Sound velocity peak

and

a dual model of cold, dense QCD

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Refs) Baym-Hatsuda-TK-Powell-Song-Takatsuka, "QHC", review on neutron stars (2018)

TK, "Stiffening of matter in quark-hadron continuity" PRD (2021)

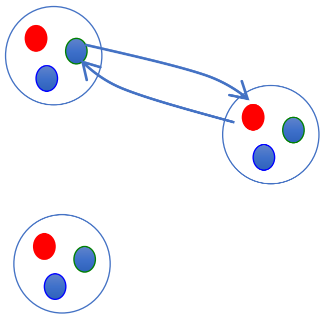
Fujimoto-TK-McLerran, "IdylliQ matter model" PRL (2024)

State of matter: overview

($n_0 = 0.16\ \mathrm{fm}^{-3}$)

[Masuda+ '12; TK+ '14]

- few meson exchange
- nucleons only

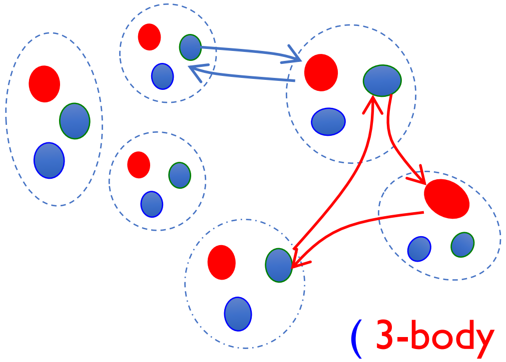


ab-initio nuclear cal.

laboratory experiments

steady progress

- many-quark exchange
- structural change,...
- hyperons, Δ, ...

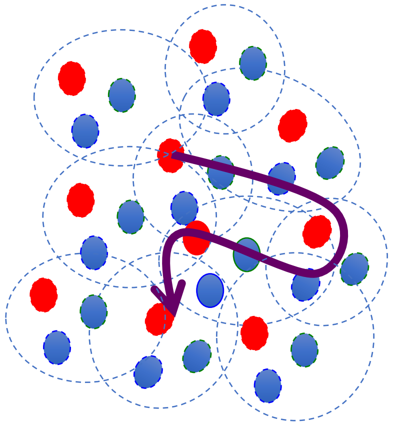


(3-body)

most difficult

(d.o.f ??)

- Baryons overlap
- Quark Fermi sea



strongly correlated

(d.o.f : quasi-particles??)

not explored well

Fujimoto-san's talk

(pQCD)

[Freedman-McLerran, Kurkela+, Fujimoto+...]

~ 1.4 $M_\odot$ ~ 2 $M_\odot$

n_B

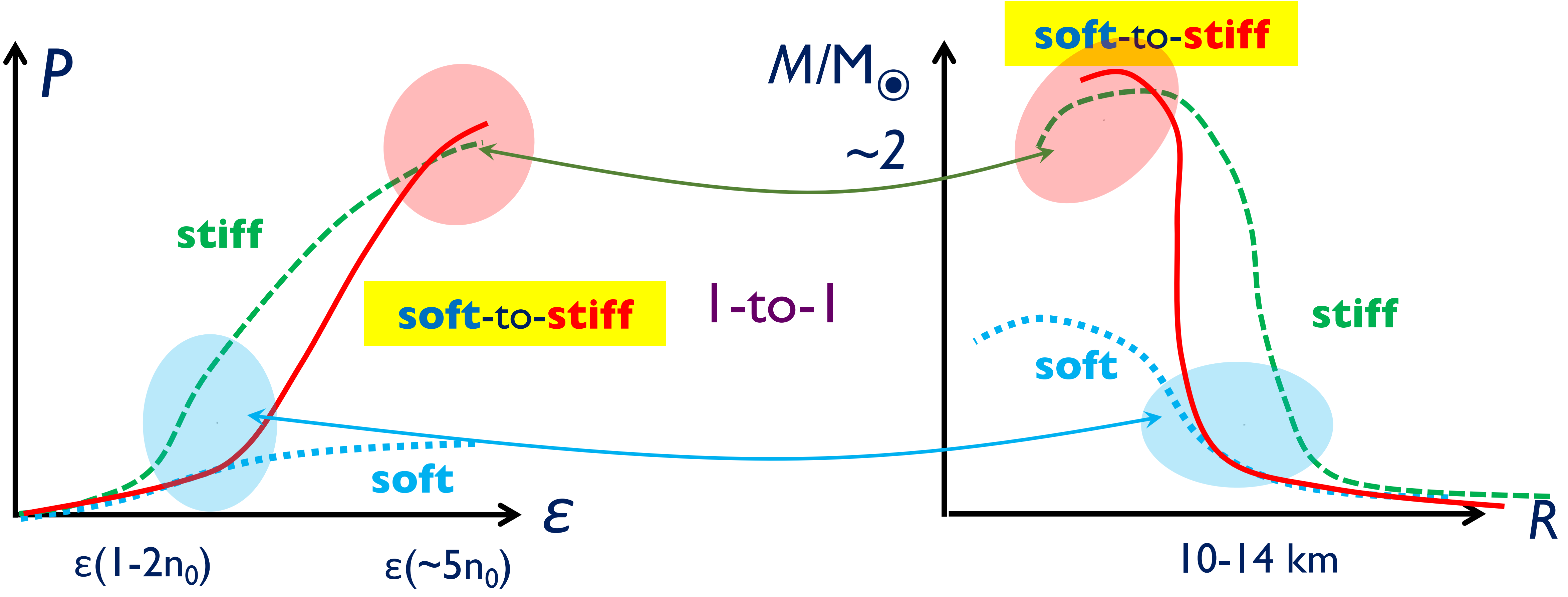
Hints from NS

~ $2n_0$ ~ $5n_0$ ~ $40n_0$

EoS & Neutron Star M-R relation

Einstein eq. : $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$

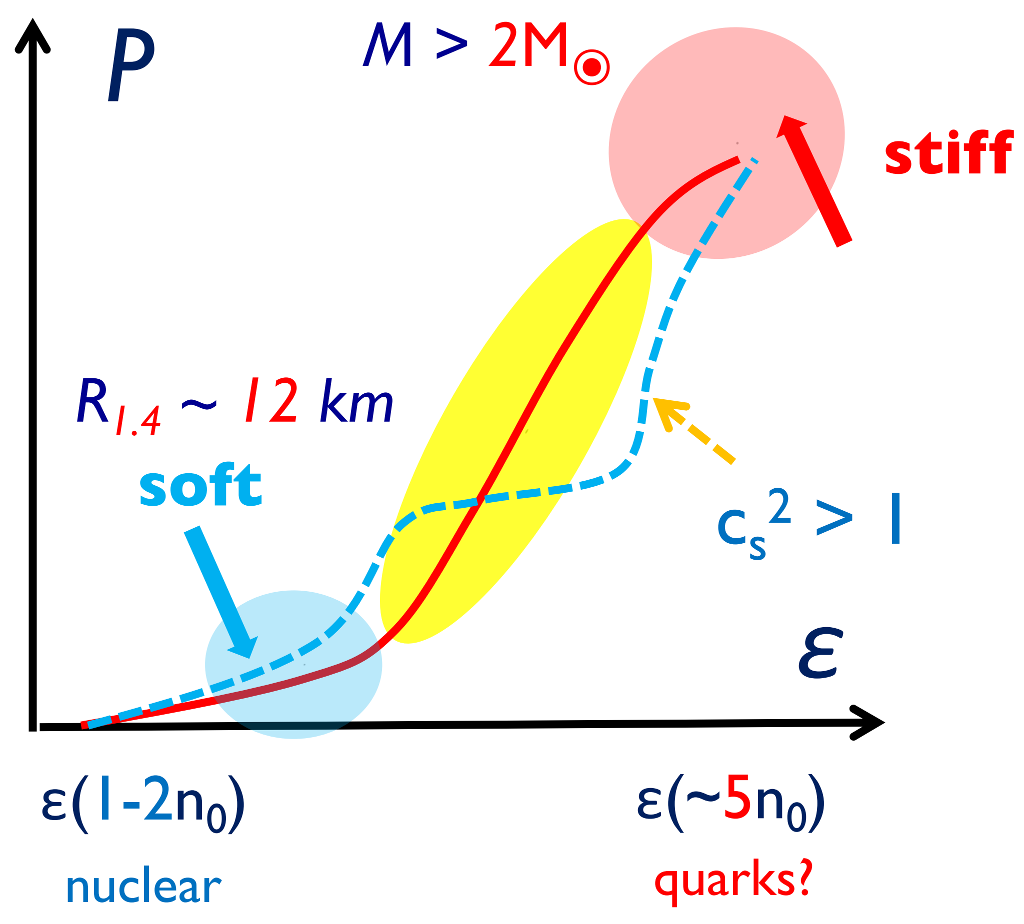
QCD (+EW) EoS

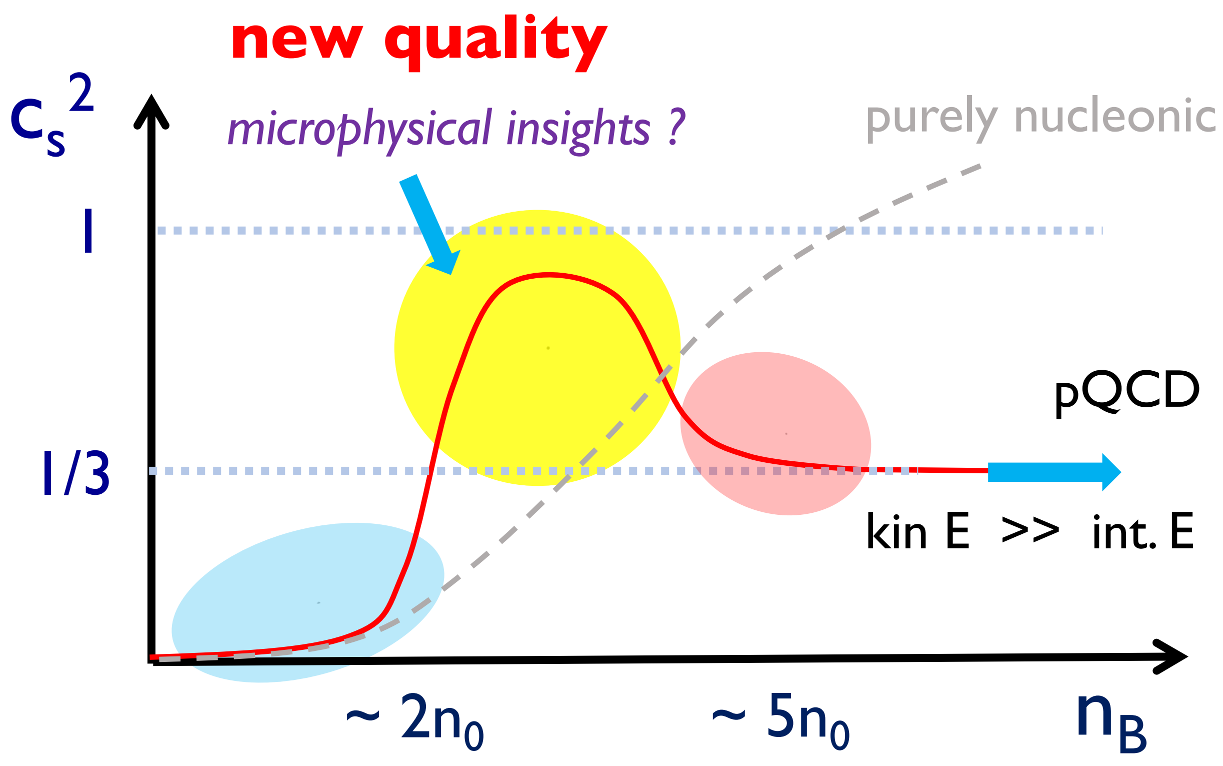


Soft to stiff is challenging:

sound velocity: $c_s^2 = dP/d\varepsilon < 1$ (*causality*)

nuclear & *quark* physics constrain each other



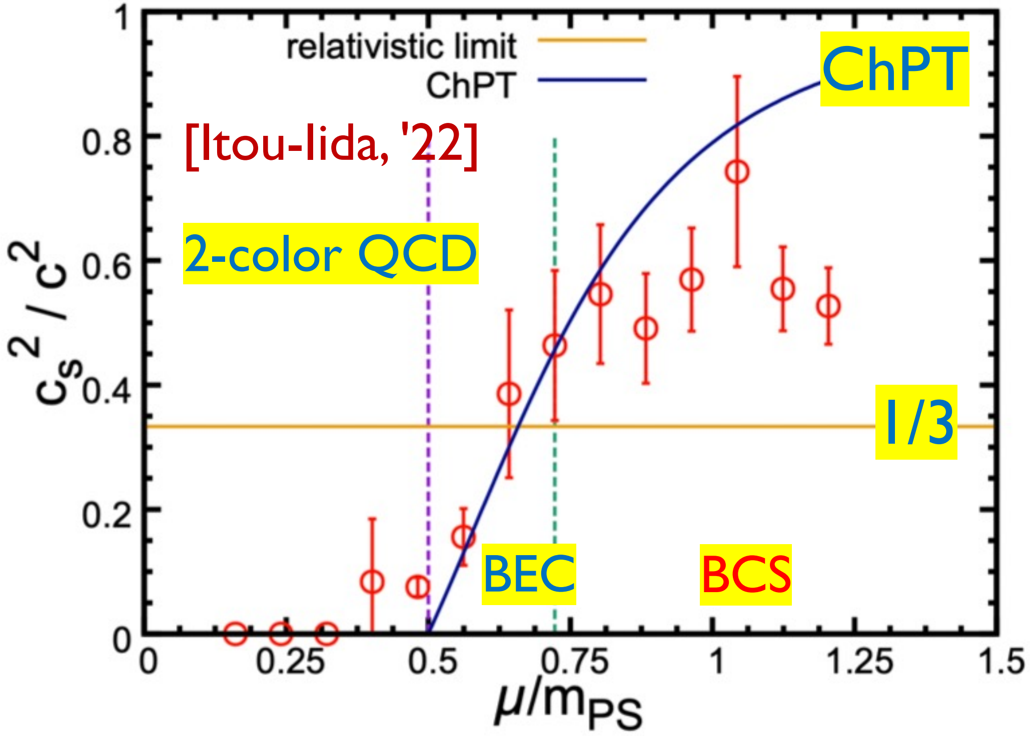


baseline: *quark-hadron continuity* **(QHC)**

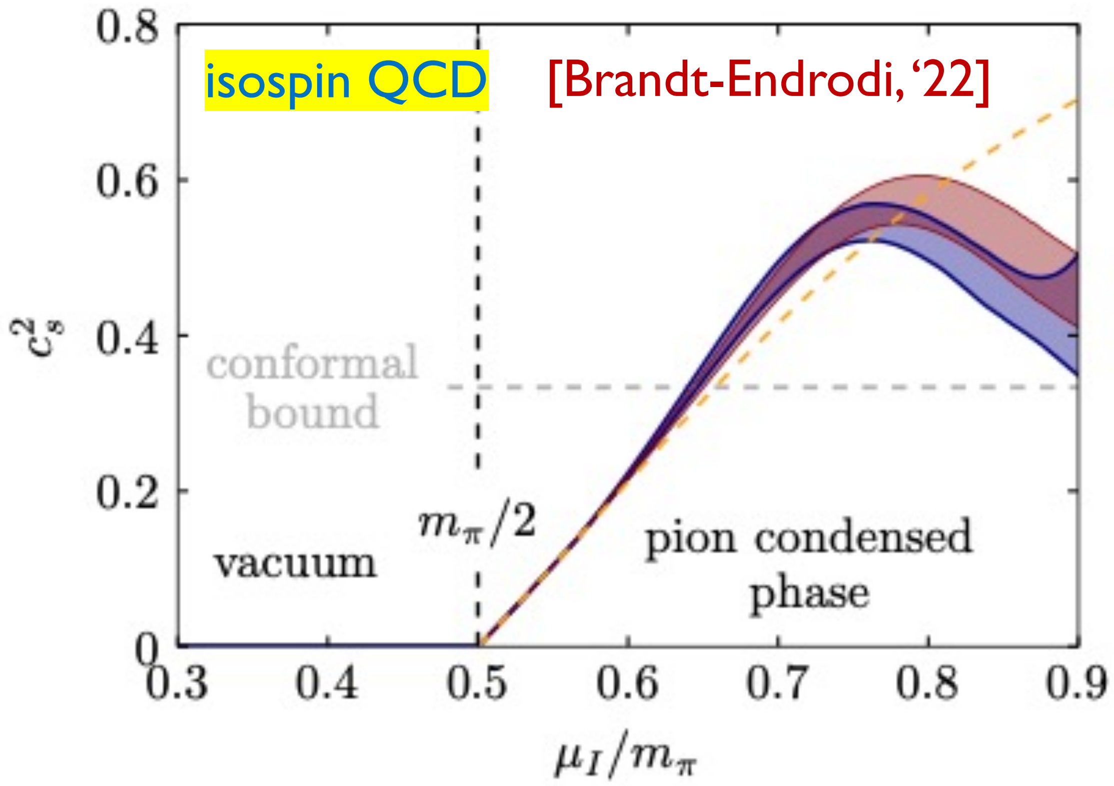
Examples from QCD-like theories

tests on the lattice

→see Itou-san's talk on Tuesday



[model study, TK-Suenaga '21]



Peak in the BEC-BCS type crossover

Contents

Pressure from $\varepsilon(n_B)$

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\underline{\frac{\varepsilon}{n_B}}\right)$$ energy per particle

e.g.)

gas of heavy particles (massive limit)

$$\varepsilon(n_B) = m_N n_B \quad \Rightarrow \quad \varepsilon/n_B = m_N \quad \Rightarrow \quad P = 0$$ soft

gas of relativistic particles (massless limit)

$$\varepsilon(n_B) = a n_B^{4/3} \quad \Rightarrow \quad \varepsilon/n_B = a n_B^{1/3} \quad \Rightarrow \quad P = \frac{\varepsilon}{3}$$ stiff

Nucleonic models & many-body forces

$$\varepsilon(n_B) = \underbrace{m_N n_B}_{\text{large (!)}} + \underbrace{a\frac{n_B^{5/3}}{m_N}}_{\text{small (!)}} + b n_B^{\alpha} \quad \Rightarrow \quad P = \underbrace{\frac{2}{3} a \frac{n_B^{5/3}}{m_N}}_{\text{small (!)}} + b(\alpha - 1) n_B^{\alpha}$$

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

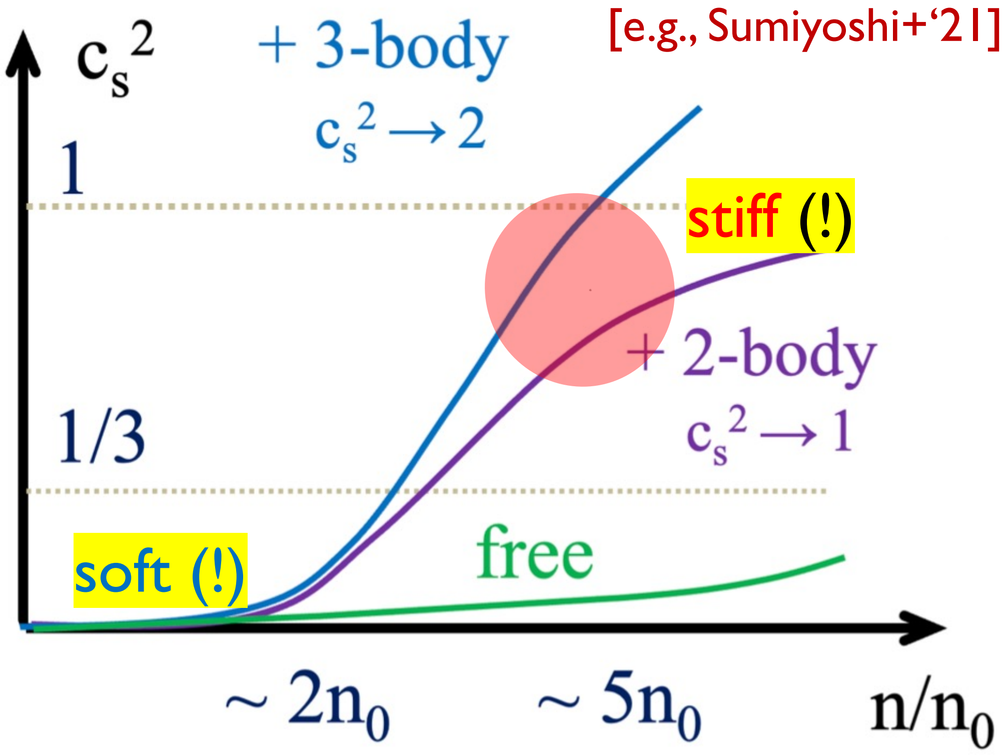
➡ p << ε (at least in dilute regime)

If interactions dominate (at large n_B):

$$P \sim (\alpha - 1)\varepsilon \;\rightarrow\; c_s^2 \sim (\alpha - 1)$$

2-body int. → α = 2 3-body int. → α = 3

- *causality & convergence ??*
- *stiffening occurs slowly (power growth)*

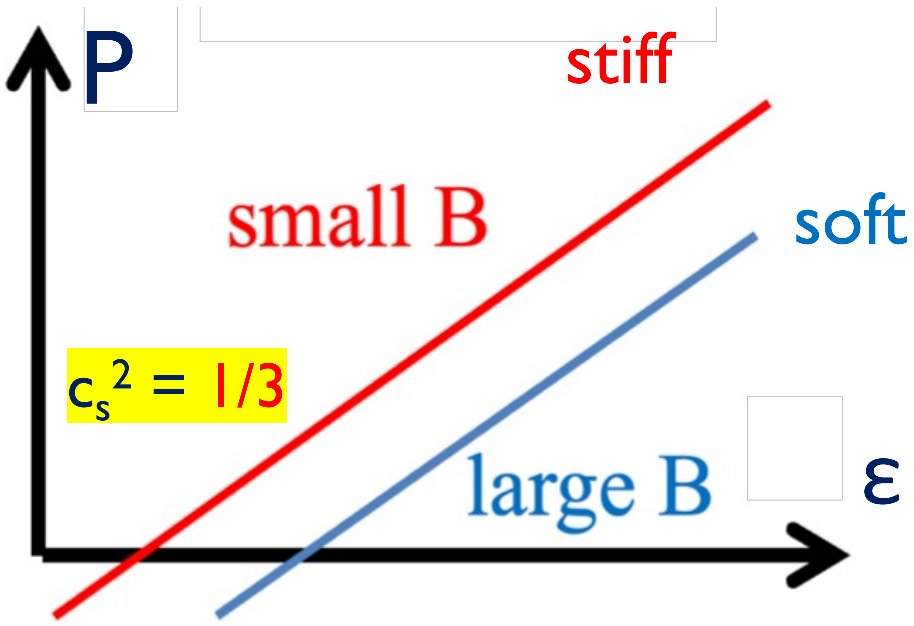


alternative: quark EOS

e.g.) free massless quarks

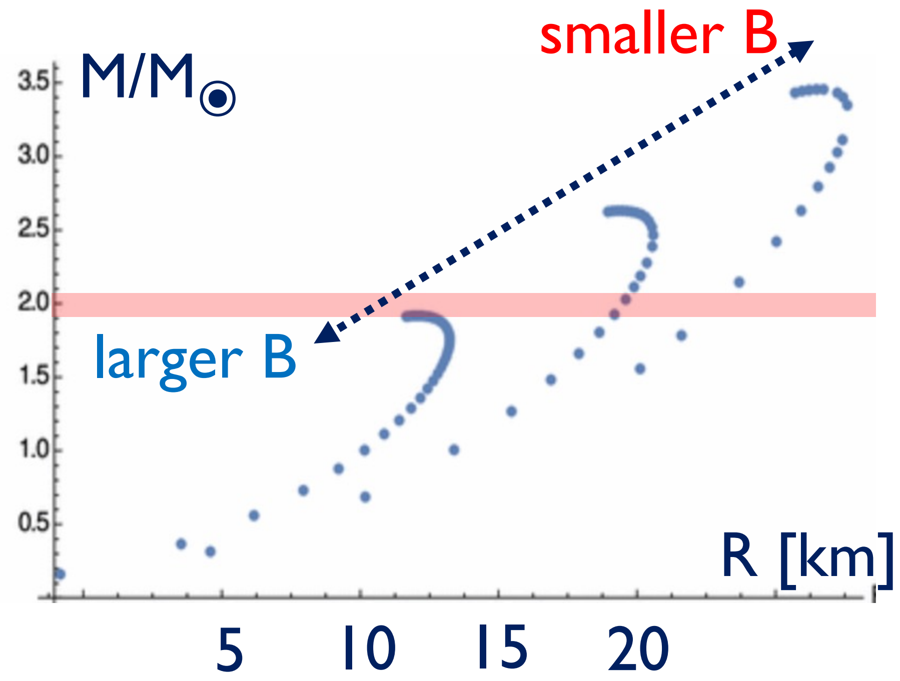
$$P = \frac{\varepsilon}{3} - B'$$

normalization



quark kin. pressure >> baryon kin. pressure

O(N_c) O($1/N_c$)

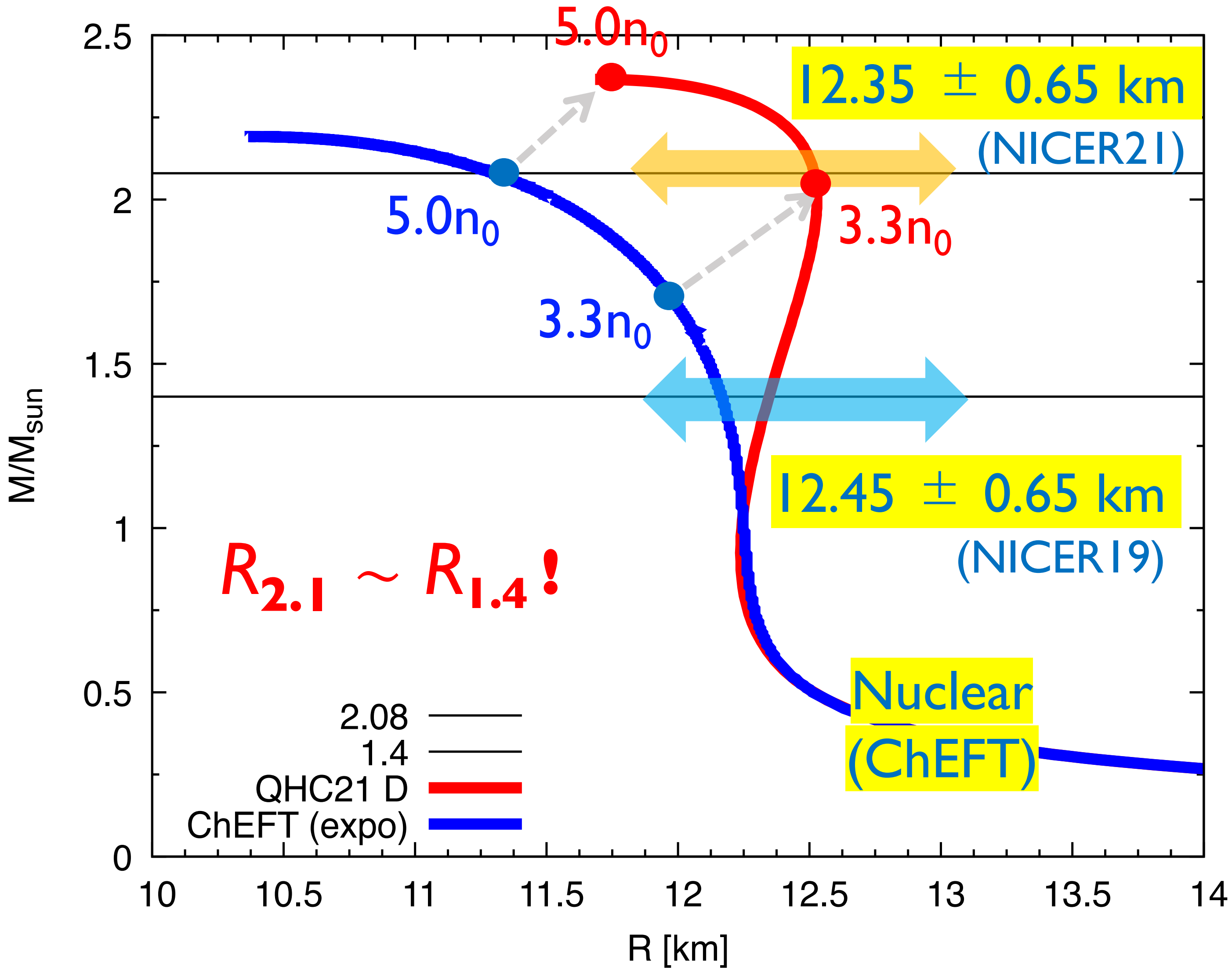


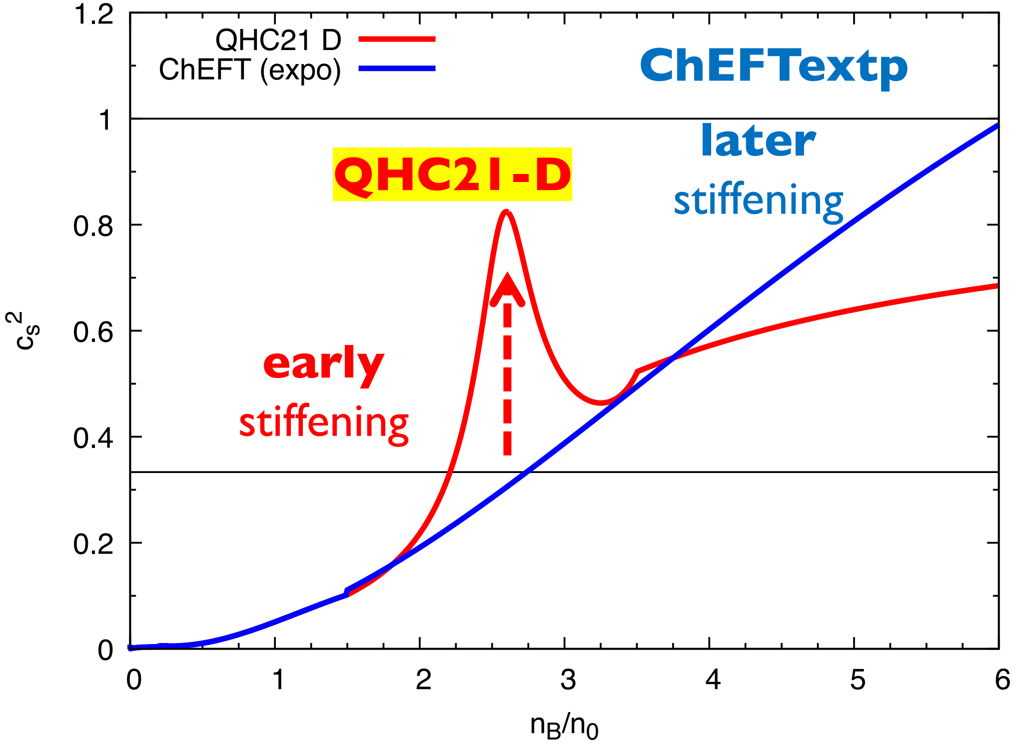
relativistic pressure → stiff EOS ?

*depends on **where** to start...*

Early vs later stiffening: QHC21 vs ChEFTextp

[TK-Hatsuda-Baym '21]





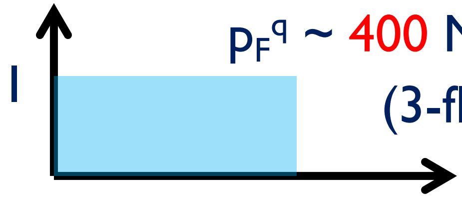
$2\text{-}3n_0$: already beyond purely nucleonic regime?

Contents

Claim

- naive estimate for quark matter formation density: $(R_B \sim 0.5\text{-}0.8\ \mathrm{fm})$

$$n_B^{\mathrm{overlap}} \sim 1/(\,4\pi R_B^3/3\,) \sim 4\text{-}7 n_0$$



- we claim the existence of **another scale**, characterizing:
  - breakdown of many-body expansion
  - ***soft-deconfinement*** [not explained today, see Fukushima-TK-Weise '20]
  - ***quark saturation*** [TK '21; Fujimoto-TK-McLerran '24]

$$n_B^{\mathrm{q\text{-}sat}} \sim 0.5 \times n_B^{\mathrm{overlap}} \sim 2\text{-}3 n_0$$

Sum rules for occupation probabilities

cf) [TK '21, TK-Suenaga '21]

occupation **probability** of **quark** state with p

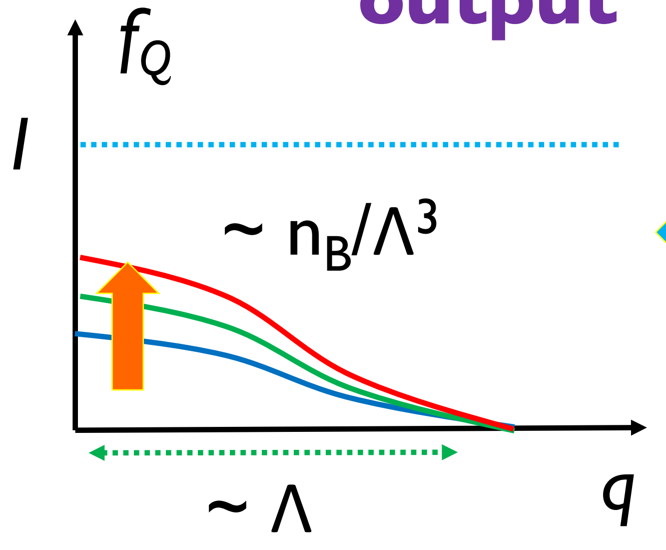
occupation **probability** of **baryon** state with P_B

quark mom. distribution **in a baryon**

$$f_{\rm Q}(\mathbf{q}) = \int_{\boldsymbol{P}_B} f_{\rm B}(\boldsymbol{P}_B)\,\varphi_Q^B(\mathbf{q} - \boldsymbol{P}_B/N_{\rm c})$$

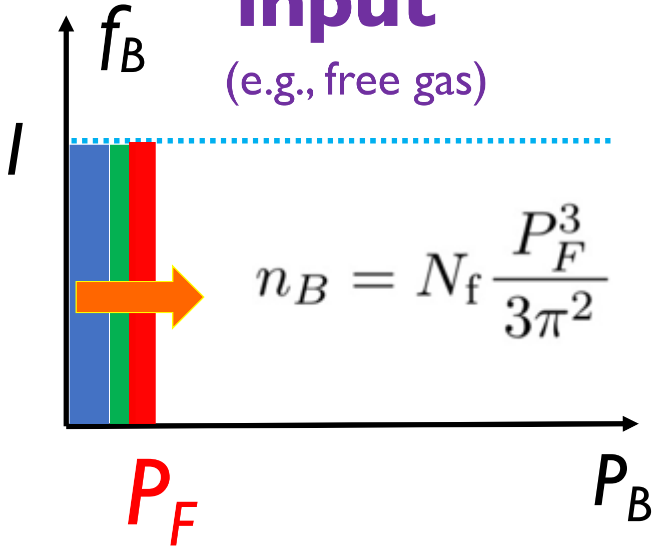
e.g.) in **ideal** baryonic matter

output



input

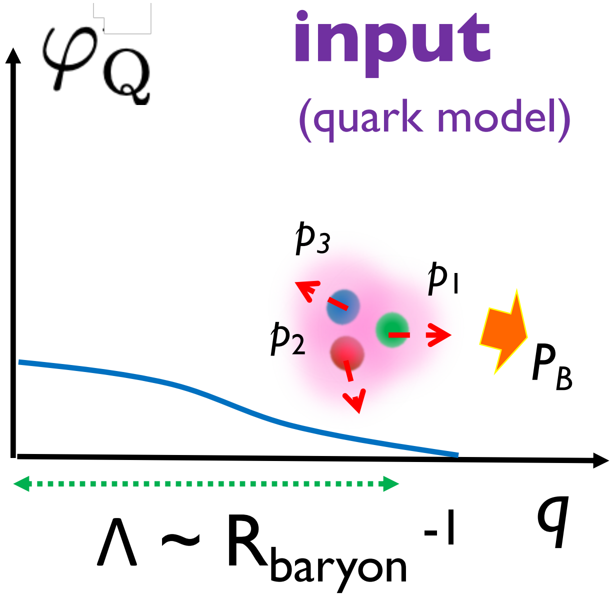
(e.g., free gas)



$\otimes$

input

(quark model)



An **ideal** model

[Fujimoto-TK-McLerran, PRL'24]

1) neglect interactions *except* confining forces

isospin, spin

e.g.) 2-flavor hamiltonian: $$\varepsilon_{\rm B}[f_{\rm B}] = 4 \int_k E_{\rm B}(k) f_{\rm B}(k)$$

2) quark distributions in a baryon remains the same (confinement persists)

3) use a special quark distribution → models become analytically solvable

$$\varphi_{\rm 3d}(\boldsymbol{q}) = \frac{2\pi^2}{\Lambda^3} \frac{e^{-q/\Lambda}}{q/\Lambda} \qquad \hat{L} = -\boldsymbol{\nabla}^2 + \frac{1}{\Lambda^2} \qquad \hat{L}[\varphi(\boldsymbol{p}-\boldsymbol{q})] = \frac{(2\pi)^3}{\Lambda^2} \delta(\boldsymbol{p}-\boldsymbol{q})$$

nontrivial output

$$f_{\rm Q}(\mathbf{q}) = \int_{\boldsymbol{P}_B} f_{\rm B}(\boldsymbol{P}_B) \varphi_Q^B(\mathbf{q} - \boldsymbol{P}_B/N_{\rm c})$$

natural at ***low*** *density*

nontrivial output

$$f_{\rm B}(N_{\rm c}\boldsymbol{q}) = \frac{\Lambda^2}{N_{\rm c}^3} \hat{L}[f_{\rm Q}(\boldsymbol{q})]$$

natural at ***high*** *density*

useful for studies of the *transient regime* (d.o.f are not clear-cut)

Variational problem with sum rule constraints

[Fujimoto-TK-McLerran, PRL'24]

constraint to fix n_B

$$\tilde{\varepsilon} = \varepsilon_{\rm B}[f_{\rm B}] - \lambda_{\rm B} n_{\rm B} \qquad E_{\rm B}(k) = \sqrt{M_{\rm B}^2 + k^2} \qquad n_{\rm B} = 4\int_k f_{\rm B}(k)$$

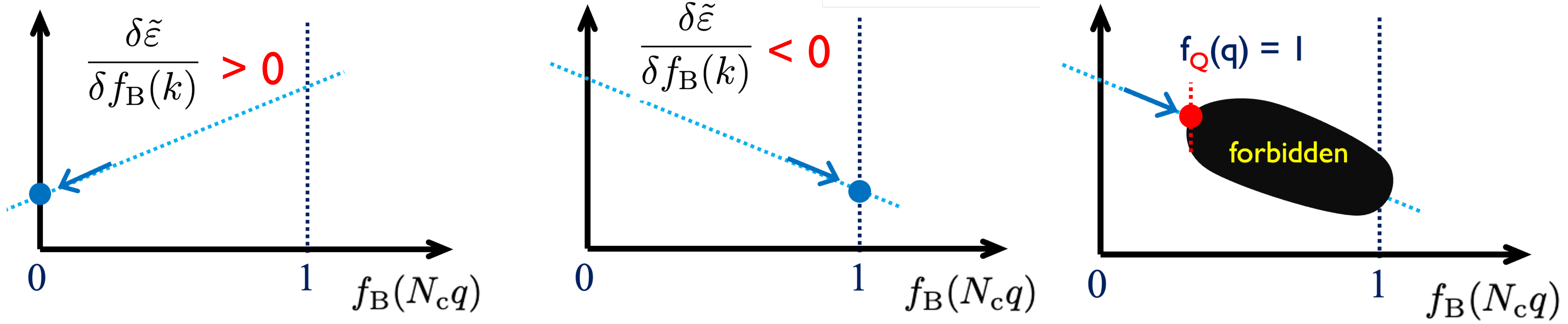
optimization: $$\frac{\delta \tilde{\varepsilon}}{\delta f_{\rm B}(k)} = E_{\rm B}(k) - \lambda_{\rm B}$$ *at a given k*

$E_B(k) > \lambda_B$

$E_B(k) < \lambda_B$

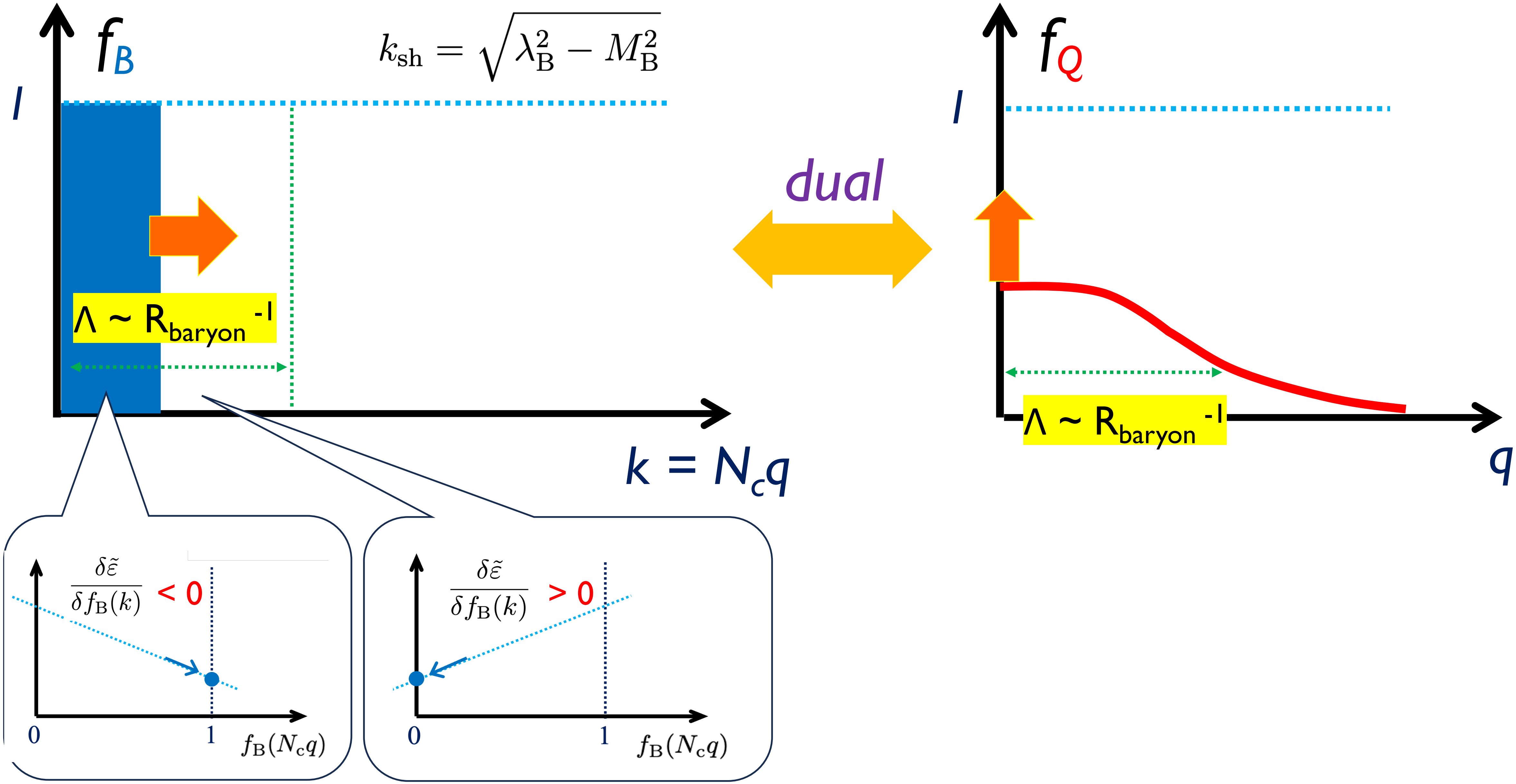
w.o. saturation

with saturation



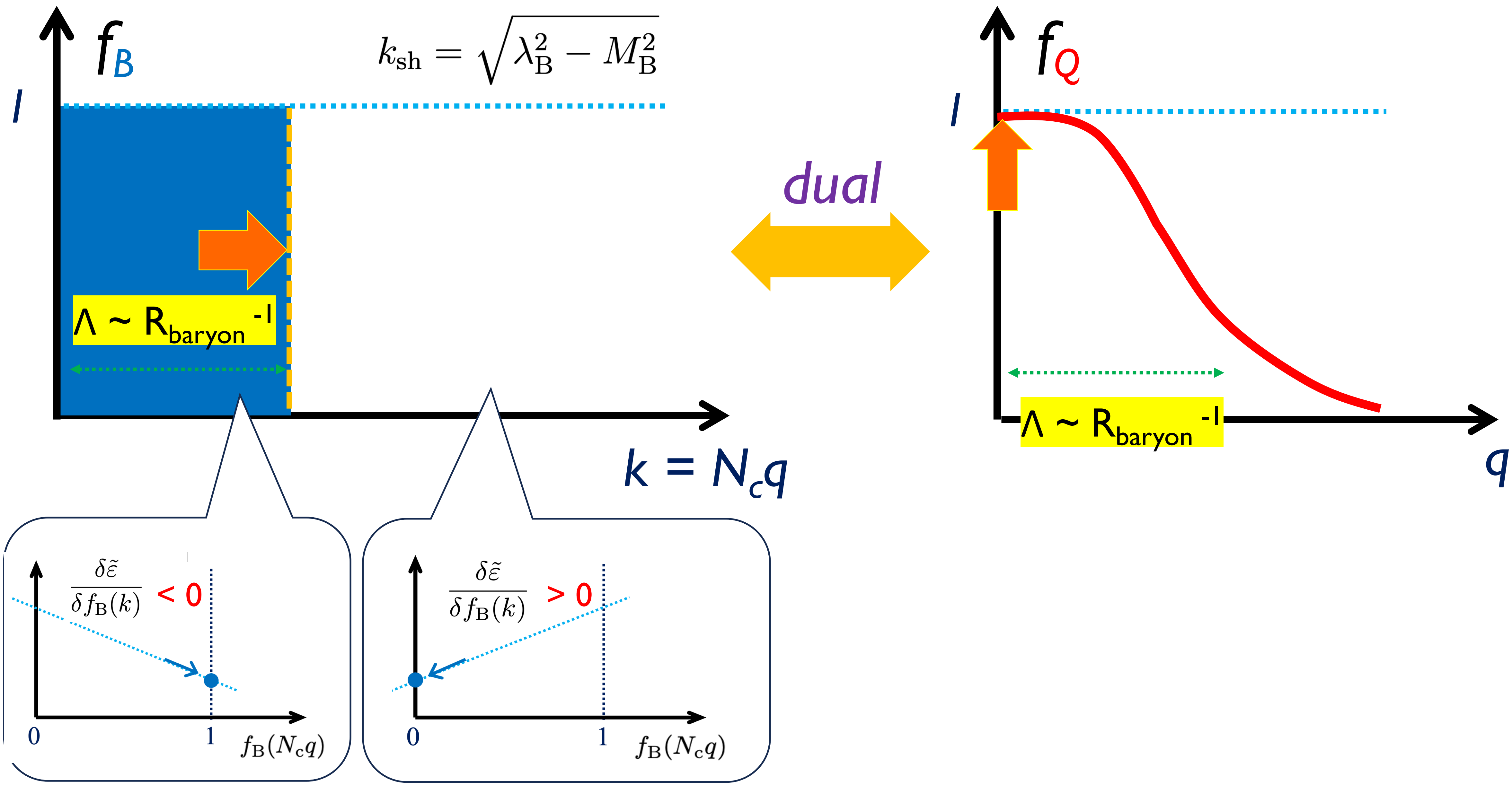
Solution (dilute regime)

[Fujimoto-TK-McLerran, PRL'24]



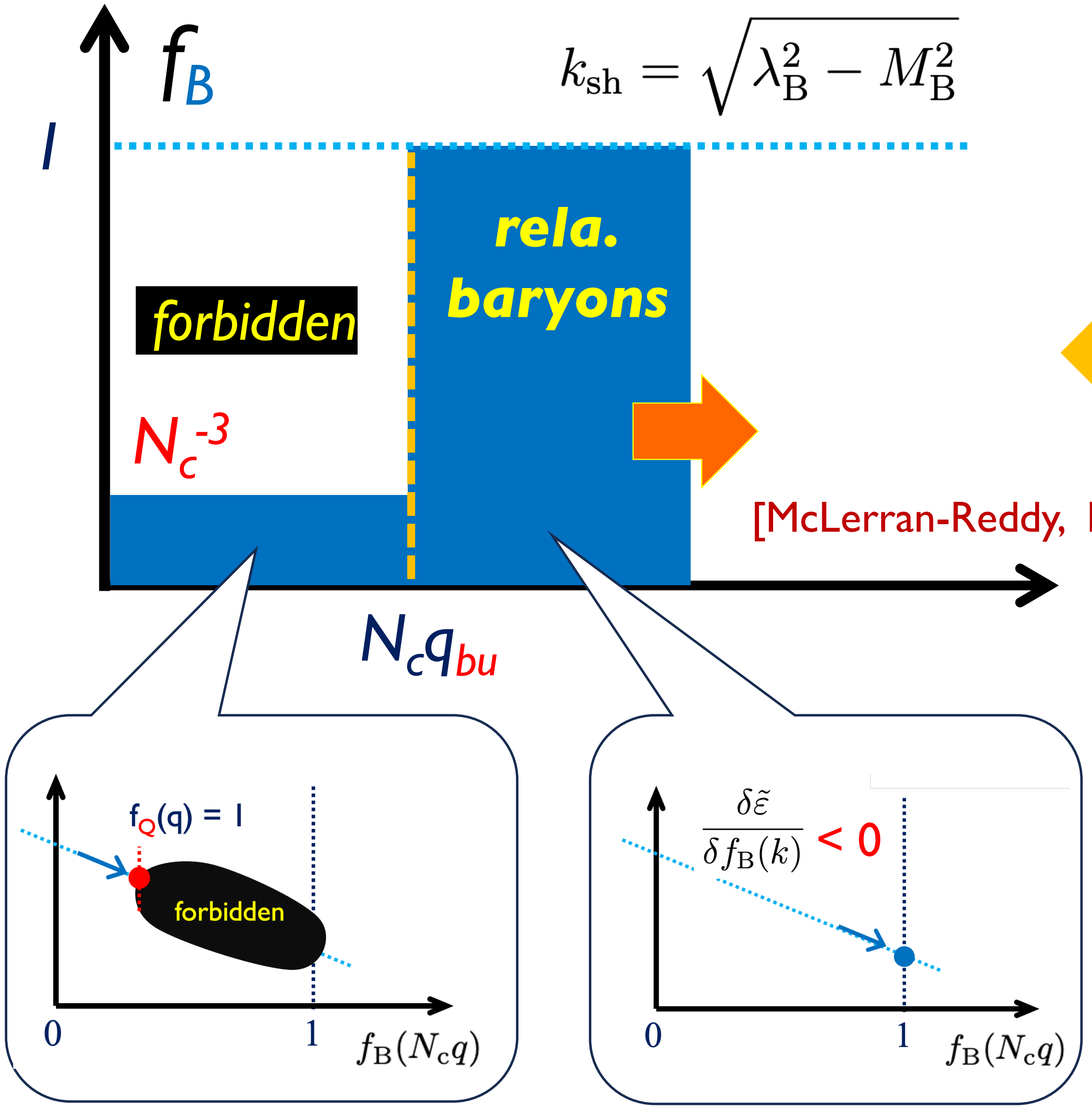
Solution (at saturation)

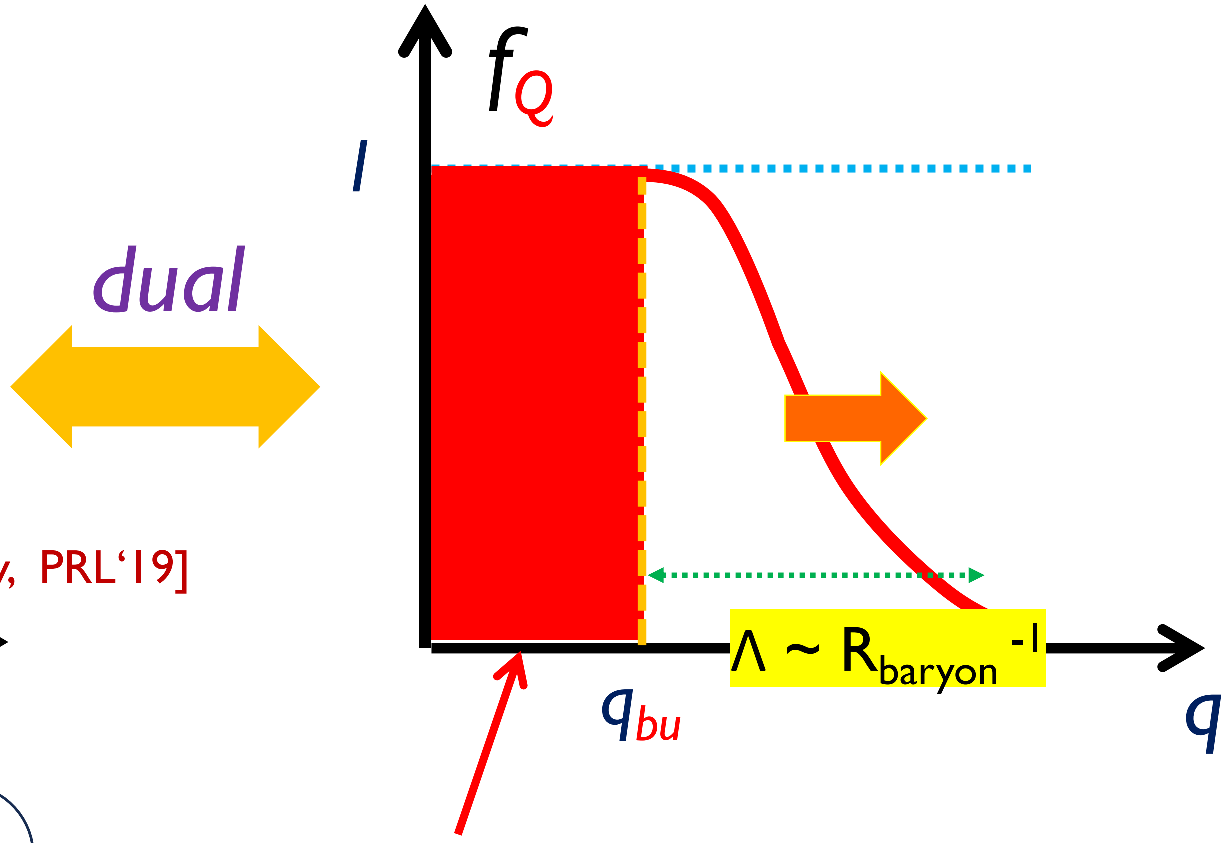
[Fujimoto-TK-McLerran, PRL'24]



Solution (post saturation)

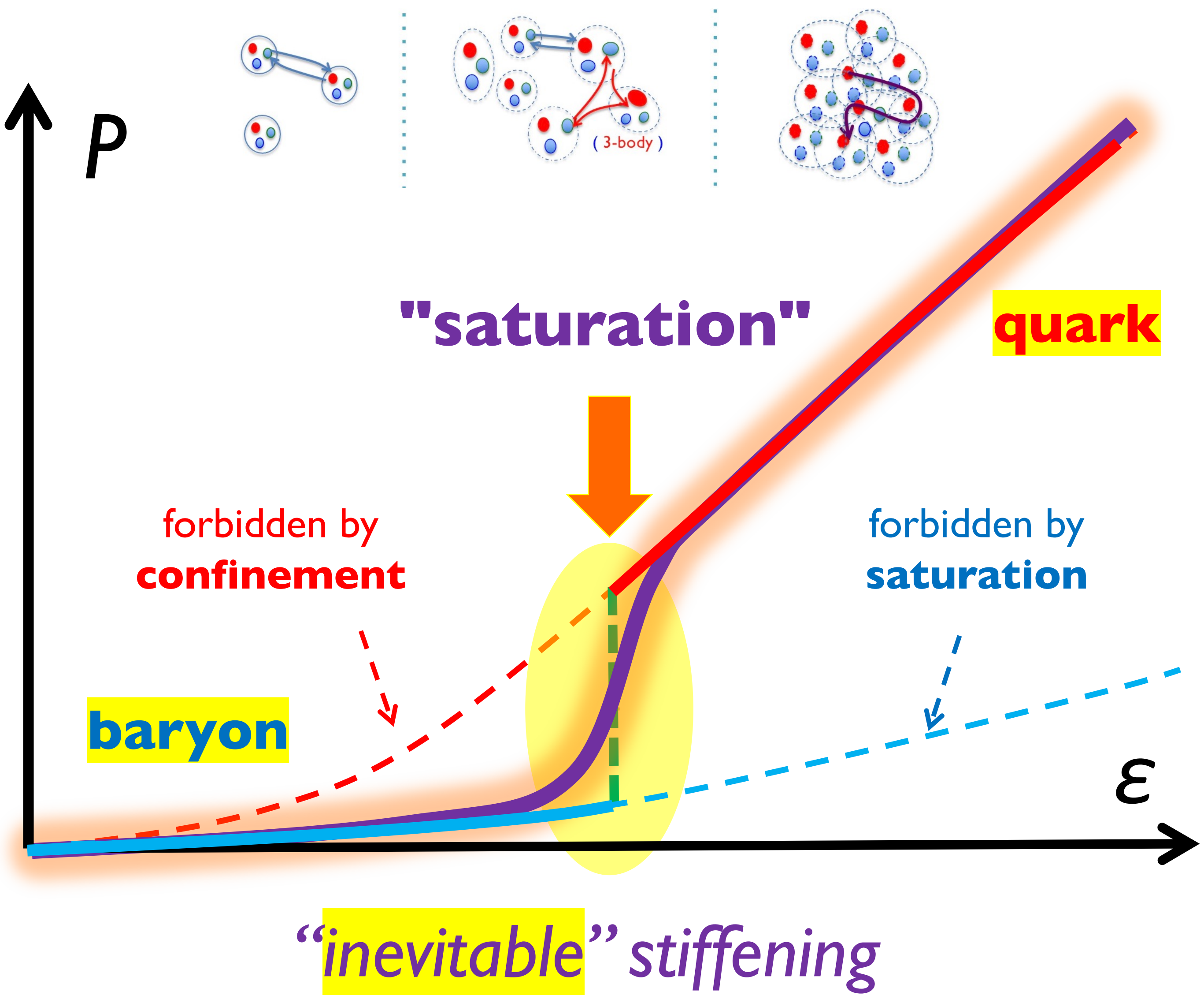
[Fujimoto-TK-McLerran, PRL'24]

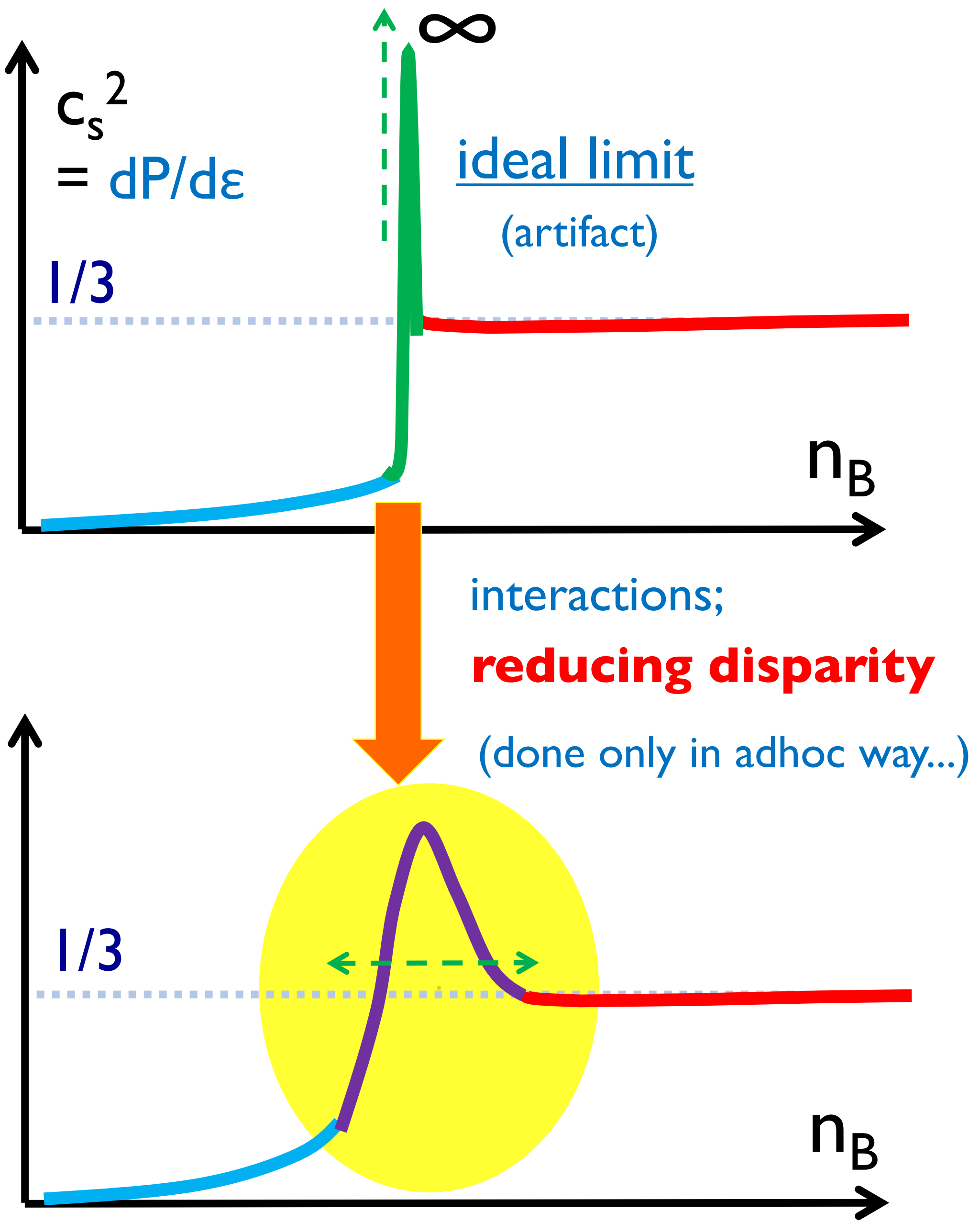




"inevitable" formation of the quark Fermi sea

Peak in sound velocity





Contents

Stiff quark matter

The appearance of c_s^2 peak is characteristic in the QHC scenarios:

good baseline, but NOT necessarily sufficient for ~ 2.1-2.3$M_\odot$ NS.

(just after the crossover, quarks are not fully relativistic.)

Can the chiral restoration stiffens EOS by making quarks relativistic?

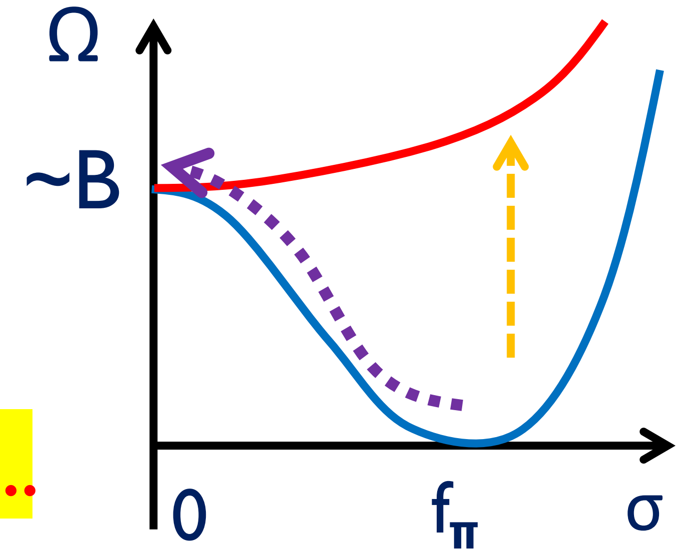
Unlikely: "*the bag constant*" from the Dirac sea

$$\varepsilon \rightarrow \varepsilon + B$$

$$P \rightarrow P - B$$

significant softening!

At this stage, we begin to discuss interactions…



Parametric analyses

[TK-Powell-Song-Baym, '14; QHC18, 19, 21]

rela. kin. energy   interactions

$$\varepsilon(n) = \underbrace{a n^{4/3}}_{\text{rela. kin. energy}} + \underbrace{b n^{\alpha}}_{\text{interactions}}$$

(n: quark density)

ideal   interactions

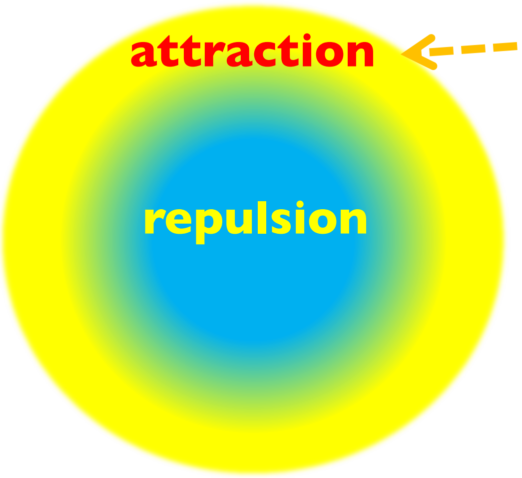
$$P = \underbrace{\frac{\varepsilon}{3}}_{\text{ideal}} + \underbrace{b\left(\alpha - \frac{4}{3}\right) n^{\alpha}}_{\text{interactions}}$$

For stiff EOS: (for large P)

for $\boldsymbol{\alpha} > \mathbf{4/3}$: b > 0 (e.g. bulk repulsion, ~ + $n_B^{\,2}/\Lambda^2$)

for $\boldsymbol{\alpha} < \mathbf{4/3}$: b < 0 (e.g. surface pairings, ~ $-\Lambda^2 n_B^{\,2/3}$)

quark Fermi sea



2- or 3-quark correlations

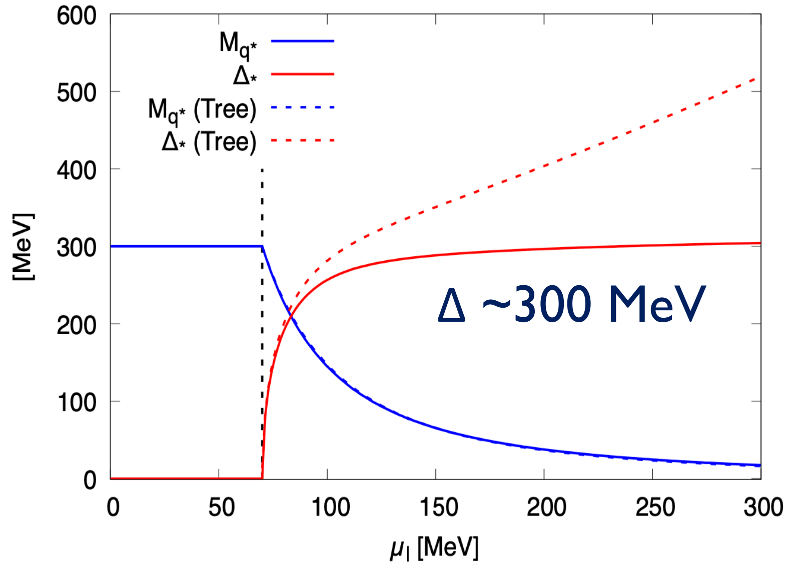
Stiff EOS from **attractive forces**

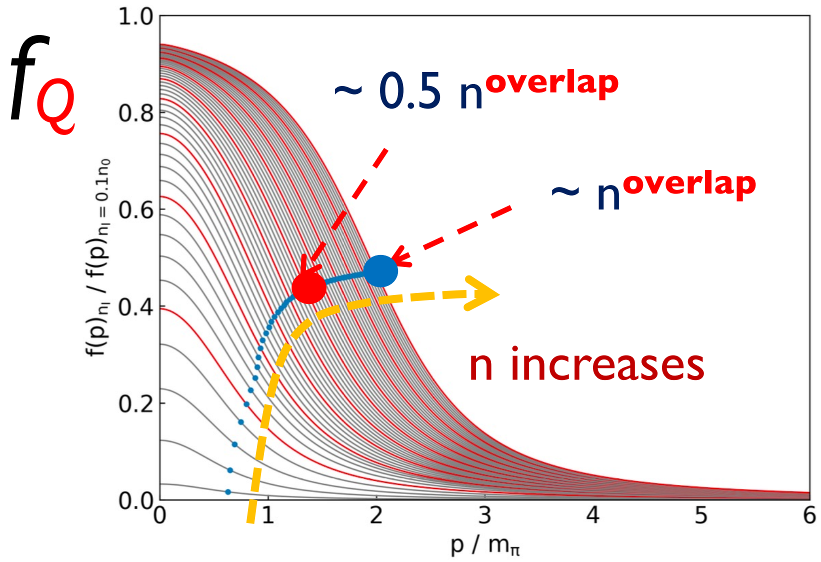
see Gordon-san's slides in the last week

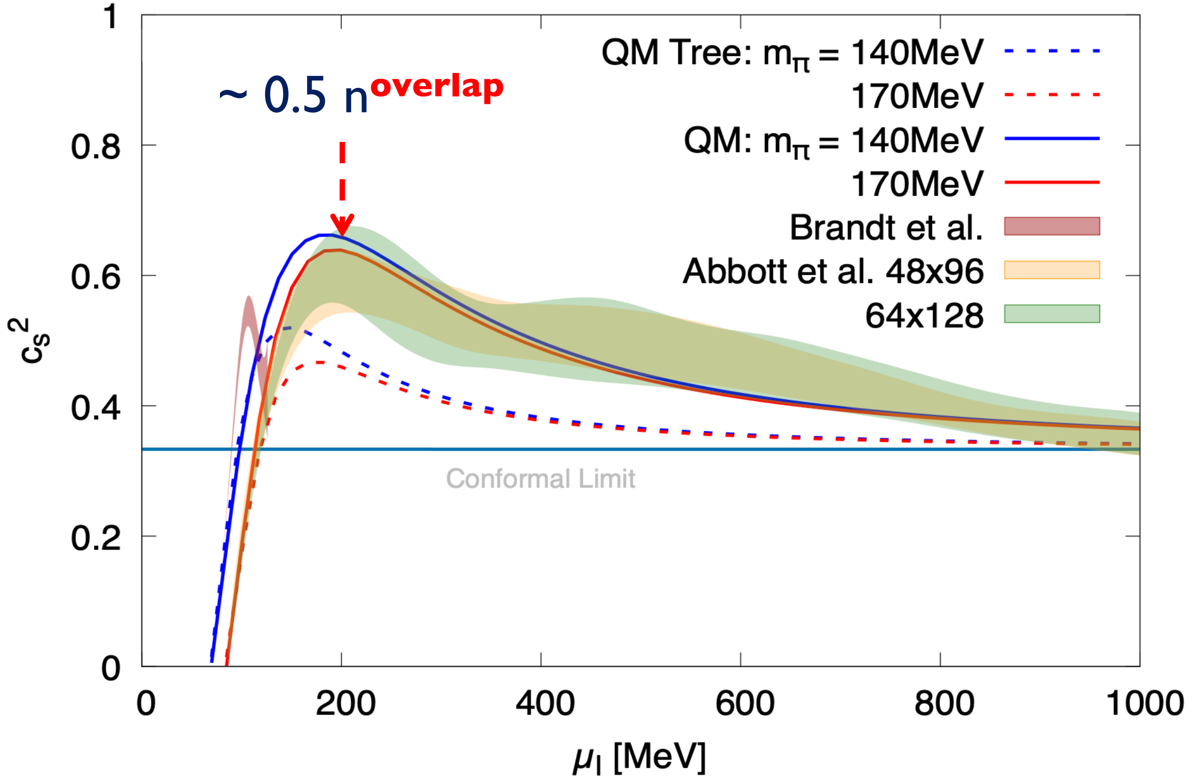
e.g. isospin QCD

quark-meson model vs lattice

[Adhikari+ '17; Chiba-TK '23, ...] [Brandt+ '22; Abbott+ '23, ...]







- $\Delta \sim 300$ MeV (!) $T_c^{QM} \sim 180$ MeV vs $T_c^{lat} \sim 170$ MeV
- $\Delta > \sim 100$ MeV → $c_s^2 > 1/3$ at $\mu_q \sim 1$GeV and beyond
- c_s^2 peak found at $n \sim 0.5\, n^{\text{overlap}}$

Summary

- *nuclear* + $R_{1.4} \sim R_{2.1}$ → *early* stiffening at ~ 0.5 $n^{overlap}$

  (beyond simple nuclear regime)

- *inevitable* quark Fermi sea formation at ~ 0.5 $n^{overlap}$

- *quark saturation* changes the trend of EOS; disparity -> c_s^2 peak

- attractive correlations near the Fermi surface stiffen EOS

- *unreasonable*(?) effectiveness of *quasi-particle* model in isospin QCD

  (as in constituent quark models in hadron spectroscopy)