Dense QCD EoS:
Conformal limit and weak-coupling results

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Based on: [1] Y. Fujimoto, K. Fukushima, L. McLerran, M. Praszalowicz, PRL129, 2207.06753 [nucl-th];
[2] Y. Fujimoto, to appear in PRD, 2312.11443 [hep-ph];
[3] Y. Fujimoto, in preparation, 2404.?????

March 18, 2024 - cond-mat.QCD @ YITP, Kyoto U

Outline

1. **Conformal limit in dense QCD**

2. **Confronting weak-coupling vs lattice QCD results at finite isospin density**

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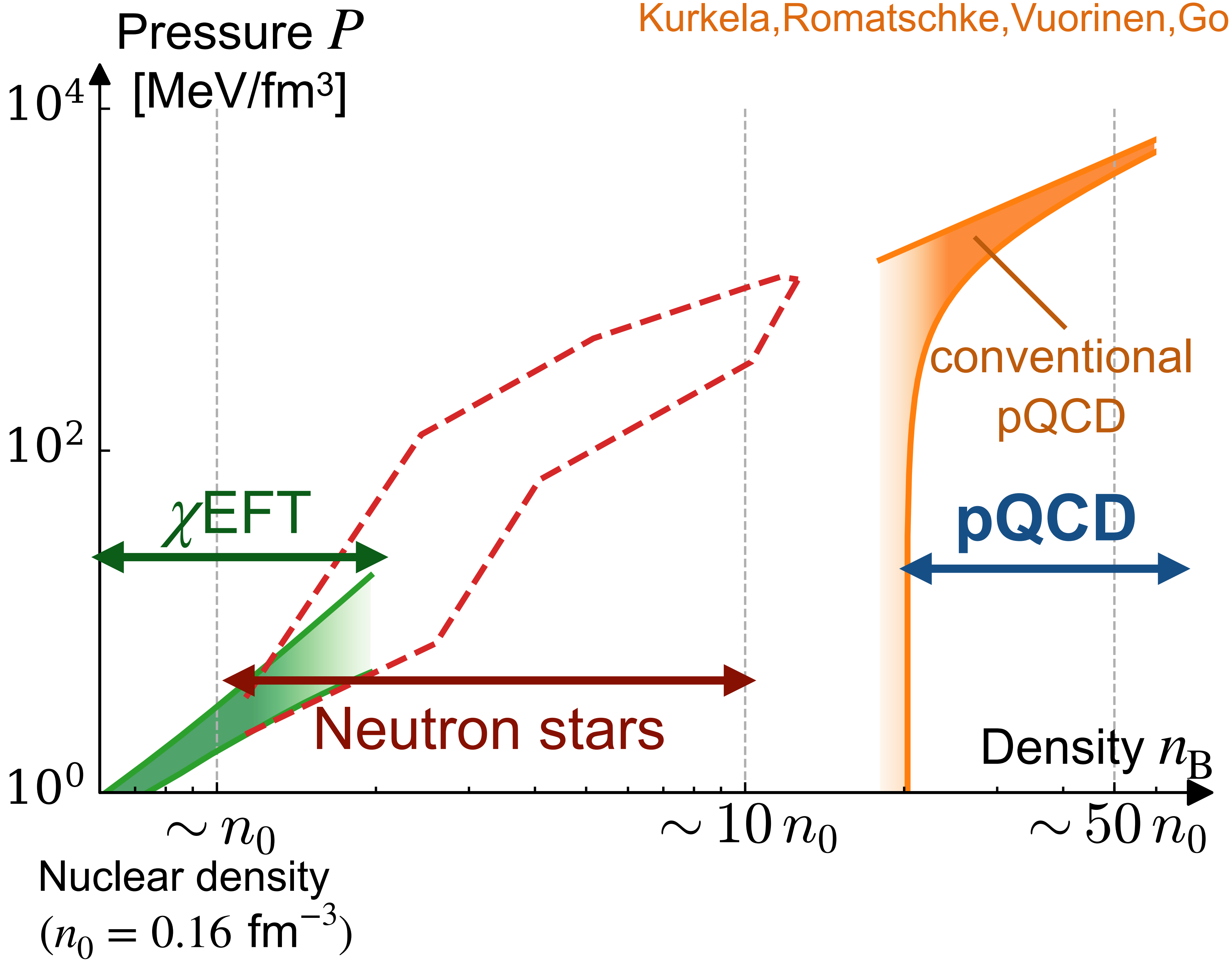
1. **Conformal limit in dense QCD**

2. Confronting weak-coupling vs lattice QCD results at finite isospin density

Dense matter equation of state (EoS)

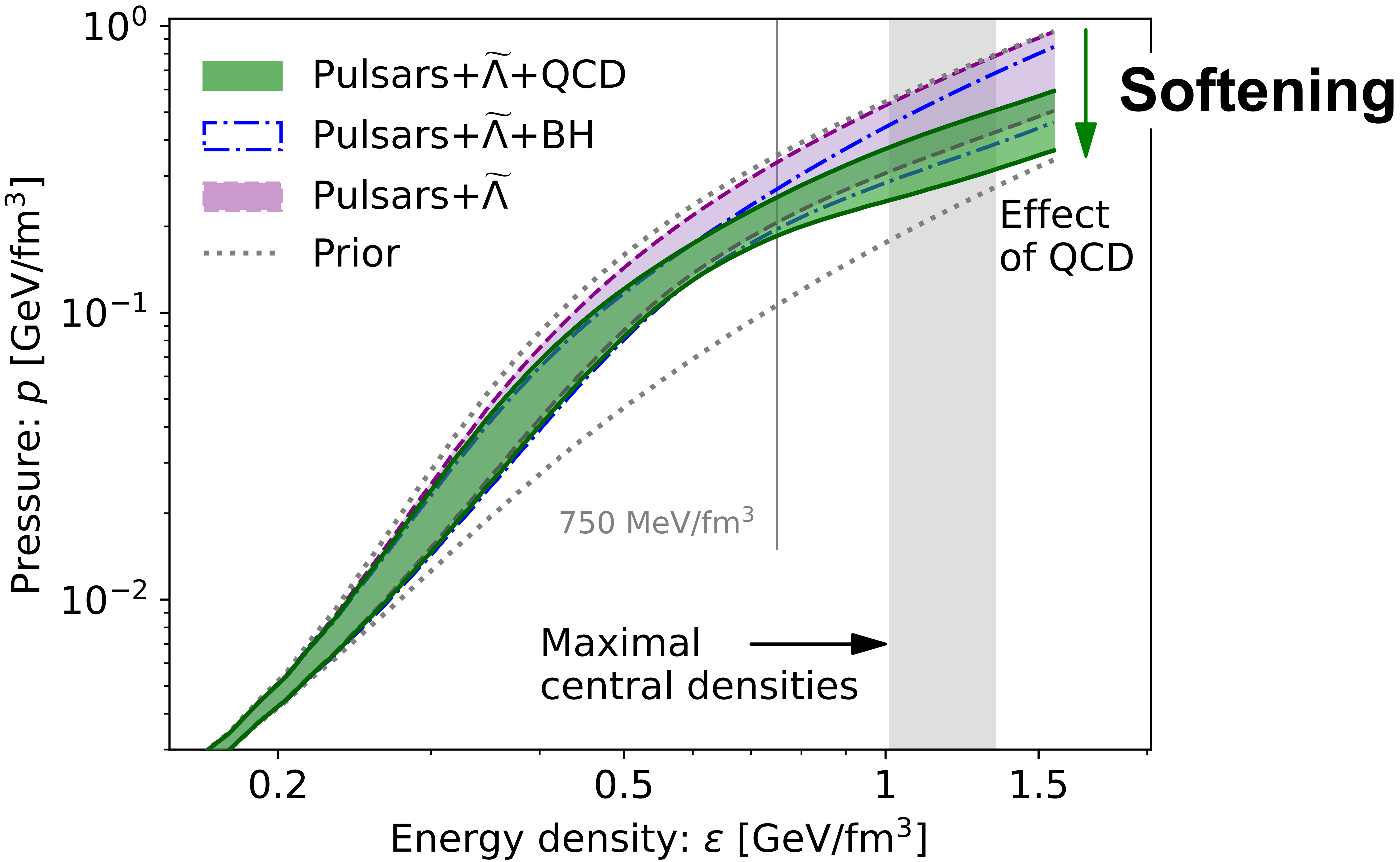
χEFT: Tews,Krüger,Hebeler,Schwenk(2013); Drischler,Furnstahl,Melendez,Philips(2020) & many others

pQCD: Freedman,McLerran(1978); Baluni(1979); Kurkela,Romatschke,Vuorinen,Gorda,Säppi+(2009-)



The effect of pQCD on the NS EoS

Gorda,Komoltsev,Kurkela (2022); Komoltsev,Somasundaram et al. (2023)



Conformal limit

Coupling goes to $\alpha_s \to 0$ when $\varepsilon \to \infty$:

$$\varepsilon - 3P \sim \beta_0 \mu^4 \left(\frac{\alpha_s}{\pi}\right)^2 \to 0$$

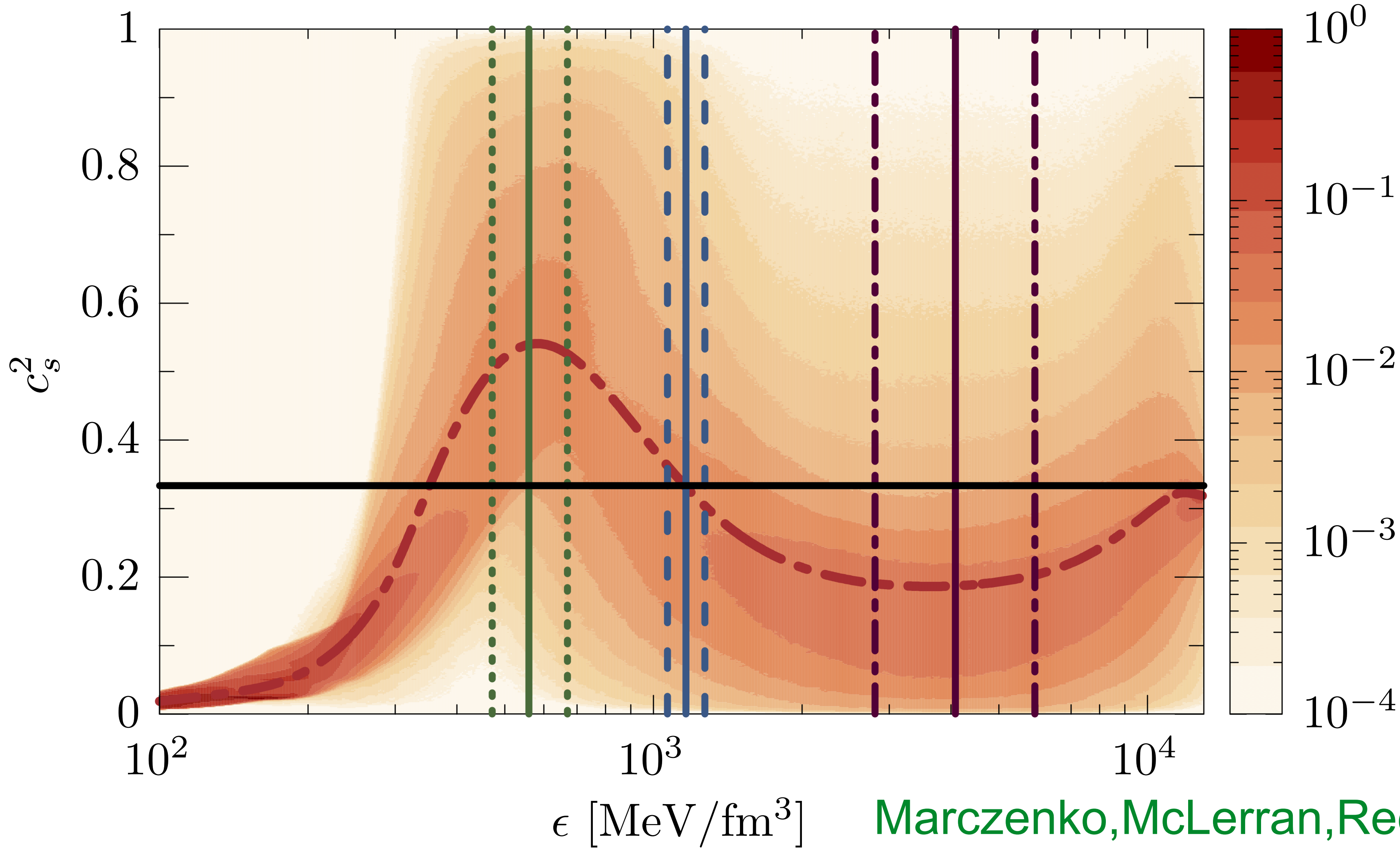
$$v_s^2 = \frac{dP}{d\varepsilon} \sim \frac{1}{3} \frac{1}{1 + \beta_0 \left(\frac{\alpha_s}{\pi}\right)^2} \to \frac{1}{3}$$

At the intermediate density, $\varepsilon - 3P = 0$ and $v_s^2 = 1/3$ are different conditions

Conformal limit for v_s^2

Bedaque,Steiner (2014); Tews,Carlson,Gandolfi,Reddy (2018); Annala+ (2019);
Altiparmak,Ecker,Rezzolla (2022); Brandes,Weise,Kaiser (2022), and so on…

Crosses the limit $v_s^2 = 1/3$ at intermediate density



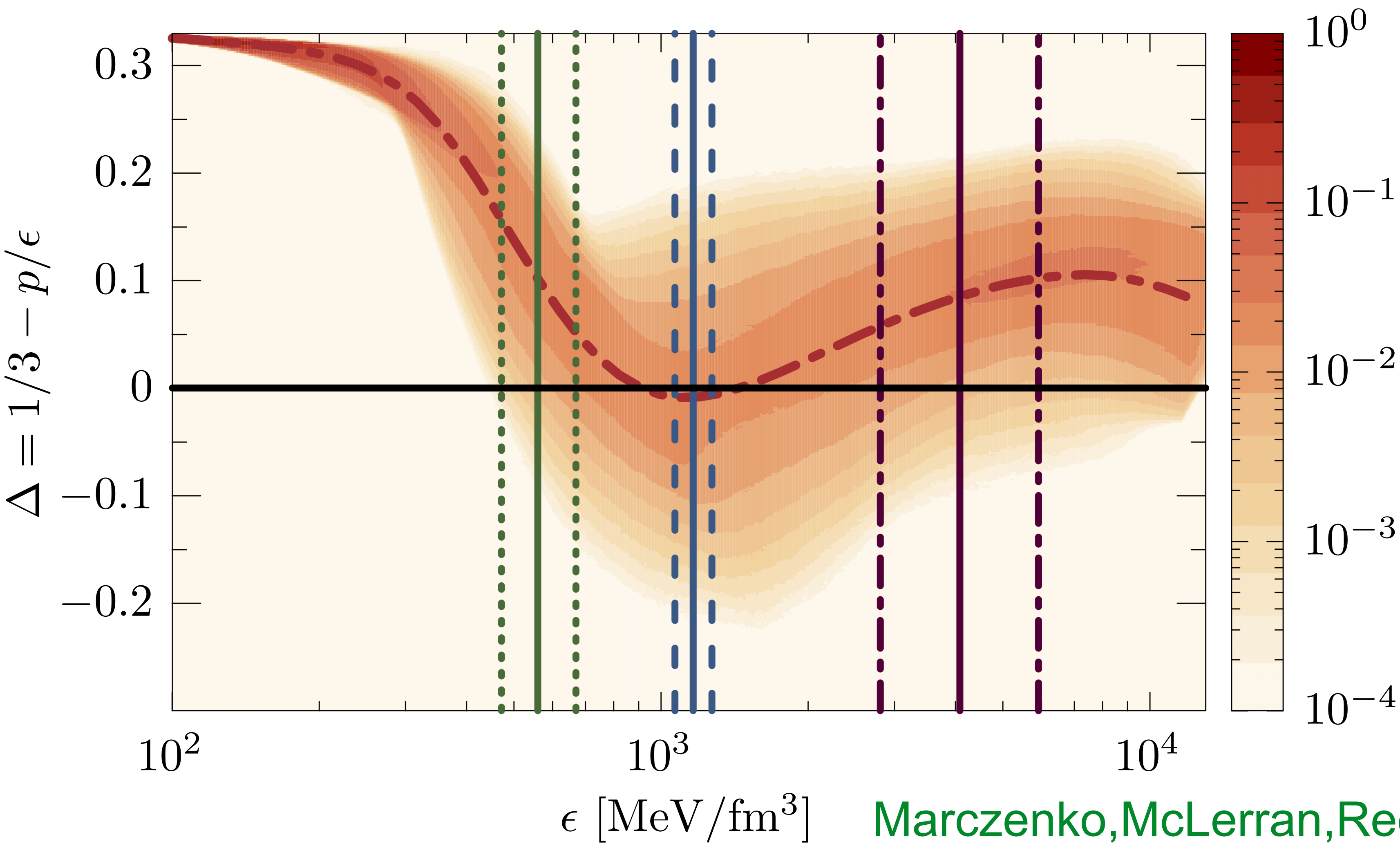
Marczenko,McLerran,Redlich,Sasaki (2022)

Ingredients: ChEFT, $2M_\odot$ & NICER pulsars, tidal deformability, pQCD

Conformal limit for $\varepsilon - 3P$

Fujimoto,Fukushima,McLerran,Praszalowicz (2022); Annala+ (2023) and so on…

Does not cross the limit $\varepsilon - 3P = 0$ at intermediate density



Marczenko,McLerran,Redlich,Sasaki (2022)

Rapid approach to $\varepsilon - 3P \to 0$ drives the peak in v_s^2
$\to$ Signature of conformality in neutron star?

Outline

1. Conformal limit in dense QCD

2. **Confronting weak-coupling vs lattice QCD results at finite isospin density**

QCD at finite isospin density

- Isospin chemical potential (conjugate to I_3):

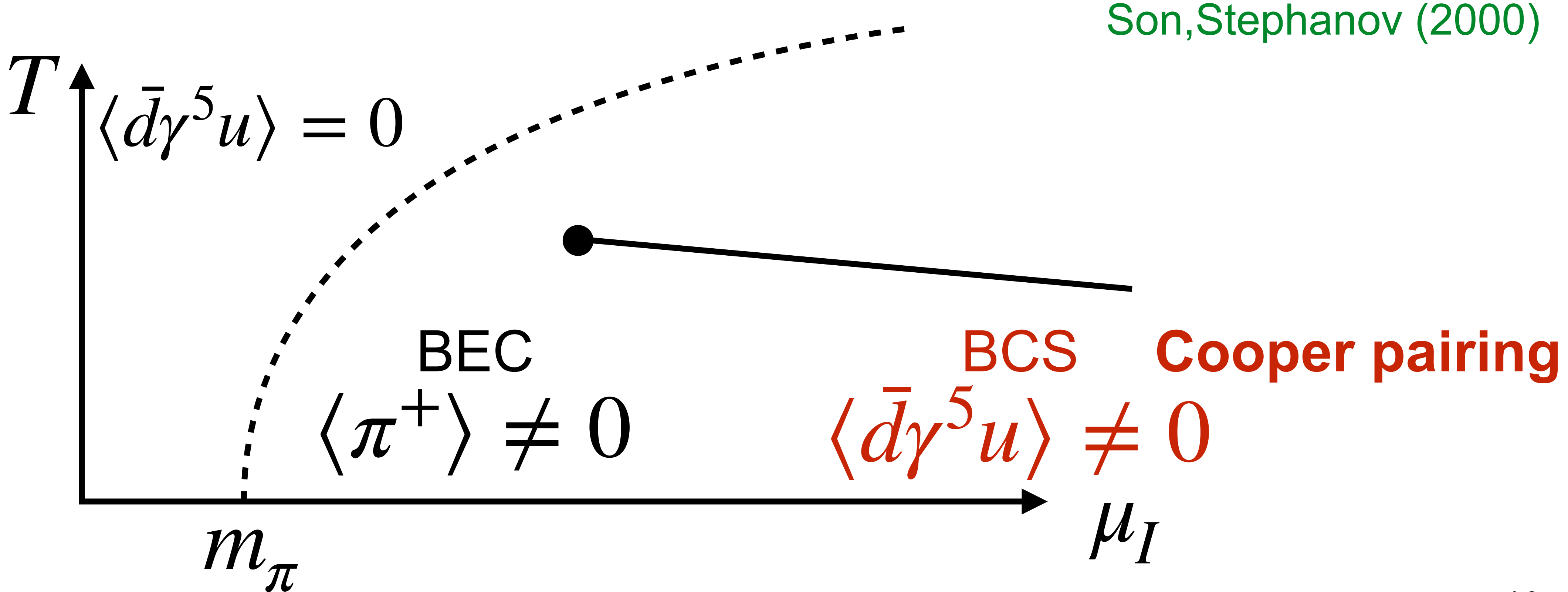
$$\mu_u = \frac{\mu_I}{2}, \quad \mu_d = -\frac{\mu_I}{2}$$ … Fermi surface of u & $\bar{d}$

Alford,Kapustin,Wilczek (1999);
Kogut,Sinclair (2002-); Beane,Detmold,Savage+ (2007-); Endrodi+ (2014-)…

- **No sign problem → can be simulated on the lattice!**

- Phases

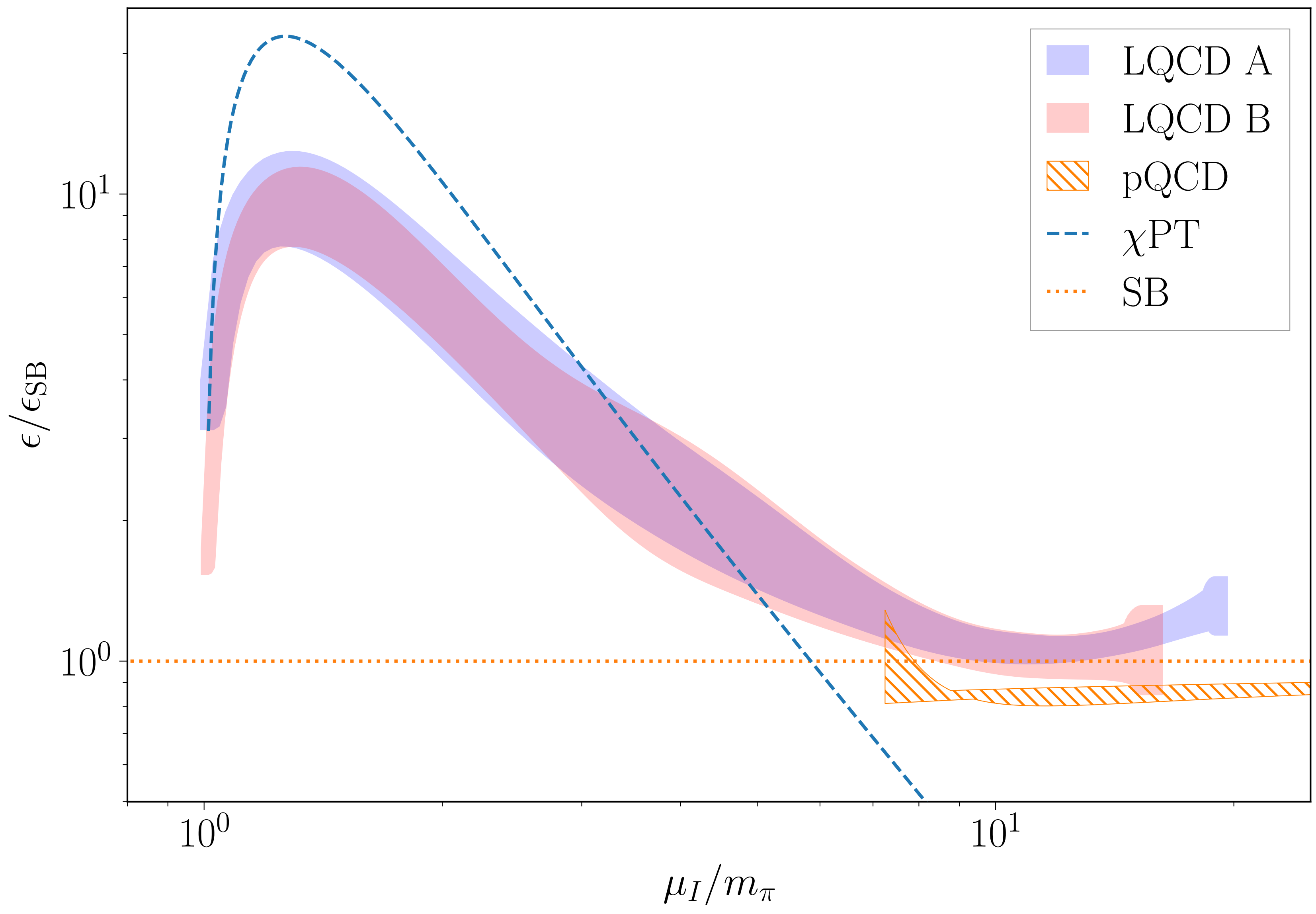
Son,Stephanov (2000)



Lattice data at finite isospin density

Abbott,Detmold,Romero-López+ (2023)

- Equation of state (EoS) is pinned down up to $\mu_I \sim 3\ \mathrm{GeV}$



Notation

- QCD_B: QCD at nonzero μ_B and zero μ_I
- QCD_I: QCD at nonzero μ_I and zero μ_B

Cooper pairing gap in weak coupling

Son (1998); Schäfer,Wilczek (1999); Pisarski,Rischke (1999);
Brown,Liu,Ren (1999); Wang,Rischke (2001); …

- Color-superconducting gap up to next to leading order:

$$\ln\left(\frac{\Delta}{\mu}\right) = -\frac{\sqrt{3}\pi}{2\sqrt{c_R}}\left(\frac{\alpha_s}{\pi}\right)^{-\frac{1}{2}} - \frac{5}{2}\ln\left(N_f\frac{\alpha_s}{\pi}\right) + \ln\frac{2^{\frac{13}{2}}}{\pi}$$

$$-\frac{\pi^2+4}{12c_R} - \zeta + \mathcal{O}(\alpha_s^{\frac{1}{2}})$$

$c_R = 2/3$ for $\bar{\mathbf{3}}$, $c_R = 4/3$ for $\mathbf{1}$ channel, $\zeta = \frac{1}{3}\ln 2$ for CFL, $\zeta = 0$ otherwise

… this formula is universal for QCD_B (color superconductivity) and QCD_I (pion condensation-like Cooper pairing) Fujimoto (2023)

Cooper pairing gap in weak coupling

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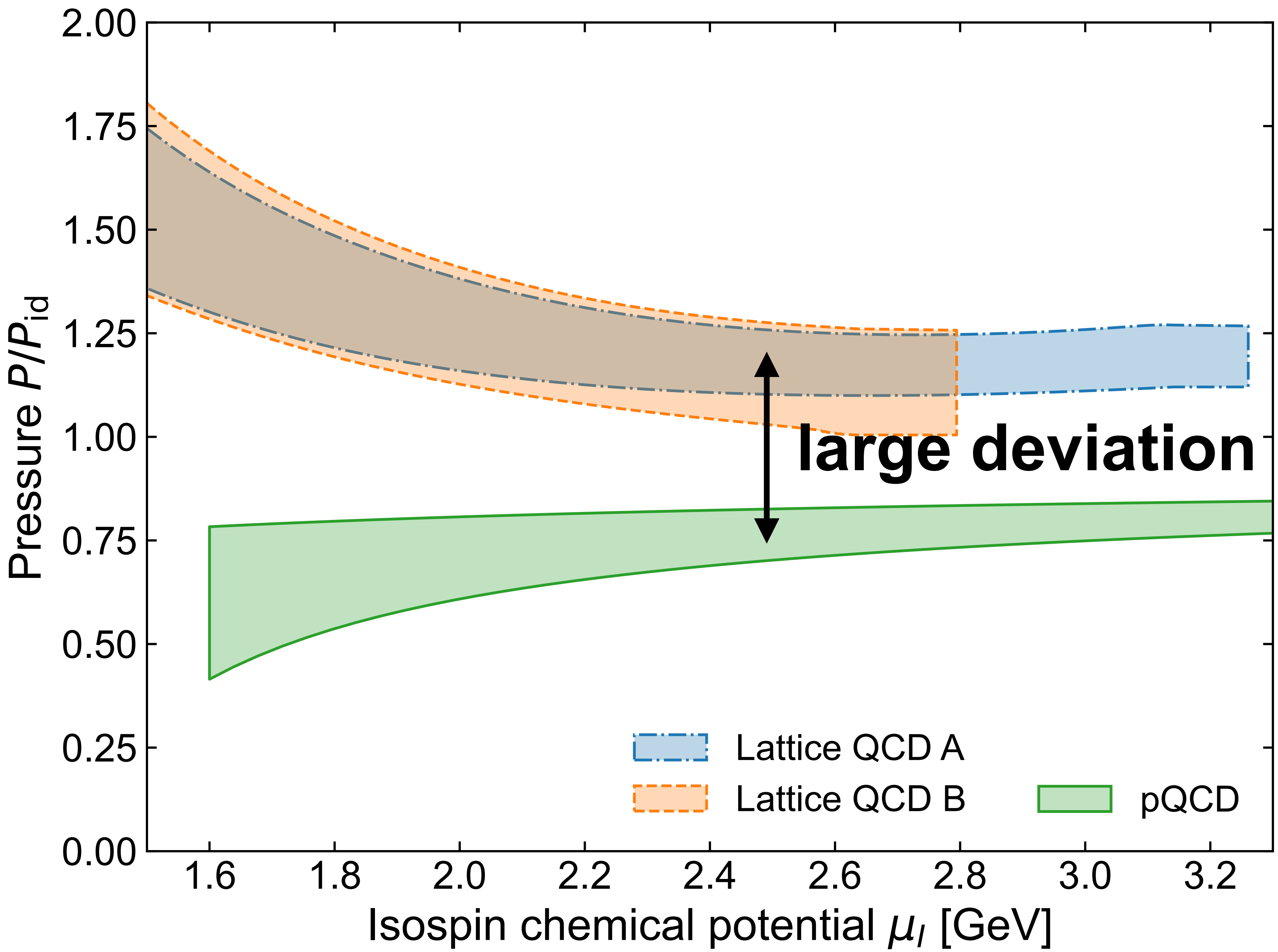
- Folklore: only applicable at very large μ e.g. $\mu \sim 10^8$ MeV [Rajagopal,Shuster (2000)]

- Standard perturbative QCD: valid up to $\mu \sim 10^3$ MeV

What is the applicability of this weak-coupling formula?

Weak-coupling results vs lattice data

Fujimoto (2023)

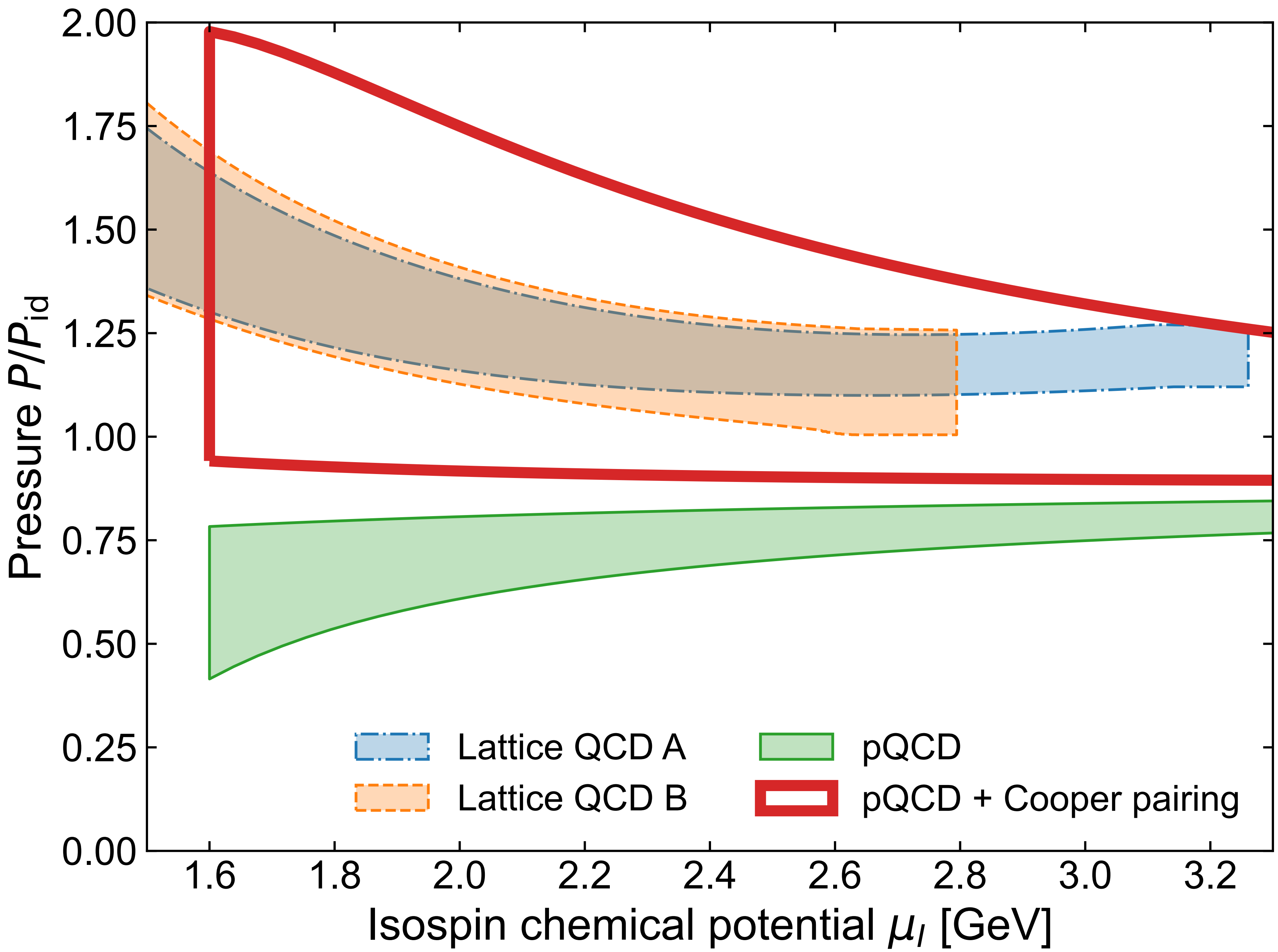


pQCD: $P/P_{\rm id} = 1 - 2\frac{\alpha_s}{\pi} - \left[2\ln\frac{\alpha_s}{\pi} + \frac{29}{6}\ln\frac{\bar{\Lambda}^2}{\mu_I^2} + 17.39\right]\left(\frac{\alpha_s}{\pi}\right)^2 + \mathcal{O}(\alpha_s^3)$

$[P_{\rm id} = \mu_I^4/(32\pi^2)]$

Weak-coupling results vs lattice data

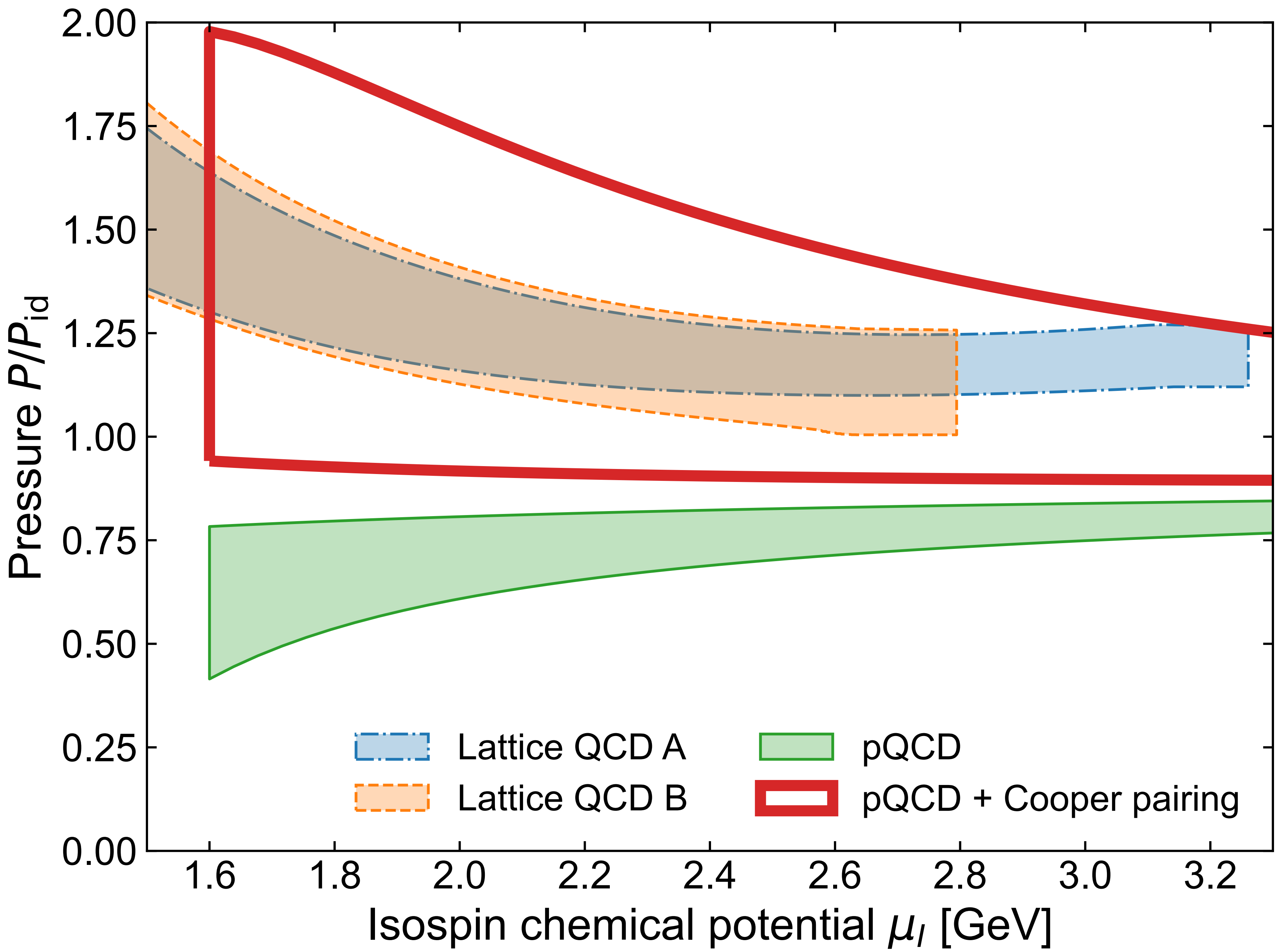
Fujimoto (2023)



+ Cooper pairing: $\delta P = \frac{3\mu_I^2}{8\pi^2}\Delta^2\left[1+\frac{\pi}{3}\left(\frac{\alpha_s}{\pi}\right)^{1/2}\right]$ … condensation energy

using the weak-coupling formula for Δ

Weak-coupling results vs lattice data



Empirical evidence for the weak-coupling Cooper pairing gap to be applicable down to $\mu \sim 10^3$ MeV

At least the magnitude is correct

Is the gap the only correction?

Alford,Braby,Paris,Reddy (2004)

$$P = a_4\mu^4 + a_2\mu^2 - B$$

- a_4: Ideal gas behavior + pQCD correction (Dominant)

- a_2: Gap correction $a_2 \propto \Delta^2$ (large),

  Quark mass $a_2 \propto -m_f^2$ (small, ~1%)

- B: Bag constant, typically $B^{1/4} \simeq 200$ MeV (small, ~0.5%)

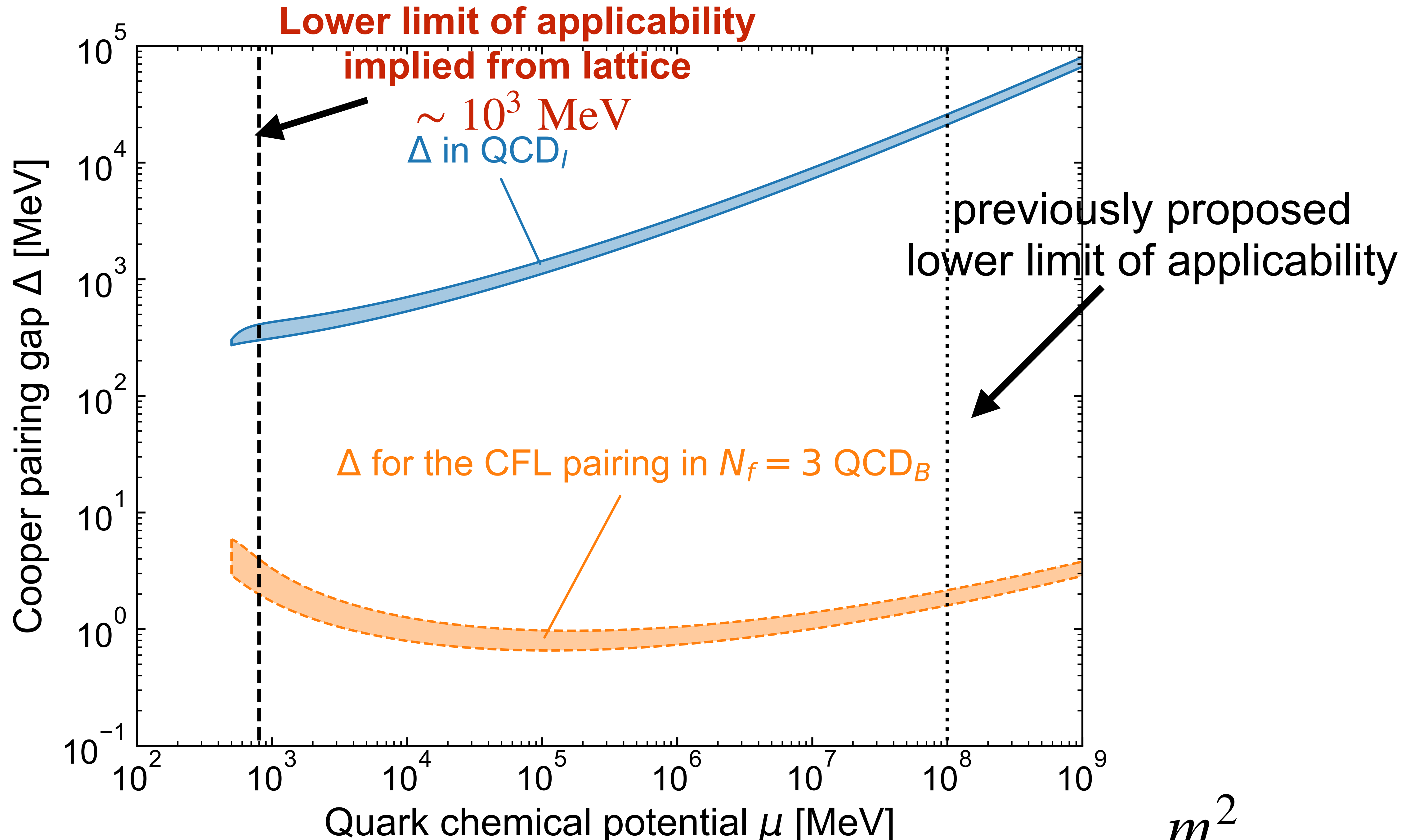
  Instantons, suppressed by $\dfrac{m_f}{\Lambda_{\mathrm{QCD}}} \sim 10^{-3}$

Shuryak; Kallman; Abrikosov; de Carvalho; Chemtob; Baluni (late 70s - early 80s)

Implication to color superconductivity

Fujimoto, *in prep.* (2024)

Weak-coupling Cooper pairing gap formula is reliable down to $\mu \sim 10^3$ MeV



$$\Delta_{\mathrm{CFL}} \sim 2 - 3 \ \mathrm{MeV} \text{ at } \mu = 800 \ \mathrm{MeV} \qquad \Delta_{\mathrm{CFL}} \sim \frac{m_s^2}{4\mu}$$

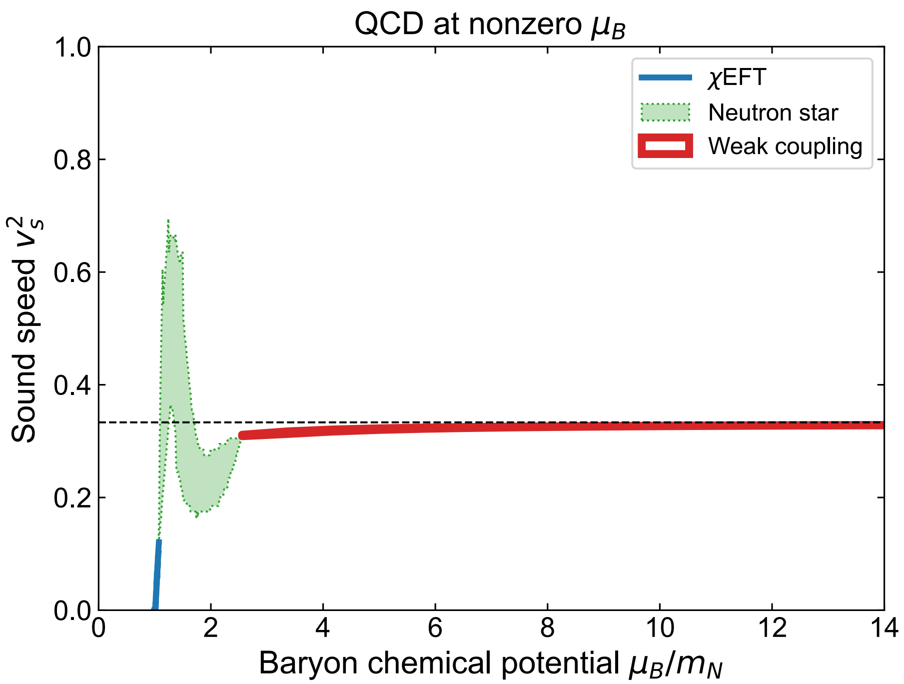
Finite-density lattice simulations

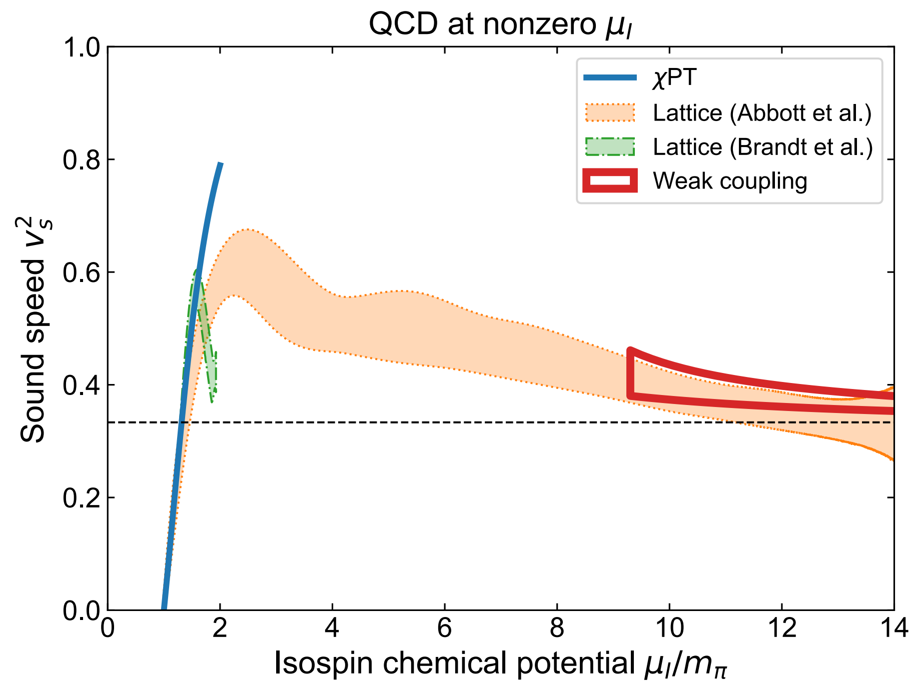
Fujimoto, *in prep.* (2024)

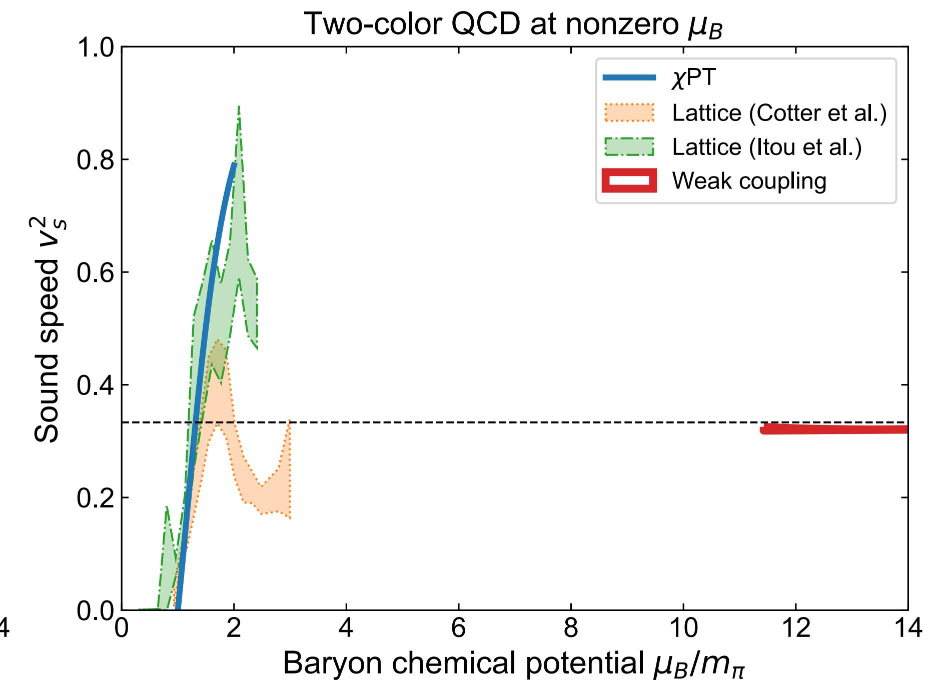
Want to know

Already known

Can in principle know







QCD at $\mu_B > 0$

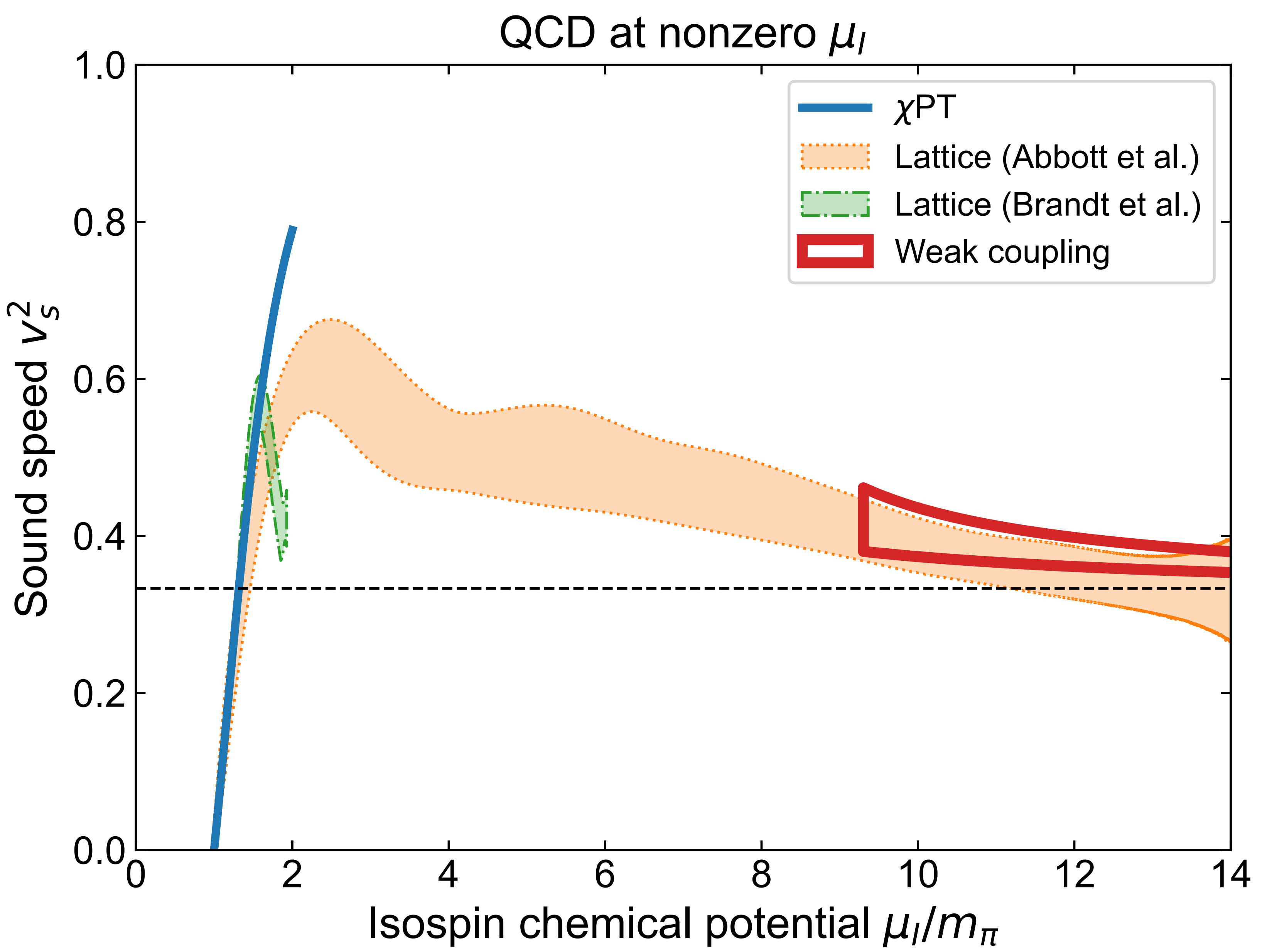
QCD at $\mu_I > 0$

$N_c = 2$ QCD at $\mu_B > 0$

Finite-density lattice simulations

Fujimoto, *in prep.* (2024)

Already known



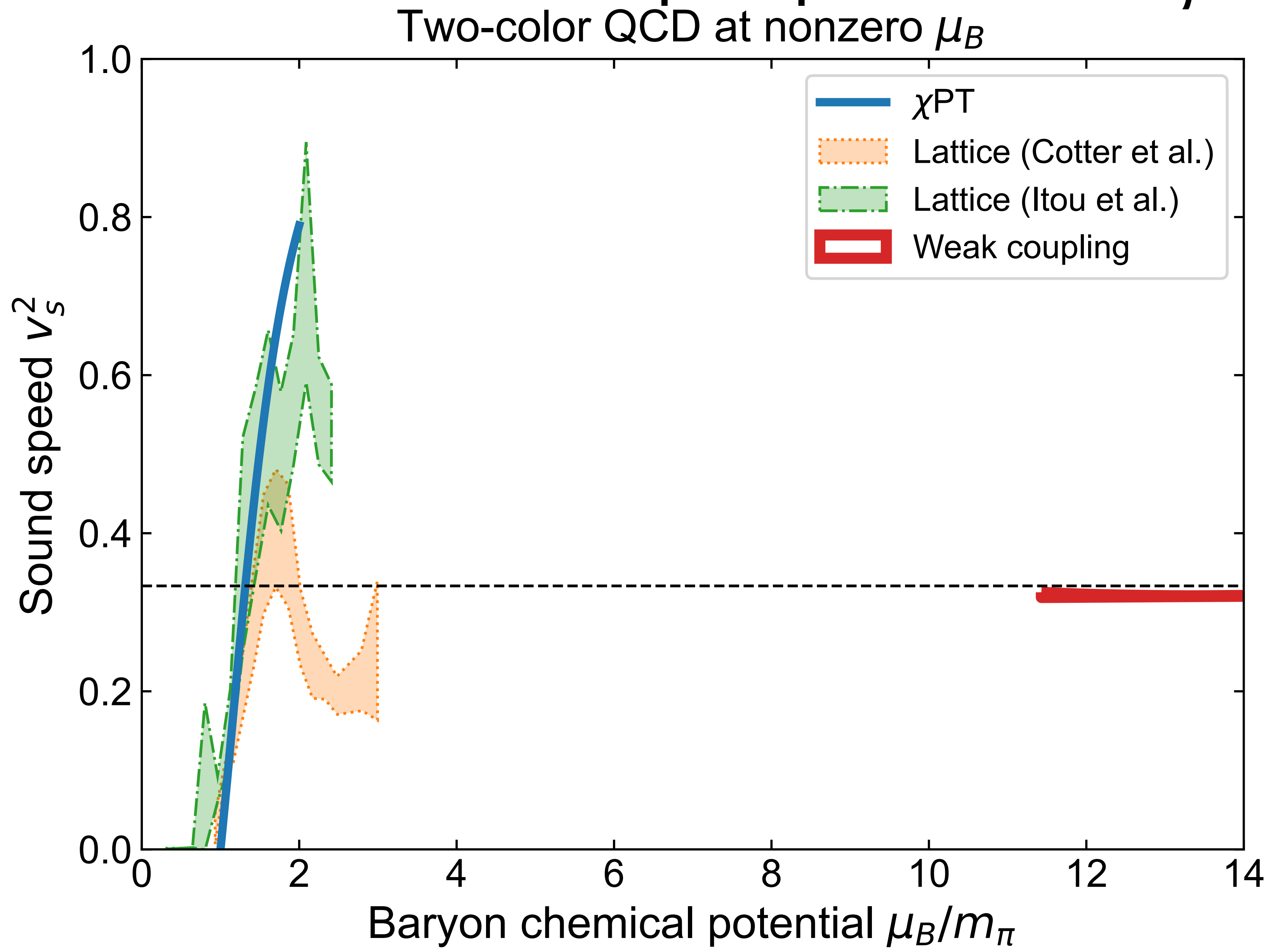
QCD at $\mu_I > 0$

Finite-density lattice simulations

Fujimoto, *in prep.* (2024)

Can in principle know

… Can the simulation extended up to perturbative μ ?



$N_c = 2$ QCD

at $\mu_B > 0$

Summary

- **QCD$_I$:** a testing ground for QCD$_B$. Lattice simulation feasible

- **Weak-coupling results** matches well with lattice QCD$_I$: Empirical evidence for the Cooper-pairing gap formula to be applicable down to $\mu \sim 10^3$ MeV

- Matching the error with lattice QCD$_I$: a prescription for fixing the ambiguity in the weak-coupling formula with less errors

- In principle, another crosscheck with lattice-QCD can be provided in $N_c = 2$