

# Gravitational waves from binary-neutron-star mergers and the equation of state

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# Plan of the talk

1. Introduction
2. Inspiral: neutron-star equation of state
3. Postmerger: crossover vs. 1st phase transition
4. Discussion and summary

# 1. Introduction

# Gravitational-wave detectors

[http://gwcenter.icrr.u-tokyo.ac.jp/wp-content/themes/lcgt/images/img\\_abt\\_lcgt.jpg](http://gwcenter.icrr.u-tokyo.ac.jp/wp-content/themes/lcgt/images/img_abt_lcgt.jpg)

KAGRA (Kamioka, Japan)

Advanced LIGO  
(Hanford/Livingston, USA)

<https://www.advancedligo.mit.edu/graphics/summary01.jpg>



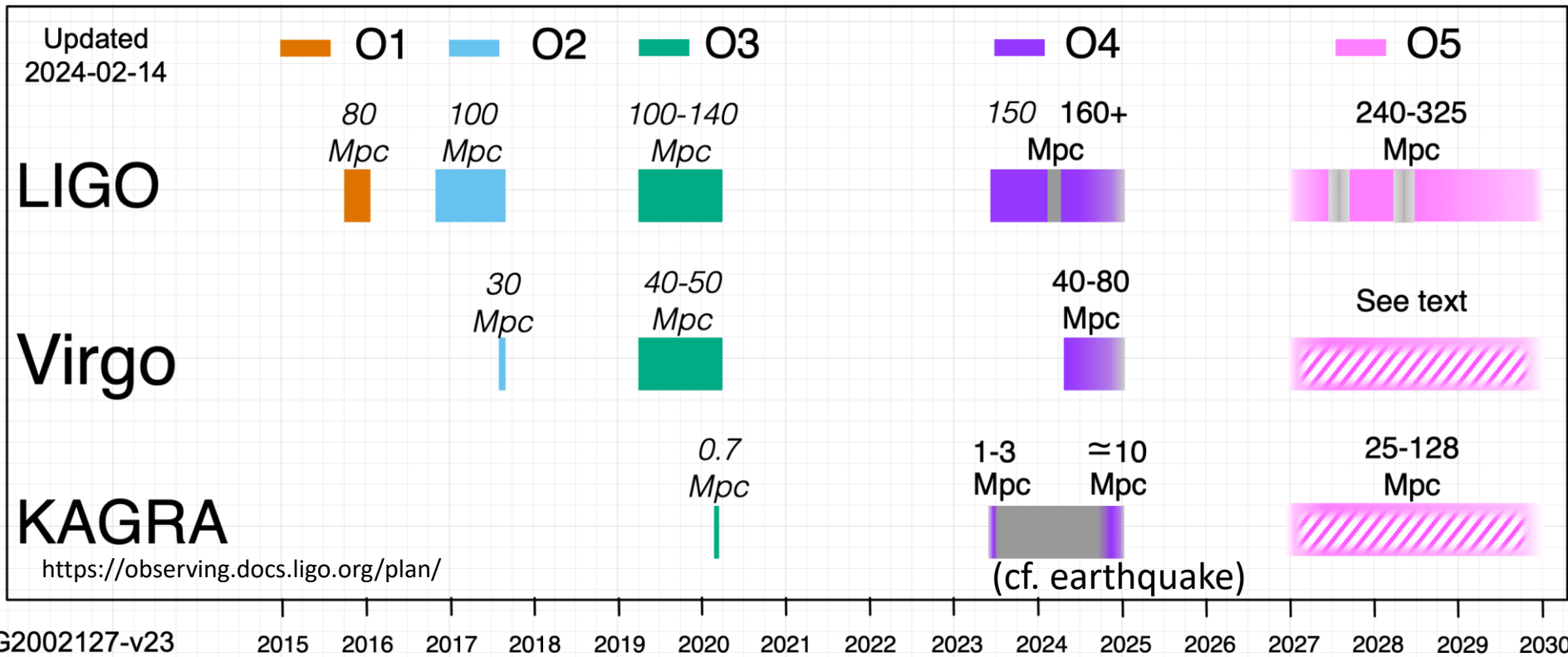
Advanced Virgo (Pisa, Italy)

<http://virgopisa.df.unipi.it/sites/virgopisa.df.unipi.it.virgopisa/files/banner/virgo.jpg>

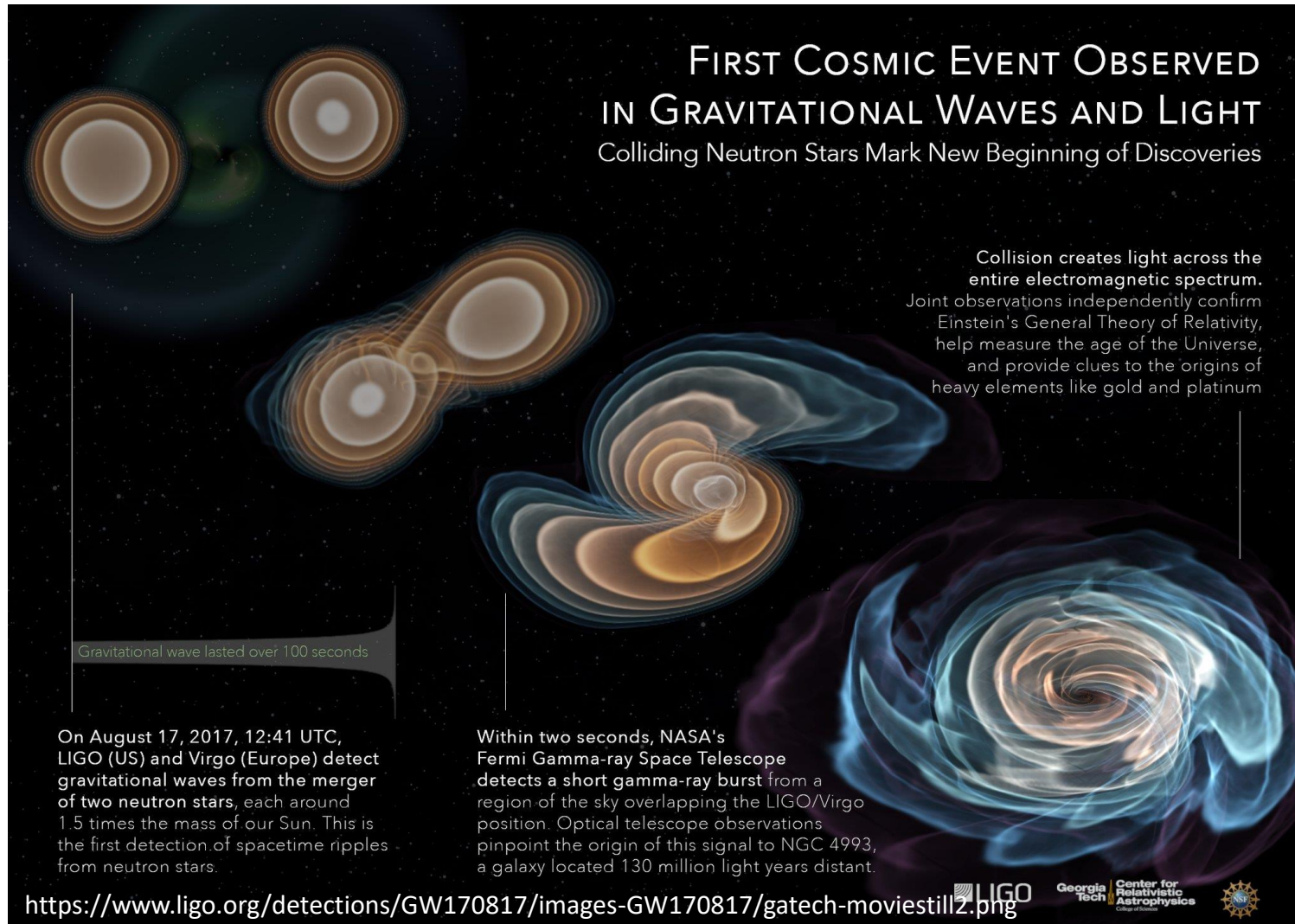
# Observation plan

O4 will continue throughout 2024 with improvement

O5 is planned to start from the beginning of 2027



# Gravitational waves: GW170817



**FIRST COSMIC EVENT OBSERVED  
IN GRAVITATIONAL WAVES AND LIGHT**  
Colliding Neutron Stars Mark New Beginning of Discoveries

Collision creates light across the entire electromagnetic spectrum. Joint observations independently confirm Einstein's General Theory of Relativity, help measure the age of the Universe, and provide clues to the origins of heavy elements like gold and platinum

Gravitational wave lasted over 100 seconds

On August 17, 2017, 12:41 UTC, LIGO (US) and Virgo (Europe) detect gravitational waves from the merger of two neutron stars, each around 1.5 times the mass of our Sun. This is the first detection of spacetime ripples from neutron stars.

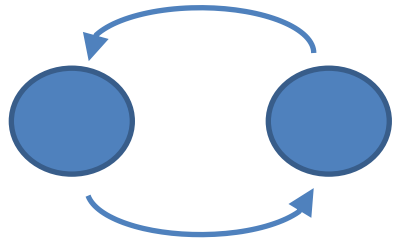
Within two seconds, NASA's Fermi Gamma-ray Space Telescope detects a short gamma-ray burst from a region of the sky overlapping the LIGO/Virgo position. Optical telescope observations pinpoint the origin of this signal to NGC 4993, a galaxy located 130 million light years distant.

<https://www.ligo.org/detections/GW170817/images-GW170817/gatech-moviestill2.png>

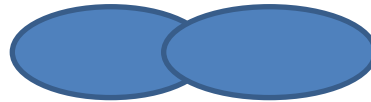
LIGO  
Georgia Tech  
Center for Relativistic Astrophysics  
College of Engineering

# Various phases of coalescence

Early inspiral: mass, spins...



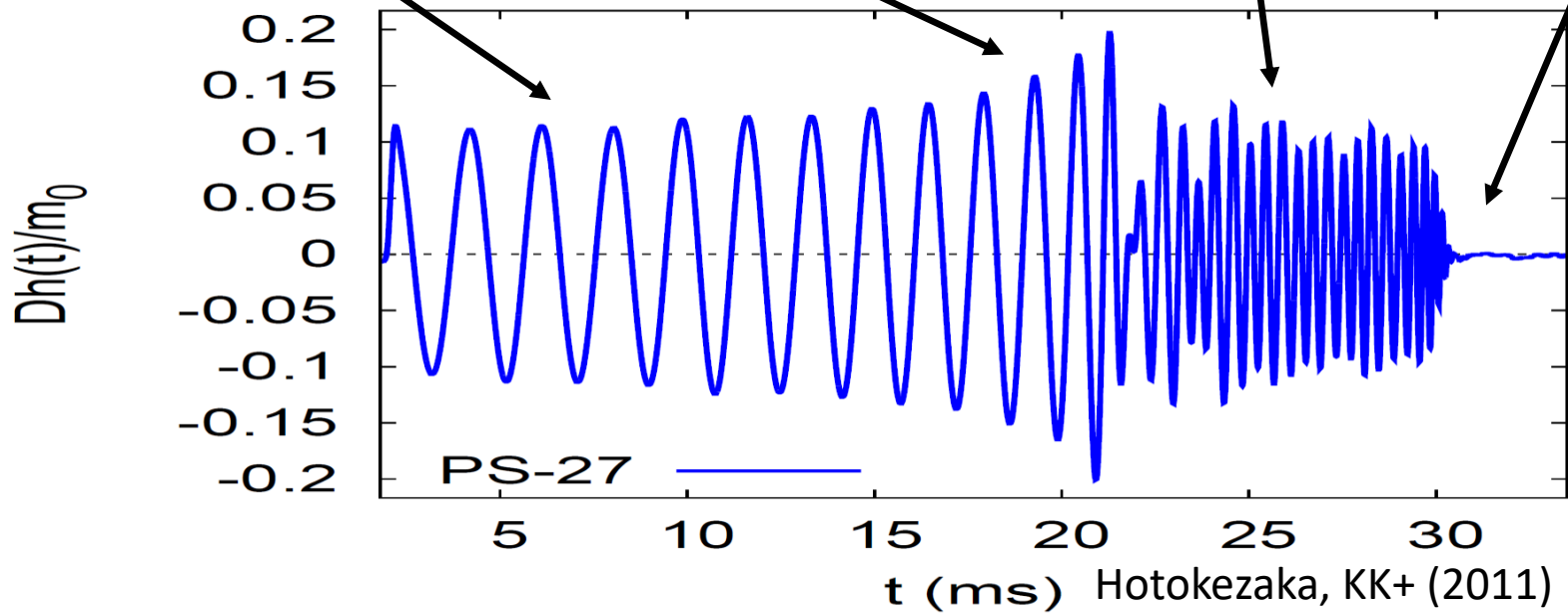
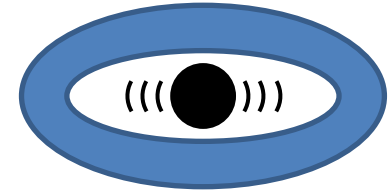
Late inspiral and merger:  
tidal deformation, NS EOS



Remnant massive NS:  
extreme temperature/density



Ringdown: GR

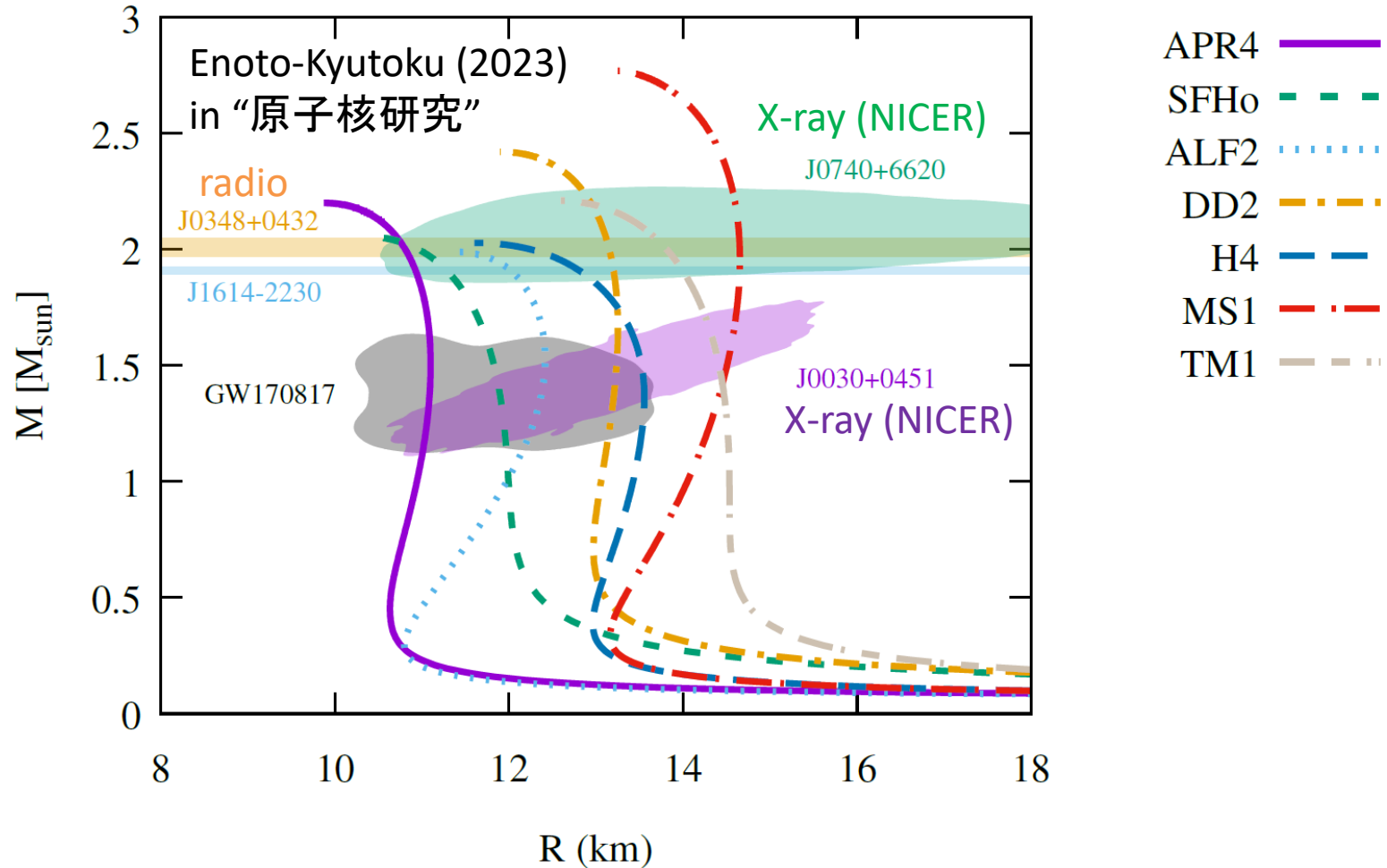


# **2. Inspiral: neutron-star equation of state**



# Current understanding

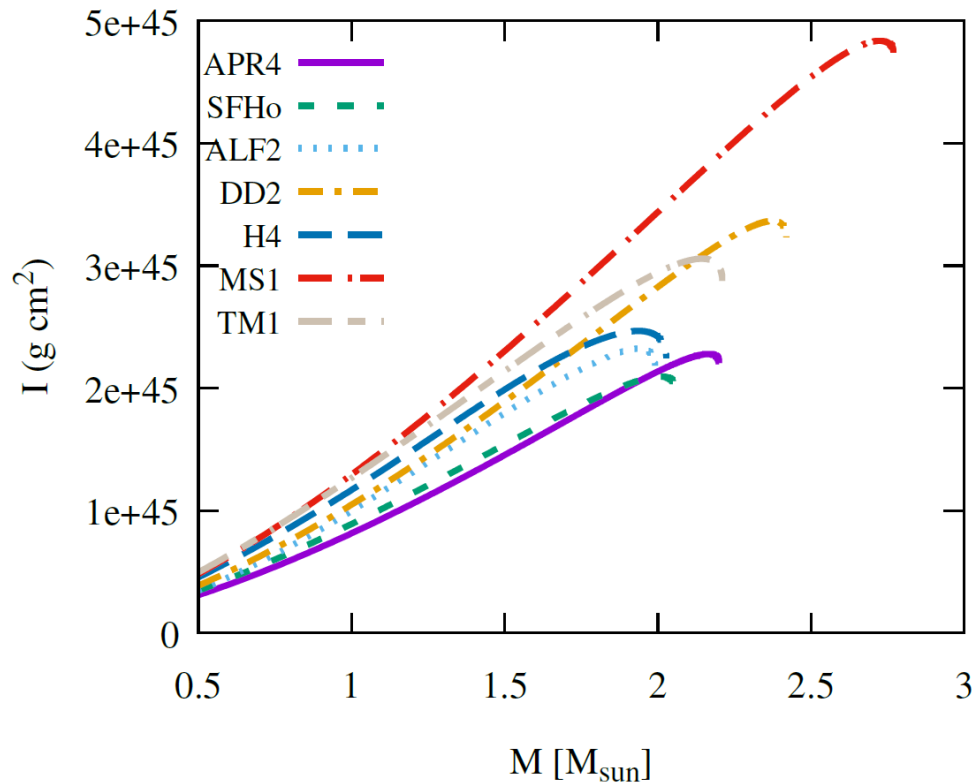
~ 11.5 – 13.5km for typical-mass neutron stars?



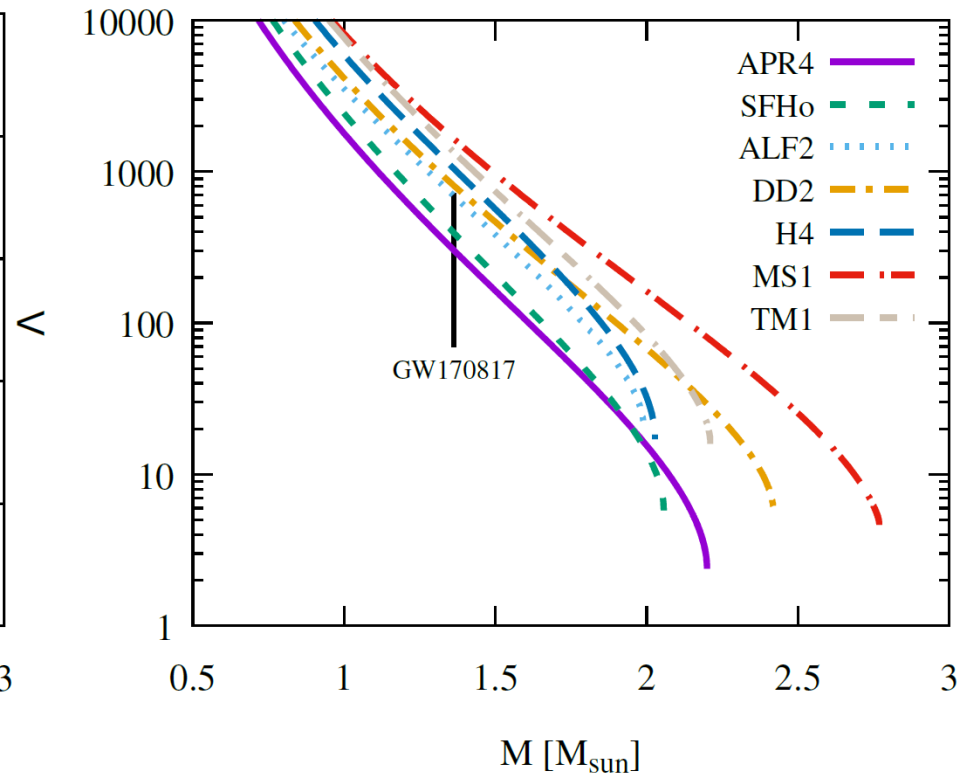
# Other macroscopic observables

The binary dynamics are affected more directly by, e.g.,

## Moment of inertia



## Tidal deformability

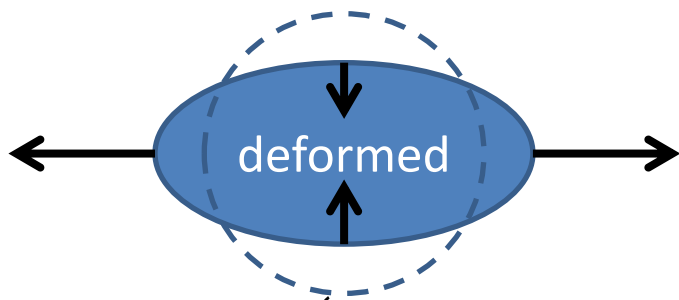


# Quadrupolar tidal deformability

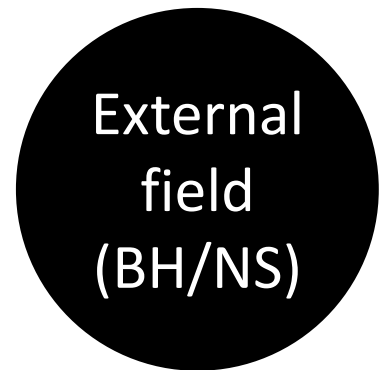
Leading-order finite-size effect on orbital evolution  
(strongly correlated with the neutron-star radius)

$$\Lambda = G\lambda \left( \frac{c^2}{GM} \right)^5 = \frac{2}{3} k \left( \frac{c^2 R}{GM} \right)^5 \propto R^5$$

$k \sim 0.1$ : (second/electric) tidal Love number



$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

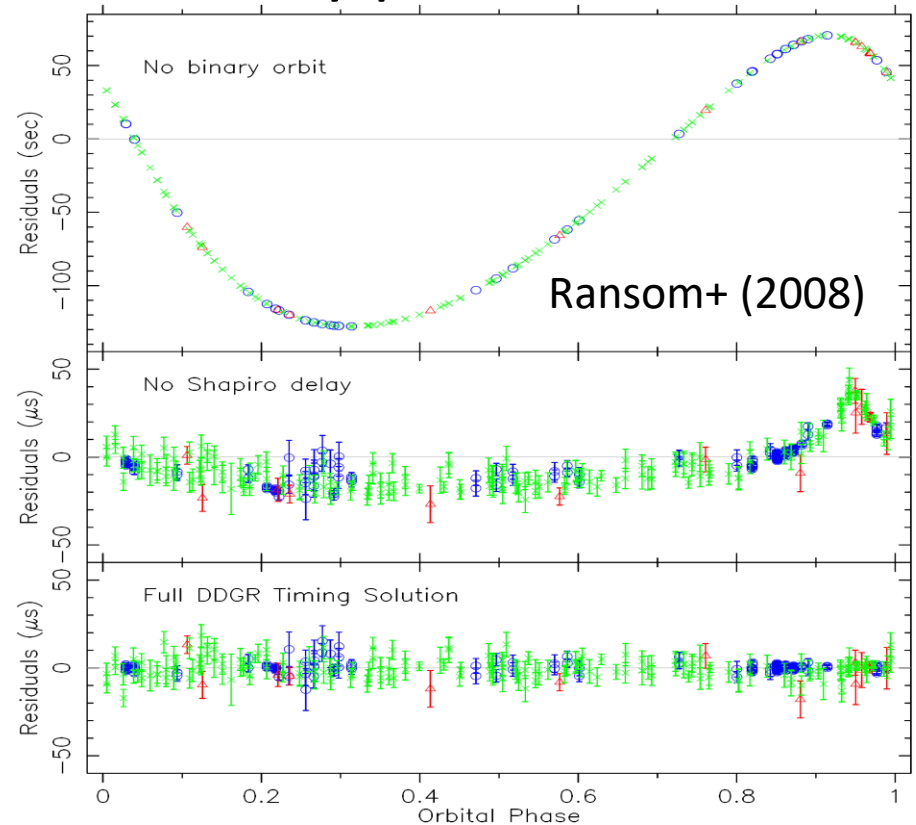
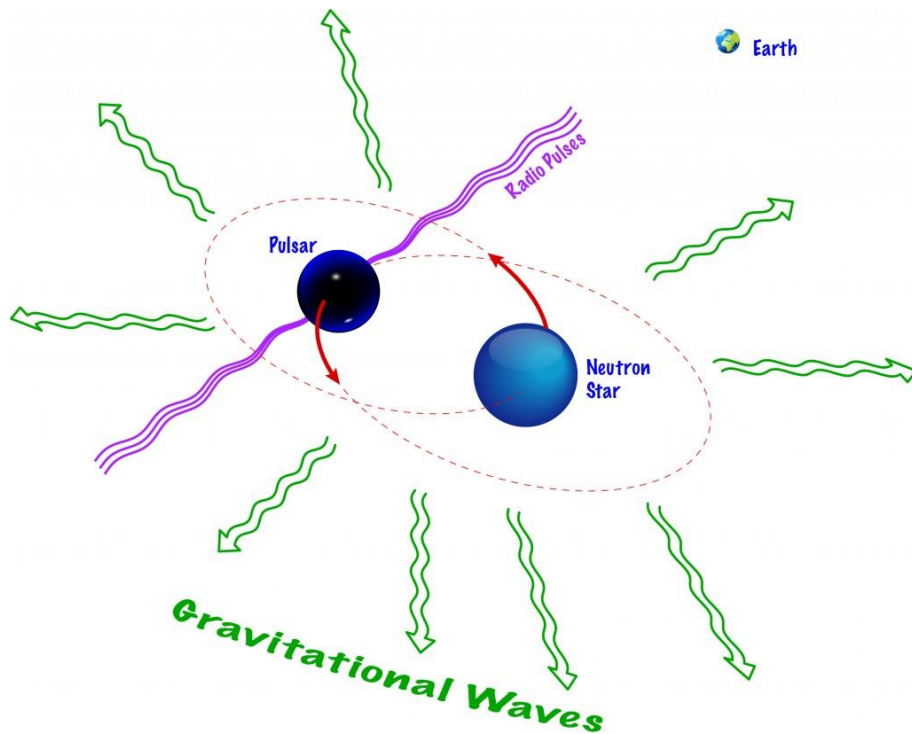


$$Q_{ij} \equiv \int \rho \left( x_i x_j - \frac{1}{3} x^2 \delta_{ij} \right) d^3 x$$

$$\mathcal{E}_{ij} \equiv \frac{\partial^2 \Phi_{\text{ext}}}{\partial x^i \partial x^j}$$

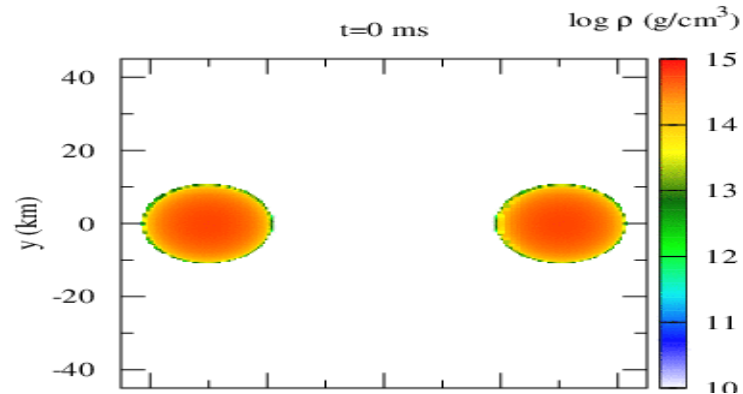
# Binary as a two-body problem

Both gravitational-wave and radio observations basically analyze gravitational two-body problems

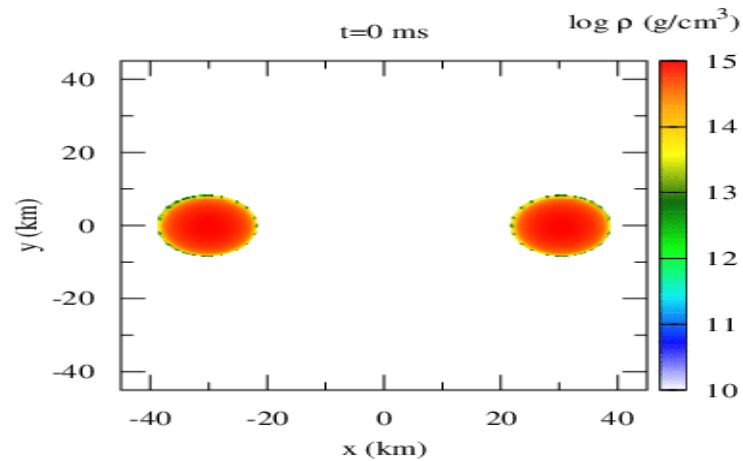
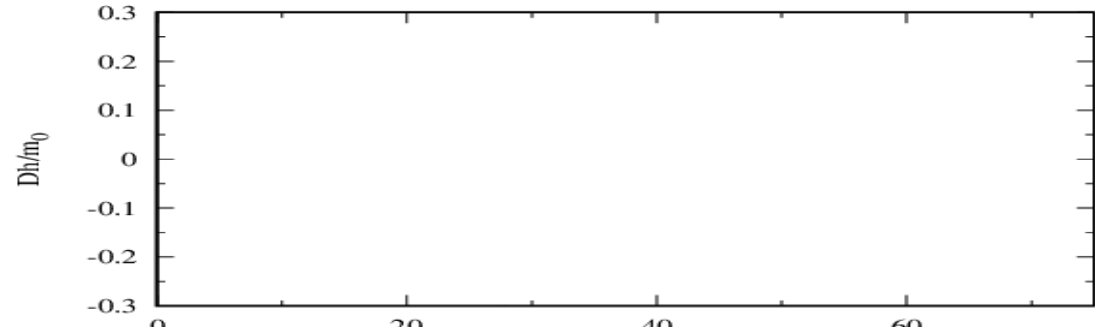


<http://asd.gsfc.nasa.gov/blueshift/wp-content/uploads/2016/02/htbinarypulsar-1024x835.jpg>

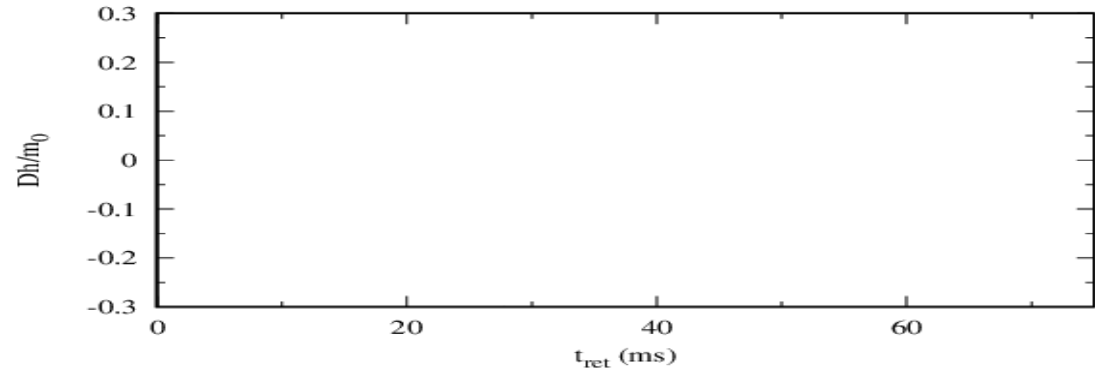
# Different orbital evolution



$$R = 13.7 \text{ km}, \Lambda = 1211$$



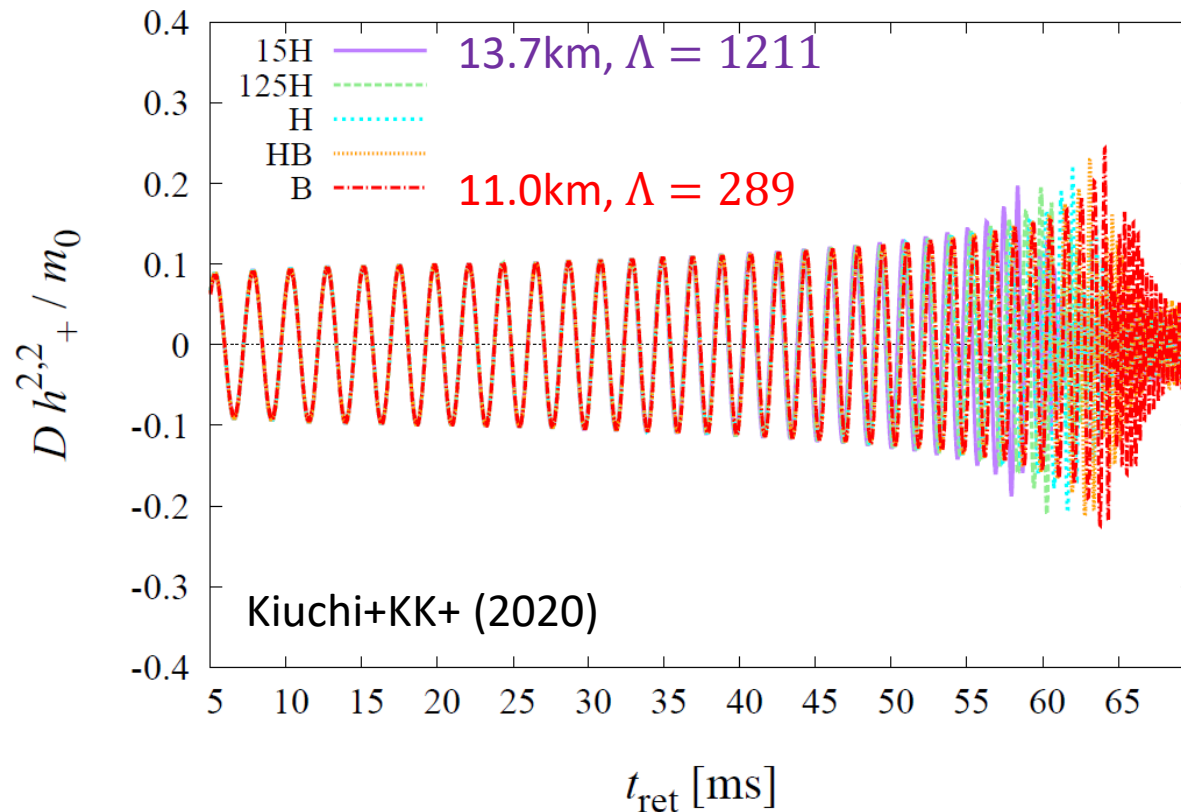
$$R = 11.0 \text{ km}, \Lambda = 289$$



# Numerical waveform

Binaries merge earlier for stiffer equations of state

This allows us to measure the tidal deformability

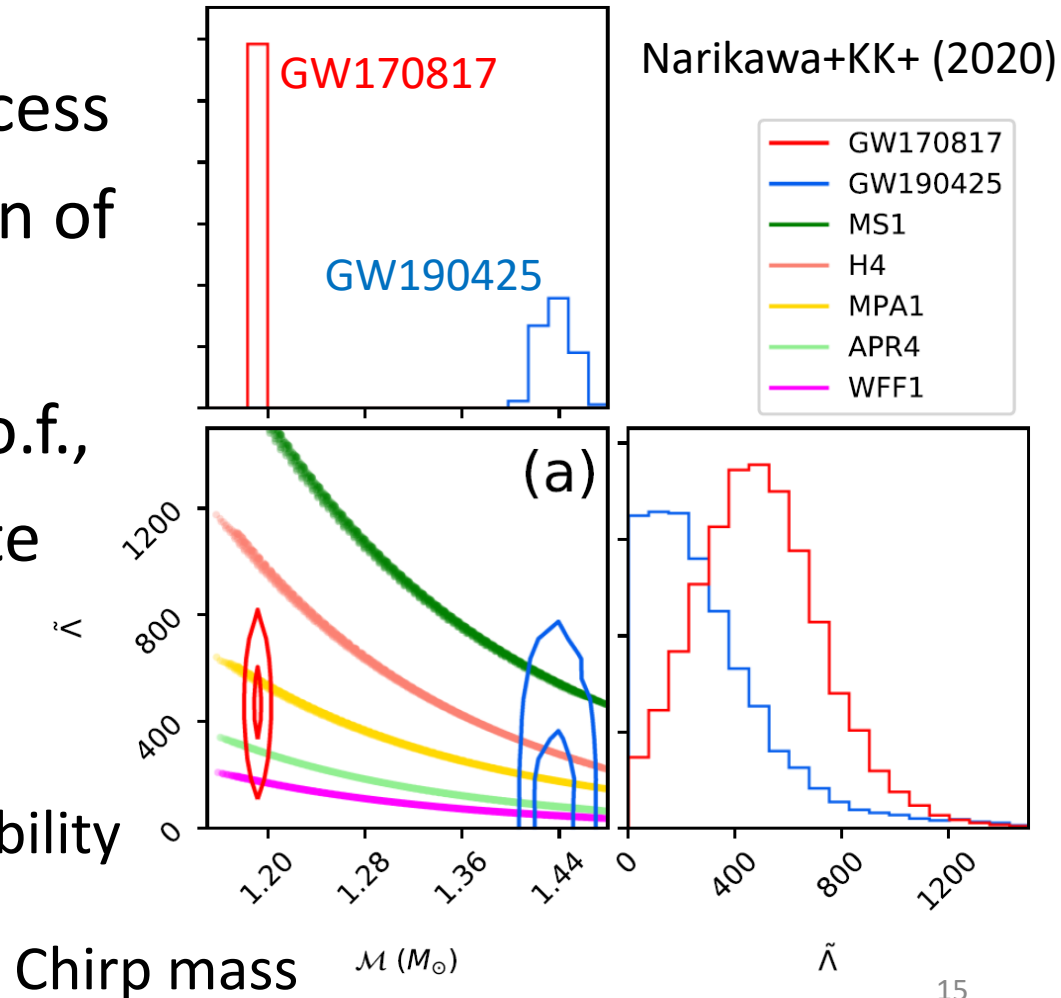


# Parameter estimation

Essentially only the tidal deformability is measured now

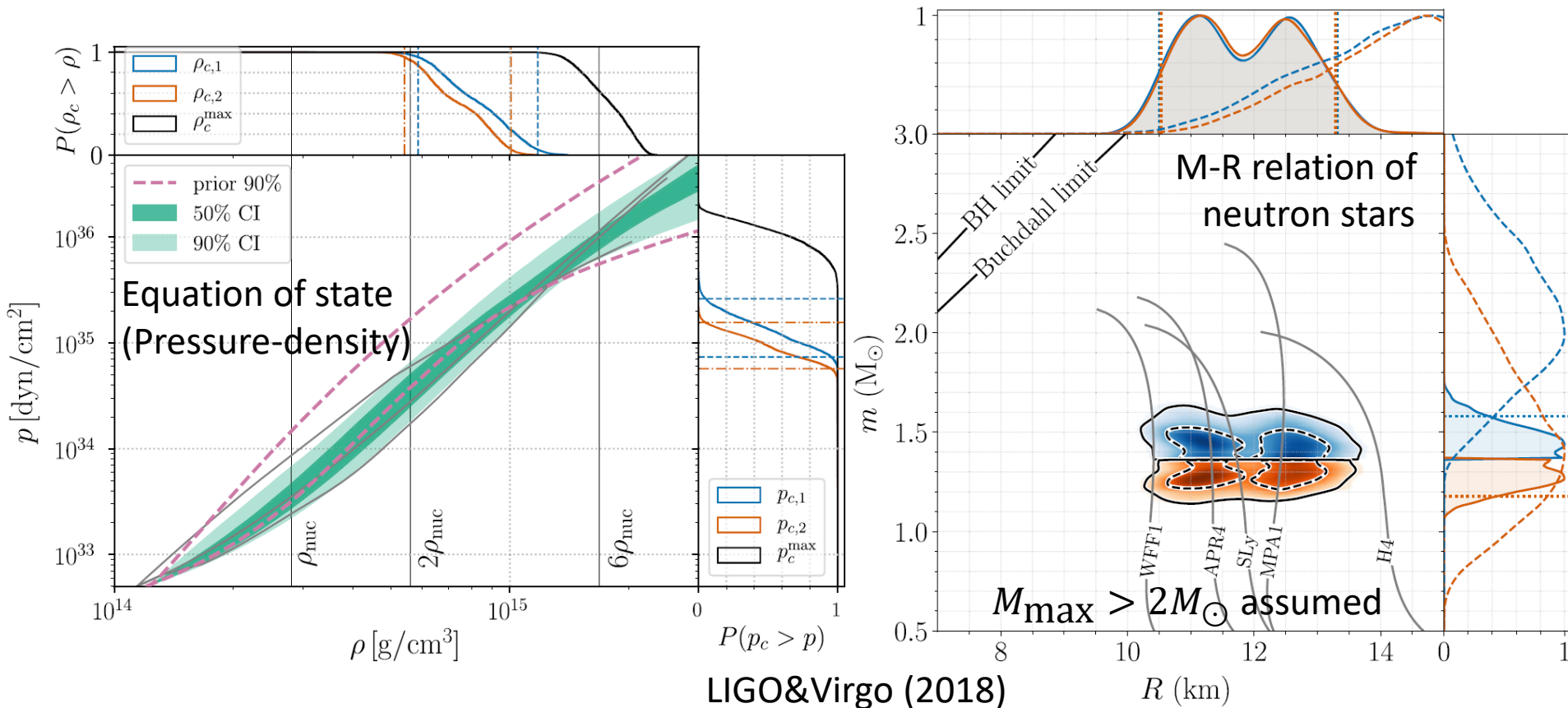
The key behind this success may be the identification of this important quantity out of the functional d.o.f., i.e., the equation of state [Flanagan-Hinderer 2008]

Binary tidal deformability



# Neutron-star equation of state

The equation of state has already been constrained and will be constrained more severely in the near future

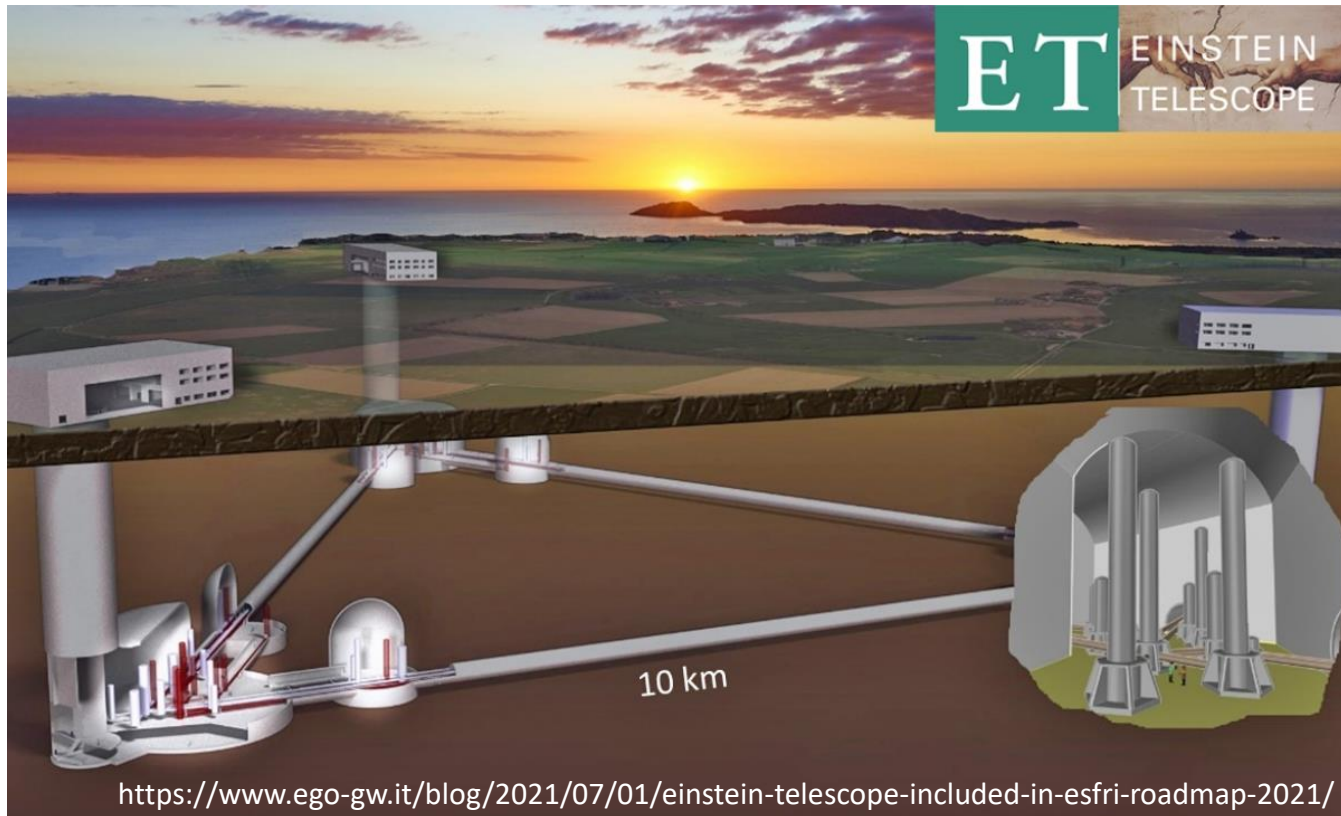




# **3. Postmerger: crossover vs. 1st-order phase transition**

# Third-generation detector

Einstein Telescope, Cosmic Explorer ... aiming at more precise understanding of already-detected binaries

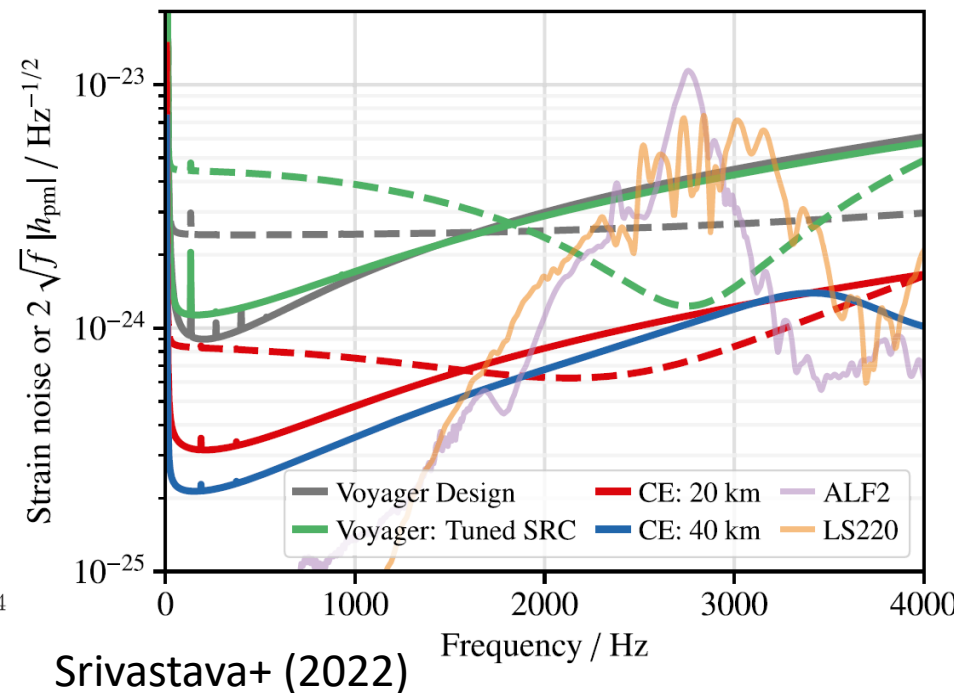
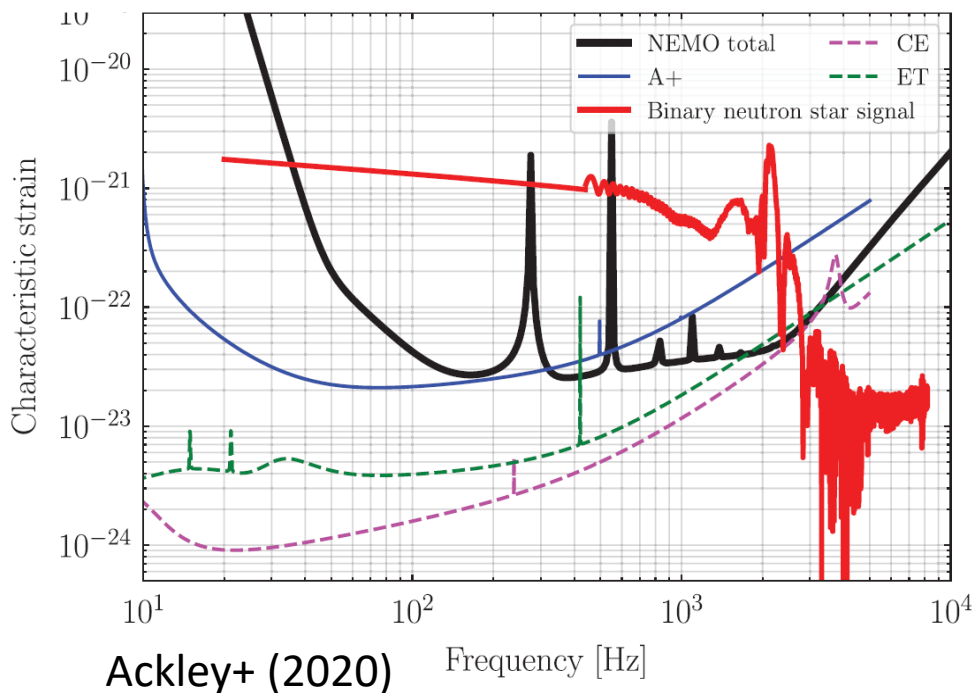


# Future high-frequency observation

The high density requires high-frequency observations

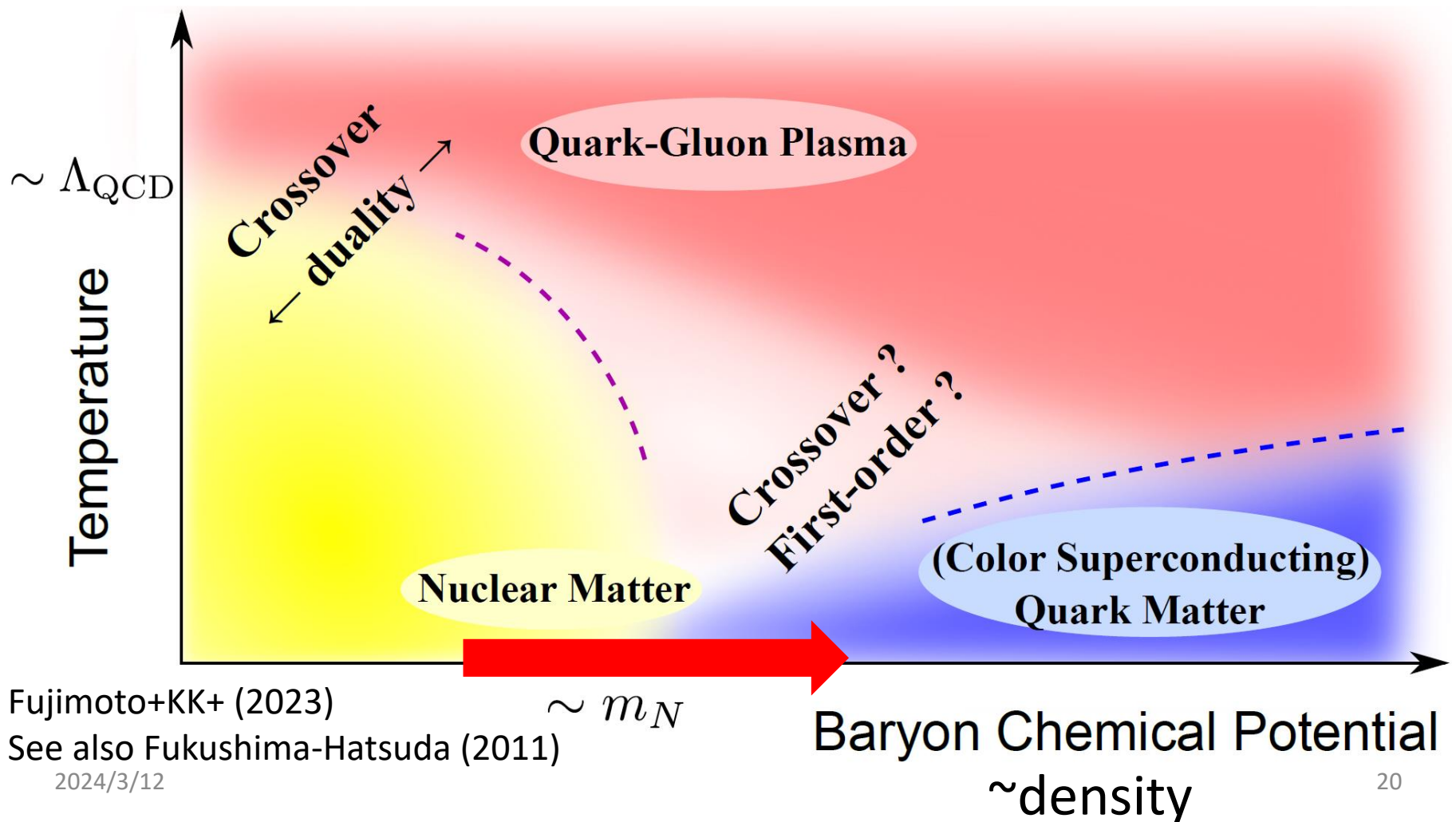
$$f \sim \sqrt{G\rho}$$

Some proposals are made for postmerger signals



# QCD phase diagram

What kind of transition occurs from hadrons to quarks?



Fujimoto+KK+ (2023)

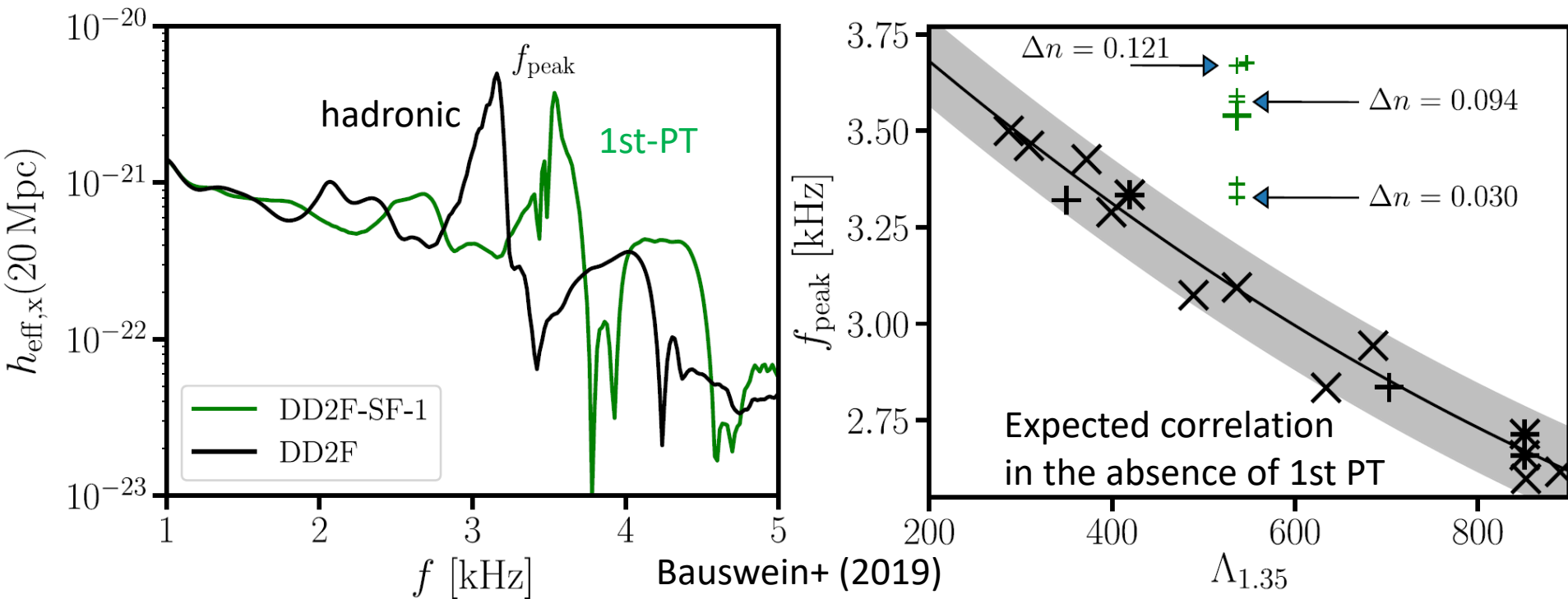
See also Fukushima-Hatsuda (2011)

2024/3/12

# 1st-order phase transition

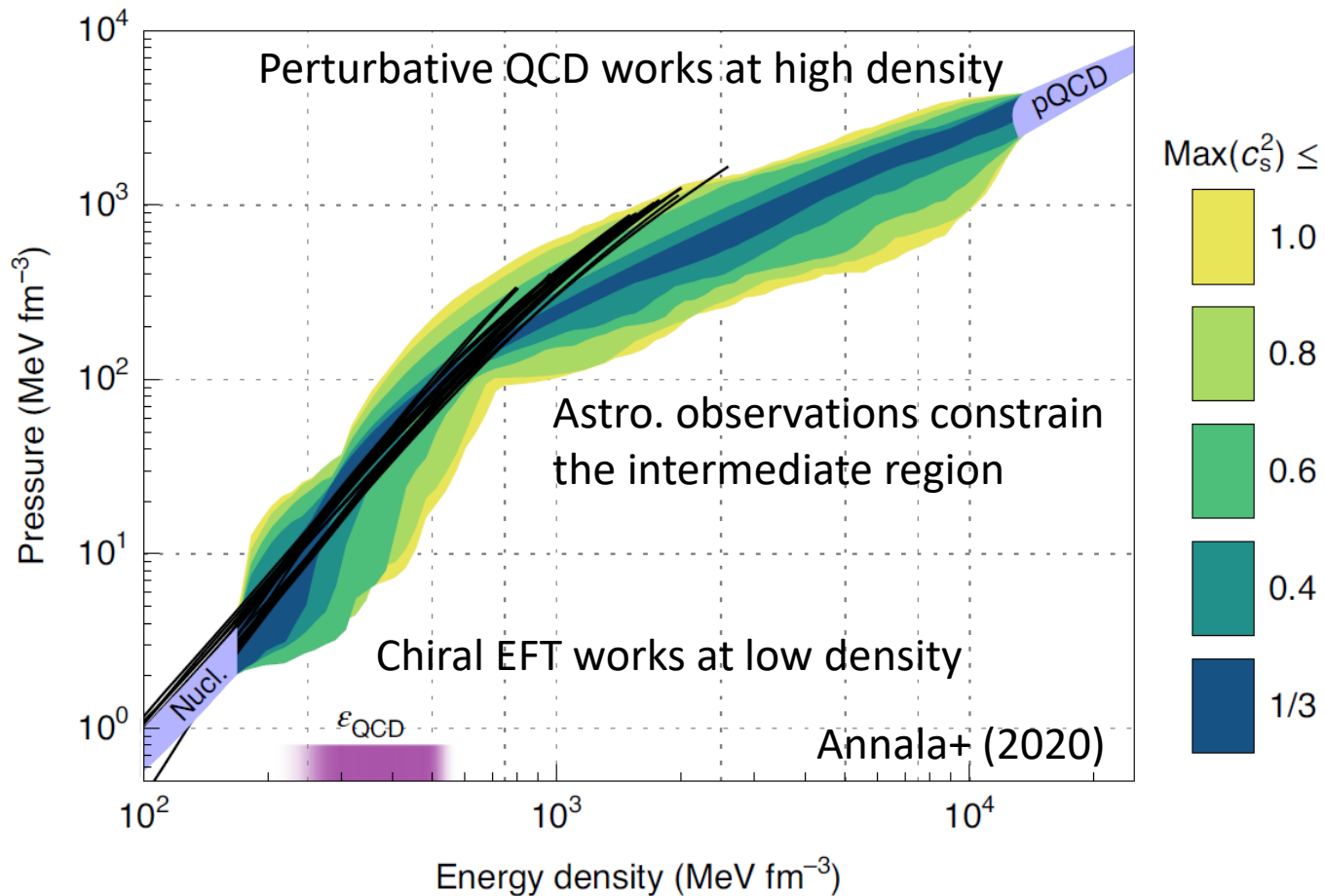
Postmerger neutron stars emit quasiperiodic signals

The shift in the peak frequency may reveal strong 1st-order phase transition at moderately high density



# Current view of the transition

Smooth crossover transition might be realistic



# Crossover vs. 1st-order PT

## Crossover

Smoothly connects two limits

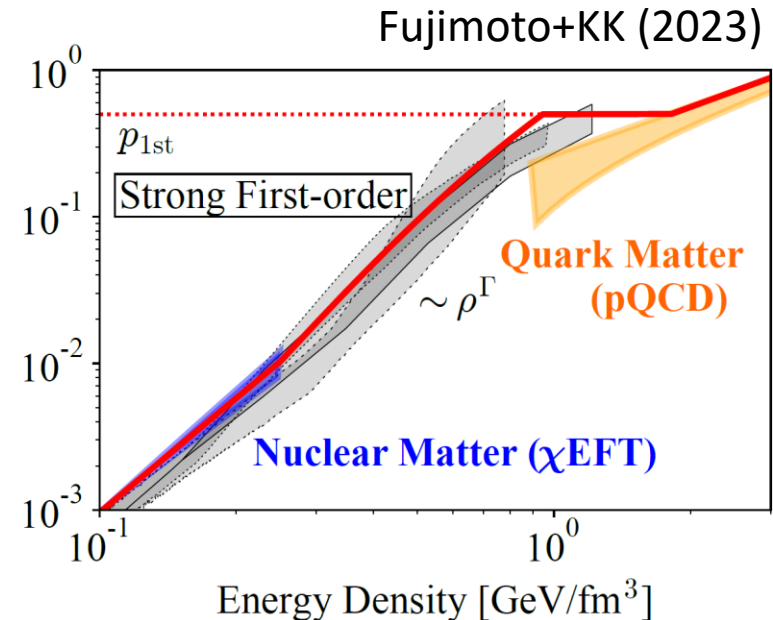
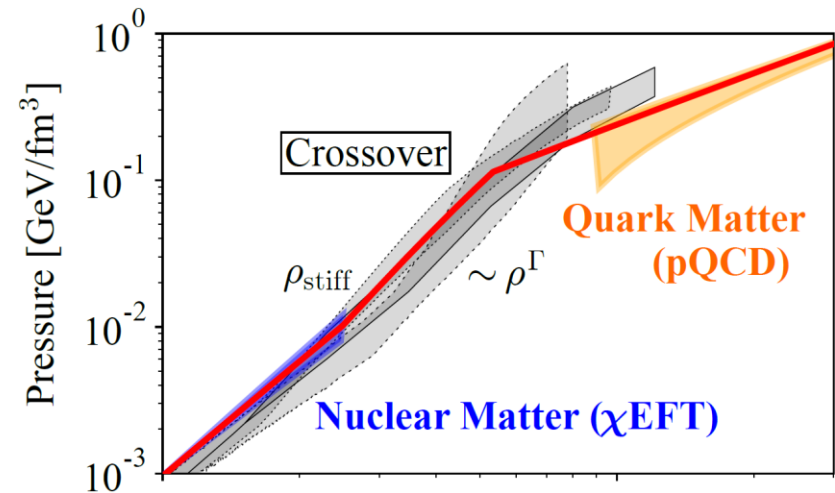
Note: we need to explain

2 solar mass neutron stars

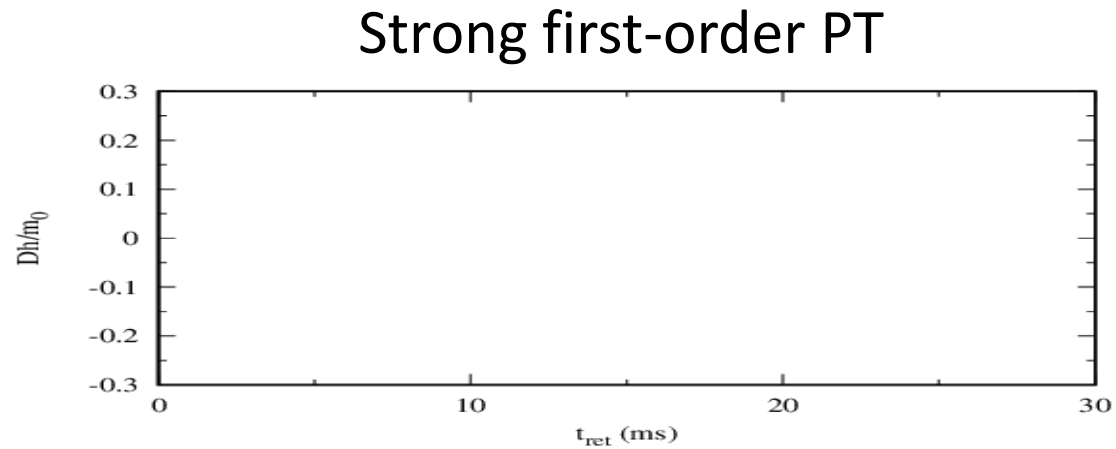
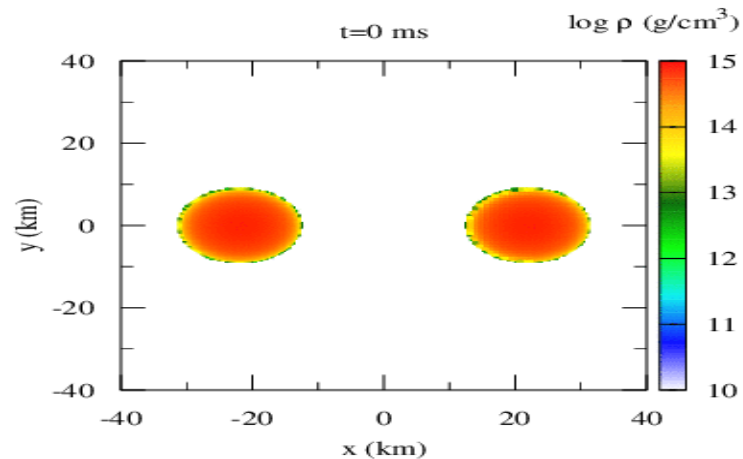
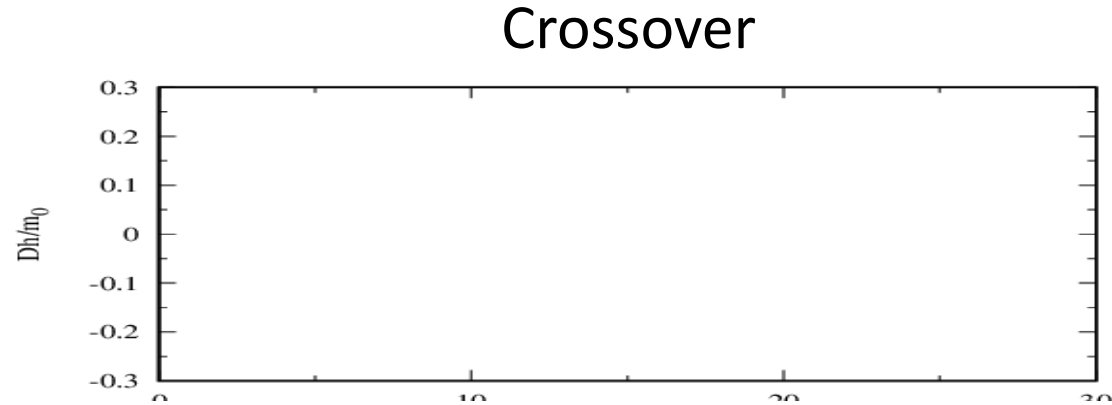
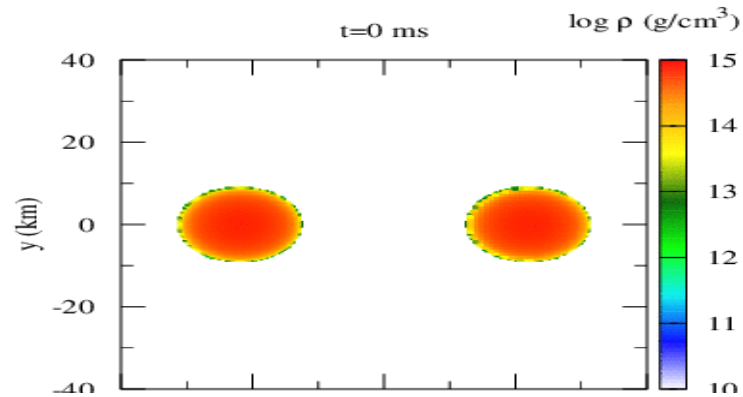
## 1st-order phase transition

Only very high density likely  
allows strong density jumps...

Invisible in astrophysics?



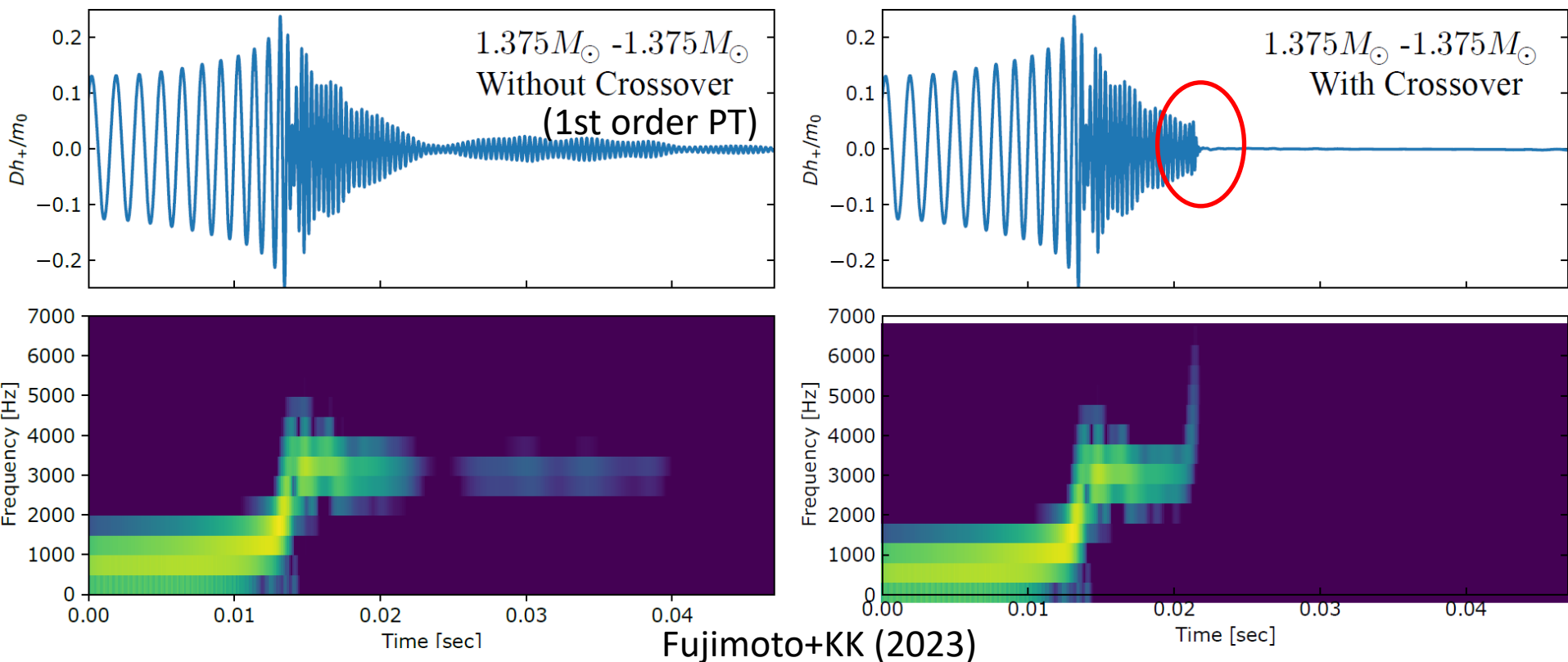
# Different postmerger evolution





# Black-hole formation as a key

Gravitational emission suddenly ends for crossover  
because of the gravitational collapse of the remnant



# Gravitational-wave spectrum

The postmerger peaks do not differ appreciably

The quasinormal-mode cutoff could be distinguishing

preliminary

# Lifetime of the merger remnant

Determined primarily by the total mass of the binary

preliminary

# Weak dependence on mass ratio

May be good news, as the mass ratio is hard to infer

preliminary

# Possible source of uncertainties

## Finite-temperature effect? (modeled by “ $\Gamma_{\text{th}}$ ”)

We vary systematically the strength of thermal pressure

## Neutrino effect? (neglected)

Its time scale is  $\sim 1\text{s}$ , much longer than our target

## Magnetic-field effect? (neglected)

Its time scale is  $\sim 0.1\text{s}$ , again longer than our target

## Grid resolution? (finite, of course)

Checked that dependence is weak, but not clean

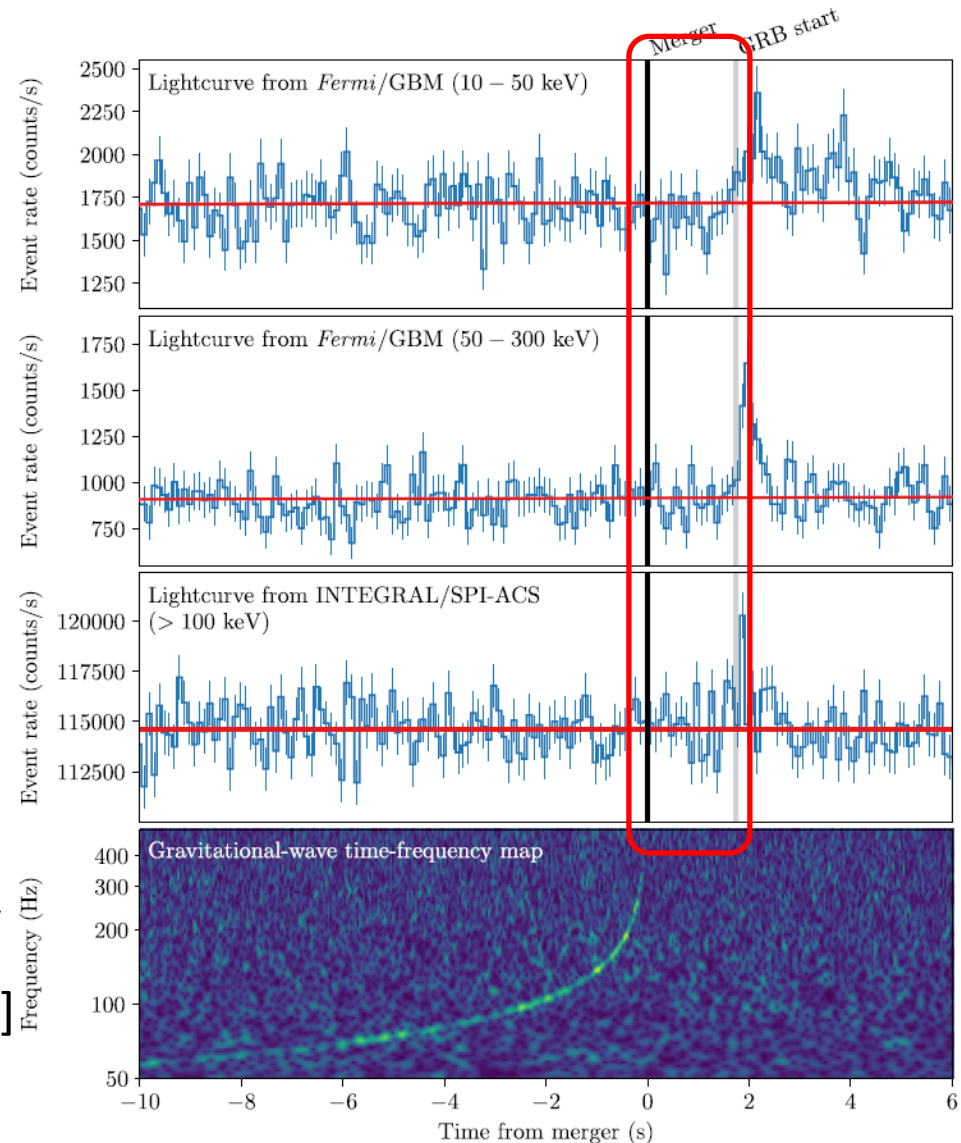
# Did GW170817 form a black hole?

Nobody knows the answer

Important for

- QCD phase structure
- gamma-ray burst
- r-process and kilonova

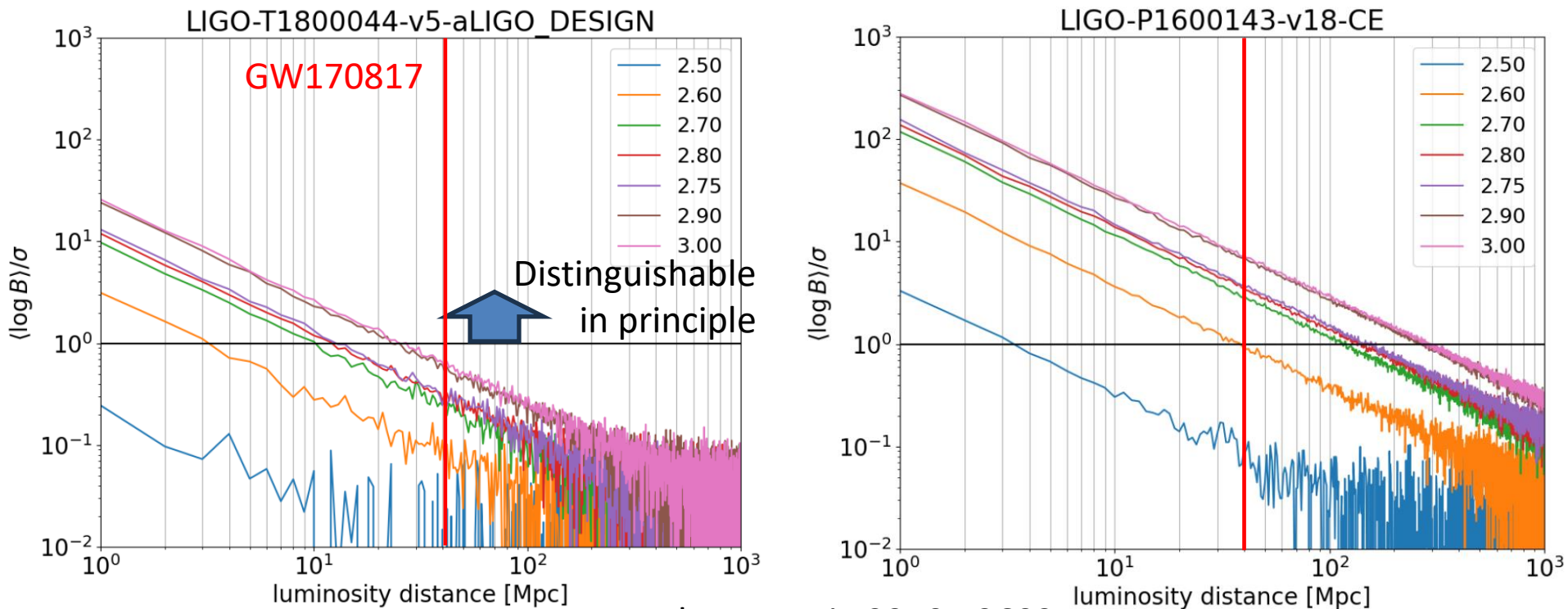
Gravitational waves are emitted for 10-100ms at  $\sim$ kHz and will be the key [neutrinos? Kyutoku-Kashiyama 2018]



# Distinguishability in data analysis

AdLIGO is insufficient even at design sensitivity (left)

Third-generation detectors may do at >100Mpc (right)



Harada+KK arXiv:2310.13603

# Which density range we can see?

The collapse is likely to set in when the central density reaches the maximum density of spherical stars

Not likely to dig into the unstable branch [cf. Ujevic+ 2024]

Various total masses

Various mass ratios

preliminary

preliminary



# 4. Discussion and summary

# Future direction and discussion

We have considered only two representative scenarios  
need systematic surveys or physics considerations

## How tiny 1st-order phase transition should be find?

- theoretically motivated strength of the transition?
- simply “as tiny as astro. observations can do?”

## Can we identify representative quantity (in general)?

- hard to infer  $P(n, T, Y_e), \mu(n, T, Y_e), \zeta(n, T, Y_e) \dots$
- $f_{\text{peak}}(m_1, m_2)$  is not very independent from  $\Lambda(m)$

# Summary

- The neutron-star equation of state is constrained by measuring tidal deformability from inspiral gravitational waveforms, particularly GW170817.
- In the future, postmerger gravitational waveforms may enable us to study the QCD phase structure via the gravitational collapse of merger remnants.
- The key toward these goals is the sensitivity at high frequency, specifically (1)  $\sim 3\text{kHz}$  for postmerger peaks, and (2)  $\sim 7\text{kHz}$  for quasinormal modes excited at the black-hole formation.

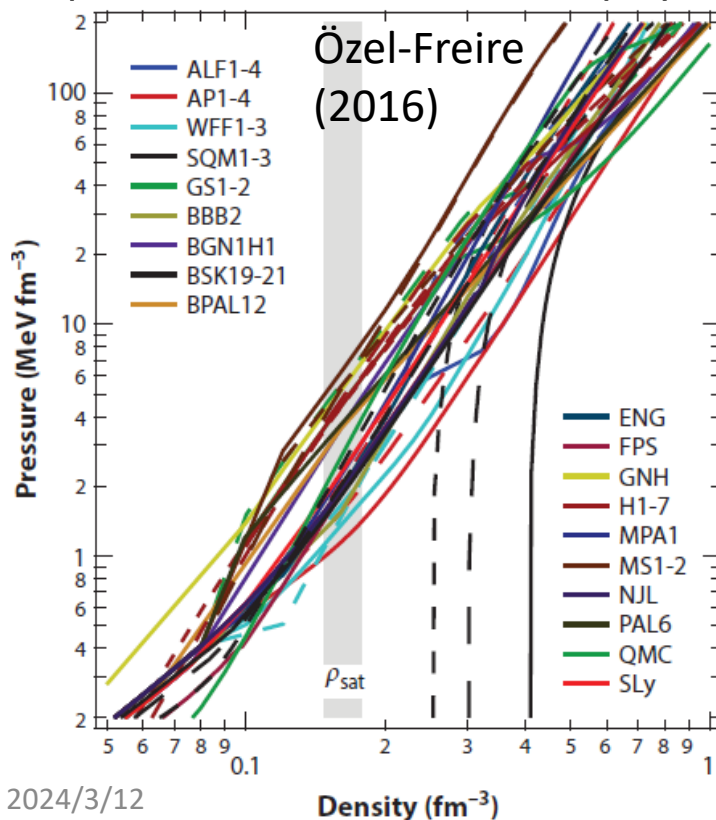


# Neutron star equation of state

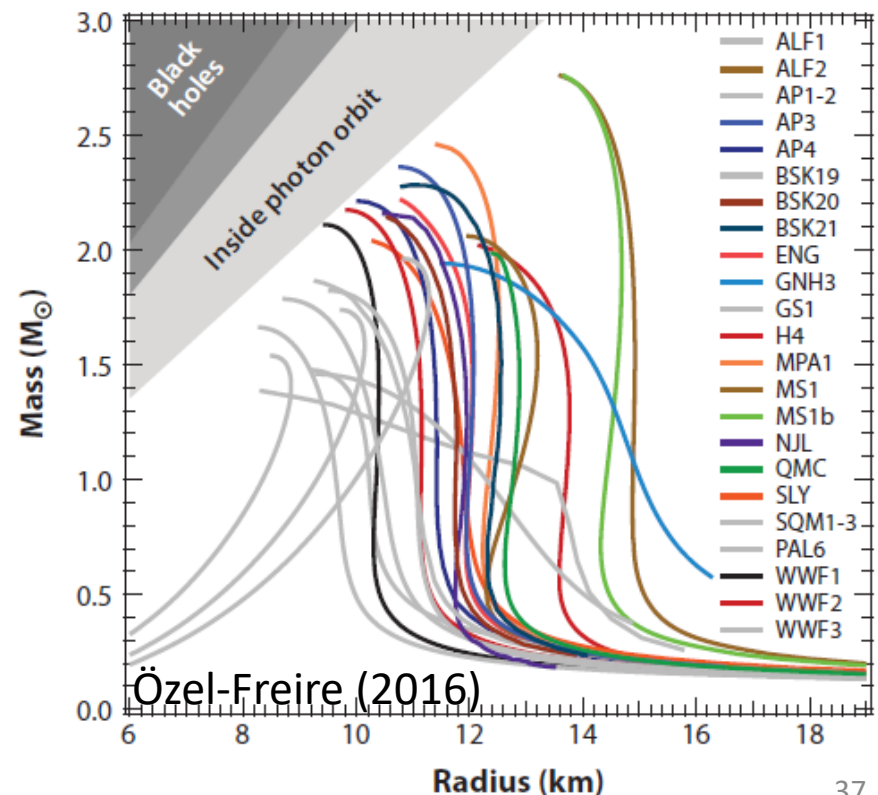
Note: not need to observe the radius, and other quantities may be fine

We want to know the realistic equation of state, that uniquely determines the mass-radius relation

Equation of state: Nuclear physics



Mass-Radius relation: Astrophysics



# Astronomical observation

**Maximum mass from radio pulsars**

J1614-2230, J3048+0432, J0740+6620

**Tidal deformability** from gravitational waves

GW170817(, GW190425: not so informative)

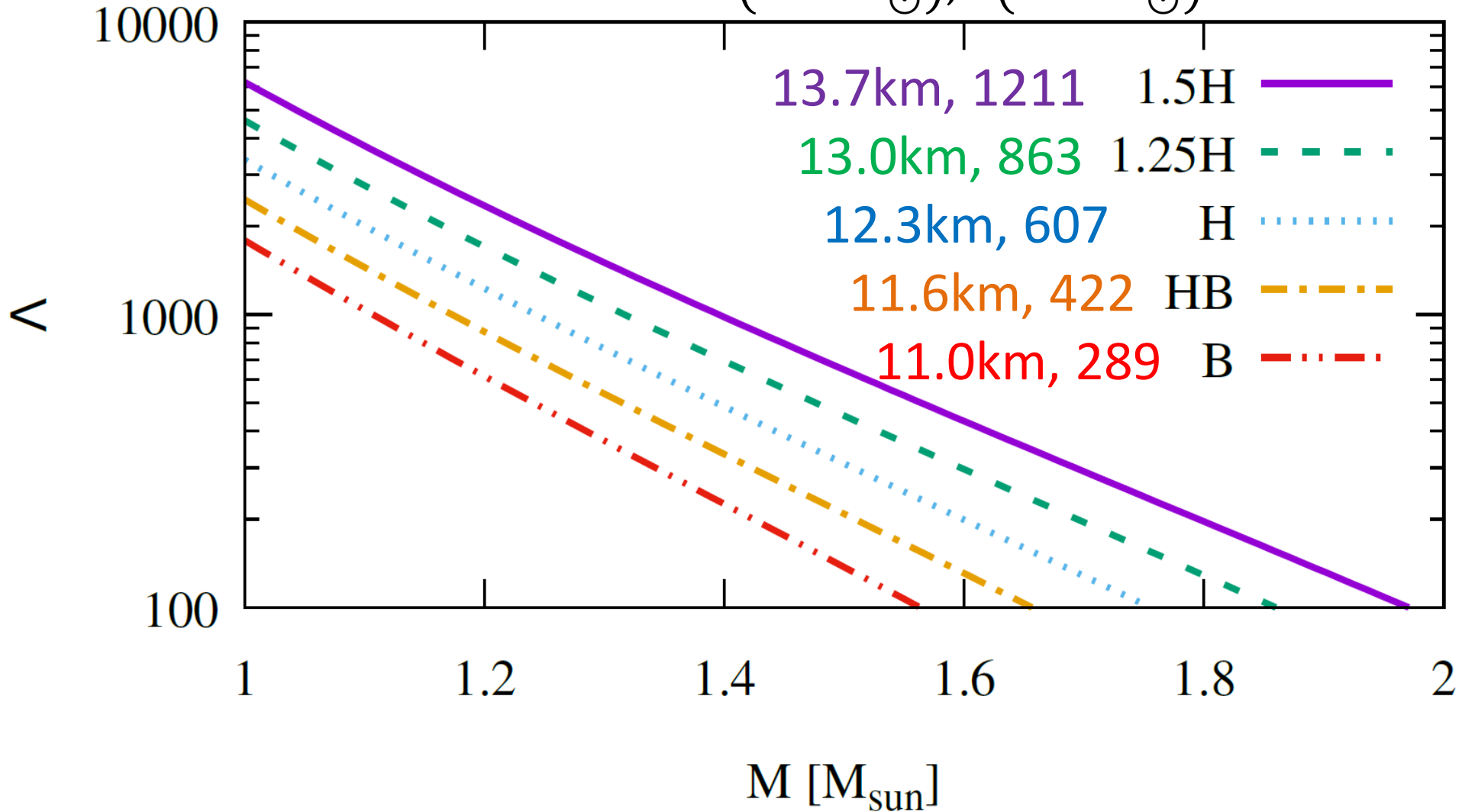
**Compactness=mass/radius from X-ray pulsations**

J0030+0451, J0740+6620

+ **moment of inertia** from radio pulsars in the future?

# $M - \Lambda$ relation and equations of state

$R(1.35M_{\odot}), \Lambda(1.35M_{\odot})$

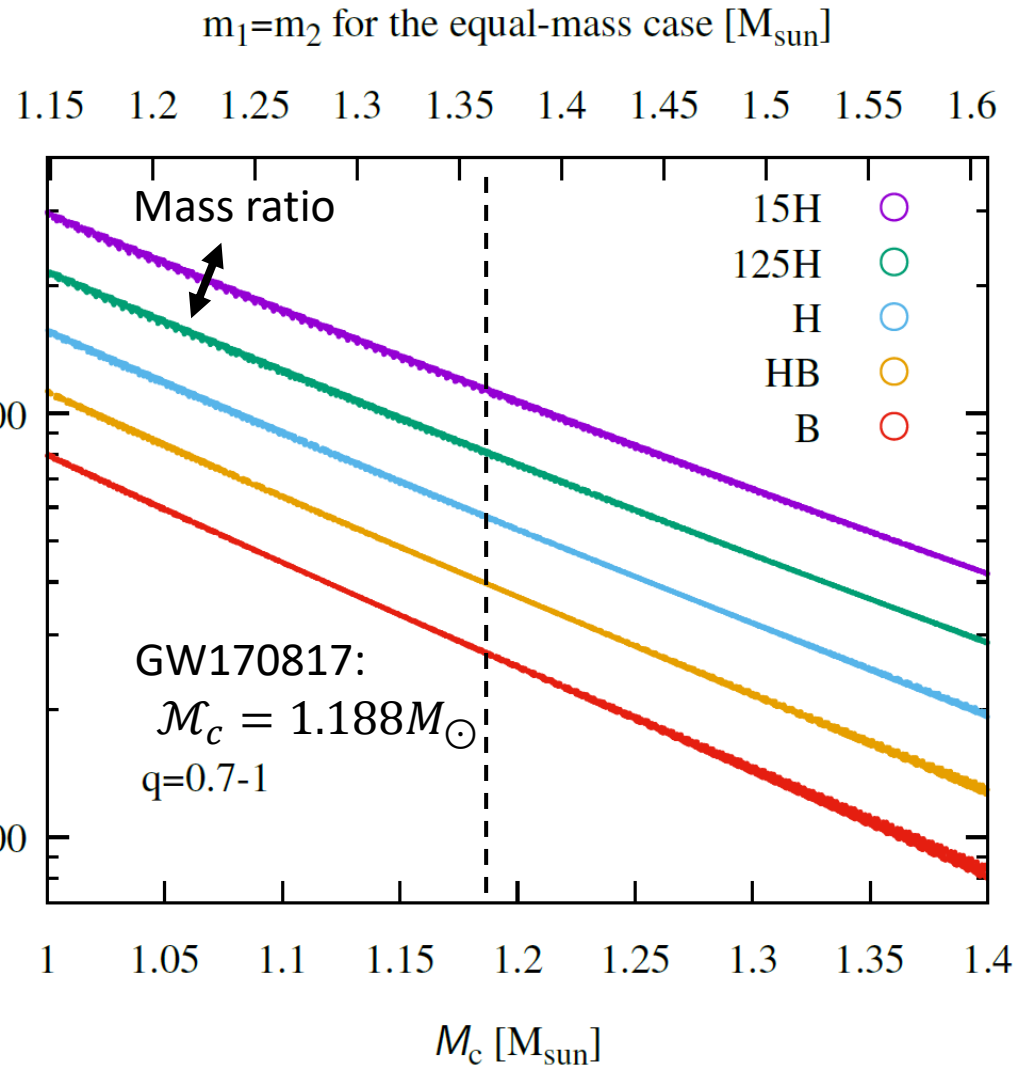


# Strong correlation of $\tilde{\Lambda} - \mathcal{M}_c$

The most measurable  $\tilde{\Lambda}$   
Is correlated strongly  
with the chirp mass  $\mathcal{M}_c$

We effectively constrain  
 $\tilde{\Lambda} <$   
 $\Lambda(M = 2^{1/5} \mathcal{M}_c)$

>13-14km is disfavored



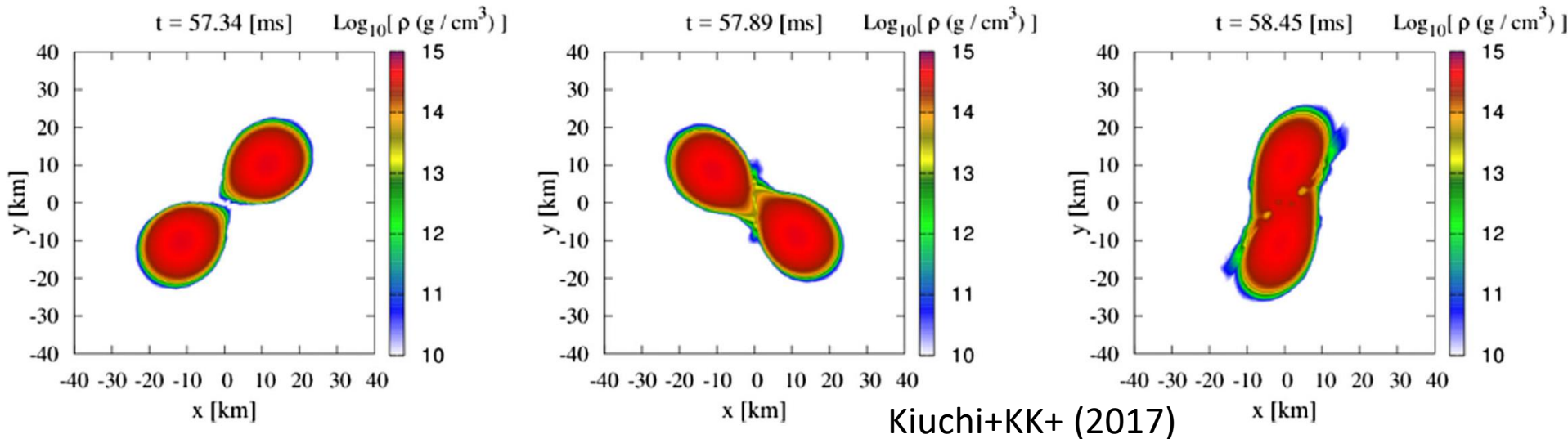


# Necessity of numerical simulations

The amplitude maximum comes after the contact

- Gravity (post-Newtonian correction) is nonlinear
- Hydrodynamics (tidal effect) is also nonlinear

Analytic computations cannot be fully accurate



# Waveform library

[https://www2.yukawa.kyoto-u.ac.jp/~nr\\_kyoto/SACRA\\_PUB/catalog.html](https://www2.yukawa.kyoto-u.ac.jp/~nr_kyoto/SACRA_PUB/catalog.html)

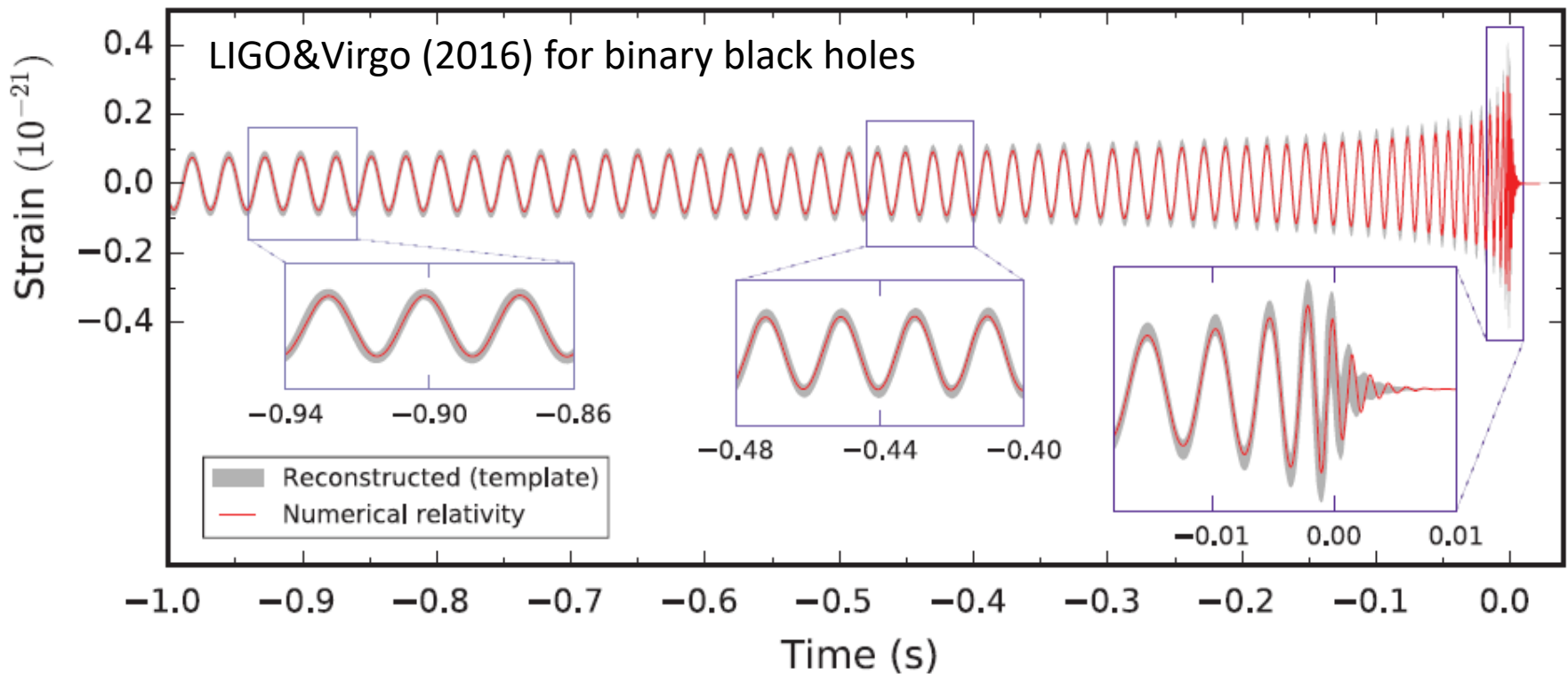
## Released Model List

Search:

Model name	$m_1$	$m_2$	$m_0$ (= $m_1+m_2$ )	$q$ (= $m_1/m_2$ )	$\eta$	$M_c$	EOS name	$\Lambda_1$	$\Lambda_2$	$\bar{\lambda}$	$m_0\Omega_0$	N	Reference
<a href="#">15H_135_135_00155_182_135</a>	1.35	1.35	2.7	1	0.25	1.17524	15H	1211	1211	1211	0.0155	182	<a href="#">Link</a>
<a href="#">15H_135_135_00155_150_135</a>	1.35	1.35	2.7	1	0.25	1.17524	15H	1211	1211	1211	0.0155	150	<a href="#">Link</a>
<a href="#">15H_135_135_00155_130_135</a>	1.35	1.35	2.7	1	0.25	1.17524	15H	1211	1211	1211	0.0155	130	<a href="#">Link</a>
<a href="#">15H_135_135_00155_110_135</a>	1.35	1.35	2.7	1	0.25	1.17524	15H	1211	1211	1211	0.0155	110	<a href="#">Link</a>
<a href="#">15H_135_135_00155_102_135</a>	1.35	1.35	2.7	1	0.25	1.17524	15H	1211	1211	1211	0.0155	102	<a href="#">Link</a>
<a href="#">15H_135_135_00155_90_135</a>	1.35	1.35	2.7	1	0.25	1.17524	15H	1211	1211	1211	0.0155	90	<a href="#">Link</a>
<a href="#">125H_135_135_00155_182_135</a>	1.35	1.35	2.7	1	0.25	1.17524	125H	863	863	863	0.0155	182	<a href="#">Link</a>
<a href="#">125H_135_135_00155_150_135</a>	1.35	1.35	2.7	1	0.25	1.17524	125H	863	863	863	0.0155	150	<a href="#">Link</a>
<a href="#">125H_135_135_00155_130_135</a>	1.35	1.35	2.7	1	0.25	1.17524	125H	863	863	863	0.0155	130	<a href="#">Link</a>
<a href="#">125H_135_135_00155_110_135</a>	1.35	1.35	2.7	1	0.25	1.17524	125H	863	863	863	0.0155	110	<a href="#">Link</a>
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<a href="#">H_135_135_00155_182_135</a>	1.35	1.35	2.7	1	0.25	1.17524	H	607	607	607	0.0155	182	<a href="#">Link</a>
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<a href="#">H_135_135_00155_130_135</a>	1.35	1.35	2.7	1	0.25	1.17524	H	607	607	607	0.0155	130	<a href="#">Link</a>
<a href="#">H_135_135_00155_110_135</a>	1.35	1.35	2.7	1	0.25	1.17524	H	607	607	607	0.0155	110	<a href="#">Link</a>
<a href="#">H_135_135_00155_102_135</a>	1.35	1.35	2.7	1	0.25	1.17524	H	607	607	607	0.0155	102	<a href="#">Link</a>
<a href="#">H_135_135_00155_90_135</a>	1.35	1.35	2.7	1	0.25	1.17524	H	607	607	607	0.0155	90	<a href="#">Link</a>
<a href="#">HB_135_135_00155_182_135</a>	1.35	1.35	2.7	1	0.25	1.17524	HB	422	422	422	0.0155	182	<a href="#">Link</a>
<a href="#">HB_135_135_00155_150_135</a>	1.35	1.35	2.7	1	0.25	1.17524	HB	422	422	422	0.0155	150	<a href="#">Link</a>
<a href="#">HB_135_135_00155_130_135</a>	1.35	1.35	2.7	1	0.25	1.17524	HB	422	422	422	0.0155	130	<a href="#">Link</a>
<a href="#">HB_135_135_00155_110_135</a>	1.35	1.35	2.7	1	0.25	1.17524	HB	422	422	422	0.0155	110	<a href="#">Link</a>
<a href="#">HB_135_135_00155_102_135</a>	1.35	1.35	2.7	1	0.25	1.17524	HB	422	422	422	0.0155	102	<a href="#">Link</a>
<a href="#">HB_135_135_00155_90_135</a>	1.35	1.35	2.7	1	0.25	1.17524	HB	422	422	422	0.0155	90	<a href="#">Link</a>
<a href="#">B_135_135_00155_182_135</a>	1.35	1.35	2.7	1	0.25	1.17524	B	289	289	289	0.0155	182	<a href="#">Link</a>
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<a href="#">B_135_135_00155_130_135</a>	1.35	1.35	2.7	1	0.25	1.17524	B	289	289	289	0.0155	130	<a href="#">Link</a>
<a href="#">B_135_135_00155_110_135</a>	1.35	1.35	2.7	1	0.25	1.17524	B	289	289	289	0.0155	110	<a href="#">Link</a>
<a href="#">B_135_135_00155_102_135</a>	1.35	1.35	2.7	1	0.25	1.17524	B	289	289	289	0.0155	102	<a href="#">Link</a>
<a href="#">B_135_135_00155_90_135</a>	1.35	1.35	2.7	1	0.25	1.17524	B	289	289	289	0.0155	90	<a href="#">Link</a>
<a href="#">15H_125_146_00155_182_135</a>	1.25	1.46	2.71	0.86	0.2485	1.17524	15H	1871	760	1200	0.0155	182	<a href="#">Link</a>
<a href="#">15H_125_146_00155_150_135</a>	1.25	1.46	2.71	0.86	0.2485	1.17524	15H	1871	760	1200	0.0155	150	<a href="#">Link</a>

# Role of theoretical templates

Parameters of binaries are estimated by measuring the match between data and theoretical waveforms  
Accurate theoretical models are indispensable

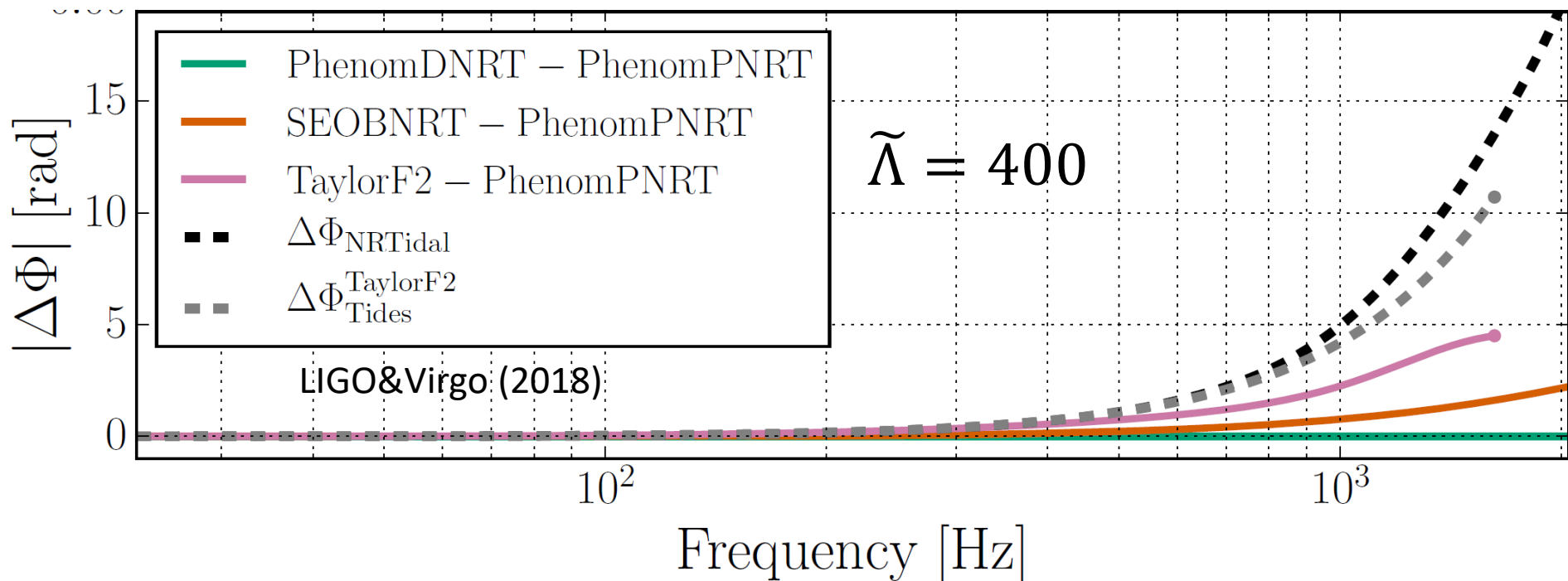


# Uncertainty in the waveform model

1 radian difference usually makes differences

Current systematic errors are larger than 1 radian

We need accurate waveforms for better estimation



# Kyoto gravitational-wave model

TaylorF2: analytic, Post-Newton phase ( $x \propto f^{2/3}$ )

$$\Psi_{\text{tidal}}^{2.5\text{PN}} = \frac{3}{128\eta} \left( -\frac{39}{2} \tilde{\Lambda} \right) x^{5/2} \left[ 1 + \frac{3115}{1248} x - \pi x^{3/2} + \frac{28024205}{3302208} x^2 - \frac{4283}{1092} \pi x^{5/2} \right]$$

+ correction terms associated w/ mass asymmetry

( $\tilde{\Lambda}$ : binary tidal deformability, i.e., weighted average)

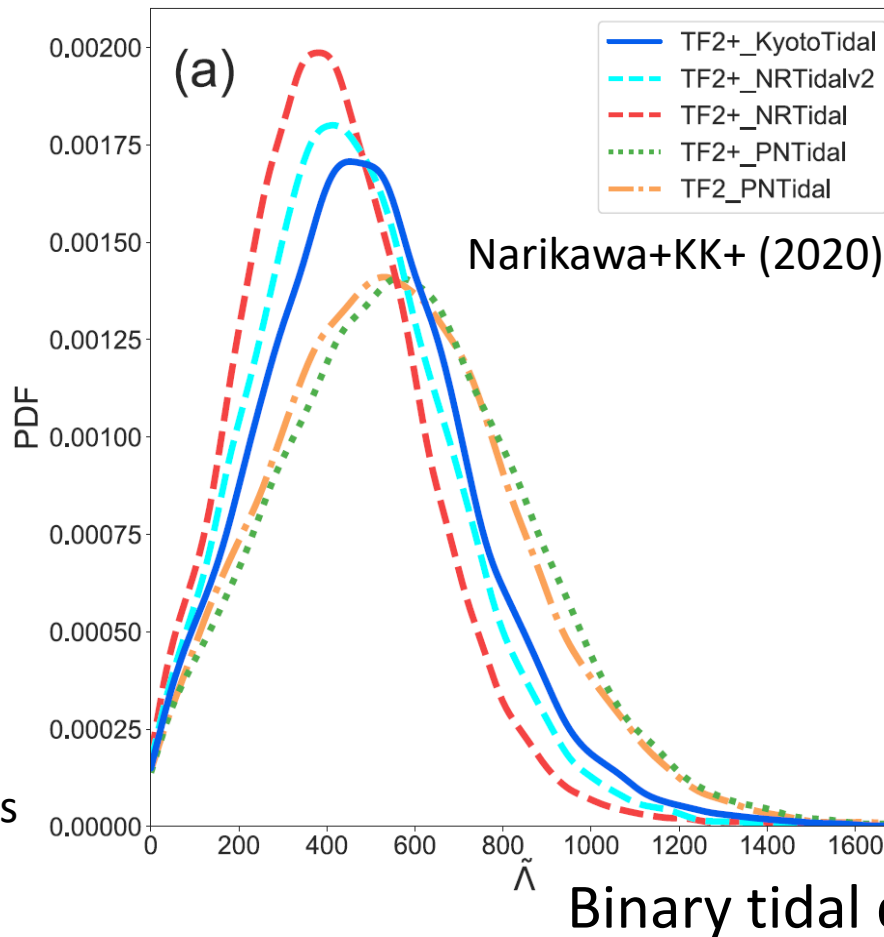
We introduce a nonlinear-in- $\tilde{\Lambda}$  term (empirically)

$$-\frac{39}{2} \tilde{\Lambda} (1 + 12.55 \tilde{\Lambda}^{2/3} x^{4.240})$$

This  $\tilde{\Lambda}^{2/3}$  term well reproduces numerical relativity

# Constraint from GW170817

Systematic bias is only  $\sim 100$  and currently negligible but may become problematic in the foreseeable future



Kyoto: our NR-based model from Kawaguchi+KK+ (2018)

NRTidal: another NR-based model used in LVC analysis

PNTidal: post Newton

# Case of GW190425

Weak constraint due to the high mass  $3.4M_{\odot}$  and the large distance 150-250Mpc

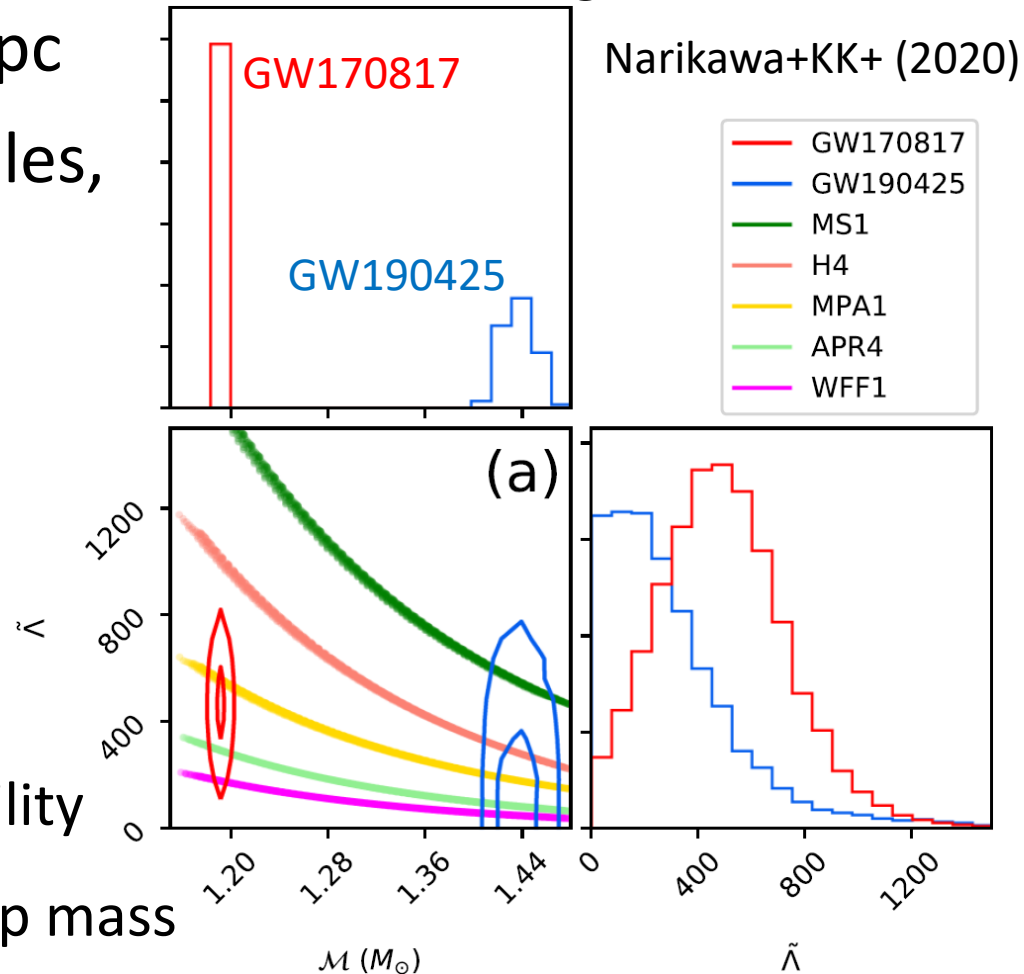
Even  $\tilde{\Lambda} = 0$ , i.e., black holes, may not be disfavored

[see also Kyutoku+ (2020)]

Simply GW170817 was extremely lucky

Binary tidal deformability

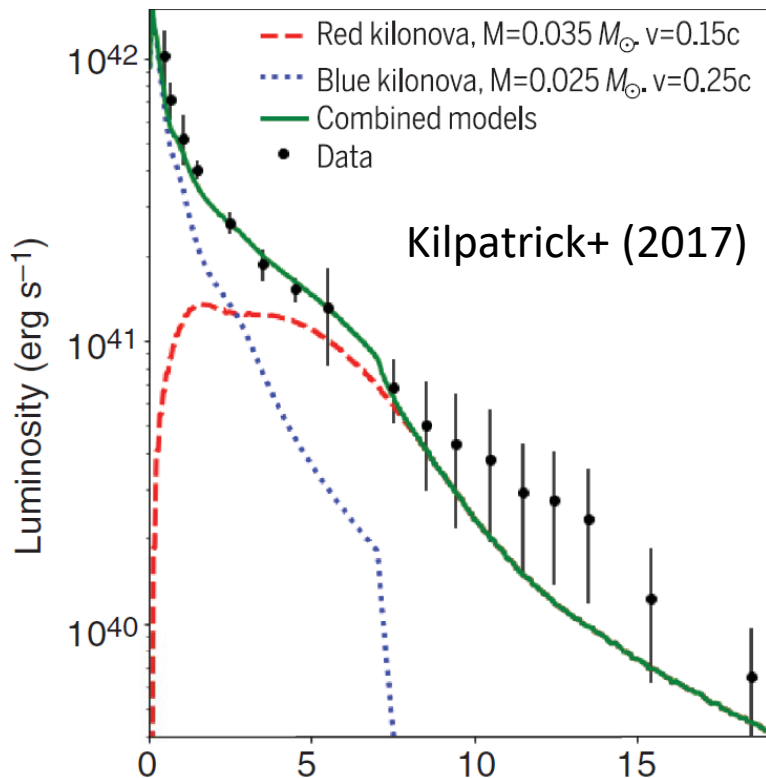
Chirp mass



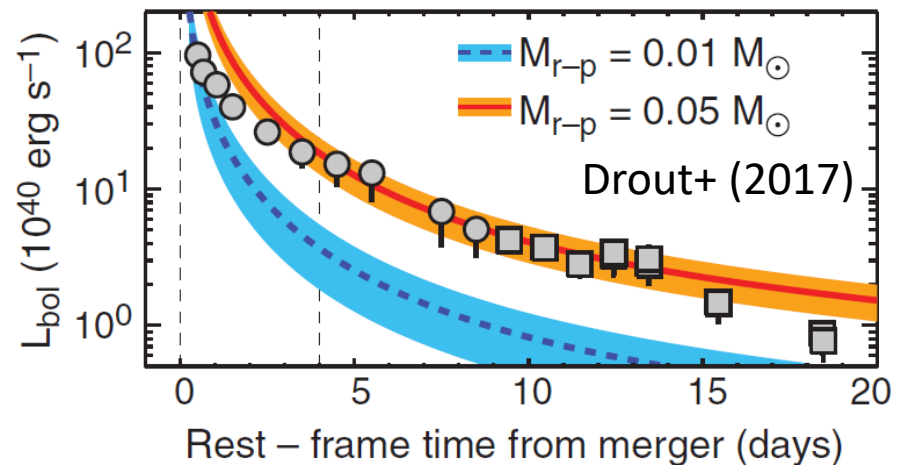
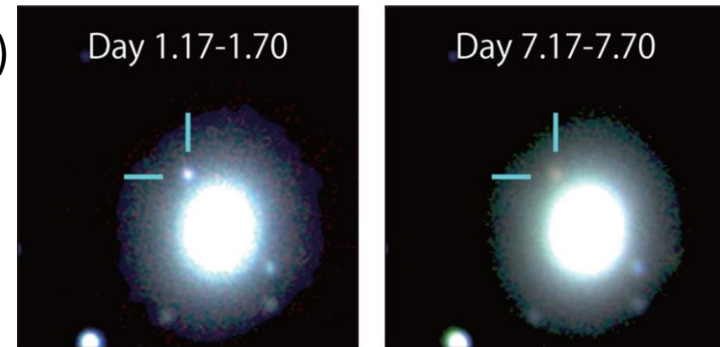
# Kilonova: AT 2017gfo

Indication of the large ejecta mass of  $\sim 0.05M_{\odot}$

It has been claimed that “this requires  $\tilde{\Lambda} > 400$ ”



Utsumi+ (2017)



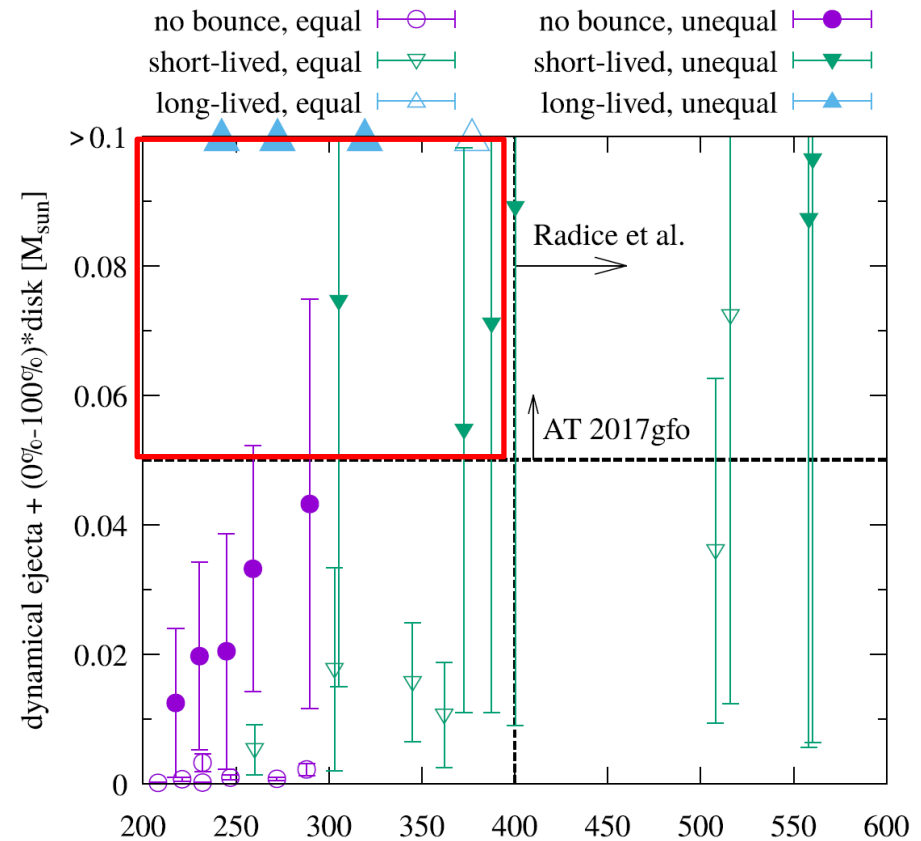


# A lot of counterexamples

Our conclusion:

Lower limits on  $\tilde{\Lambda}$  can be derived only under restrictive assumptions

(vertical bars denote mass ejection efficiency from the disk, not errors)

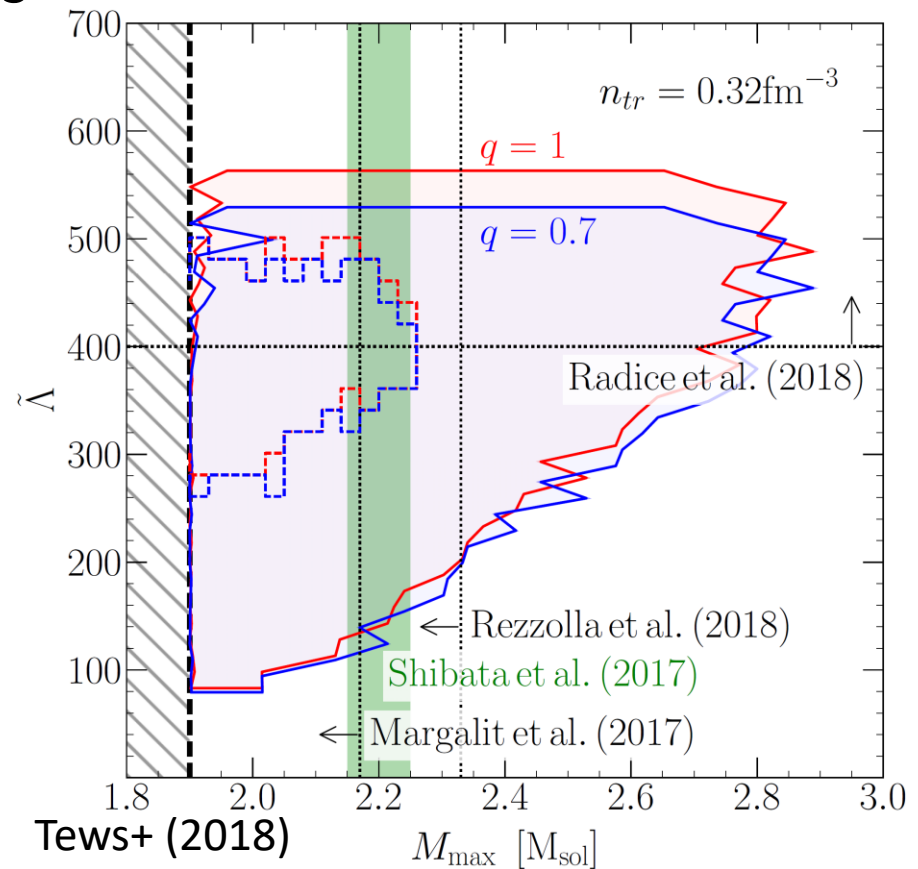


Kiuchi, KK+ (2019) binary tidal deformability  $\tilde{\Lambda}$

# Reason?

$M_{\max}$  may not be strongly correlated with  $\tilde{\Lambda} \propto R^{\sim 6}$   
of typical-mass neutron stars

If the remnant survived  
moderately long due to  
the large value of  $M_{\max}$ ,  
there should be no reason  
that mass ejection is weak

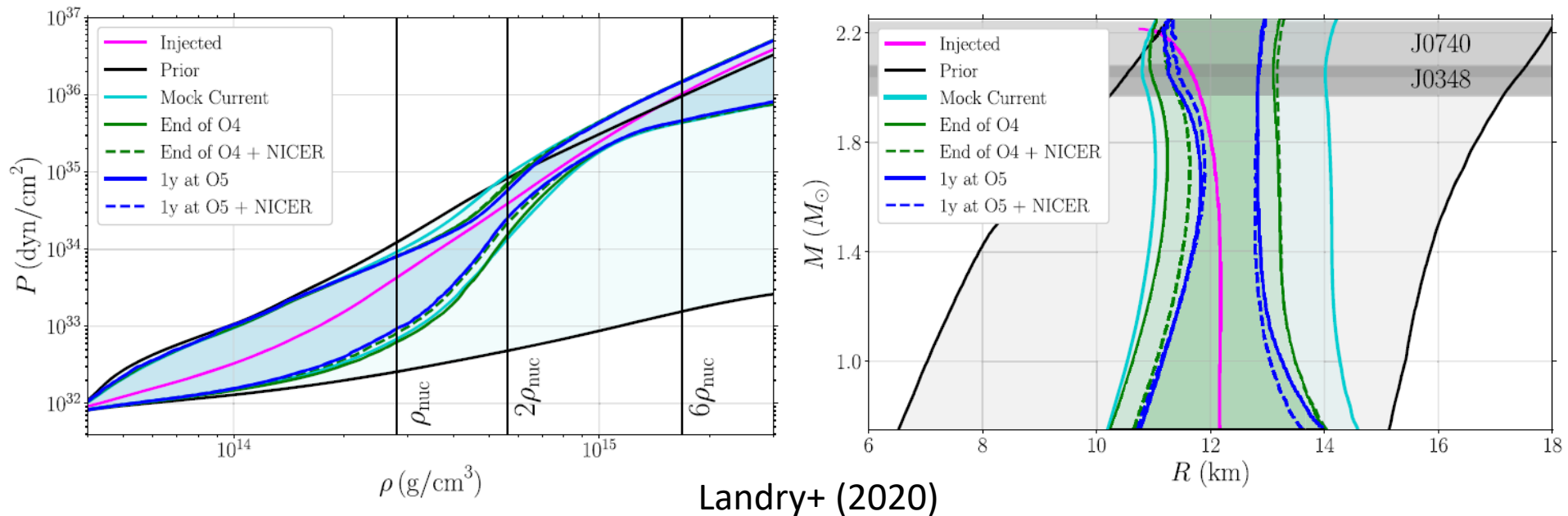




# What should we understand then?

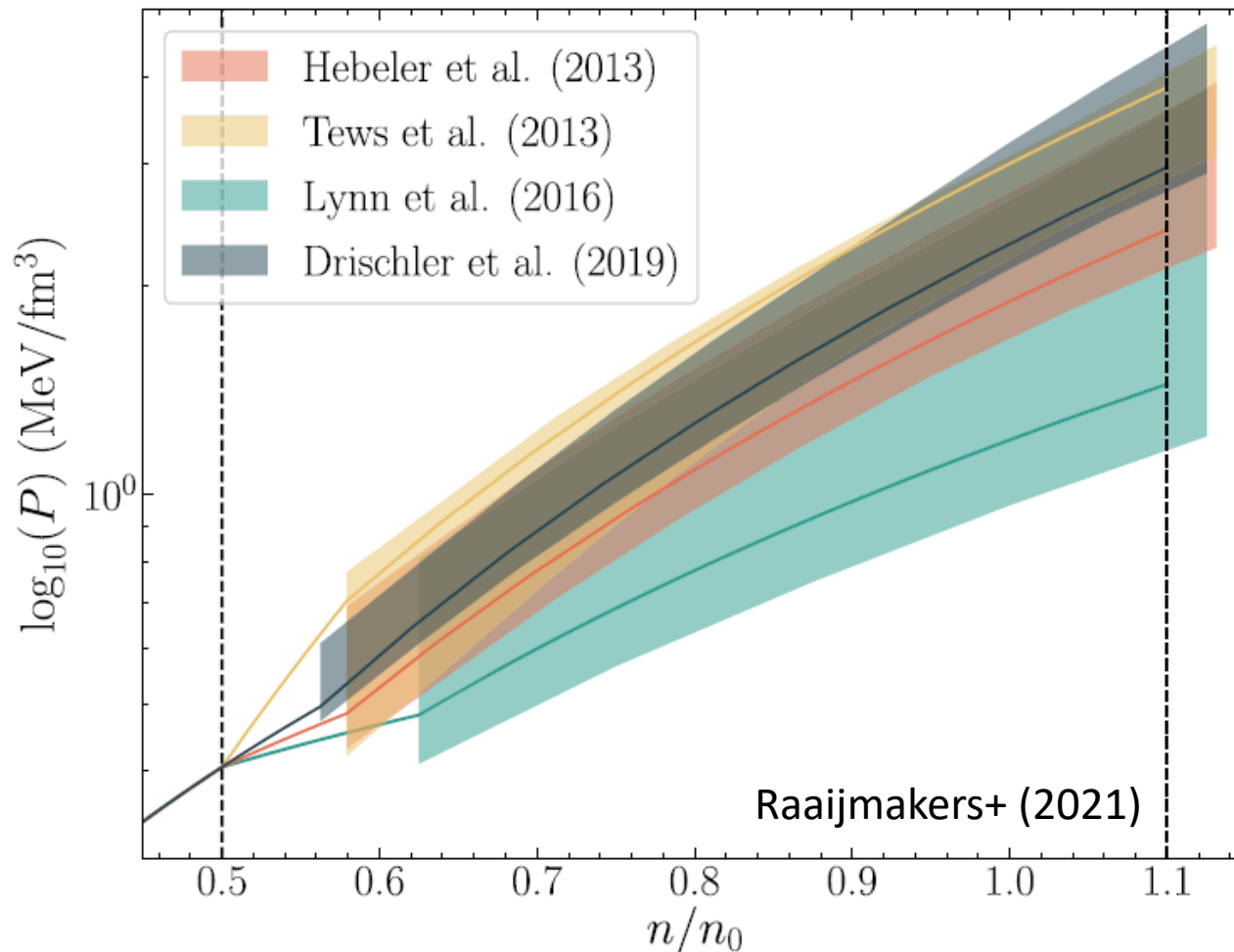
Moderate-density (around twice the saturation density) will be understood precisely by a lot of observations

On the basis of this idea, we would like to understand properties of ultrahigh-density matter



# Uncertainty in chiral EFT

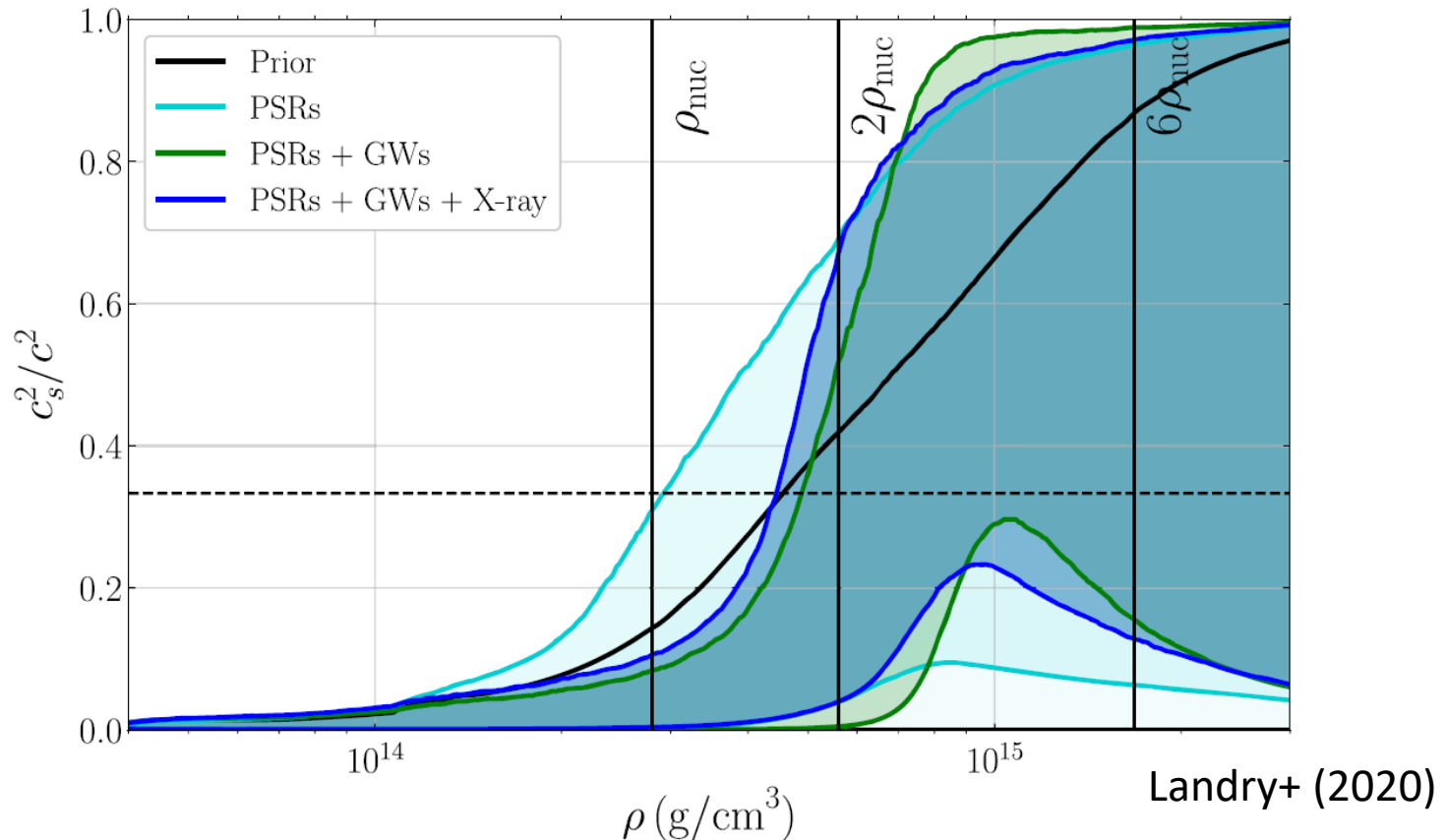
The validity range is crucial for strength of constraints



# Current view on the sound speed

Not stiff at low density, but  $2M_{\odot}$  must be supported.

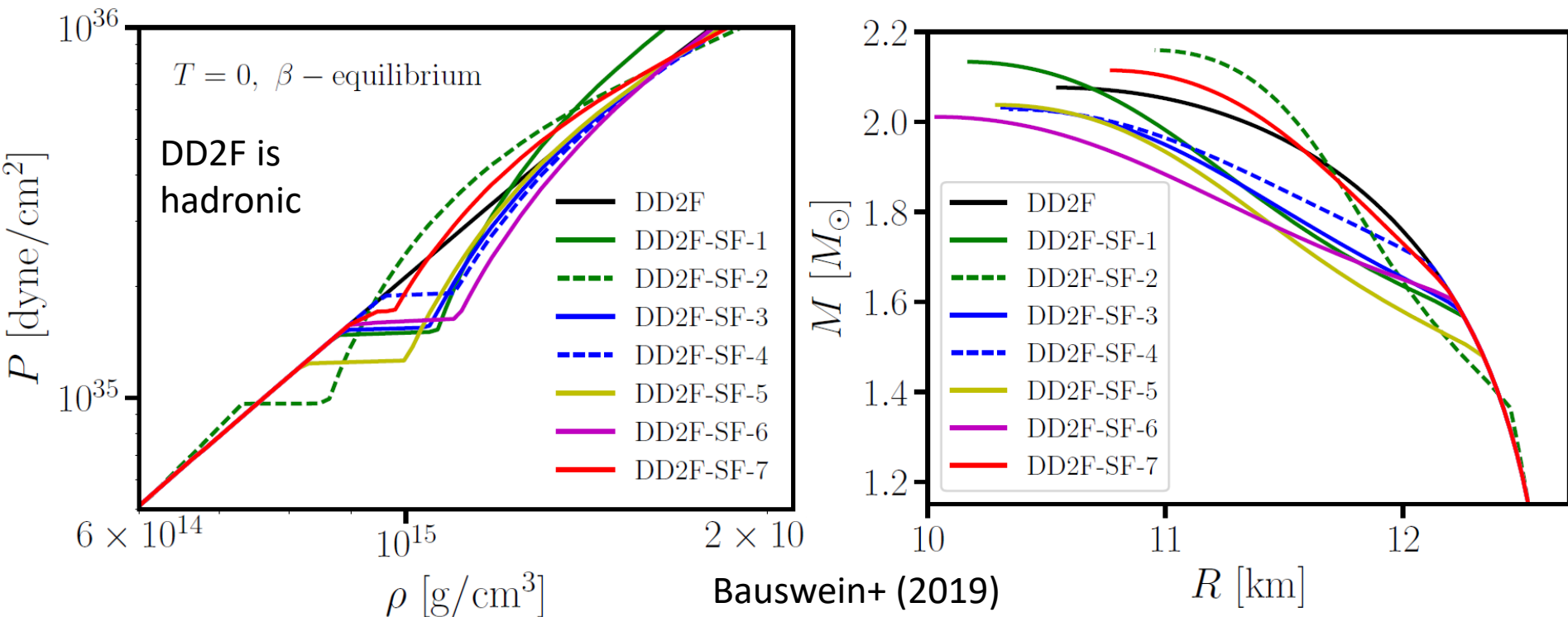
Conformal limit ( $c_s^2/c^2 = 1/3$ ) is likely to be exceeded



# First-order phase transition

The mass-radius relation breaks suddenly

An extreme case results in the so-called “twin star”

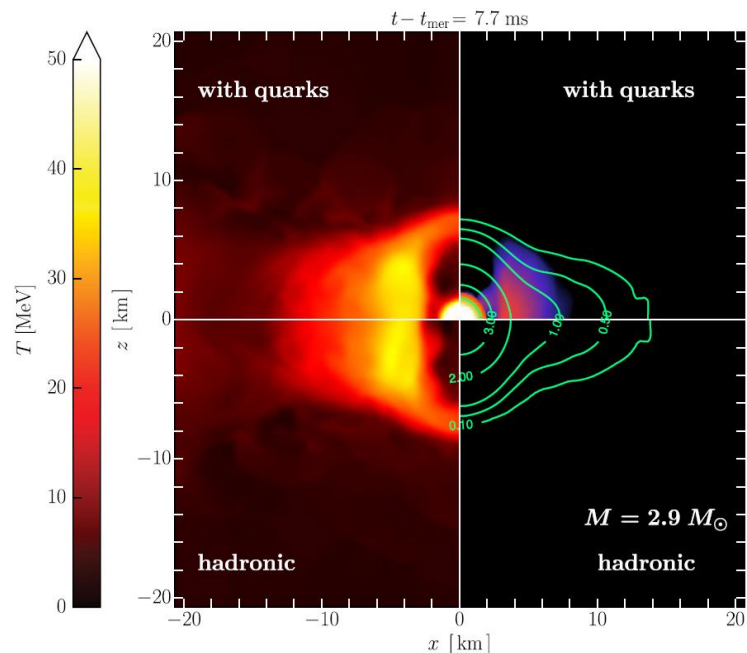


# Structure of the merger remnant

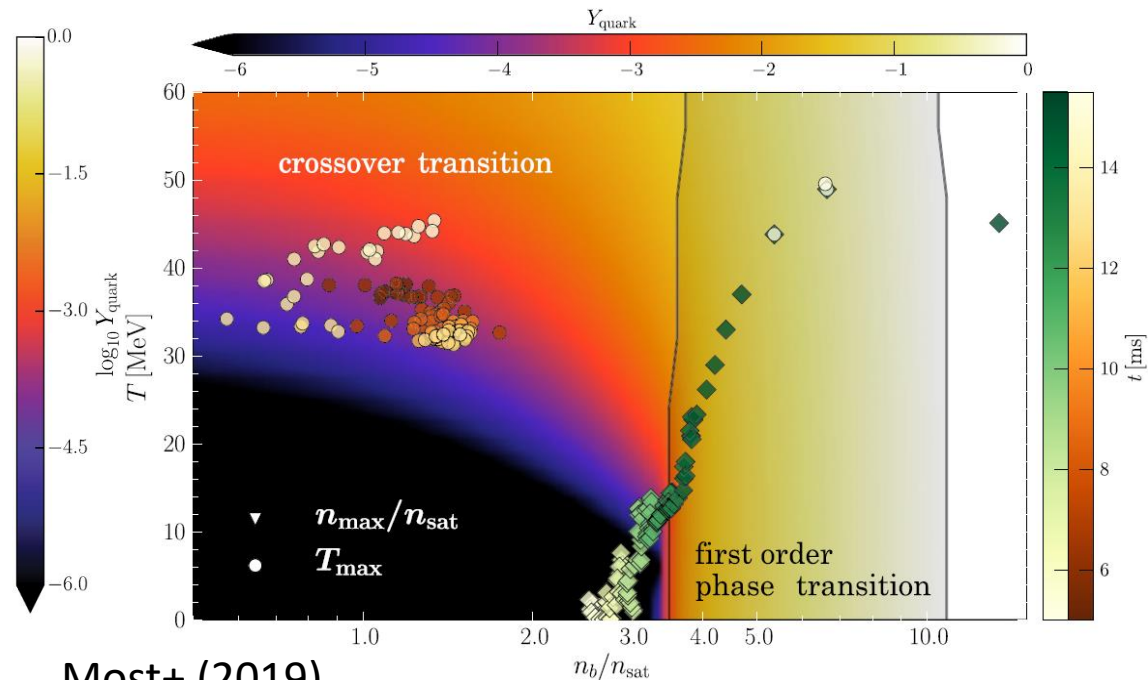
Density/temperature structures are not very different

Quarks appear at the high- $n$  core and high- $T$  envelope

Top: w/ quark, bottom: hadron only



Time evolution of maximum  $n$  and  $T$



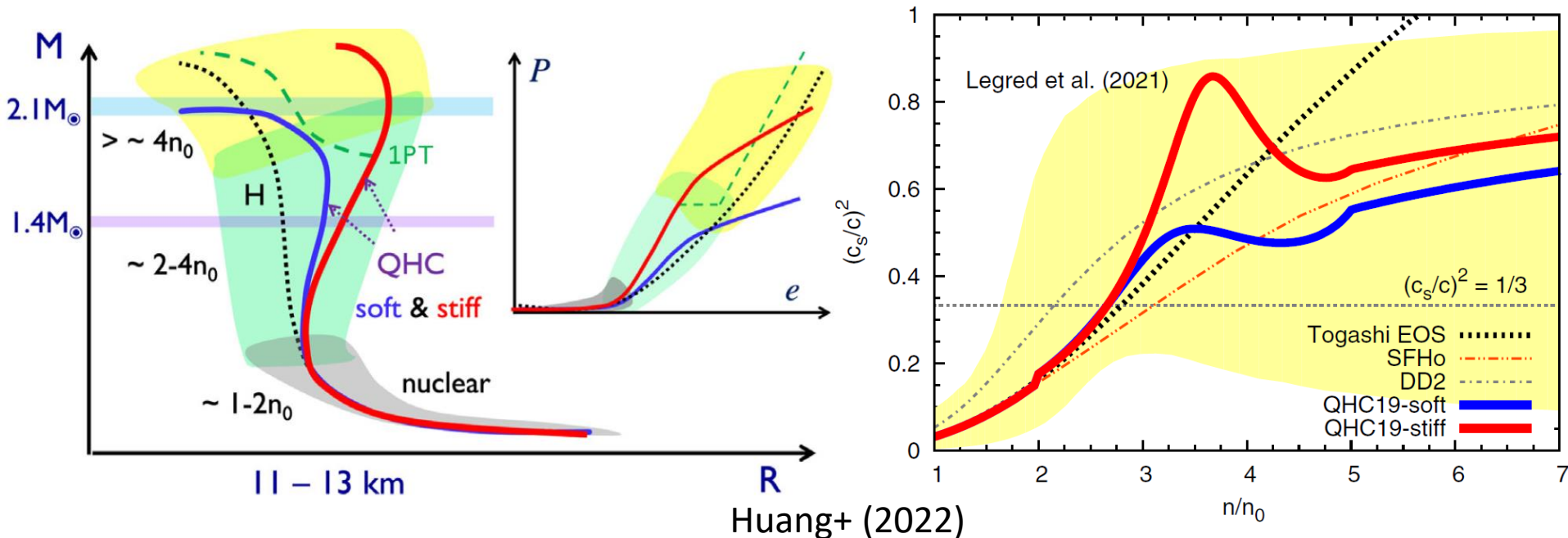
Most+ (2019)



# Relation to independent studies

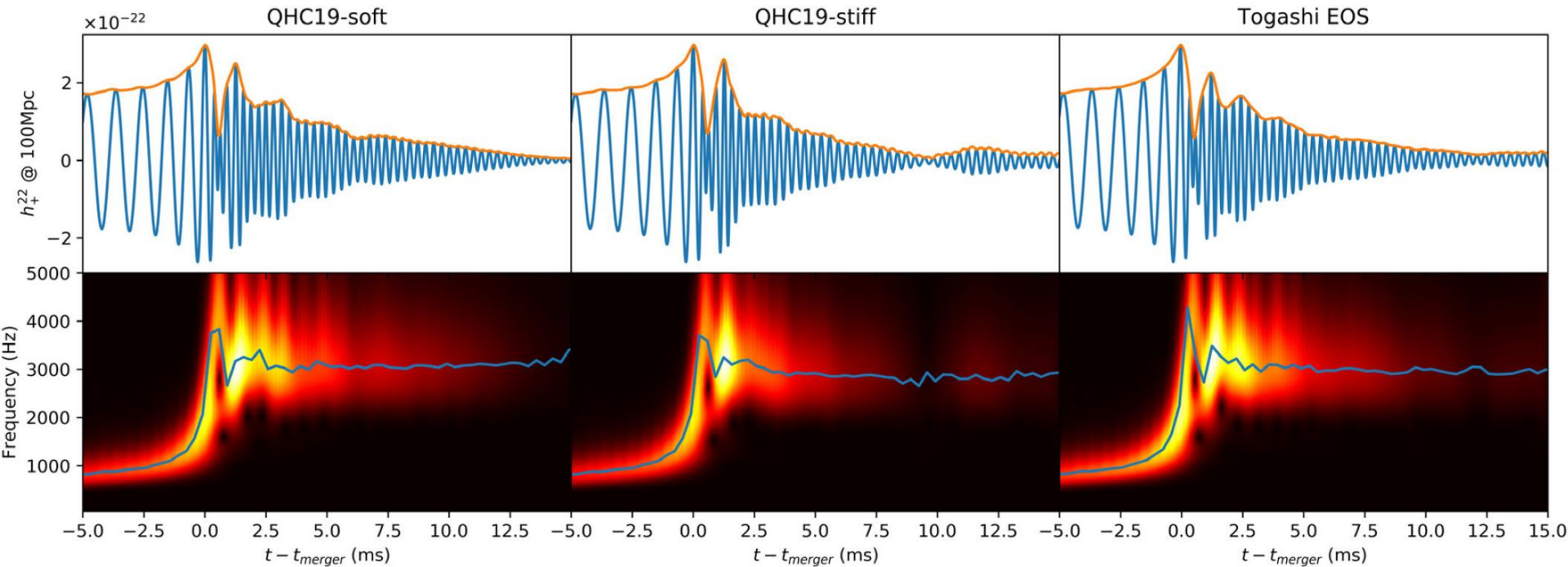
There exists other studies, e.g., those based on QHC

We require explicitly that the perturbative QCD regime is realized after the crossover from hadronic matter



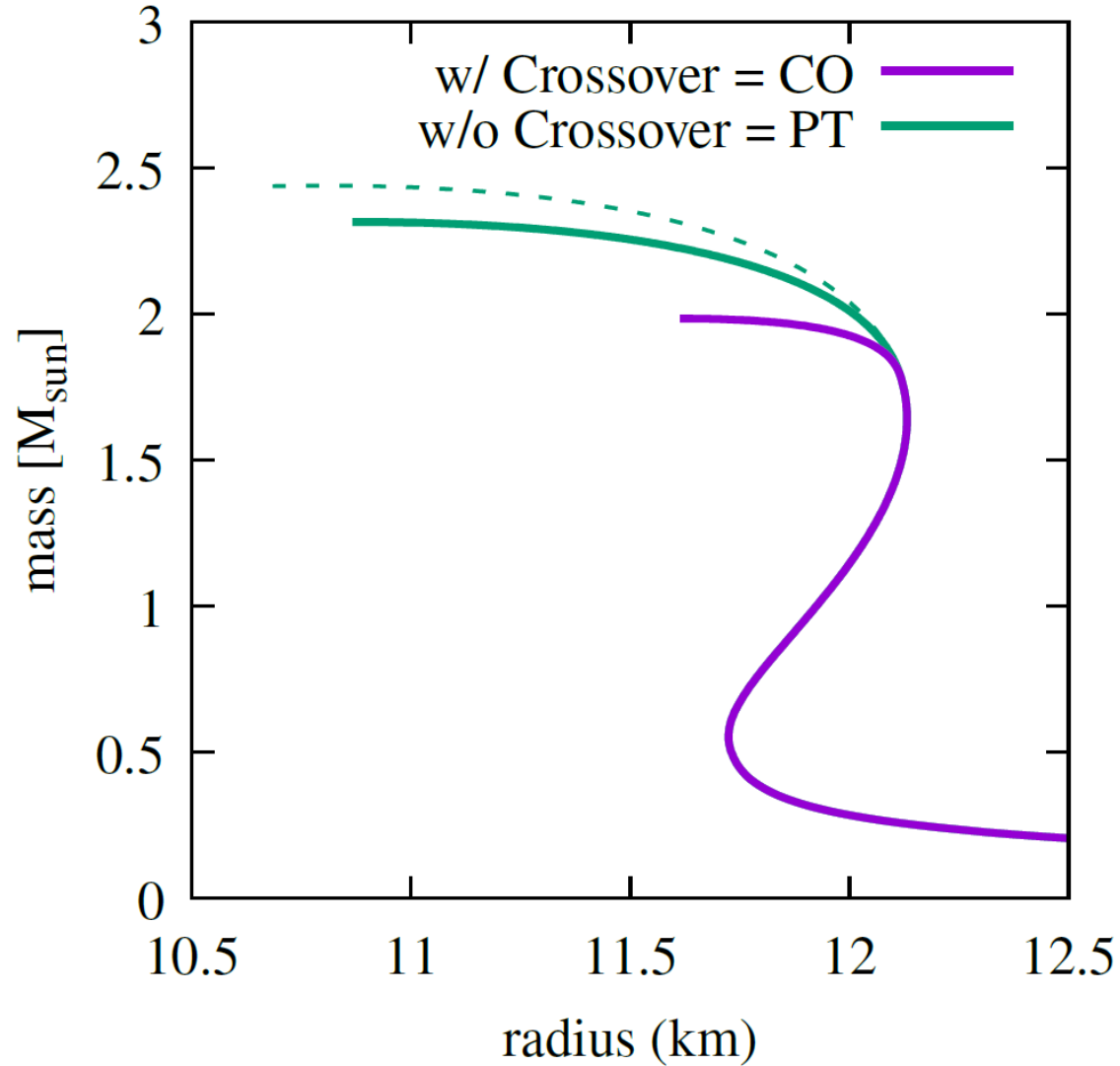
# Results derived with QHC

Soft equations of state at high density derive high postmerger frequency: also consistent with our results



Huang+ (2022)

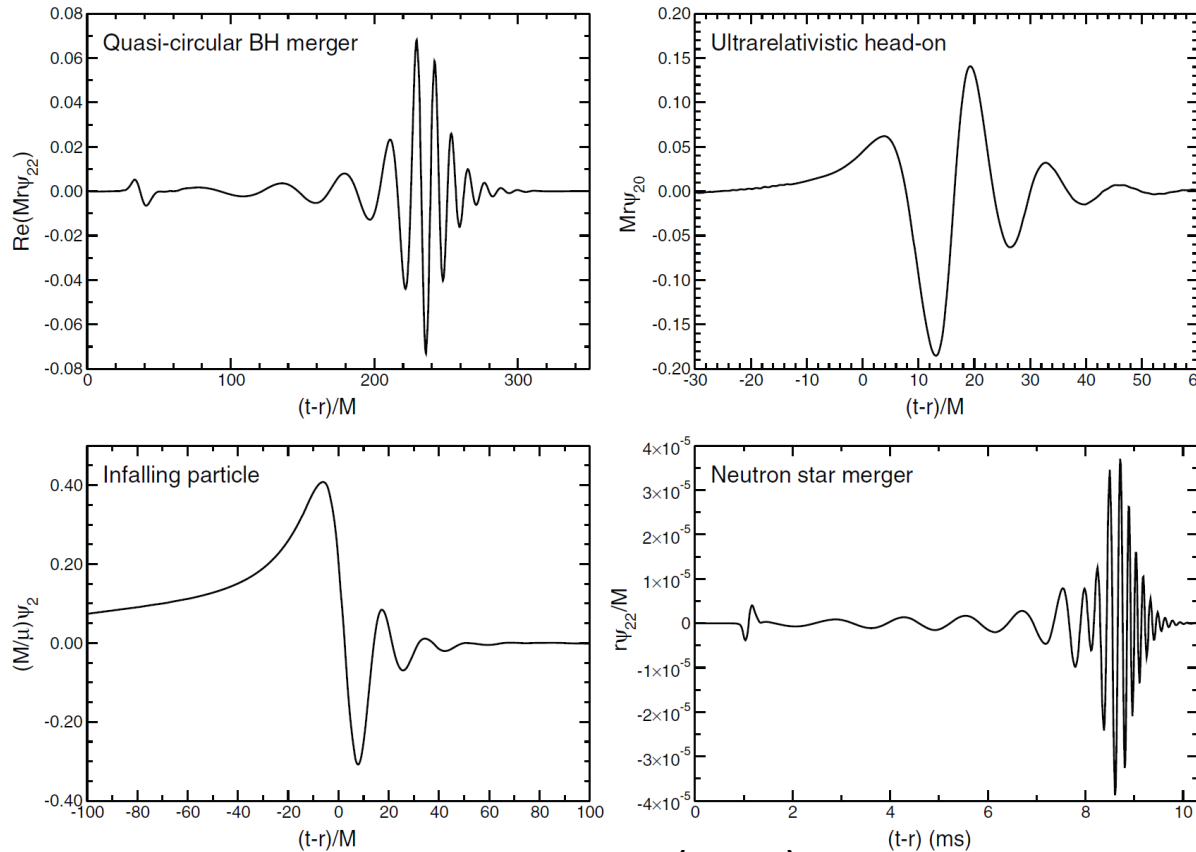
# Mass-radius relation



# Quasinormal modes of black holes

Damped oscillations governed by the mass and spin

Excited when they are formed in gravitational collapse



Berti+ (2009)

# Multimessenger observation

If the collapse is too early, no material is left outside and the kilonova cannot be as bright as AT 2017gfo

Our crossover model may be pass this test with mass asymmetry (1s-order PT trivially passes this test because no gravitational collapse)

