

# Testing Gravity: Connecting Theoretical developments to forthcoming Observations

[Testing Gravity: Th  $\times$  Obs]

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# Our review paper is now accessible!

**PTEP**

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## Cosmological gravity probes: Connecting recent theoretical developments to forthcoming observations

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Ryotaro Kase<sup>11</sup>, **Taishi Katsuragawa<sup>12</sup>**, Yosuke Kobayashi<sup>13</sup>, Toshiya Namikawa<sup>4</sup>,  
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- Prog. Theor. Exp. Phys. (2023)7, 072E01
- arXiv:2212.09094

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6	Outlook				
7	Summary				

Concrete predictions from theories by analytic computations

Numerical tools

How the effect of gravity are captured in observations

Our high-priority subjects

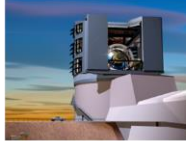
# 1. Current status

# Golden Age of Observational Cosmology

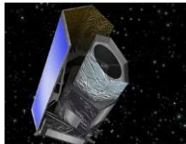
2022 2024 2026 2028 2030



Subaru  
(Optical • IR)



Vera C. Rubin  
(Optical)



Euclid  
(Optical • NIR)



Roman  
(NIR)



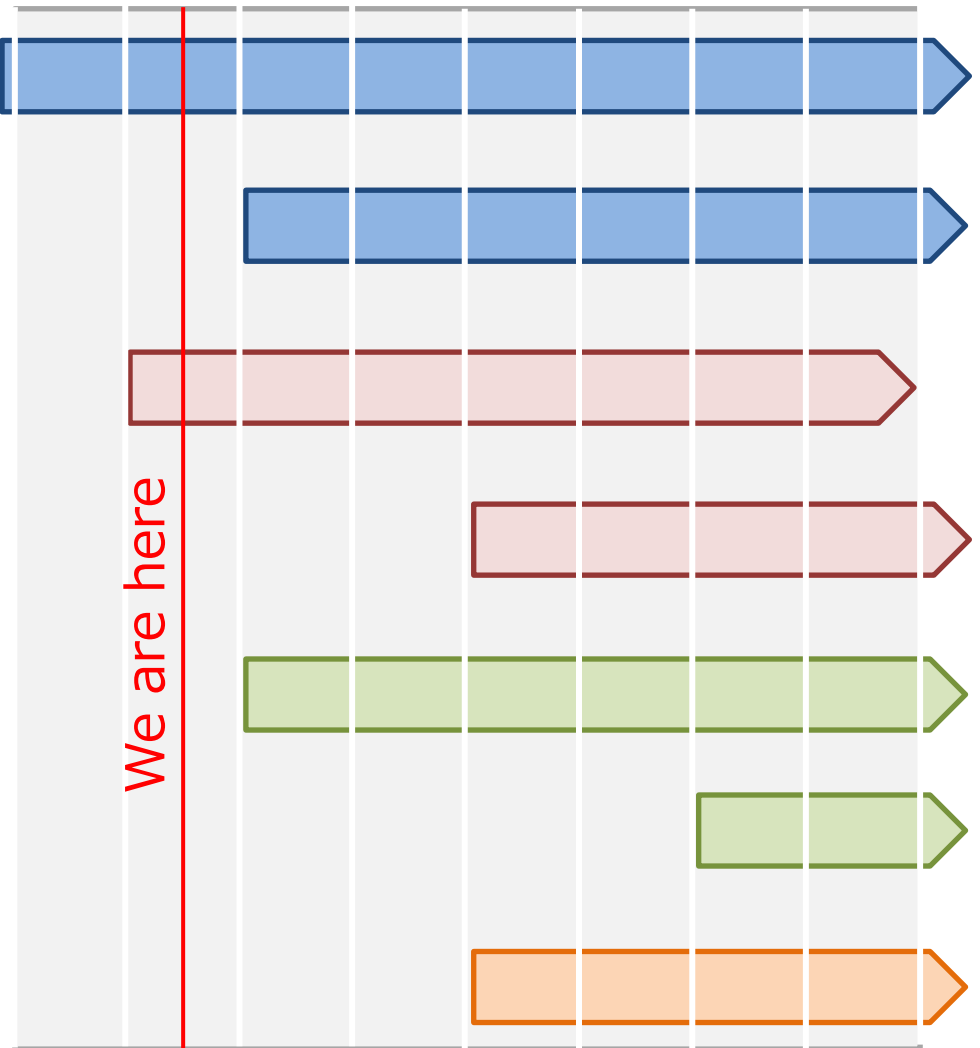
Simons Observatory  
(CMB)



LiteBIRD  
(CMB)

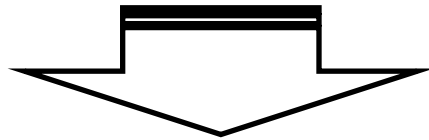


Square Kilometre  
Array (radio)



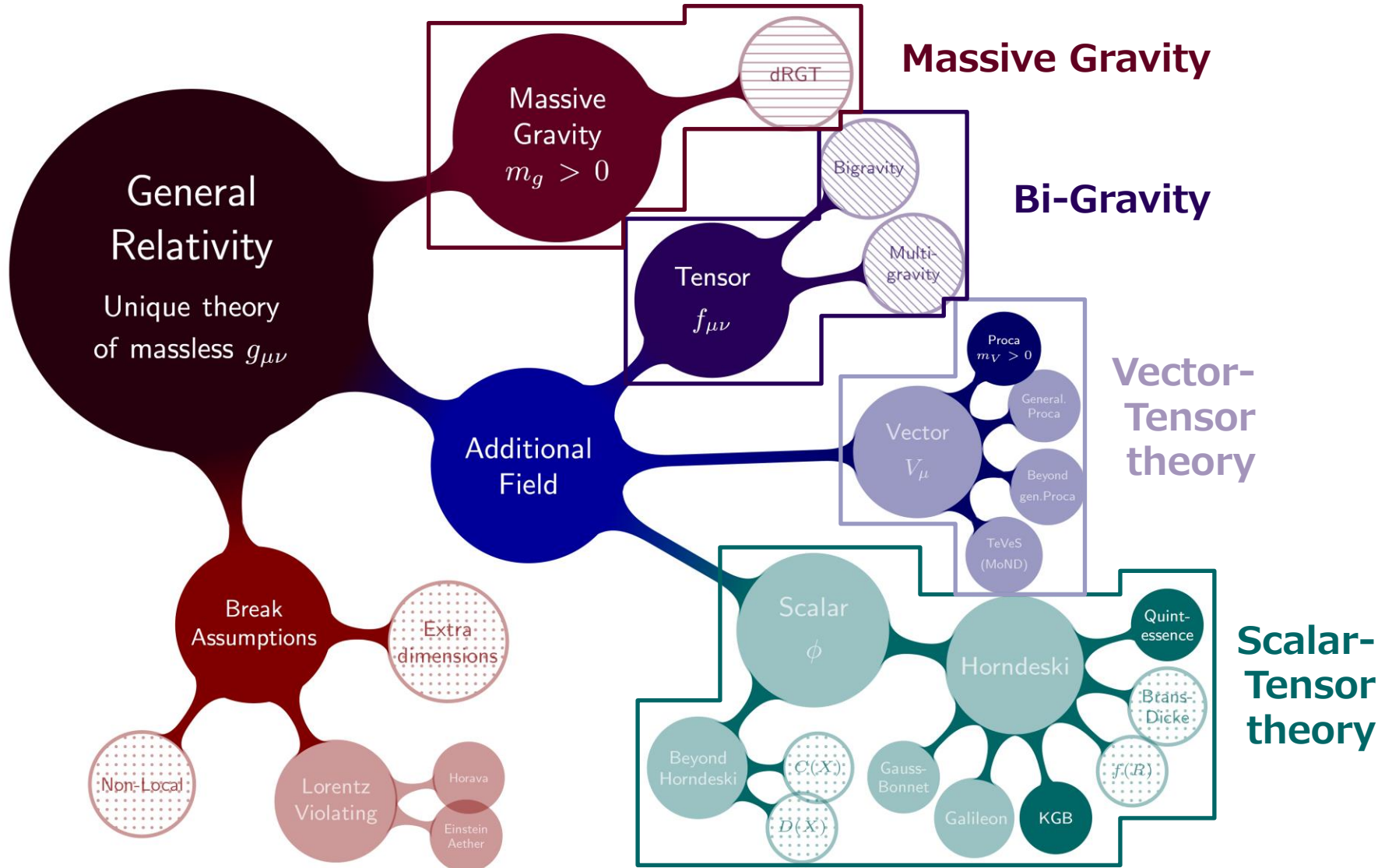
# Golden Age of Observational Cosmology

Near-future observations will provide vast high-quality data suitable for proving gravity theory on large scales.

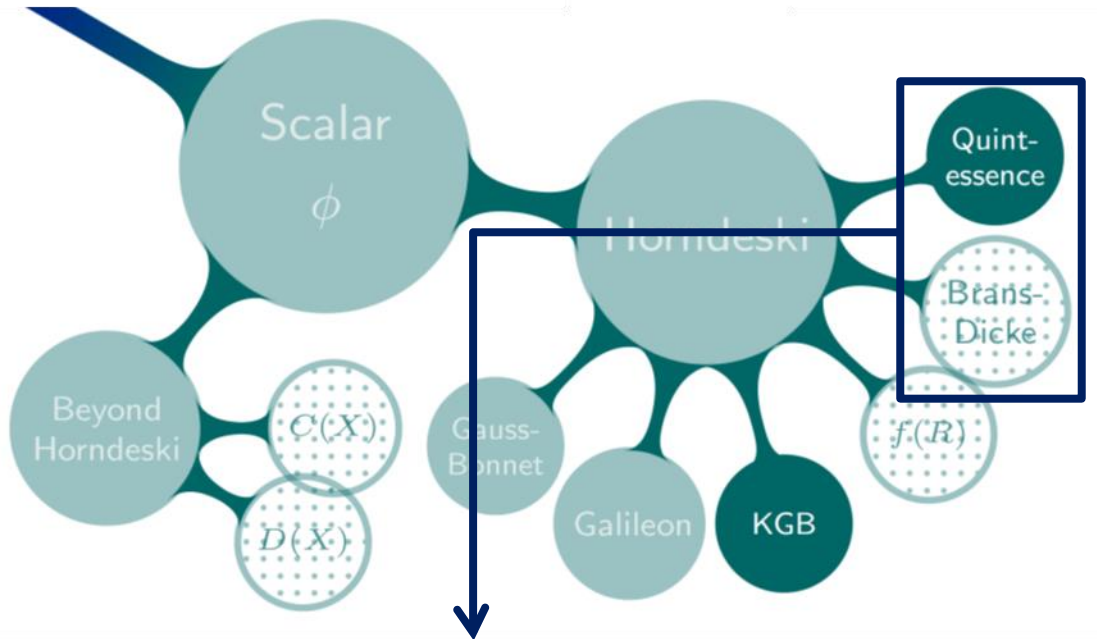


We now need to prepare well-motivated theory and appropriate observables that can indicate any signs beyond GR!

# Landscape of Gravity Theory



# Example: Scalar-Tensor Theories



- ◆ (Old) well-known theories: Only one parameter

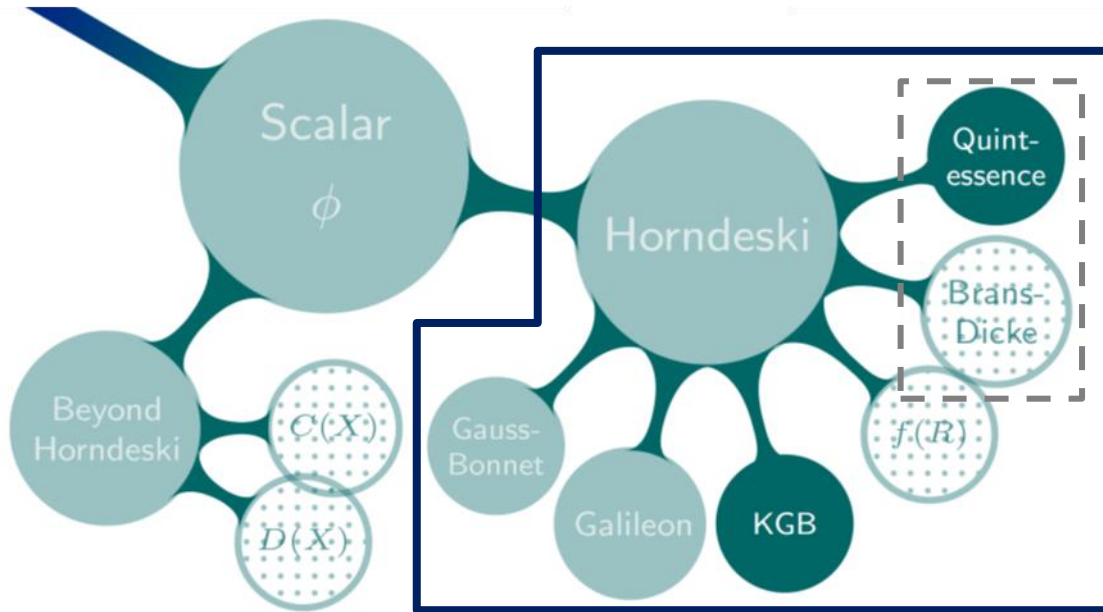
$$\mathcal{L} = \frac{1}{2} \left[ \phi R - \frac{\omega}{\phi} (\partial\phi)^2 \right]$$

[Brans+Dicke, Phys.Rev.124,925(1961), ...]

The feasibility can be discussed on a **model-by-model** basis.



# Example: Scalar-Tensor Theories



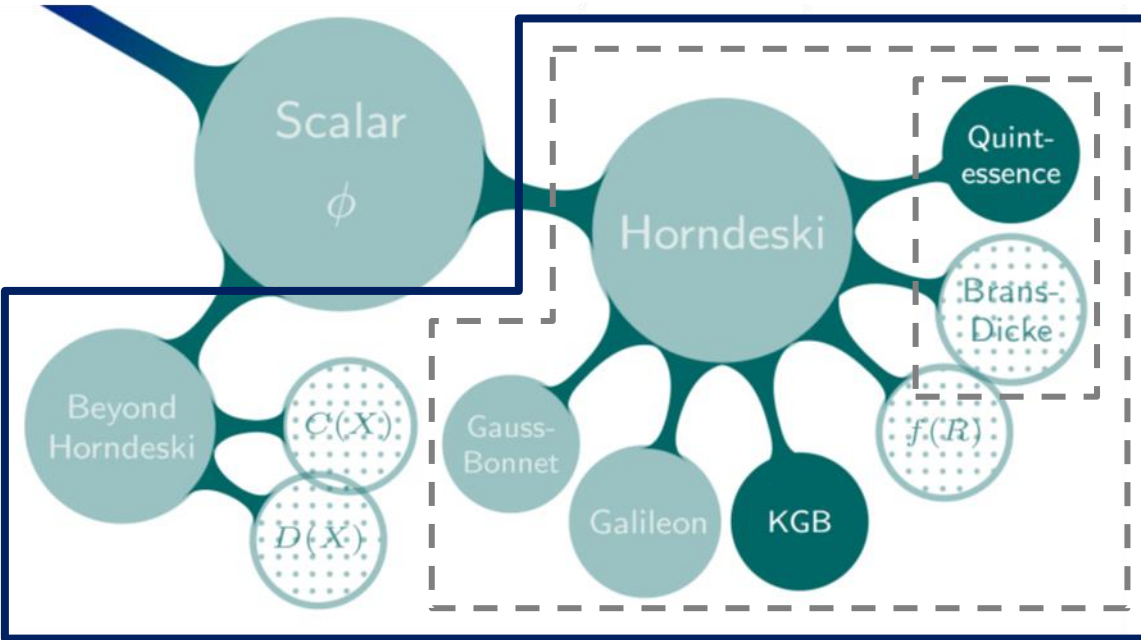
This provides  
**a bird's eye view**  
understanding.

## ◆ Horndeski theory: 4 arbitrary functional DoF

$$\mathcal{L} = P(\phi, X) - Q(\phi, X)\square\phi + G_4(\phi, X)R - \frac{\partial G_4}{\partial X} (\nabla_\mu \nabla_\nu \phi)^2 + \dots$$

[Horndeski, Int. J. Theor. Phys. 10, 363 (1974),  
Deffayet+, PRD84, 063039 (2011),  
Kobayashi+Yamaguchi+Yokoyama, PTP126, 511 (2011)]

# Example: Scalar-Tensor Theories



This gives a deeper understanding of its **stability**.

◆ **DHOST** (Degenerate Higher-Order Scalar-Tensor theory) : **15 DoF!**

$$\mathcal{L} = \dots + f(\phi, X)R + A_1(\phi, X) (\nabla_\mu \nabla_\nu \phi)^2 + \dots$$

[Langlois+Noui, JCAP02,034(2015),  
Crisostomi+, JCAP04,044(2016),  
Ben Achour+, PRD93,124005(2016)]



# Take-Home Message

Theoretical

(Th1) Theory

(Th2) Effective Theory

(Obs3) Pheno. model

(Obs2) Pheno. parameter

(Obs1) Observable

Observational

- ◆ Main message: A **hierarchical structure** exists in the cosmological test of gravity.



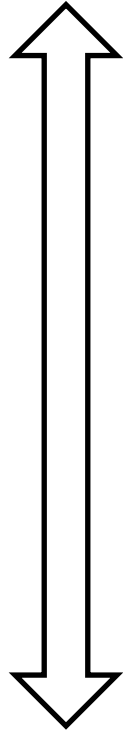
It is essential to link each hierarchy appropriately:

The key is to connect **Theoretical studies with** **Observational ones!**

2. Hierarchical  
structure from theory  
to observation  
(or vice versa)

(Th1) **Theory**

$F(R)$  , Horndeski, ...

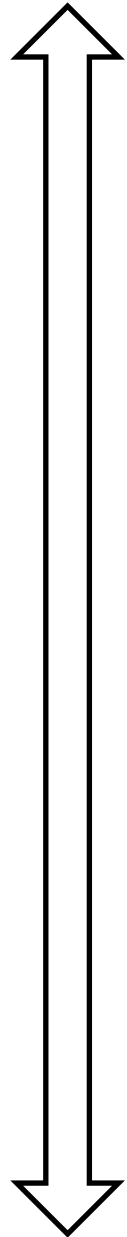


Can we compare  
(full-)theory of gravity  
to  
observational data  
directly ?

(Obs1) **Observable**

CMB, LSS, GW, ...

Theoretical



(Th1) **Theory**

**Unknown  
Theory**

(Th2) **Effective  
Theory**

modifying gravity  
theory at level of  
**perturbed action**

(Obs3) **Pheno.  
model**

modifying gravity  
model at level of **EoM**

(Obs2) **Pheno.  
parameter**

to explore gravity  
**w/o changing  
 $\Lambda$ CDM observables**

(Obs1) **Observable**

CMB, LSS, GW, ...

Observational

# (Obs2) Pheno. parameter

: w/o changing  $\Lambda$ CDM observables



- ◆ Background level: Hubble expansion rate

$$H^2(a) = H_0^2 \left[ \frac{\Omega_{m,0}}{a^3} + \Omega_{DE,0} \exp \left( -3 \int_1^a [1 + \boxed{w_{DE}(a')}] d \ln a' \right) \right]$$

Dark Energy Equation-of-State

- ◆ Perturbed level: Growth rate of density fluct.

$$\delta(a, \mathbf{k}) = \exp \left( \int_0^a \boxed{f(a')} d \ln a' \right) \delta_*(\mathbf{k})$$

Linear growth rate



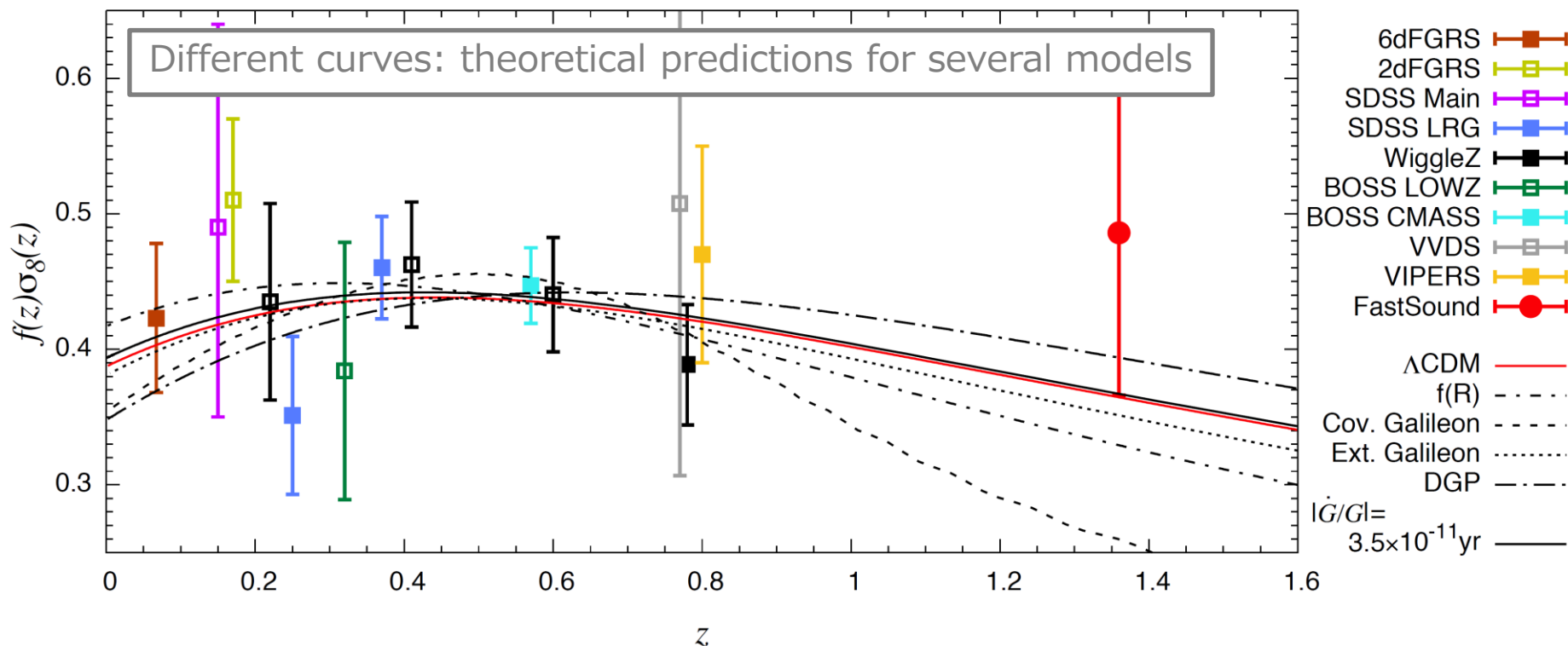
# (Obs2) Pheno. parameter

: w/o changing  $\Lambda$ CDM observables



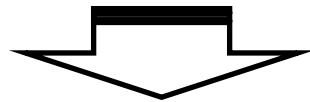
- ◆ Measuring  $f$  from RSD is frequently used for test of gravity responsible for current acceleration.

FastSound [Okumura+] PASJ68,3,38(2016)



Q. Are  $w_{\text{DE}}$  &  $f$  enough to test gravity?

A. **NOT** enough. Even if  $w_{\text{DE}} = -1$ ,  $f = f_{\text{GR}}$ , it is **NOT** necessary that our Universe is described by  $\Lambda\text{CDM}$  with GR.



**Nonlinear growth** can carry new information that is not included in linear-order.

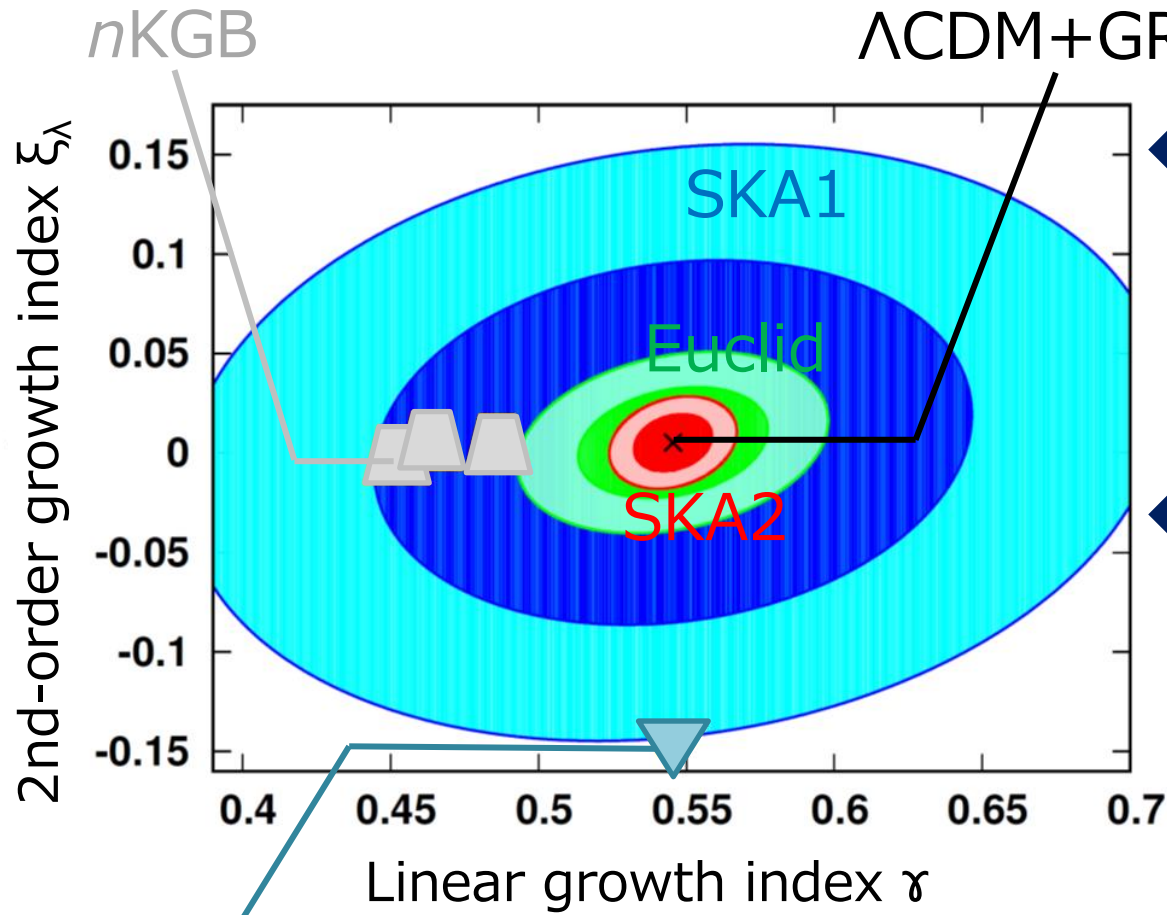
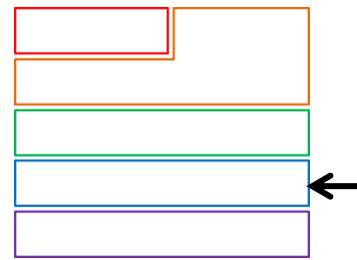
$$\delta(a, \mathbf{k}) = \delta_1(a, \mathbf{k}) + \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \boxed{F_2(a, \mathbf{p}, \mathbf{k} - \mathbf{p})} \delta_1(a, \mathbf{p}) \delta_1(a, \mathbf{k} - \mathbf{p}) + \dots$$

(quasi-)nonlinear growth

[Takushima+(2014,2015),**DY**+(2017),Namikawa+(2018),  
Hirano+(2018),Hirano+**DY**+(2020),**DY**+Sugiyama(2022),  
**DY**+(2023),Sugiyama+**DY**+(2023a,b),Yamashita+**DY**+(in prep.)]

# (Obs2) Nonlinear growth

: to extract higher-order contributions



- ◆ Nonlinear growth index ( $\xi_\lambda$ ) can be used to distinguish various models!
- ◆ Note: Mapping of these parameters to specific theories is **not** fully understood.

A model from Horndeski theory  
(specific model with  $w_{DE} = -1$ ,  $f = f_{GR}$ )

DY+, PRD96, 123516 (2017)  
Namikawa+, PRD98, 043530 (2018)

# (Obs3) Pheno. model

: modifying gravity at level of EoM



## ◆ Non-relativistic matter feels

$$\ddot{\delta} + 2H\dot{\delta} + \frac{1}{a^2} \nabla^2 \Phi = 0$$

Gravitational potential ( $\delta g_{00}$ )

This term depends on gravity model via **Poisson equation:**

$$\nabla^2 \Phi = 4\pi G a^2 \rho \delta$$

We add

phenomenological functional DoF  $\mu(a, k)$

# (Obs3) Pheno. model

: modifying gravity at level of EoM



## ◆ Relativistic matter feels:

$$\nabla^2(\Phi + \Psi) = 8\pi G a^2 \rho \delta$$

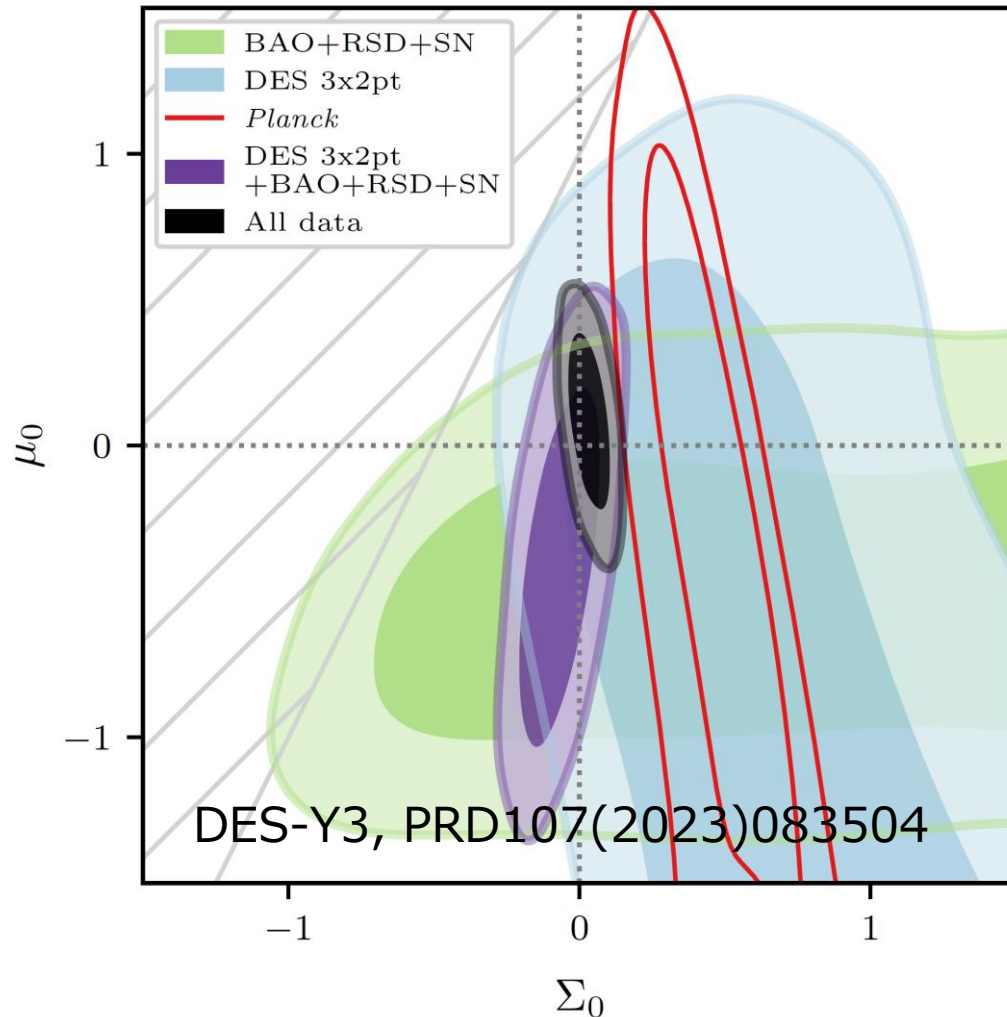
Gravitational potential ( $\delta g_{ij}$ )

Phenomenological  
functional DoF  $\Sigma(a, k)$

- ✓ Note: To practically obtain the constraints, a functional form of  $\mu$  &  $\Sigma$  should be specified.

# (Obs3) Pheno. model

: modifying gravity at level of EoM



- ◆ A specific choice of functional forms:

$$\mu(a) = 1 + \mu_0 \frac{\Omega_{\text{DE}}(a)}{\Omega_{\text{DE},0}}$$

$$\Sigma(a) = 1 + \Sigma_0 \frac{\Omega_{\text{DE}}(a)}{\Omega_{\text{DE},0}}$$

- ◆ Pheno. model is useful to investigate how signals deviate from standard ones, although the mapping is not fully understood.

# (Th2) Effective Theory

: modifying at level of perturbed action



- ◆ We consider the perturbed action so that the physical meaning of pert. is obvious:

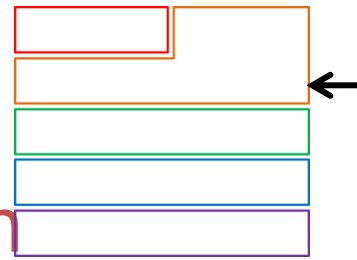
[Example] Effective 2nd-order Lagrangian

$$\mathcal{L} = \frac{1}{16\pi G} \left[ (1 + \alpha_T) {}^{(3)}R + \delta K^i_j \delta K^j_i - \delta K^2 + \dots \right]$$

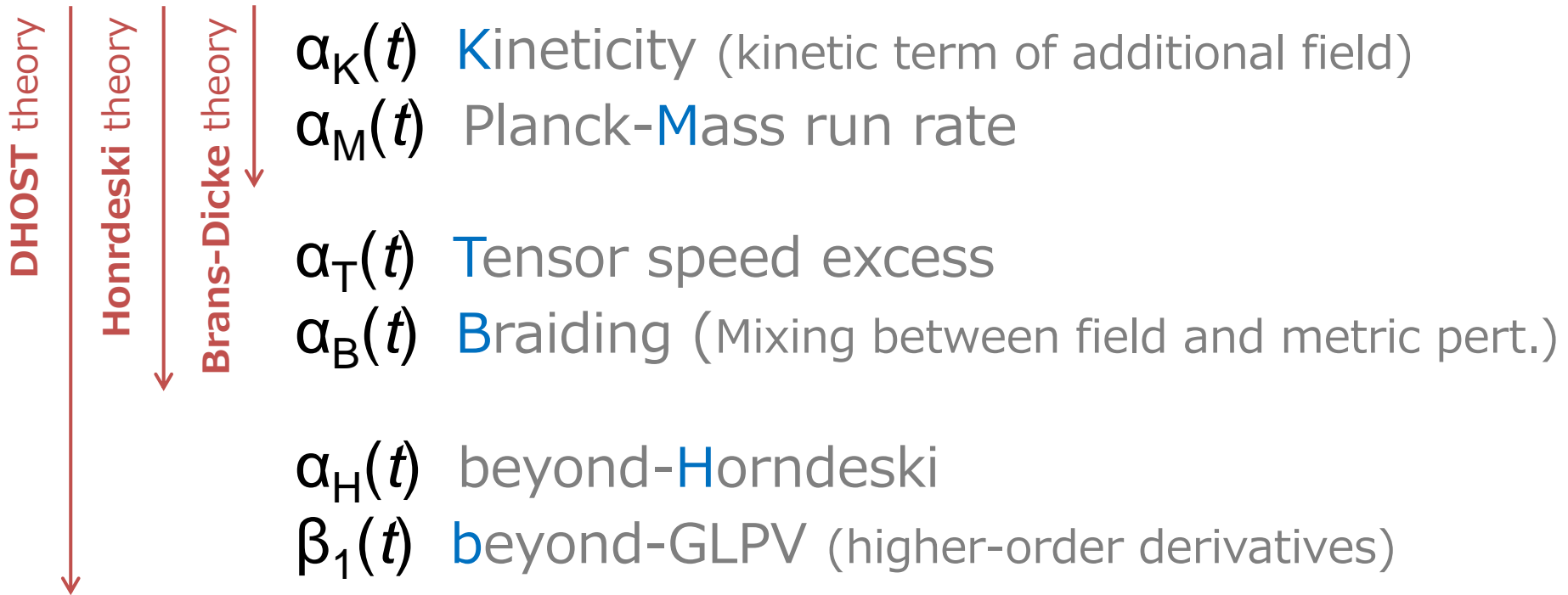
Independently of the details of original theory,  
this term always represents  
sound speed of GWs !

# (Th2) Effective Theory

: modifying at level of perturbed action



- ◆ Even complex full theories can be described by linear theories with **a few** EFT parameters:



Scalar-Tensor Theories: Bellini+, JCAP07,050(2014), Langlois+, JCAP05,033(2017), ...  
Vector-Tensor Theories: Aoki+, JCAP01,056(2022), Fluid: Aoki+, JCAP08,072(2022)

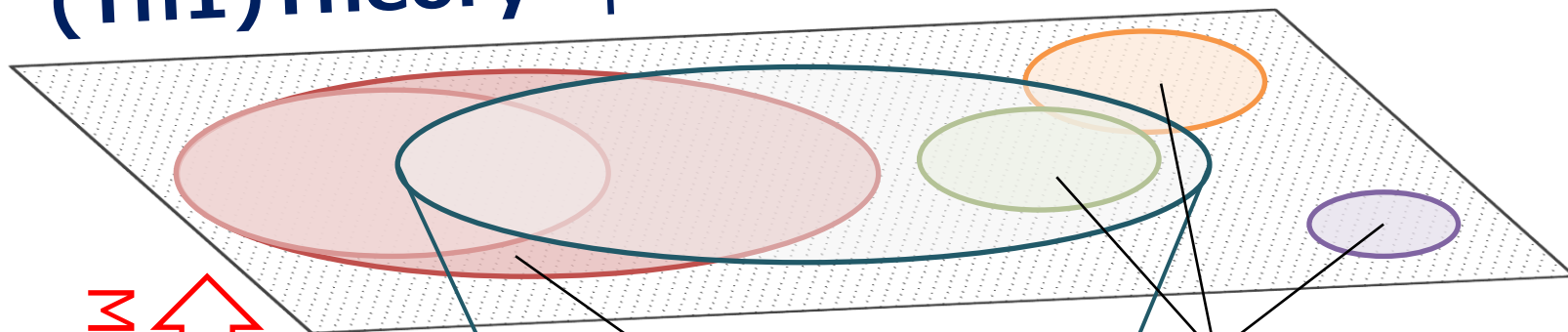


# (Th2) Effective Theory

: modifying at level of perturbed action



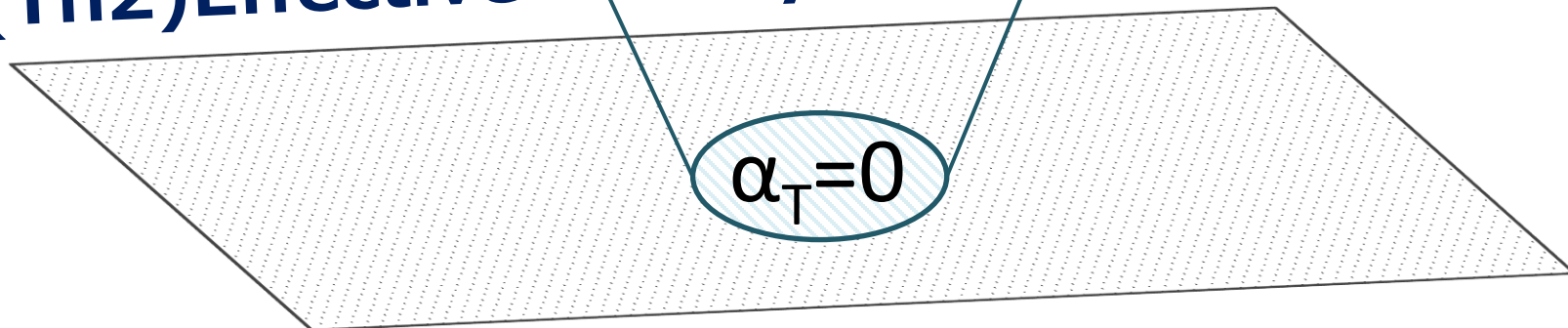
“(Th1)Theory” parameter space



Mapping

Known models

“(Th2)Effective Theory” parameter space



# 3. Connecting Theoretical studies with Observations (Th × Obs)

General Relativity  
of massless  $g_{\mu\nu}$

**Vast theory-space  
of gravity (Th1)**

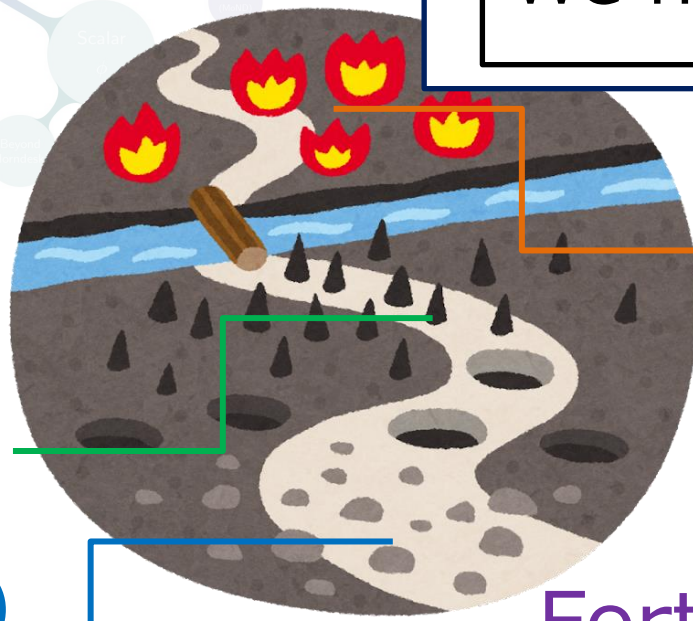


A mind map centered on 'Gravity' with branches for 'General Relativity of massless  $g_{\mu\nu}$ ', 'Massive Gravity  $m_g > 0$ ', 'Break Assumptions', 'Non Local', 'Lorentz Violating', 'Extra dimensions', 'Scalar', 'Bigravity', 'dRGT', 'Tensor-Vector-Scalar', 'Proca  $m_{\nu} > 0$ ', and 'Fierz-Pauli'.

Many challenges  
lie between  
**Theory & Observation...**

We need a **highway!**

(Obs3)  
Pheno. model



A circular illustration of a landscape with a river, a path, a forest, and a volcano. A green box highlights a specific area of the landscape.

(Obs2)  
Pheno. parameter

(Th2)  
Effective Theory

Forthcoming  
cosmological  
**observations (Obs1)**



A vertical stack of four small images showing astronomical observations: a telescope, a galaxy cluster, a galaxy, and a nebula.

# (Th2) Effective Theory → (Obs3) Pheno. model



We have already created a **dictionary** to connect (Th2) and (Obs3) in specific gravity theories.

- Ex) Interaction between metric and scalar-field pert.

$$\mathcal{L}_{\text{int}} = -2M_{\text{Pl}}^2 a H \alpha_B \Phi \nabla^2 \varphi$$

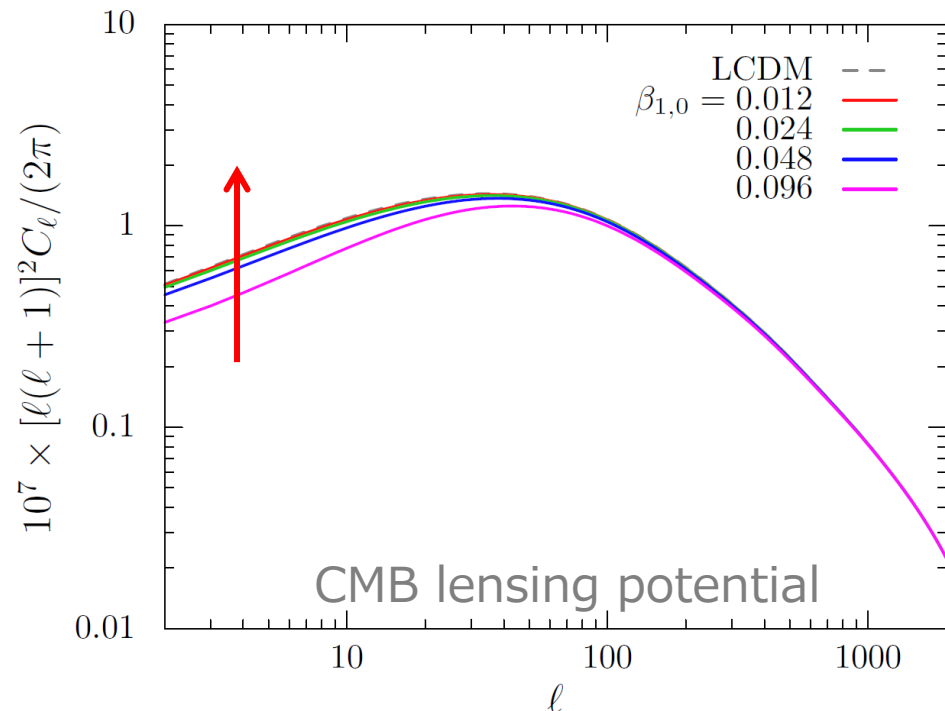
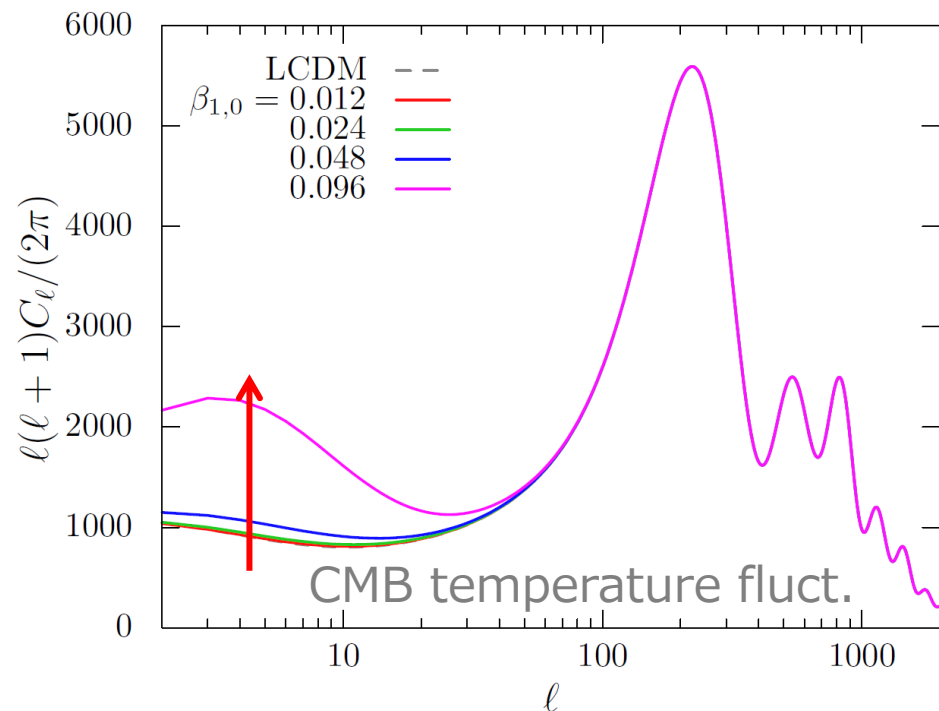


$$\mu = \Sigma = 1 - \frac{\alpha_B^2}{\frac{\dot{H}}{H^2} + \frac{3}{2}\Omega_m + \alpha_B(1 + \alpha_B) + \frac{(H\alpha_B)'}{H^2}}$$

Horndeski theory : Pogosian+, PRD94,104014(2016), Gleyzes+, JCAP02,056(2016)  
DHOST theory : Hirano+DY+, PRD99,104051(2019)/PRD102,103505(2020)  
Vector-Tensor theory : Aoki+, JCAP01,056(2022), ...

(Th2) Eff. Theo./ (Th1) Theory  
 → (Obs1) Observation[CMB]

- ◆ T. Hiramatsu (Rikyo U) developed the novel **Boltzmann code “CMB2nd”** that is utilized for computing CMB in the context of **DHOST**.



Hiramatsu+DY, PRD102, 083525 (2020), Hiramatsu, JCAP10(2022)035  
 Hiramatsu+Kobayashi, JCAP07, 040 (2022), ...

We have started  
new Japanese Working Group:  
“Testing Gravity: **Th** × **Obs**”

- Start : Aug. 2020
- Aim : Several multi-wavelength wide-field cosmological surveys have been conducted and planned, hence immediate validation system of gravity model needs to be established. For this reason, the aim of this working group is **to strongly connect theoretical and observational studies in Japan.**
- Chairs : Miyatake, Yokoyama, Arai(Nagoya), **DY(OUS)**
- Members : >50

# Our review paper is now accessible!

**PTEP**

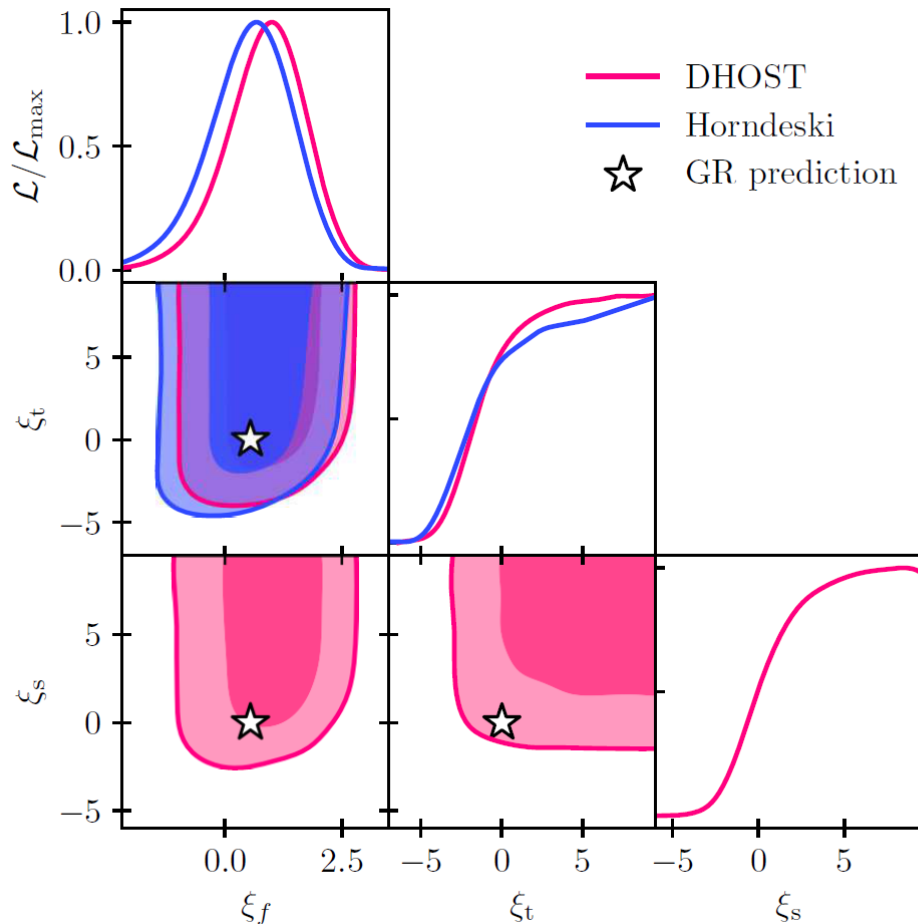
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# New Th×Obs project has started!



Sugiyama+DY+, MNRAS523, 2, 3133  
Sugiyama+DY+, MNRAS524, 2, 1651

[Analysis based on DY+Sugiyama,PRD105,063515(2022)]

◆ Requests from **Obs** side :  
Novel technique  
“anisotropic galaxy 3PCF”

+

◆ Requests from **Theo** side :  
Theoretical prediction of  
nonlinear growth



New **Th**×**Obs** project  
“Test of gravity from  
anisotropic galaxy 3PCF”



# What we have to do are...

## □ Coupling between matter and gravity

- ✓ Most calculations are based on minimal coupling.
- ✓ **Nonminimal (e.g. disformal) coupling** may lead to strange phenomena in observables  
[Kimura+DY+Yamaguchi+(2018),Chibana+DY+Yamaguchi(2019)]

## □ Nonlinearity

- ✓ **Screening mechanism** should be considered. Beyond-Horndeski class such as DHOST leads to partial breaking of screening mechanism  
[Kobayashi+DY+(2015),Langlois+DY+(2018)]
- ✓ Deeper understanding of **nonlinear growth** of structure is also needed.

# Summary

Theoretical

(Th1) Theory

(Th2) Effective Theory

(Obs3) Pheno. model

(Obs2) Pheno. parameter

(Obs1) Observable

Observational

- ◆ Main message:  
A **hierarchical structure** exists in the cosmological test of gravity.

The key is to connect  
**Theoretical studies with**  
**Observational ones!**

- ◆ Japanese working group  
"Testing Gravity: Th×Obs"
- ◆ Our review paper is now accessible!