Non-Abelian chiral soliton lattice in rotating QCD matter: Nambu-Goldstone and excited modes

Kentaro Nishimura (Hiroshima Univ)
In a collaboration with
Minoru Eto (Yamagata Univ) and Muneto Nitta (Keio Univ)

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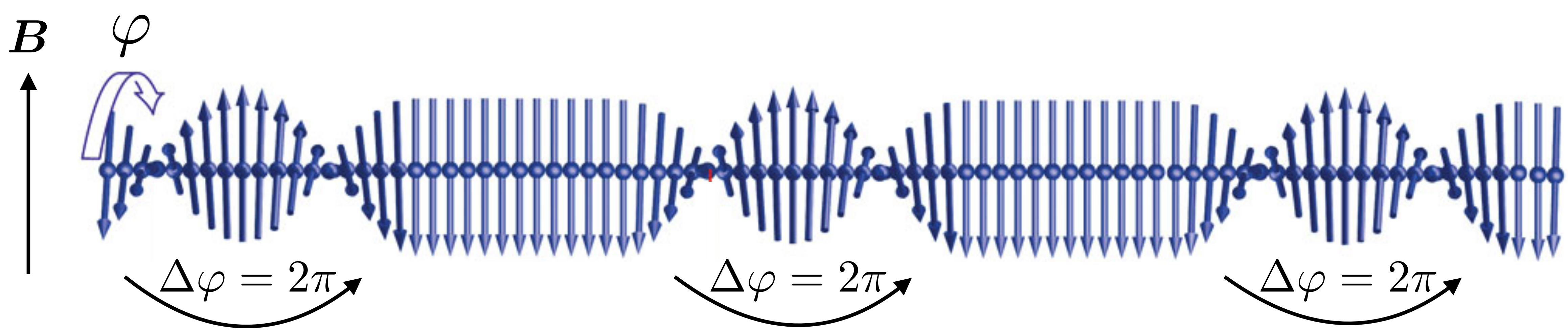
Chiral Soliton Lattice (CSL)

- CSL = A periodic array of topological solitons
- Chiral magnet Togawa et al., PRL (2012); See also Kishine and Ovchinnikov (2015) for review



The twisting spin structure is spatially localized = soliton

- Ferromagnetic interaction→Aligning neighboring spins.
- Zeeman effect→Spins align with the direction of the magnetic field
- Dzyaloshinskii・Moriya interaction→Twisting neighboring spins.

Universality of CSL

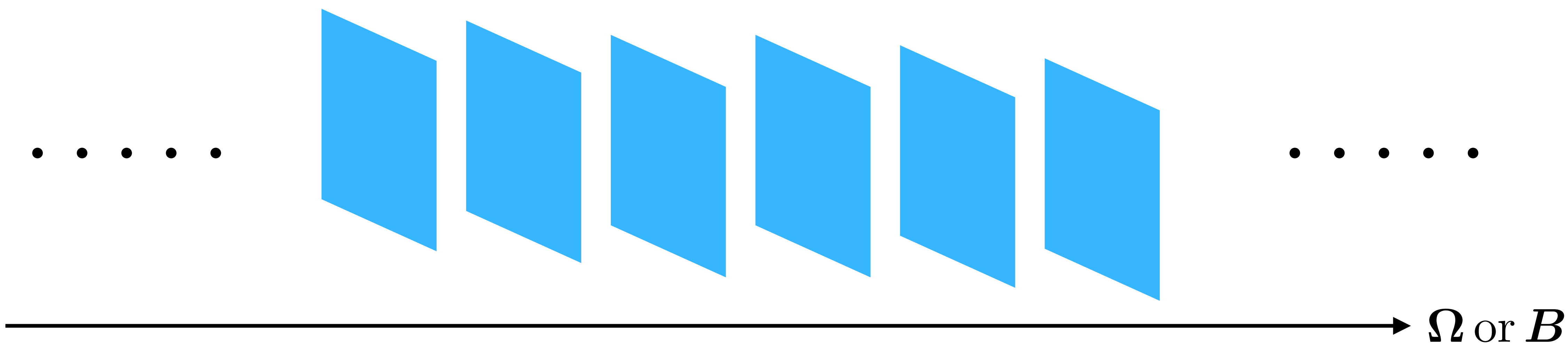
	NG mode	Explicitly breaking	Surface term
Chiral magnet [1]	Magnon	Magnetic Field	Dzyaloshinskii-Moriya Interaction
QCD in magnetic field [2]	Pion	Quark mass	Chiral anomaly
QCD under rotation [3]	Pion	Quark mass	Chiral anomaly

These systems can be described by the sine-Gordon theory with a total derivative term.

[1] Togawa et al., PRL (2012)
[2] Son and Stephanov, PRD (2008); Brauner and Yamamoto, JHEP (2017)
[3] Huang, Nishimura and Yamamoto, JHEP (2018); Nishimura and Yamamoto, JHEP (2020)

Motivation and summary

- Conventional CSL = a periodic array of the solitons with no internal d.o.f



- New CSL = A periodic array of the topological solitons with S^2 moduli
  - We consider QCD under rotation at finite μ_B. Eto, KN and Nitta, JHEP (2022)
  - What is the ground state?

- Excitations from this new state due to the new SSB ☜ NEW!

Eto, KN and Nitta, JHEP (2024)

Chiral perturbation theory

- D.o.f in the case of two flavor = pions + η : $U = \mathrm{e}^{\mathrm{i}\phi_0}\Sigma\,, \qquad \Sigma = \mathrm{e}^{\mathrm{i}\phi_a \tau_a}$

- Kinetic term : $\mathcal{L}_{\mathrm{kin}} = \frac{f_\pi^2}{4} g^{\mu\nu} \mathrm{tr}(\partial_\mu \Sigma \partial_\nu \Sigma^\dagger) + \frac{f_\eta^2}{2} g^{\mu\nu} \partial_\mu \phi_0 \partial_\nu \phi_0$

- Mass term : $\mathcal{L}_{\mathrm{mass}} = \frac{B}{2} \mathrm{tr}(MU + MU^\dagger)$

- When masses of up and down quarks are equal, this term is invariant under SU(2)$_V$.

- Topological term : $\mathcal{L}_{\mathrm{topo}} = \frac{\mu_{\mathrm{B}}^2}{2\pi^2} \boldsymbol{\Omega} \cdot \boldsymbol{\nabla} \phi_0$

- Effective Lagrangian is related to the chiral anomaly.

  Son and Surowka, PRL (2008); Landsteiner, Megias and Pena-Benitez, PRL (2011);
  Son and Yamamoto, PRL (2012); Stephanov and Yin, PRL (2012)

- This term is a rotational analog of $\frac{\mu_{\mathrm{B}}}{2\pi^2} \boldsymbol{\nabla} \pi_0 \cdot \boldsymbol{B}$

  Huang, Nishimura and Yamamoto, JHEP (2018); Nishimura and Yamamoto, JHEP (2020)
  See also Son and Zhitnitsky, PRD (2004); Son and Stephanov, PRD 2008 for the magnetic field case

Dimensionless Hamiltonian

- We can diagonalize the matrix U in the two flavor symmetric case.

$$U = \exp(\mathrm{i}\phi_0 + \mathrm{i}\tau_3\phi_3)$$

- Dimensionless effective Hamiltonian $(C \equiv 4mB\,, \boldsymbol{\Omega} = \Omega\hat{\boldsymbol{z}})$

$$\frac{\mathcal{H}}{C} = \frac{1}{2}\left(\frac{\mathrm{d}\phi_0}{\mathrm{d}\zeta}\right)^2 + \frac{1-\epsilon}{2}\left(\frac{\mathrm{d}\phi_3}{\mathrm{d}\zeta}\right)^2 - \frac{\cos(\phi_0+\phi_3)+\cos(\phi_0-\phi_3)}{2} - S\frac{\mathrm{d}\phi_0}{\mathrm{d}\zeta}$$

Inhomogeneous configuration of ϕ_0 is favored by this term!

- Dimensionless coordinates and parameters :

$$\zeta \equiv \frac{\sqrt{C}z}{f_\eta}\,, \qquad \epsilon \equiv 1 - \left(\frac{f_\pi}{f_\eta}\right)^2\,, \qquad S \equiv \frac{\Omega\mu_{\mathrm{B}}^2}{2\pi^2 N_{\mathrm{c}} f_\eta \sqrt{C}}$$

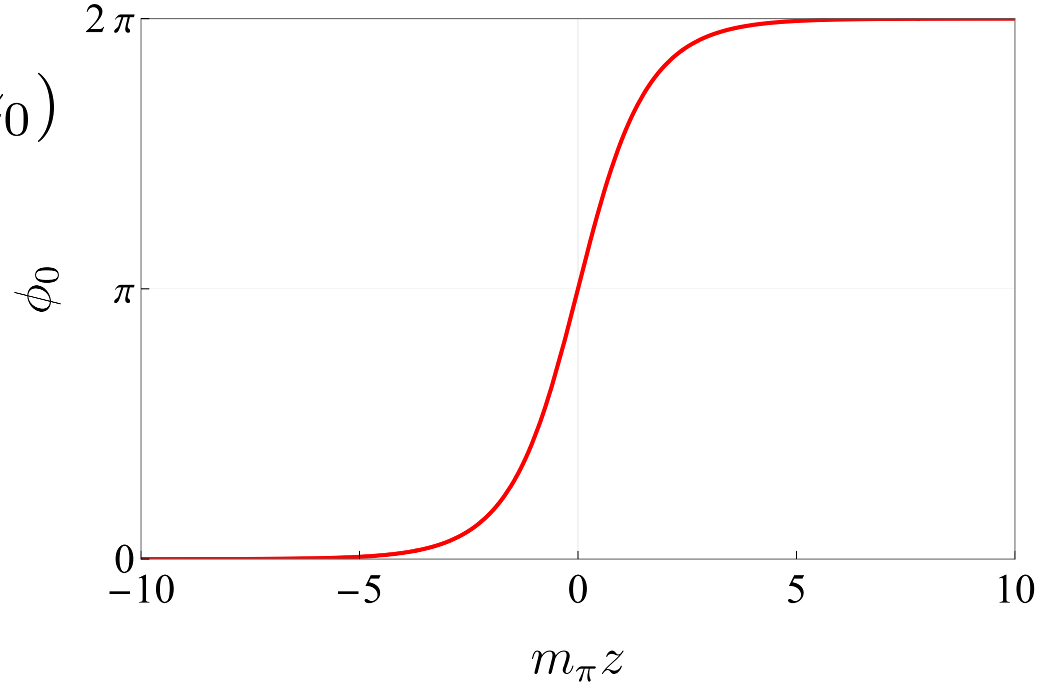
- The difference from the Hamiltonian describing the usual CSL： $\varepsilon \neq 0$

$$\epsilon_{\mathrm{hadron}} \sim 0.17\,, \qquad \epsilon_{\mathrm{CFL}} \sim 0.24$$

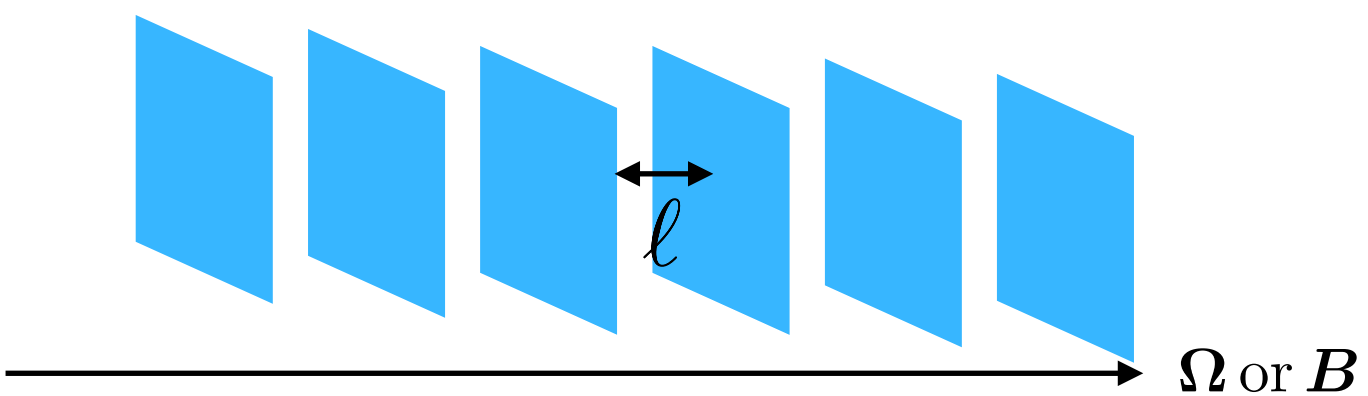
Donoghue, Golowich and Holstein (2022); Son and Stephanov, PRD (2000)

$\varepsilon = 0,\ \phi_3 = 0$

- **Hamiltonian and EOM :** $\frac{\mathcal{H}}{C} = \frac{1}{2}\left(\frac{\mathrm{d}\phi_0}{\mathrm{d}\zeta}\right)^2 + (1 - \cos\phi_0) - S\frac{\mathrm{d}\phi_0}{\mathrm{d}\zeta}$, $\partial_z^2\phi_0 = m_\pi^2 \sin\phi_0$
- **Sine-Gordon soliton:** $\phi_0 = 4\tan^{-1}\exp m_\pi (z - z_0)$
- **Energy :** $E = \int_{-\infty}^{\infty} \mathrm{d}z\, \mathcal{H} = 8m_\pi^2 f_\pi - \mu_\mathrm{B}^2 \Omega/(2\pi)$
- **Critical angular velocity :** $\Omega_\mathrm{c} = 8\pi f_\pi m_\pi^2/\mu_\mathrm{B}^2$



- **A periodic array of the topological solitons is energetically more stable.**



- **A value of ℓ that minimizes the energy density per unit length** $\rightarrow \ell = \ell(\mu_\mathrm{B}, \Omega)$

Huang, Nishimura and Yamamoto, JHEP (2018); Nishimura and Yamamoto, JHEP (2020)

$\varepsilon = 0,\ \phi_3 \neq 0$

- Effective Hamiltonian in the case of $\varepsilon = 0$, $\phi_3 \neq 0$:

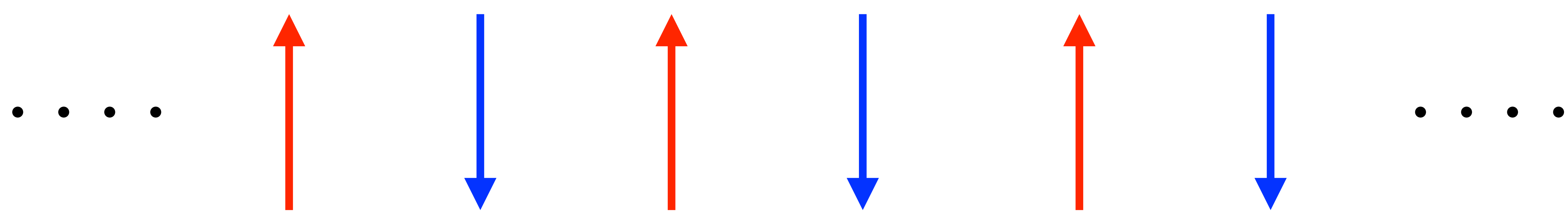
$$\frac{\mathcal{H}}{C} = \frac{1}{2}\left[\frac{1}{2}\left(\frac{\mathrm{d}\phi_+}{\mathrm{d}\zeta}\right)^2 + (1-\cos\phi_+) - S\frac{\mathrm{d}\phi_+}{\mathrm{d}\zeta}\right] + \frac{1}{2}\left[\frac{1}{2}\left(\frac{\mathrm{d}\phi_-}{\mathrm{d}\zeta}\right)^2 + (1-\cos\phi_-) - S\frac{\mathrm{d}\phi_-}{\mathrm{d}\zeta}\right]$$

- Useful coordinates : $\phi_\pm = \phi_0 \pm \phi_3$

$\Phi_\pm$ are completely decoupled, and each Hamiltonian is half of that of ϕ_0

- $\phi_\pm$ and Ω_c has the same values as that of $\phi_3=0$ case.

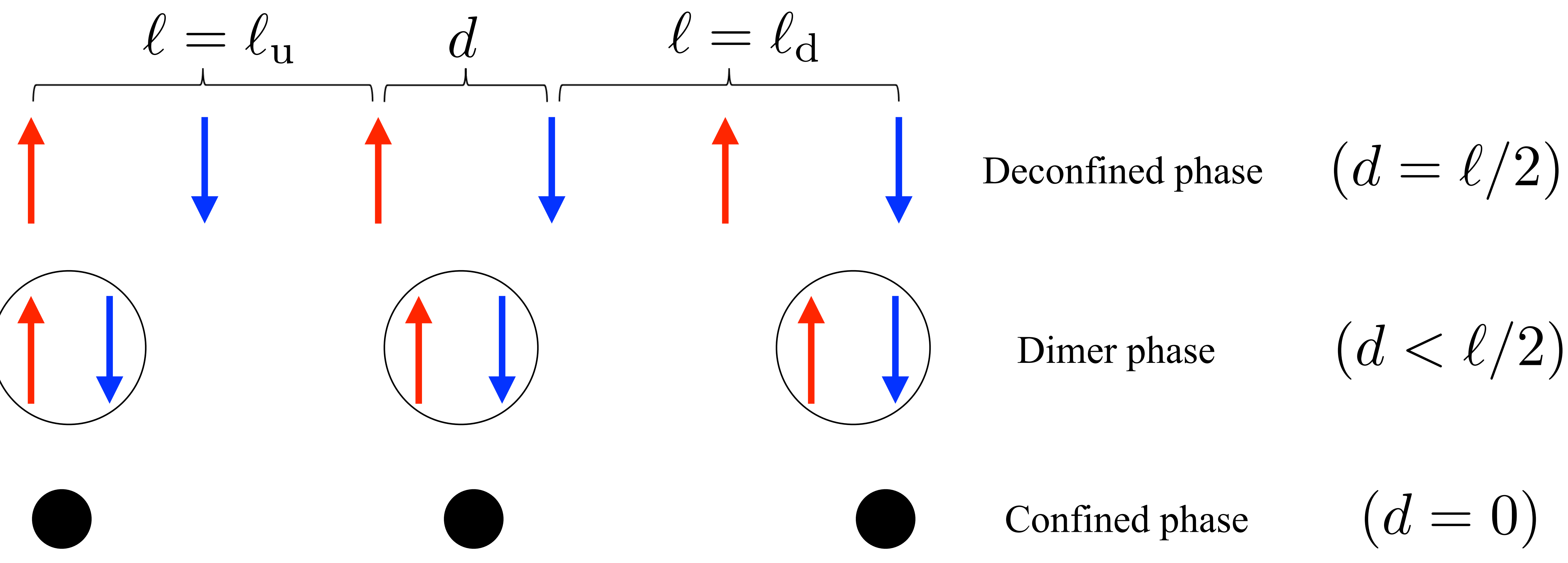
- The ground state is the CSL of both $\phi_\pm$.



ϕ_+ soliton = up soliton ϕ_- soliton = down soliton

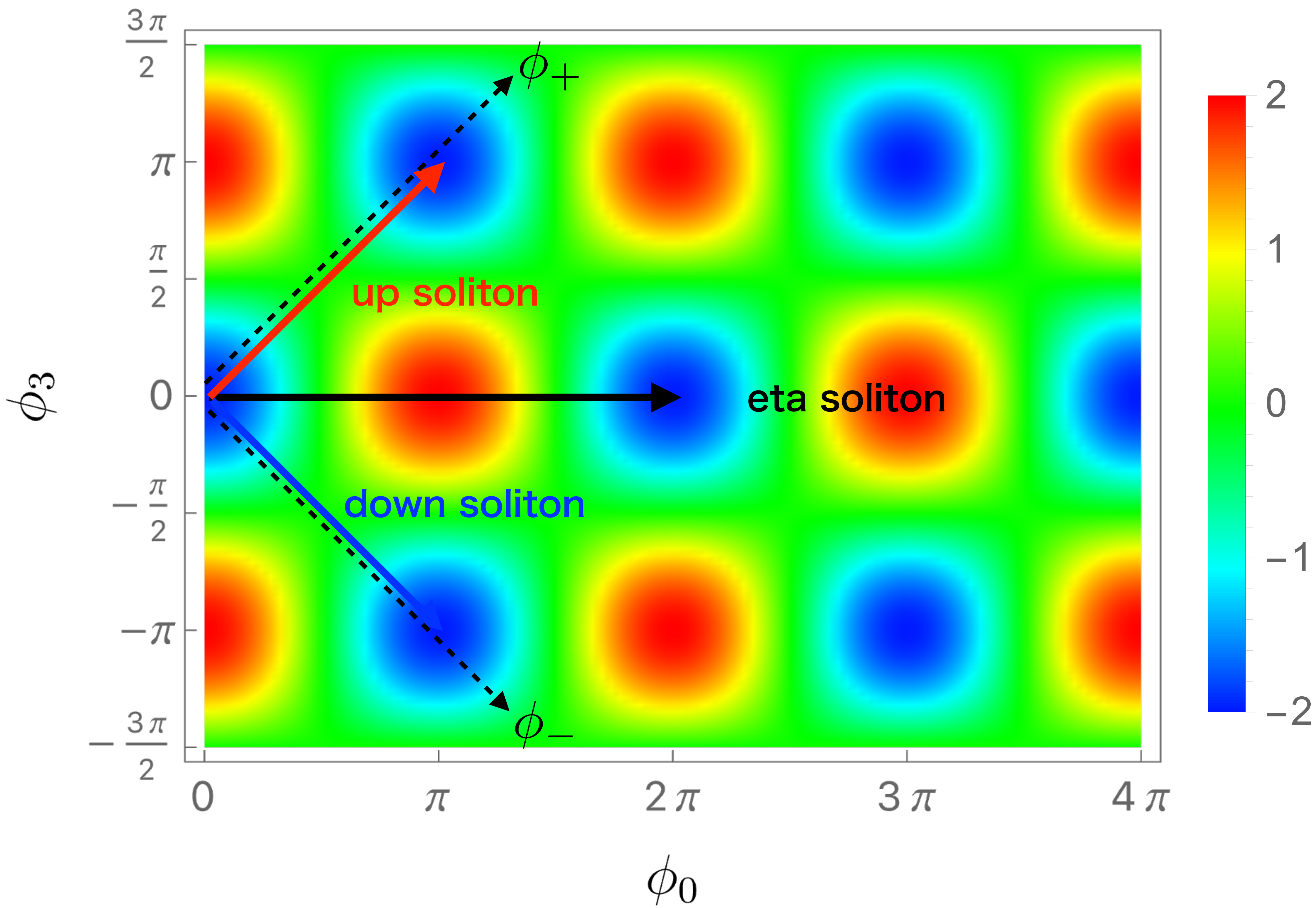
d dependence

- How do the up and down solitons arrange?
- ℓ is determined by minimization of the total energy density per unit length.
- d is a free parameter at ε=0.

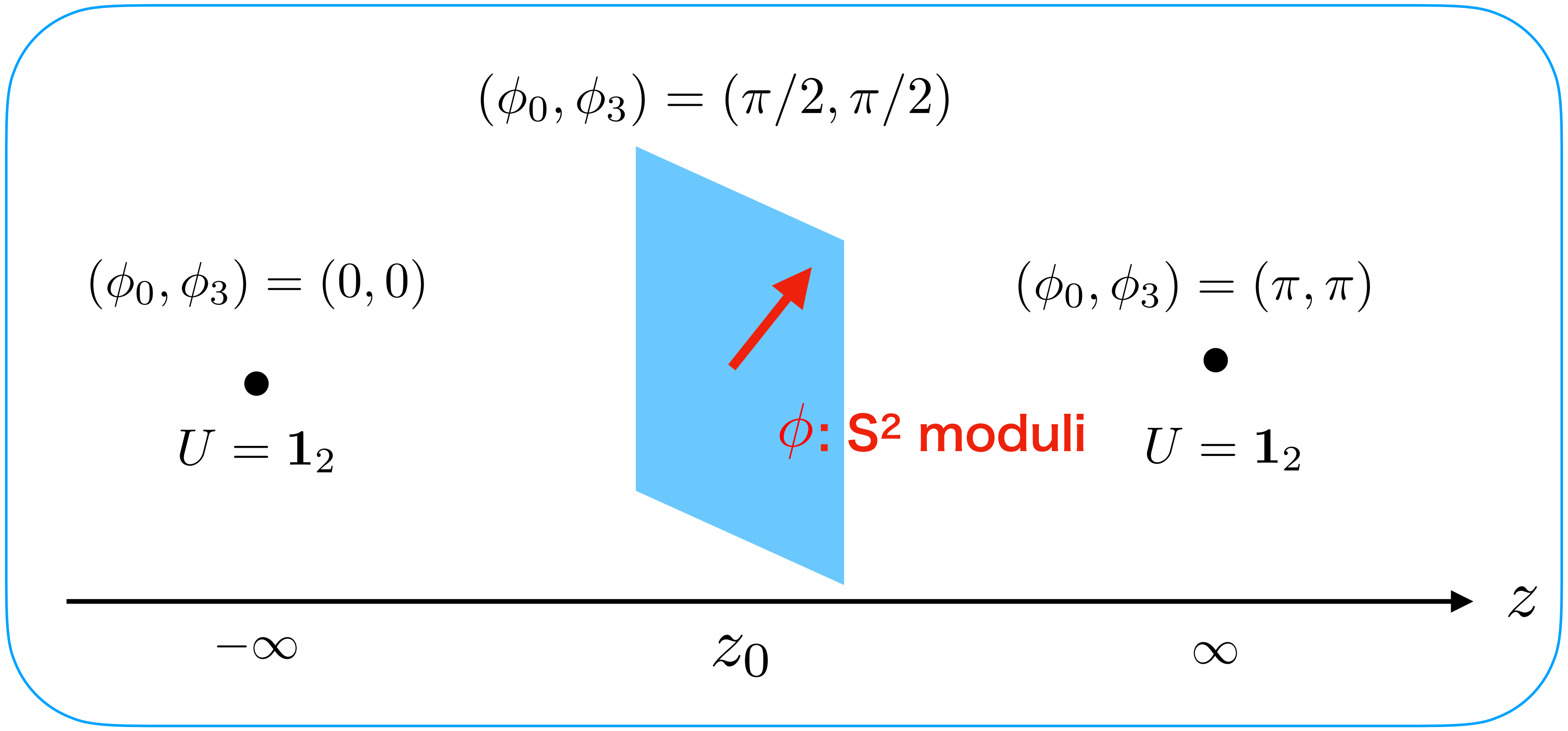


Mass term

- The solitons are field configurations connecting between two vacua.



Non-Abelian soliton



- The position z_0 can be chosen freely→z_0 is moduli from the translation.

- $U = \mathrm{diag}(\mathrm{e}^{\mathrm{i}\phi_+}, 1)$ is invariant only when $g = \mathrm{e}^{\mathrm{i}\sigma_3\theta}$
  - U transforms under SU(2)$_V$ as $gUg^\dagger$

- Moduli space : $\mathrm{SU}(2)/\mathrm{U}(1) \cong S^2$

Nitta, Nucl. Phys.B (2015); Eto and Nitta, PRD (2015)

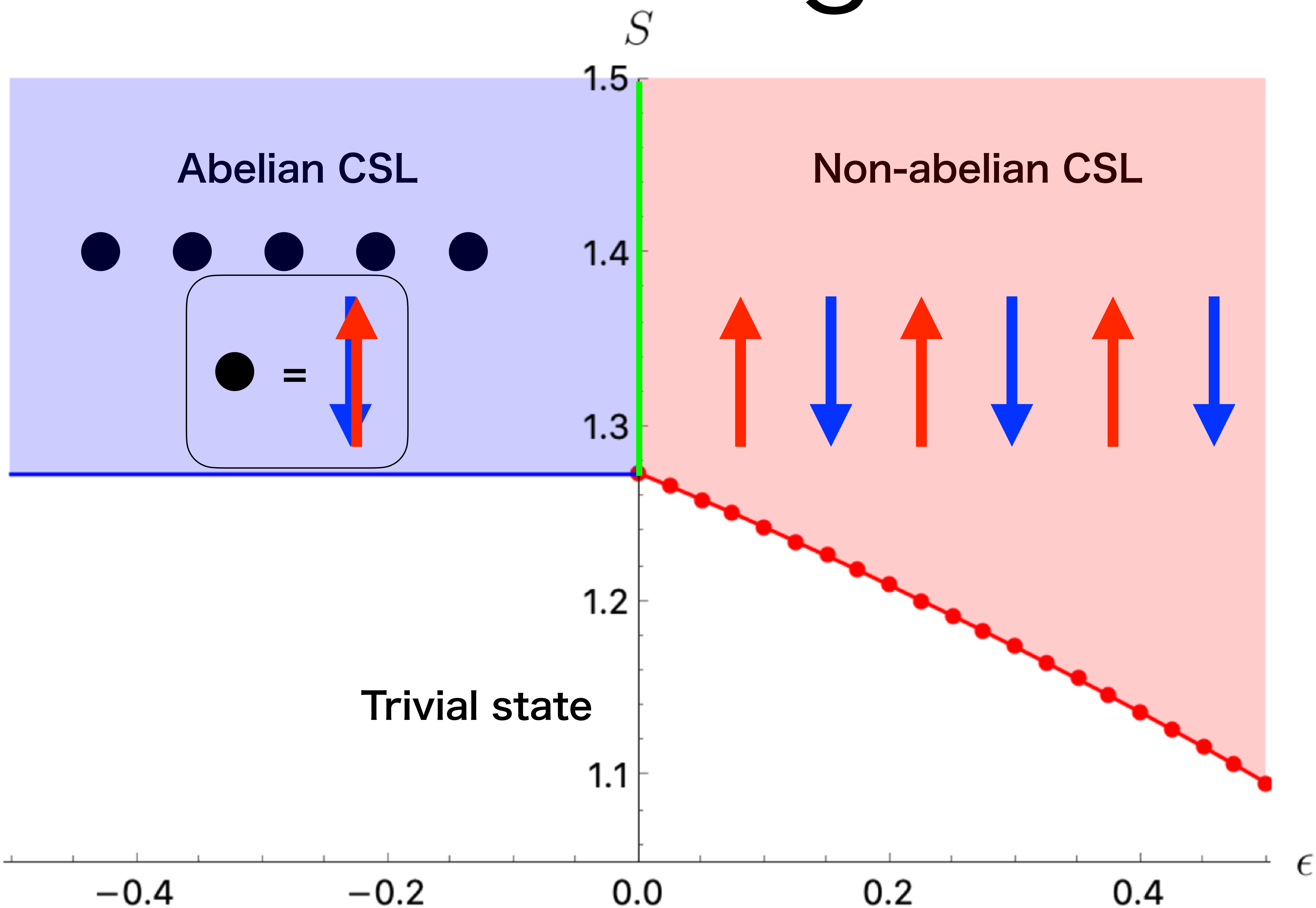
Interaction between $\phi_\pm$

- **The potential depending on both $\phi_\pm$:** $V_\epsilon = \frac{\epsilon}{4}\frac{\mathrm{d}\phi_+}{\mathrm{d}\zeta}\frac{\mathrm{d}\phi_-}{\mathrm{d}\zeta}$
- **For the solitonic configurations,** $\frac{\mathrm{d}\phi_\pm}{\mathrm{d}\zeta}$ **has a peak the center of the kink.**

Distance between up and down solitons	Small	Large	
$\varepsilon > 0$	Positive	~0	Repulsive
$\varepsilon < 0$	Negative	~0	Attractive

- **V_ε at $\varepsilon > 0$ leads to the repulsive interaction between up and down.**
- **V_ε at $\varepsilon < 0$ leads to the attractive interaction between up and down.**

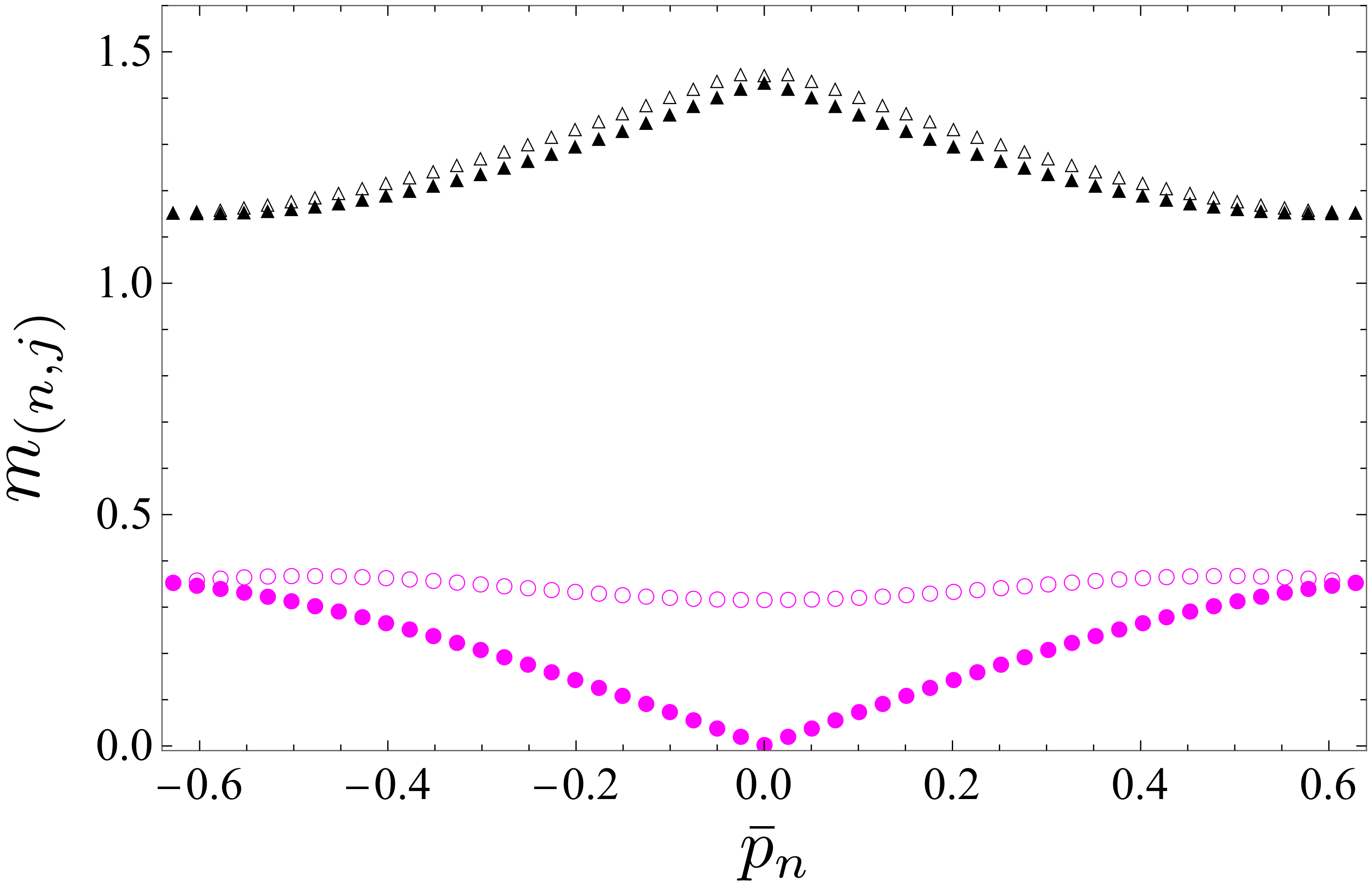
Phase diagram



Eto, KN and Nitta JHEP (2024)

Dispersion relation (1)

- **Excitation from the non-Abelian CSL**
- **Fluctuations of the neutral part :** $\delta\phi_{0,3} = \phi_{0,3} - \bar{\phi}_{0,3}$



$m_{(n,1)} \leq m_{(n,2)} \leq \cdots$

- $m_{(n,1)}$ ←**NG mode (Phonon)!**
- $m_{(n,2)}$
- $m_{(n,3)}$ **Gapped**
- $m_{(n,4)}$

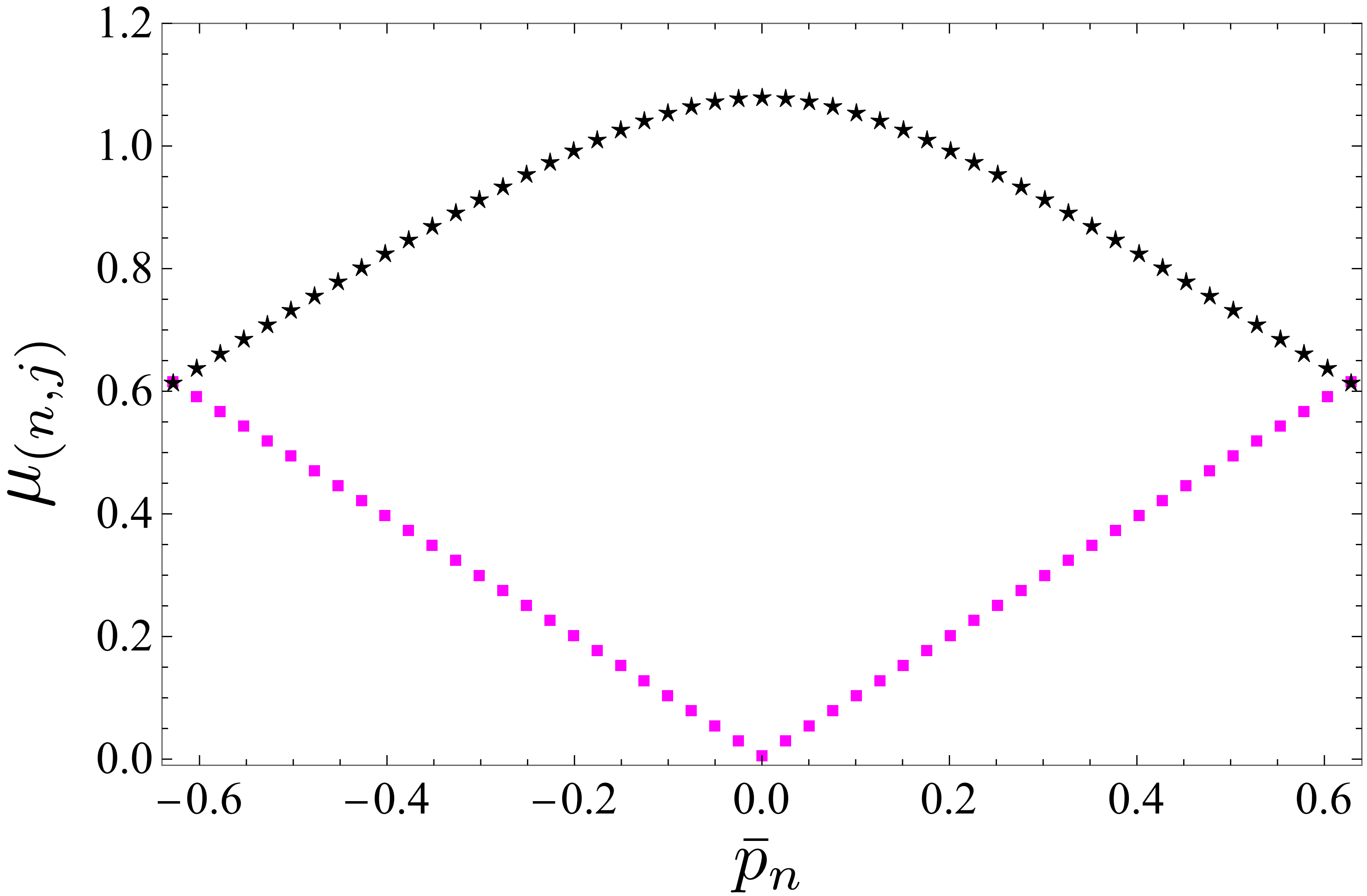
This NG mode is associated with SSB of the translation.

of the phonon = 1

Eto, KN and Nitta JHEP (2024)

Dispersion relation (2)

- **Fluctuations of the charged part :** $\delta\phi_\pm = \delta\phi_1 \pm \mathrm{i}\delta\phi_2$
- **The EOMs of $\delta\phi_\pm$ has the mathematical same form.**



$\mu_{(n,1)} \leq \mu_{(n,2)} \leq \cdots$

■ $\mu_{(n,1)}$ ←NG mode!

★ $\mu_{(n,2)}$ **Gapped**

This NG mode is associated with SSB of $SU(2)_V \to U(1)$.

of the isospinon = 2

Eto, KN and Nitta JHEP (2024)

Summary and future direction

- New type of CSL = A stack of the topological solitons with S^2 moduli

- New NG mode from SSB of $SU(2)_V \to U(1)$

- Non-Abelian CSL in CFL phase
  - ChPT like EFT can be applied to CFL phase.