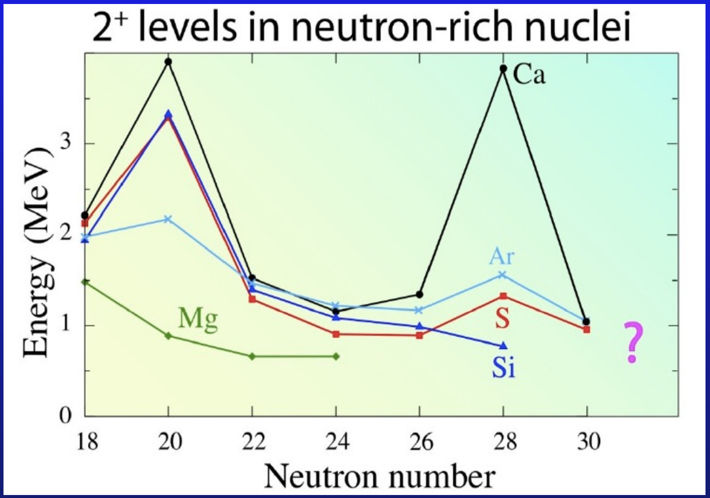
Shell structure (spin-orbit forces) for very neutron rich nuclei?

Do N=50, 82 remain neutron magic numbers?

Proton shell structure?

Being explored at rare isotope accelerators: RIKEN Rare Ion Beam Facility (RIBF), FRIB (MSU) + soon RAON 라온 (KoRIA)

Modification of shell structure for N >> Z



Usual shell closings
(N ~ Z) at 20, 28, 50, 82, 126

No shell effect for Mg(Z=12), Si(14), S(16), Ar(18) at N=20 and 28

Oxygen has new shell closure at N=16

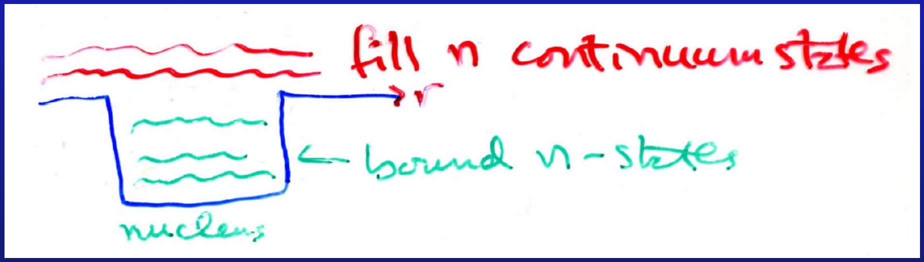
Calcium has shell closure at N=34

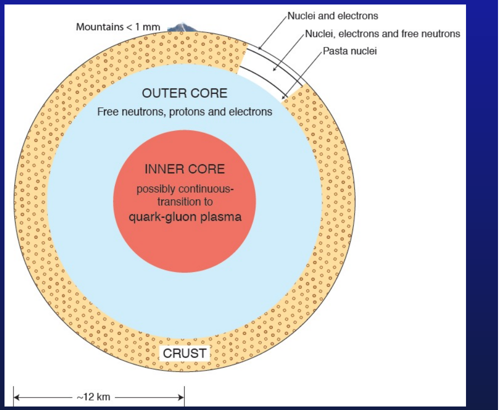
Recently observed: O(28) with N=20
Na(39) with N=28

Spin-orbit forces and hence shell structure modified by tensor and 3-body forces in neutron-rich nuclei

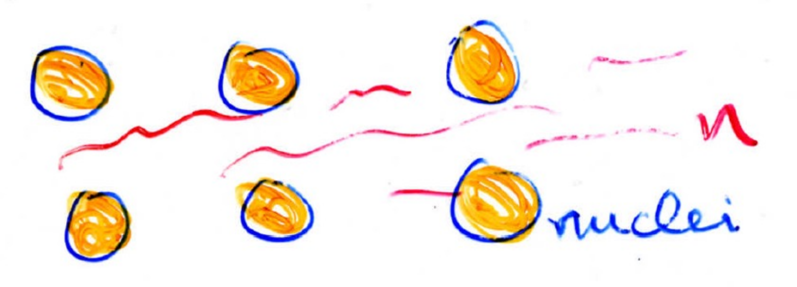
Neutron drip

Beyond density ρ_{drip} ~ 4.3 X 10^{11} g/cm^3 neutron bound states in nuclei become filled. Further neutrons must go into continuum states. Form degenerate neutron Fermi sea.





Neutrons in neutron sea are in equilibrium with those inside nucleus (common μ_n)



How do the nuclei turn into liquid at higher densities?

Proton drip!! Protons preferentially remain in bound states, until shortly before nuclei merge in liquid interior.

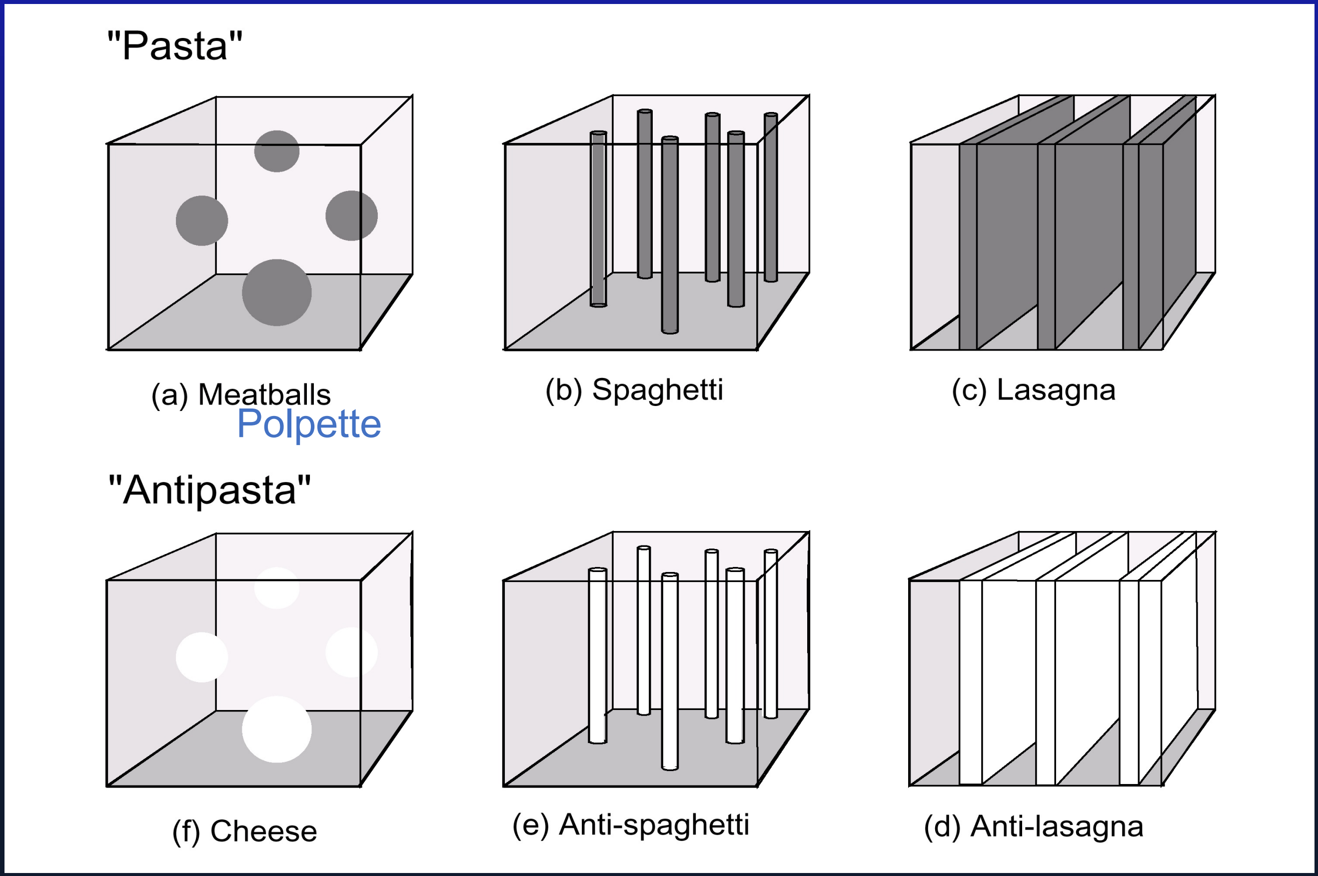
J. Keller, K. Hebeler, C. J. Pethick, A. Schwenk arXiv:2401.13461

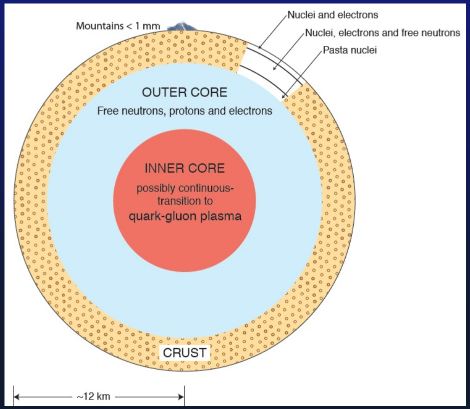
Pasta Nuclei in inner crust? $n > 0.1\, n_0$

D. G. Ravenhall, C. J. Pethick, and J. R. Wilson, PRL 50, 2066 (1983)

When Coulomb wins over surface energies:
as in Bohr-Wheeler criterion for nuclear fission ($Z^2/A > 50$)

F K Lamb





F.K. Lamb

Involves over half the mass of the crust!! Effects on crust bremsstrahlung of neutrinos, pinning of n vortices, modes of crust, ... ??

The liquid interior

Neutrons (likely superfluid) ~ 95%	Non-relativistic
Protons (likely superconducting) ~ 5%	Non-relativistic
Electrons (normal, $T_c \sim T_f\, e^{-137}$) ~ 5%	Fully relativistic

Eventually muons, hyperons??, quark matter and possible exotica:

- pion or kaon condensation?
- matter in transition from nucleonic to quark liquid (“quarkyonic” matter)

Phase transition from crust to liquid at $n_b \simeq 0.7\, n_0 \simeq 0.09\ \mathrm{fm}^{-3}$ (mass density ~ $2 \times 10^{14}\mathrm{g/cm}^3$). 10% uncertainty!

$n_0 \simeq 0.16\ \mathrm{fm}^{-3}$

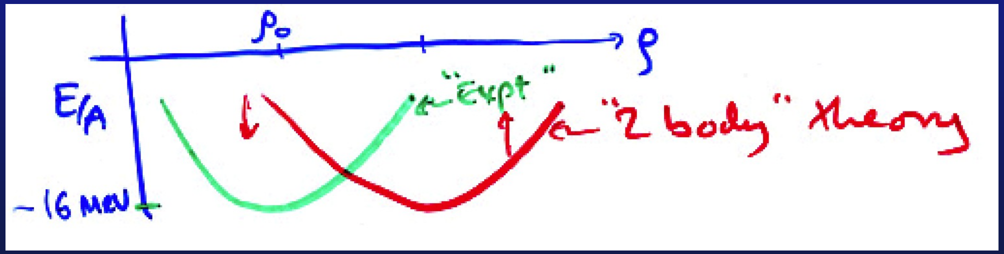
Uncertainties in nuclear matter liquid.

Near nuclear matter density

1) Determine N-N potentials (e.g., Argonne V-18 2-body potential) from scattering expts E < 300 MeV, +deuteron, 3 body nuclei (^{3}He, ^{3}H)

Solve Schrödinger equation by variational techniques

Two body potential alone fails!!
Fix with three body forces.



Underbind ^{3}H: Exp = -8.48 MeV, Theory = -7.5 MeV
^{4}He: Exp = -28.3 MeV, Theory = -24.5 MeV

2) Chiral effective theory approach

3) Relativistic mean field models

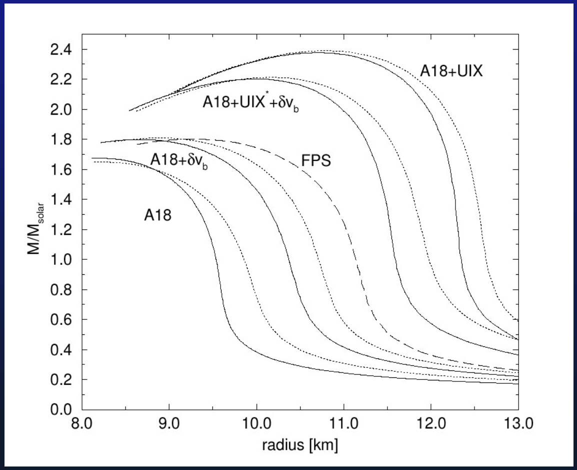
Theoretical extrapolation from low energy laboratory nuclear physics near nuclear matter density up to $2n_0$ or higher

Large pressure in neutron star matter

Nuclear regime: densities up to ~1.5 n_0

Variational calculations from empirical nucleon-nucleon forces with three body corrections do not give large enough pressure

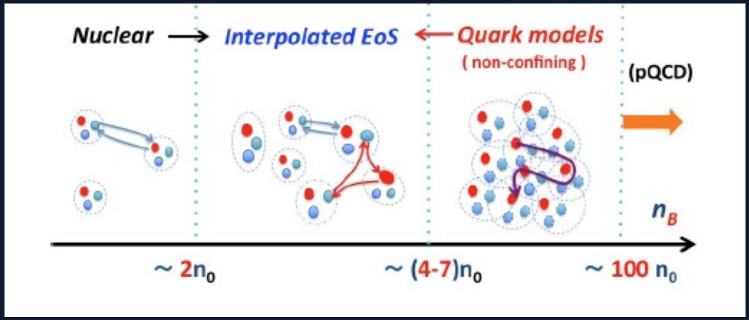
– don’t get radii above 11.5 km.



Nuclear regime better described by Chiral Effective Theory

Higher densities

Allow for onset of quark degrees of freedom

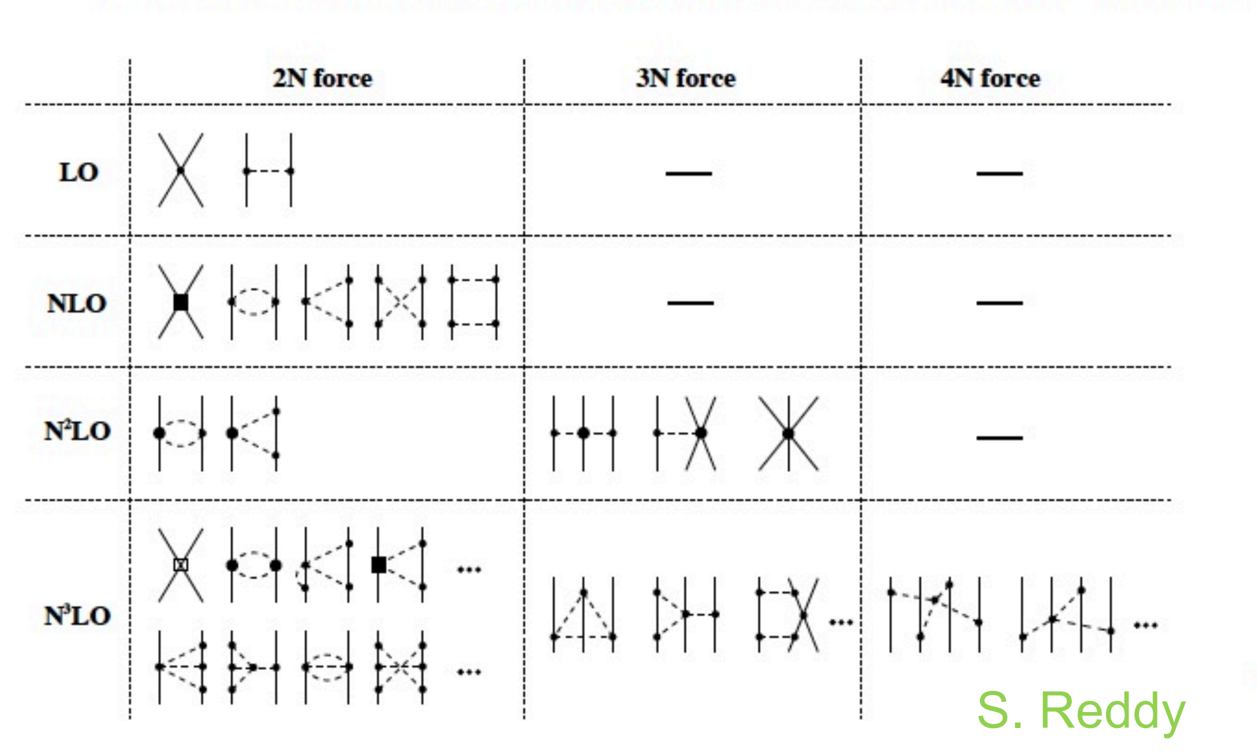


crossover to quarks

Chiral effective field theory (Weinberg)

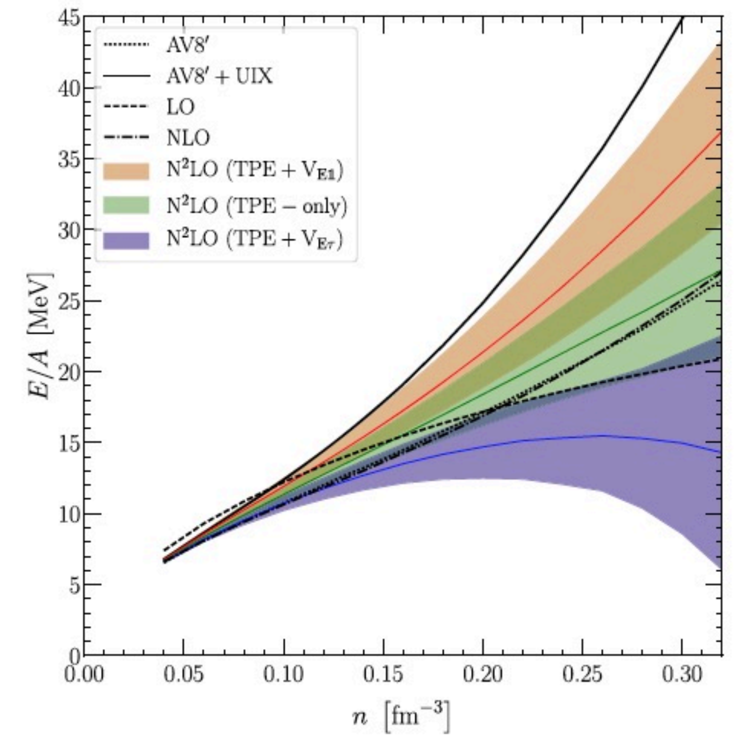
Expand E/A in powers of momentum scale Q.

Fit to NN phase shifts and deuteron; 3N and 4N binding energies and charge radii



S. Reddy

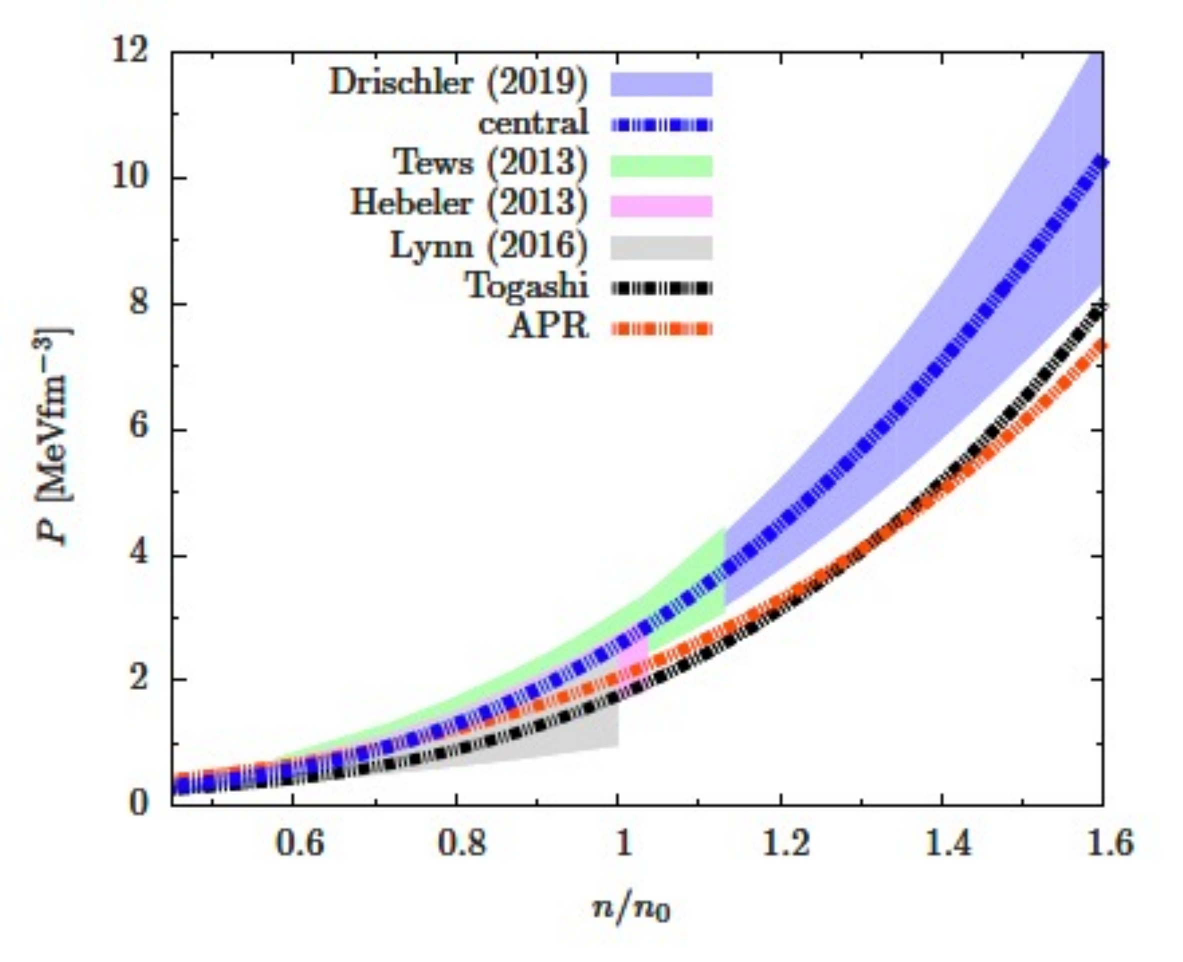
Beane, Bedaque, Epelbaum, Kaplan, Machliedt, Meißner, Phillips, Savage, van Kolck, Weinberg, Wise



Range of energy per baryon in XEFT

Completely breaks down by $2n_0$

Chiral Effective Theory equation of state (central) significantly stiffer than variational calculations with empirical forces (Akmal, Pandharipande and Ravenhall (APR), Togashi)



Significant because XEFT predicts larger stellar radii

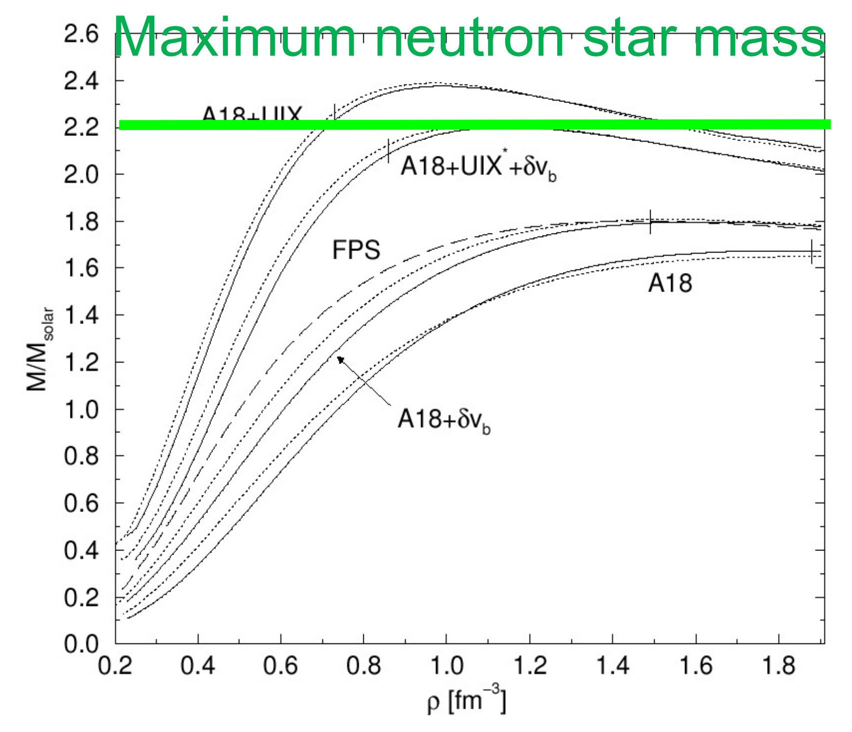
Construct neutron star models

E = energy density = ρc^2
n_b = baryon density
P(r) = pressure = $n_b^2\, d(E/n_b)/dn_b$

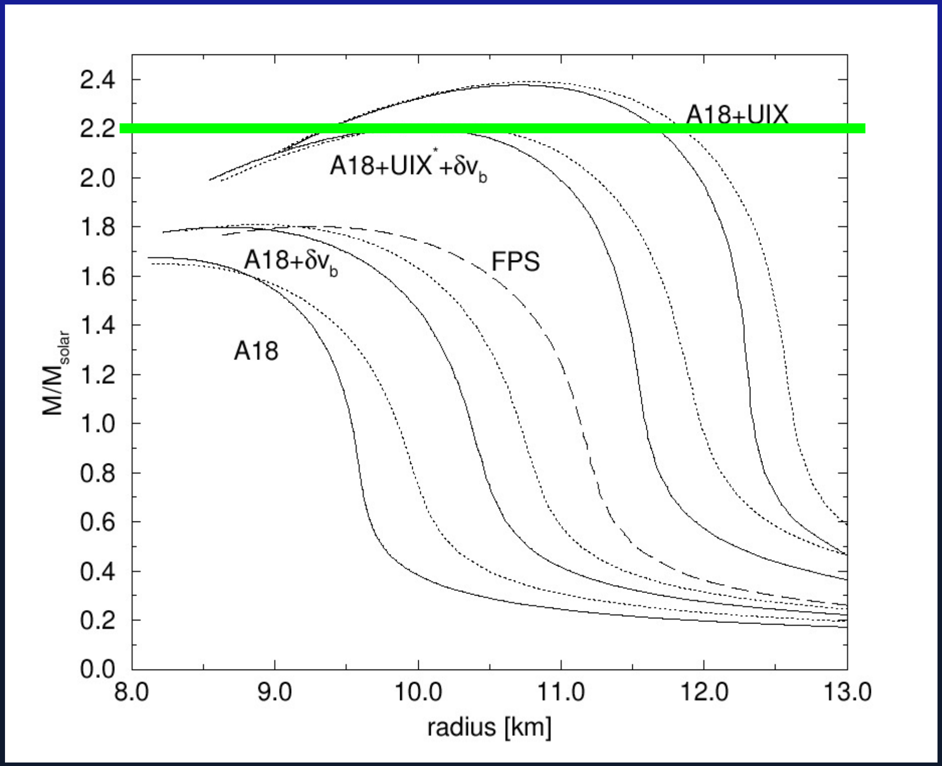
$$\frac{\partial P(r)}{\partial r} = -G\frac{\rho(r)+P(r)/c^2}{r\,(r-2Gm(r)/c^2)}\left[m(r)+4\pi r^3 P(r)/c^2\right]$$

TOV equation

$$M=\int_0^R 4\pi r^2\,dr\,\rho(r)$$


Maximum neutron star mass

Mass vs. central density



Mass vs. radius

Akmal, Pandharipande and Ravenhall (APR) 1998 nuclear equation of state

Fundamental limitations of eq. of state based on NN interactions

Accurate for n~ n_0. But for n >> n_0:

-sound speed becomes greater than speed of light

-importance of 3 (4,5...) body forces grows with n

-chiral effective theory breaks down above n ~ 1.5-2n_0.

-can forces be described with static few-body potentials?

-can one even describe system in terms of well-defined “asymptotic" laboratory particles?

Given all information on Nb+Nb atomic scattering could one predict that Nb is a superconductor?

Early percolation of nucleonic volumes! New degrees of freedom enter!!

Squeeze:

=>

Remember quarks

Equation of state based on NN interactions alone
yields nice looking models, but with faulty input physics
Wrong degrees of freedom at high density

That it seems to work does not make it right!

Cancelling the six’s

$$\frac{16}{64} = \frac{1\cancel{6}}{\cancel{6}4} = \frac{1}{4}$$

$$\frac{19}{95} = \frac{1\cancel{9}}{\cancel{9}5} = \frac{1}{5}$$

$$\frac{18}{85} = \frac{1\cancel{8}}{\cancel{8}5} \simeq \frac{1}{5}$$

The Golden Age of Neutron Stars:

Wealth of new observational data, and even Gold

The Golden Age of Neutron Stars:

Wealth of new observational data, and even Gold

Detections of heavy neutron stars in pulsars
Equation of state relatively stiff

Masses and radii of neutron stars
NICER measuring M, R directly for a few pulsars

Gravitational waves from ns-ns and ns-black hole mergers explore masses, radii, and tidal deformabilities
Production of heavy elements via r-process: Au, Pt, Ag, U, Sr

LIGO/Virgo/KAGRA
Eventually Cosmic Explorer, Einstein Telescope, LISA

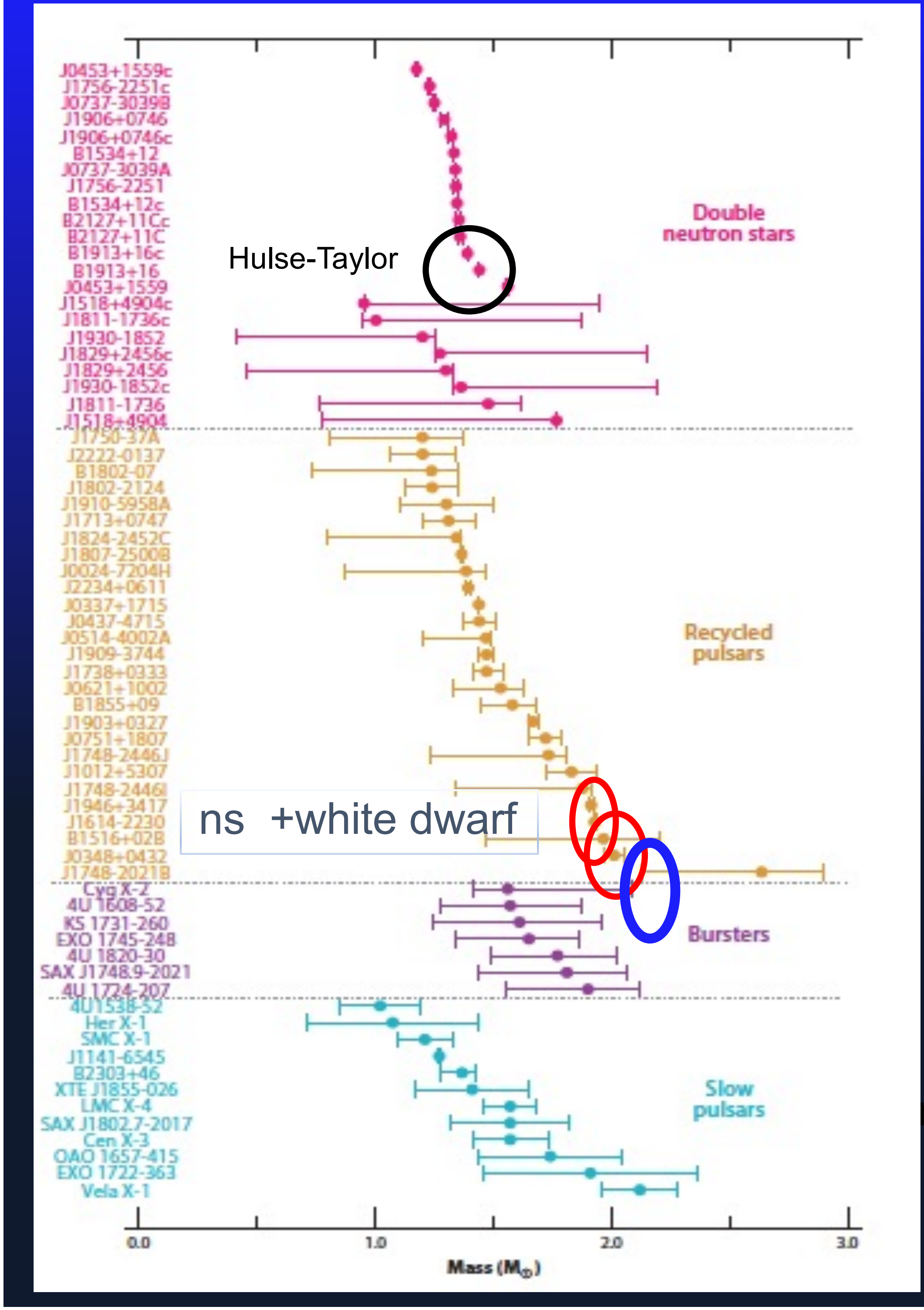
Pulsar timing arrays -- probe low frequency gravitational radiation

Glitches: probe n,p superfluidity and crust

Cooling of n-stars: search for rapid cooling from exotic states, measuring equation of state in crust

Neutron star masses

Özel & Freire, Ann Rev AA (2016)



PSR J1614-2230 :

$M_{nstar} = 1.928 \pm 0.017 M_\odot$

PSR J0348+0432:

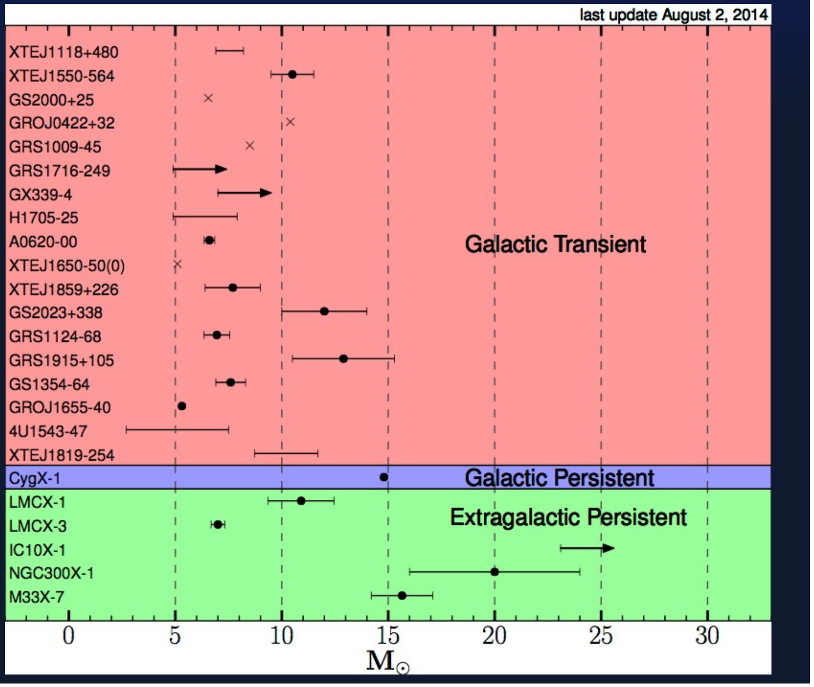
$M_{nstar} = 2.01 \pm 0.04 M_\odot$

PSR J0740+6620 :

$M_{nstar} = 2.08 \pm 0.07 M_\odot$

rs

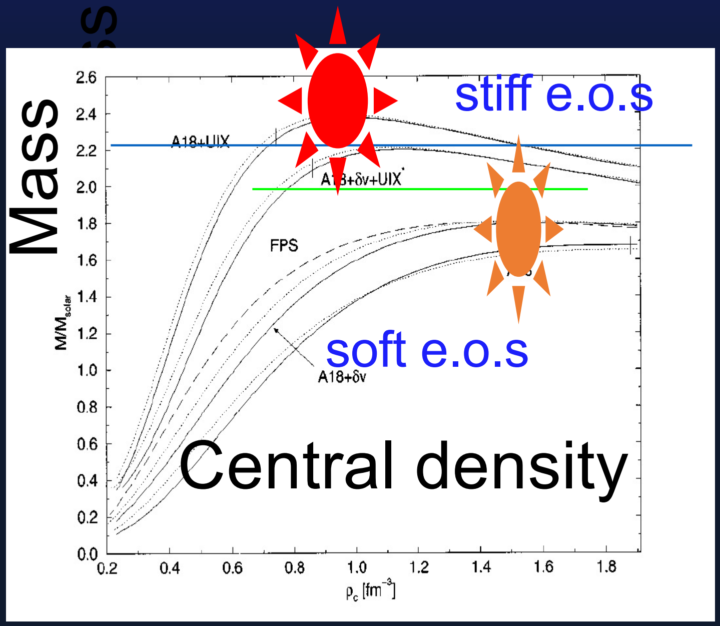
Galactic black hole masses



Mass determinations of three high mass neutron stars (pulsars in binaries)

PSR J0348+0432:	$M_{\text{neutron star}} = 2.01 \pm 0.04 M_{\odot}$	2013
PSR J1614-2230 :	$1.93 \pm 0.02 M_{\odot}$	2016
PSR J0740+6620 :	$2.08 \pm 0.07 M_{\odot}$	2019

=> the equation of state is stiff



Softer equation of state =>
lower maximum mass and
higher central density

Binary neutron stars ~ 1.4 $M_{\odot}$:
consistent with soft eq. of state

Can quarks support two solar masses?

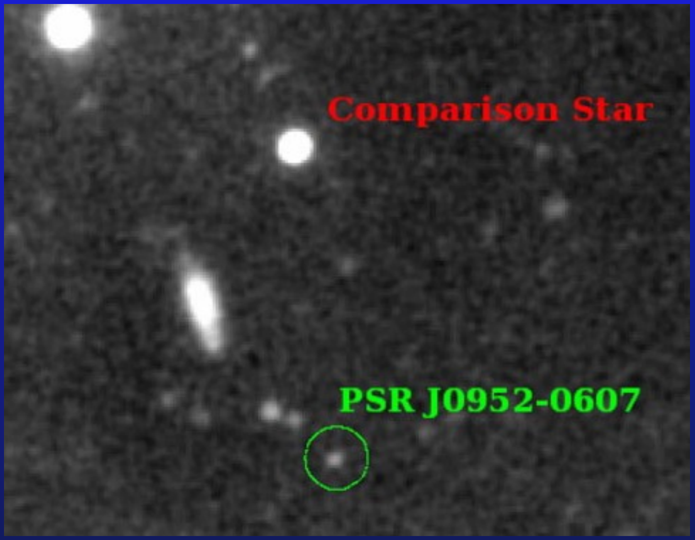
Further massive neutron stars

PSR J0952-0607

(R. Romani, Ap.J. Lett. 934:L17, 2022)

Black widow pulsar, P =14.1 msec.
Mass of companion = 0.032 ± 0.002 M_{sun}

M = 2.35 ± 0.17 M_{sun}



PSR 1748-2021B (Arecibo, 1998)

Binary pulsar in globular cluster M5. P = 16.7 msec.
Companion light and small. Awaiting detection by JWST. M<2.5 M_{sun}

PSR J0514–4002E MeerKAT (arXiv:2401.09872)
Companion is compact object with $2.09 < M < 2.71 M_\odot$
Neutron star or black hole??

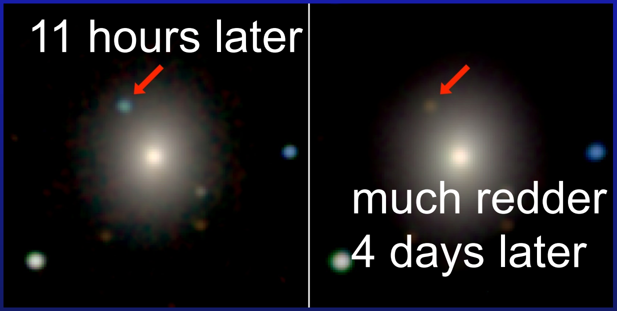
Gravitational radiation: new window on neutron stars

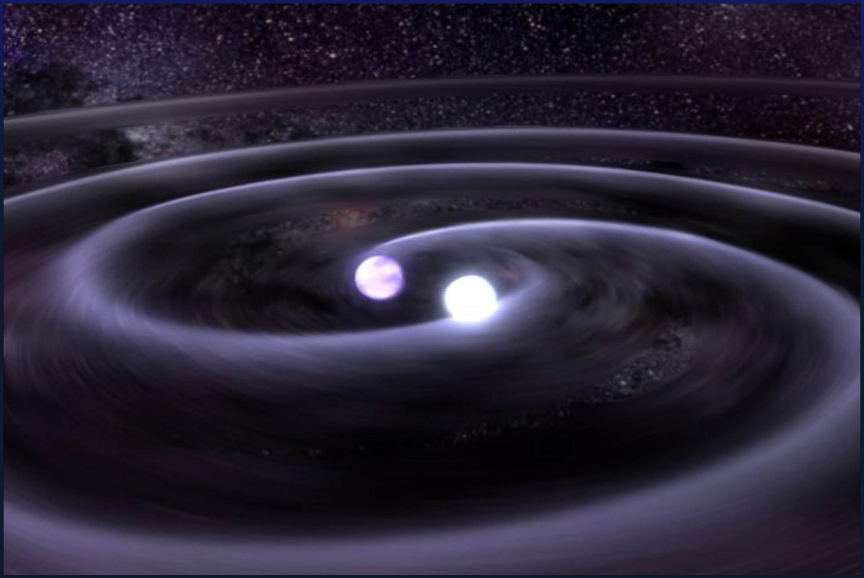
GW170817: Initial spectacular event – Multi-messenger astronomy

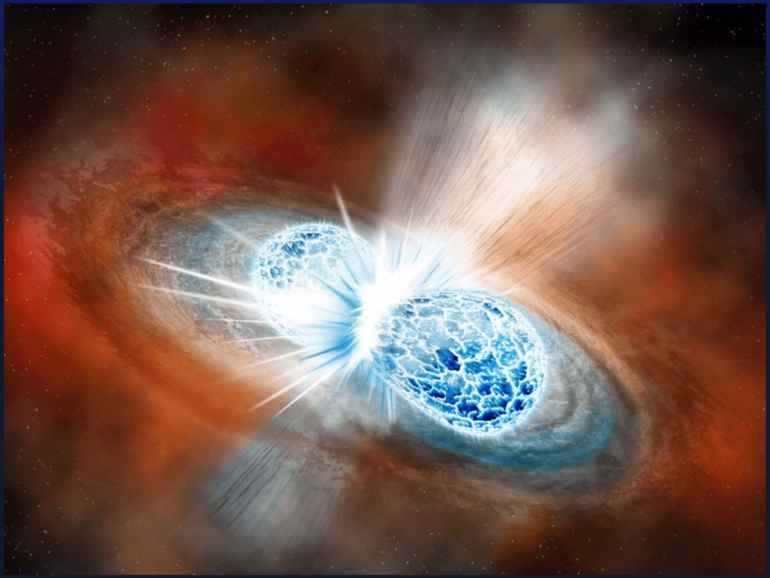
Neutron star – neutron star (BNS) merger observed on 17 Aug. 2017 by LIGO and Virgo (gravitational radiation), FERMI (gamma ray telescope) and ~ 70 other electromagnetic observatories.

$m_1 \sim 1.36\text{-}1.60\ M_\odot$, $m_2 \sim 1.17\text{-}1.36\ M_\odot$

radii ~ 11.9 ± 0.7 km





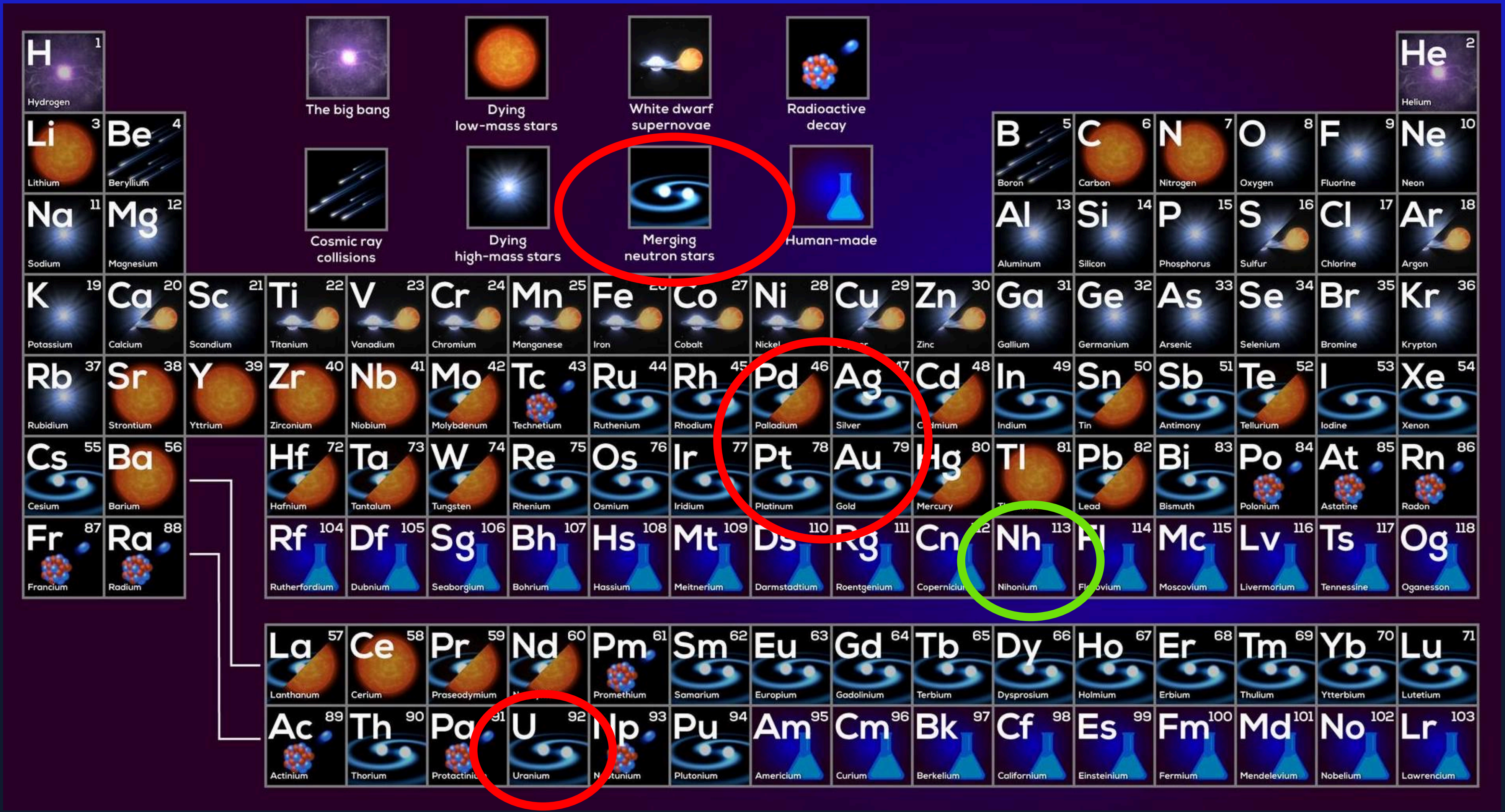


Two neutron stars merging, emitting gravitational radiation
And, post-merger, forming:

→

Kilonova: neutron-rich site of r-process

Binary neutron star mergers likely site of heavy element production (via r-process)



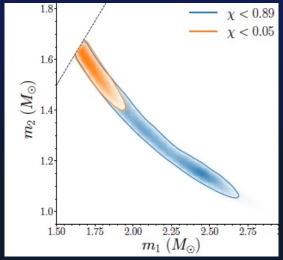
Periodic table of the elements with their “origins”

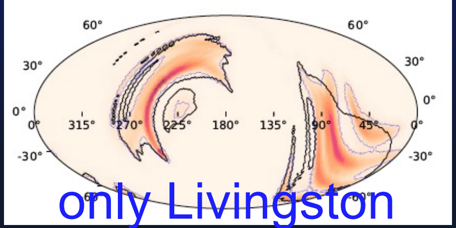
Gravitational radiation: new window on neutron stars

GW170817: Initial spectacular event – Multi-messenger astronomy

Neutron star – neutron star (BNS) merger observed on 17 Aug. 2017
by LIGO and Virgo (gravitational radiation),
FERMI (gamma ray telescope) and
~ 70 other electromagnetic observatories.
$m_1 \sim 1.36\text{-}1.60\ M_\odot$, $m_2 \sim 1.17\text{-}1.36\ M_\odot$
radii ~ 11.9 ± 0.7 km

GW190425: BNS $m_1 \sim 1.62\text{-}2.52\ M_\odot$
$m_2 \sim 1.12\text{-}1.68\ M_\odot$
Unlocalized, and no e.m. signals



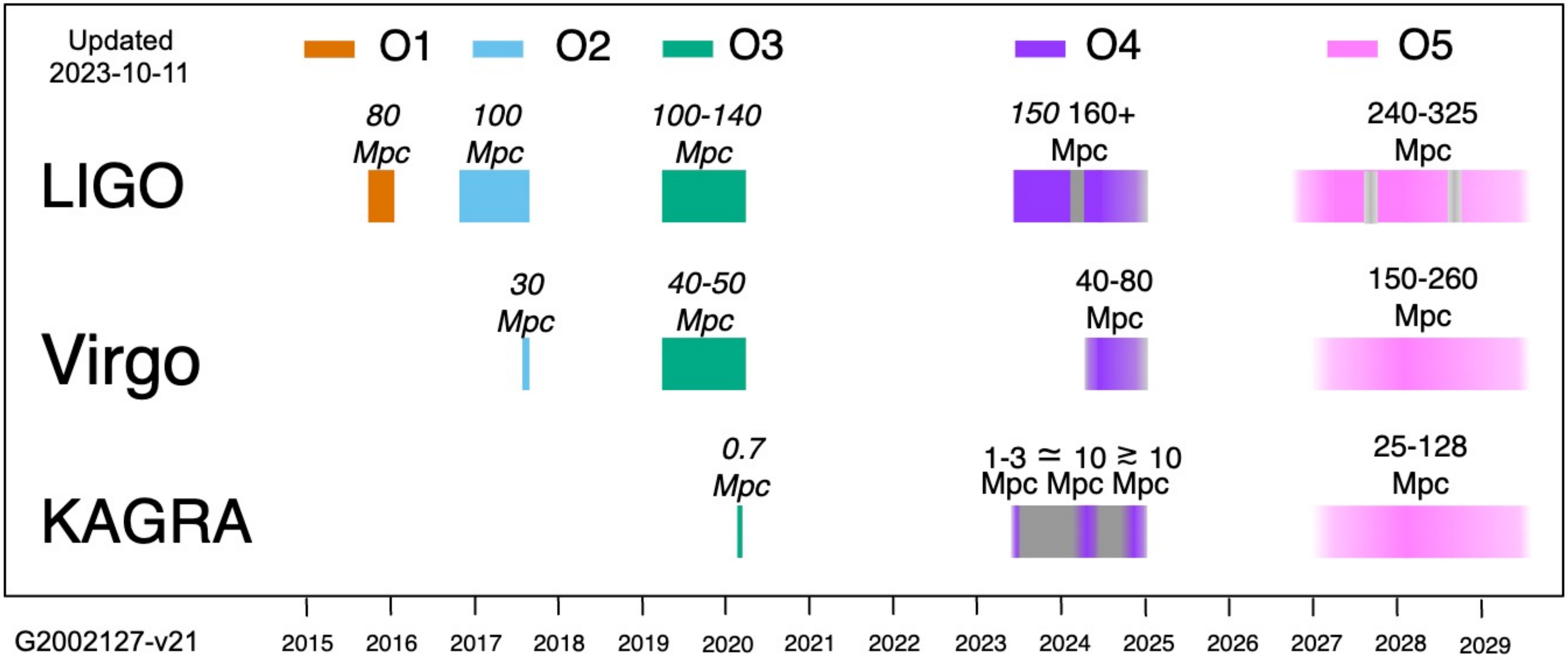


only Livingston

GW190814: $m_1 \sim 23.1\ M_\odot$ = black hole No e.m.
$m_1 \sim 2.5\text{-}2.67\ M_\odot$ = rotating neutron star or black hole??

GW190426: $m_1 \sim 5.7\ M_\odot$, $m_2 \sim 1.5\ M_\odot$ No e.m.

3rd generation detectors, to 400 Mpc => ~ 10^2 BNS mergers/year



Observing Run 4 (O4) started on May 24, 2023, and will continue for 20 calendar months from that date. We expect that up to four facilities (LHO, LLO, Virgo, and KAGRA) will contribute data during O4.

- Only LHO and LLO are currently operating, with BNS range of 140-165 Mpc. These two facilities will be offline for commissioning for about 2 months starting January 16, 2024.
- Virgo anticipates joining O4 in March 2024 with a BNS range of 40-45 Mpc.
- KAGRA expects to join O4 in Spring 2024 with a BNS range of about 10 Mpc.

NICER = **N**eutron star **I**nterior **C**omposition **E**xplore**R**

X-ray timing (to 300 nsec)
& spectroscopy (0.12-12 KeV)


Delivered to Int'l Space Station 03 June 2017

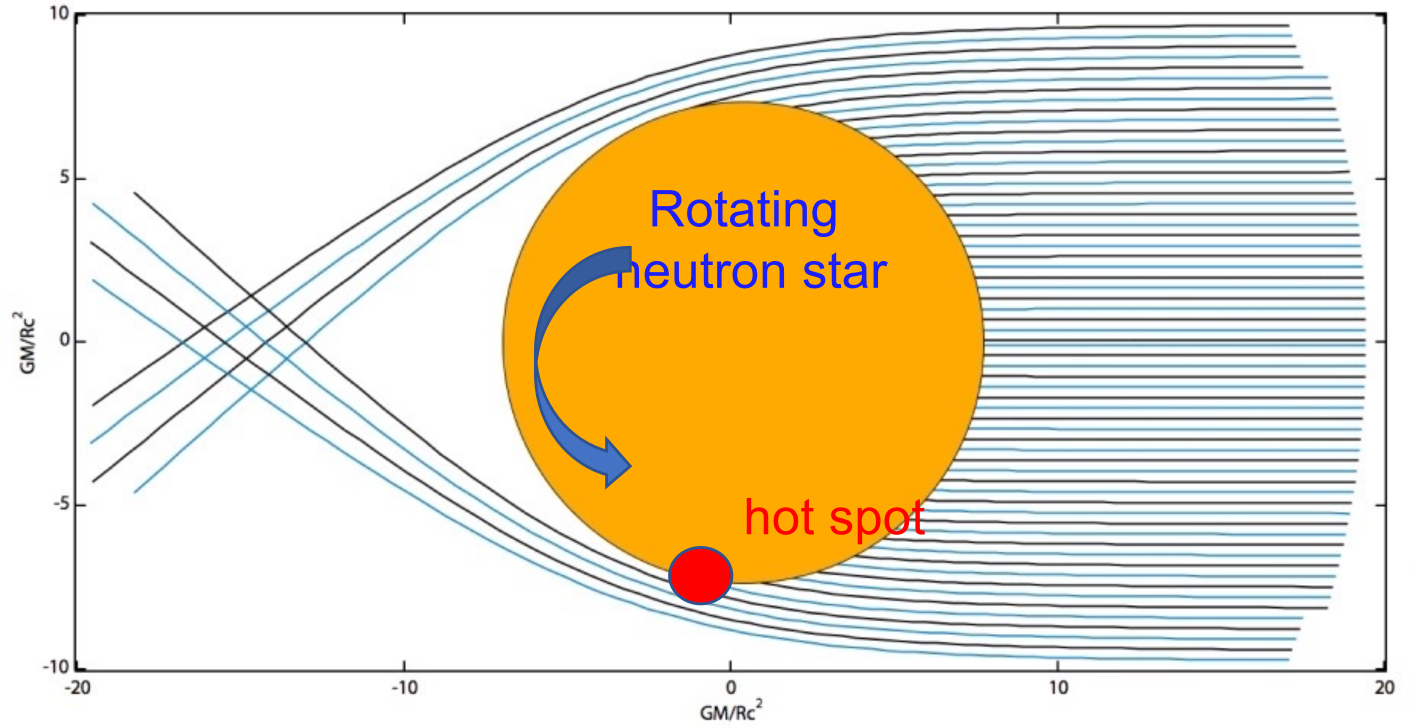
Measure masses and radii (5%) by monitoring X-ray pulse profiles of nearby neutron stars (J0437, ...)

Properties of n.s. crusts via astroseismology

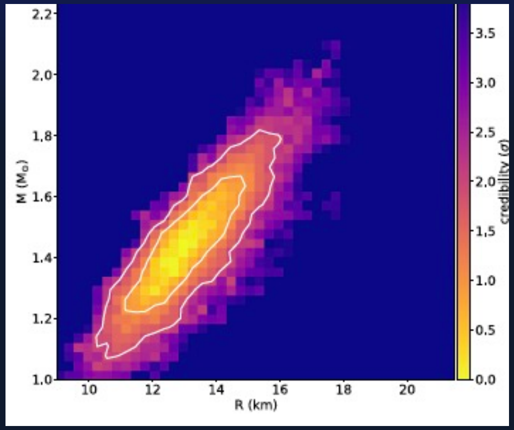
Periodic pulsations from transient & steady systems

Results for two neutron stars
PSR J0740+6620 : $M_{nstar} = 2.08 \pm 0.07\ M_{\odot}$, R_{nstar} ~12-13 km
PSR J0030+0451 : $= 1.44 \pm 0.15\ M_{\odot}$, ~13 km

Track hot spots on neutron star. Light bending by star enables one to see spot "behind" star. Bending depends on M and R.



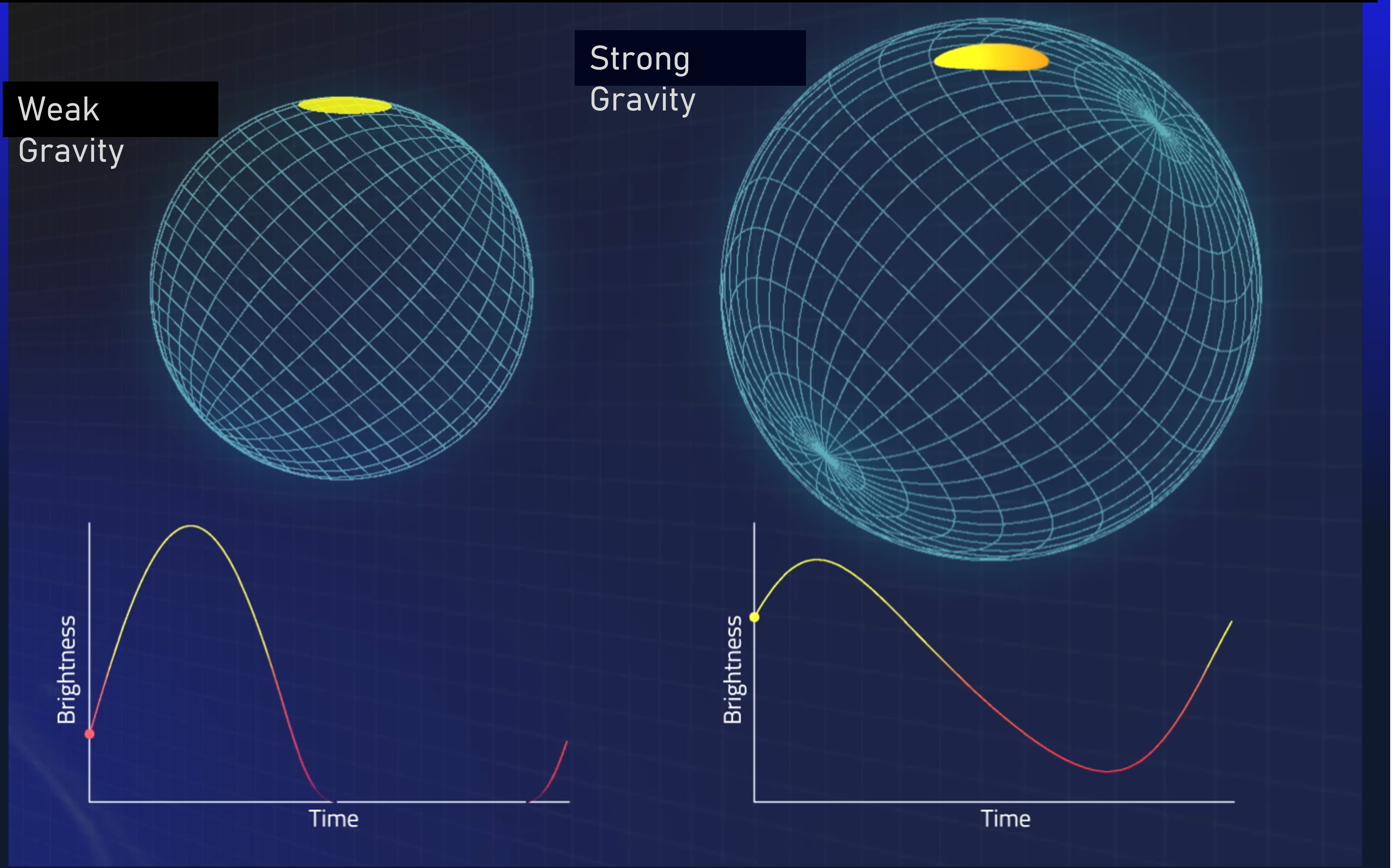
Observer = NICER



PSR J0030+0451
Mass vs Radius
Miller et al.,Ap.J. (2019)

Measure amplitudes and phases in different frequencies, construct model of hot spots to interpret data

Pulse Profile Modeling (PPM)



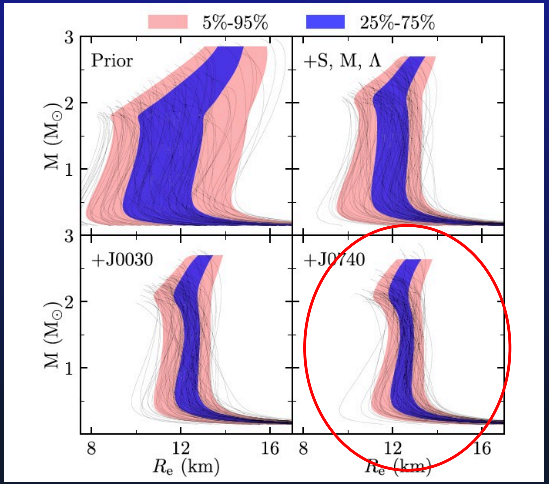
Credit: Morsink/Moir/Arzoumanian/NASA-GSFC

NICER determinations of radius and mass of PSR J0740+6620

$M = 2.08 \pm 0.07 M_\odot$ Greenbank/CHIME
radio pulsar measurements

M.C. Miller et al., UMd/UIUC

Ap. J. Lett., 918, L28 (2021)



$R_{eq} = 13.7^{+2.4}_{-1.5}$ km (1σ)
$M = 2.062^{+0.090}_{-0.091}$ $M_\odot$
from NICER/XMM data alone

(R ⇨ 12.3 ± 0.7 km (1σ)
with nuclear + other astro data)

$R(1.4\ M_\odot) = 12.4 \pm 0.6$ km

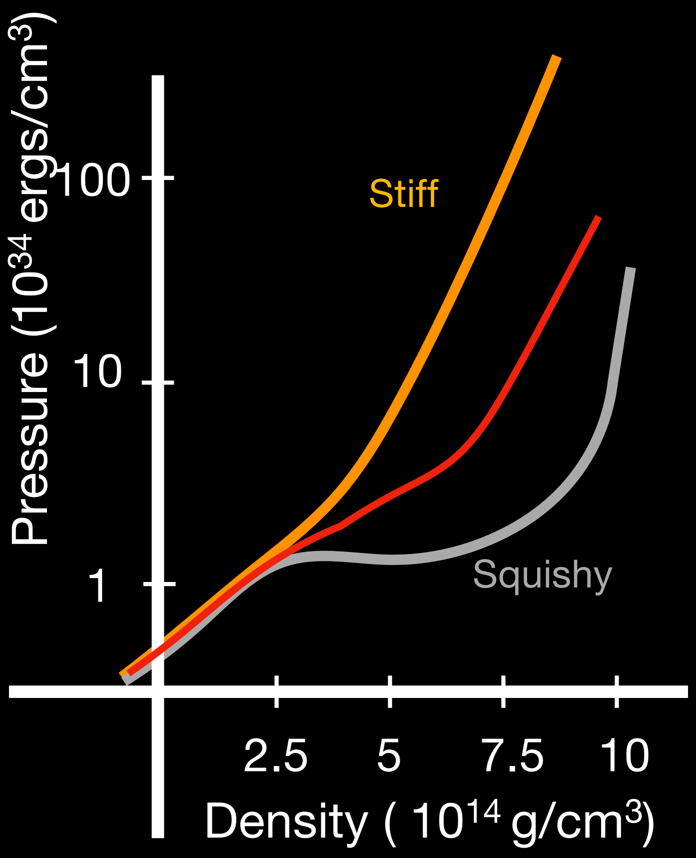
T. E. Riley et al., UVAmsterdam

Ap. J. Lett. 918, L27 (2021)

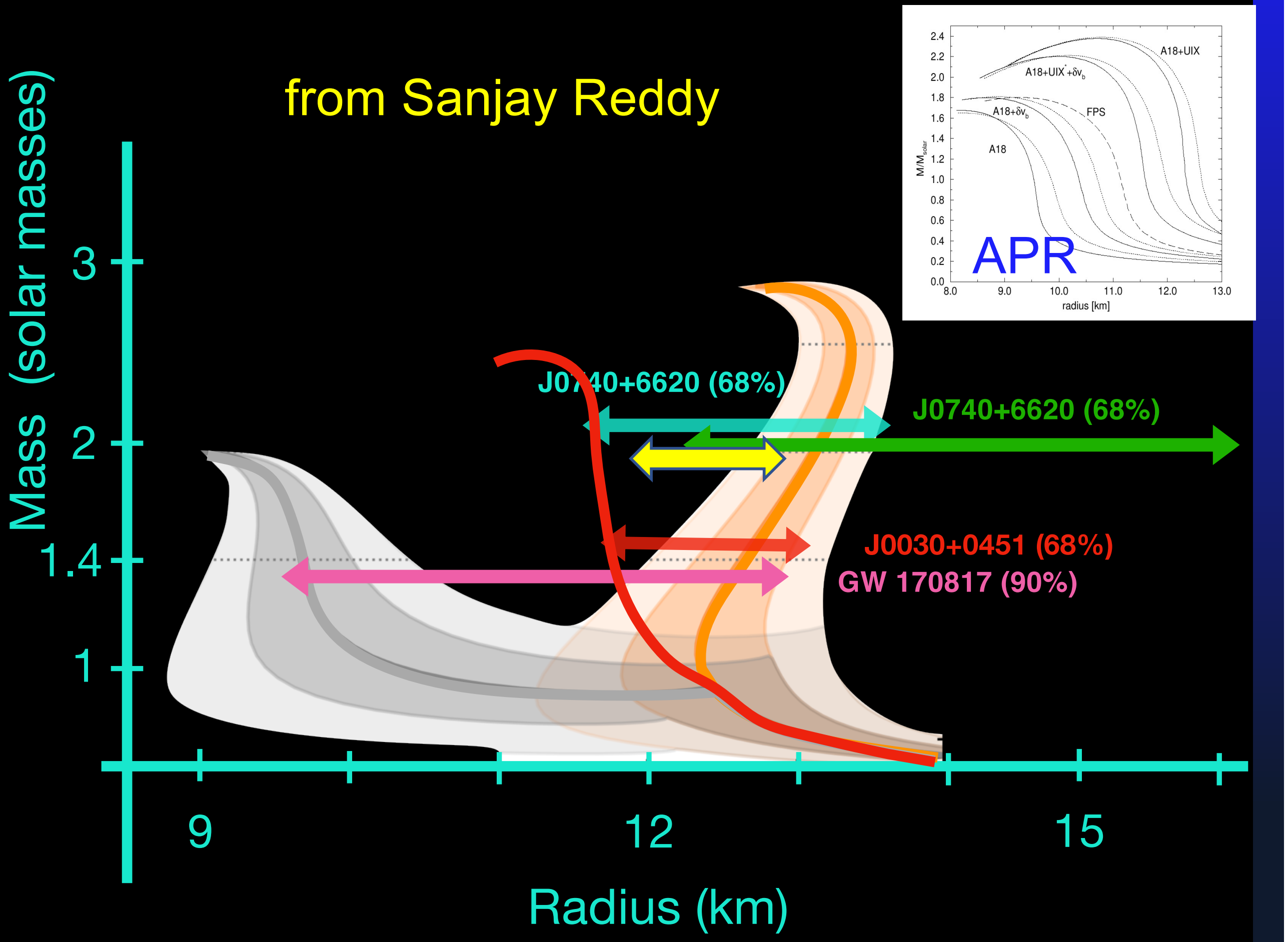
R_{eq} = 12.39 (+1.30-0.98) km

M = 2.072 (+0:067- 0.066) $M_\odot$

The size of neutron stars ⟷ Pressure of dense matter



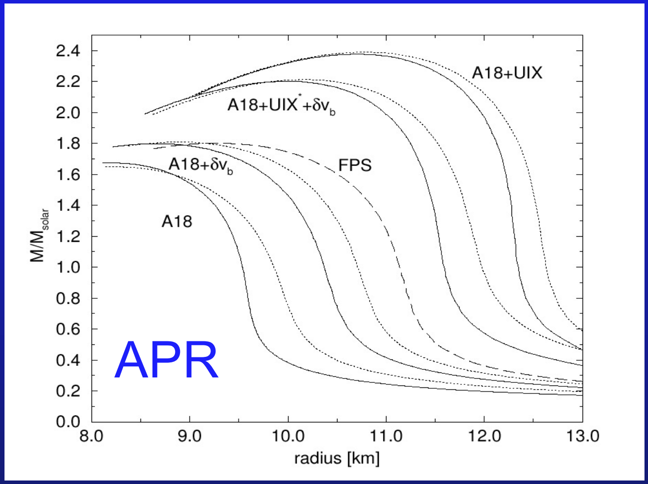
from Sanjay Reddy

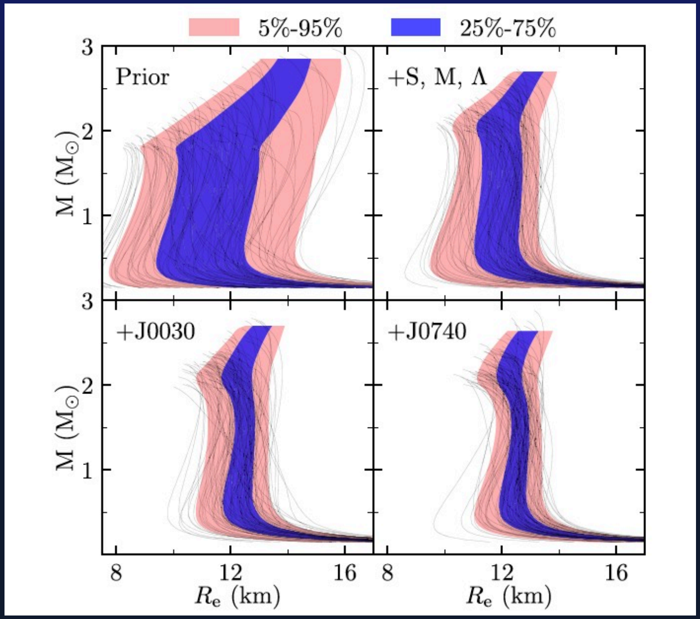


NICER radius range with LIGO, nuclear and other astro information excludes soft equations of state and large first order phase transition

Messages from NICER

Would expect that adding mass to neutron star decreases its radius. All eqns of state based on interacting nucleons show this behavior. But inferred eqn of state shows radius from ~1.4-2.1 M_{sun} changes little.



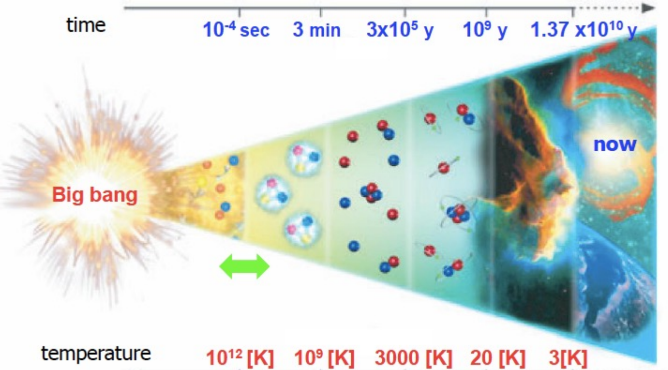


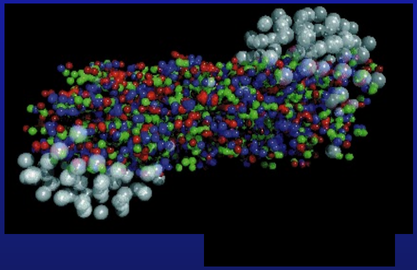
Points to rapid stiffening of nuclear matter, and onset of higher momentum degrees of freedom.

Nucleons beginning transition to quark matter. Pauli blocking of quarks pushes quarks to become relativistic, and quarks start to contribute directly to the pressure, well before quark Fermi sea develops.

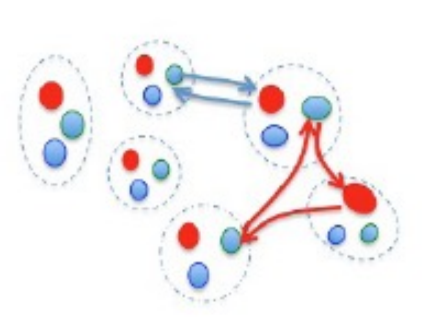
Quarks in dense matter

The early universe before one microsecond after the big bang -- hot quark gluon plasma

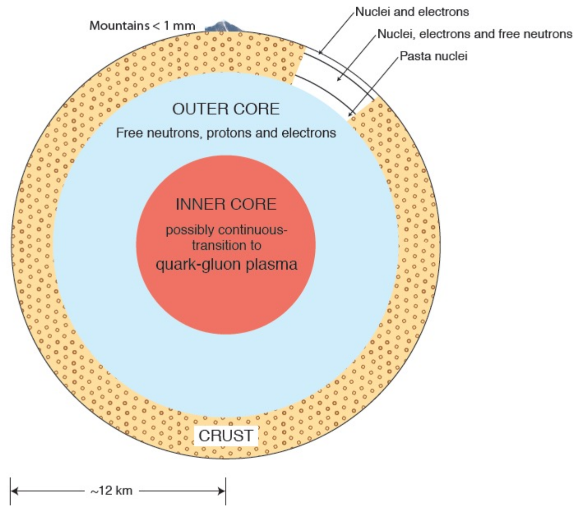




and created in ultrarelativistic heavy ion collisions



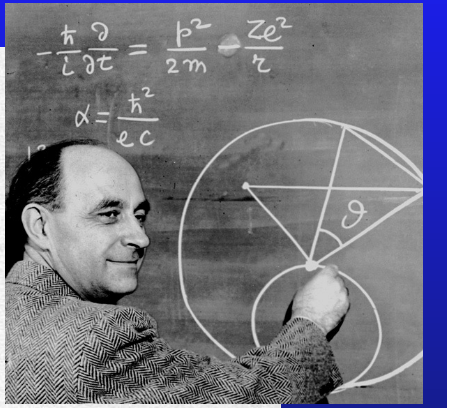
Quarks (and gluons) in nuclei will be mapped by future Electron-Ion Collider



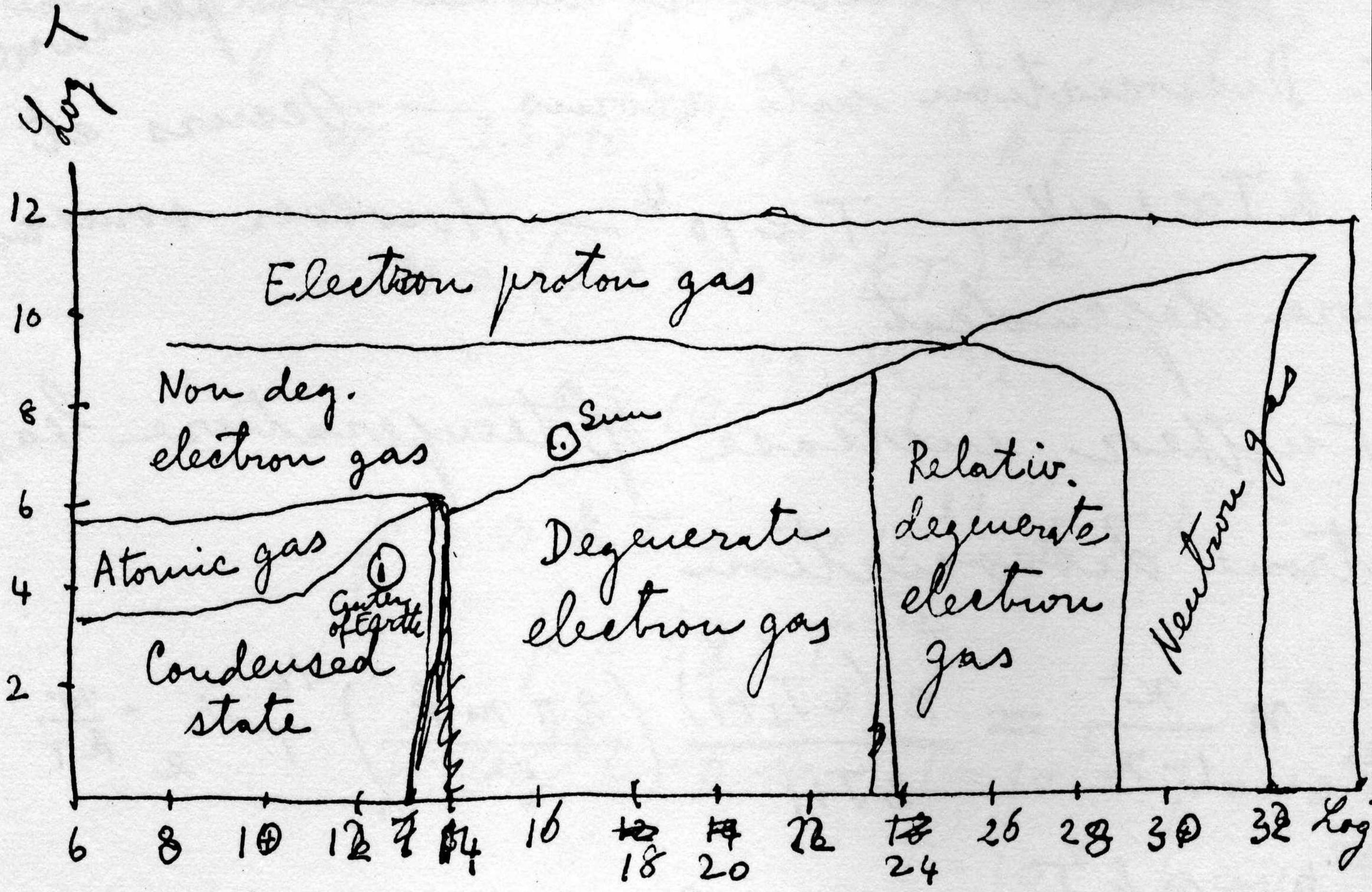
Cold quark matter cores of high mass neutron stars –

Strongly interacting system: cannot do lattice QCD simulations at finite density, zero temperature, owing to fermion sign problem.

E. Fermi: *Notes on Thermodynamics and Statistics* (1953))



70 – Matter in unusual conditions



Start from ordinary condensed matter with ~~domina~~ equation of state controlled by ordinary chemical forces.

Earliest phase diagram of dense qcd matter

“Quark liberation”

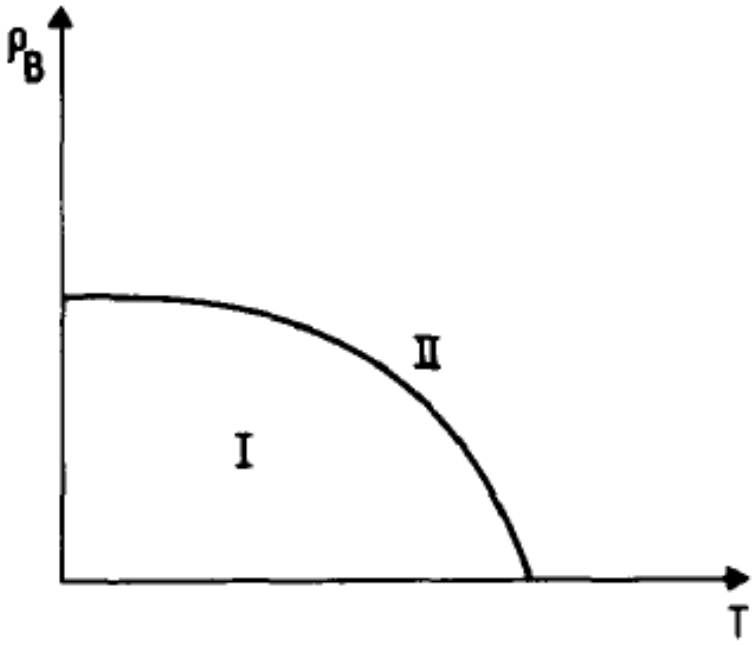
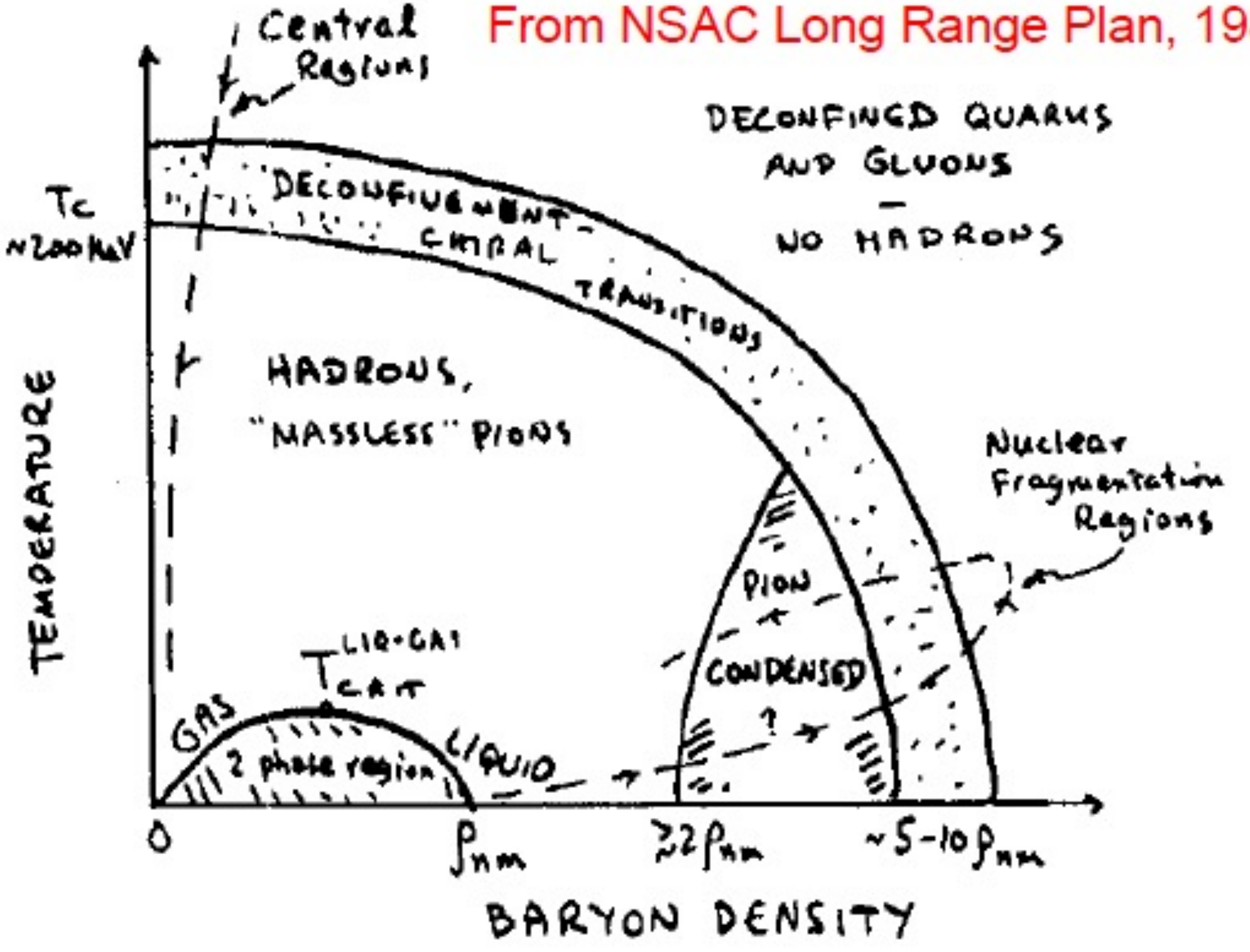


Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

N. Cabbibo and G. Parisi, Phys. Lett. B58, 67 (1975)

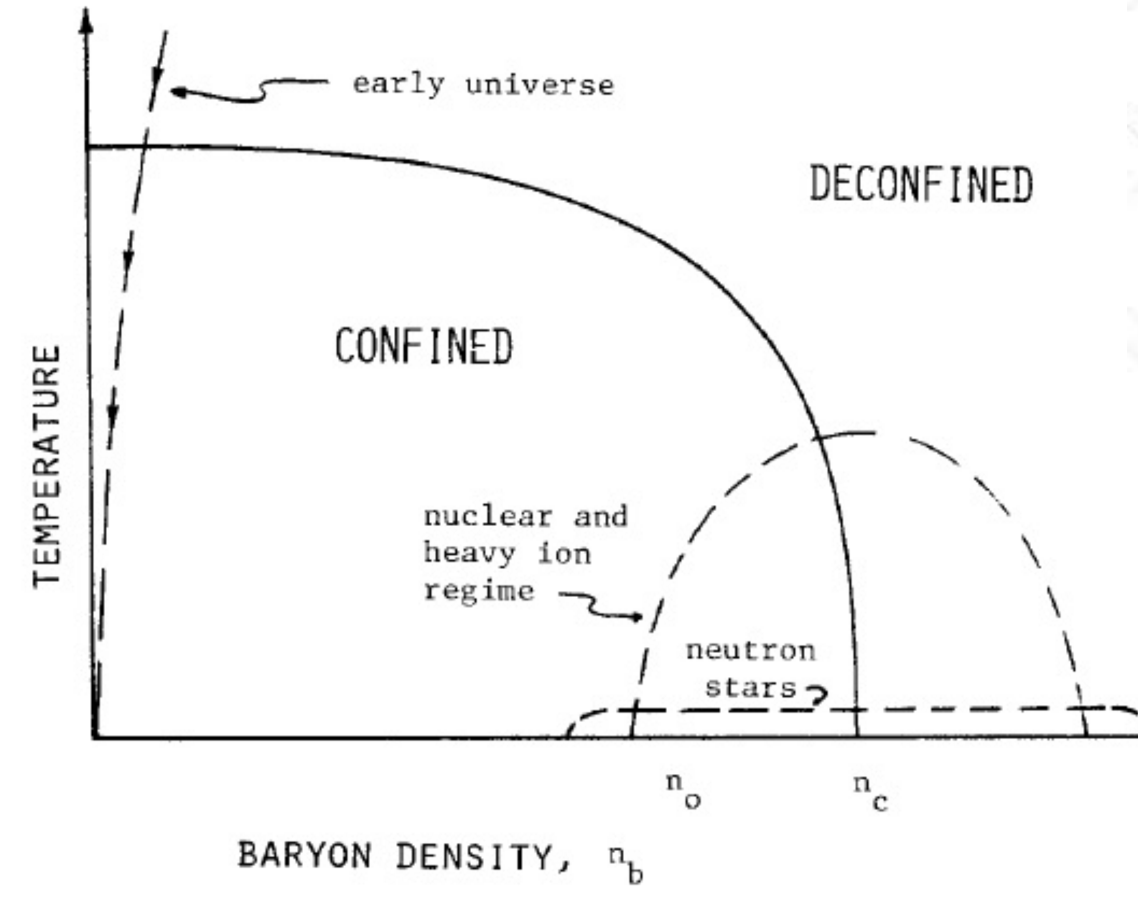
PHASE DIAGRAM OF NUCLEAR MATTER

From NSAC Long Range Plan, 1983

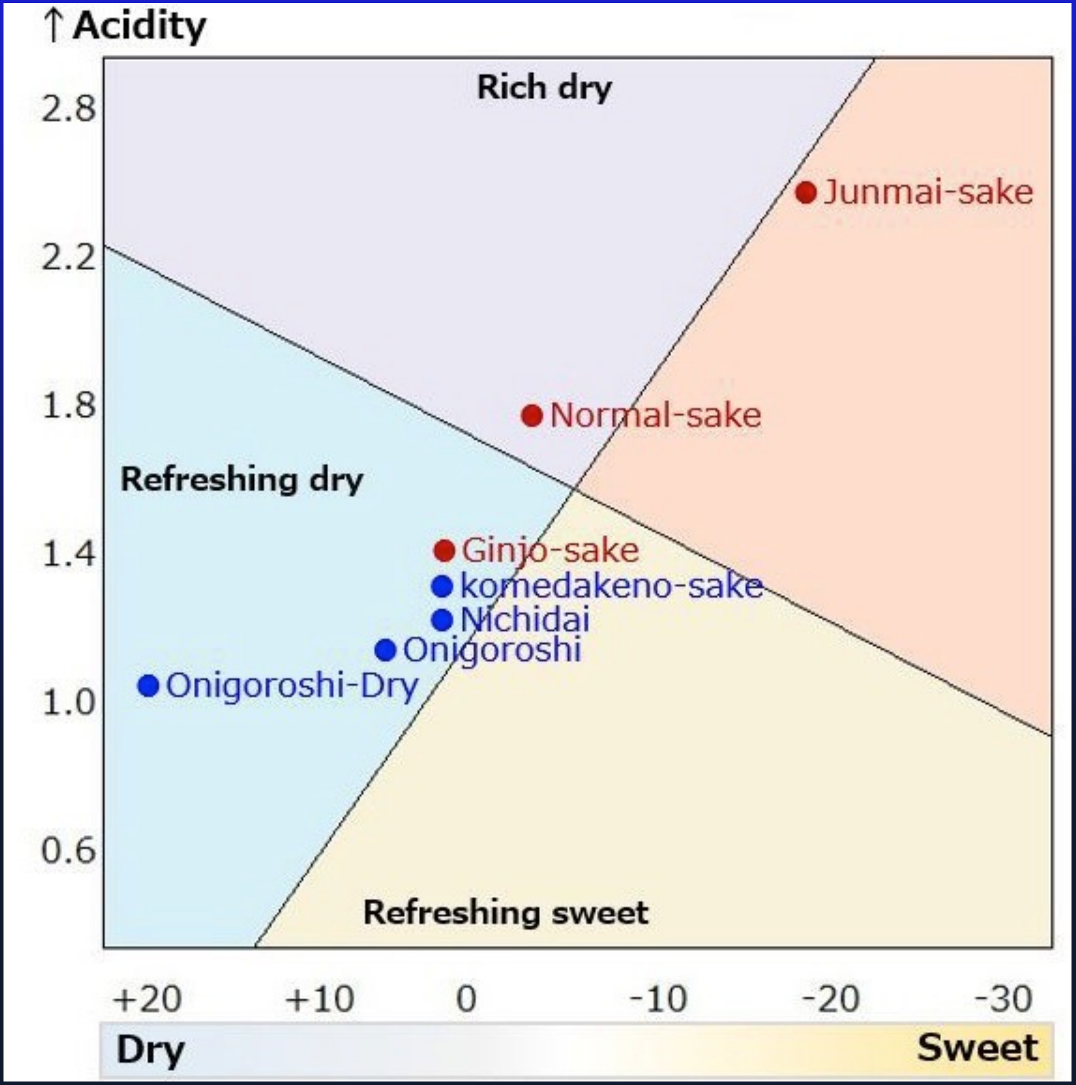


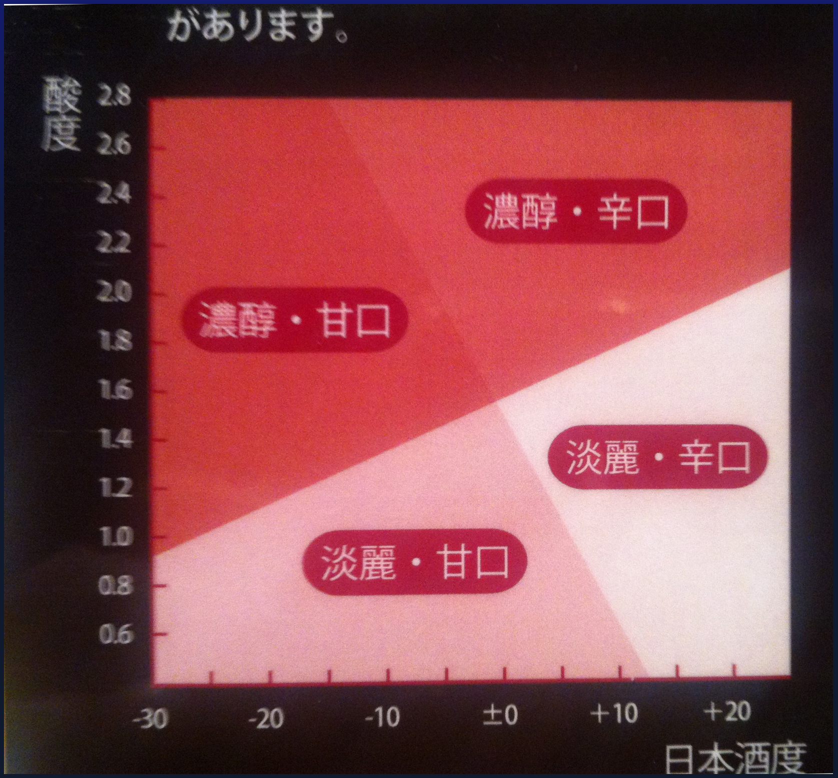
“My first” QCD phase diagram

GB, April 1981 Erice School of Nuclear Physics, Prog, in Part. and Nucl. Physics 8, 73 (1982)

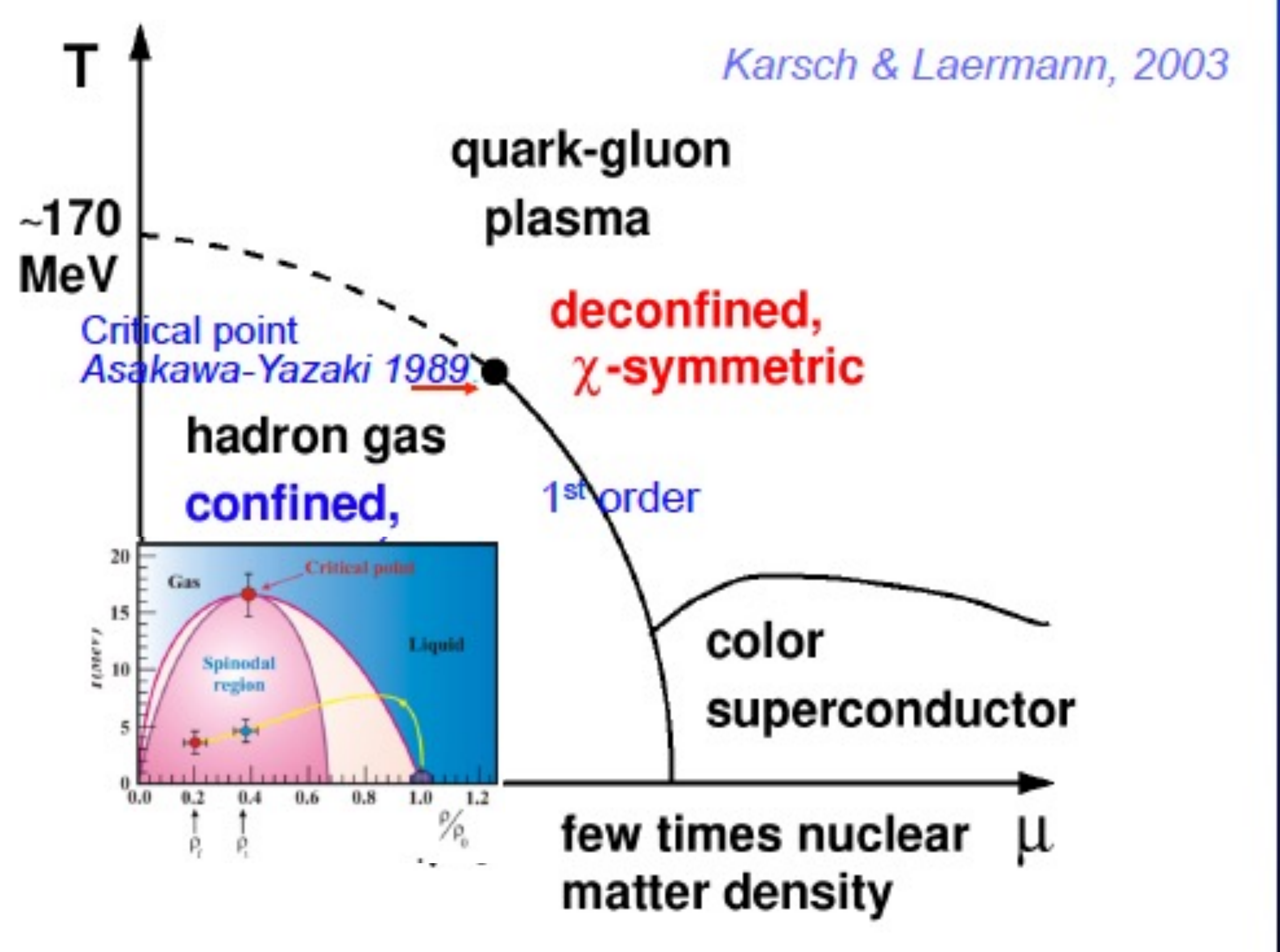


The phase diagram of sake



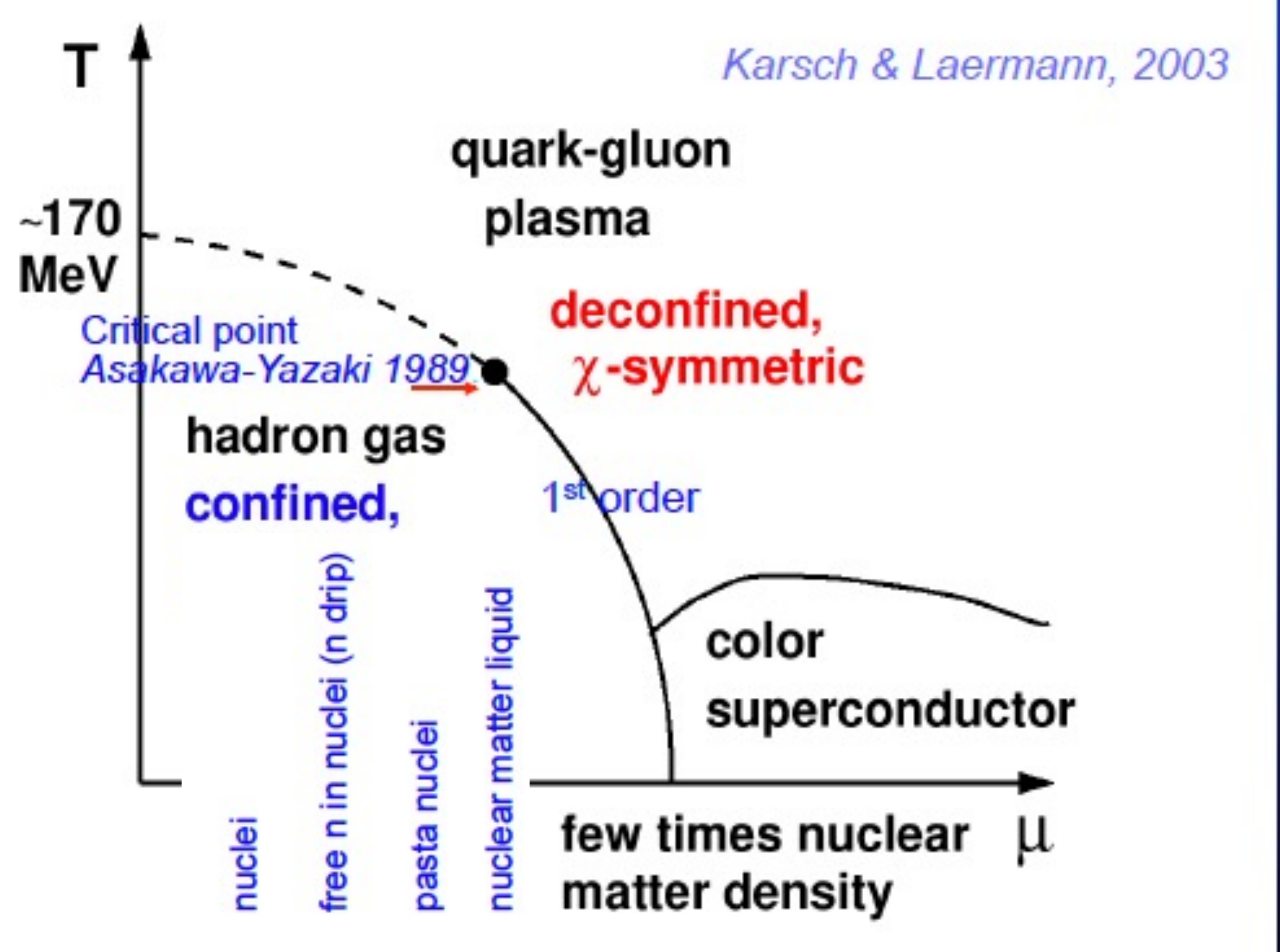


Arrival of the Asakawa-Yazaki critical point



Chemical potential replaces the density

Richer structure at low density

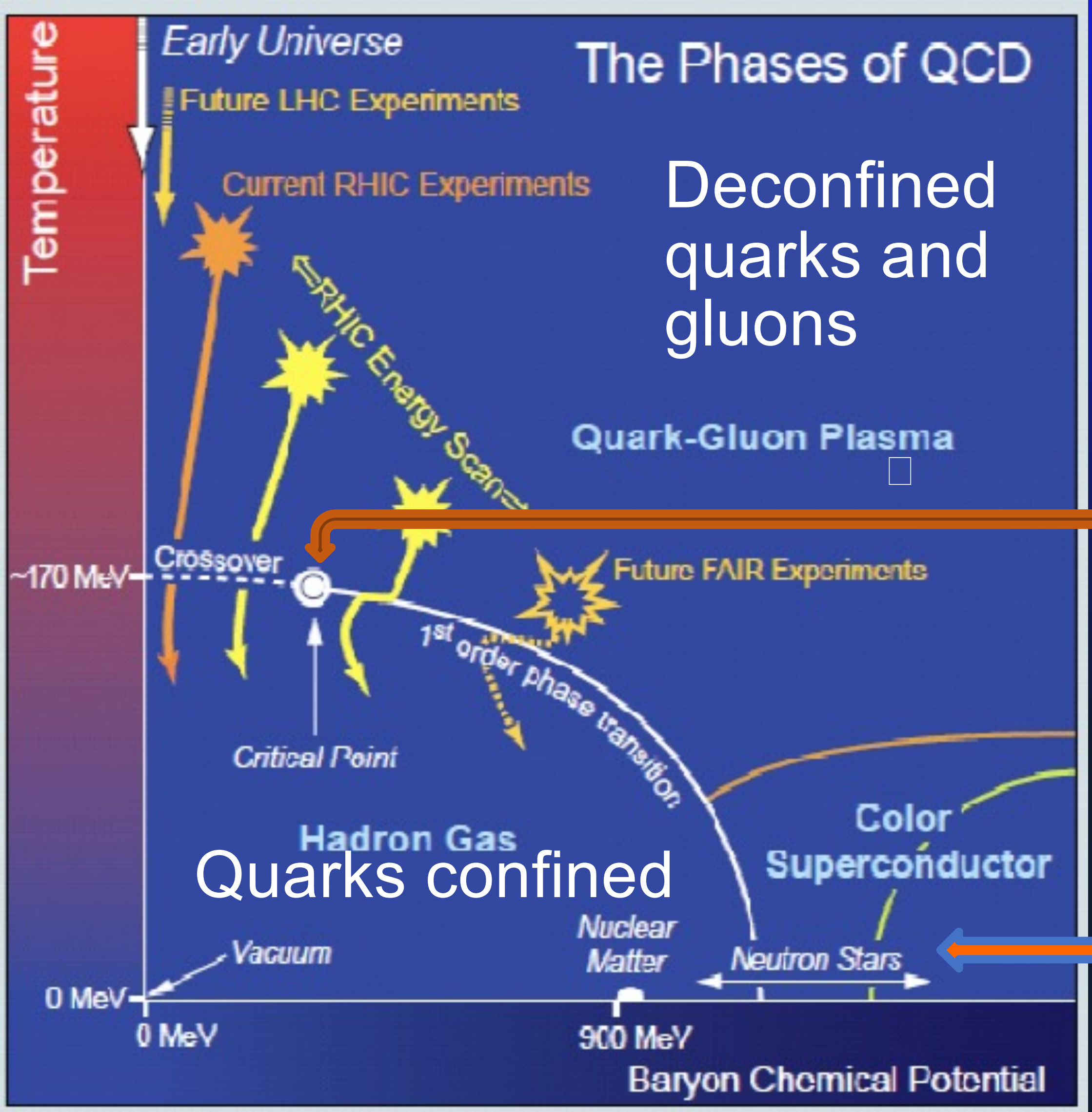


Chemical potential replaces the density

More modern phase diagram

K. Fukushima (I-Pad)

Crossover at zero net baryon density



Deconfined quarks and gluons

Quarks confined

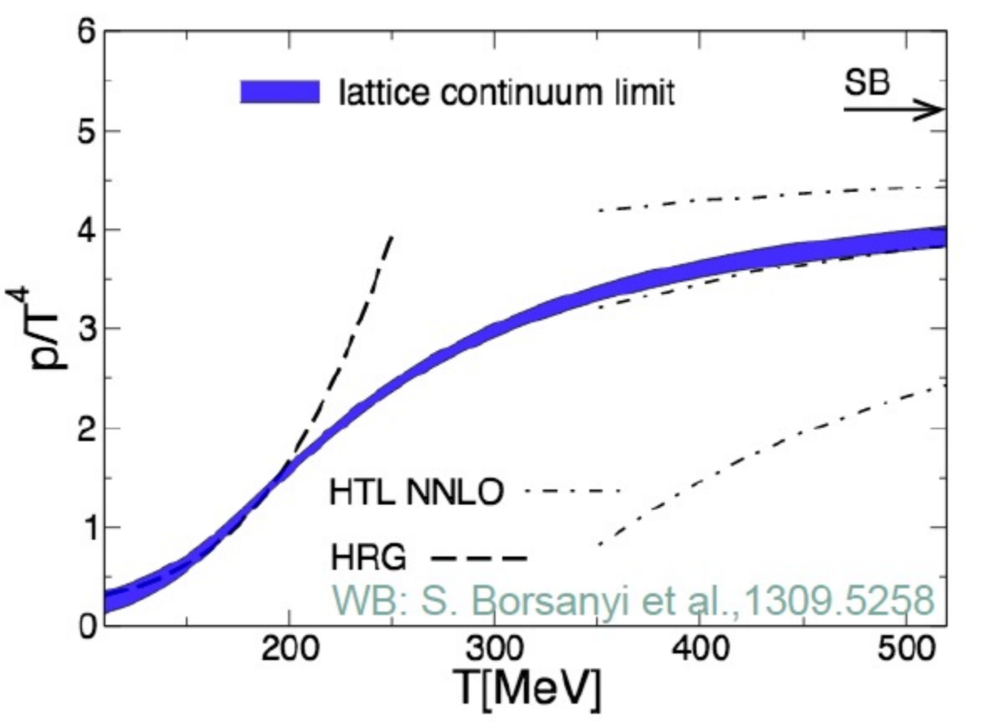
Asakawa-Yazaki critical point (1989)

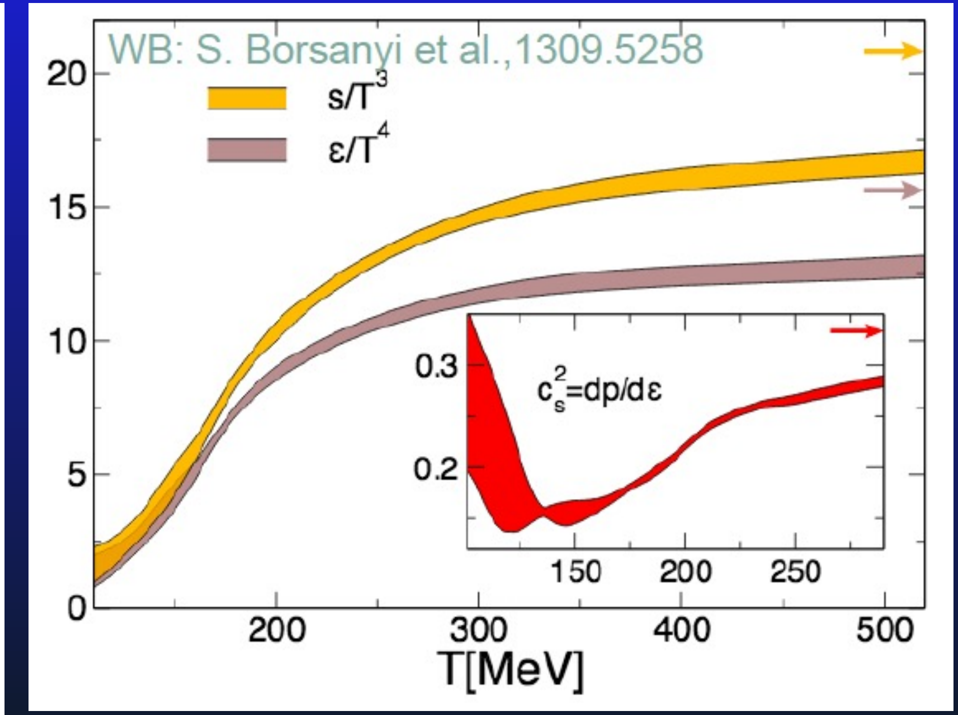
Search in RHIC & SPS energy scans

States of color superconductivity – diquark BCS pairing

2SC / Color Flavor Locked
(Alford, Rajagopal, Wilczek, ...)

Crossover at zero net density: see no evidence of phase transition in pressure, entropy, or energy density.

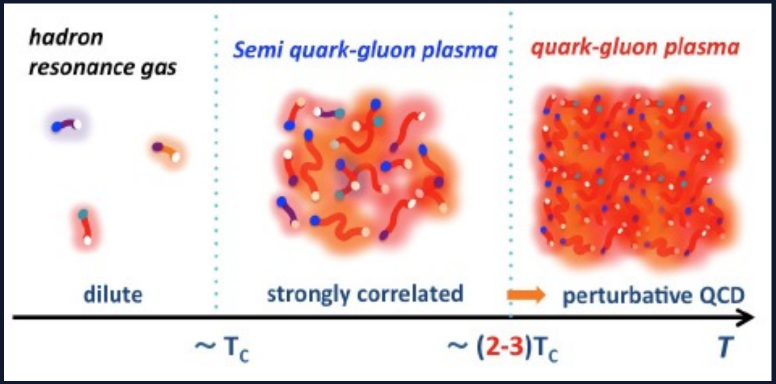




Wuppertal-Budapest lattice collaboration

WB: S. Borsanyi et al., PLB (2014)

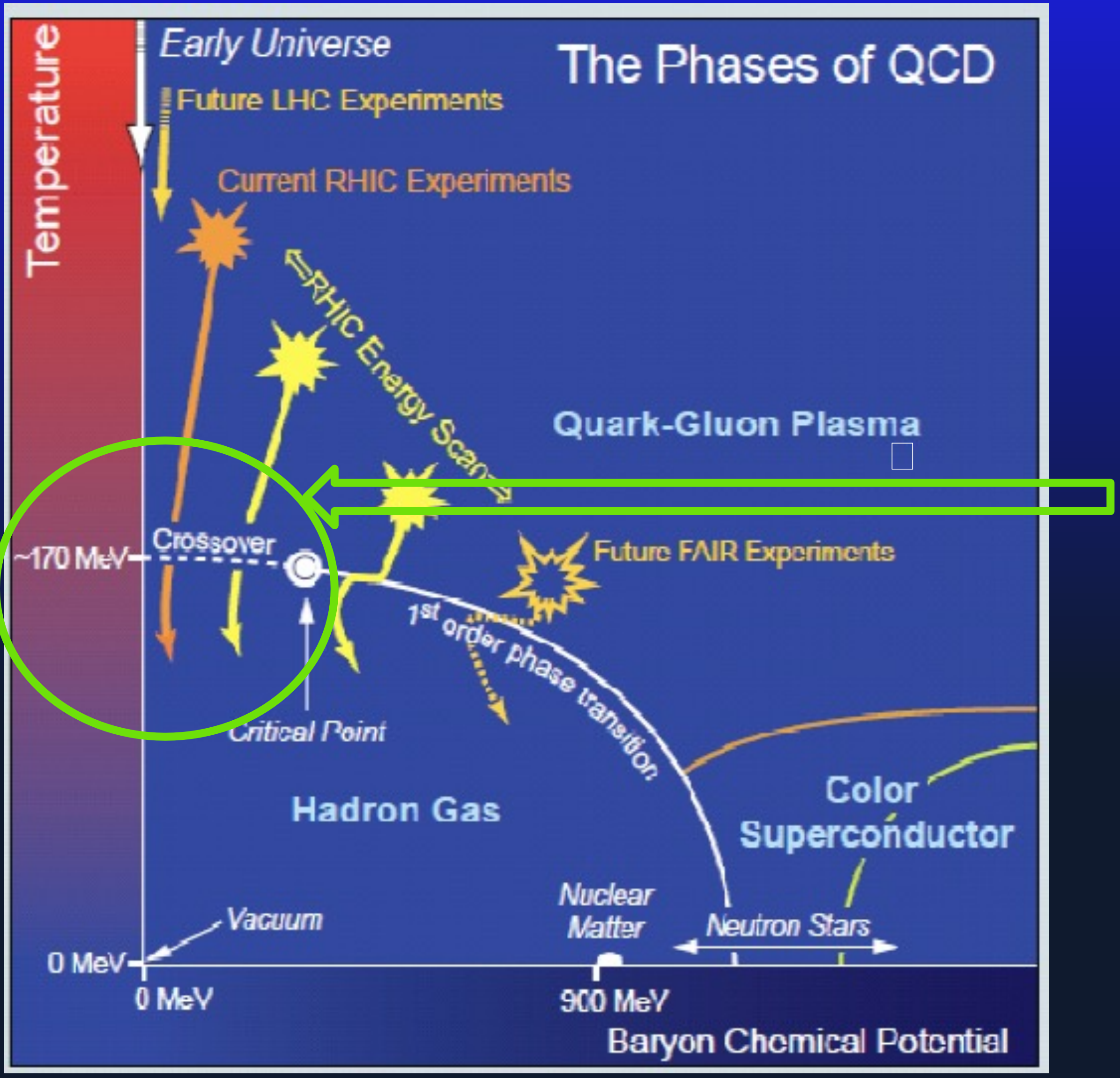
HotQCD: A. Bazavov et al., PRD (2014)



Lattice gauge theory not yet well implemented for finite baryon density!!

Fermion sign problem

Crossover at zero net baryon density

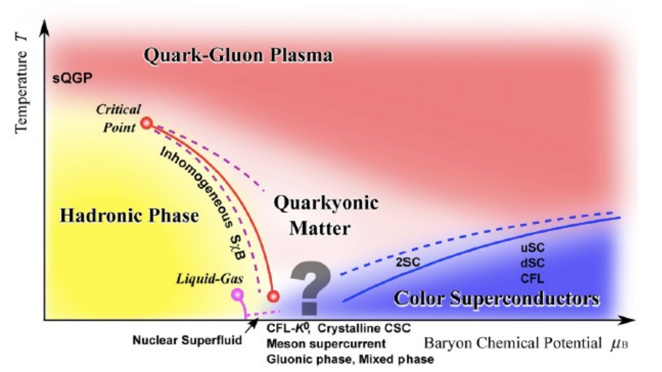


QCD lattice gauge theory -- for finite light quark masses -- predicts crossover from confined phase at lower T to deconfined phase at higher T.

Do quarks roam freely in the deconfined phase? If so, they must also roam freely at lower T.

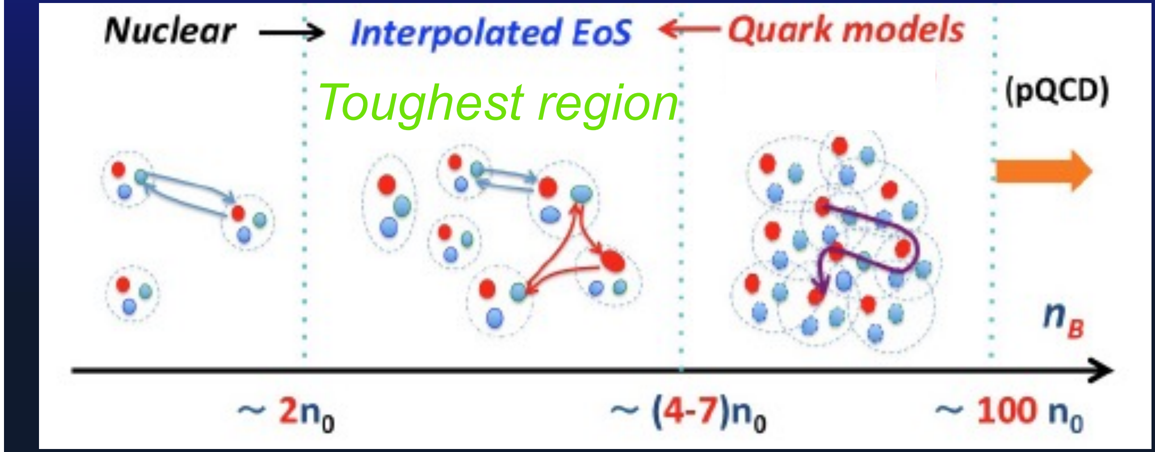
Are there really quarks running about freely in the room?

No free quarks even above the crossover!



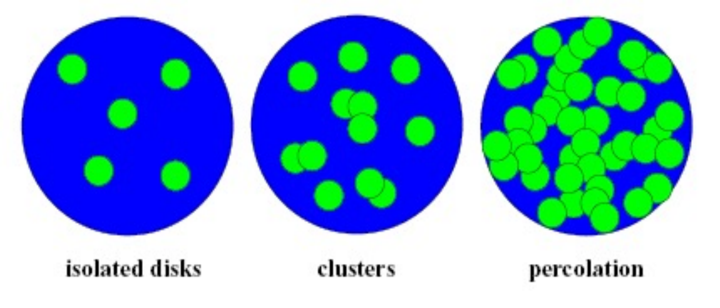
In confined region quarks are inside hadrons. Also have quarks and antiquarks in the QCD forces between hadrons. Form larger clusters, at higher density or temperature, which percolate at the crossover. Like electrons in metals, percolation depends on quantum states of quarks.

In deconfined regime clusters extend across all of available space.



Percolation of clusters along the density axis, at zero temperature.

Quarks can still be bound even if deconfined.



$n_{perc} \sim 0.34\ (3/4\pi\ r_n^3)\ \ \mathrm{fm}^{-3}$

r_n = nucleon radius

n_0 = nuclear saturation density

$n=5n_0$ =, r_n = 0.8 fm, considerable nucleon overlap

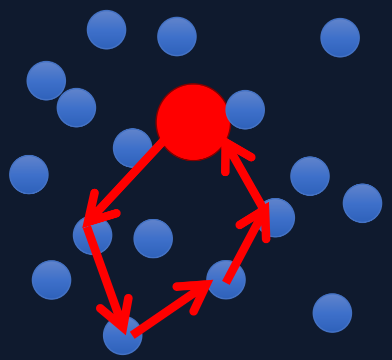
Classical vs. quantum percolation

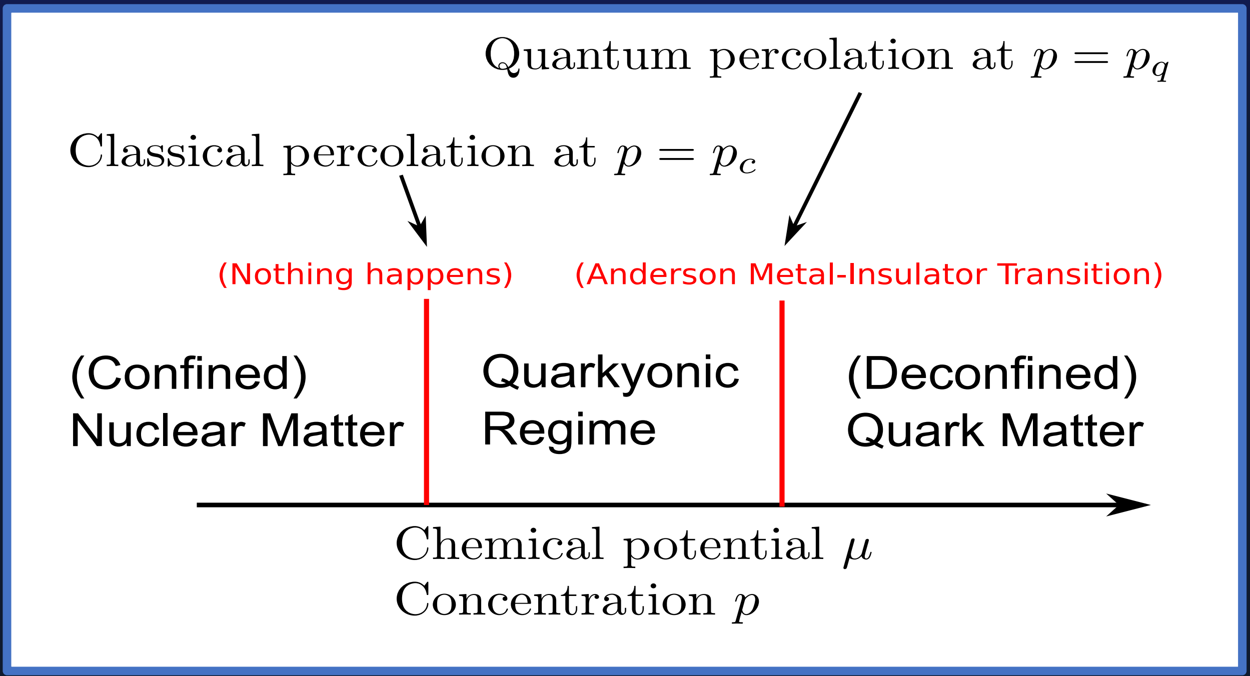
But aren’t nucleons, with long distance cloud of mesons always overlapping?

Does anything actually happen at classical percolation transition? No obvious lattice calculation to do!

Distinguish classical (geometric) percolation
from quantum percolation in terms of wave functions

Deconfinement as (inverse) Anderson localization
(K. Fukushima):





Quantum percolation at $p = p_q$

Classical percolation at $p = p_c$

(Nothing happens)

(Anderson Metal-Insulator Transition)

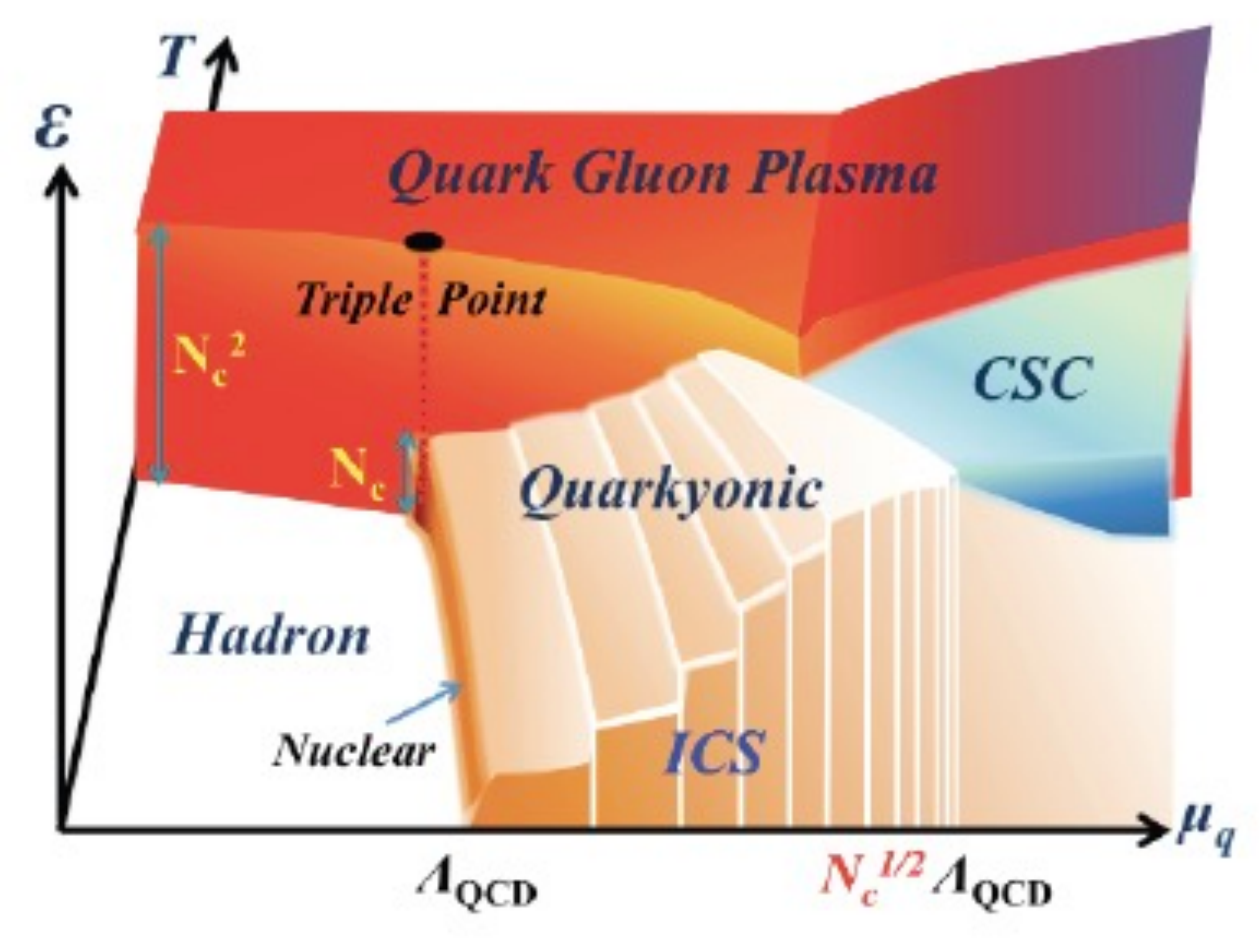
(Confined)
Nuclear Matter

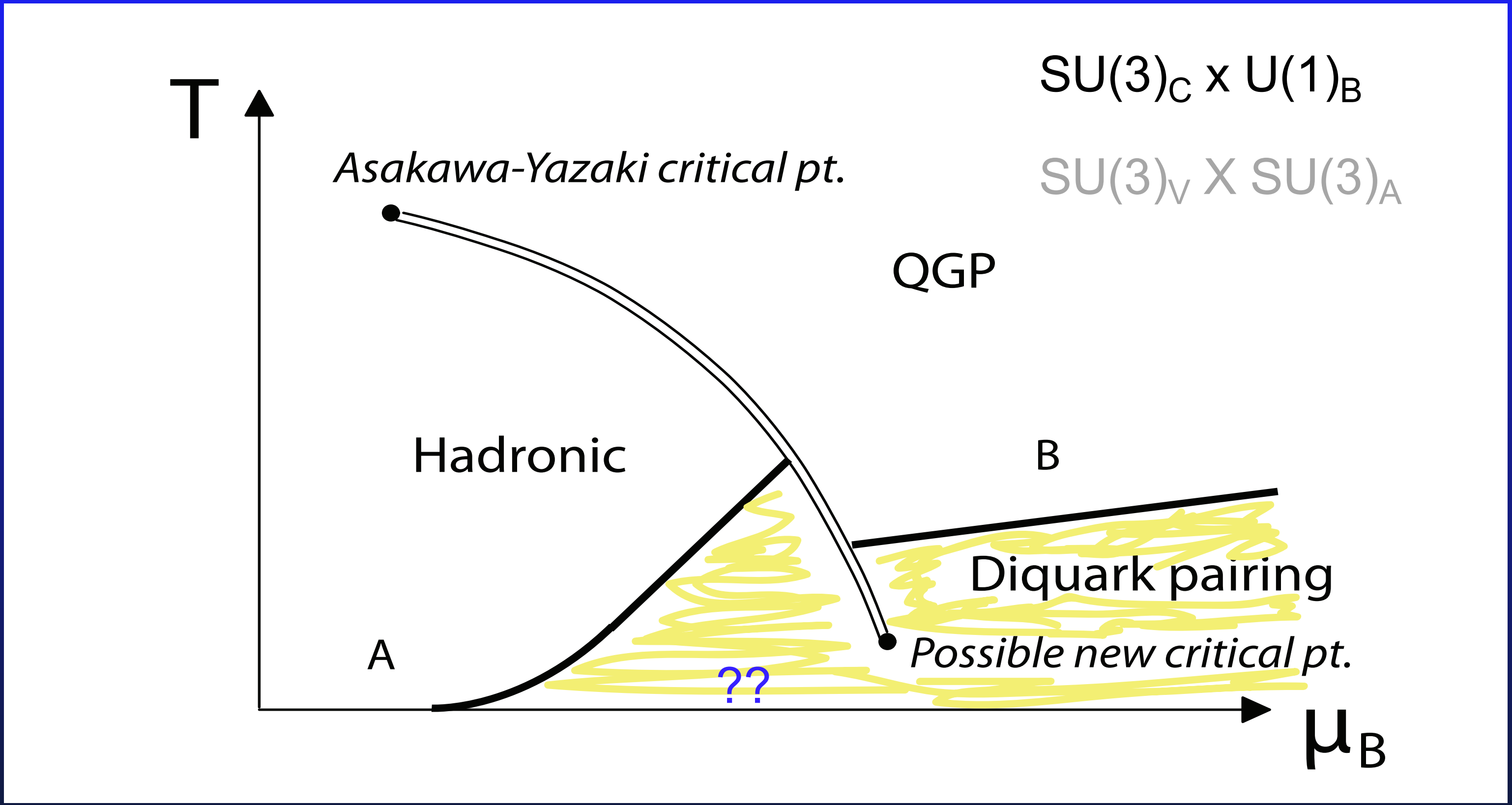
Quarkyonic
Regime

(Deconfined)
Quark Matter

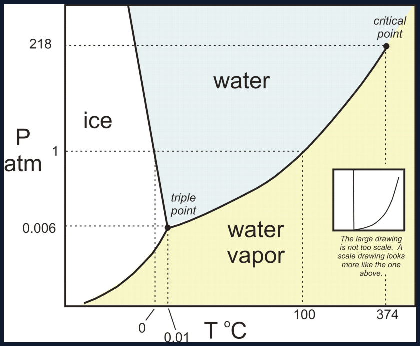
Chemical potential μ
Concentration p

Quarkyonic phases



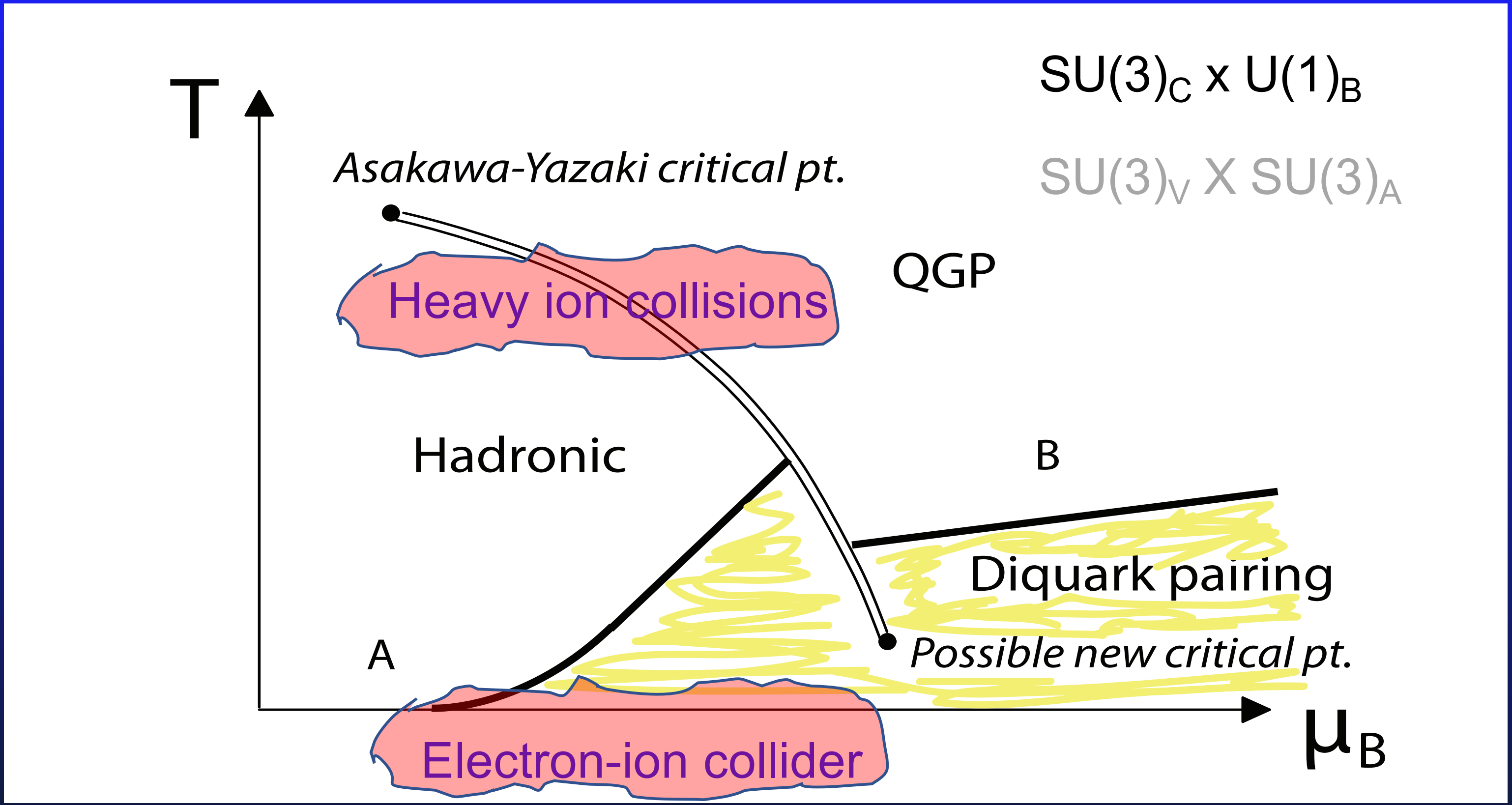


Critical points similar to those in liquid-gas phase diagram (H_2O). Neither critical point necessary!!



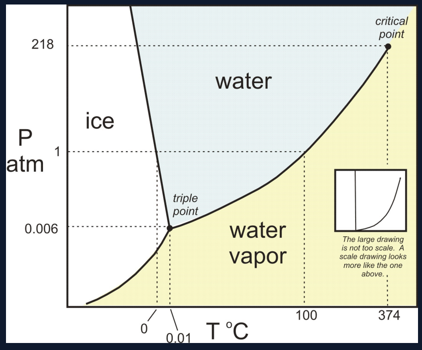
Can go continuously from A to B around the upper critical point. Liquid-gas phase transition.

In lower shaded region have BCS pairing of nucleons, of quarks, and possibly other states (meson condensates, quarkyonic). Different symmetry structure than at higher T.

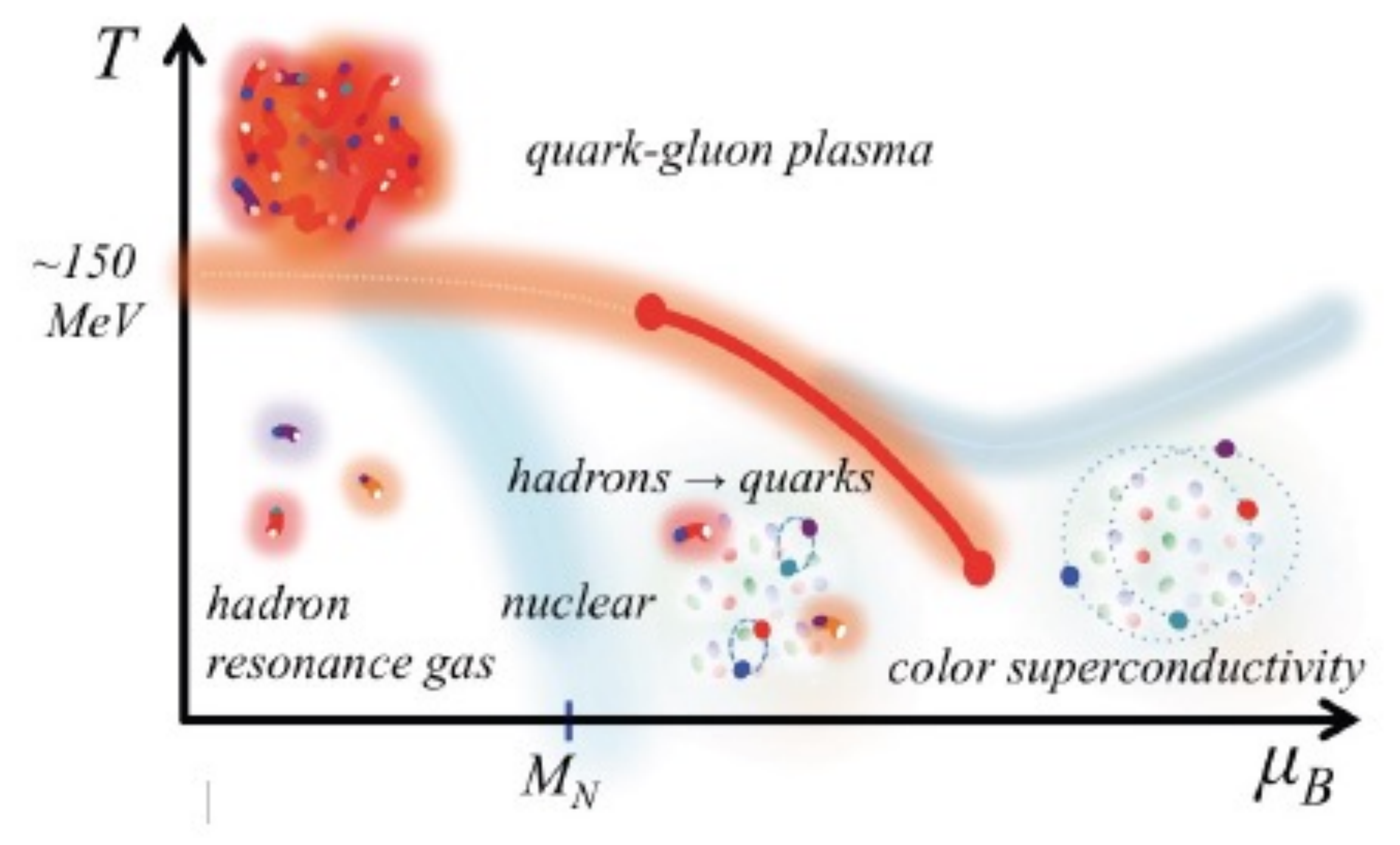


Critical points similar to those in liquid-gas phase diagram (H_2O). Neither critical point necessary!!

Can go continuously from A to B around the upper critical point. Liquid-gas phase transition.

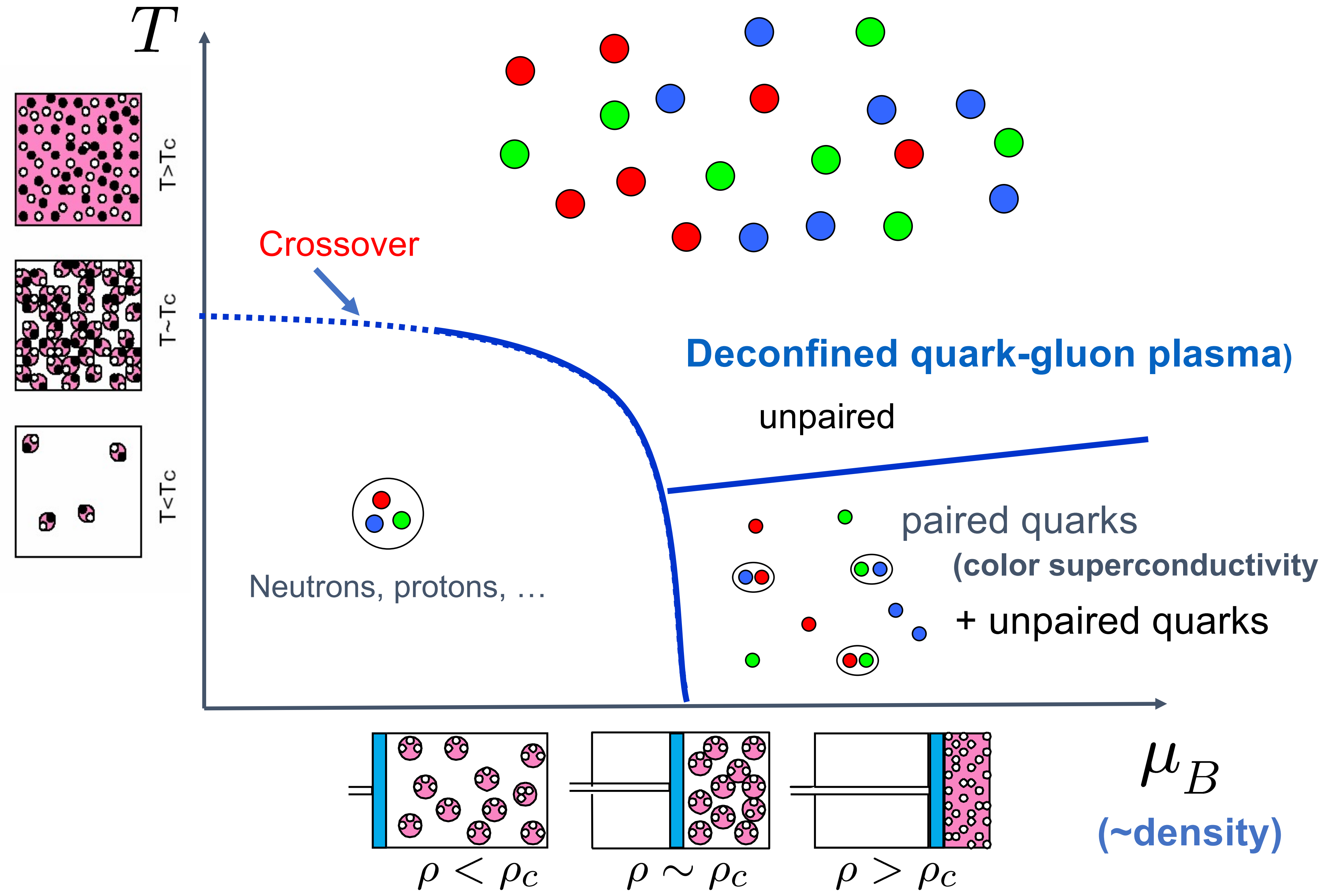


In lower shaded region have BCS pairing of nucleons, of quarks, and possibly other states (meson condensates, quarkyonic). Different symmetry structure than at higher T.


T
quark-gluon plasma
~150
MeV
hadrons → quarks
hadron
resonance gas
nuclear
color superconductivity
M_N
μ_B

Phase diagram of dense nuclear matter

T. Hatsuda



Crossover

Deconfined quark-gluon plasma)

unpaired

paired quarks

(color superconductivity

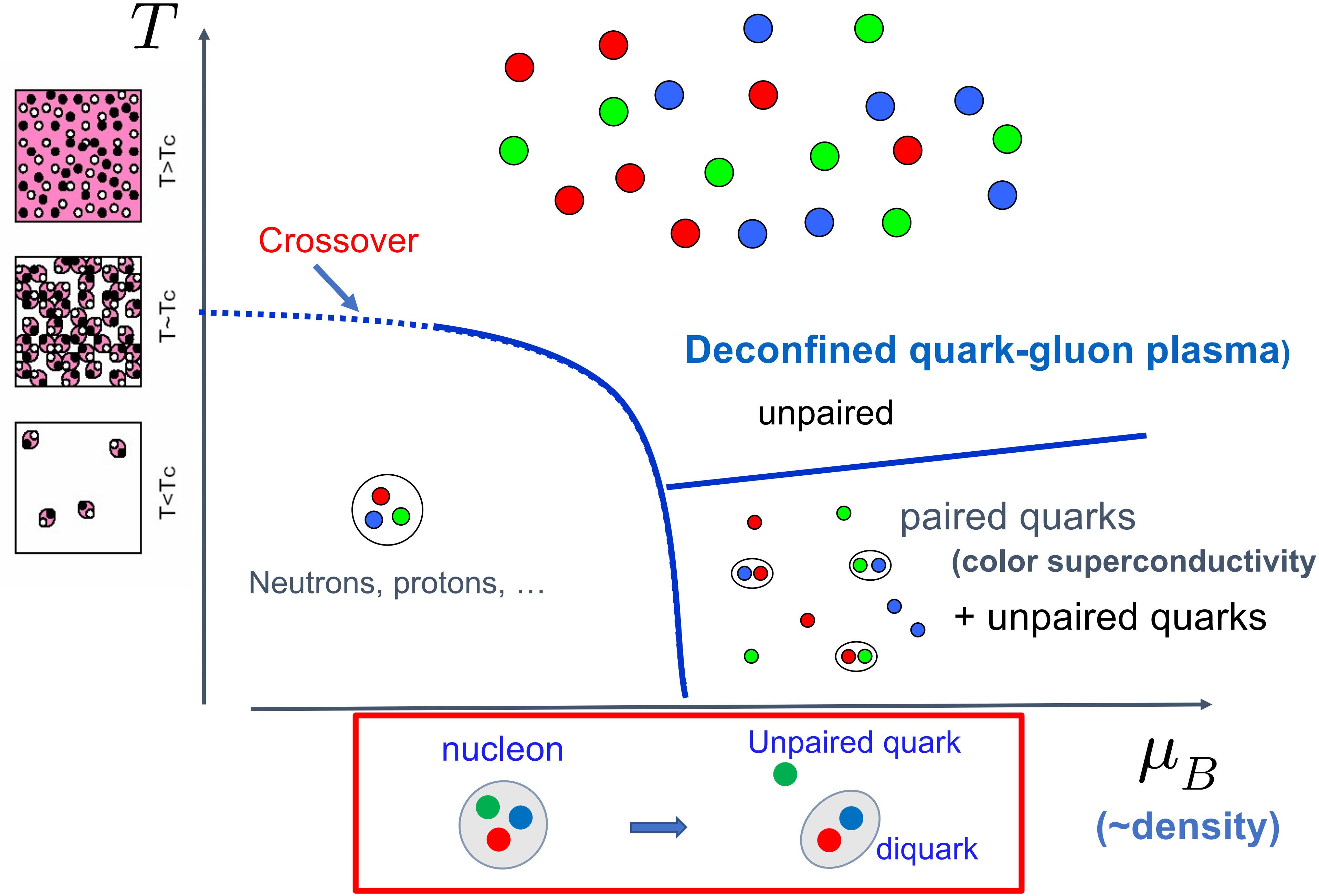
+ unpaired quarks

Neutrons, protons, …

(~density)

Phase diagram of dense nuclear matter

T. Hatsuda



T

T>Tc

T~Tc

T<Tc

Crossover

Deconfined quark-gluon plasma)

unpaired

paired quarks

(color superconductivity

+ unpaired quarks

Neutrons, protons, …

μ_B

(~density)

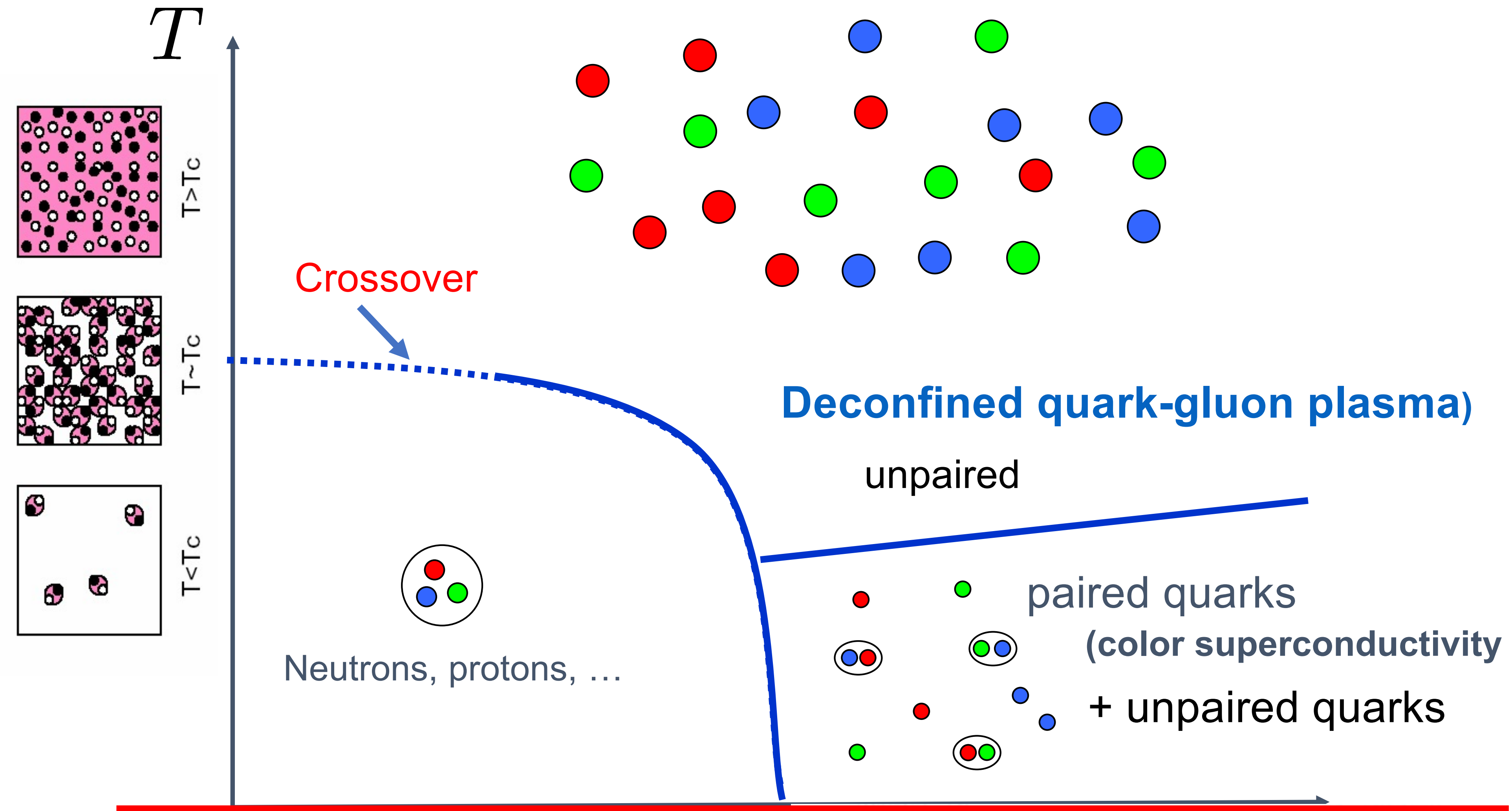
nucleon

Unpaired quark

diquark

Phase diagram of dense nuclear matter

T. Hatsuda



Crossover

Deconfined quark-gluon plasma)

unpaired

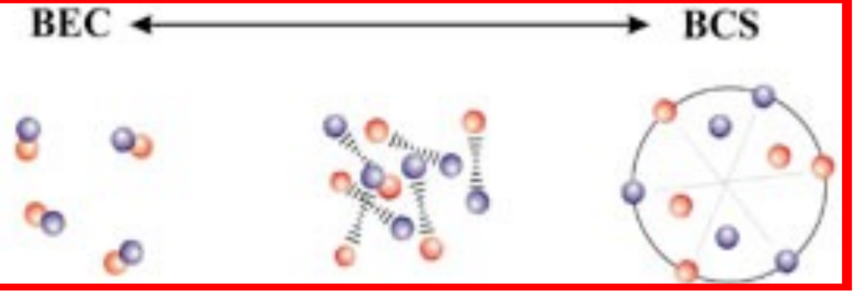
Neutrons, protons, …

paired quarks

(color superconductivity

+ unpaired quarks

sea of diquarks (BEC)
becomes BCS paired



Interplay between BCS pairing and chiral condensate

T. Hatsuda, M. Tachibana, N. Yamamoto & GB, PRL 97, 122001 (2006)

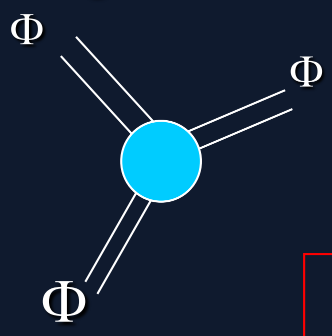
Hadronic phase breaks chiral symmetry, producing chiral (particle-antiparticle) bosonic condensate: $\Phi \sim \langle \bar{q} q \rangle$

Analogous to polarization in two component Fermi gases

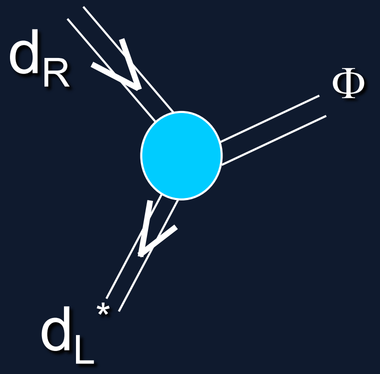
Color superconducting phase has particle-particle (diquark) pairing:

$$d \sim \langle q q \rangle$$

Spontaneous breaking of the axial $U(1)_A$ symmetry of QCD (axial anomaly) leads to attractive (Kobayashi-Miskawa-‘t Hooft) 6-quark interaction between the chiral condensate and pairing fields. Each encourages the other!



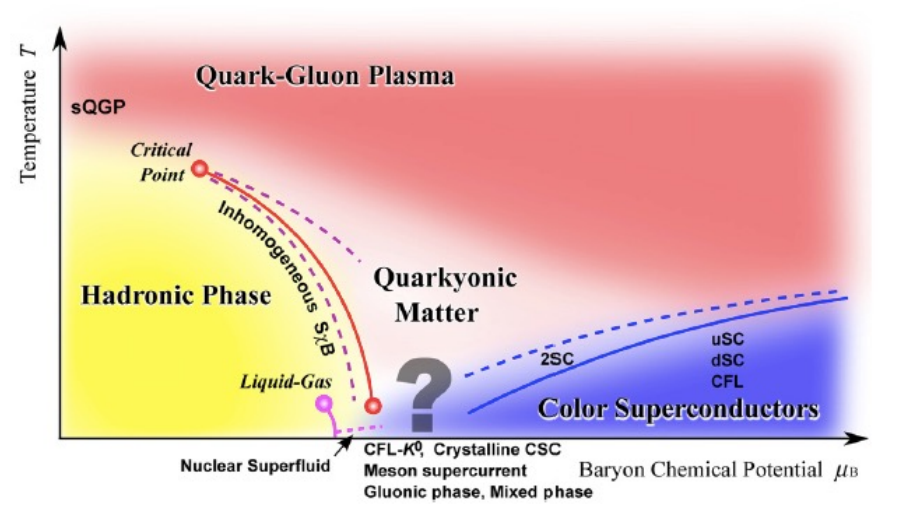
$\sim \Phi^3$



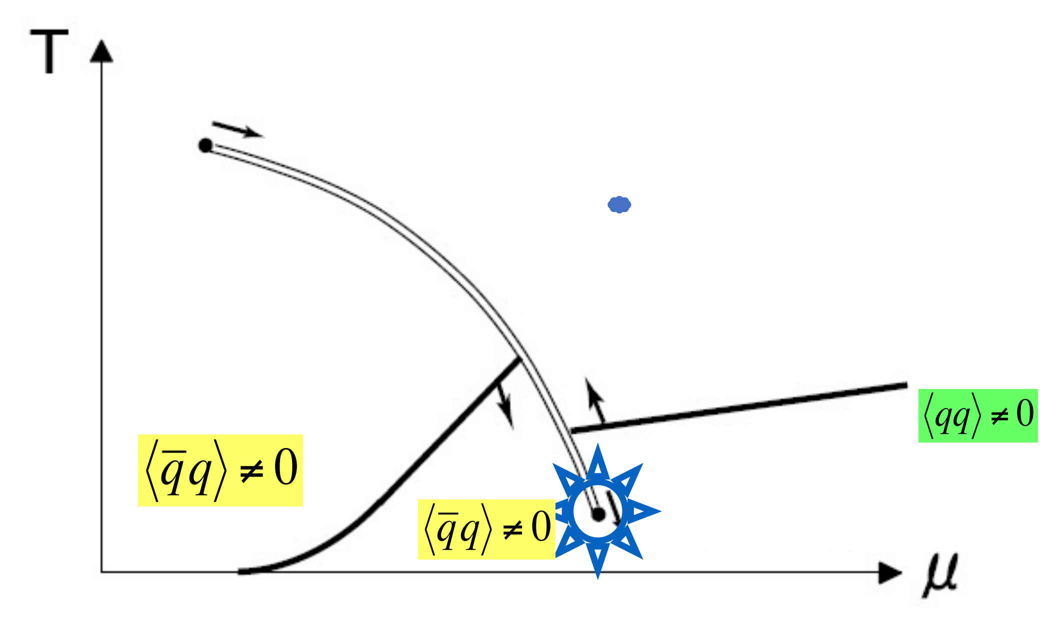
$\sim d_L^* d_R \Phi$

Possible new low temperature critical point
induced by anomaly coupling of chiral and diquark condensates

Hatsuda, Tachibana, Yamamoto & GB, PRL 97, 122001 (2006); PRD 76, 074001 (2007)



Fukushima & Hatsuda,
Rep. Prog. Phys. **74** (2011)



Too cold to be accessible experimentally at RHIC. Possibly at FAIR.

Consistent with quark-hadron continuity (*Schäfer-Wilczek 1999*) – cf. quarks in neutron stars

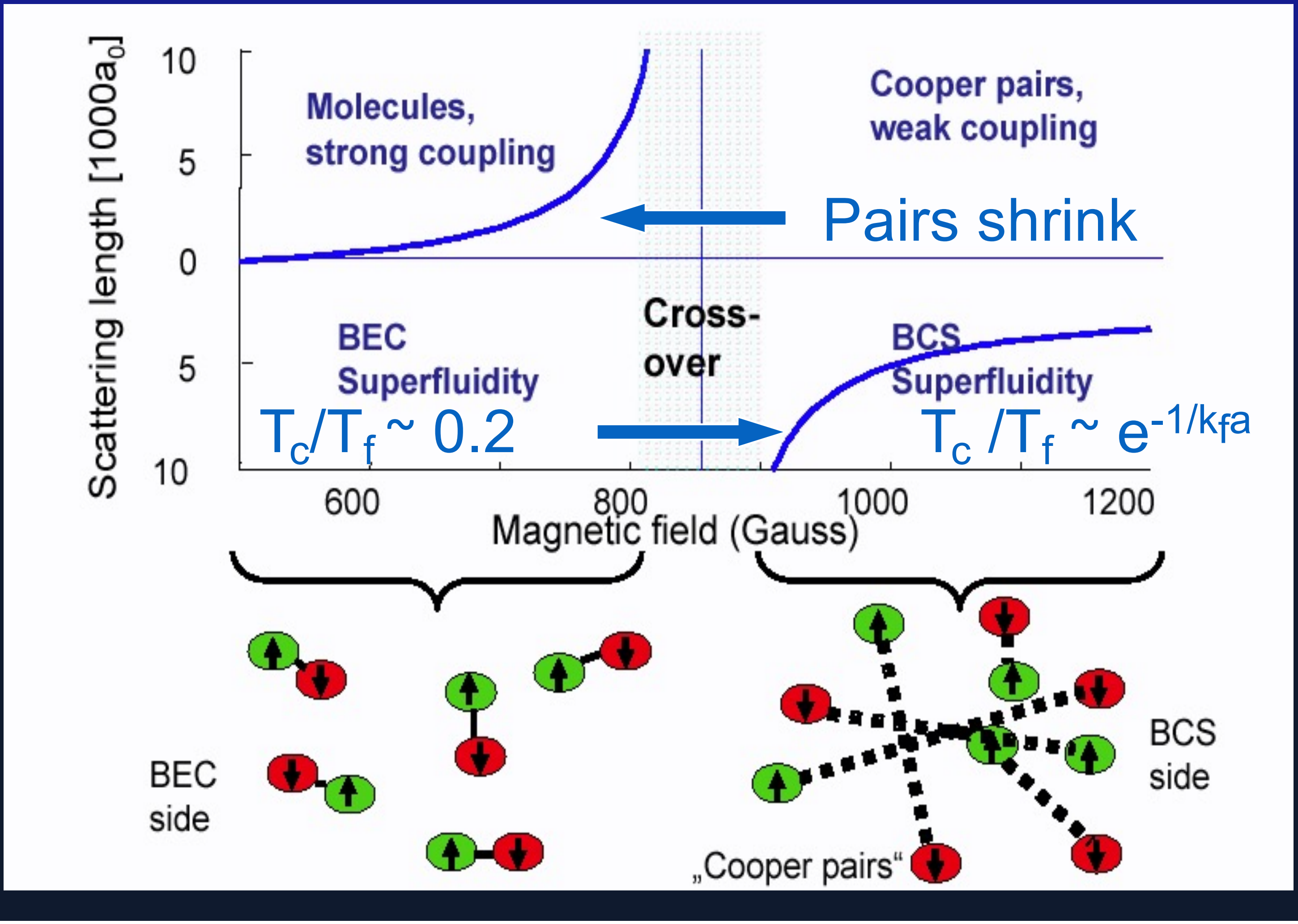
BEC-BCS crossover in Fermi systems

Continuously transform from molecules to Cooper pairs:

D.M. Eagles (1969)

A.J. Leggett, J. Phys. (Paris) C7, 19 (1980)

P. Nozières and S. Schmitt-Rink, J. Low Temp Phys. 59, 195 (1985)



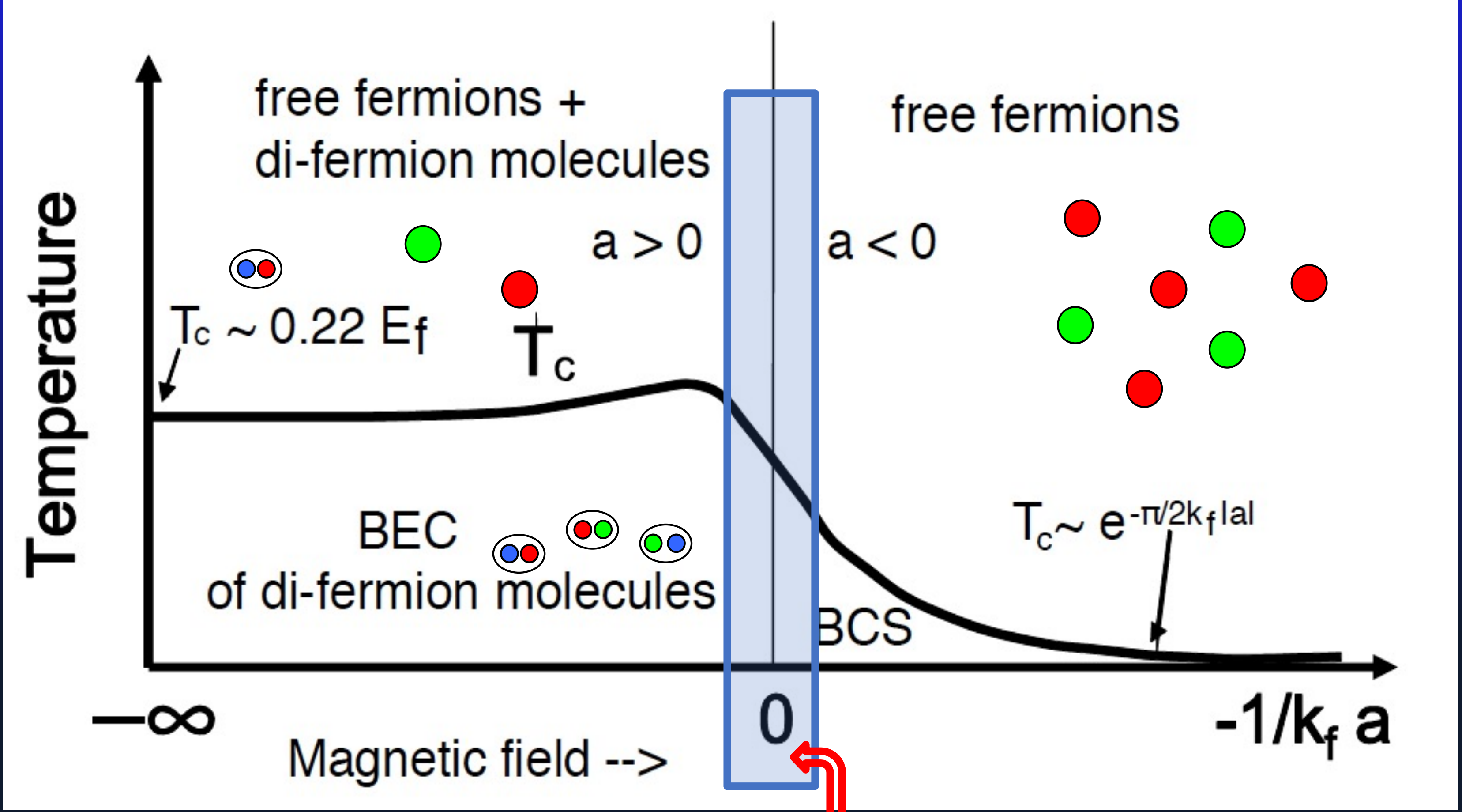
Pairs shrink

$T_c/T_f \sim 0.2$

$T_c/T_f \sim e^{-1/k_f a}$

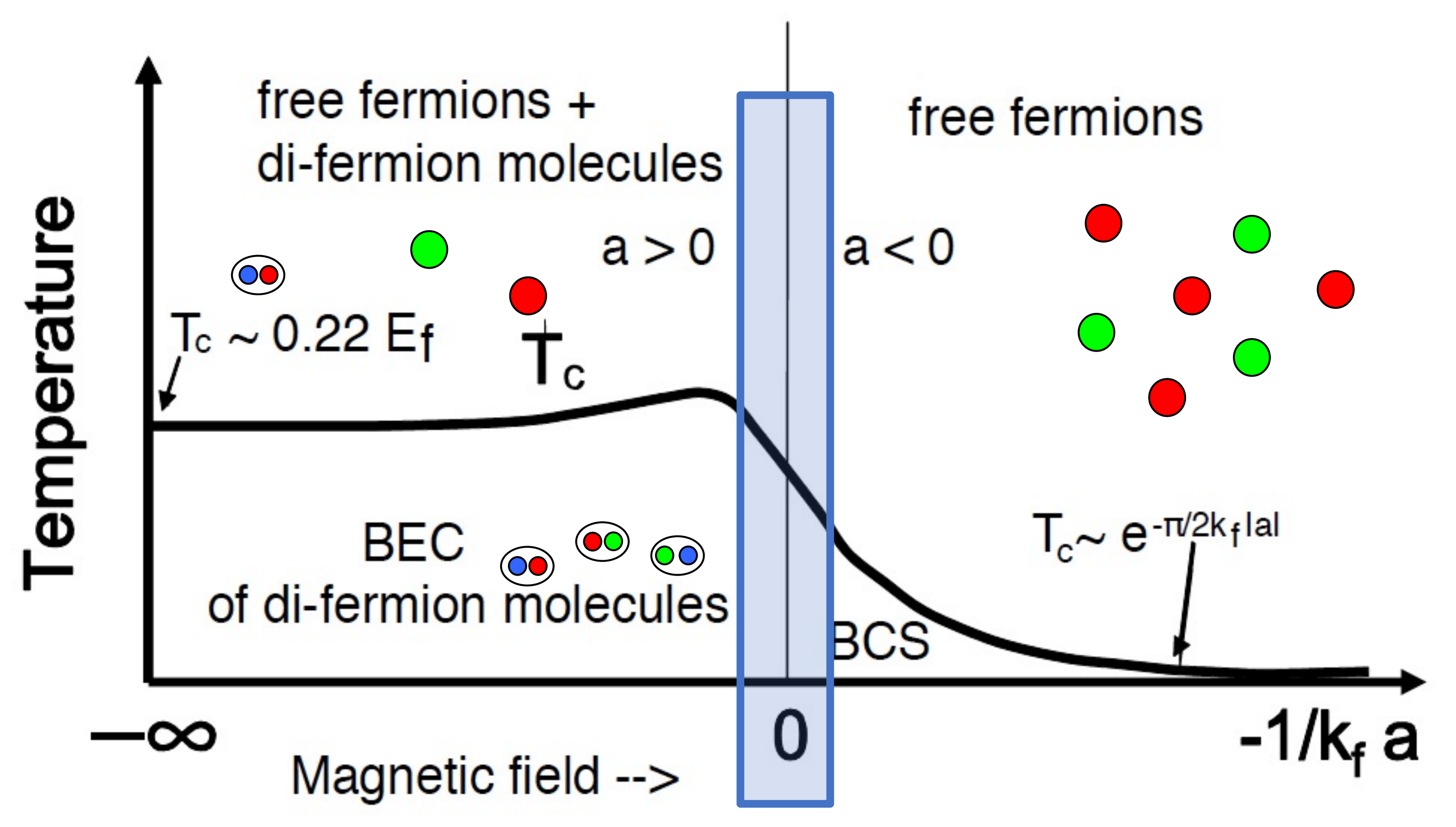
^{6}Li

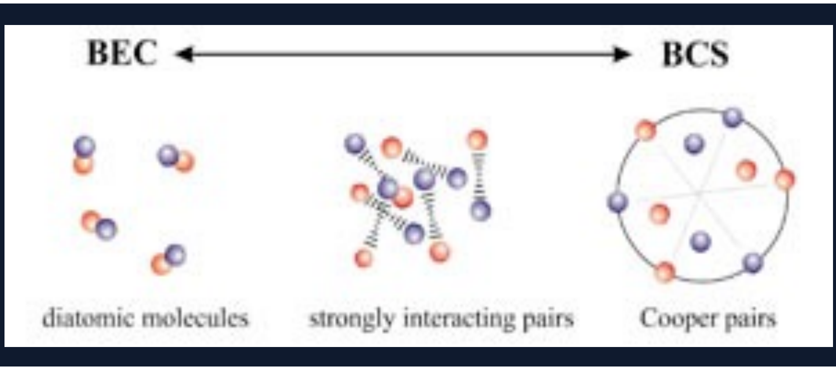
Phase diagram of ultracold atomic fermion gases: in temperature and strength of the particle interactions



Unitary regime (Feshbach resonance) – BEC-BCS crossover. No phase transition through crossover

Phase diagram of ultracold atomic fermion gases: in temperature and strength of the particle interactions

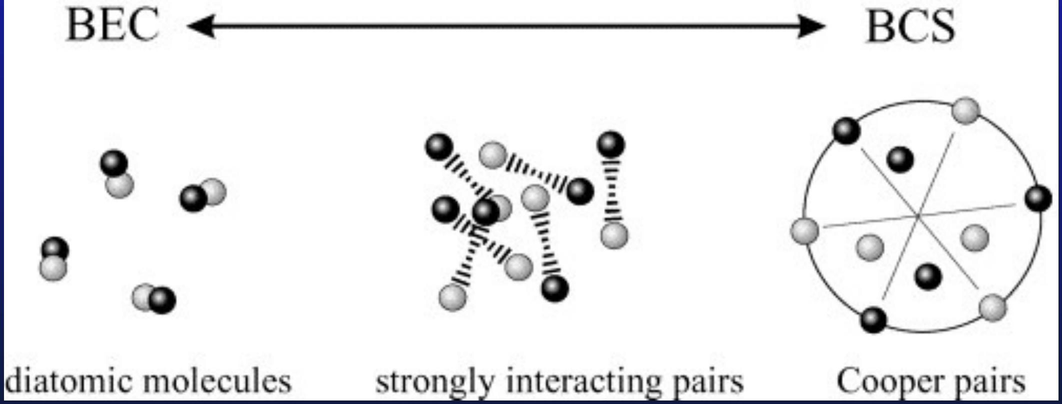




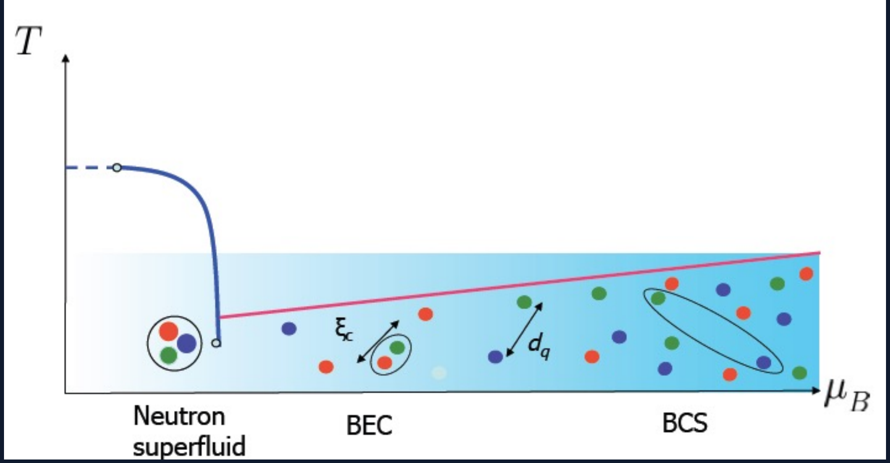
Similar evolution of states in atomic clouds and nuclear matter

GB, T.Hatsuda, M.Tachibana, & Yamamoto. J. Phys. G: Nucl. Part. 35, 10402 (2008)
H. Abuki, GB, T. Hatsuda, & N. Yamamoto, Phys. Rev. D81, 125010 (2010)

Evolution of Fermi atoms with weakening attraction between atoms:



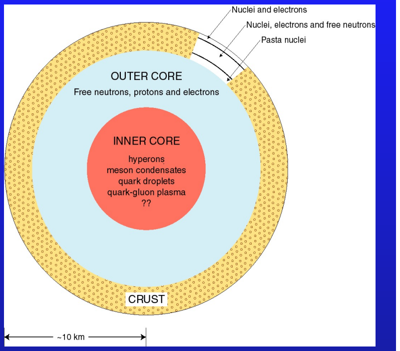
Similarly, as nuclear matter becomes denser have "continuous" evolution from hadrons (nucleons) to quark pairs (diquarks) to quark matter:



denser →

K. Masuda, T. Hatsuda, & T. Takatsuka, Ap. J.764, 12 (2013)

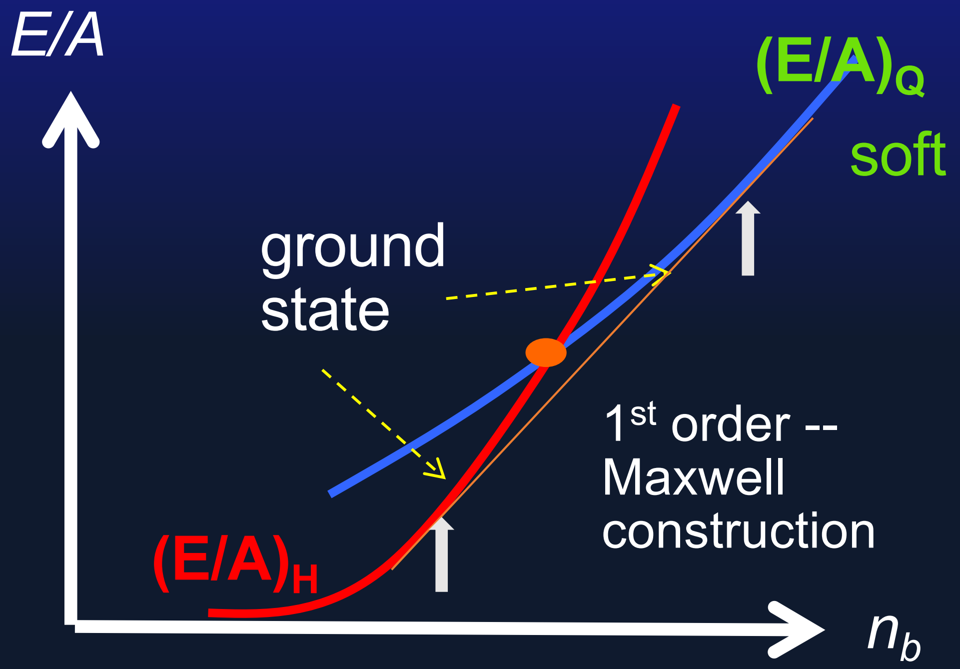
Quark matter cores in neutron stars



Canonical picture: compare calculations of eqs. of state of hadronic matter and quark matter. *GB & S.A. Chin (1976)*

Crossing of thermodynamic potentials
=> first order phase transition.

ex. nuclear matter using hadronic interactions,
vs. perturbative expansion or bag models.

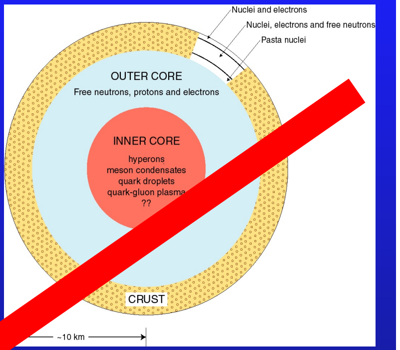


Assumes hadronic state at high densities – not possible when hadrons substantially overlap

Allows only quark equations of state lying under hadronic at high density. Soft only and therefore can't support two solar mass stars.

Conclude typically transition at n ~ 10n$_0$ -- would not be reached even in high mass neutron stars => at most small quark matter cores

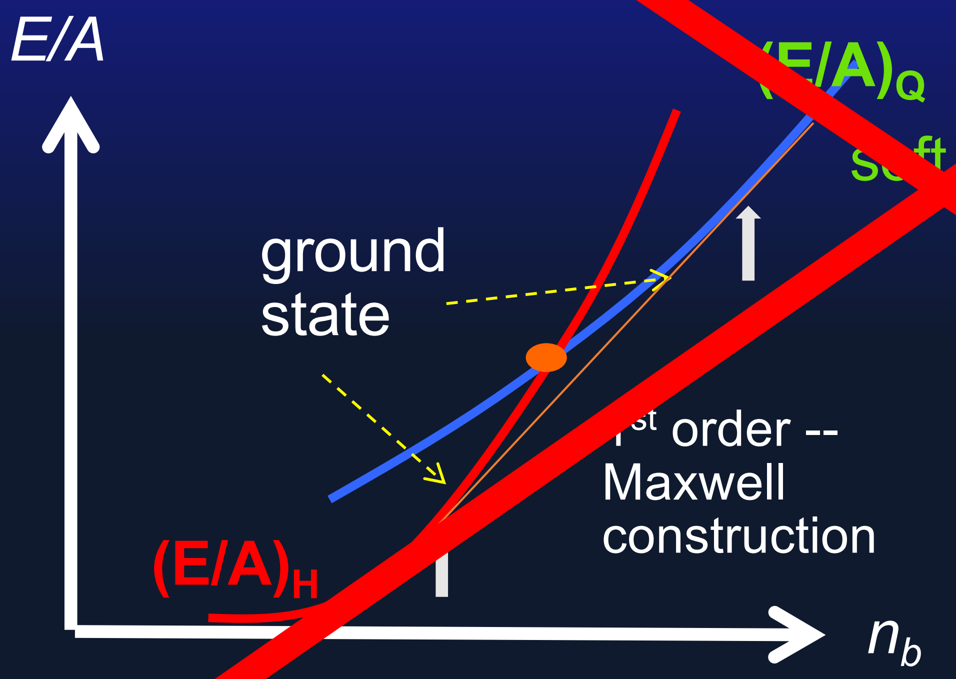
Quark matter cores in neutron stars



Canonical picture: compare calculations of eqs. of state of hadronic matter and quark matter. *GB & S.A. Chin (1976)*

Crossing of thermodynamic potentials
=> first order phase transition.

ex. nuclear matter using hadronic interactions,
vs. perturbative expansion or bag models.



Assumes hadronic state at high densities – not possible when hadrons substantially overlap

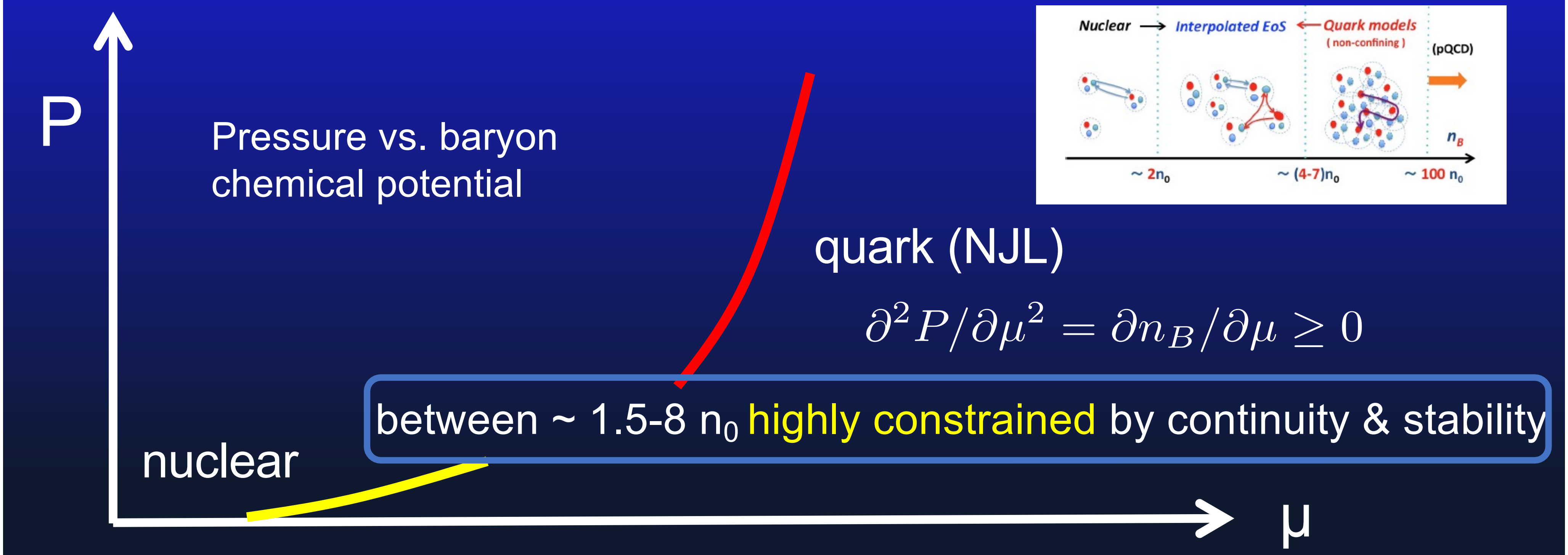
Allows only quark equations of state lying under hadronic at high density. Soft only and therefore can't support two solar mass stars.

Conclude typically transition at n ~ $10n_0$ -- would not be reached even in high mass neutron stars => at most small quark matter cores

QHC21 (quark-hadron crossover) equation of state:

QHC19 --GB, S. Furusawa, T. Hatsuda, T. Kojo & H. Togashi, Ap.J. 885:42 (2019)
QHC21 --T. Kojo, GB, &T. Hatsuda,, Ap.J. 934:46 (2022). arXiv:2111.11919

Have good idea of equation of state at nuclear and at high densities.



Pressure vs. baryon chemical potential

$$\partial^2 P/\partial\mu^2 = \partial n_B/\partial\mu \geq 0$$

between ~ 1.5-8 n_0 highly constrained by continuity & stability

Quarks in Nambu-Jona-Lasinio (NJL) model with universal repulsive short-range qq coupling (*Kunihiro*) and diquark (BCS) pairing interaction

$$\mathcal{L}_V^{(4)} = -g_V\left(\bar{q}\gamma^\mu q\right)^2$$

$$\mathcal{L}_d^{(4)} = H \sum_{A,A'=2,5,7} [(\bar{q} i\gamma_5\tau_A\lambda_{A'}C\bar{q}^T)(q^T C i\gamma_5\tau_A\lambda_{A'}q)$$

Nuclear matter equation of state below 1.5 n_0

NJL model n above ~ $5n_0$

NJL Lagrangian $\mathcal{L} = \bar{q}(i\gamma_\mu\partial^\mu - m_q + \mu\gamma_0)q + \mathcal{L}^{(4)} + \mathcal{L}^{(6)}$

$$\mathcal{L}_\chi^{(4)} = G\sum_{a=0}^{8}[(\bar{q}\tau_a q)^2 + (\bar{q}i\gamma_5\tau_a q)^2]$$

chiral interactions, coupling = G

$$\mathcal{L}_d^{(4)} = H\sum_{A,A'=2,5,7}[(\bar{q}i\gamma_5\tau_A\lambda_{A'}C\bar{q}^T)(q^T C i\gamma_5\tau_A\lambda_{A'}q)]$$

BCS pairing interactions

$\mathcal{L}^{(6)}$ = Kobayashi-Maskawa-‘t Hooft six quark axial anomaly

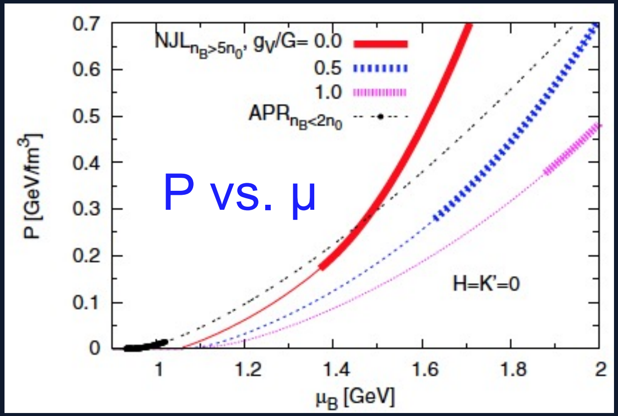
plus universal repulsive quark-quark vector coupling

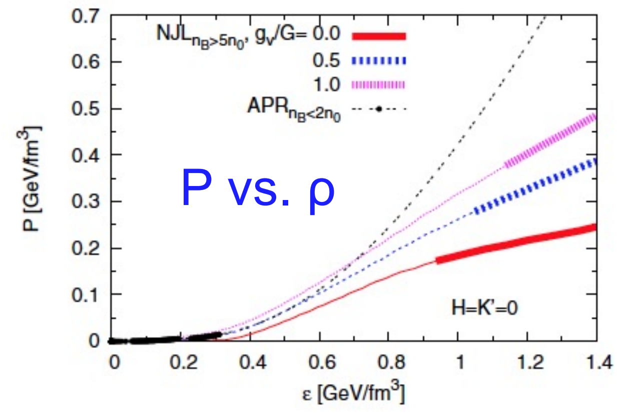
$$\mathcal{L}_V^{(4)} = -g_V\left(\bar{q}\gamma^\mu q\right)^2$$

T. Kunihiro

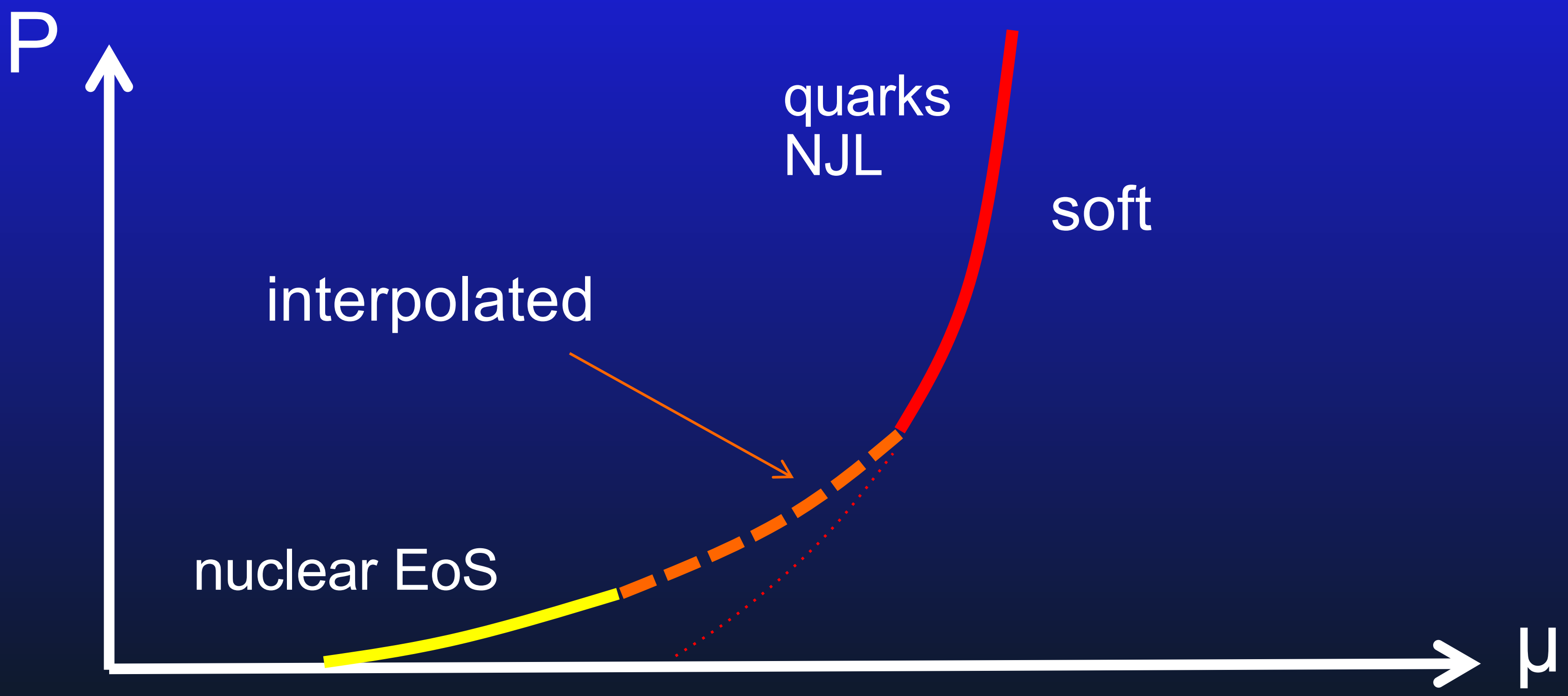
Include u,d, and s quarks

Reviewed in:
GB, T. Kojo, T. Hatsuda,
T. Takatsuka, & Y. Song
ROPP 81 (2018) 056902



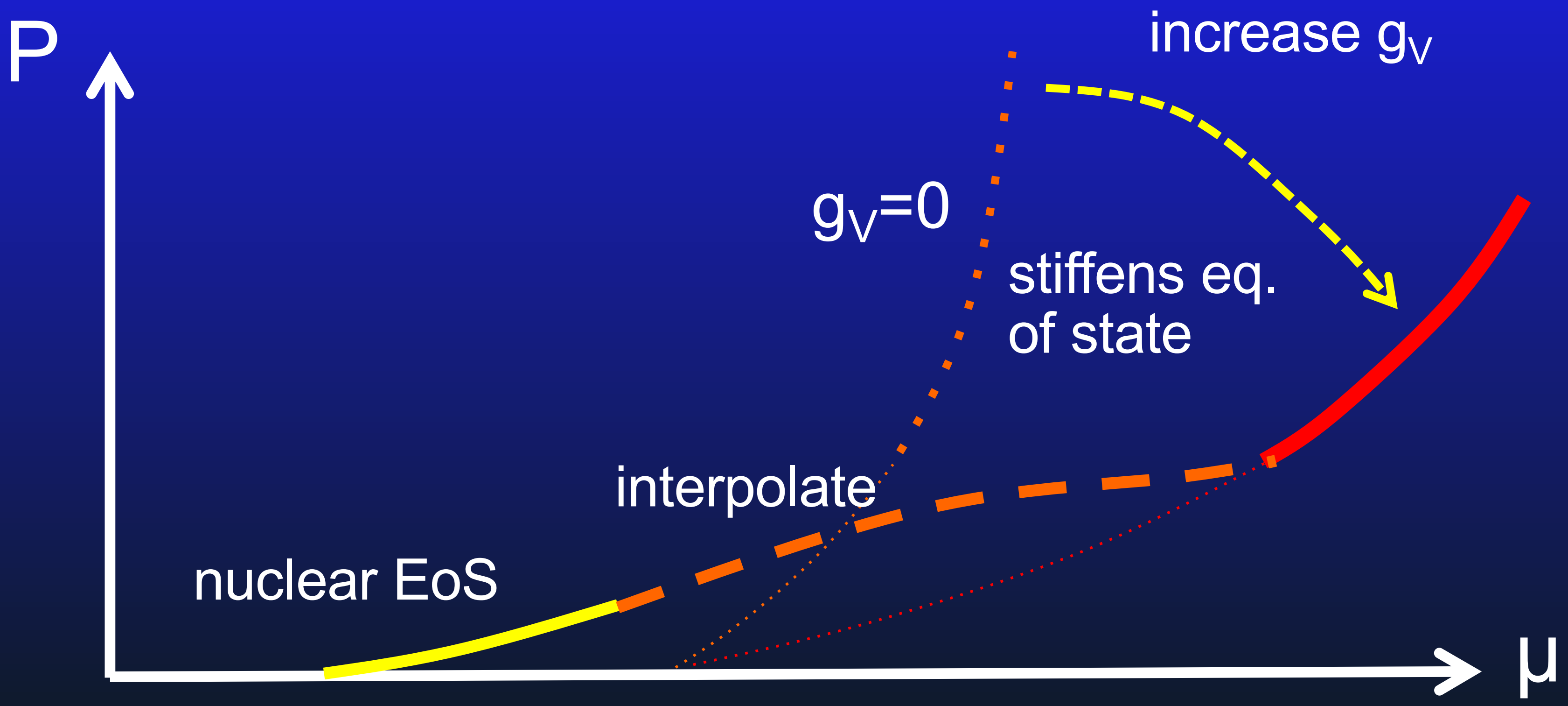


Minimal model: $g_V = 0$



Soft quark equation of state does not allow high mass neutron stars. Need large quark repulsion g_V to reach high mass.

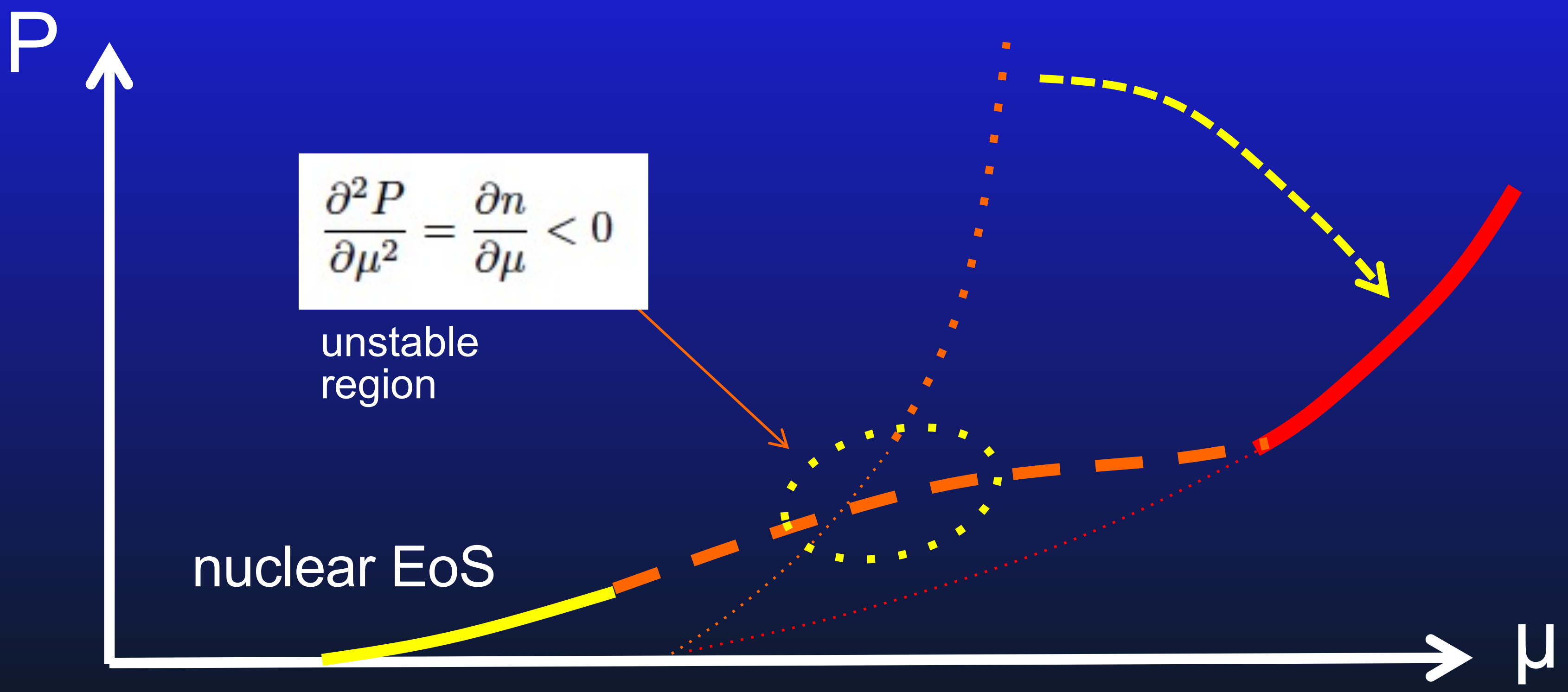
Vector interaction stiffens eq. of state



$$\mathcal{L}_V^{(4)} = -g_V \left(\bar{q}\gamma^\mu q\right)^2$$

Shift of pressure in quark phase towards higher μ

Vector interaction stiffens eq. of state



$$\frac{\partial^2 P}{\partial \mu^2} = \frac{\partial n}{\partial \mu} < 0$$

unstable
region

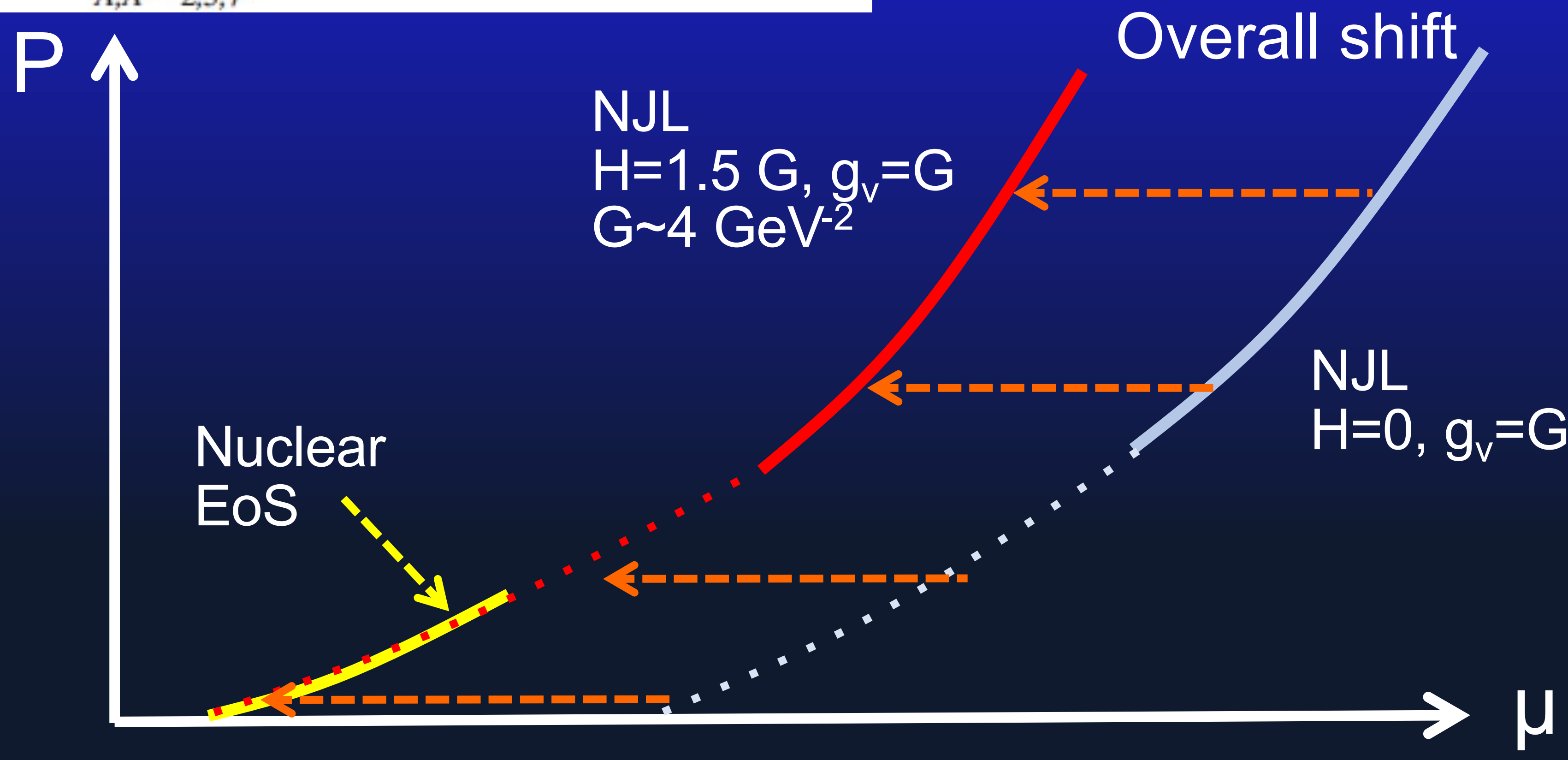
nuclear EoS

But larger g_V leads to unphysical thermodynamic instability


In this house, we obey
the laws of thermodynamics!

Thermodynamic stability requires large diquark (BCS) pairing interaction, H

$$\mathcal{L}_d^{(4)} = H \sum_{A,A'=2,5,7} [(\bar{q} i\gamma_5 \tau_A \lambda_{A'} C \bar{q}^T)(q^T C i\gamma_5 \tau_A \lambda_{A'} q)$$

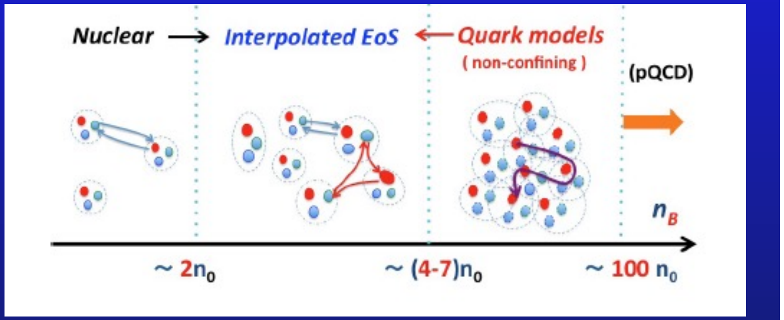


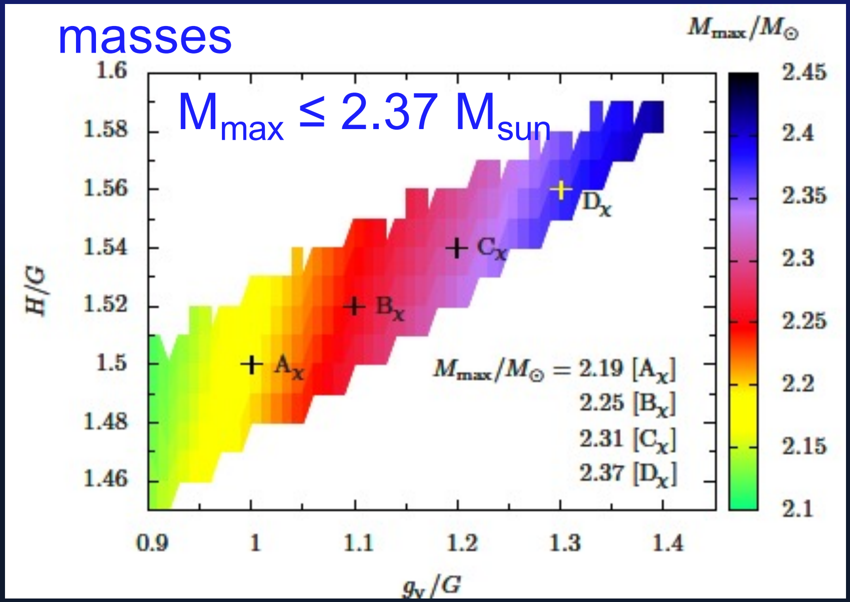
Increased BCS pairing (onset of stronger 2-body correlations) as quark matter comes nearer to becoming confined

QHC21 (quark-hadron crossover) eqn of state

T. Kojo, GB, & T. Hatsuda, Ap. J. 934:46 (2022) *https://compose.obspm.fr.*

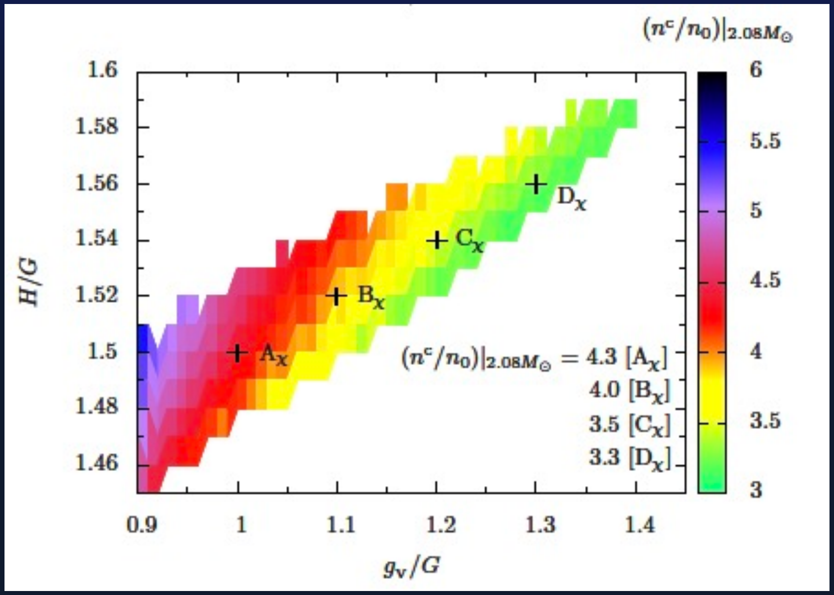
Parameters g_v and H must be in colored region so that
speed of sound ≦ speed of light.




masses

$M_{max} \lesssim 2.37\ M_{sun}$

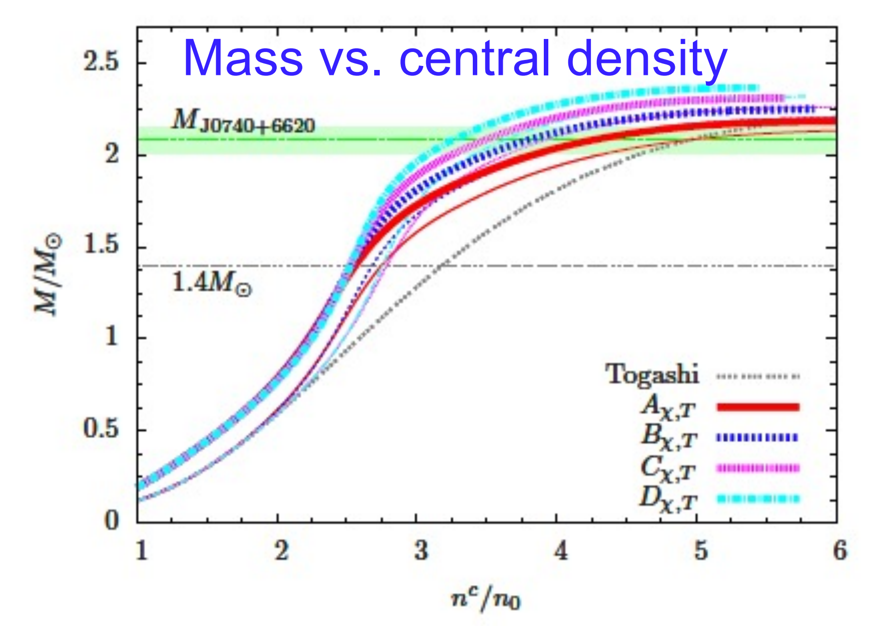
Further restricted by maximum neutron star mass > 2.08 M_{sun}



Strange quarks included

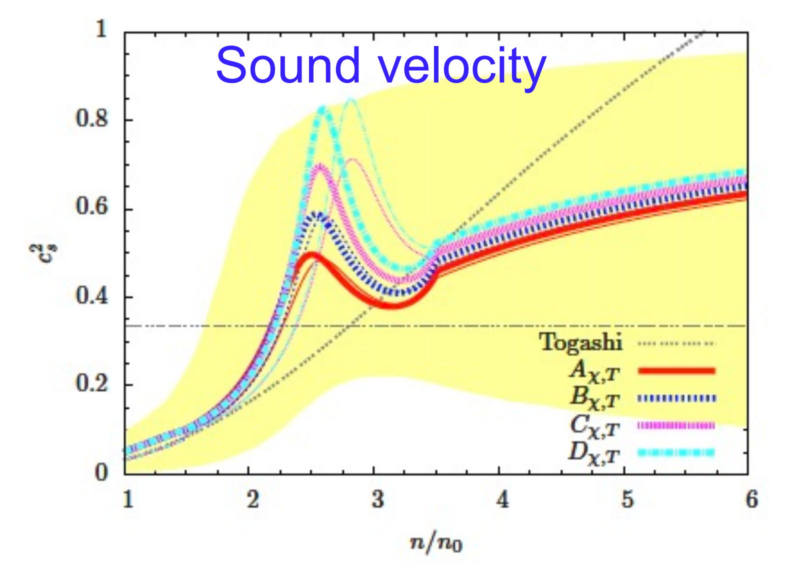
2.08 M_{sun} => $g_v \gtrsim 0.9$ G

Quarks beginning to enter neutron stars

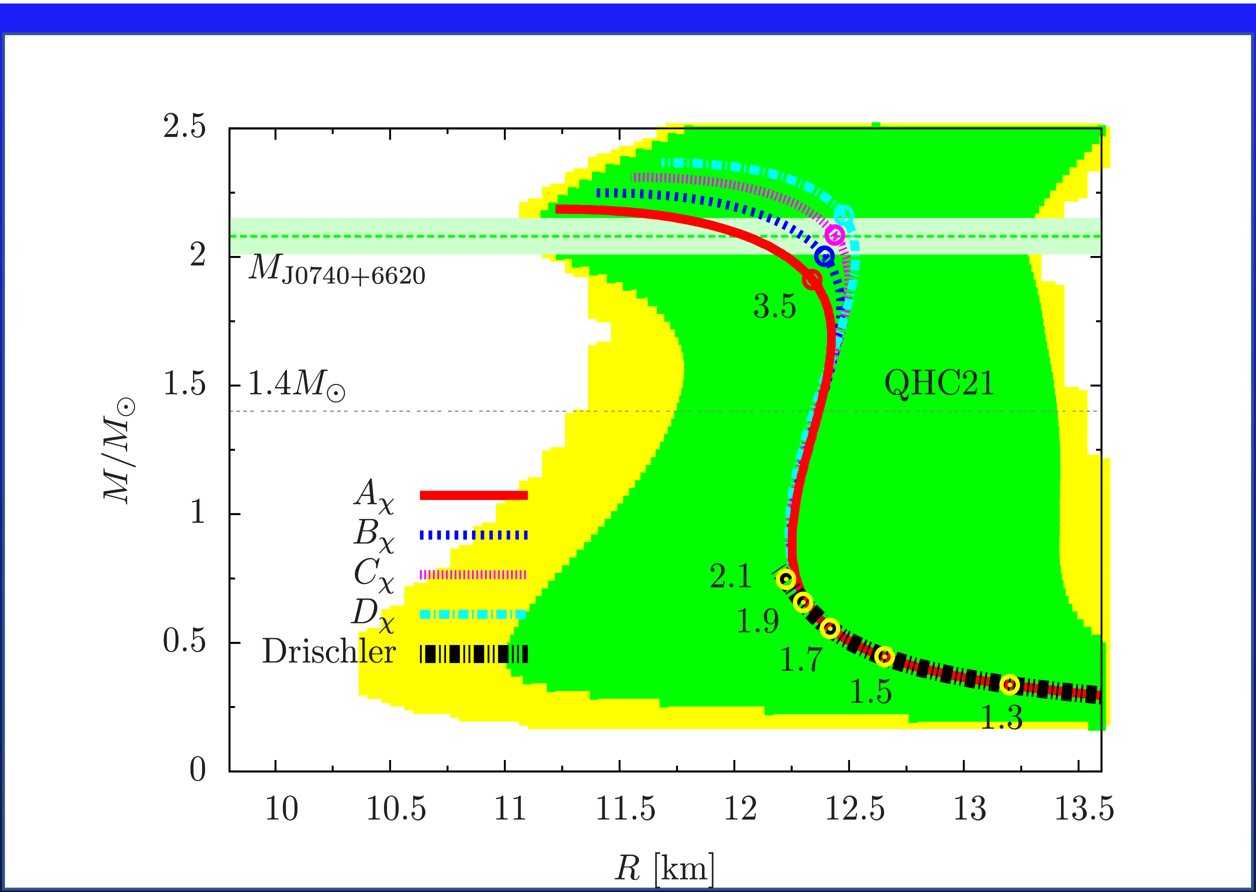


Central density of PSR J0740+6620 ~5n_0. Well above densities where pure hadronic calculations are valid. Entering transition to strongly interacting quark matter.

Cores of higher mass stars could reach beyond transition. Fully developed quark matter in cores? Need fully microscopic calculations of matter undergoing transition from nucleonic to quark degrees of freedom.



Peak not seen in nucleonic eq. of state

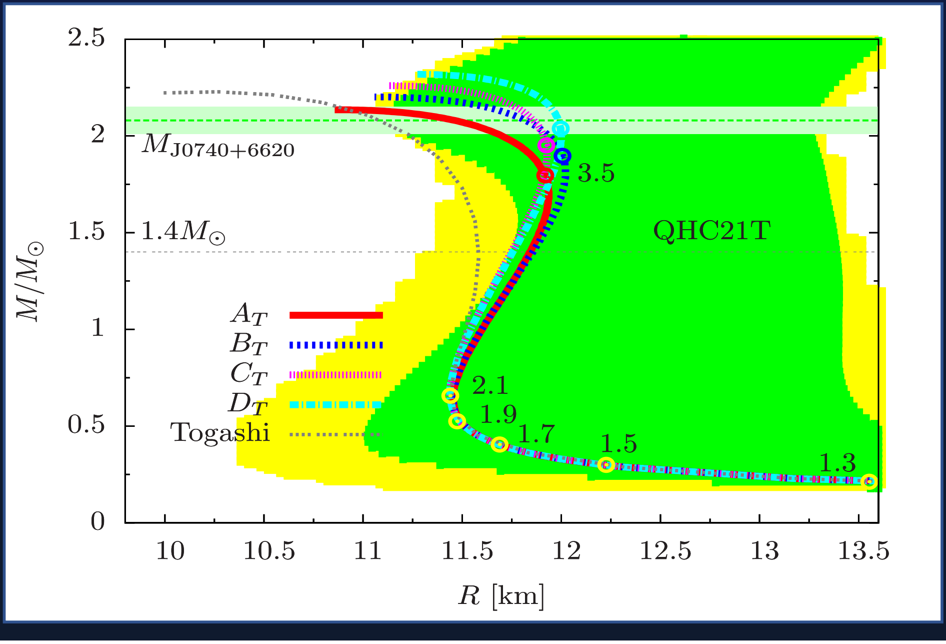


Chiral EFT + transition to quark matter in excellent agreement with NICER inferences of radii. Rapid pressure rise!

Central density for 2.08 M_{sun} ~ 3.6 n_0

QHC21

Nucleonic APR-Togashi too soft in nuclear regime.



NICER has provided first empirical tests of theories of neutron rich nuclear matter

1) Chiral Effective Field Theory of nuclear matter points toward stiff neutron rich matter in nuclear regime (up to 1.5 n_0), adequate to allow 12-13 km neutron stars.

2) Effective equality of radii of neutron stars of 1.4 and 2.08 M_{sun}

3) Looking forward to future NICER measurements of radii of neutron stars of intermediate masses, and improved precision.

Challenges for future:

Build consistent phenomenological picture of matter above nuclear matter density from both gravitational radiation and NICER data.

Develop microscopic pictures of transition from hadronic to quark degrees of freedom in the regime 1.5-8 n_0.

(Not good enough to draw a curve $P(\rho)$ that fits data. Must understand microscopic physics at QCD level.)

What are the lightest and heaviest neutron stars, and lightest black holes? (cf. LIGO/Virgo's eight compact objects)

Future NICER data and eventual gravitational wave data
(3rd generation detectors, to 400 Mpc => ~ 10^2 BNS mergers/year)
will continue to clarify physics of matter under extreme conditions.

Superfluids in neutron stars

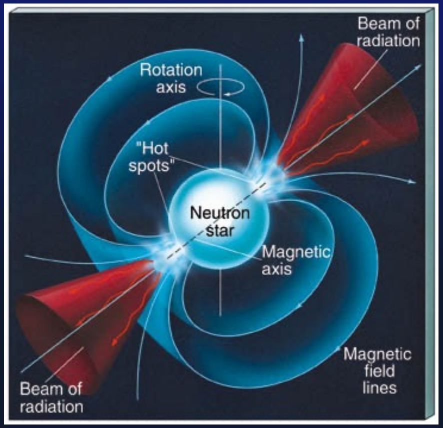
Mass ~ 1-2+ M_{sun}

Radius ~ 12-13 km

Temperature
~ 10^6-10^9 K

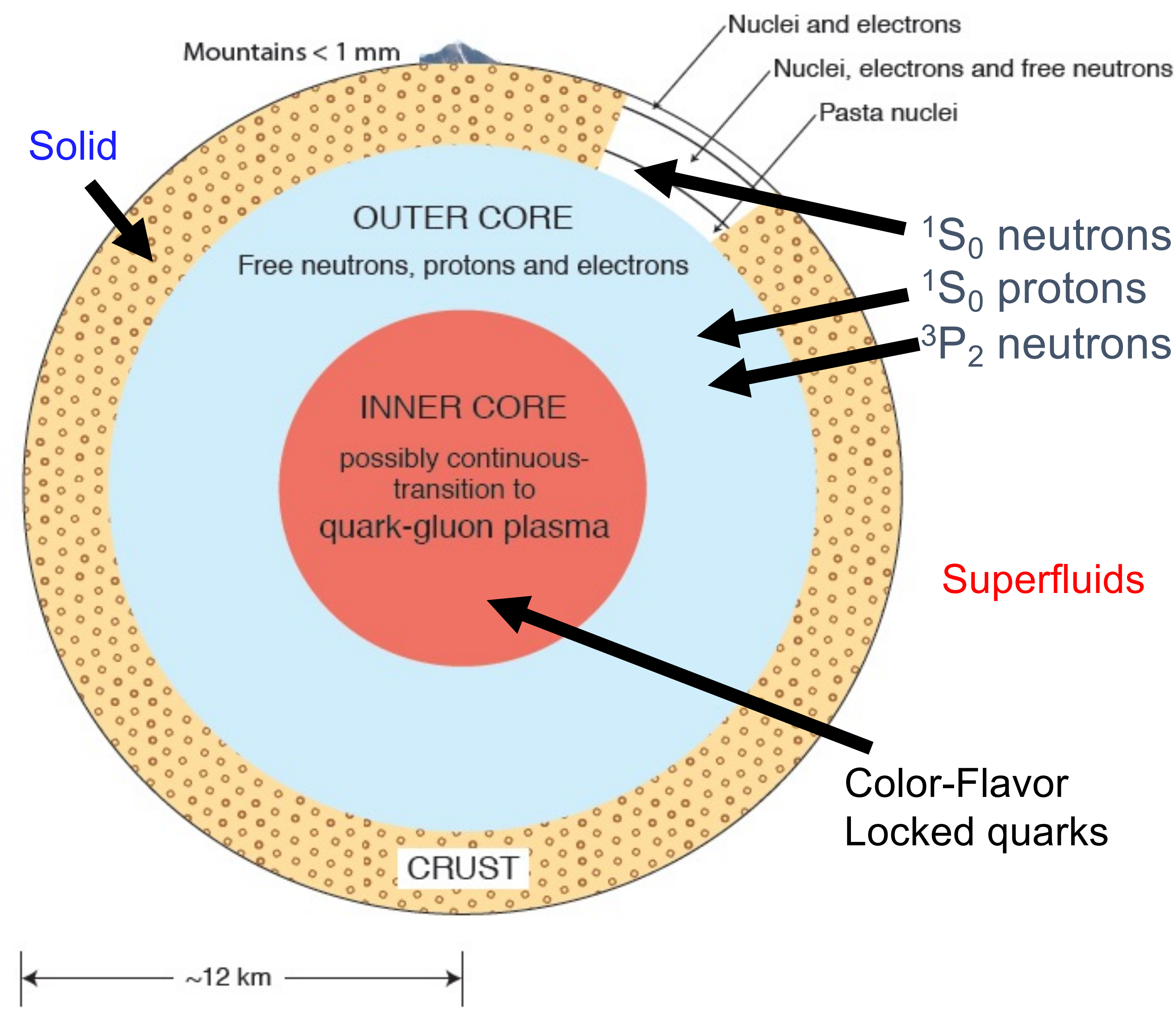
Baryon no. ~ 10^{57}

Magnetic fields
~ 10^6 – 10^{16} G



Surface gravity
~10^{11} that of Earth

Surface binding
~ 1/10 mc^2



Solid

$^1S_0$ neutrons

$^1S_0$ protons

$^3P_2$ neutrons

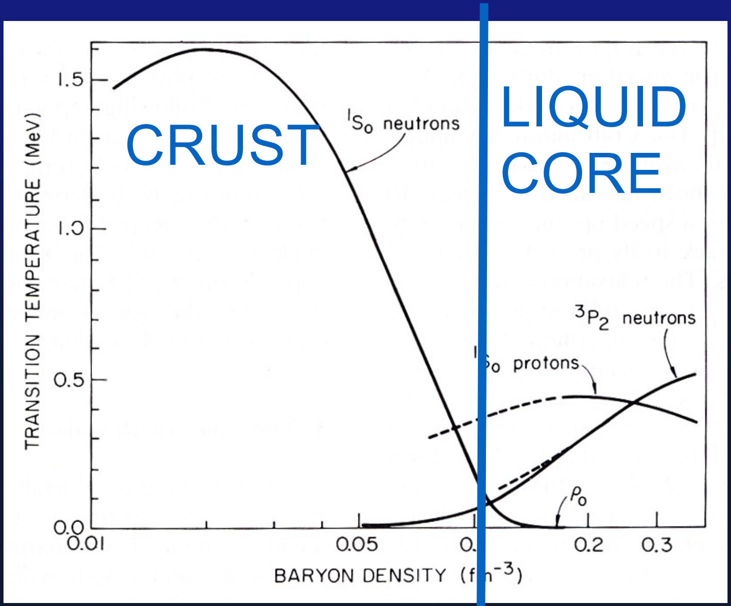
Superfluids

Color-Flavor
Locked quarks

Superfluidity of nuclear matter in neutrons stars

Neutron stars (very big Dewars) have the preponderence of superfluids in the universe, and with the highest T_c's ~ $10^{10\text{-}11}$ K

Estimate pairing gaps and T_c's from scattering phase shifts



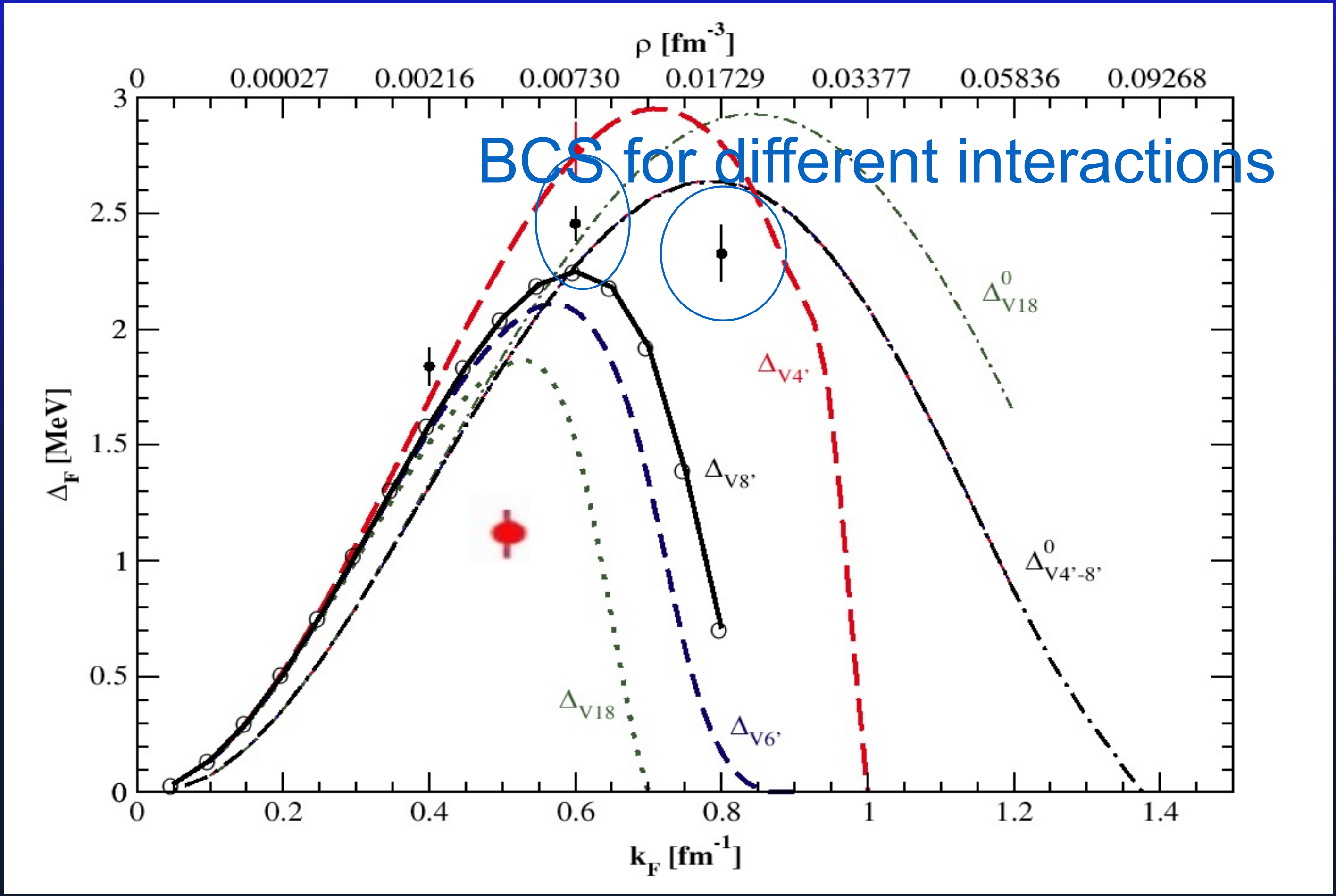
Neutron fluid in crust BCS-paired in relative $^1S_0$ states (singlet spin)

Neutron fluid in core $^3P_2$ paired (triplet spin)

Proton fluid $^1S_0$ paired

Quantum Monte Carlo (AFDMC) $^1S_0$ nn gap in crust:

Fabrocini et al, PRL 95, 192501 (2005)

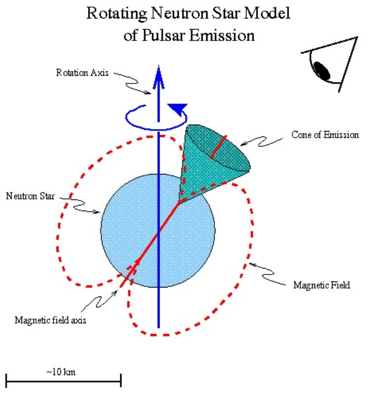


BCS for different interactions

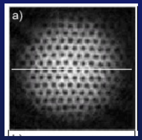
QMC (black points) close to standard BCS (upper curves)
Green’s function Monte Carlo (Gezerlis 2007)

Superconducting protons in neutron star magnetic fields, ~$10^{12\text{-}16}$G

Even though superconductors expel magnetic flux, for magnetic field below critical value, flux diffusion times in neutron stars are >> age of universe.
Electric conductivity >>> Cu at room temp.
Proton superconductivity forms with field present.



Proton fluid threaded by triangular (Abrikosov) lattice of vortices parallel to magnetic field (for Type II superconductor)



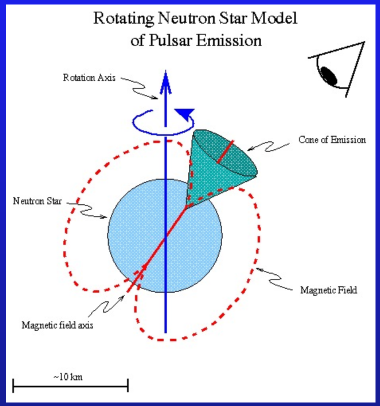
Quantized magnetic flux per vortex:

$$\oint_C \mathbf{B} \cdot d\ell = \frac{2\pi\hbar c}{2e} = \phi_0 = 2 \times 10^{-7}\ \text{G cm}^2$$

Vortex core ~ 10 fm,

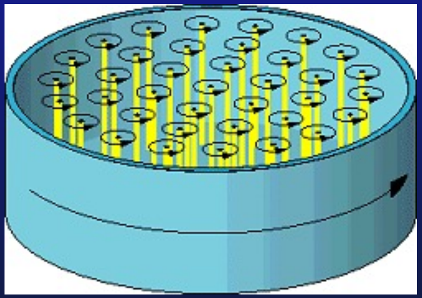
$n_{vort} = B/\phi_0$ => spacing ~ 5×10^{-10} cm $(B/10^{12}\text{G})^{-1/2}$

Rotating superfluid neutrons,

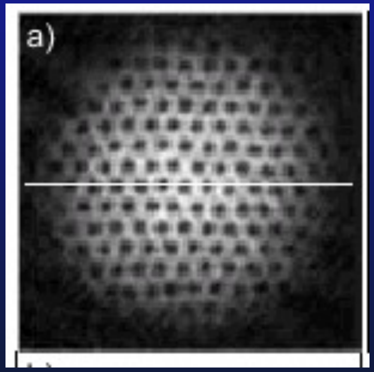


(Rotation periods from few seconds to > msec.)

Rotating superfluid threaded by triangular lattice of vortices parallel to stellar rotation axis



Bose-condensed ^{87}Rb atoms
Schweikhard et al., PRL92 040404 (2004)

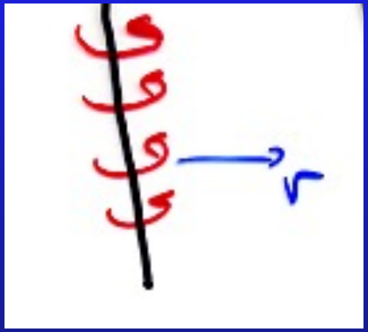


Quantized circulation of superfluid velocity about vortex:

$$\oint_C \mathbf{v_s} \cdot d\ell = \frac{2\pi\hbar}{2m_n}$$

Vortex core ~ 10 fm. Vortex separation ~ $0.01P(s)^{1/2}$cm. P=89 ms
Vela pulsar (PSR0833-45) ~ 10^{17} vortices

Vortices in superfluids: quantized circulation



Order parameter $\Psi(\vec{r}) = |\psi| e^{i\phi(\vec{r})}$

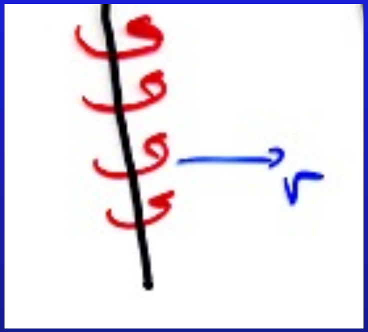
Superfluid velocity $\vec{v}(\vec{r}) = (\hbar/m)\nabla\phi$

Quantized circulation $\oint_C \vec{v}\cdot d\vec{\ell} = \frac{2\pi\hbar n}{m}$ n = integer

Singly quantized (n=1) vortex flow: $v_\varphi(r) = \hbar/mr$

But what m should one use in an interacting system?

Vortices in superfluids: quantized circulation



Order parameter $\Psi(\vec{r}) = |\psi| e^{i\phi(\vec{r})}$

Superfluid velocity $\vec{v}(\vec{r}) = (\hbar/m)\nabla\phi$

Quantized circulation $\oint_C \vec{v}\cdot d\vec{\ell} = \frac{2\pi\hbar n}{m}$ n = integer

Singly quantized (n=1) vortex flow: $v_\varphi(r) = \hbar/mr$

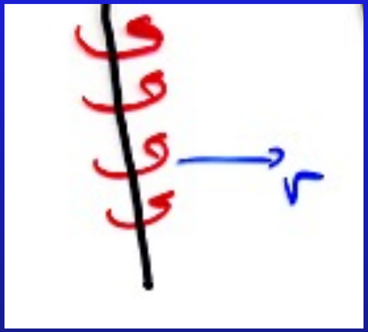
But what m should one use in an interacting system?

How is the superfluid velocity related to the momentum?

$$\phi = \vec{p}\cdot\vec{r} - \mu t \Rightarrow \vec{v} = \vec{p}/\mu$$

μ = chemical potential including rest mass

Vortices in superfluids: quantized circulation



Order parameter $\Psi(\vec{r}) = |\psi| e^{i\phi(\vec{r})}$

Superfluid velocity $\vec{v}(\vec{r}) = (\hbar/m)\nabla\phi$

Quantized circulation $\oint_C \vec{v}\cdot d\vec{\ell} = \dfrac{2\pi\hbar n}{m}$ n = integer

Singly quantized (n=1) vortex flow: $v_\varphi(r) = \hbar/mr$

But what m should one use in an interacting system?

How is the superfluid velocity related to the momentum?

$$\phi = \vec{p}\cdot\vec{r} - \mu t \Rightarrow \vec{v} = \vec{p}/\mu$$

μ = chemical potential including rest mass

In circulation, replace m by μ more correctly

$$\oint_C \vec{v}\cdot d\vec{\ell} = \frac{2\pi\hbar n}{\mu}$$

In superfluid ^{4}He, -7.17K correction to m_4 is only ~ 1 : 6 X10^{12}

Pulsar glitches

Sudden speedups in rotation period, relaxing back in days to years, with no significant change in pulsed electromagnetic emission: ~500 glitches detected in > 100 pulsars

Vela (PSR0833-45) Period=1/Ω =0.089sec

>15 glitches since discovery in 1969

$\Delta\Omega/\Omega \sim 10^{-6}$ Largest = 3.14 X 10^{-6} on Jan. 16, 2000

Moment of inertia ~ 10^{45} g-cm^2 => $\Delta E_{rot} \sim 10^{43}$ erg

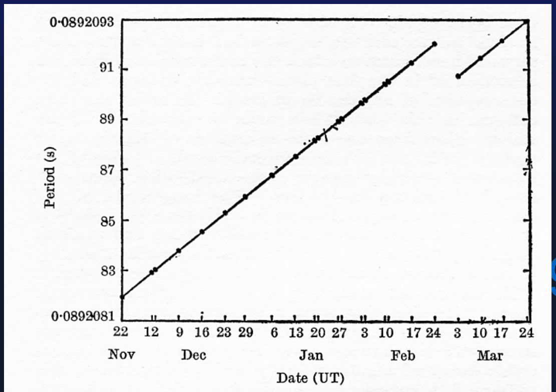


Fig. 1. The barycentric period of *PSR* 0833–45 as observed from November 22, 1968, to March 24, 1969, showing the 134 ns decrease between February 24 and March 3.

9

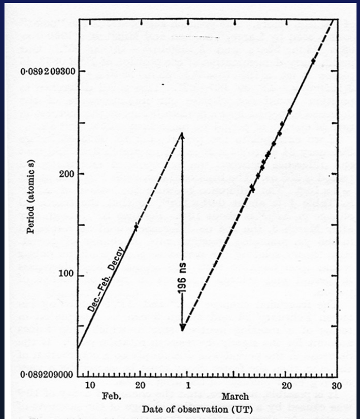
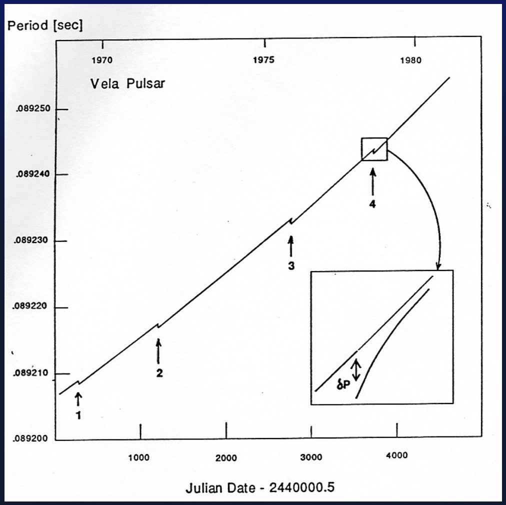


Fig. 1. Heliocentric period of *PSR* 0833–45 observed in February and March 1969, based on position α 08 h 33 m 39·0 s, δ −45° 00′ 05·0″ (epoch 1950·0) (ref. 3). The rate of increase of the period was 10·69 ± 0·20 ns day⁻¹ between December 8, 1968, and February 19, 1969. Since March 13, 1969, the rate of decay has been 10·64 ± 0·20 ns day⁻¹. At some time between February 19 and March 13 the period decreased by 196 ns.



(2018)

Reichley and Downs, Nature 1969

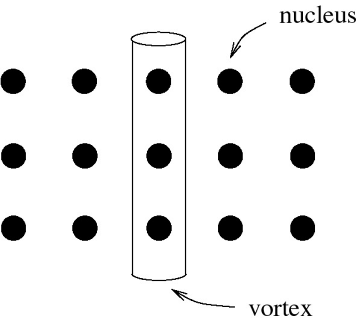
Radhakrishnan and Manchester, Nature 1969

Crab (PSR0531+21) P = 0.033sec 25 glitches since 1969

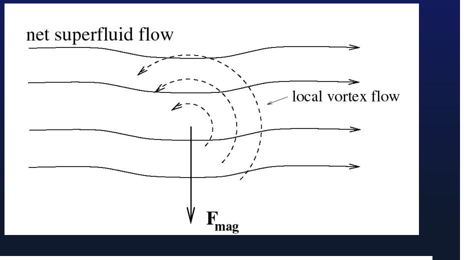
$\Delta\Omega/\Omega \sim 10^{-9}$ to 0.5 X 10^{-6} (in 2018)

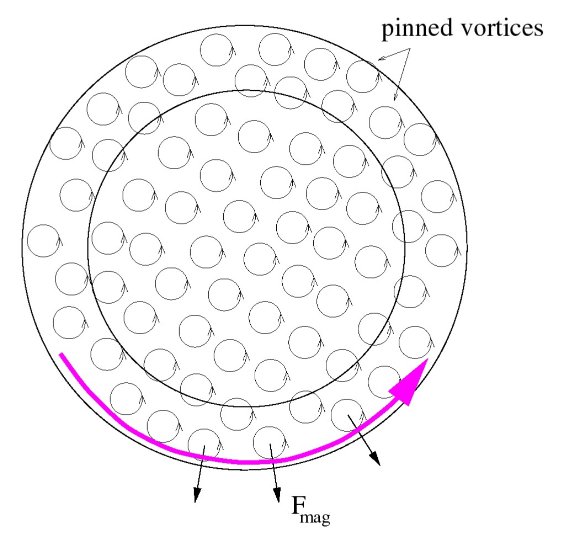
Vortex model of glitches

Pin vortices on nuclei in inner crust.
E ~ few Mev/nucleus.
(Bogoliubov- de Gennes calculations suggest pinning between nuclei)

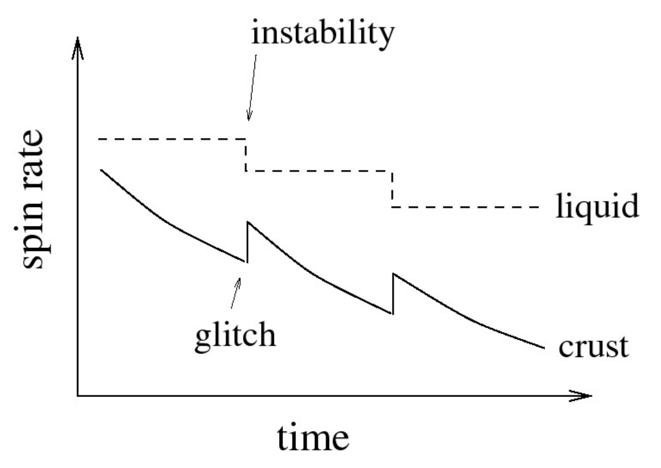


$n_{vortices}$ fixed => $\Omega_{superfluid}$ fixed; Ω_{crust} decreases as star radiates.
As $\Omega_{sf} - \Omega_{crust}$ grows, Magnus force = $\rho_s\ \Omega \times (\mathbf{v}_{vortex} - \mathbf{v}_{superfl})$ drives unpinning (glitch) and outward relaxation.





Collective outward motion of many (~10^{14}) Vortices produces large glitch



BCS pairing in Color Flavor Locked (CFL) phase

In free equally populated up, down, and strange quark matter have $SU(3)_F$ symmetry in flavor (uds) and $SU(3)_C$ symmetry in color (rgb)

Most favored BCS pairing state is anti-symmetric in spin, flavor (i), and color (α):

$$\Phi_{\alpha i} \propto \epsilon_{ijk}\epsilon_{\alpha\beta\gamma}\langle q_{\beta j} C \gamma_5 q_{\gamma k}\rangle \chi_{\mathrm{spin-singlet}}$$

$$\Phi = \begin{pmatrix} \Phi^{\bar r \bar u} & 0 & 0 \\ 0 & \Phi^{\bar g \bar d} & 0 \\ 0 & 0 & \Phi^{\bar b \bar s} \end{pmatrix} \chi \to \begin{pmatrix} \Delta & 0 & 0 \\ 0 & \Delta & 0 \\ 0 & 0 & \Delta \end{pmatrix} \chi$$

flavor ->

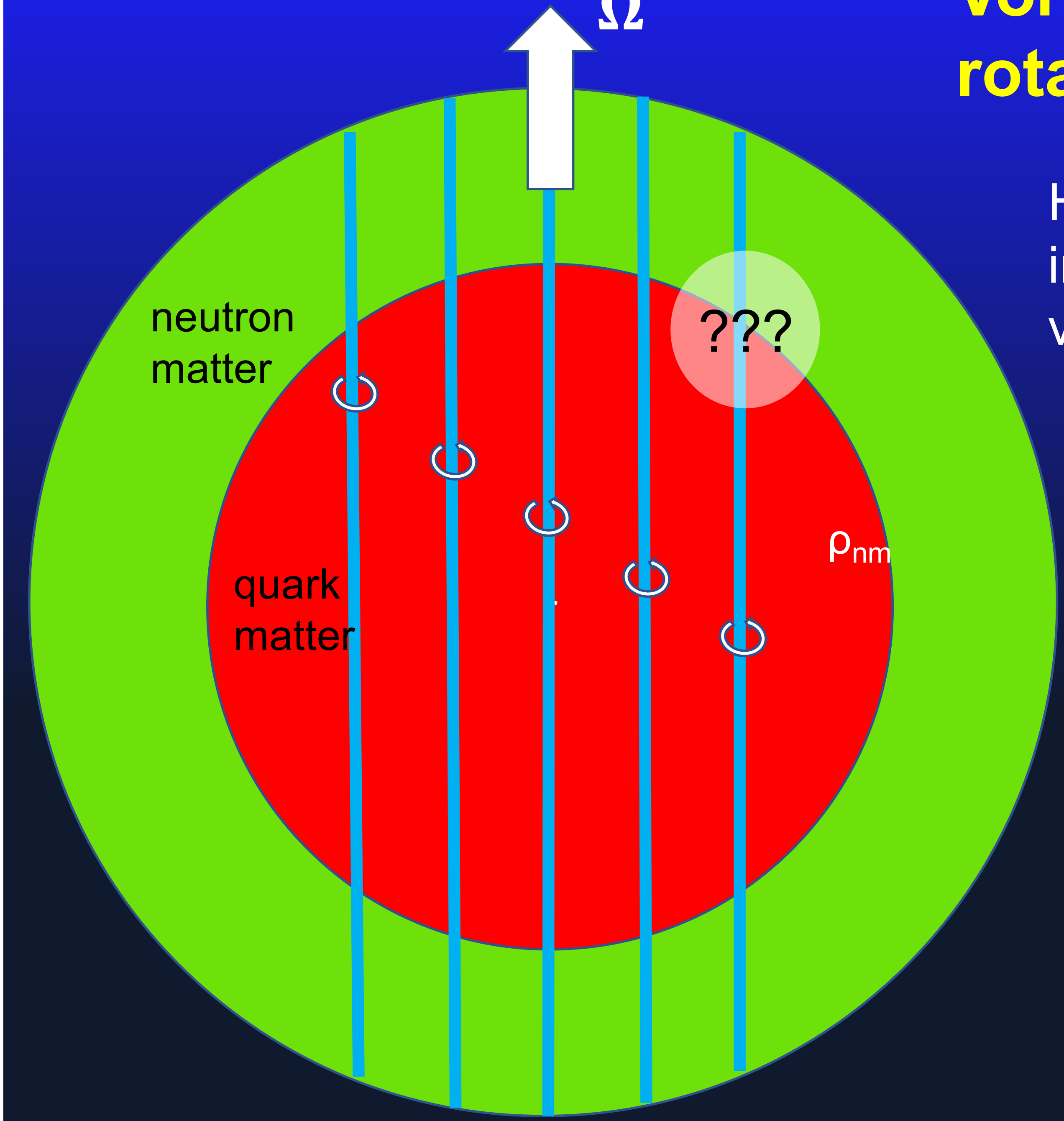
color ->

CFL order parameter in ground state

$$\langle u\, d \rangle = \langle d\, s \rangle = \langle s\, u \rangle$$

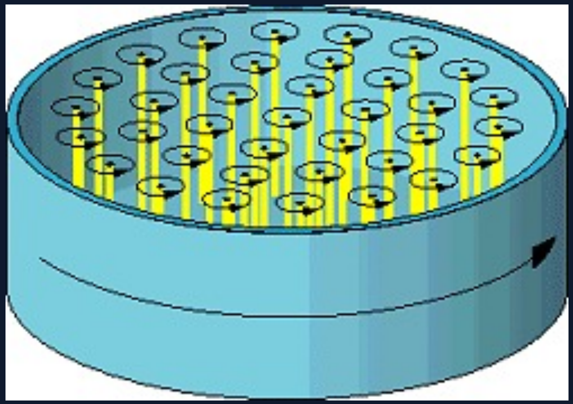
Pairing with correlation of color and flavor reduces symmetry from $SU(3)_C \times SU(3)_F \times U(1)_B$ to $SU(3)_{C+F}$

Vortices threading rotating neutron star



How do neutron vortices interface with quark (CFL) vortices??

M. Alford, GB, K. Fukushima, T. Hatsuda, & M. Tachibana, PR D 99, 036004 (2019).



Try to match circulations

Circulation: $C = \oint_{\mathcal{C}} \vec{v} \cdot d\vec{\ell} = \dfrac{2\pi\hbar n}{\mu}$ v = superfluid velocity p/μ

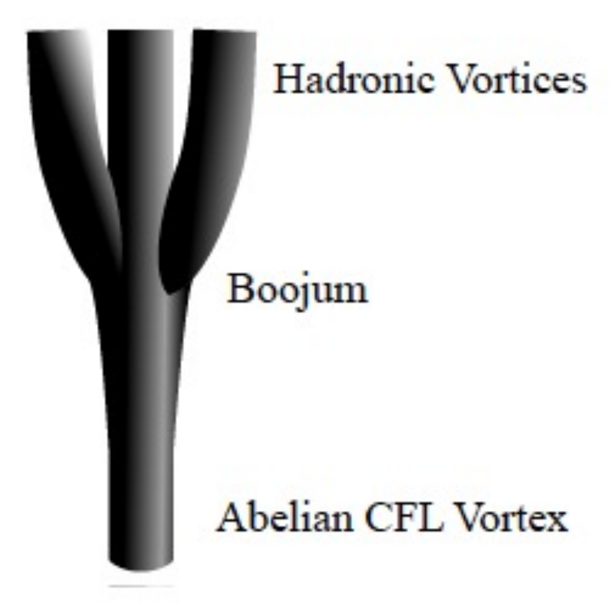
In paired hadronic phase $\mu = 2\mu_n$ (μ_n=neutron chemical potential).

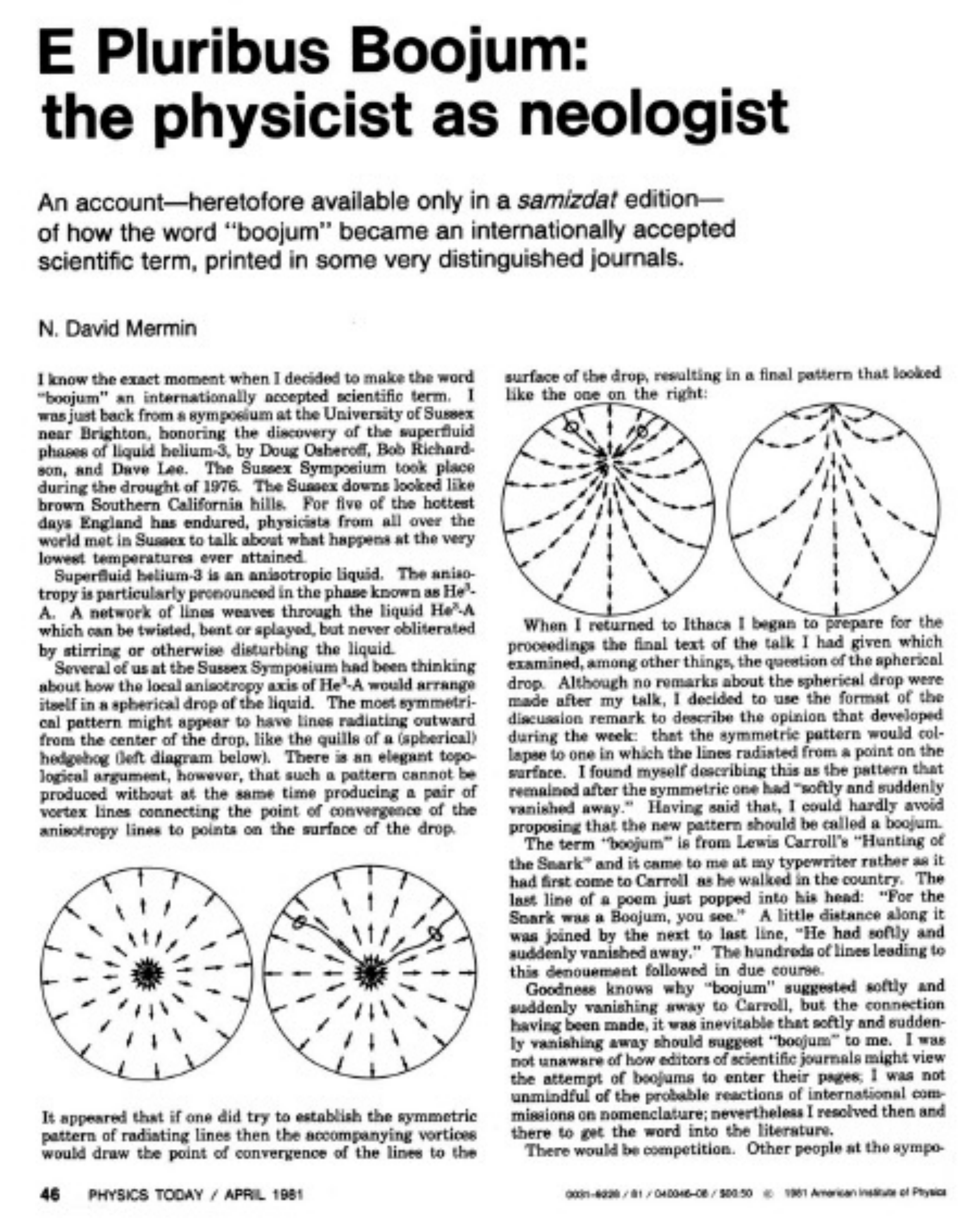
In paired quark phase $\mu = 2\mu_q = 2\mu_n/3$ (μ_q= quark chemical pot.), since nucleon is made of 3 quarks, $\mu_n = 3\mu_q$

=> quark phase superfluid velocity = 3X velocity in hadronic phase.

Continuity in flow states in neutron star would require 3 hadron vortices merging into a single quark vortex.

A boojum!





E Pluribus Boojum: the physicist as neologist

An account—heretofore available only in a *samizdat* edition—of how the word "boojum" became an internationally accepted scientific term, printed in some very distinguished journals.

N. David Mermin

I know the exact moment when I decided to make the word "boojum" an internationally accepted scientific term. I was just back from a symposium at the University of Sussex near Brighton, honoring the discovery of the superfluid phases of liquid helium-3, by Doug Osheroff, Bob Richardson, and Dave Lee. The Sussex Symposium took place during the drought of 1976. The Sussex downs looked like brown Southern California hills. For five of the hottest days England has endured, physicists from all over the world met in Sussex to talk about what happens at the very lowest temperatures ever attained.

Superfluid helium-3 is an anisotropic liquid. The anisotropy is particularly pronounced in the phase known as He^3-A. A network of lines weaves through the liquid He^3-A which can be twisted, bent or splayed, but never obliterated by stirring or otherwise disturbing the liquid.

Several of us at the Sussex Symposium had been thinking about how the local anisotropy axis of He^3-A would arrange itself in a spherical drop of the liquid. The most symmetrical pattern might appear to have lines radiating outward from the center of the drop, like the quills of a (spherical) hedgehog (left diagram below). There is an elegant topological argument, however, that such a pattern cannot be produced without at the same time producing a pair of vortex lines connecting the point of convergence of the anisotropy lines to points on the surface of the drop.

It appeared that if one did try to establish the symmetric pattern of radiating lines then the accompanying vortices would draw the point of convergence of the lines to the surface of the drop, resulting in a final pattern that looked like the one on the right:

When I returned to Ithaca I began to prepare for the proceedings the final text of the talk I had given which examined, among other things, the question of the spherical drop. Although no remarks about the spherical drop were made after my talk, I decided to use the format of the discussion remark to describe the opinion that developed during the week: that the symmetric pattern would collapse to one in which the lines radiated from a point on the surface. I found myself describing this as the pattern that remained after the symmetric one had "softly and suddenly vanished away." Having said that, I could hardly avoid proposing that the new pattern should be called a boojum.

The term "boojum" is from Lewis Carroll's "Hunting of the Snark" and it came to me at my typewriter rather as it had first come to Carroll as he walked in the country. The last line of a poem just popped into his head: "For the Snark was a Boojum, you see." A little distance along it was joined by the next to last line, "He had softly and suddenly vanished away." The hundreds of lines leading to this denouement followed in due course.

Goodness knows why "boojum" suggested softly and suddenly vanishing away to Carroll, but the connection having been made, it was inevitable that softly and suddenly vanishing away should suggest "boojum" to me. I was not unaware of how editors of scientific journals might view the attempt of boojums to enter their pages; I was not unmindful of the probable reactions of international commissions on nomenclature; nevertheless I resolved then and there to get the word into the literature.

There would be competition. Other people at the sympo-

46 PHYSICS TODAY / APRIL 1981

0031-9228 / 81 / 040046-06 / $00.50

David Mermin
Physics Today
April 1981

A boojum tree

Abelian vortex in CFL phase

Order parameter matrix of Abelian CFL vortex.

$$\Phi(r,\phi) = \Delta \cdot f(r)e^{i\varphi}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

Circulation = $2\pi\hbar/2\mu_q = 3(2\pi\hbar/2\mu_n)$

But this vortex is unstable against decay into three color flux tubes with lower kinetic energy *(A. P. Balachandran et al. PR D 73 (2006); E. Nakano et al., PR D 78, 045002 (2008). Phys. Lett. B 672 (2009)):*

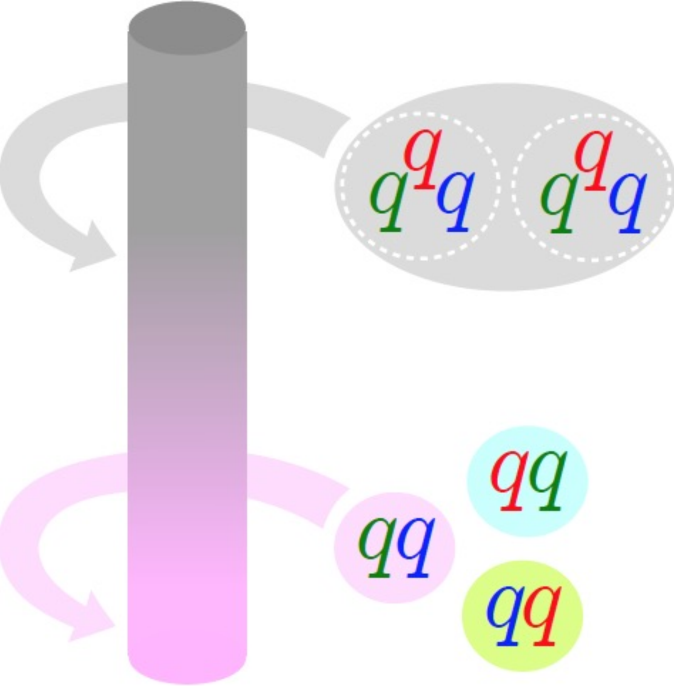
Color flux tube

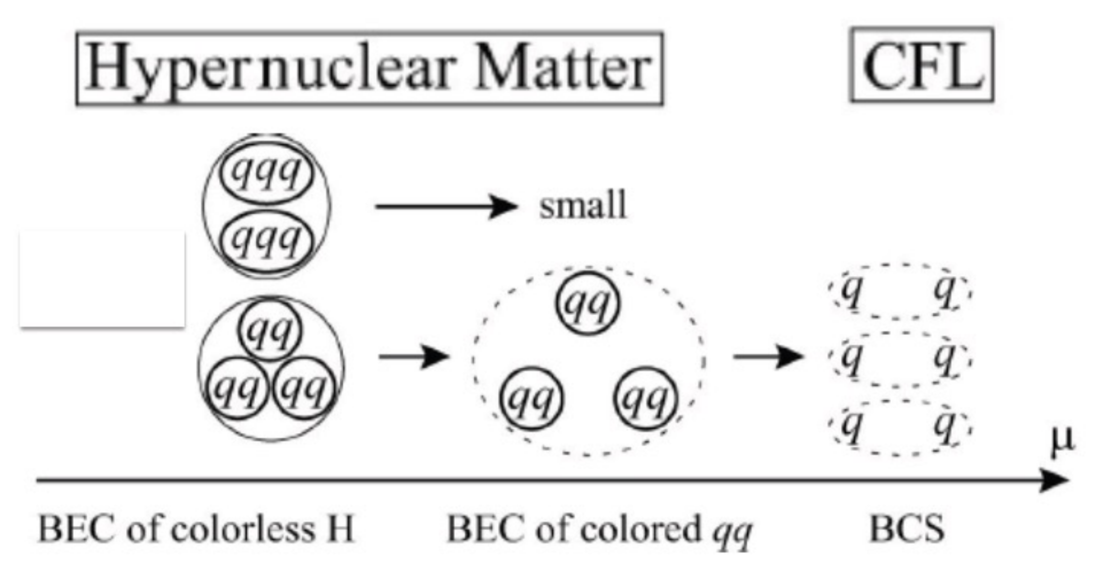
$$\Phi^R_{\alpha i} = \Delta\begin{pmatrix} e^{i\varphi}f(r) & 0 & 0\\ 0 & g(r) & 0\\ 0 & 0 & g(r)\end{pmatrix} = \Delta e^{\frac{i}{3}\varphi}\begin{pmatrix} e^{\frac{2i}{3}\varphi}f(r) & 0 & 0\\ 0 & e^{-\frac{i}{3}\varphi}g(r) & 0\\ 0 & 0 & e^{-\frac{i}{3}\varphi}g(r)\end{pmatrix}$$

"red" flux tube order parameter

Leading phase (1/3) is $U(1)_B$. Phases within => color rotation do not contribute to circulation = $\frac{1}{3}2\pi\hbar/2\mu_q = 2\pi\hbar/2\mu_n$

Single color flux tube has circulation 1/3 that of initial (unstable) Abelian CFL vortex – same as a single original hadronic vortex.





Pairing continuity

K. Fukushima, PRD (2004)

Conclude that three hadronic vortices can turn into three non-Abelian CFL vortices, with no discontinuity in circulation. But: gauge invariance???

Gauge invariant description of flux tubes

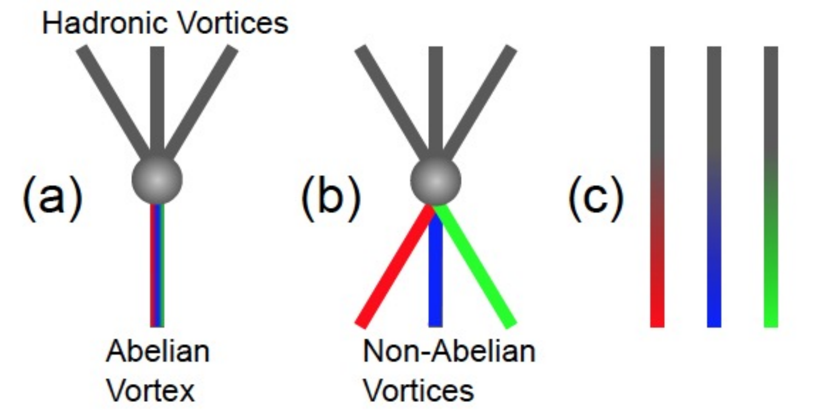
$$\Phi^R_{\alpha i} = \Delta \begin{pmatrix} e^{i\varphi} f(r) & 0 & 0 \\ 0 & g(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix} = \Delta e^{\frac{i}{3}\varphi} \begin{pmatrix} e^{\frac{2i}{3}\varphi} f(r) & 0 & 0 \\ 0 & e^{-\frac{i}{3}\varphi} g(r) & 0 \\ 0 & 0 & e^{-\frac{i}{3}\varphi} g(r) \end{pmatrix}$$

red flux tube order parameter

Then $$\boxed{\Upsilon(\vec{r}) = \frac{1}{6}\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}\Phi_{\alpha i}\Phi_{\beta j}\Phi_{\gamma k}} = e^{i\varphi}\Delta^3 f(r) g^2(r)$$

is gauge invariant order parameter, independent of choice of color of the gauge fixed $\Phi^R_{\alpha i}$. Only one gauge invariant physical object.

Quark-hadron continuity



Can envision continuous evolution of vortices from nuclear (hadronic) phase to quark phase provided order parameter in hadronic phase is anti-symmetric in flavor.

BCS pairs in neutron gas have 6 quarks: ddu + ddu.

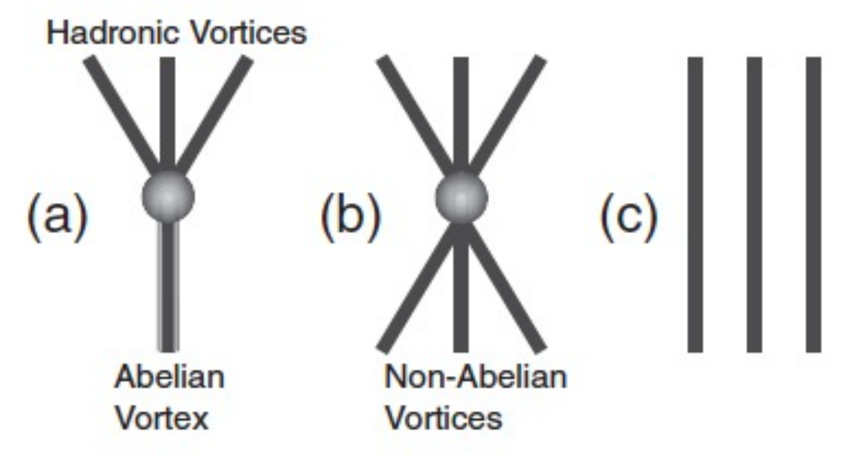
$$\langle nn \rangle \rightarrow \langle ud \rangle \langle ud \rangle \langle dd \rangle$$

Cannot arrange into flavor anti-symmetric quark pairing.
But in SU(3)$_{\text{flavor}}$ invariant hadronic matter with equal mass
n, p, Λ, Σ, and Ξ baryons can have flavor antisymmetric pairings

$$\langle -\sqrt{\frac{1}{8}}[\Lambda\Lambda] + \sqrt{\frac{3}{8}}[\Sigma\Sigma] + \sqrt{\frac{4}{8}}[N\Xi] \rangle$$

Connecting neutron matter to usual CFL quarks requires transition.
Other quark matter pairings, e.g., $^3P_2$, pairing could work.

Quark-hadron continuity



Can envision continuous evolution of vortices from nuclear (hadronic) phase to quark phase provided order parameter in hadronic phase is anti-symmetric in flavor.

BCS pairs in neutron gas have 6 quarks: ddu + ddu.

$$\langle nn\rangle \rightarrow \langle ud\rangle\langle ud\rangle\langle dd\rangle$$

Cannot arrange into flavor anti-symmetric quark pairing.
But in SU(3)$_{\text{flavor}}$ invariant hadronic matter with equal mass
n, p, Λ, Σ, and Ξ baryons can have flavor antisymmetric pairings

$$\langle -\sqrt{\frac{1}{8}}[\Lambda\Lambda]+\sqrt{\frac{3}{8}}[\Sigma\Sigma]+\sqrt{\frac{4}{8}}[N\Xi]\rangle$$

Connecting neutron matter to usual CFL quarks requires transition.
Other quark matter pairings, e.g., $^3P_2$, pairing could work.

Further QCD issues

Stability of core of gauge invariant flux tube against (1D) spontaneous flavor symmetry breaking:

M. Eto, Y. Hirono, M. Nitta, and S. Yasui, PTEP 2014, 012D01 (2014)

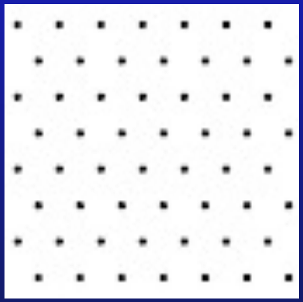
Fate at interface with hadronic matter of color flux in cores of gauge invariant flux tubes? Should not penetrate into hadronic phase

Our understanding of (de)confinement is challenged by vortex continuity problem

Talks of Muneto Nitta and Yui Hayashi on Wednesday

A few open physics issues

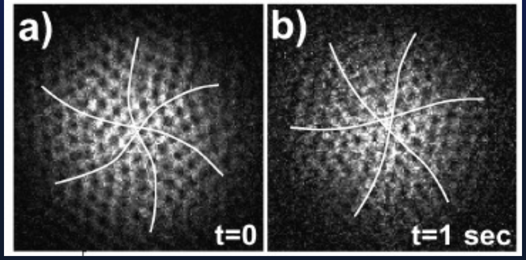
Pairing states for $m_s > m_u$ and m_d? Spatially inhomogeneous Fulde-Ferrel-Larkin-Ovchinnikov states.

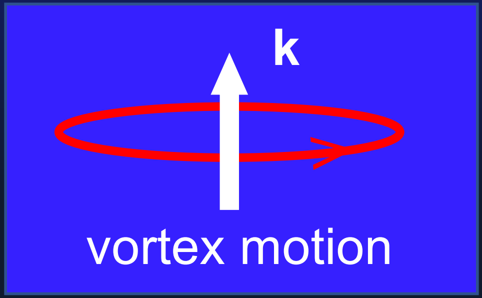


Geometry of vortex arrays
in rotating CFL matter. Triangular?

Modes of quark matter vortex arrays?

Tkachenko modes



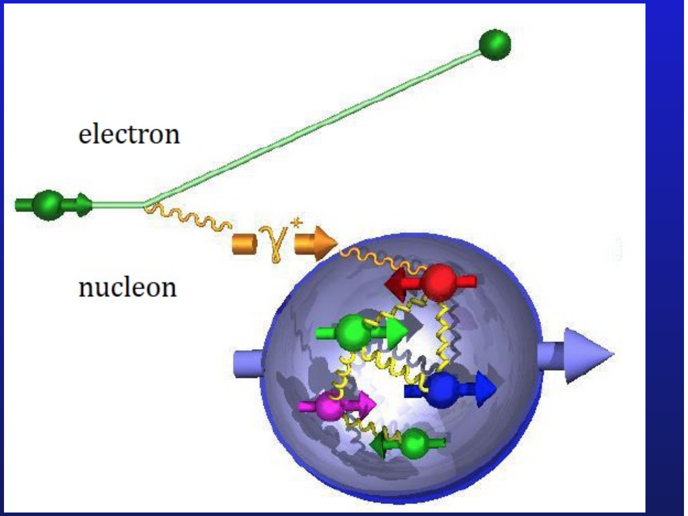


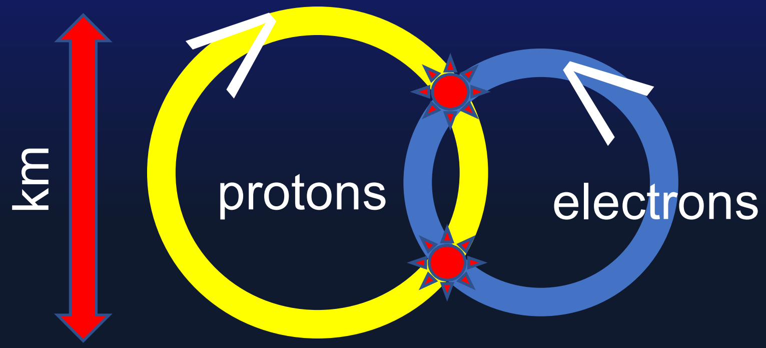
Schweikhard et al., PRL92 040404 (2004)

Effect of vortices in quark matter on glitches?

The Electron-Ion Collider

A very big accelerator (1-2 G$) -- colliding beams of electrons with beams of protons or heavier ions (atomic nuclei). Collide electron beams with beams of protons, and heavier ions. A giant electron microscope for peering at the quarks and gluons deep inside the nucleon, as well as atomic nuclei. QCD machine.





Electron-ion center of mass energy:
$\sqrt{s}$ ~ 20~140 GeV.
High luminosity (event rate) and spin polarized beams!

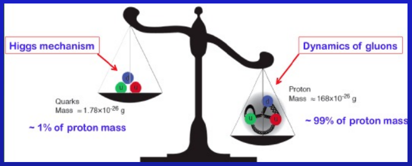
Under construction in RHIC at Brookhaven.

Electron microscope Invented 1931

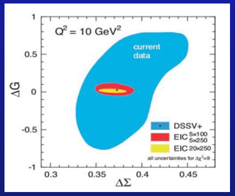
ca. 1940

Basic science questions for an EIC

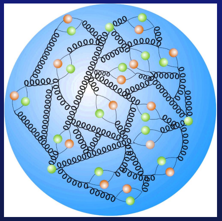
How does the nucleon get its mass?



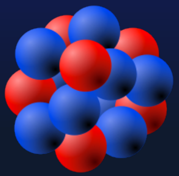
How does the spin of the nucleon arise from its elementary quark and gluon constituents?



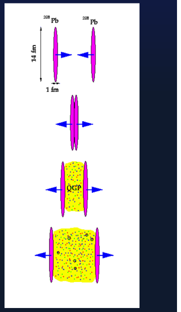
What are the emergent properties of dense systems of gluons?



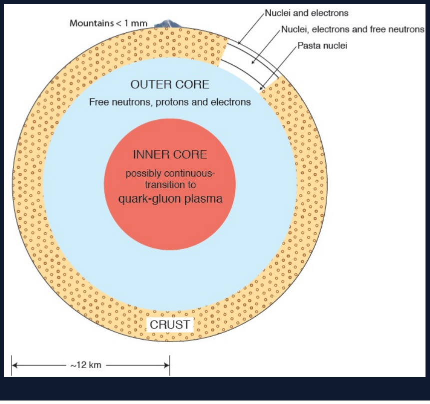
What is the internal (QCD) structure of nuclei?

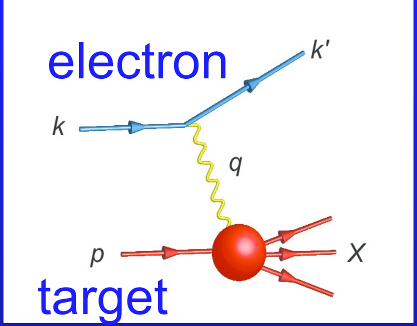


What is the initial state in ultrarelativistic heavy-ion collisions?

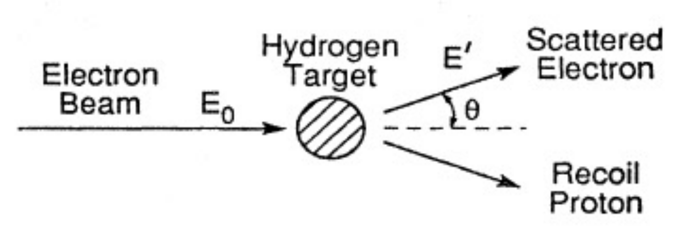


How does dense matter crossover from nucleonic degrees of freedom to quark degrees of freedom at higher density
-- application to neutron stars





Kinematic variables in electron scattering from nucleons and nuclei



Electron scattering angle: θ

Electron energy transfer in lab: $\nu = E_e - E_e'$

3-momentum transfer from electron: $\mathbf{q}$

4-momentum transfer squared: $Q^2 = \mathbf{q}^2 - \nu^2$
Larger Q^2 = higher (transverse) resolution

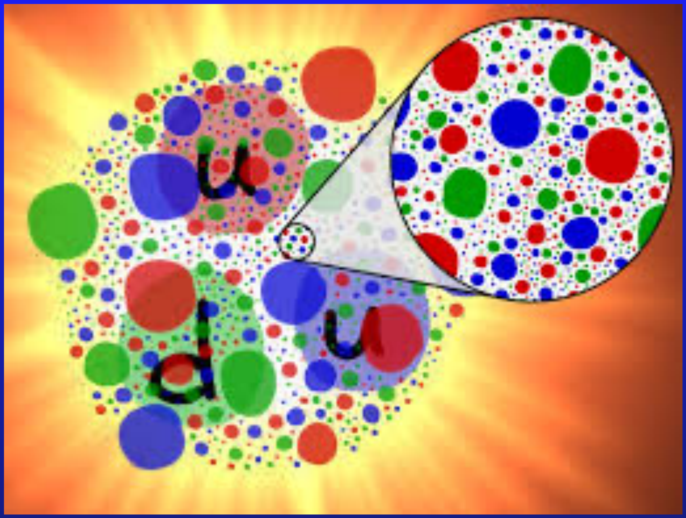
Elastic scattering on target of mass m:
Energy conservation $(m^2 + q^2)^{1/2} = m + \nu$ => $\nu = Q^2/2m$

Bjorken scaling variable: $x = Q^2/2m_{proton}\,\nu$
x ~ inverse shutter speed

The variables Q^2 and x define the landscape of electron scattering

Parton model (1969)

R.P. Feynman and J.D. Bjorken:



Understand electron scattering in terms of *partons* -- quasiparticles

$\nu = E_{el} - E'_{el}$

Given parton carries momentum p (in beam direction) -- a fraction x of the total target proton momentum p_{proton} . Same x

$$\frac{p_{parton}}{p_{proton}} = x = \frac{Q^2}{2m_{proton}\nu}$$

energy conservation in scattering on parton $\nu = \frac{Q^2}{2m_{parton}}$

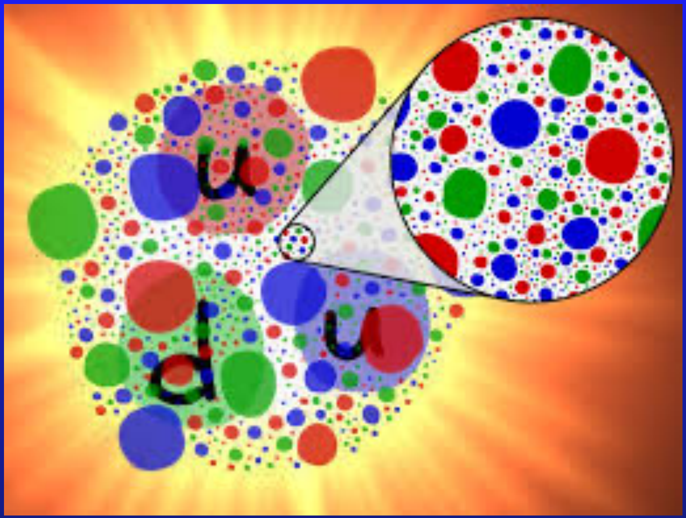
$$m_{parton} = x\, m_{proton}$$

Measure x in "infinite momentum" frame, i.e., proton moving at (nearly) speed of light.

Point partons soon identified with quarks, antiquark, and gluons -- all governed by QCD, with asymptotic freedom.

Parton model (1969)

R.P. Feynman and J.D. Bjorken:

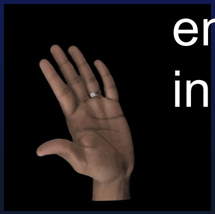


Understand electron scattering in terms of *partons* -- quasiparticles

$\nu = E_{el} - E'_{el}$

Given parton carries momentum p (in beam direction) -- a fraction x of the total target proton momentum p_{proton} . Same x

$$\frac{p_{parton}}{p_{proton}} = x = \frac{Q^2}{2m_{proton}\nu}$$



energy conservation in scattering on parton

$$\nu = \frac{Q^2}{2m_{parton}}$$

$$m_{parton} = x\, m_{proton}$$

Measure x in "infinite momentum" frame, i.e., proton moving at (nearly) speed of light.

Point partons soon identified with quarks, antiquark, and gluons -- all governed by QCD, with asymptotic freedom.

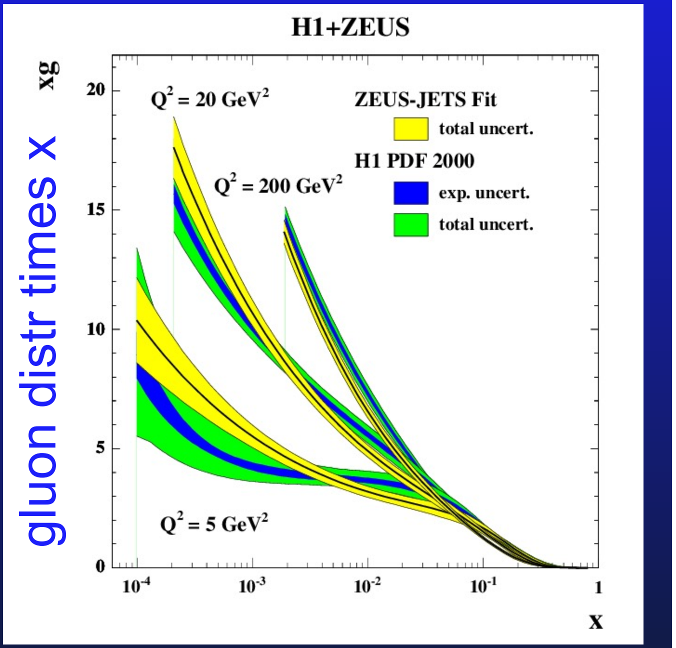
1991-2007 e scattering at HERA

(Hadron-Electron Ring Accelerator, Hamburg)

High-energy collisions of 27.5 GeV electron and positron beams (polarizable) with 920 GeV proton beams (unpolarized).

HERMES (spin) fixed-target experiment.

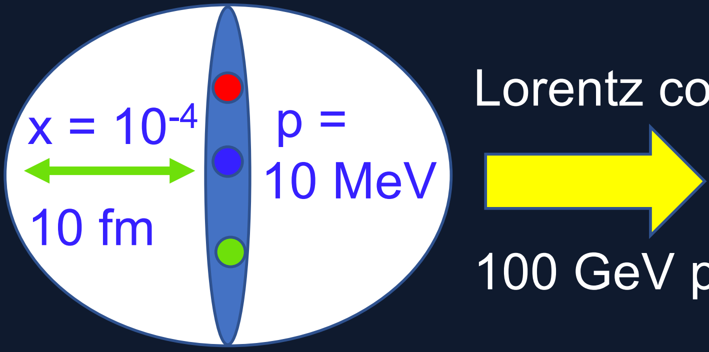
HERA => great abundance of low x gluons within nucleon.



How can electrons probe electrically neutral gluons?

$g \rightarrow q + \bar{q}$ causing an effective electric dipole moment

View proton in frame
in which proton is "slower"
than ∞ momentum.
Heisenberg =>
"wee" partons stick out.



$x = 10^{-4}$

10 fm

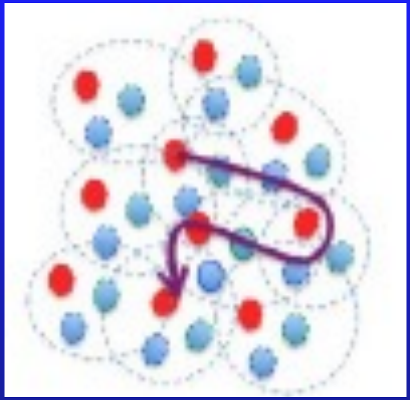
p = 10 MeV

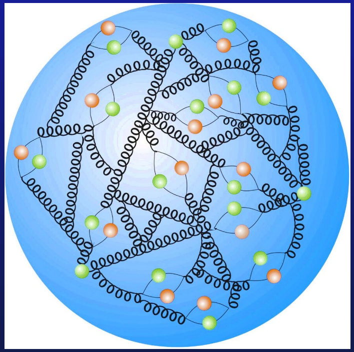
Lorentz contracted

100 GeV proton

Gluon physics

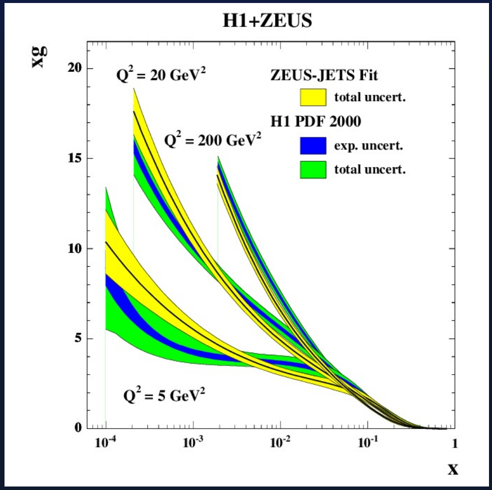
Gluons in nucleons and nuclei (as well as other hadrons) are like dark matter in the universe– unseen but crucial in holding matter together.



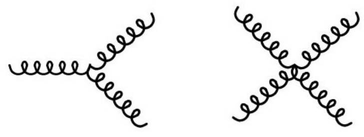

DESY

"The most precise picture of the proton"

Nucleons and nuclei are in fact complex interacting many-body systems -- beyond bags of free quarks and free gluons. Ex., nuclei exhibit composite fermions. Confinement!



HERA => huge numbers of low momentum gluons in the nucleon -- at low x ($<10^{-4}$).
Low momentum sector (wee partons) dominated by strongly interacting gluons!. The gluon field is highly non-linear!

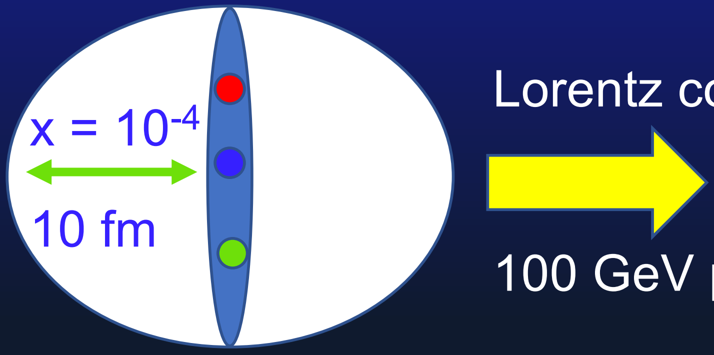


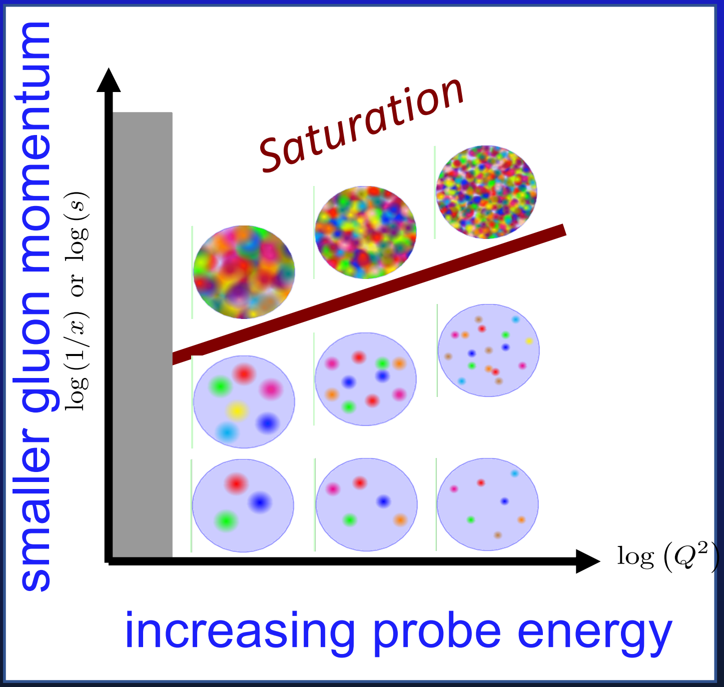
A new many-body system! New emergent phenomena?

Gluon self-interactions become important when there are enough: at small x

Scale of saturated gluonic matter: Q_s

At HERA (318 GeV c.m.) $Q_s \sim 1$ GeV

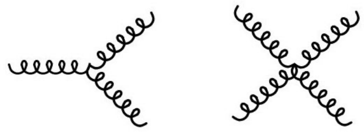




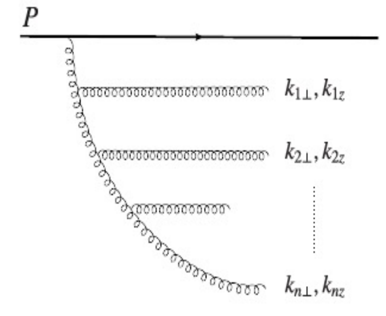
First approximation, dense cloud of gluons forms a Bose condensate – “color glass condensate.”

Excitations of saturated gluonic matter? Topology?

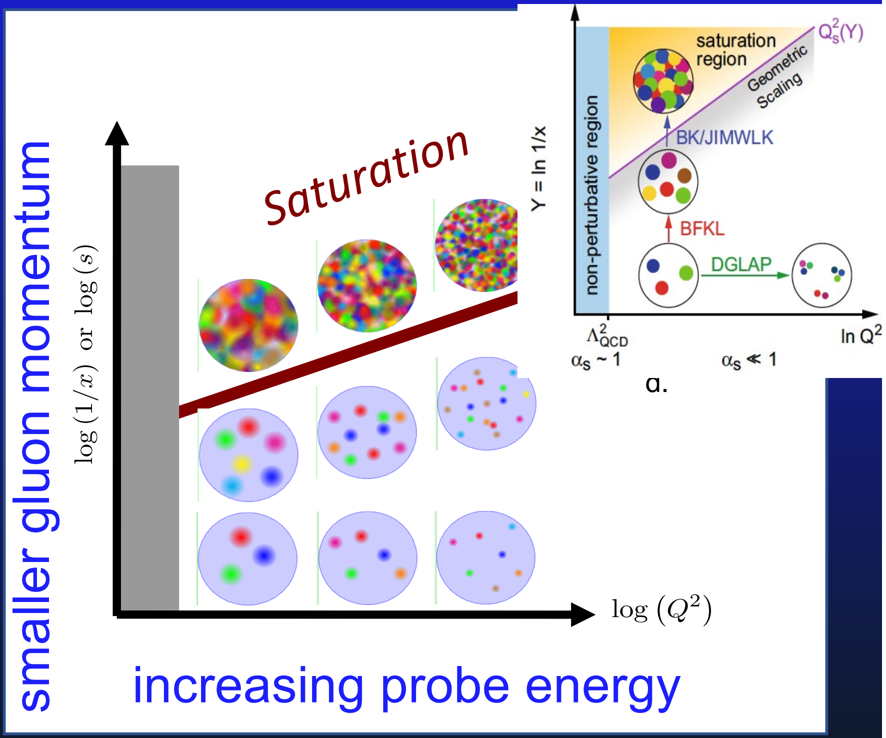
Physics of non-linearity of gluon field



-- Saturated gluonic matter. Onset at gluon saturation scale Q_s: [at HERA (318 GeV cm) $Q_s \sim 1$ GeV]



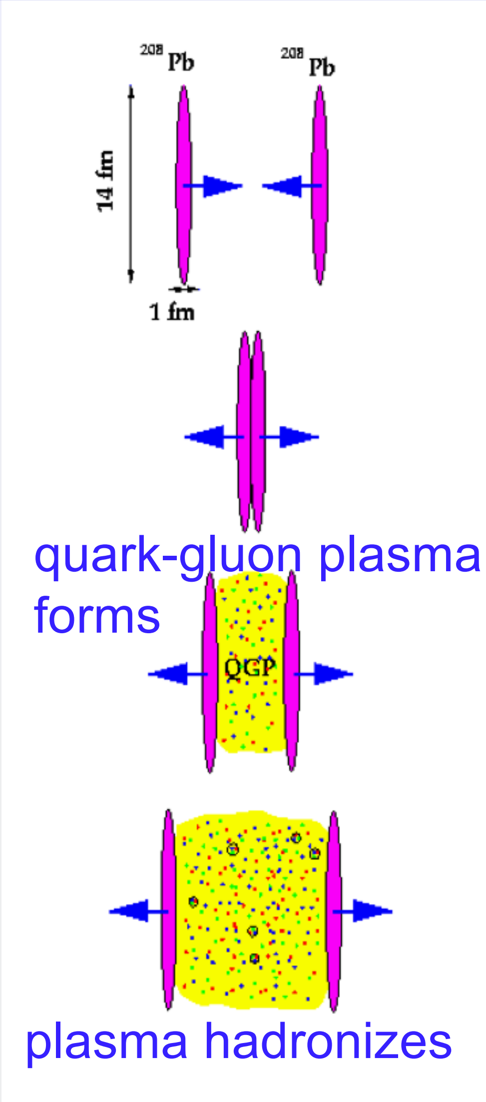
-- metastable state, decays into quark-gluon plasma. New far-from equilibrium quantum transition. Strongly coupled many body phenomenon.



-- Cf. early universe -- formation of topological defects, e.g., handedness asymmetry of produced q q-bar pairs (chiral magnetic effect) related to the structure of the color field in saturated gluonic matter.

-- expect useful interaction between EIC/QCD physics and condensed matter physics.

Connections to heavy ion collisions:

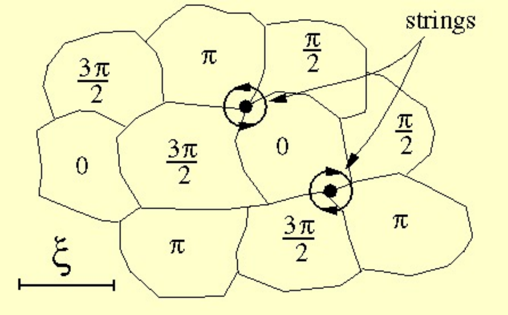


quark-gluon plasma forms

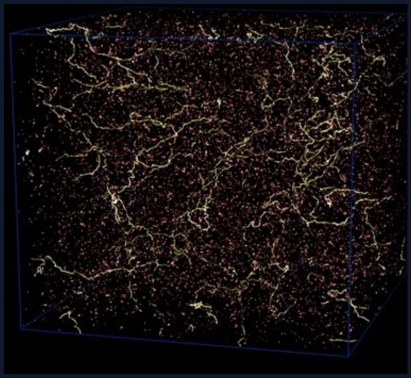
plasma hadronizes

Saturated gluonic matter reachable at a sufficiently energetic EIC. Describes initial state in ultrarelativistic heavy ion collisions. Bose-condensed gluonic matter (color-glass condensate, ...). Condensate is metastable, decaying into quark-gluon plasma.

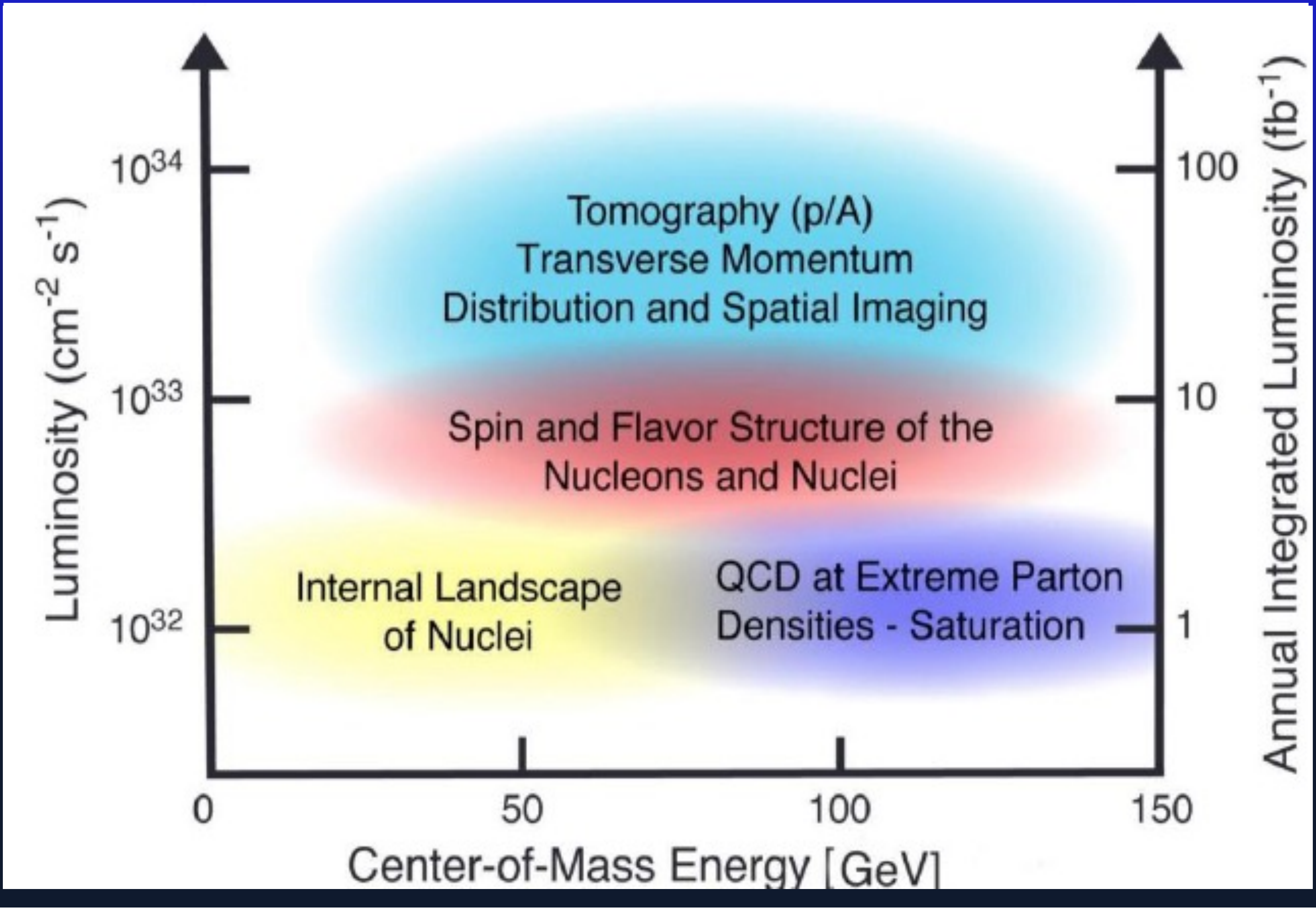
As in early universe, form topological defects, e.g., handedness asymmetry of produced $q\bar{q}$ pairs (chiral magnetic effect) related to the structure of the color field in saturated gluonic matter.



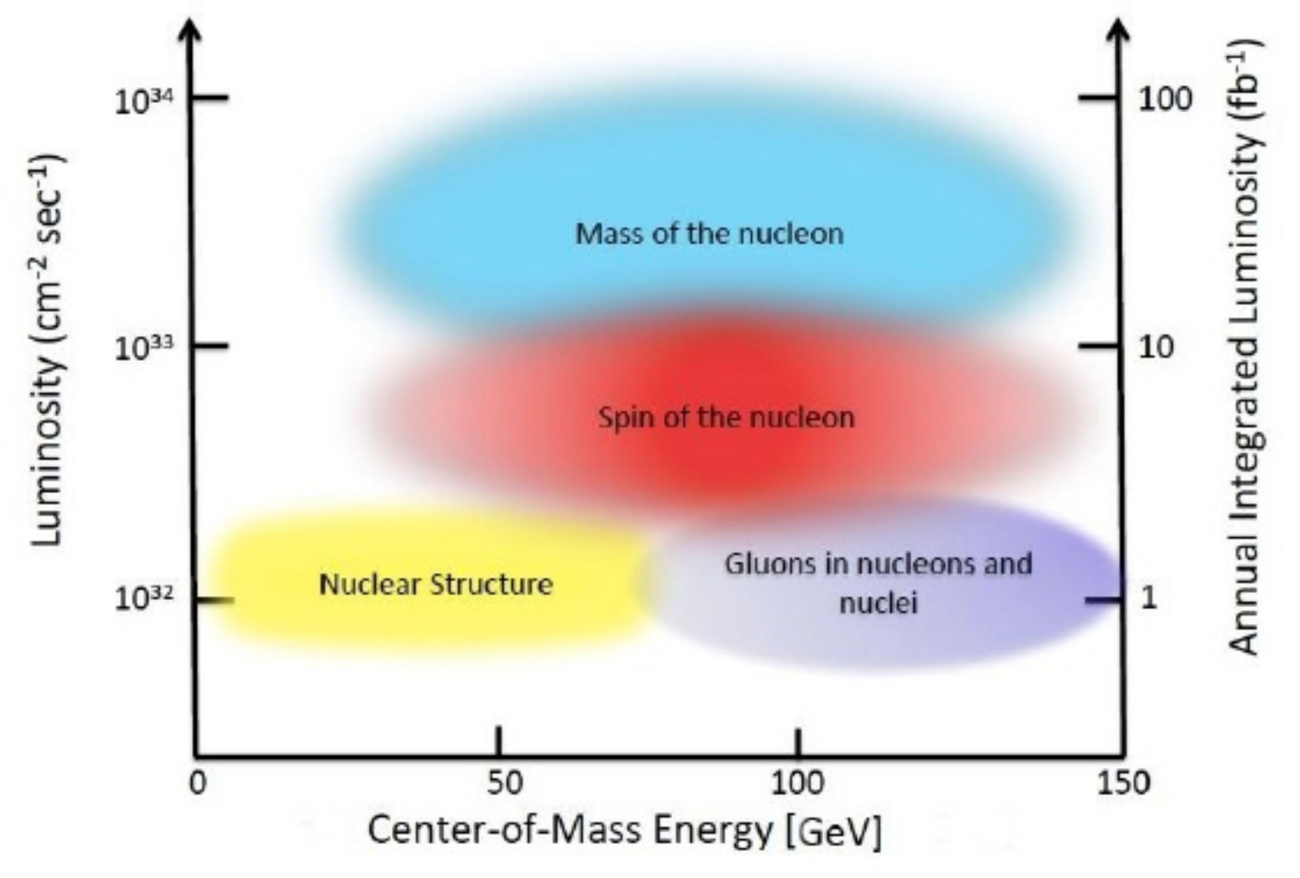
cosmic strings

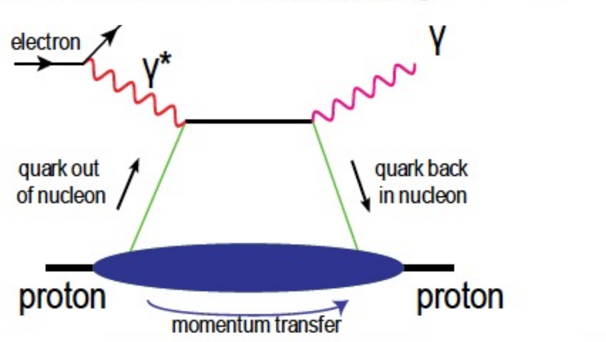


Machine requirements:
in (ep) cm. energy - luminosity landscape

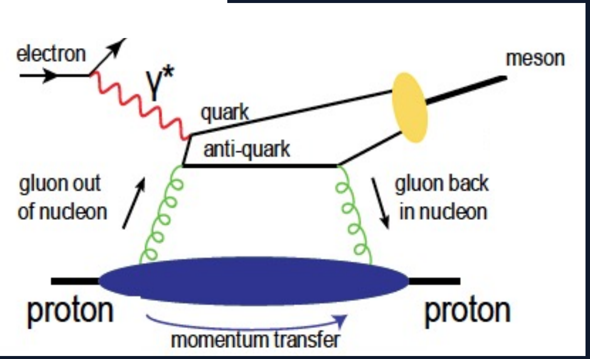


Basic experiments in c.m. energy - luminosity landscape





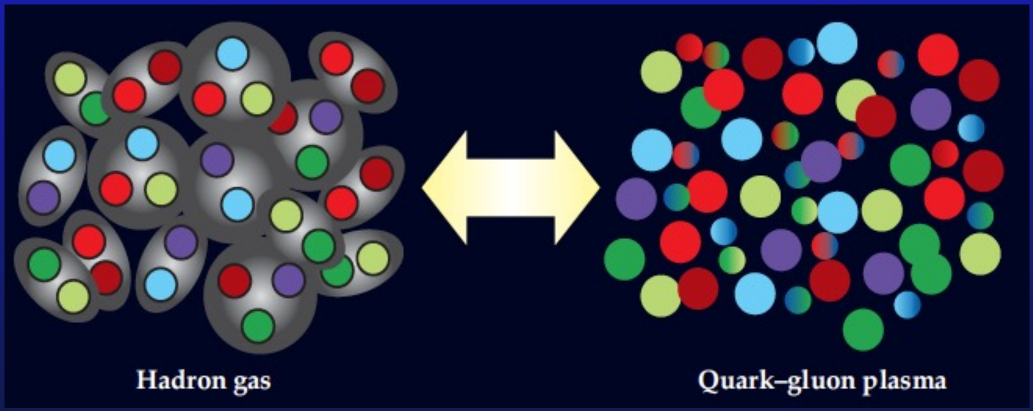
Deeply virtual Compton scattering

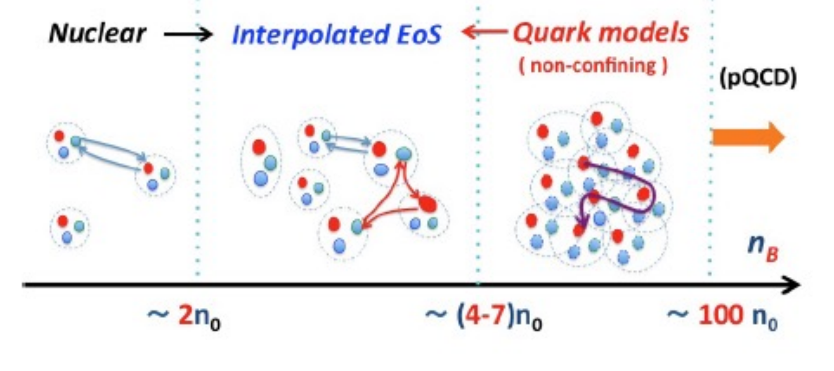


Deeply virtual meson production

Dense matter and neutron stars:

Study transition from cold nuclear matter to quark matter – vital for neutron stars. What is energy density vs. baryon density?





Expect "smooth" transition from nucleons to quarks

Gluon (and quark) distributions in nuclei at finer and finer scales should shed light on transition from nucleonic to quark degrees of freedom as density of matter increases.

Can mapping of energy-momentum tensor (stress-energy tensor) in eA collisions reveal pressure vs. baryon density in dense matter?

Deducing stress tensor $T_{\mu\nu}$ of QCD matter from deeply virtual Compton scattering

Measured form factors in DVCS contain information on $T_{\mu\nu}$ in nucleon

$\langle T_{00} \rangle = \rho$ = energy density
and in stationary spherically symmetric system

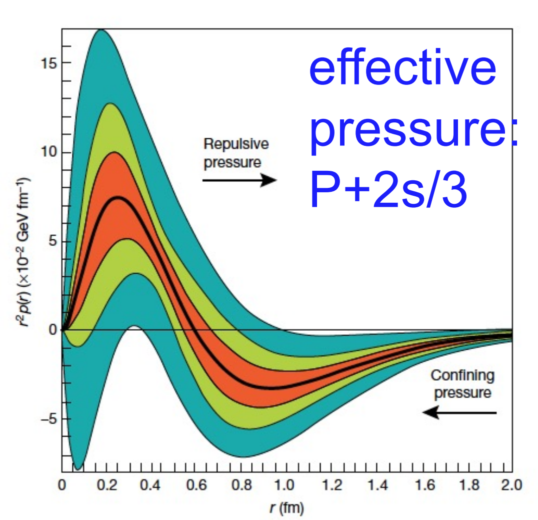
$$\boxed{\langle T_{ij} \rangle = (\hat{r}_i \hat{r}_j - \frac{1}{3}\delta_{ij})s(r) + \delta_{ij}P(r)}$$

In equilibrium:
$\partial_r(P + 2s/3) + 2s/r = 0$

P = pressure
s = transverse stress, ~ surface tension in 3D.

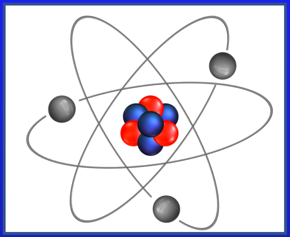
Theory: M.V. Polyakov & P. Schweitzer,
Int.J.Mod.Phys. A33, 1830025 (2018),
C. Lorcé, H. Moutarde, & A. P. Trawinski,
arXiv:1810.09837. Eur. Phys. J. C79 89 (2019).

Expt: V. D. Burkert, L. Elouadrhiri, & F. X. Girod,
Nature 557, 396 (2018).



effective pressure: P+2s/3

In an atom



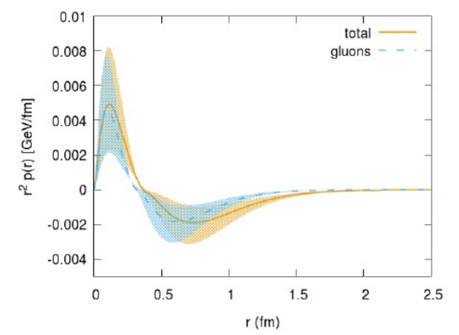
Pressure: $$P(r) = \frac{1}{3}\sum_i \langle T_{ii}\rangle = P_e + \frac{\langle \vec{E}^{\,2}\rangle}{24\pi}$$

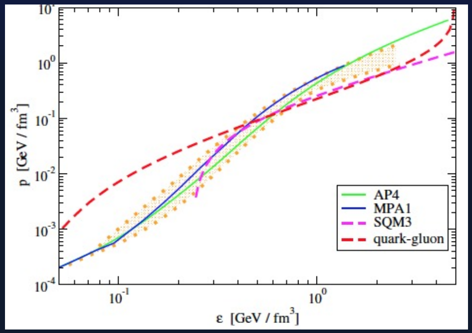
Electron pressure: $$P_e = \frac{1}{3m}\left(\langle \nabla\psi^\dagger \cdot \nabla\psi\rangle - \frac{1}{4m}\nabla^2\langle\psi^\dagger\psi\rangle\right)$$

Transverse stress $$s(r) = \frac{1}{2m}\left[3\langle\nabla_r\psi^\dagger\cdot\nabla_r\psi\rangle - \langle\nabla\psi^\dagger\cdot\nabla\psi\rangle\right] - \frac{\langle E_r^2\rangle}{4\pi}$$

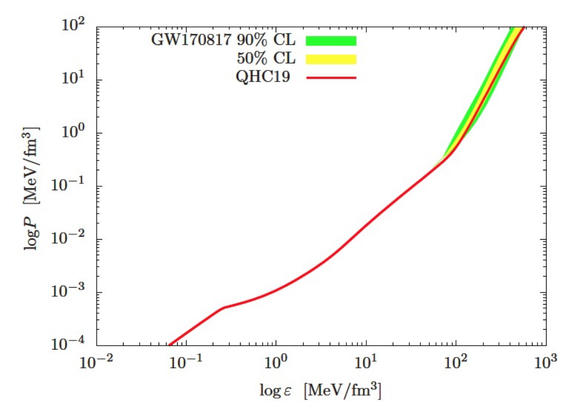
Thomas-Fermi atom: $$n_e = \frac{p_f(r)^3}{3\pi^2},\quad P_e(r) = \frac{p_f^2 n_e}{5m_e},\quad s(r) = -\langle E_r^2\rangle/4\pi$$

Can one go from nucleon to dense matter in neutron stars?





S. Liuti, A. Rajan, & K. Yagi, arXiv:1812.01479v2



GB et al. QHC19

Possibly out to a few fm., but certainly not where transverse stresses (or the “pion” cloud) become important.

In QCD

Lagrangian $$L_{QCD} = \bar{q}(\gamma_\mu D^\mu - m)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}$$

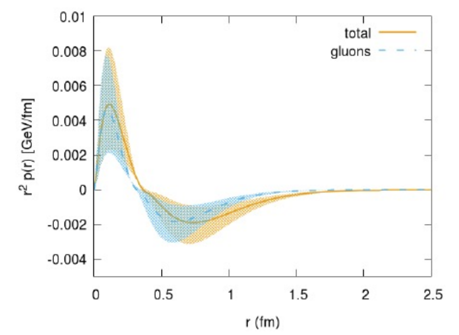
implies stress tensor $$T_{\mu\nu} = \bar{q}(\gamma_\mu D_\nu + \gamma_\nu D_\mu)q - \frac{1}{2}G_{\mu\beta}G_\nu^{\ \beta} - g_{\mu\nu}L$$

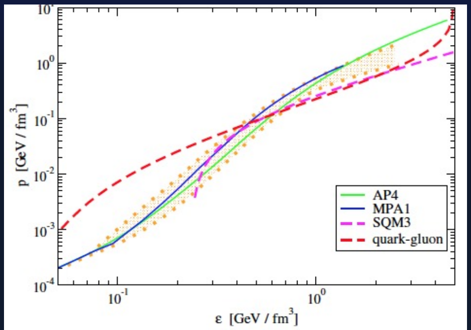
Transverse stress and pressure

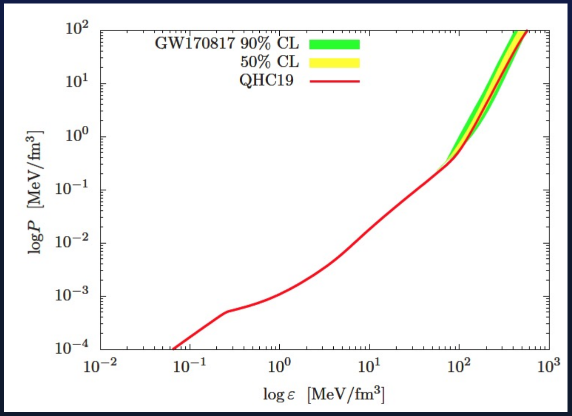
$$s(r) = 3\langle q(\hat{r}\cdot\vec{\gamma})D_r q\rangle - \frac{3}{4}\langle G_{r\beta}G_r^\beta\rangle$$

$$P = \frac{1}{3}\sum_i \langle T_{ij}\rangle = \frac{2}{3}\langle \bar{q}(\vec{\gamma}\cdot\vec{D})q\rangle - \frac{1}{2}\langle \vec{G}_\beta\cdot\vec{G}^\beta\rangle + \langle L\rangle$$

Can one go from nucleon to dense matter in neutron stars?







S. Liuti, A. Rajan, & K. Yagi, arXiv:1812.01479v2

GB et al. QHC19

Possibly out to a few fm., but certainly not where transverse stresses (or the “pion” cloud) become important.

どうもありがとう