Higgs-confinement continuity in light of particle-vortex statistics

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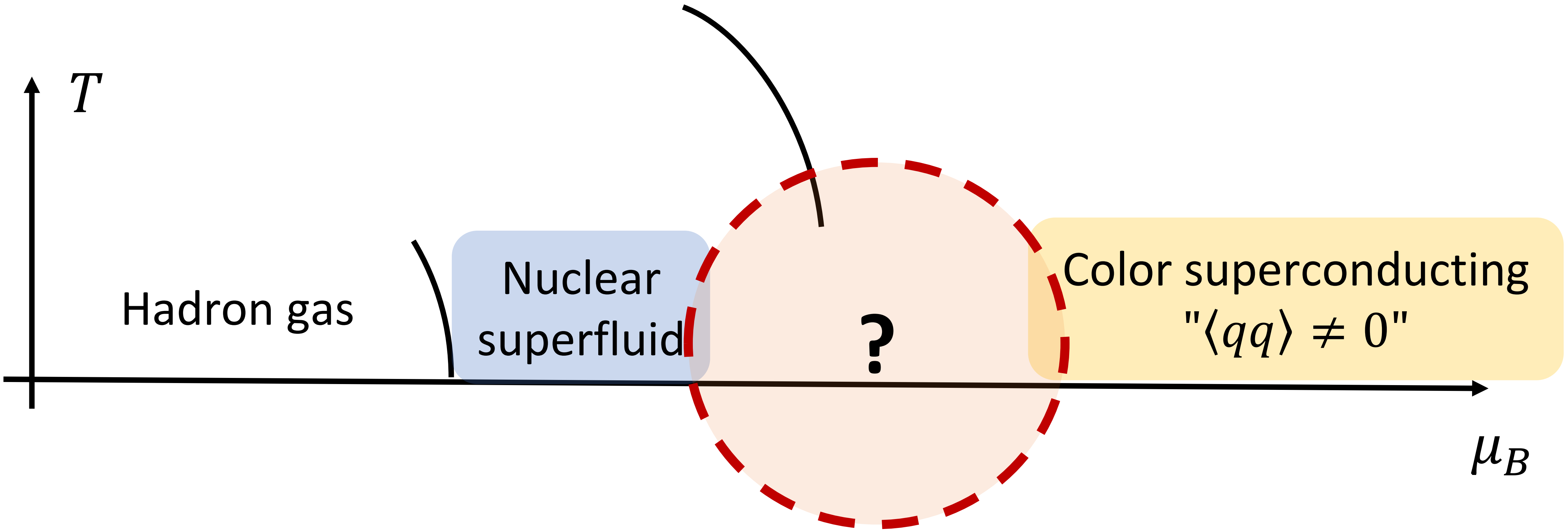
Dense QCD matter



same symmetry breaking pattern → No transition?
("quark-hadron continuity") [Schäfer-Wilczek '99]

(vague) main question

In general context:

Higgs phase (CFL phase) vs. **confinement phase (nuclear superfluid phase)**

Is **gauge-variant** (Higgs) **condensate** meaningful?

Generally, No. [Elizur '75]

- In particular, in fundamental gauge-Higgs systems, the Higgs and confining regimes are connected (**Higgs-confinement continuity**).
- However, for some models, the Higgs and confining phases are **distinguishable**.

→ recent discussion on quark-hadron continuity.

Higgs-confinement continuity

An idea behind quark-hadron continuity:

Higgs-confinement continuity

[Osterwalder-Seiler '78][Fradkin-Shenker '79][Banks-Rabinovici '79]

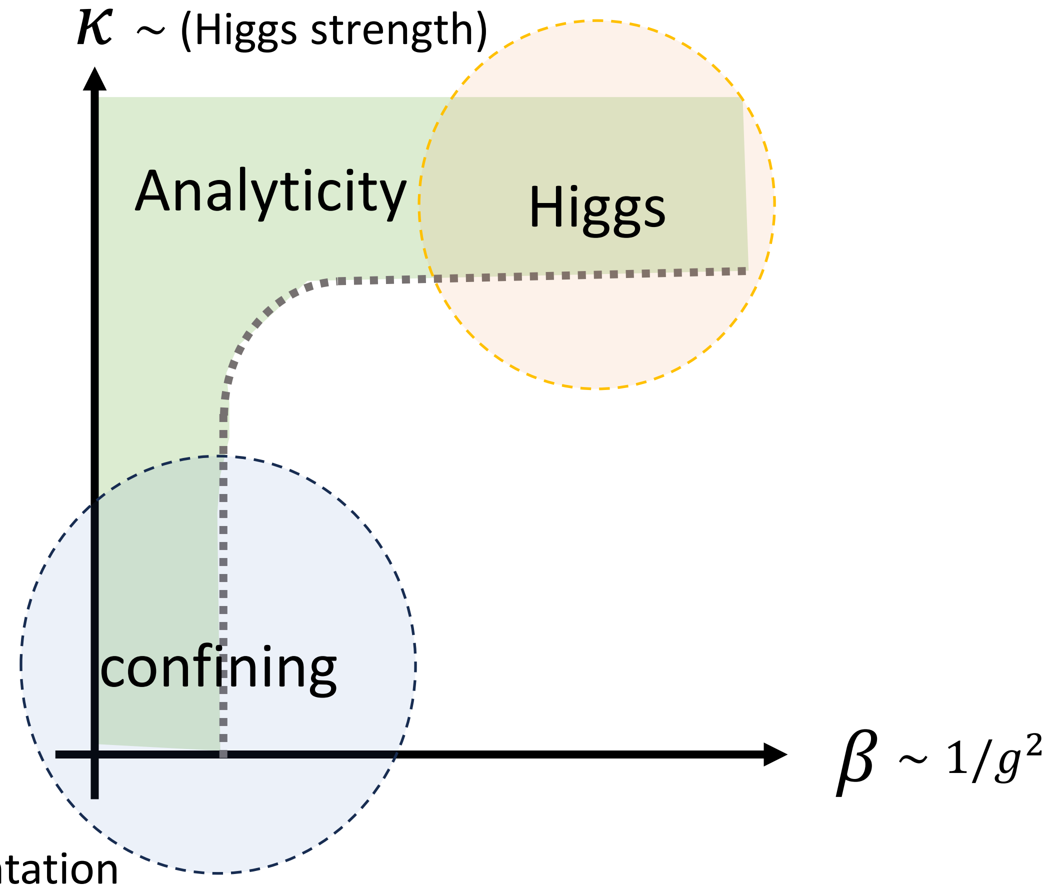
For fundamental gauge-Higgs systems, the confining regime (small β, κ) and Higgs regime (large β, κ) are connected by an analyticity region (without any transition).

⇨

Condensation of fundamental matter
"$\langle\phi\rangle \neq 0$" does not distinguish phases

The CFL diquark condensation is in (anti-)fundamental representation

e.g. charge-1 Abelian Higgs model
[U(1) gauge + charge-1 U(1)-valued scalar on lattice]
$S = \beta \sum_{\square:\text{ plaquette}} U_\square + \kappa \sum_{\ell:link} \phi_x^* U_\ell \phi_{x'} + c.c.$

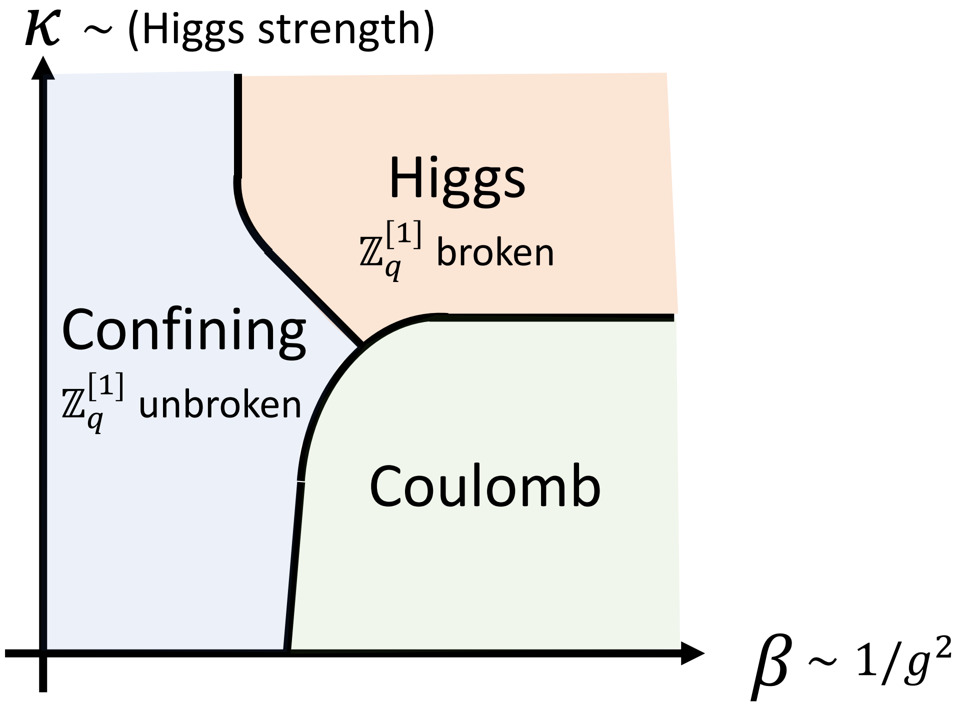


When distinguishable? to introduce the recent discussion……

e.g.) **charge-q Abelian Higgs Model** $(q > 1)$

The Higgs/confining phases are distinguished by perimeter-law/area-law of the Wilson loop

(equivalently $\mathbb{Z}_q^{[1]}$ broken/unbroken)

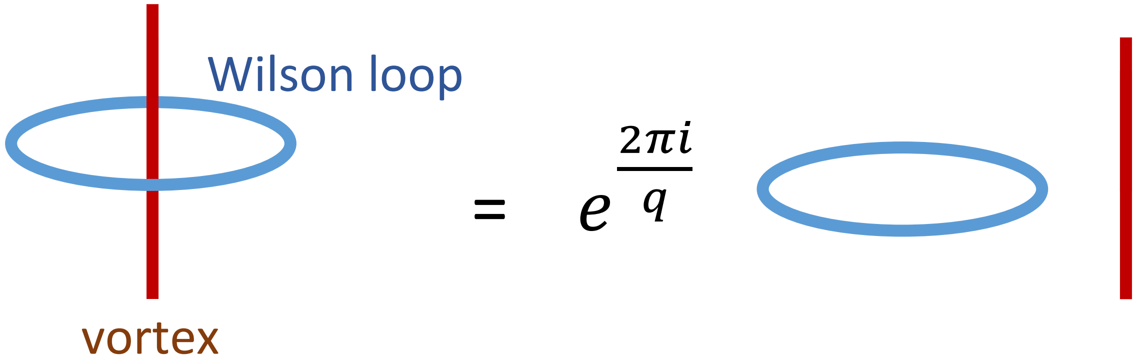


Another characterization: **topological ordering**

In the (low-energy effective theory of) Higgs phase, we have two topological operators, showing nontrivial mutual statistics:

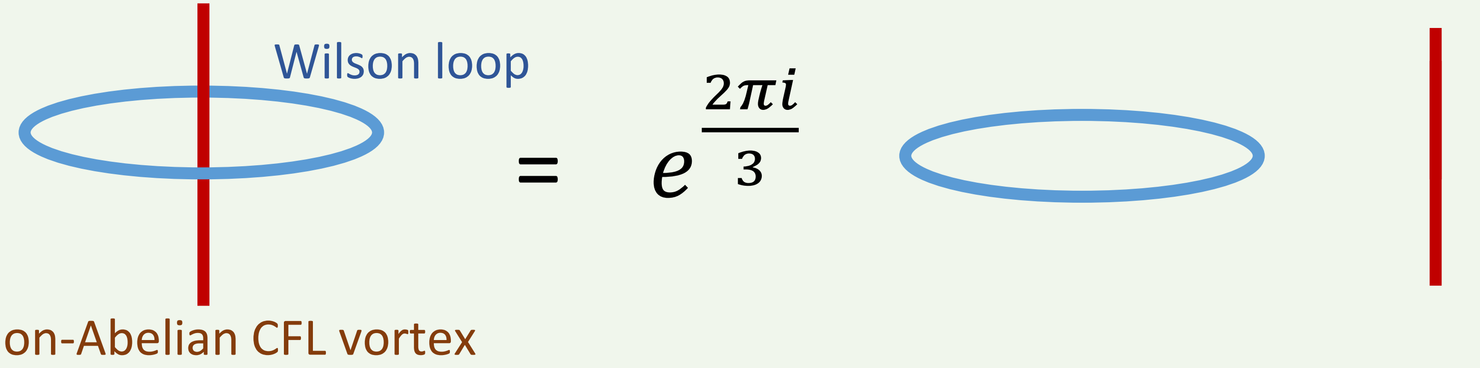
- Wilson loop: $W(C)$
- Vortex worldsheet: $V(S)$

$$\langle W(C)V(S)\rangle = e^{\frac{2\pi i}{q}\mathrm{Link}(C,S)}\langle W(C)\rangle\langle V(S)\rangle$$



Recent discussion

- Particle-vortex statistics in the CFL phase [Cherman-Sen-Yaffe ‘18]



Does this nontrivial AB phase signal a quark-hadron transition?

how connected, concretely?

- This AB phase does *not* mean topological order [Hirono-Tanizaki ‘18 ‘19]

(∵ The CFL vortex is (partially) a global vortex, which does not become topological)

At least, the previous logic does not apply here → continuity?

More constructively?

- Still, it was conjectured that **the AB phase can be an order parameter** for a Higgs-confinement transition, by studying an Abelian toy model (detailed later) [Cherman-Jacobson-Sen-Yaffe ‘20]. → transition?

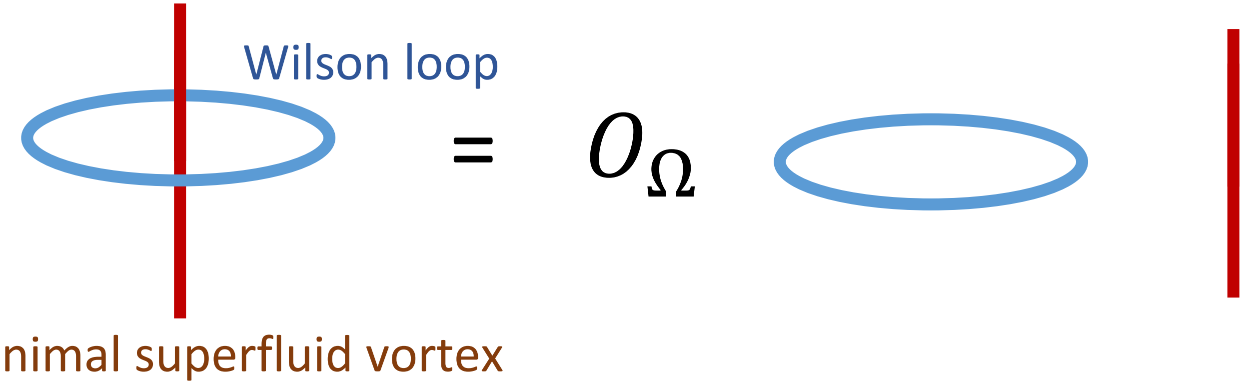
An order parameter?: Aharonov-Bohm phase

Conjectured order parameter: **AB phase $\boldsymbol{O_\Omega}$ around a minimal superfluid vortex**

In the low-energy effective theory,

$$O_\Omega := \frac{\langle W(C) V(S) \rangle}{\langle W(C) \rangle \langle V(S) \rangle}$$

We shall compute **phase of a large Wilson loop $\boldsymbol{W(C)}$ in the presence of vortex.**

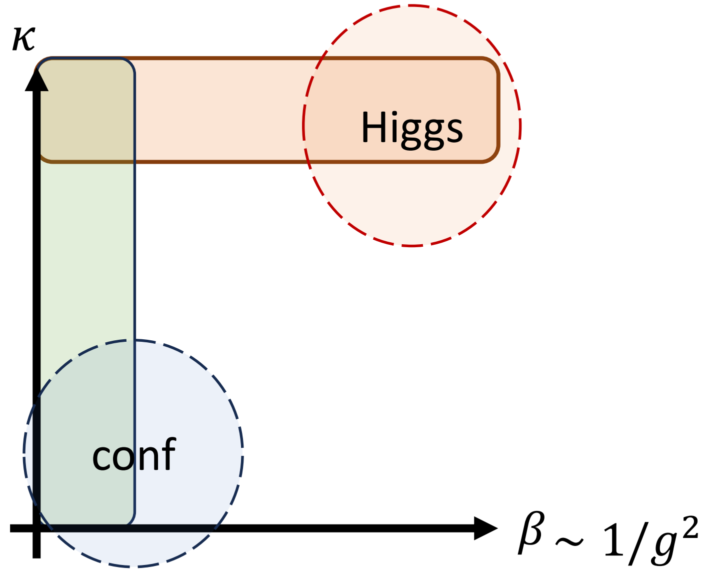


Results by a Fradkin-Shenker-like analysis

For superfluid fundamental gauge-Higgs systems, the **Aharonov-Bohm phase around the vortex** is **continuous** (or constant, if protected by symmetry) in the strong-coupling and deep-Higgs regions, connecting confining and Higgs regimes

Below, we illustrate this claim in the following two lattice models analogous to:

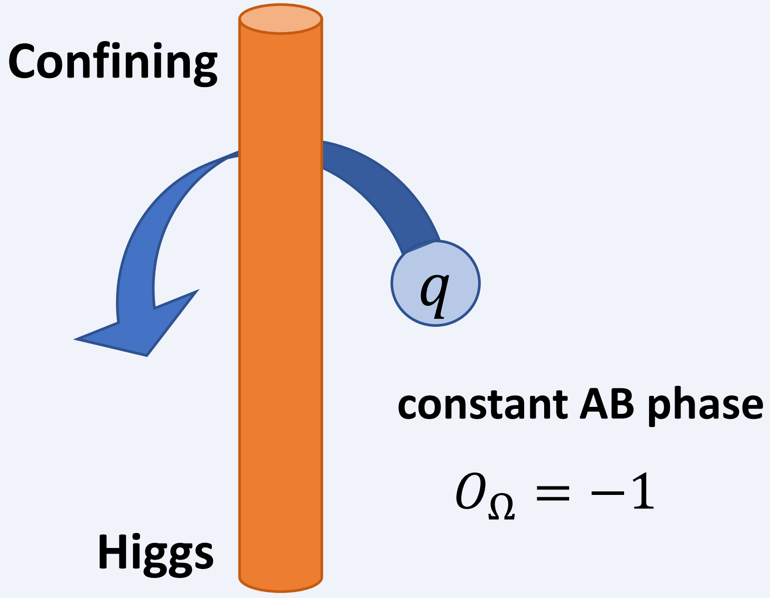
1) The Abelian toy model
2) Ginzburg-Landau model for CFL diquark



Results

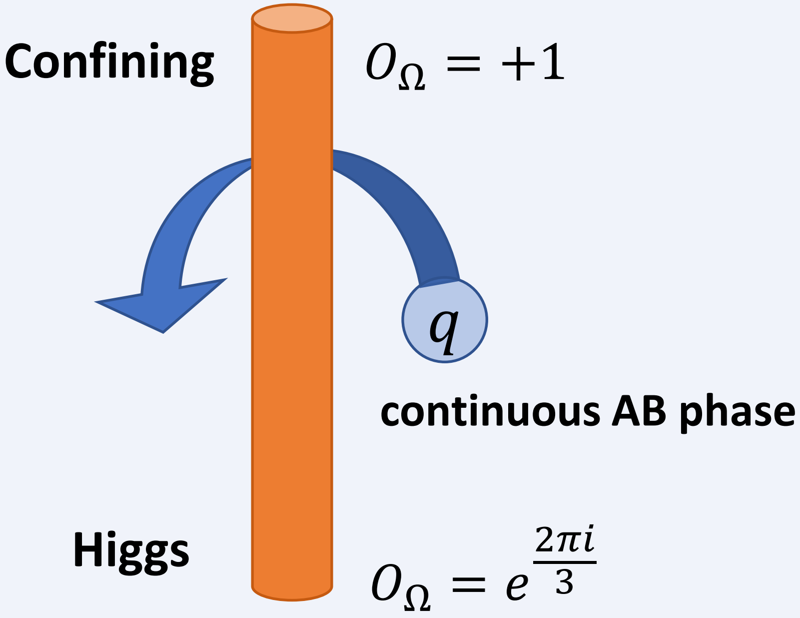
(1) Abelian toy model

($^{\exists}$ symmetry restricting $\boldsymbol{O_\Omega = \pm 1}$)



(2) GL-type model for CFL diquark

(+ superfluid "dibaryon" scalar)



cf.) discussion on vortex continuity
[Alford-Baym-Fukushima-Hatsuda-Tachibana '19]
[Chatterjee-Nitta-Yasui '19]

Example 1: the Abelian toy model

used to argue a Higgs-confinement transition [Cherman-Jacobson-Sen-Yaffe '20].

Field contents:

3d compact U(1) gauge a + charge-(± 1) matters (ϕ_+, ϕ_-) + neutral scalar ϕ_0

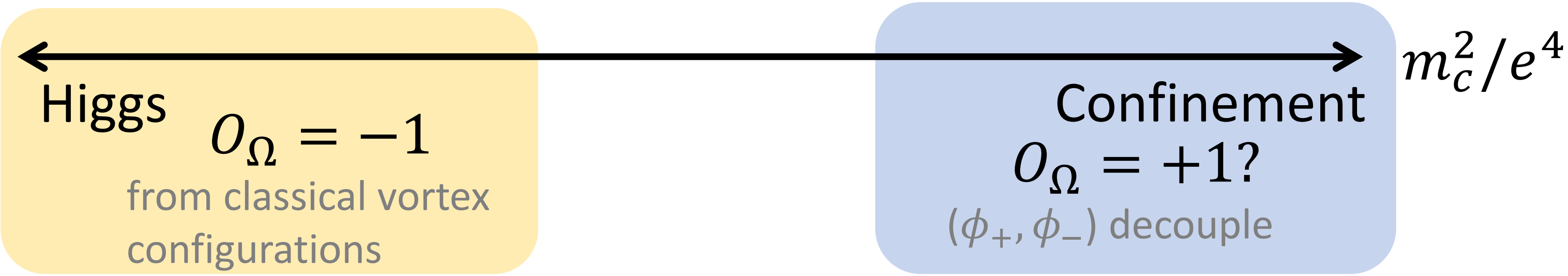
Action:

$$S = \int \frac{1}{2e^2}|da|^2 + |D\phi_+|^2 + |D\phi_-|^2 + |d\phi_0|^2 + V_c(\phi_+) + V_c(\phi_-) + V_0(\phi_0) + \epsilon\phi_+\phi_-\phi_0 + c.c.$$

$V_0(\phi_0)$: wine bottle potential → ϕ_0 condensation (superfluidity)

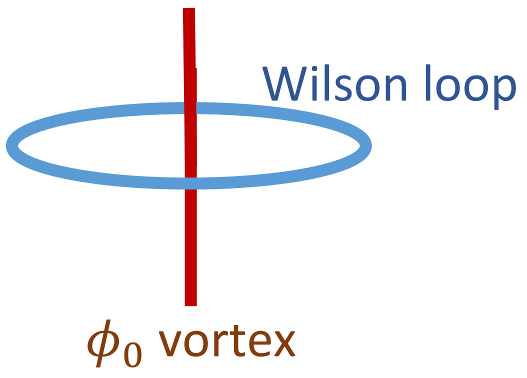
$V_c(\phi_\pm) = m_c^2|\phi_\pm|^2 + \lambda_c|\phi_\pm|^4$: identical potential for charged matters (ϕ_+, ϕ_-)

AB phase as an order parameter?:



The AB phase must be ± 1 due to $\mathbb{Z}_2$ symmetry $[\phi_\pm \to \phi_\mp, a \to -a]$ → **transition somewhere?**

Example 1: the Abelian toy model



Result: ***the AB phase is -1 (constant!) in both regions*** (skip lattice details)

- Deep Higgs region

In the deep Higgs limit, the gauge field a is frozen to be $\frac{d\varphi_+ - d\varphi_-}{2}$ (with $\phi_\pm = v\, e^{i\,\varphi_\pm}$).
The minimal ϕ_0 vortex rotates $(\phi_+\phi_-)$ by 2π asymptotically.

$$\langle W(C)V(S)\rangle \sim \left\langle e^{i\int_C \frac{d\varphi_+ - d\varphi_-}{2}}\right\rangle_{\mathrm{vortex}} \sim -1$$

in accordance with [Cherman-Jacobson-Sen-Yaffe ‘20].

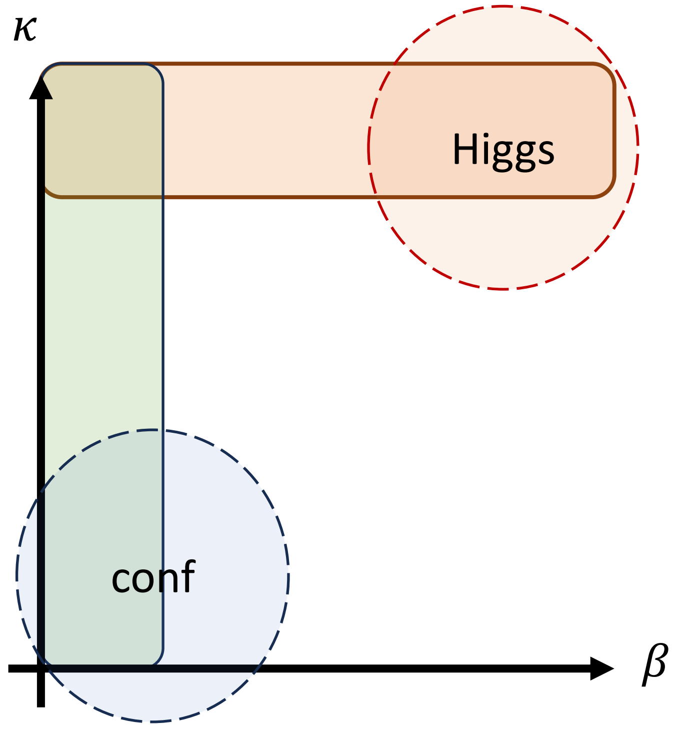
- Strong coupling region

Even if charged matters are heavy, **an asymptotically large Wilson loop is dominated by screened perimeter-law part**, which can be affected by ϕ_0 vortex.

For example, in the “deep confining regime” ($\beta \to +0$, small κ),

$$\langle W(C)V(S)\rangle = \left\langle \prod_{\ell \in C} U_\ell \right\rangle_{\mathrm{vortex}} \sim \left\langle \prod_{\ell\in C} [(\phi_+ \text{ hopping})^* + (\phi_- \text{ hopping})] \right\rangle_{\mathrm{vortex}}$$

$$e^{i\,\theta_1} + e^{i\,\theta_2} = (e^{i\,\theta_1}\sqrt{e^{i\,(-\theta_1+\theta_2)}})|e^{i\,\theta_1} + e^{i\,\theta_2}| \to \sim \left\langle e^{i\int_C \frac{d\varphi_+ - d\varphi_-}{2}}\right\rangle_{\mathrm{vortex}} \sim -1 \text{ (matched!)}$$

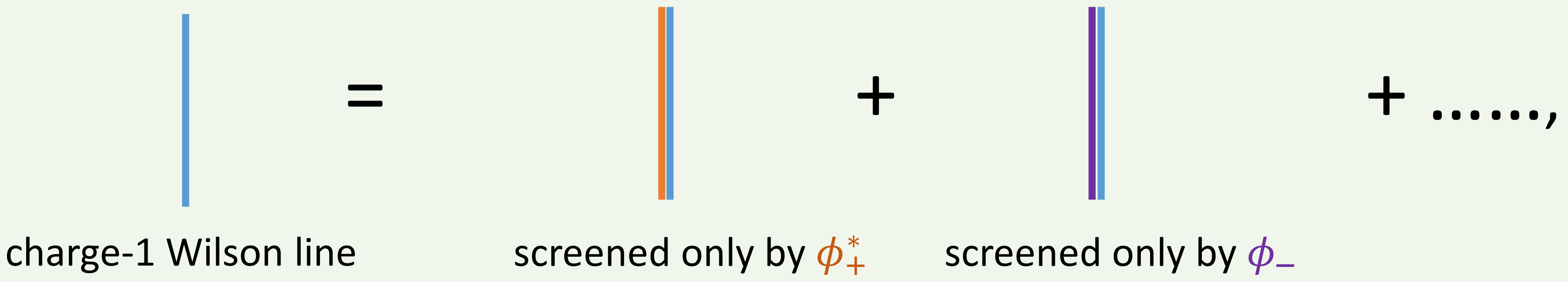


(Actually, $O_\Omega = -1$ without κ expansion)

Aside: Aharonov-Bohm phase as Berry phase

<u>Paradox (in deep confining limit) ?</u>

When $\epsilon\phi_+\phi_-\phi_0$ is weak, the screening contribution (in worldline representation) looks like:



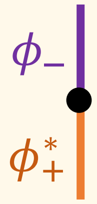
which leads to a trivial phase $O_\Omega = +1$? (cf. [Cherman-Jacobson-Sen-Yaffe '24]).

Short answer: NO.
For $\epsilon \neq 0$ (even if it is small), mixing configurations (alternating ϕ_+^* and ϕ_-) are significant due to the combinatorial factor since #links of the Wilson loop is arbitrary large.

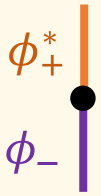
Aside: Aharonov-Bohm phase as Berry phase

<u>3-point interaction:</u> $\epsilon\phi_+\phi_-\phi_0$ + c.c. (in terms of (ϕ_+,ϕ_-) worldline)

⟷ **mixing mass** term (when ϕ_0 is fixed)



$\tilde{\epsilon}\,\phi_0$,



$\tilde{\epsilon}\,\phi_0^*$

($\tilde{\epsilon} = e^{-1/2\epsilon}$ in the Villain lattice formulation given in [Cherman-Jacobson-Sen-Yaffe '24]).)

The phase factor of the Wilson loop (for **slowly-varying** $\phi_0 = v\, e^{i\,\varphi_0}$) can be written as,

$$W(C) \sim \mathrm{Tr}\left(\prod_{x\in C}\begin{bmatrix} 1 & \tilde{\epsilon}\,\phi_0(x) \\ \tilde{\epsilon}\,\phi_0^*(x) & 1 \end{bmatrix}\right) \sim \mathrm{Tr}\left(\prod_{x\in C}\left[(1+\tilde{\epsilon}\,v)|0_x\rangle\langle 0_x| + (1-\tilde{\epsilon}\,v)|1_x\rangle\langle 1_x|\right]\right)$$

$$\sim \prod_{\ell\in C}\langle 0_x|0_{x'}\rangle \sim e^{\frac{i}{2}\int d\varphi_0} = -1$$

Berry phase of worldline quantum mechanics on loop C!

$|0_x\rangle$ is the ground state of "Hamiltonian"

$$H = -\tilde{\epsilon}\,v\,(\cos\varphi_0\,\sigma_X - \sin\varphi_0\,\sigma_Y)$$

($\rightarrow$ Berry connection $A = -\frac{1}{2}\,\mathrm{d}\varphi_0$)

Example 2: Ginzburg-Landau model for diquark

(anti-)fundamental
CFL diquark $\Phi^{ai} \sim \epsilon^{abc}\,\epsilon^{ijk}\,q^t_{bj} C\gamma^5 q_{ck}$

We add superfluid "dibaryon"
for nuclear superfluid phase

Field contents:

SU(3) gauge a + (3×3)-matrix-valued fundamental matter Φ + neutral scalar ϕ_0

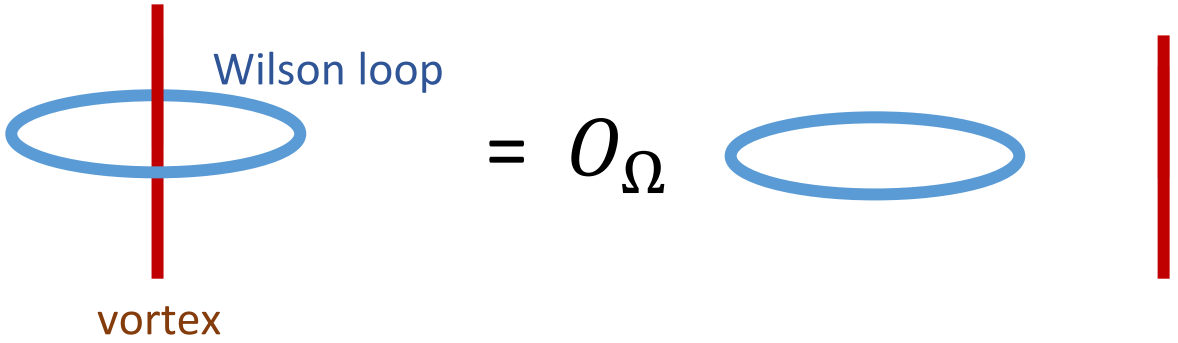
Action:

$$S = \int |f|^2 + |D\Phi|^2 + |D\phi_0|^2 + V\left(\mathrm{tr}\,\Phi^\dagger\Phi\right) + V_0(\phi_0) + \epsilon\,\phi_0^*(\det\Phi) + c.c.$$

$V_0(\phi_0)$: wine bottle potential → ϕ_0 condensation (superfluidity)

By tuning $V\left(\mathrm{tr}\,\Phi^\dagger\Phi\right)$, this model has superfluid confining regime [nuclear superfluidity] and Higgs regime [CFL].

(apparent) mismatch of AB phase:



$$, \quad O_\Omega = \begin{cases} +1\ ? & \text{(Confining limit)} \\ e^{\frac{2\pi i}{3}} & \text{(Higgs limit)} \end{cases}$$

Example 2: Ginzburg-Landau model for diquark

We can perform the similar analysis on an analogous lattice model

- **Deep Higgs region**

$$\langle W(C)V(S)\rangle \sim e^{\frac{2\pi i}{3}}$$

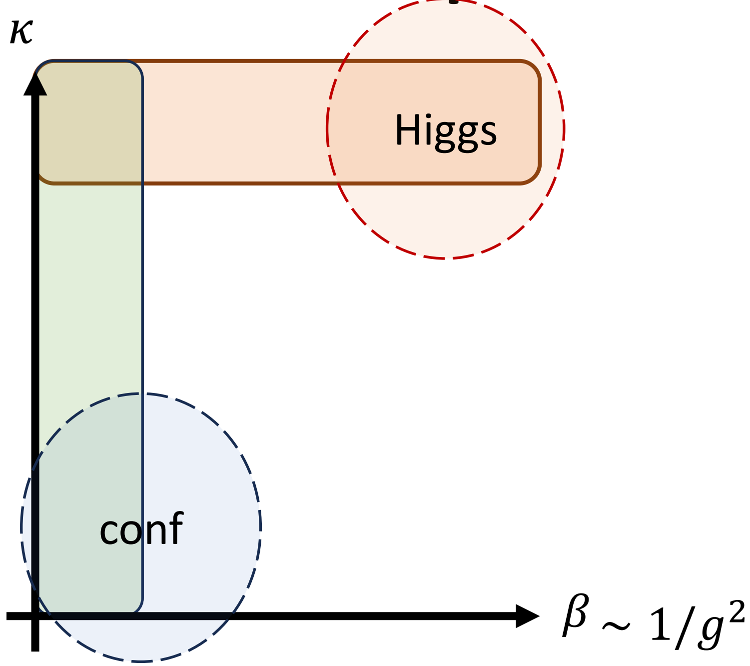
reproducing [Cherman-Sen-Yaffe '18]

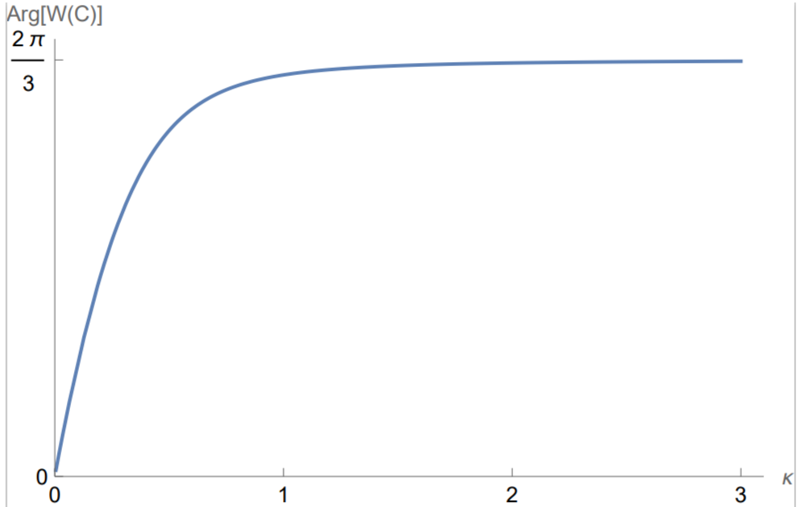
- **Strong coupling region**

The AB phase is trivial in the deep confining limit ($\beta \to 0$, small κ)

$$\langle W(C)V(S)\rangle \sim 1$$

Still, the AB phase is **continuous** and smoothly interpolates between 1 ($\kappa \to +0$) and $e^{\frac{2\pi i}{3}}$ ($\kappa \to +\infty$) in the strong coupling limit ($\beta \to 0$):



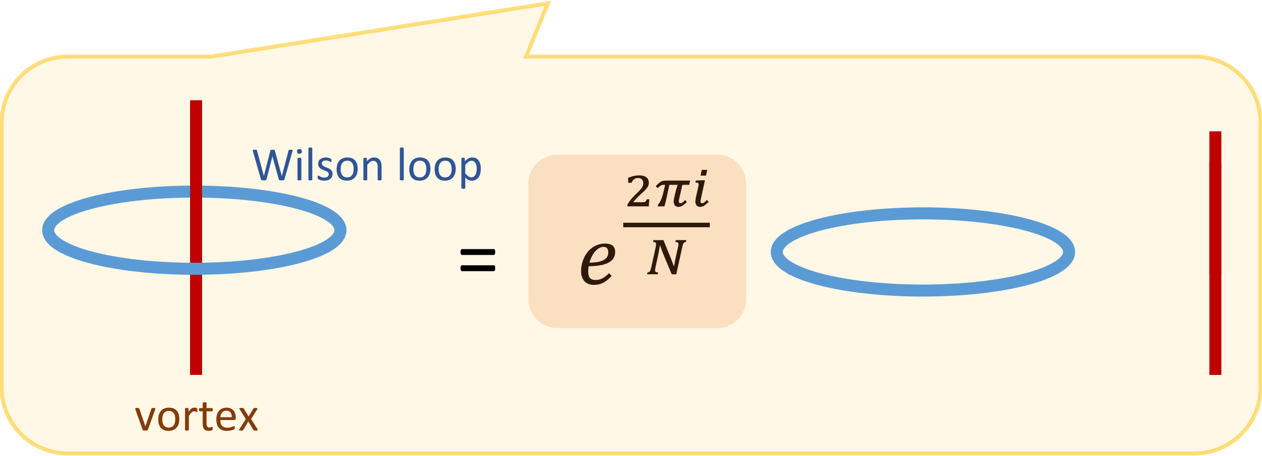


AB phase can smoothly vary!

Summary

e.g.) diquark condensation in dense QCD

In some superfluid gauge-Higgs systems,



<u>Recently-debated issue</u>:
Does the nontrivial AB phase necessitate a transition?

<u>Claim:</u> For fundamental superfluid gauge-Higgs systems,
the AB phase respects the Higgs-confinement continuity.
⇒ the quark-hadron continuity is still a possible scenario.