Inference of the neutron star matter equation of state

Condensed Matter Physics of QCD

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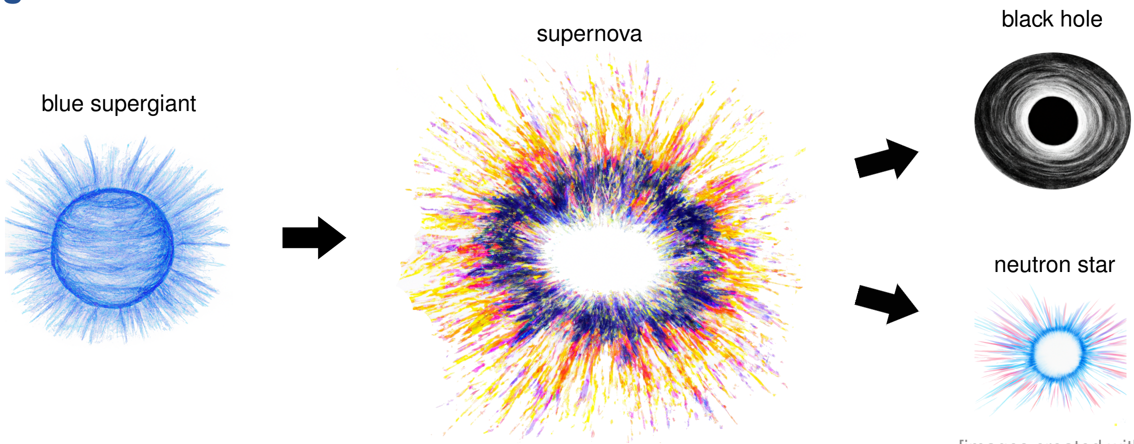
Neutron stars



[images created with DALL-E, openAI]

- Masses $M \sim 1 - 2\,M_{\odot}$, radii $R \sim 11 - 13\,\mathrm{km}$

  → **High baryon densities** in core beyond terrestrial experiments

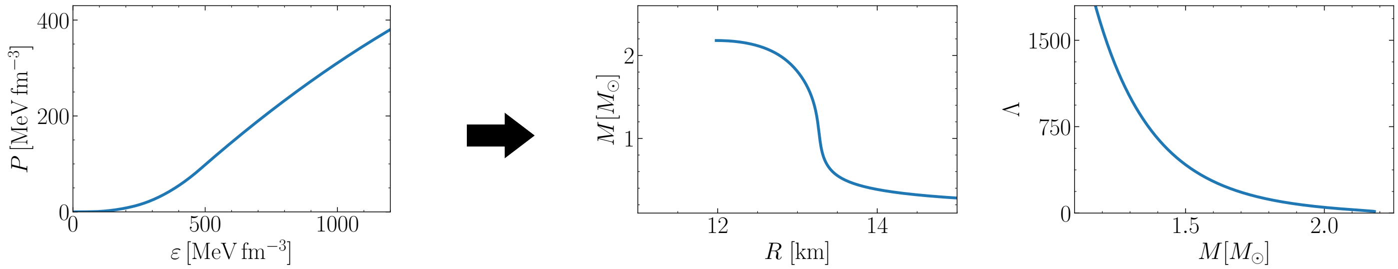
- Recent substantial extension of observational data base

  → **Phase transition** in dense neutron star matter?

Equation of state

- Internal structure described by Tolman-Oppenheimer-Volkoff (TOV) equations [Tolman, Phys. Rev. 55 (1939)] [Oppenheimer and Volkoff, Phys. Rev. 55 (1939)]
- Solved given **equation of state** (EoS) $P(\varepsilon)$ and central energy density $\varepsilon(r=0)=\varepsilon_c$

  → Solution for different ε_c yields (M, R)-relation
- Each EoS has maximum density $\varepsilon_{c,\mathrm{max}}$ corresponding to **maximum supported mass** M_{max}
- Simultaneously solve for **tidal deformability** Λ [Flanagan and Hinderer, Phys. Rev. D 77 (2008)]



Speed of sound

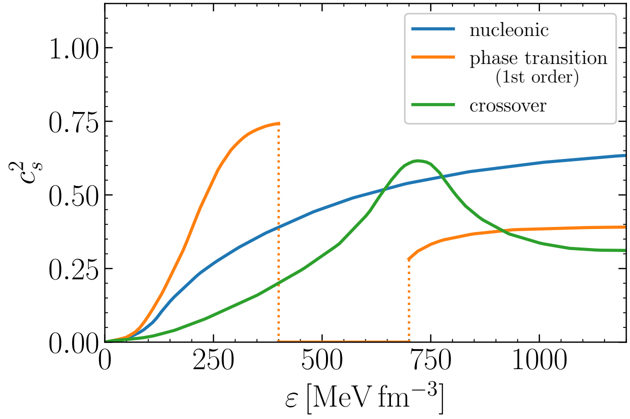
- Determine EoS from **speed of sound**

$$c_s^2(\varepsilon) = \frac{\partial P(\varepsilon)}{\partial \varepsilon}$$

- Causality & thermodynamic stability: $0 \leq c_s \leq 1$

- Characteristic signature of phase structure:
  - **Nucleonic**: monotonically rising sound speed
  - **First-order phase transition**: coexistence interval with zero sound speed $c_s^2 \sim 0$
  - **Crossover**: peaked behaviour

[McLerran and Reddy, Phys. Rev. Lett. 122 (2019)]



- Introduce **general parametrization** $c_s^2(\varepsilon, \theta)$ to model diverse scenarios

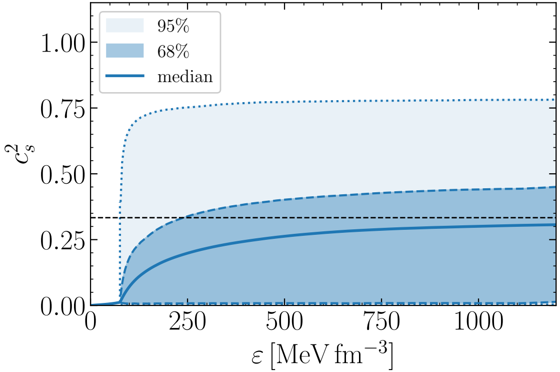
[Annala *et al.*, Nature Phys. 16, 907 (2020)]

Bayesian inference

- Constrain parameters of $c_s^2(\varepsilon, \theta)$ via **Bayesian inference** based on data $\mathscr{D}$

$$p(\theta|\mathscr{D}) \propto p(\mathscr{D}|\theta)\, p(\theta)$$

- Compute posterior probability $p(\theta|\mathscr{D})$ for parameters θ:
  - Compute likelihood $p(\mathscr{D}|\theta)$ for each data $\mathscr{D}$
  - Choose prior for parameters $p(\theta)$



[LB, Weise and Kaiser, Phys. Rev. D 108 (2023)]

- Quantify evidence for hypothesis H_0 vs. H_1 with **Bayes factors**

$$\mathscr{B}^{H_1}_{H_0} = \frac{p(\mathscr{D}|H_1)}{p(\mathscr{D}|H_0)}$$

→ Compare to classification scheme for statistical conclusions

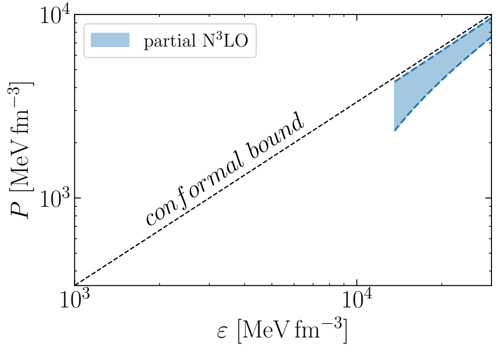
[Lee and Wagenmakers, *Bayesian Cognitive Modeling* (Cambridge University Press, 2014)]
[Jeffreys, *Theory of Probability* (Oxford University Press, 1961)]

Theory constraints

- **Perturbative QCD** calculations provide asymptotic boundary condition at $n \gtrsim 40\, n_0$

→ Interpolate EoS at smaller densities with $0 \leq c_s \leq 1$

[Komoltsev and Kurkela, Phys. Rev. Lett. 128 (2022)]

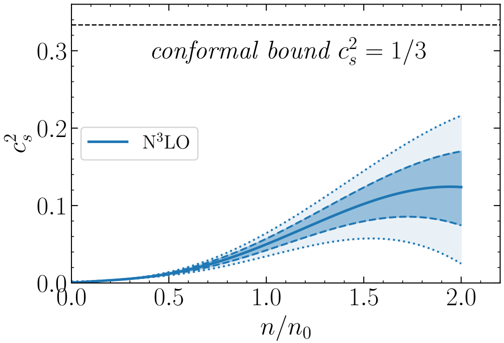


[Gorda *et al.*, Phys. Rev. Lett. 127 (2021)]

- **Chiral effective field theory** provides constraint at low densities

→ Employ up to $n \leq 1.3\, n_0$

[Essick *et al.*, Phys. Rev. C 102 (2020)]



[Drischler, Han and Reddy, Phys. Rev. C 105 (2022)]

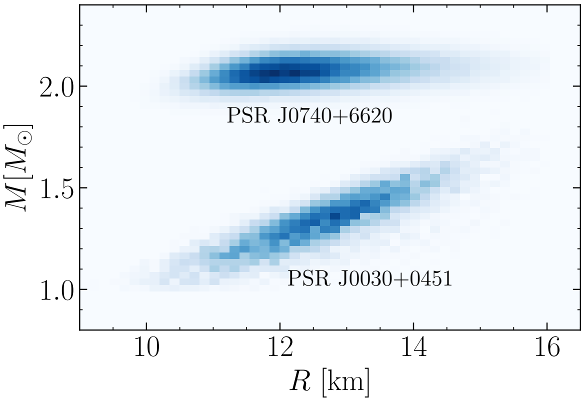
- Intermediate density regime unconstrained, use astrophysical data in addition

Astrophysical data

- **Shapiro time delay:** in binary systems gravitational field of companion changes pulsar signal

  → Extract neutron star masses with high precision:
  [Antoniadis *et al.*, Science 340 (2013)] [Fonseca *et al.*, Astrophys. J. Lett. 915 (2021)]

PSR J0348+0432	$M = 2.01 \pm 0.04\, M_\odot$
PSR J0740+6620	$M = 2.08 \pm 0.07\, M_\odot$



- **NICER:** hot spots on magnetic polar caps of neutron stars

  → Thermal X-ray emission modulated by gravitational field

- Infer **mass and radius** :

PSR J0030+0451	$R = 12.71^{+1.14}_{-1.19}$ km	$M = 1.34^{+0.15}_{-0.16}\, M_\odot$	[Riley *et al.*, Astrophys. J. Lett. 887 (2019)]
PSR J0740+6620	$R = 12.39^{+1.30}_{-0.98}$ km	$M = 2.072^{+0.067}_{-0.066}\, M_\odot$	[Riley *et al.*, Astrophys. J. Lett. 918 (2021)]

Astrophysical data

- Binary neutron star mergers produce **gravitational waves**
- Waveform depends on M_2/M_1 and combination of **tidal deformabilities**

$$\bar{\Lambda} = \frac{16}{13}\frac{(M_1+12M_2)M_1^4\Lambda_1+(M_2+12M_1)M_2^4\Lambda_2}{(M_1+M_2)^5}$$

GW170817 $\bar{\Lambda} = 320^{+420}_{-230}$ [Abbott *et al.* (LIGO and Virgo Collaborations), Phys. Rev. X 9 (2019)]

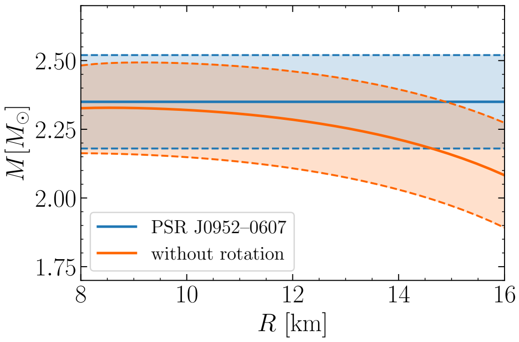
GW190425 $\bar{\Lambda} \leq 600$ [Abbott *et al.* (LIGO and Virgo Collaborations), Astrophys. J. Lett. 892 (2020)]

- **Black widow pulsars** accrete most of mass from companion
- PSR J0952-0607 **heaviest neutron star** observed so far

$$M = 2.35 \pm 0.17\, M_\odot$$ [Romani *et al.*, ApJL 934 (2022)]

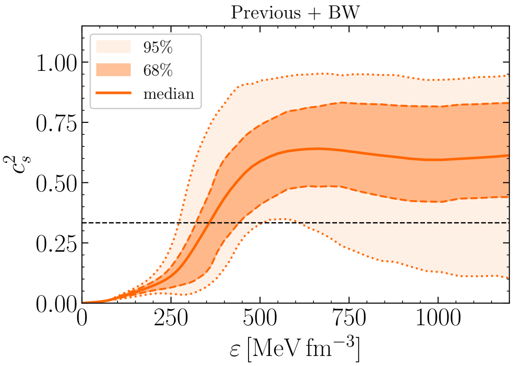
- Second fastest known pulsar $T = 1.41\,\mathrm{ms}$, rotation correction via empirical formula

[Konstantinou and Morsink, Astrophys. J. 934, 139 (2022)]

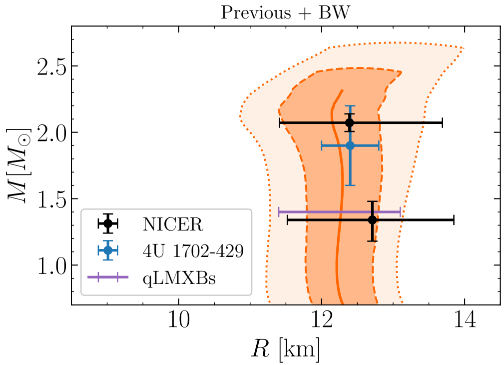


Posterior results

- **Steep increase** of speed of sound around $\varepsilon \sim 250 - 600\,\mathrm{MeV\,fm^{-3}}$
  [LB, Weise and Kaiser, Phys. Rev. D 108 (2023)]
- Conformal bound $c_s^2 \leq 1/3$ exceeded inside neutron stars
  [Altiparmak, Ecker and Rezzolla, Astrophys. J. Lett. 939 (2022)]
  [Legred, Chatziioannou, Essick, Han and Landry, Phys. Rev. D 104 (2021)]
- Slight **tension** between ChEFT at $n \simeq 2\,n_0$ and astro data
  [Essick *et al.*, Phys. Rev. C 102 (2020)]



- Median with **almost constant radius** $R \sim 12.3\,\mathrm{km}$
- Good agreement with data **not included** in Bayesian analysis:
  - Thermonuclear burster 4U 1702-429
    [Nättilä *et al.*, Astron. & Astrophys. 608 (2017)]
  - $R(M = 1.4\,M_\odot)$ from quiescent low mass X-ray binaries (qLMXBs)
    [Steiner *et al.*, MNRAS 476 (2018)]



Maximum coexistence interval

- Significantly increased pressure compared to previous EoS
- Maxwell construction of first-order phase transition: constant pressure in **phase coexistence region**

  → Width Δn measure of phase transition 'strength'
- **Maximum possible interval** within posterior credible band

$$\left(\frac{\Delta n}{n}\right)_{\max} \leq 0.2 \qquad \text{at 68\% level}$$

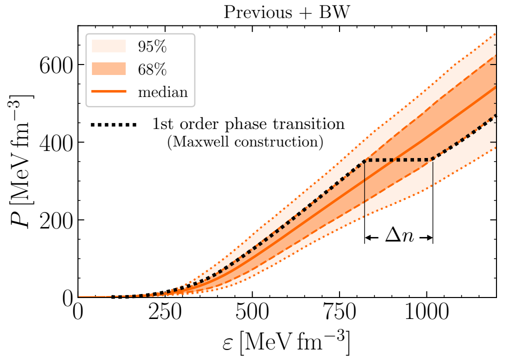
[LB, Weise and Kaiser, Phys. Rev. D 108 (2023)]

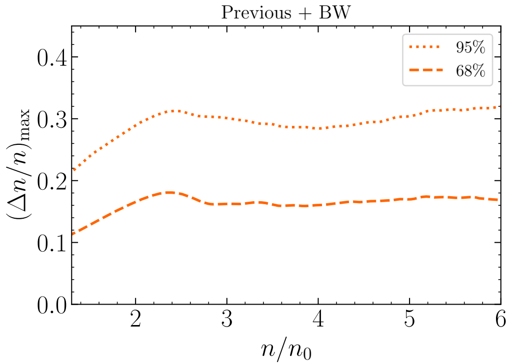
- Compare to 'strong' nuclear liquid-gas phase transition

$$\frac{\Delta n}{n} > 1$$

[Fiorilla, Kaiser and Weise, Nucl. Phys. A 880 (2012)]

→ Only **weak first-order phase transitions** possible





Evidence against small sound speeds

- Quantify **evidence of small sound speeds** inside neutron star cores with Bayes factor

$$\mathscr{B}^{c^2_{s,\mathrm{min}}>0.1}_{c^2_{s,\mathrm{min}}\leq 0.1}$$

→ $c^2_{s,\mathrm{min}} \leq 0.1$ prerequisite for first-order phase transition

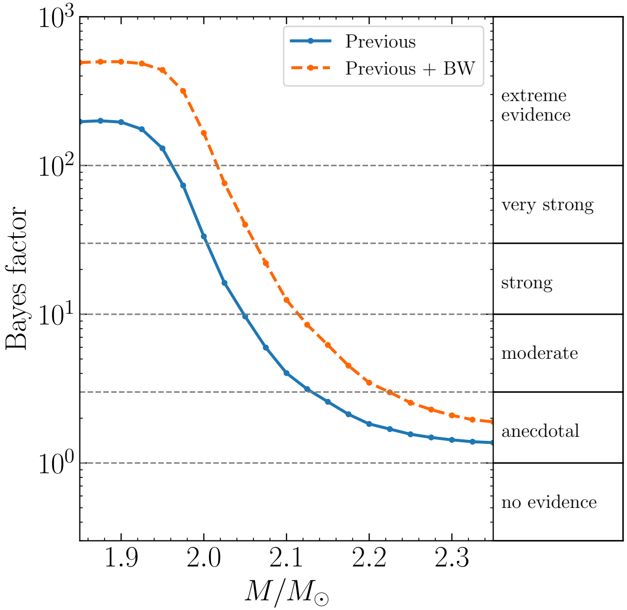
- Previous analyses: $c_s^2 > 0.1$ in neutron stars with $M \leq 2\,M_\odot$

[Ecker and Rezzolla, Astrophys. J. Lett. 939 (2022)]
[Annala *et al.*, Nat. Commun. 14 (2023)]

- **Heavy mass measurement** increases Bayes factor

- Strong evidence against $c^2_{s,\mathrm{min}} \leq 0.1$ in cores of neutron stars with $M \leq 2.1\,M_\odot$ [LB, Weise and Kaiser, Phys. Rev. D 108 (2023)]

→ **Strong first-order phase transitions unlikely** based on empirical data



Low-energy nucleon structure

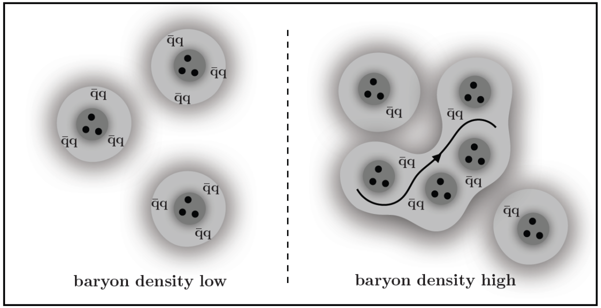
- **Central densities** in neutron stars (68%):

$$n_c(1.4\,M_\odot) = (2.6 \pm 0.4)\,n_0 \qquad n_c(2.3\,M_\odot) = (3.8 \pm 0.8)\,n_0$$

→ Average distance between baryons $d > 1\,\mathrm{fm}$

- **Two scales** in nucleons:
  - Compact hard core contains valence quarks $R_{\mathrm{core}} \sim 1/2\,\mathrm{fm}$
  - Surrounding soft quark-antiquark cloud $R_{\mathrm{cloud}} \sim 1\,\mathrm{fm}$

[Fukushima, Kojo and Weise, Phys. Rev. D 102 (2020)]



[LB and Weise, Symmetry 16 (2024)]

- At $n > 2\,n_0$ percolation of quark-antiquark pairs

→ **Cores begin to touch** and overlap at $n > 5\,n_0$

Chiral symmetry restoration

- **Chiral nucleon-meson model:** nucleons interacting via exchange of effective mesons

  [Floerchinger and Wetterich, Nucl. Phys. A 890–891 (2012)]

- *Mean-field (MF)*: **first-order order phase transition** to chirally restored phase

- *Extended mean-field (EMF):* fermionic vacuum **fluctuations stabilize order parameter**

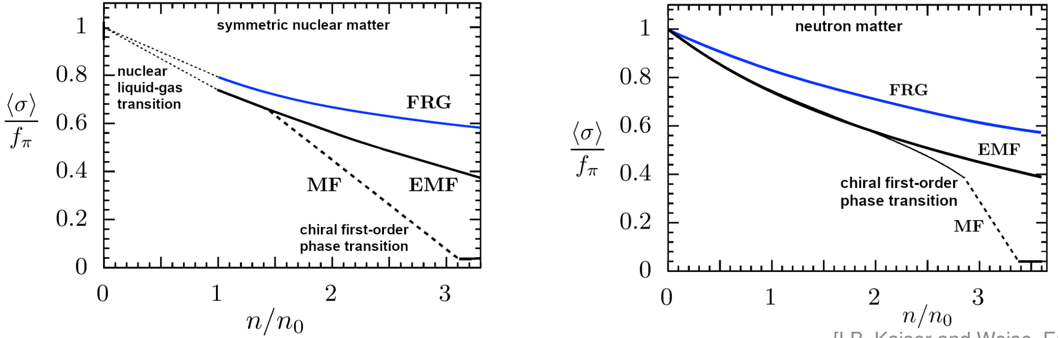
  [Skokov et al., Phys. Rev. D 82 (2010)]

- *Functional renormalization group (FRG):* further stabilization through additional pion and nucleon loops

  [Drews and Weise, Prog. Part. Nucl. Phys. 93 (2017)]

  → Comparable impact of fluctuations in alternative chiral models

  [Gupta and Tiwari, Phys. Rev. D 85 (2012)]
  [Zacci and Schaffner-Bielich, Phys. Rev. D 97 (2018)]



[LB, Kaiser and Weise, Eur. Phys. J. A 57 (2021)]

Neural likelihood estimation

- qLMXBs: **X-ray spectra** s in telescopes depend on (M,R) and nuisance parameters $\nu = (N_H, d, \log T_{eff})$
- Likelihood $p(s|\theta,\nu)$ analytically intractable

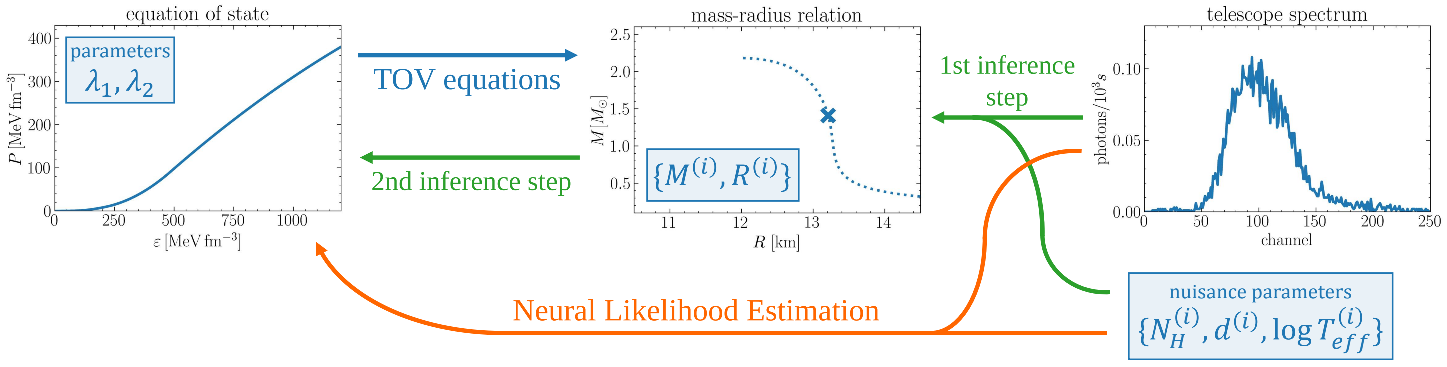
  → **Two steps:** infer neutron star mass and radius and use (M,R) posteriors as likelihood

  [Riley, Raaijmakers and Watts, MNRAS 478 (2018)]

- **Neural likelihood estimation:** *train neural network to approximate likelihood based on simulations*

$$q_\Phi(s|\theta,\nu) \approx p(s|\theta,\nu)$$

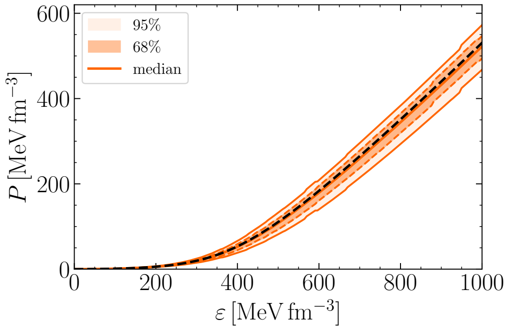
[Papamakarios, Sterratt and Murray, AISTAT (2019)]

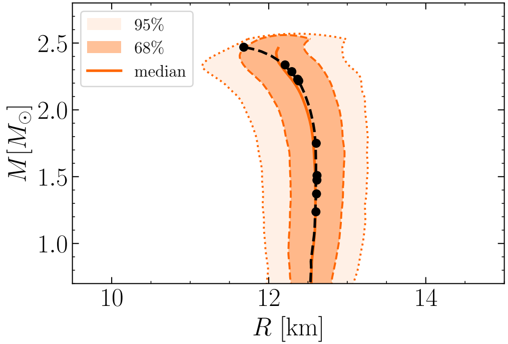


Posterior results

- Spectral EoS model with only two parameters $\theta = (\lambda_1, \lambda_2)$
- Simulate telescope spectra with XSPEC
  [Arnaud, ASP Conf. Ser. 17 (1996)]
- Likelihood differentiable → can use Hamiltonian Monte Carlo sampling
- **Full posterior** $p(\theta, \nu | s)$ **directly from** (simulated) **telescope spectra**

  → Spectrum can contain additional EoS information not captured by (M, R)
  [Elshamouty *et al.*, Astrophys. J. 826 (2016)]
- On simulated test data **surpass accuracy of all previous methods**
  [Farrell *et al.*, JCAP 02 (2023)] [Farrell *et al.*, JCAP 12 (2023)]





[LB *et al.*, arXiv:2403.00287 (2024)]

Scaling to more observations

- **Amortization:** need to train neural network only once

  → Inexpensive likelihood evaluation

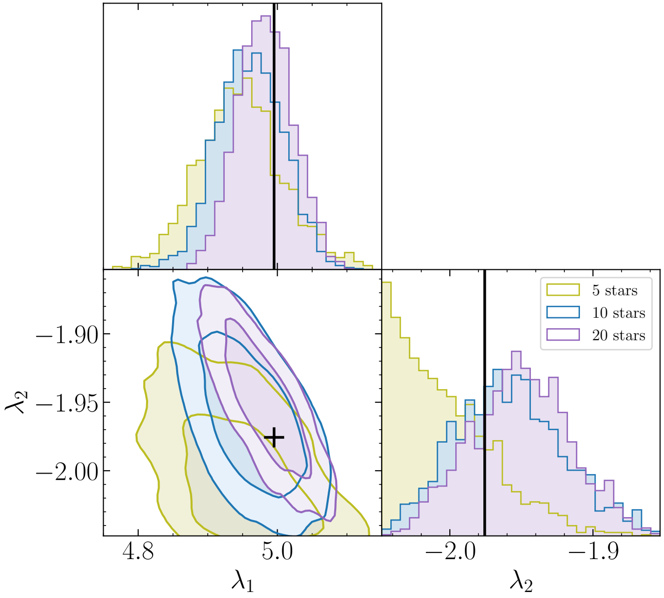
  → **Inclusion of additional observations straight-forward**

  → Expect many more measurements in the future

- Future: apply to real telescope spectra

- Extend approach to NICER or GW data

[Dax *et al.*, Phys. Rev. Lett. 127 (2021)]



[LB *et al.*, arXiv:2403.00287 (2024)]

Summary

- **Bayesian inference** of sound speed in neutron star matter based on theory constraints and astro data
- Maximum possible phase coexistence interval $(\Delta n/n)_{\mathrm{max}} \leq 0.2$
- Strong evidence against $c^2_{s,\mathrm{min}} \leq 0.1$ in cores of neutron stars with $M \leq 2.1\,M_\odot$

  → **Strong first-order phase transitions unlikely** based on empirical data
- Central densities $n_c < 5\,n_0$ for $M \leq 2.3\,M_\odot$: average distance between baryons still $> 1\,\mathrm{fm}$

  → Fluctuations stabilize hadronic phase?
- **Full posterior directly from telescope spectra** with neural likelihood estimation

  → Naturally scales to growing number of observations expected in coming years

Supplementary material

Outlook

- Fourth observation run of LIGO, Virgo and KAGRA started on May 4$^{\text{th}}$
- Four more objects to be measured by NICER telescope [Greif *et al.*, MNRAS 485 (2019)]
- Moment-of-inertia measurement of PSR J0737-3039 in next few years [Landry and Kumar, Astrophys. J. 868 (2018)]
- Extract more information with novel statistical tools from Machine Learning [LB *et al.*, arXiv:2403.00287 (2024)]

→ **Many more future measurements** will put even **tighter constraints** on phase structure at high densities


nature

NEWS FEATURE | 04 March 2020

The golden age of neutron-star physics has arrived

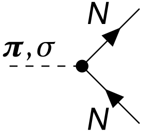
These stellar remnants are some of the Universe's most enigmatic objects – and they are finally starting to give up their secrets.

Adam Mann

Chiral nucleon-meson model

- Interactions of fermions via the **exchange of effective mesons**: Nambu-Goldstone boson $\boldsymbol{\pi}$ and heavy σ

$$\mathscr{L} = \bar{N}\gamma_\mu \partial_\mu N + \frac{1}{2}\left(\partial_\mu \sigma \partial_\mu \sigma + \partial_\mu \boldsymbol{\pi} \cdot \partial_\mu \boldsymbol{\pi}\right) +$$



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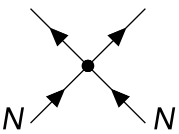


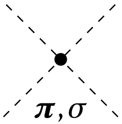
$$- \mathscr{U}(\boldsymbol{\pi}, \sigma)$$

→ Short distance dynamics modelled by massive vector fields

[Floerchinger and Wetterich, Nucl. Phys. A 890–891 (2012)]

- Boson self-interactions and explicit symmetry breaking term

$$\mathscr{U}(\sigma, \boldsymbol{\pi}) =$$



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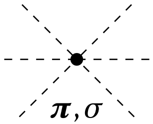


$$+ \cdots - m_\pi^2 f_\pi \left(\sigma - f_\pi\right)$$

- Expectation value $\langle\sigma\rangle$ dynamically creates nucleon mass

→ $\langle\sigma\rangle / \langle\sigma\rangle_{\text{vac}} = \langle\sigma\rangle / f_\pi$ **order parameter for chiral symmetry**

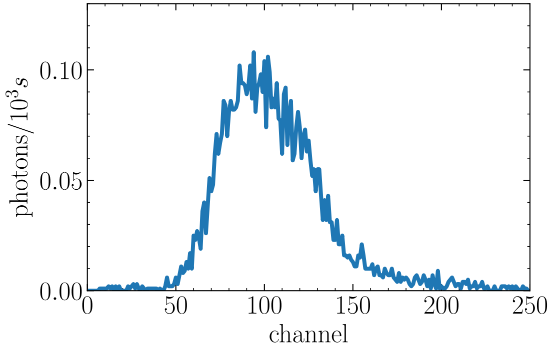
Mass-radius data

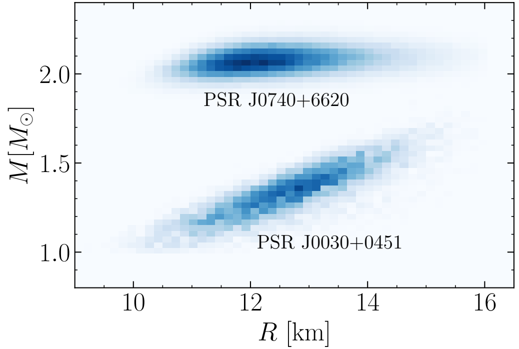
- Mass-radius data of neutron stars:
  - Pulse profile modelling [Riley *et al.*, Astrophys. J. Lett. 887 (2019)]
  - Thermonuclear bursters [Nättilä *et al.*, Astron. & Astrophys. 608 (2017)]
  - Quiescent low mass X-ray binaries [Steiner *et al.*, Mon. Not. Roy. Astron. Soc. 476 (2018)]
- Measurements of **X-ray spectra in telescopes**

  → Depends on (M, R) and **nuisance parameters** $\nu = (N_H, d, \log T_{\text{eff}})$

  → Information from other observations $\tilde{p}(\nu)$
- Infer neutron star mass and radius using e.g. `XSPEC` package [Arnaud, Jacoby and Barnes, ASP Conf. Series volume 101 (1996)]
- Use (M, R) **posteriors as likelihood**, valid for flat priors

$$p(\mathscr{D}|M,R) \propto p(M,R|\mathscr{D})$$





Mass-radius data

- Complicated (M, R) posterior distributions available via samples

  → Approximate with **Kernel Density Estimation** (KDE)

$$\hat{f}_h(x) = \frac{1}{Nh} \sum_{i=1}^{N} K\left(\frac{x - x_i}{h}\right) \qquad K(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)$$

  → Depends on bandwidth parameter h

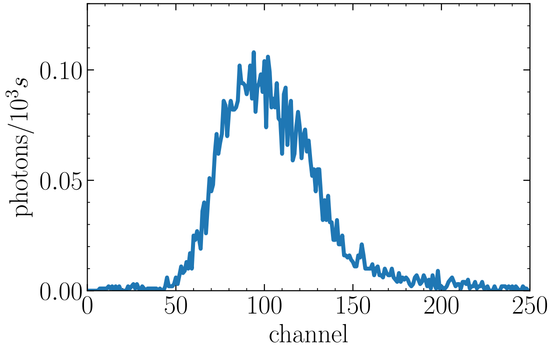
- Solve TOV equations to obtain $R(M, \lambda)$

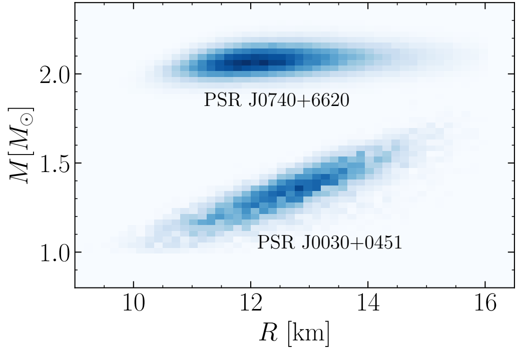
- **Likelihood**:

$$p(M, R|\mathscr{D}) = \int_{M_{\min}}^{M_{\max}(\lambda)} \mathrm{d}M \ \mathrm{KDE}(M, R(M, \lambda))p(M)$$

[LB, Weise and Kaiser, Phys. Rev. D 108 (2023)]

- Mass prior $p(M)$ of neutron star mass population





Implicit assumptions

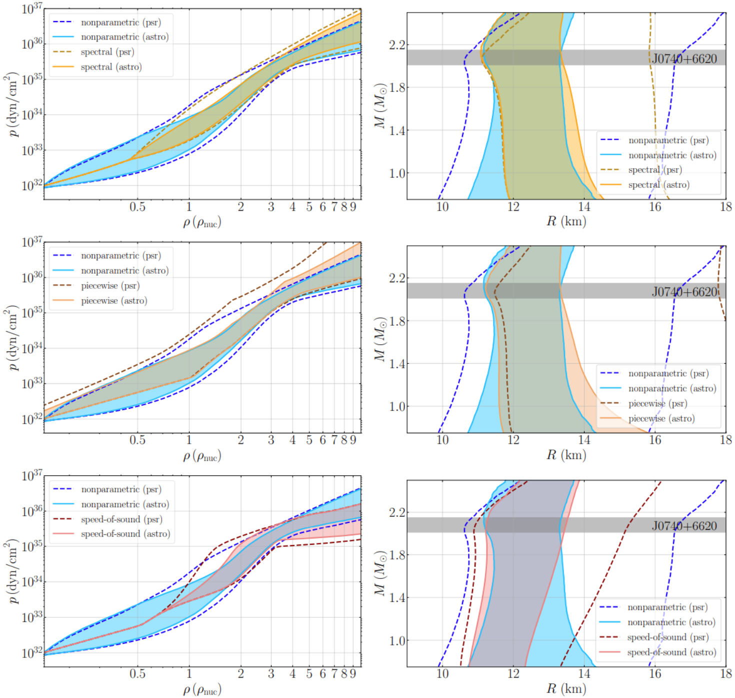
- Only few measurements available so far

  → Large uncertainties, results **prior dependent**

- Results depend on implicit assumptions:
  - Bandwidth of KDE
  - Posteriors as likelihood
  - Unknown neuron star mass population
  - Modelling of neutron star atmospheres
  - …

→ Complimentary **Machine Learning** analysis

[Fujimoto, Fukushima and Murase, Phys. Rev. D 101 (2020)]



[Legred *et al.*, Phys. Rev. D 105 (2022)]

Neural Likelihood Estimation

- Train neural density estimator (normalizing flow) to **approximate likelihood** based on **telescope spectra** s

$$q_\phi(s|\lambda,\nu) \approx p(s|\lambda,\nu)$$

[Papamakarios, Sterratt and Murray, AISTAT (2019)]

- **Sample** (λ_i, ν_i, s_i) from $p(\lambda, \nu, s)$

$$\lambda_i \sim p(\lambda)$$
$$\nu_i \sim p(\nu)$$
$$s_i \sim p(s|\lambda_i,\nu_i) \quad \text{via simulator based on } \texttt{XSPEC} \text{ plus noise}$$

- **Training** q_ϕ based on (λ_i, ν_i, s_i) equals maximizing $\sum_i \log q_\phi(s_i|\lambda_i,\nu_i)$

$$\mathbb{E}_{p(\lambda,\nu,s)}\left[\log q_\phi(s|\lambda,\nu)\right] = -\mathbb{E}_{p(\lambda,\nu)}\left[D_{KL}\left(p(s|\lambda,\nu)\middle|\middle|q_\phi(s|\lambda,\nu)\right)\right] + \text{const}$$

$\rightarrow$ Kullback-Leibler divergence zero for $q_\phi(s|\lambda,\nu) = p(s|\lambda,\nu)$

- Insert real telescope spectrum s_0

$$p(s_0|\lambda,\nu) \approx q_\phi(s_0|\lambda,\nu)$$

Neural Likelihood Estimation

- Multiply with priors and **sample from posterior**

$$p(\lambda, \nu | s_0) \propto q_\phi(s_0|\lambda, \nu_0)\tilde{p}(\nu_0)p(\nu)p(\lambda)$$

→ **Different scenarios for prior information** on nuisance parameters $\tilde{p}(\nu_0)$ (true, tight, loose)

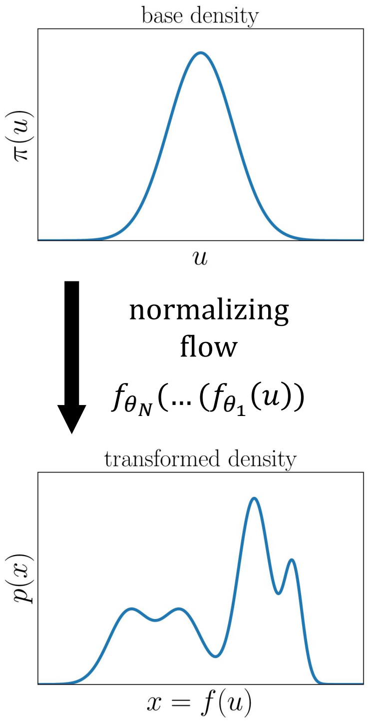
parameter	true	tight	loose
d	exact	5%	20%
N_H	exact	30%	50%
$\log(T_{eff})$	exact	± 0.1	± 0.2

- Can use Hamiltonian Monte Carlo (HMC) for sampling

- **Normalizing flows**: distribution $p(x)$ via *invertible and differentiable transformations* f_{θ_i} of base distribution $\pi(u)$

→ Compute probability density and generate samples

- Here: distribution of spectra s conditioned on λ and ν



NLE for neutron stars

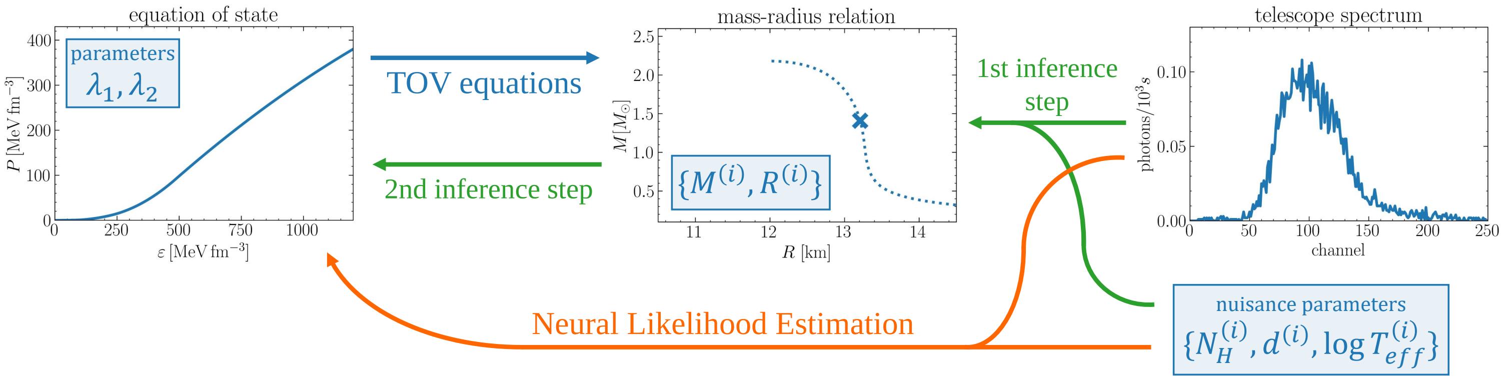
1. **Train 100 neural density estimators** with (s, λ, ν) with different hyperparameters

   → Use only top 5 $\quad \bar{q}_\phi(s|\lambda,\nu) = \frac{1}{N}\sum_j q_{\phi_j}(s|\lambda,\nu)$

2. Here: combine likelihoods for 10 simulated test spectra

3. Multiply with priors $p(\lambda)$, $p(\nu)$ and nuisance parameter information $\tilde{p}(\nu_i)$

4. **Sample from posterior** using HMC $\quad p(\lambda,\nu|s) \propto \left(\prod_i \bar{q}_\phi(s_i|\lambda,\nu_i)\tilde{p}(\nu_i)\right)p(\lambda)p(\nu)$



Posterior distribution

- **Posterior directly from telescope spectra**

  → Here based on 10 simulated spectra for one example EoS

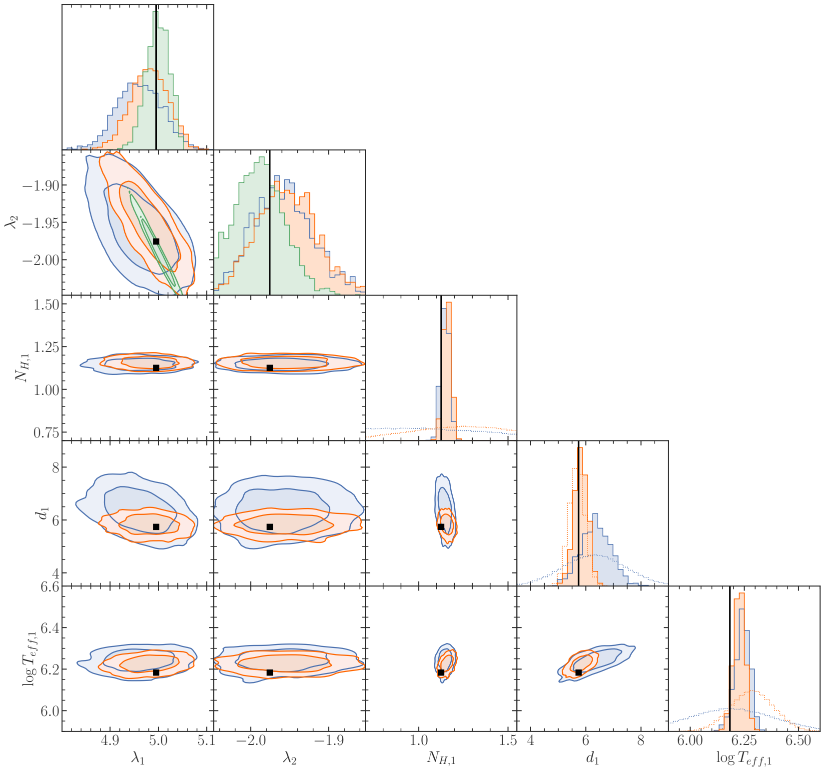
- **Three scenarios**: true (green), tight (orange) and loose (blue)

  → Uncertainties much smaller in true case

- Constrain N_H and $\log T_{eff}$ much **better than prior information**

  → In tight case d posterior similar to prior

- Transform into $P(\varepsilon)$ and $R(M)$ constraints



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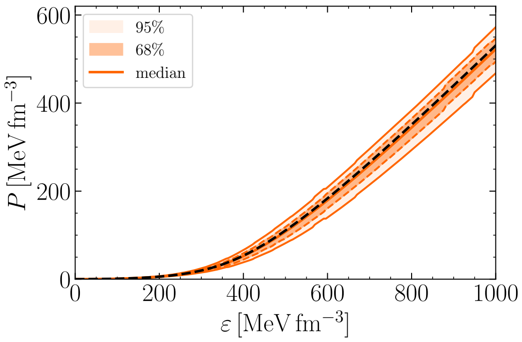
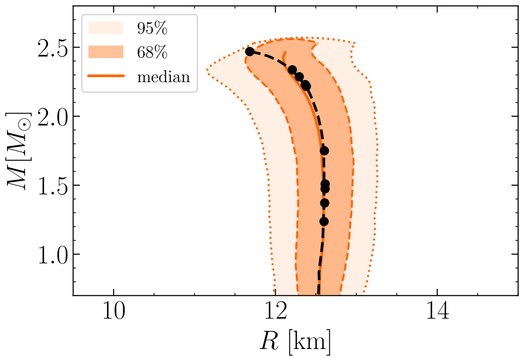




Posterior distribution

- **Posterior directly from telescope spectra**

  → Uncertainty propagated via normalizing flow

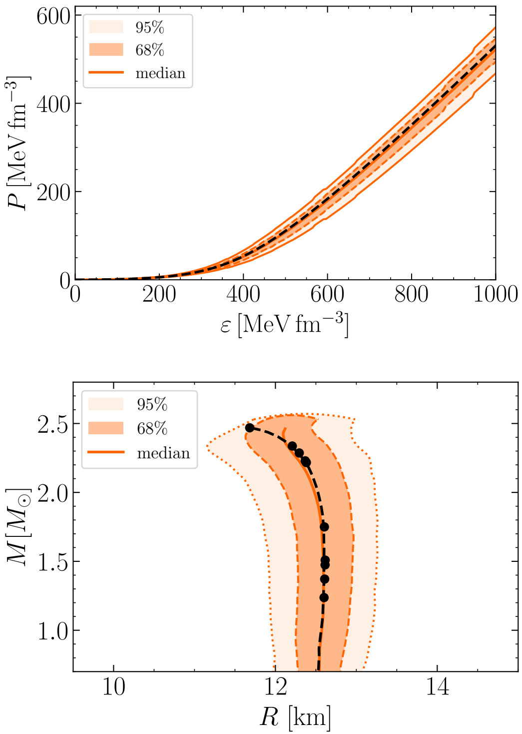
- Possible to combine with likelihoods from other data (e.g. gravitational waves)

- **Amortization**: need to train the density estimator only once

  → Faster likelihood computation

  → Easy to include new measurements, analyse impact of future measurements

- Expected to extend easily to more complex EoS model



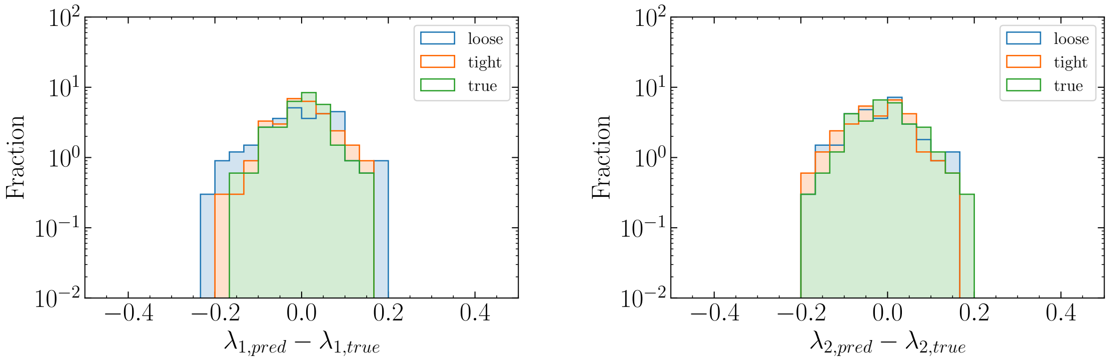
Performance

- Compute **maximum posterior estimate** $(\lambda_{1,pred}, \lambda_{2,pred})$ for 100 random EoS

  → Again 10 simulated spectra for each EoS

- Compare to ground truth values $(\lambda_{1,true}, \lambda_{2,true})$

  → Distribution centered around 0, **no systematic bias**



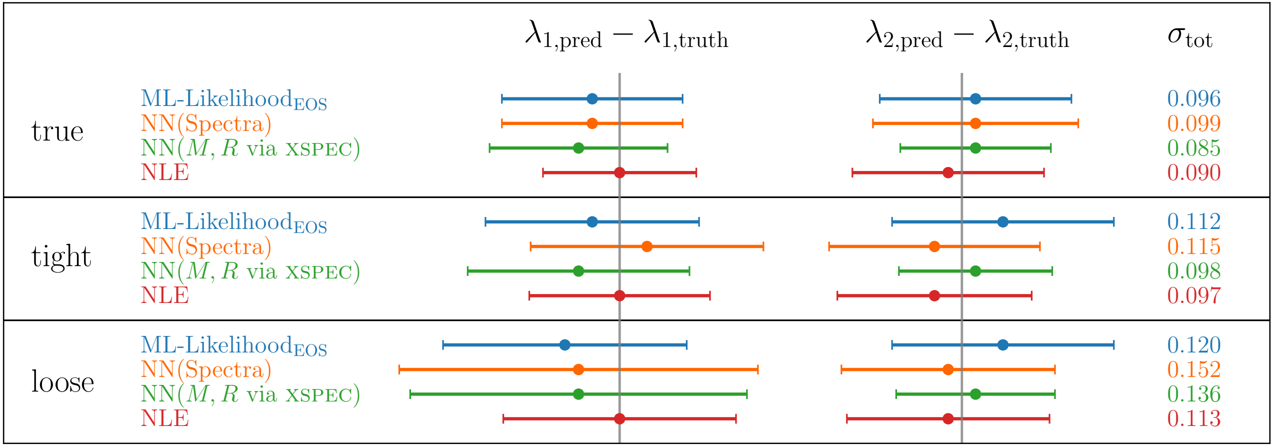
Performance

- Compare mean and standard deviation to previous approaches

$$\sigma_{tot} = \sqrt{\sigma^2_{\lambda_1} + \sigma^2_{\lambda_2}}$$

→ **Better performance** than previous approaches!

[Farrell *et al.*, JCAP 02 (2023)] [Farrell *et al.*, JCAP 12 (2023)]



Work in progress

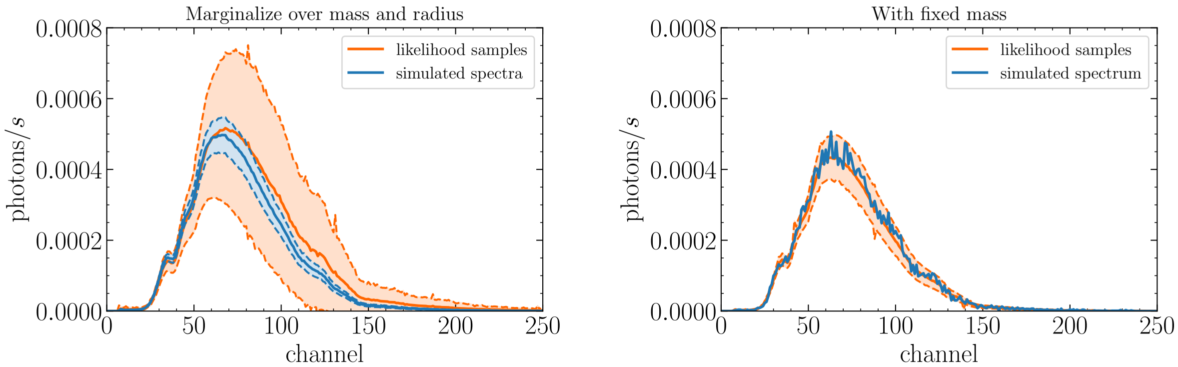
- **Sample spectra from likelihood** and compare to simulations for given λ and ν

  → Need to marginalize over mass and radius (left)

- Large differences, use **mass as additional nuisance parameter?**

  → Much better agreement (right), can infer mass and radius of NS from spectra

- But: parameter space increases, multimodal distributions, sampling problems



Nucleon radii

- Axial radius from neutrino-deuteron scattering

$$\langle r_A^2 \rangle = (0.46 \pm 0.22)\,\mathrm{fm}^2$$

[Hill *et al.*, Rep. Prog. Phys. 81 (2018)]

$\rightarrow$ Subtract dominant cloud contribution from a_1 meson

$$\sqrt{\langle r_A^2 \rangle_{\mathrm{core}}} = \left[\langle r_A^2 \rangle - \delta \langle r_A^2 \rangle\right]^{1/2} \simeq 0.55\,\mathrm{fm}$$

- Mass radius of proton from J/Ψ photoproduction ($\rightarrow$ dominated by gluon dynamics at nucleon center)

$$\sqrt{\langle r^2 \rangle_{\mathrm{mass}}} = (0.55 \pm 0.03)\,\mathrm{fm}$$

[Kharzeev, Phys. Rev. D 104 (2021)]

- Baryonic core radius from isoscalar and isovector radii

$$\sqrt{\langle r_B^2 \rangle} \simeq 0.47\,\mathrm{fm}$$

- Cloud range given e.g. by proton charge radius

$$\sqrt{\langle r_p^2 \rangle} = (0.840 \pm 0.003)\,\mathrm{fm}$$

[Lin, Hammer and Meißner, Phys. Rev. Lett. 128 (2022)]

Twin stars

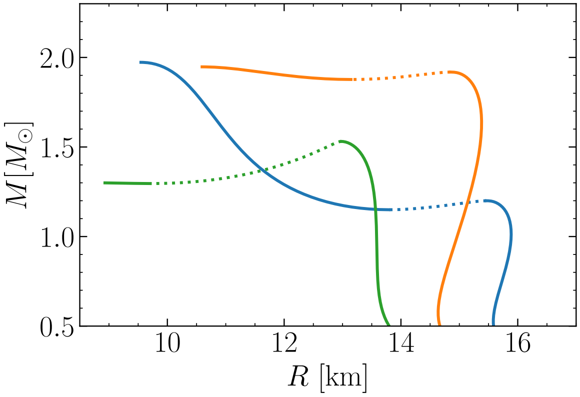
- Strong phase transitions can lead to mass-radius relations with **multiple stable branches** ('twin stars')

- Bayes factor gives **extreme evidence** against multiple stable branches
  [Gorda *et al.*, arXiv:2212.10576 (2022)]

- Without likelihood from **ChEFT** 'only' strong evidence:

$$\mathscr{B}^{N_{\text{branches}}=1}_{N_{\text{branches}}>1} = 12.97$$

[Essick, Legred, Chatziioannou, Han and Landry, arXiv:2305.07411 (2023)]



- Disconnection takes place at $M \sim 0.8\,M_\odot$

→ Unlikely based on nuclear phenomenology

Possible impact of HESS J1731-347

- Central compact object within supernova remnant HESS J1731-347:

$$M = 0.77^{+0.20}_{-0.17}\,M_\odot$$
$$R = 10.4^{+0.86}_{-0.78}\,\text{km}$$

[Doroshenko *et al.*, Nat. Astron. 6 (2022)]

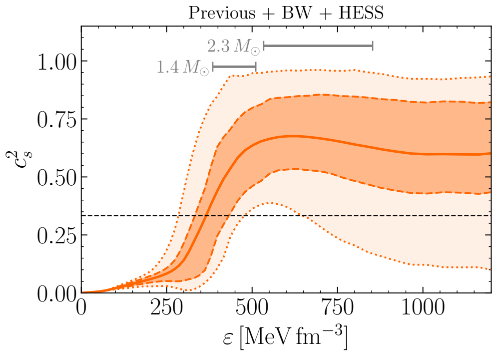
- Unusually **light neutron star** with very low radius

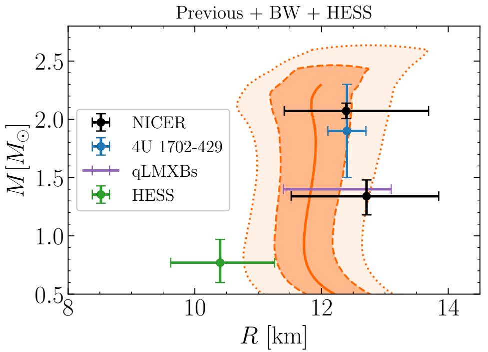
→ Neutron star mass $M < 1.17\,M_\odot$ in contradiction with known formation mechanisms
[Suwa *et al.*, MNRAS 481 (2018)]

→ Strange star?

- Systematic uncertainty: larger masses and radii might be possible
[Alford and Halpern, Astrophys. J. 944 (2023)]

- **Tension** between HESS and current astrophysical data
[Jiang, Ecker and Rezzolla, arXiv:2211.00018 (2022)]





General EoS parametrization

- Determine EoS from **speed of sound**

$$c_s^2(\varepsilon) = \frac{\partial P(\varepsilon)}{\partial \varepsilon}$$

- Parametrize by **segment-wise linear interpolations**

$$c_s^2(\varepsilon,\theta) = \frac{(\varepsilon_{i+1}-\varepsilon)c_{s,i}^2 + (\varepsilon-\varepsilon_i)c_{s,i+1}^2}{\varepsilon_{i+1}-\varepsilon_i}$$

[Annala *et al.*, Nature Phys. 16, 907 (2020)]

- Matching to BPS crust at low densities $(c_{s,0}^2, \varepsilon_0) = (c_{s,\mathrm{crust}}^2, \varepsilon_{\mathrm{crust}})$ [G. Baym, C. Pethick, and P. Sutherland, Astrophys. J. 170 (1971)]
- Constant speed of sound $c_s^2(\varepsilon,\theta) = c_{s,N}^2$ beyond last point $\varepsilon > \varepsilon_N$
- Choose $N = 5$ corresponding to 7 segments and 10 free parameters
- **Priors** sampled logarithmically

$$c_{s,i}^2 \in [0,1] \quad \varepsilon_i \in [\varepsilon_{crust}, 4\,\mathrm{GeV\,fm^{-3}}]$$

- Parametrizations with only 4 segments leads to comparable results as non-parametric Gaussian process

[Annala *et al.*, arXiv:2303.11356 (2023)]

Bayesian inference

- Bayes theorem:

$$p(\theta|\mathscr{D},\mathscr{M}) = \frac{p(\mathscr{D}|\theta,\mathscr{M})\,p(\theta|\mathscr{M})}{p(\mathscr{D}|\mathscr{M})}$$

- Choose **Priors** for parameters $p(\theta|\mathscr{M})$

- **Likelihood** $p(\mathscr{D}|\theta,\mathscr{M})$: probability of data $\mathscr{D}$ to occur for θ and model $\mathscr{M}$

- (M,R,Λ) can be deterministically determined for θ

$$p(\mathscr{D}|\theta,\mathscr{M}) = p(\mathscr{D}|M,R,\Lambda,\mathscr{M})$$

→ For computational feasibility assume (valid for flat Priors in (M,R,Λ))

$$p(\mathscr{D}|M,R,\Lambda,\mathscr{M}) \propto p(M,R,\Lambda|\mathscr{D},\mathscr{M})$$

[Riley, Raaijmakers and Watts, Mon. Not. Roy. Astron. Soc. 478 (2018)] [Raaijmakers *et al.*, ApJL 918 (2021)]

Bayesian inference

- Bayes theorem:

$$p(\theta|\mathscr{D},\mathscr{M}) = \frac{p(\mathscr{D}|\theta,\mathscr{M})\,p(\theta|\mathscr{M})}{p(\mathscr{D}|\mathscr{M})}$$

- **Evidence** $p(\mathscr{D}|\mathscr{M})$: determined via normalization of the posterior

$$p(\mathscr{D}|\mathscr{M}) = \int d\theta \;\; p(\mathscr{D}|\theta,\mathscr{M})\,p(\theta|\mathscr{M})$$

→ High-dimensional integral, use sampling techniques

- **Credible bands**: determine $P(\varepsilon_i,\theta)$ on grid $\{\varepsilon_i\}$ for posterior samples to get $p\big(P\big|\varepsilon_i,\mathscr{D},\mathscr{M}\big)$

→ Compute credible interval $[a,b]$ with probability α at ε_i

$$\alpha = \int_a^b dP \;\; p\big(P\big|\varepsilon_i,\mathscr{D},\mathscr{M}\big)$$

→ Combine credible intervals at all ε_i to posterior credible band $P(\varepsilon)$

Trace anomaly measure

- Trace anomaly measure as signature of **conformality**

$$\Delta = \frac{g_{\mu\nu} T^{\mu\nu}}{3\varepsilon} = \frac{1}{3} - \frac{P}{\varepsilon}$$

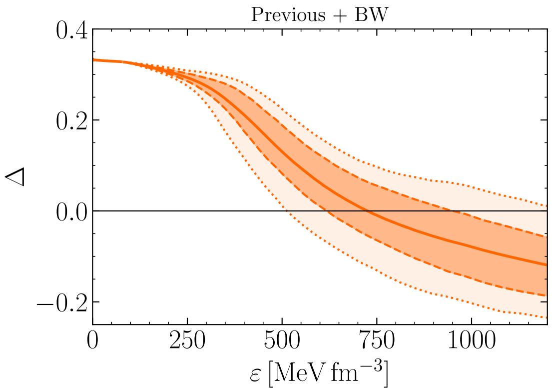
[Fujimoto, Fukushima, McLerran and Praszałowicz, Phys. Rev. Lett. 129 (2022)]

- Median becomes negative around $\varepsilon \sim 700\,\mathrm{MeV\,fm^{-3}}$

→ Moderate evidence for Δ turning **negative** inside neutron stars

Bayes factor $\mathscr{B}^{\Delta<0}_{\Delta\geq 0} = 8.11$

[Ecker and Rezzolla, Astrophys. J. Lett. 939 (2022)] [Annala *et al.*, arXiv:2303.11356 (2023)]
[Marczenko, McLerran, Redlich and Sasaki, Phys. Rev. C 107 (2023)]



- At higher energy densities again positive Δ to reach asymptotic pQCD limit

Chiral effective field theory

- Previous analyses: only EoS within ChEFT band a priori

- ChEFT uncertainties represent variety of **empirical nuclear data**

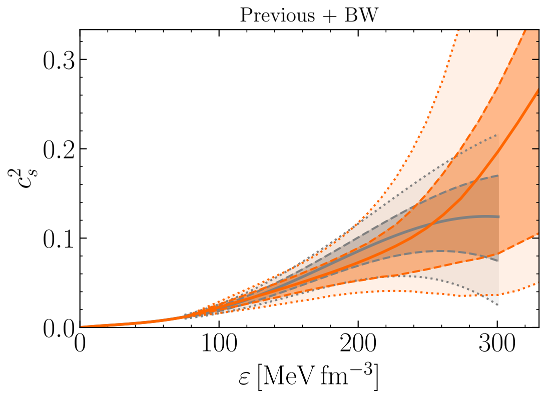
  → Similar treatment to astro data allows balancing between constraints and consistent Bayes factor analysis

- **Likelihood** via Bayesian linear regression:

$$p\big(\mathscr{D}_{\mathrm{ChEFT}}\big|c_s^2(n,\theta)\big) \propto \exp\left[-\frac{1}{2}\int_{0.5\,n_0}^{n_{\mathrm{ChEFT}}=1.3\,n_0} \mathrm{d}n \left(\frac{\langle c_s^2(n)\rangle - c_s^2(n,\theta)}{\sigma(n)}\right)^2\right]$$

- Slight **tension** between ChEFT at $n \simeq 2\,n_0$ and astro data

[Essick *et al.*, Phys. Rev. C 102 (2020)]



[Drischler, Han and Reddy, Phys. Rev. C 105 (2022)]

Impact of pQCD

- Matching to pQCD at $n_{c,\mathrm{max}}$ has only **negligible impact**

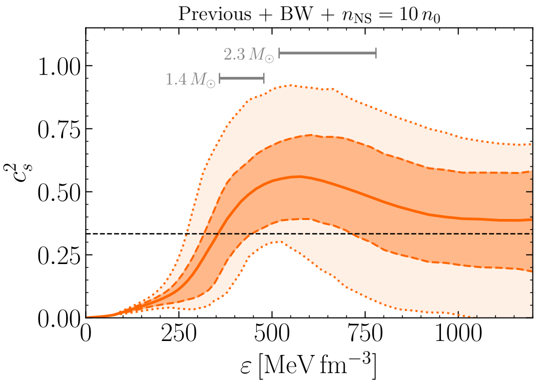
  [Somasundaram, Tews and Margueron, arXiv:2204.14039 (2022)]

- **Change matching** to asymptotic pQCD from $n_{c,max}$ to $10\,n_0$

  → Much smaller c_s^2 at high energy densities

  [Gorda, Komoltsev, and Kurkela, arXiv:2204.11877 (2022)]
  [Komoltsev and Kurkela, Phys. Rev. Lett. 128 (2022)]

  → Few changes in mass-radius, properties of $2.3\,M_\odot$ neutron star change only slightly



- EoS beyond $n_{c,\mathrm{max}}$ no longer constrained by astrophysical data

  → Impact depends **unconstrained interpolation** to high densities

  [Essick, Legred, Chatziioannou, Han and Landry, arXiv:2305.07411 (2023)]

Perturbative QCD

- Connection of $(\mu_{\rm NS}, n_{\rm NS}, P_{\rm NS})(\theta)$ to $(\mu_{\rm pQCD}, n_{\rm pQCD}, P_{\rm pQCD})$

$$\int_{\mu_{\rm NS}}^{\mu_{\rm pQCD}} d\mu\ n(\mu) = P_{\rm pQCD} - P_{\rm NS} = \Delta P$$

- Causality and thermodynamic stability imply minimum and maximum values

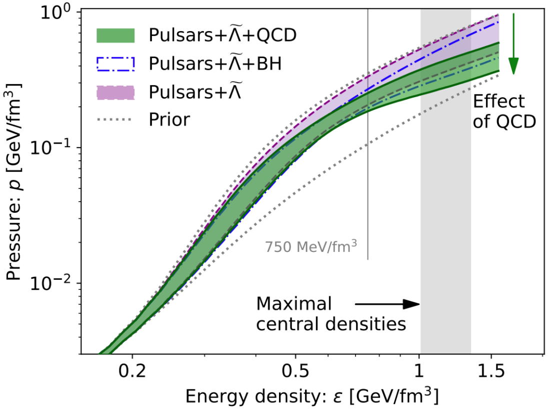
$$\Delta P_{\min} = \frac{\mu_{\rm pQCD}^2 - \mu_{\rm NS}^2}{2\mu_{\rm NS}} n_{\rm NS} \qquad \Delta P_{\max} = \frac{\mu_{\rm pQCD}^2 - \mu_{\rm NS}^2}{2\mu_{\rm pQCD}} n_{\rm pQCD}$$

- **Likelihood**

$$p\left(\mathcal{D}_{\rm pQCD} \middle| \Delta P(\theta), \mathcal{M}\right) = \begin{cases} 1 & \text{if } \Delta P(\theta) \in [\Delta P_{\min}(\theta), \Delta P_{\max}(\theta)] \\ 0 & \text{else} \end{cases}$$

→ Take logarithmic average over renormalisation scale $X \in [1/2, 2]$

[Komoltsev and Kurkela, Phys. Rev. Lett. 128 (2022)]



[Gorda, Komoltsev, and Kurkela, arXiv:2204.11877 (2022)]

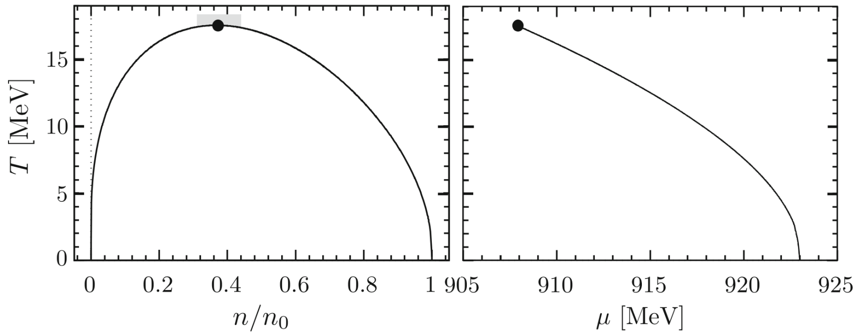
Mean-field approximation

- **Mean-field (MF)** approximation: replace chiral boson fields by expectation values $\langle\sigma\rangle$ and $\langle\boldsymbol{\pi}\rangle = 0$

  → Diverging fermionic vacuum contribution: $$\delta\Omega_{\text{vac}} = -4\int \frac{d^3p}{(2\pi)^3}\,E$$

- Compute with dimensional regularisation in **extended mean-field (EMF)** approach [Skokov et al., Phys. Rev. D 82 (2010)]

- Adjust model parameters to reproduce empirical nuclear properties, i.e., liquid-gas phase transition
  [Elliot et al., Phys. Rev. C 87 (2013)]



[LB, Kaiser and Weise, Eur. Phys. J. A 57 (2021)]

Functional Renormalization Group

- Additional fluctuations beyond vacuum contribution (chiral boson and nucleon loops)

  → Include using non-perturbative **Functional Renormalization Group (FRG)** approach

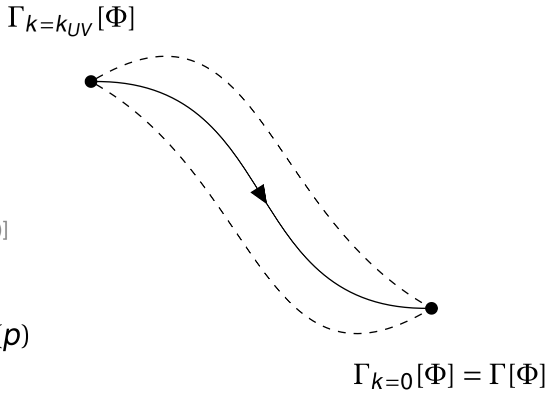
  [Drews and Weise, Prog. Part. Nucl. Phys. 93 (2017)]

- Initialize scale-dependent effective action $\Gamma_k[\Phi]$ at $k_{UV} \sim 4\pi f_\pi$

- Evolution $k \to 0$ governed by Wetterich's flow equation

$$k\frac{\partial \Gamma_k[\Phi]}{\partial k} = \frac{1}{2}\mathrm{Tr}\left[k\frac{\partial R_k}{\partial k}\cdot\left(\Gamma_k^{(2)}[\Phi]+R_k\right)^{-1}\right]$$

[Wetterich, Phys. Lett. B 301 (1993)]



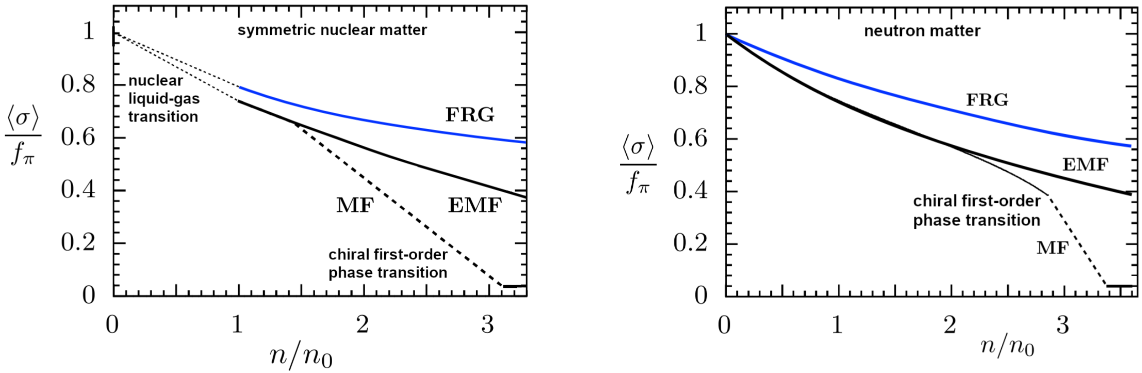
- $\Gamma_k[\Phi]$ contains all **fluctuations** with $p^2 \geq k^2$ through regulator $R_k(p)$

Phase structure

- *Mean-field:* unphysical **first-order order phase transition** to chirally restored phase
- *Extended mean-field:* vacuum contribution stabilizes order parameter
- *FRG:* further stabilization through additional fluctuations

→ **Smooth crossover** at densities $n > 6\,n_0$ (with $n_0 = 0.16\,\mathrm{fm}^{-3}$)

→ *No phase transition in neutron star matter?*



[LB, Kaiser and Weise, Eur. Phys. J. A 57 (2021)]

Likelihoods

- EoS supports masses between $M_{\min}$ and $M_{\max}(\theta)$
- Choose flat **mass prior** and $M_{\min} = 0.5\,M_\odot$

$$p(M(\theta)) = \begin{cases} \frac{1}{M_{\max}(\theta) - M_{\min}} & \text{if } M \in [M_{\min}, M_{\max}(\theta)] \\ 0 & \text{else} \end{cases}$$

[Landry, Essick and Chatziioannou, Phys. Rev D 101 (2020)]

- When number of data increases incorporate mass population

→ Wrong population model causes a bias

[Mandel, Farr and Gair, Mon. Not. Roy. Astron. Soc. 486 (2019)]

- Assume **Shapiro** mass measurements Gaussian to compute likelihood

$$\begin{aligned} p\big(M(\theta)\big|\mathscr{D}_{\text{Shapiro}}, \mathscr{M}\big) &= \int_{M_{\min}}^{M_{\max}(\theta)} \mathrm{d}M \; \mathcal{N}\left(M, \langle M \rangle, \sigma_M\right) p(M(\theta)) \\ &\approx \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{M_{\max}(\theta) - \langle M \rangle}{\sqrt{2}\sigma_M}\right)\right] p(M(\theta)) \end{aligned}$$

Likelihoods

- Data available as samples, approximate underlying probability with Kernel Density Estimation (KDE)
- Solve TOV equations to obtain $R(M,\theta)$ and $\Lambda(M,\theta)$
- **NICER likelihood**:

$$p\big((M,R)(\theta)\big|\mathcal{D}_{\mathrm{NICER}},\mathcal{M}\big) = \int_{M_{\min}}^{M_{\max}(\theta)} \mathrm{d}M\ \mathrm{KDE}\big(M,R(M,\theta)\big)p(M(\theta))$$

- **GW likelihood**:

$$p\big((M,\Lambda)(\theta)\big|\mathcal{D}_{\mathrm{GW}},\mathcal{M}\big) = \int \mathrm{d}M_1 \int \mathrm{d}M_2\ \mathrm{KDE}\big(M_1,M_2,\Lambda(M_1,\theta),\Lambda(M_2,\theta)\big)$$

- Do not assume neutron star-neutron star merger events

  → GW likelihood not weighted by mass prior and $\Lambda(M) = 0$ for black holes

- Do not assumed fixed chirp mass $M_{\mathrm{chirp}} = (M_1 M_2)^{3/5}(M_1 + M_2)^{-1/5}$

Conformal limit

- Derived from naive dimensional analysis and asymptotic limit

$$\mu \gg \Lambda_{\text{QCD}} \qquad \Longrightarrow \qquad \begin{aligned} &P \propto \mu^{d+1} \\ &c_s^2 = \frac{\partial P}{\partial \varepsilon} \sim \frac{1}{d} \end{aligned}$$

[Hippert, Fraga and Noronha, Phys. Rev. D 104 (2021)]

- Expected to hold in all **conformal field theories**

[Bedaque and Steiner, Phys. Rev. Lett. 114 (2015)]

- Recent Bayesian analyses found speeds of sound $c_s^2 > 1/3$ inside neutron stars

[Landry, Essick and Chatziioannou, Phys. Rev. D 101 (2020)] [Gorda, Komoltsev, and Kurkela, arXiv:2204.11877 (2022)]
[Altiparmak, Ecker, and Rezzolla, arXiv:2203.14974 (2022)] [Leonhardt *et al.*, Phys. Rev. Lett. 125 (2020)]

→ Also $c_s^2 > 1/3$ in recent $N_C = 2$ **lattice QCD**

[Iida and Itou, PTEP 2022 (2022)]

- Hard Dense Loop resummation methods: conformal limit may be approached asymptotically from above

[Fujimoto and Fukushima, Phys. Rev. D 105 (2022)]

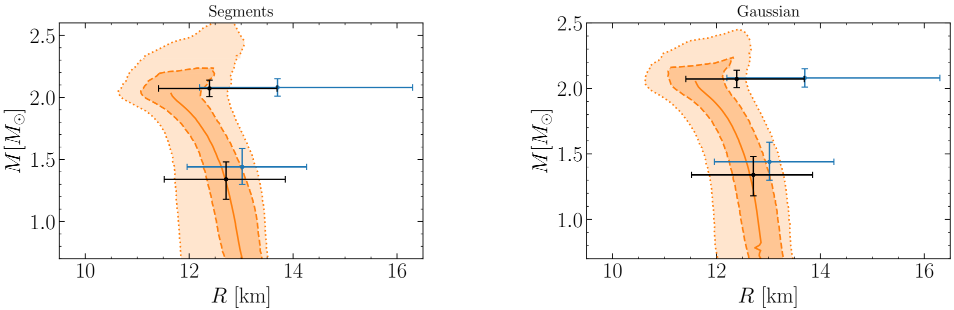
Parametrization dependence

- 'Old' segment-wise parametrisation: different ChEFT constraint, $c_s^2 = 1/3$ reached asymptotically from below
- Compared to skewed Gaussian plus logistic function to reach asymptotic limit $c_s^2 = 1/3$

$$c_s^2(x,\theta) = a_1 \exp\left[-\frac{1}{2}\frac{(x-a_2)^2}{a_3^2}\right]\left(1+\operatorname{erf}\left[\frac{a_6}{\sqrt{2}}\frac{x-a_2}{a_3}\right]\right) + \frac{1/3 - a_7}{1+\exp\left[-a_5(x-a_4)\right]} + a_7$$

[Greif *et al*,, MNRAS 485, 5363 (2019)] [Tews, Margueron and Reddy, EPJA 55, 97 (2019)]

- Very similar findings, results **robust against change** of parametrization and Prior



[LB, Weise and Kaiser, Phys. Rev. D 107 (2023)]