Tensor clustering fossils
(in modified gravity)

Based on **DY** 2407.10450

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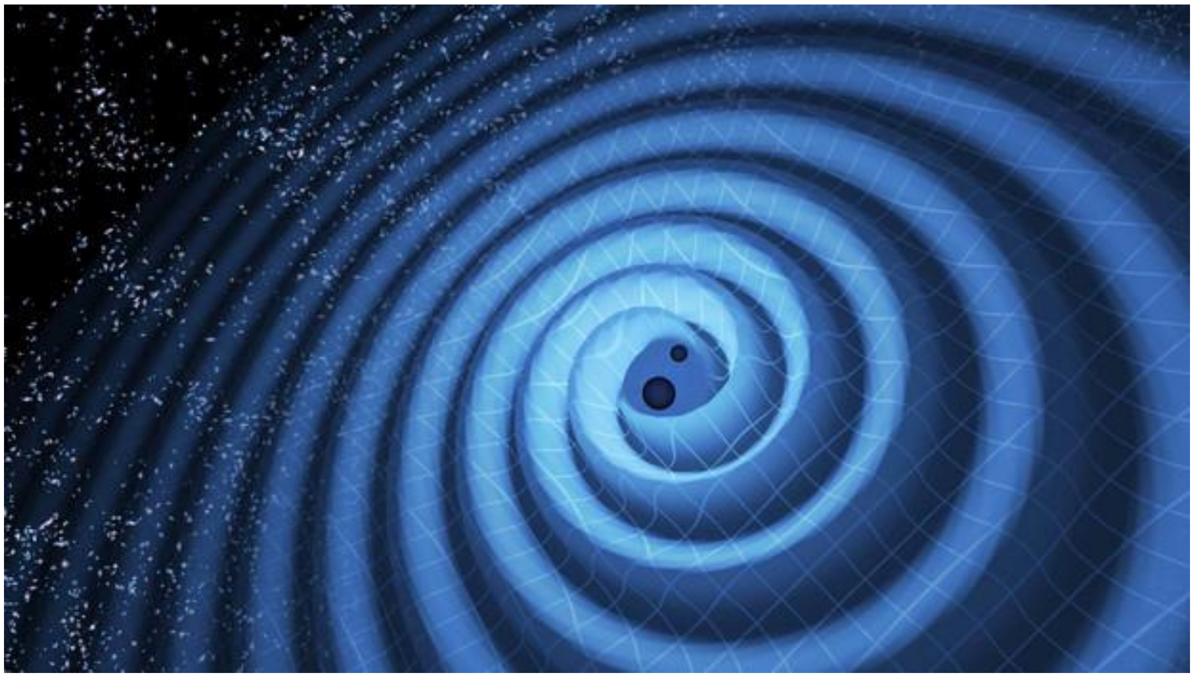
2024/09/04 Fourth Mini-workshop on the Early-Universe

Today's topic:
Gravitational Waves



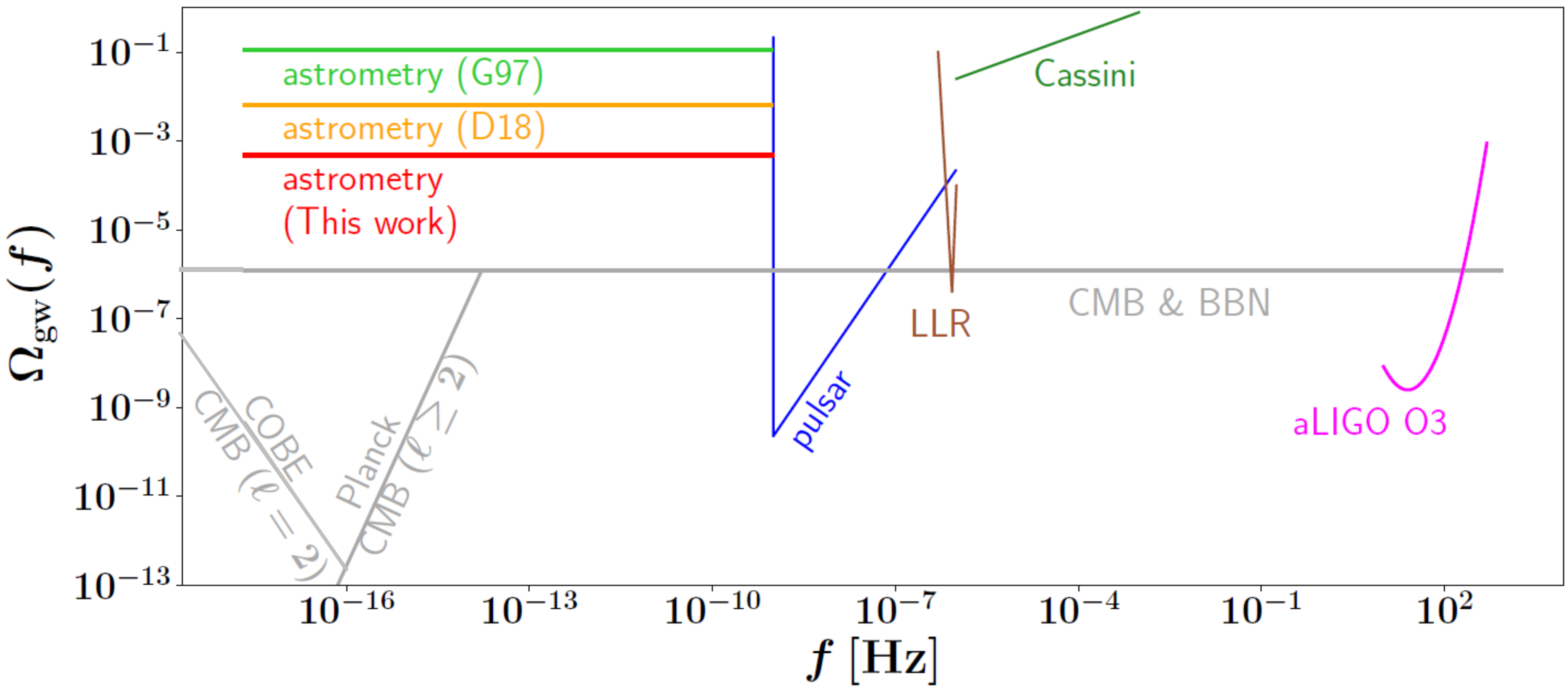
How do we observe gravitational waves?

Constraints on GWs

- Cosmic Microwave Background
  - ✓ B-mode polarization [$f<10^{-16}$Hz] (LiteBIRD,⋯)
  - ✓ Curl-lensing [10^{-16}Hz$<f<10^{-13}$Hz] (CMB S4,⋯)
- Pulser Timing Array [10^{-9}Hz$<f<10^{-5}$Hz] (NANOGrav,⋯)
- Direct detection experiments [10^{-3}Hz$<f<10^{2}$Hz] (LIGO, KAGRA, LISA,⋯)
- Astrometry [10^{-18}Hz$<f<10^{-9}$Hz] (Gaia, SKA, ngVLA,⋯)
- Matter clustering [$f<10^{-18}$Hz]: Today's topic

Constraints on Ω_{GW}

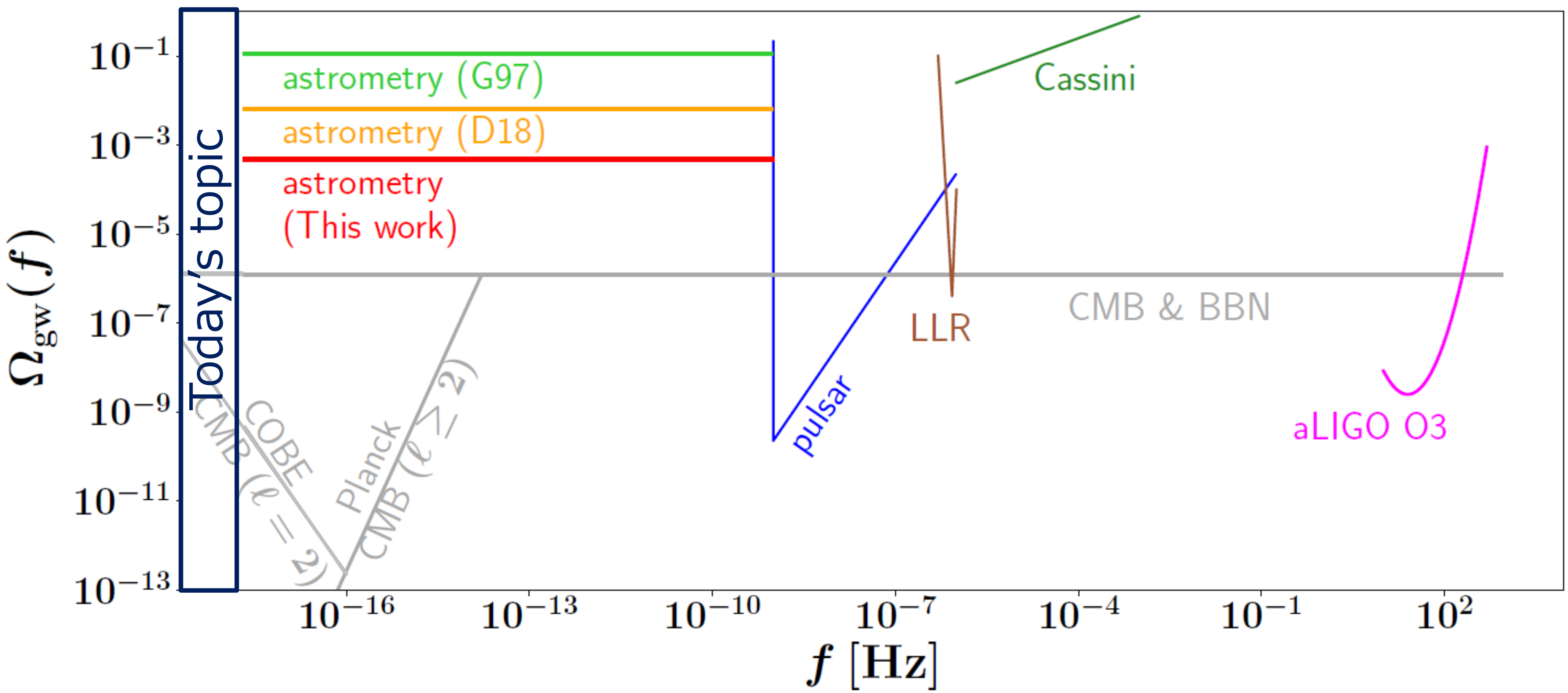
$$\Omega_{\mathrm{GW}} \equiv \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\ln k} \sim \frac{f}{H_0^2}\langle h^2\rangle$$



[Figure from Aoyama+**DY**+Shiraishi+Ouchi 2105.04039]

Constraints on $\Omega_{\rm GW}$

$$\Omega_{\rm GW} \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln k} \sim \frac{f}{H_0^2}\langle h^2\rangle$$

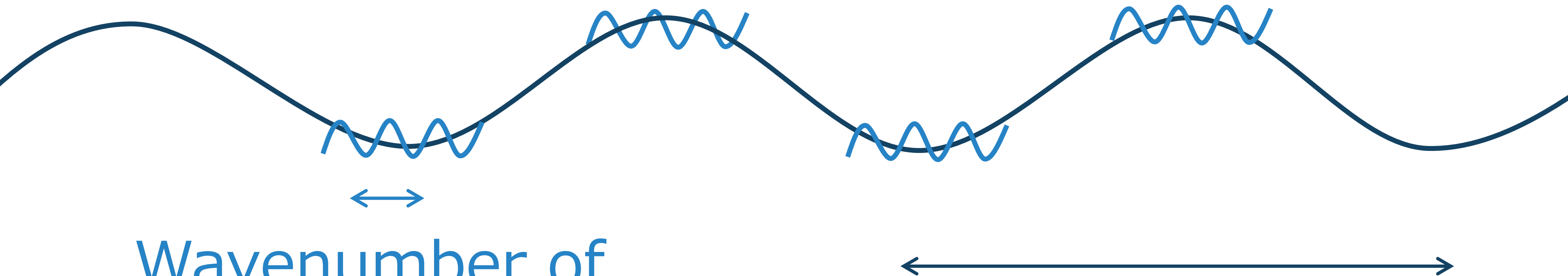


[Figure from Aoyama+**DY**+Shiraishi+Ouchi 2105.04039]

What is
“tensor clustering *fossils*”?

Mechanism: Rough sketch 1/6

[Masui+Pen PRL105(2010)161302]

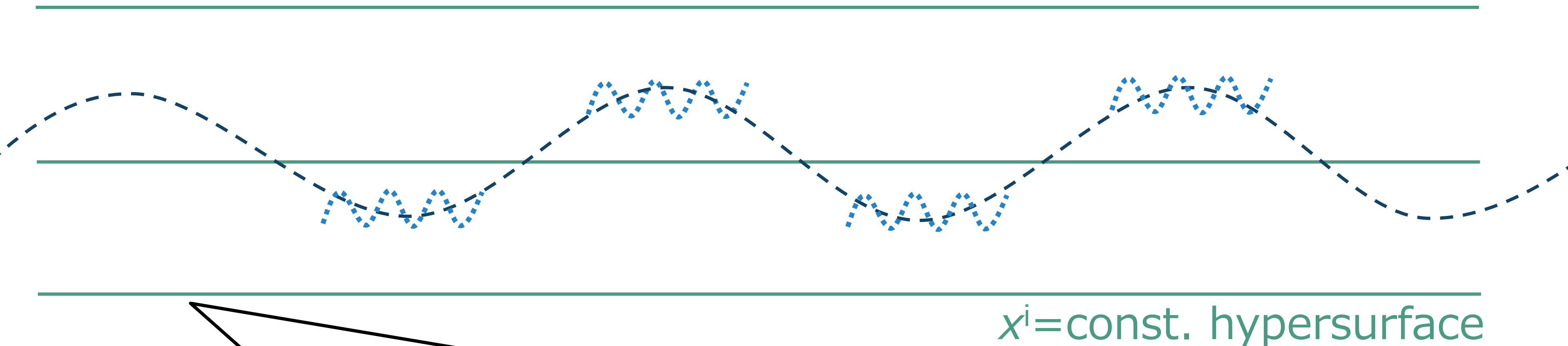


Wavenumber of density contrast: k

Wavenumber of GWs: k_{GW} ($\ll k$)

Mechanism: Rough sketch 2/6

[Masui+Pen PRL105(2010)161302]

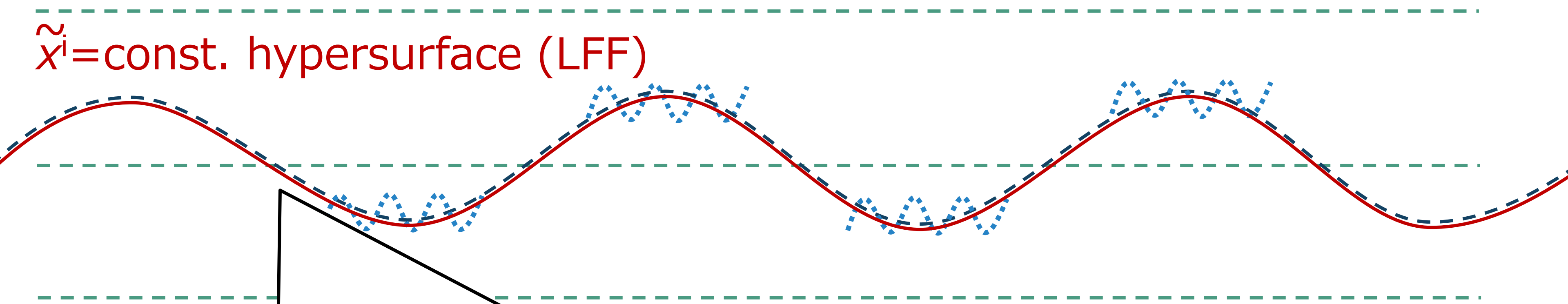


□ Metric: FLRW + large-scale GWs (cosmological frame)

$$\mathrm{d}s^2 = a^2(\eta)\left[-\mathrm{d}\eta^2 + (\delta_{ij} + h_{ij})\,\mathrm{d}x^i\mathrm{d}x^j\right]$$

Mechanism: Rough sketch 3/6

[Masui+Pen PRL105(2010)161302]



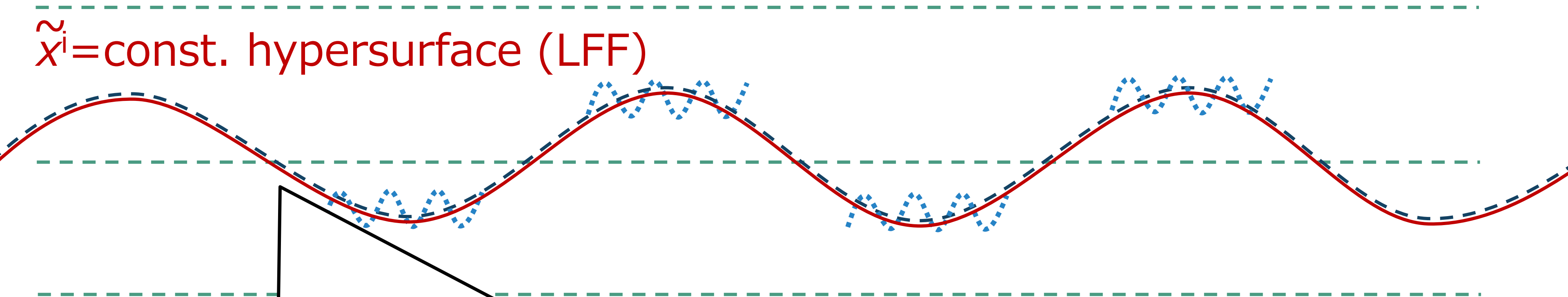
We can perform a coordinate transformation that the spacetime appears locally Minkowski at a point:

$$\tilde{x}^i = x^i + \frac{1}{2} h^i{}_j x^j$$

(Equivalent principle)

Mechanism: Rough sketch 4/6

[Masui+Pen PRL105(2010)161302]



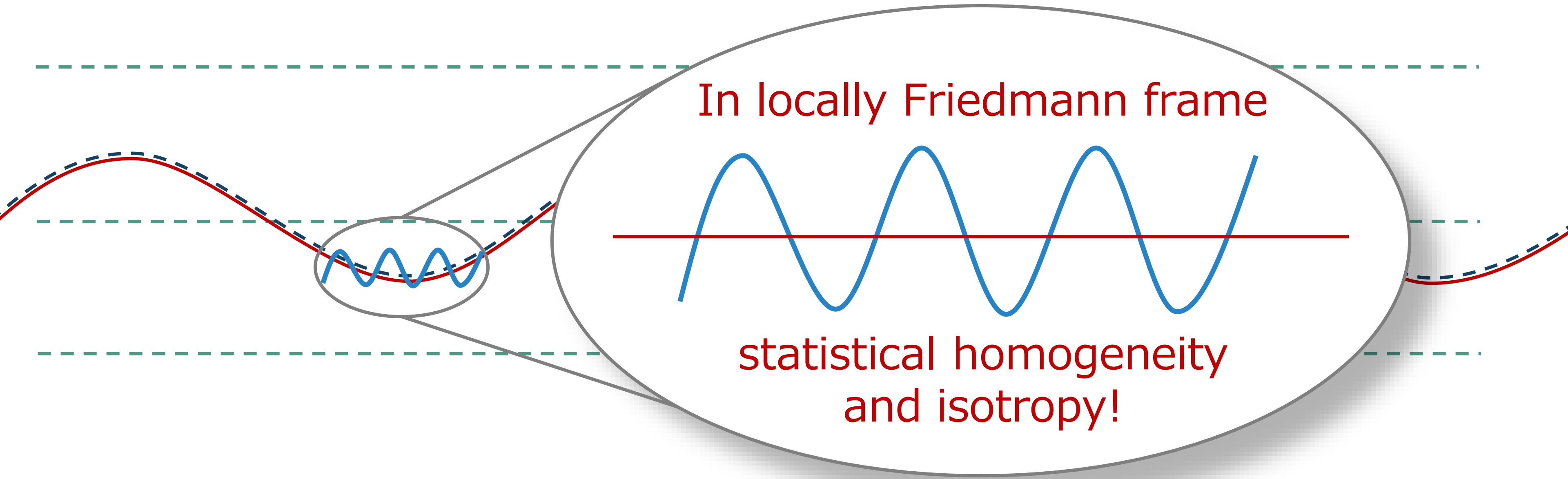
- Locally Friedmann Frame (LFF):

$$\tilde{x}^i = x^i + \frac{1}{2} h^i{}_j x^j$$

$$ds^2 = a^2(\eta)\left[-d\eta^2 + \delta_{ij} d\tilde{x}^i d\tilde{x}^j - \tilde{x}^k \partial_i h_{jk} d\tilde{x}^j d\tilde{x}^k\right]$$

Mechanism: Rough sketch 5/6

[Masui+Pen PRL105(2010)161302]



- Small-scale scalar modes must be **uncorrelated** with the long-wavelength GWs.

⇨ Statistical homogeneity/isotropy should be **defined in locally Friedmann frame**.

Mechanism: Rough sketch 6/6

[Masui+Pen PRL105(2010)161302]

- When the long-wavelength GWs are considered, the isotropy **in the cosmological frame** is broken!

$$\widetilde{k}_i = k_i - \frac{1}{2} k_j h_i{}^j$$

anisotropy

$$\Rightarrow \quad \widetilde{P}_\delta(\widetilde{\boldsymbol{k}})\Big|_{h_{ij}} = P_\delta^{\mathrm{LFF}}(k) - \boxed{\frac{1}{2}\frac{\mathrm{d}P_\delta}{\mathrm{d}\ln k} h_{ij} \widehat{k}^i \widehat{k}^j} + \mathcal{O}(h^2)$$

GWs ***tidally*** imprint **local anisotropy** in the small-scale distribution of matter!

Detail analysis of tensor clustering fossils

Detail analysis: Setup

[Dai+Jeong+Kamionkowski PRD88(2013)043507
Schmidt+Pajer+Zaldarriaga PRD89(2014)083507
DY 2407.10450]

- Metric: FLRW background in the Newton gauge

$$\mathrm{d}s^2 = -\Big[1 + 2\Phi(t,\boldsymbol{x})\Big]\mathrm{d}t^2 + a^2(t)\Big\{\Big[1 - 2\Psi(t,\boldsymbol{x})\Big]\delta_{ij} + h_{ij}(t,\boldsymbol{x})\Big\}\mathrm{d}x^i\mathrm{d}x^j$$

- Matter: pressureless non-relativisitic matter

$$T^0{}_0 = -\rho_{\mathrm{m}}(t)\Big[1 + \delta(t,\boldsymbol{x})\Big]\,,$$

$$T^0{}_i = a(t)\rho_{\mathrm{m}}(t)v_i(t,\boldsymbol{x})\,,$$

$$T^i{}_0 = -\frac{1}{a(t)}\rho_{\mathrm{m}}(t)\Big[\delta^{ij} - h^{ij}(t,\boldsymbol{x})\Big]v_j(t,\boldsymbol{x})\,,$$

$$T^i{}_j = 0\,.$$

Matter equations

[Dai+Jeong+Kamionkowski PRD88(2013)043507
Schmidt+Pajer+Zaldarriaga PRD89(2014)083507
DY 2407.10450]

- Continuity and Euler equations for pressureless non-relativistic matter:

$$\dot{\delta} + \frac{1}{a}\left(\delta^{ij} - \boxed{h^{ij}}\right)\partial_i v_j = 0\,,$$

$$\dot{v}_i + H v_i + \frac{1}{a}\partial_i \Phi = 0\,,$$

NOTE: We only keep **the *cross* interaction terms** between the scalar and tensor modes.

1st-order EoM of δ

[Dai+Jeong+Kamionkowski PRD88(2013)043507
Schmidt+Pajer+Zaldarriaga PRD89(2014)083507
DY 2407.10450]

- At **1st order**, the growth equation of the matter is the same as the standard one:

$$\ddot{\delta}^{(1)} + 2H\dot{\delta}^{(1)} - \frac{3}{2}H^2\delta^{(1)} = 0$$

For simplicity, we assume the matter-domination and the subhorizon limit ($k \gg 1/H$).

2nd-order EoM of δ

[Dai+Jeong+Kamionkowski PRD88(2013)043507
Schmidt+Pajer+Zaldarriaga PRD89(2014)083507
DY 2407.10450]

- At **2nd order**, the matter is affected by **the tidal interaction** due to GWs:

$$\ddot{\delta}^{(2)} + 2H\dot{\delta}^{(2)} - \frac{3}{2}H^2\delta^{(2)} = \boxed{-\dot{h}^{ij}\frac{\partial_i\partial_j}{\partial^2}\dot{\delta}^{(1)}}$$

↓ ***tidal*** interaction

For simplicity, we assume the matter-domination
and the subhorizon limit ($k \gg 1/H$).

$\delta^{(2)}$ induced by tidal interaction

[Dai+Jeong+Kamionkowski PRD88(2013)043507
Schmidt+Pajer+Zaldarriaga PRD89(2014)083507
DY 2407.10450]

- The solution of the **2nd-order density contrast** can be obtained by the Green function method:

$$\delta^{(2)}(\eta) = \int_0^\eta \mathrm{d}\bar{\eta} \left[-h^{ij\prime}(\bar{\eta}) \frac{\partial_i \partial_j}{\partial^2} \delta^{(1)\prime}(\bar{\eta}) \right] G_{\mathrm{ret}}(\eta, \bar{\eta})$$

↓ Source term from ***tidal*** interaction

For simplicity, we assume the matter-domination and the subhorizon limit ($k \gg 1/H$).

$\delta^{(2)}$ induced by tidal interaction

[Dai+Jeong+Kamionkowski PRD88(2013)043507
Schmidt+Pajer+Zaldarriaga PRD89(2014)083507
DY 2407.10450]

- The solution of the **2nd-order density contrast** can be obtained by the Green function method:

(Using EoM of GWs and integrating by part)

$$\delta^{(2)}(\eta) = -\, h^{ij}(\eta)\frac{\partial_i\partial_j}{\partial^2}\delta^{(1)}(\eta) + \int_0^\eta \mathrm{d}\bar{\eta}\left[\partial^2 h^{ij}(\bar{\eta})\frac{\partial_i\partial_j}{\partial^2}\delta^{(1)}(\bar{\eta})\right]G_{\rm ret}(\eta,\bar{\eta})$$

For simplicity, we assume the matter-domination and the subhorizon limit ($k \gg 1/H$).

Final compact expression

[Dai+Jeong+Kamionkowski PRD88(2013)043507
Schmidt+Pajer+Zaldarriaga PRD89(2014)083507
DY 2407.10450]

$$\delta(\eta) = \left[1 - h^{ij}(\eta)\frac{\partial_i\partial_j}{\partial^2}\right]\delta^{(1)}(\eta) + \int_0^\eta \mathrm{d}\bar{\eta}\left[\partial^2 h^{ij}(\bar{\eta})\frac{\partial_i\partial_j}{\partial^2}\delta^{(1)}(\bar{\eta})\right]G_{\mathrm{ret}}(\eta,\bar{\eta})$$

Final compact expression

[Dai+Jeong+Kamionkowski PRD88(2013)043507
Schmidt+Pajer+Zaldarriaga PRD89(2014)083507
DY 2407.10450]

First correction term leads to the **anisotropic** matter power spectrum in the infinite wavelength limit.

$$\delta(\eta) = \left[1 - h^{ij}(\eta)\frac{\partial_i\partial_j}{\partial^2}\right]\delta^{(1)}(\eta) + \int_0^\eta \mathrm{d}\bar{\eta}\left[\partial^2 h^{ij}(\bar{\eta})\frac{\partial_i\partial_j}{\partial^2}\delta^{(1)}(\bar{\eta})\right]G_{\mathrm{ret}}(\eta,\bar{\eta})$$

Second correction term gives the nonzero contributions even **AFTER** the tensor mode itself decays away!
= **“Tensor Clustering Fossil”**

Infinite wavelength limit 1/2

- For $k_{GW} \to 0$, the 2nd-order density contrast is given by

Linear transfer function $\delta^{(1)}(\eta, \boldsymbol{k}) = T_\delta(\eta, k)\Phi_{\rm prim}(\boldsymbol{k})$

$$\delta(\eta, \boldsymbol{k}) = \boxed{\mathcal{T}_\delta(k)}\Phi_{\rm prim}(\boldsymbol{k})\left[1 - \frac{1}{2}\boxed{\frac{\mathrm{d}\ln \mathcal{T}_\delta(k)}{\mathrm{d}\ln k}} h_{ij}^{\rm prim}\,\widehat{k}^i\widehat{k}^j\right]$$

In the subhorizon limit, $k^2\Phi \propto \delta \Rightarrow \mathrm{d}(\ln T_\delta)/\mathrm{d}(\ln k) = 2$.

- The primordial correlation between large-scale tensor and small-scale scalar modes leads to

$$\widetilde{P}_\Phi(\widetilde{\boldsymbol{k}})\Big|_{h_{ij}} = P_\Phi(k)\left[1 - \frac{1}{2}\frac{\mathrm{d}\ln P_\Phi}{\mathrm{d}\ln k} h_{ij}^{\rm prim}\,\widehat{k}^i\widehat{k}^j\right]$$

Infinite wavelength limit 2/2

- Matter power spectrum in **cosmological frame**

$$\widetilde{P}_\delta(\widetilde{\boldsymbol{k}})\Big|_{h_{ij}} = \mathcal{T}_\delta^2(k)\,\widetilde{P}_\Phi(\widetilde{\boldsymbol{k}})\Big|_{h_{ij}}\left[1-\frac{1}{2}\frac{\mathrm{d}\ln\mathcal{T}_\delta^2}{\mathrm{d}\ln k}h_{ij}^{\mathrm{prim}}\widehat{k}^i\widehat{k}^j\right]$$

$$P_\Phi(k)\left[1-\frac{1}{2}\frac{\mathrm{d}\ln P_\Phi}{\mathrm{d}\ln k}h_{ij}^{\mathrm{prim}}\widehat{k}^i\widehat{k}^j\right]$$

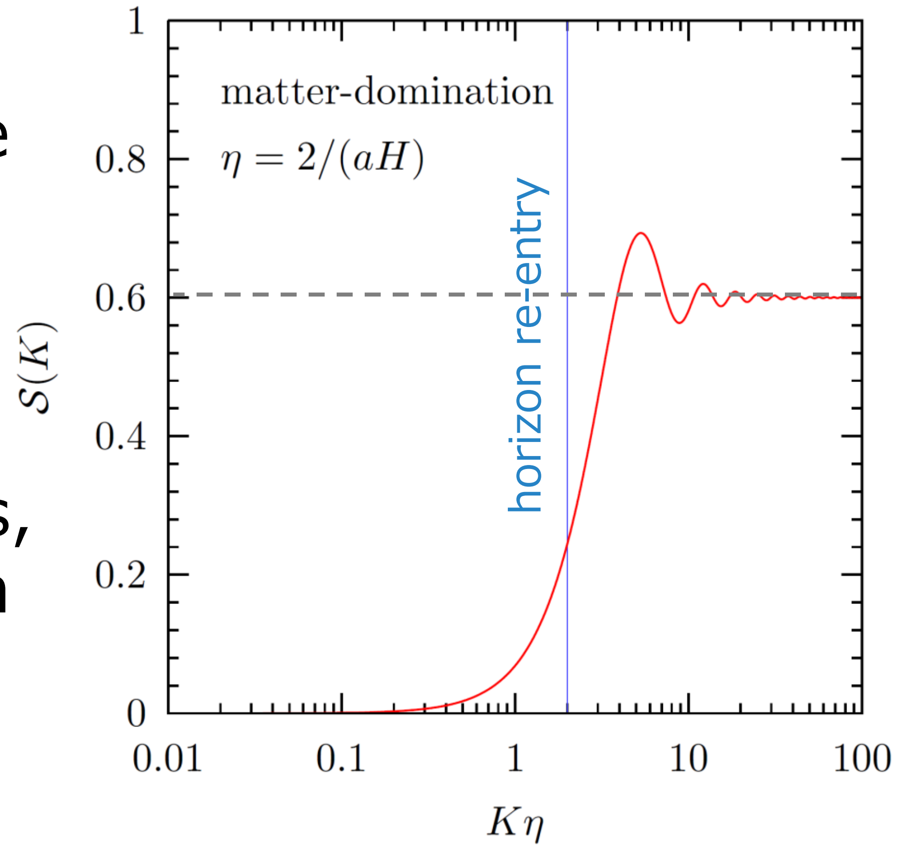
$$= P_\delta(k)\left[1-\frac{1}{2}\frac{\mathrm{d}\ln P_\delta}{\mathrm{d}\ln k}h_{ij}^{\mathrm{prim}}\widehat{k}^i\widehat{k}^j\right]$$

This is the same as the expression in the rough sketch.

Fossil effect

$$\delta_{\rm fossil}(\eta, \boldsymbol{k}) = -h_{ij}^{\rm prim}(\boldsymbol{K})\widehat{k}^i\widehat{k}^j\delta^{(1)}(\eta, \boldsymbol{k})\,\mathcal{S}(K\eta)$$

- When tensor mode re-enters horizon (Kη~1), tensor mode **interacts** with small-scale scalar modes.
- The interaction dies out as the tensor mode itself decays, but **this effect persists** even after the horizon re-entry.



[Figure from Dai+Jeong+Kamionkowski(2013)]

Detectability of tensor clustering fossils

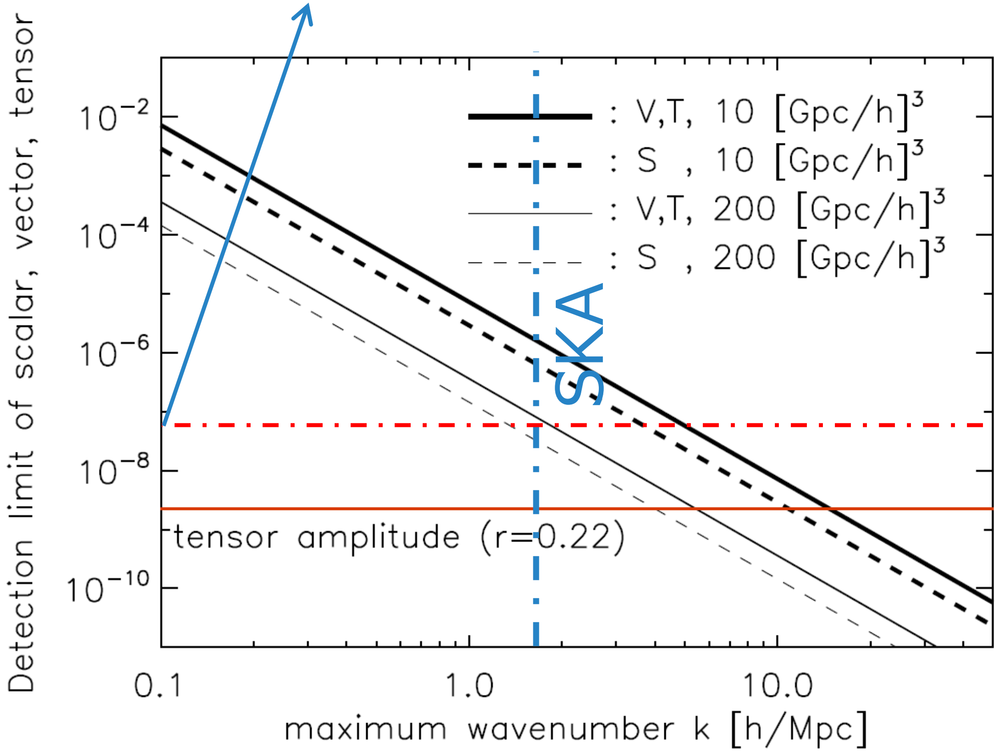
Detectability of tensor fossils ①

- Detection of primordial GWs is probably **beyond** the reach of current galaxy surveys [k_{max}/k_{min}<O(10^3)].

⇩

Measurement with a **huge dynamic range** [k_{max}/k_{min}>10^4] is needed!

$r_{3\sigma}$=7.3 (SKA) [Masui+Pen(2010)]

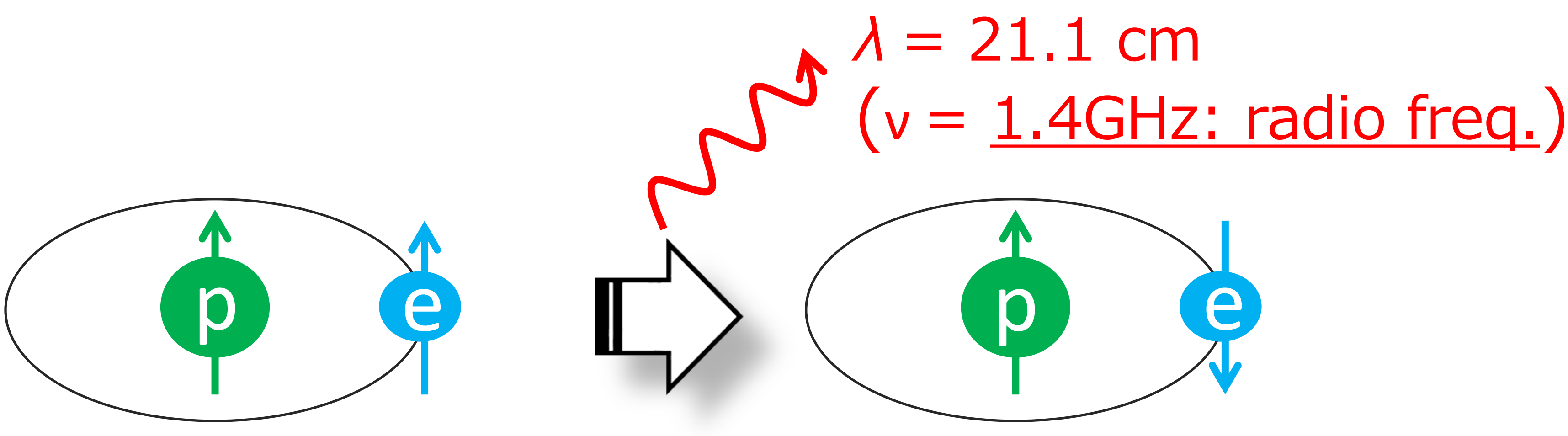


[Figure from Jeong+Kamionkowski PRL108(2012)251301]

What observations could measure tensor fossils?

◻ 21-cm line

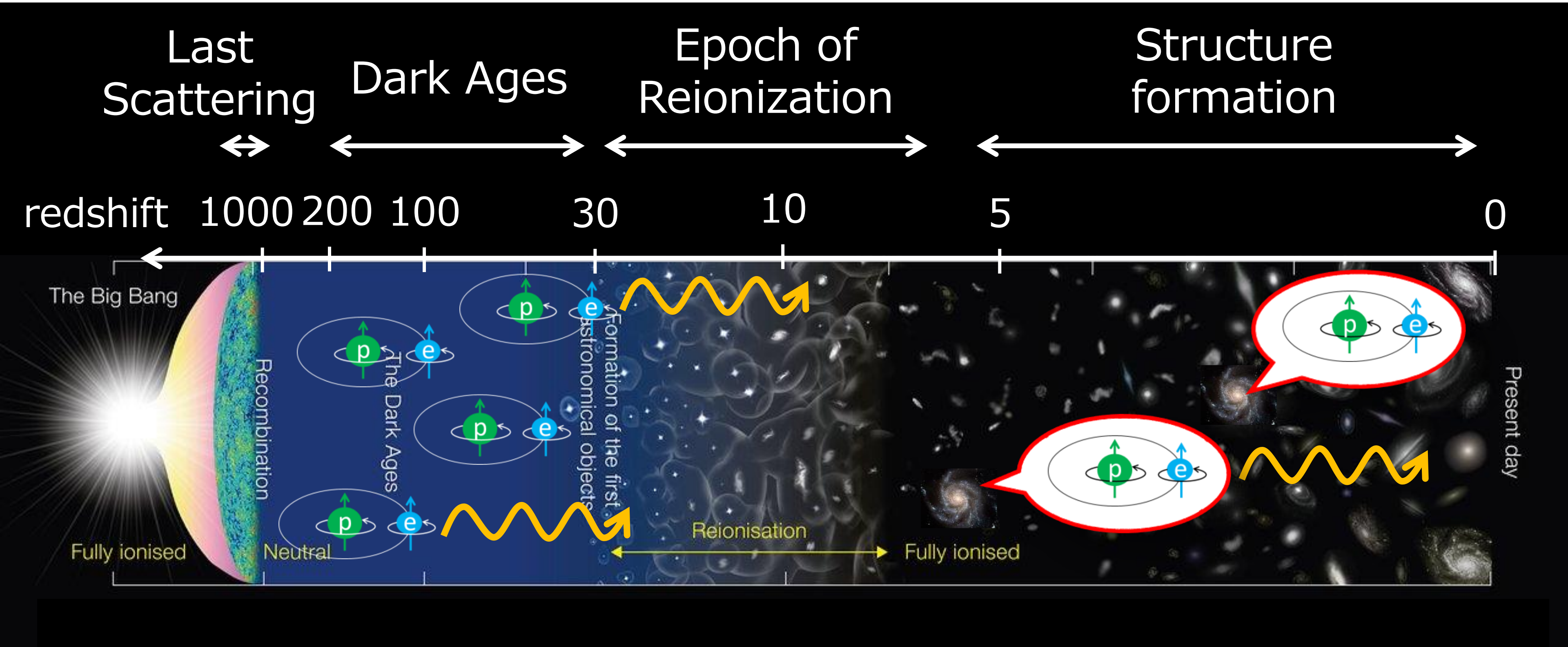
- ✓ Spectral line that is created by a hyperfine (spin-flip) transition in neutral hydrogen atoms.
- ✓ Wavelength = 21.1 cm, which is frequently observed in radio astronomy



Why 21-cm line?

- Neutral hydrogen is the most **ubiquitous** baryonic matter in the high-redshift Universe.

  ⇨ We can observe large survey volume!



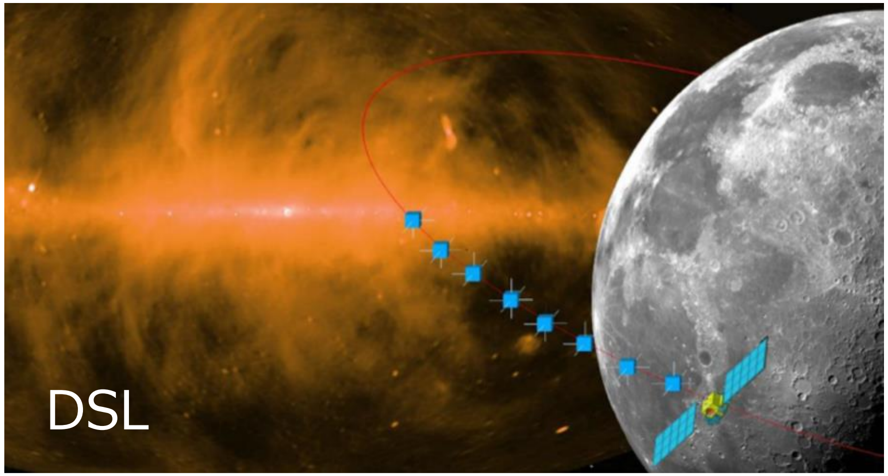
Moon-based observations

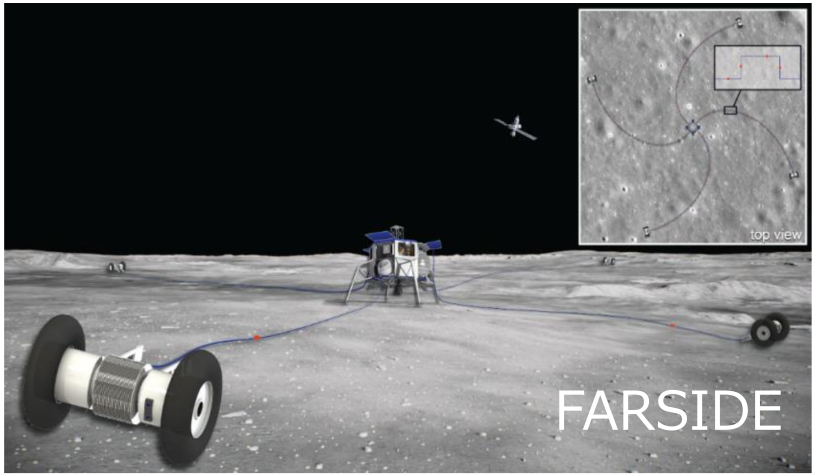
Lunar orbit

- DARE/DAPPER (NASA)
- DSL (China)
- NCLE (Netherland+China)
- CoDex (ESA)

Farside of the Moon

- FARSIDE (NASA)
- LCRT (NASA)
- **TSUKUYOMI (JAXA)**





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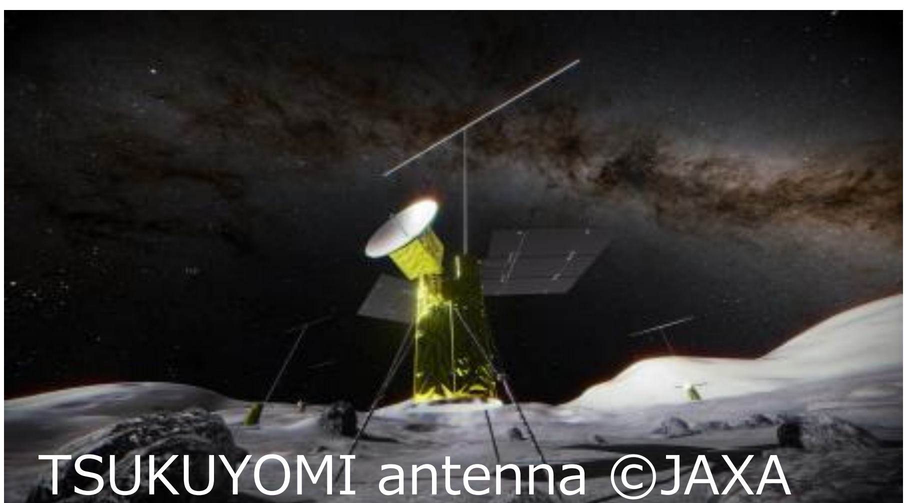
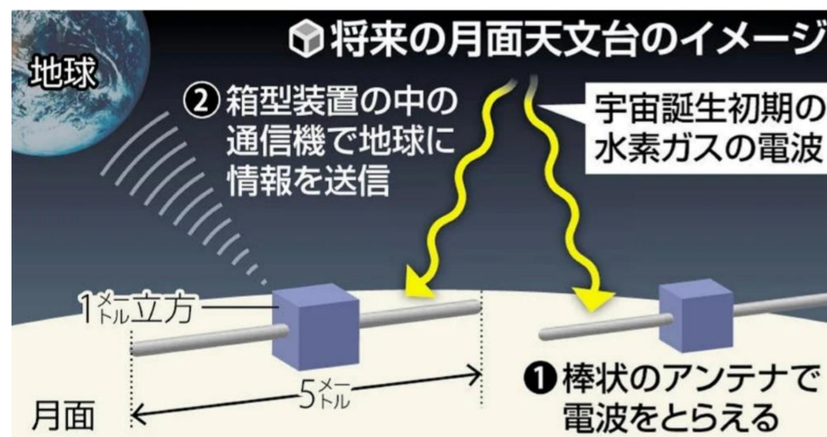


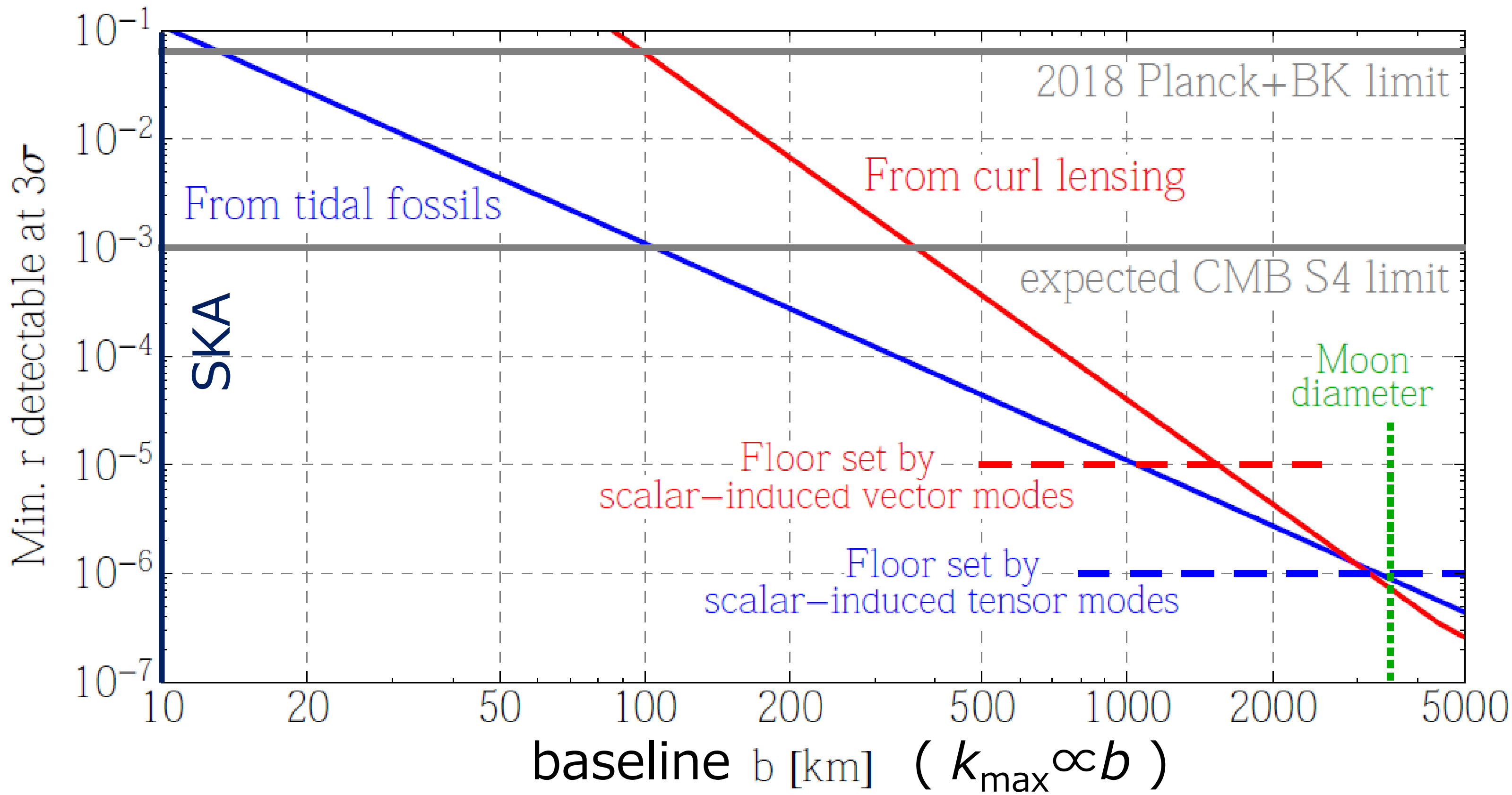
TSUKUYOMI antenna ©JAXA

[MAINICHI newspaper online, 2022/11/26]



Detectability of tensor fossils ②

The **Moon**-based 21-cm line observations of neutral hydrogen during the dark ages can detect the primordial GWs with small *r*.



[Figure from Cosmic Vision 21cm Collaboration 181009572]

What should we do next?

- Generic coupling between inflaton and spectator field [Jeong+Kamionkowski, PRL108(2012)251301]

$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle|_{h_p(\mathbf{K})} = f_p(\mathbf{k}_1, \mathbf{k}_2) h_p^*(\mathbf{K}) \epsilon_{ij}^p k_1^i k_2^j \delta_{\mathbf{k}_{123}}^D$$

- Parity-odd gravitational waves [Masui+Pen+Turok, PRL118(2017)22,221301]

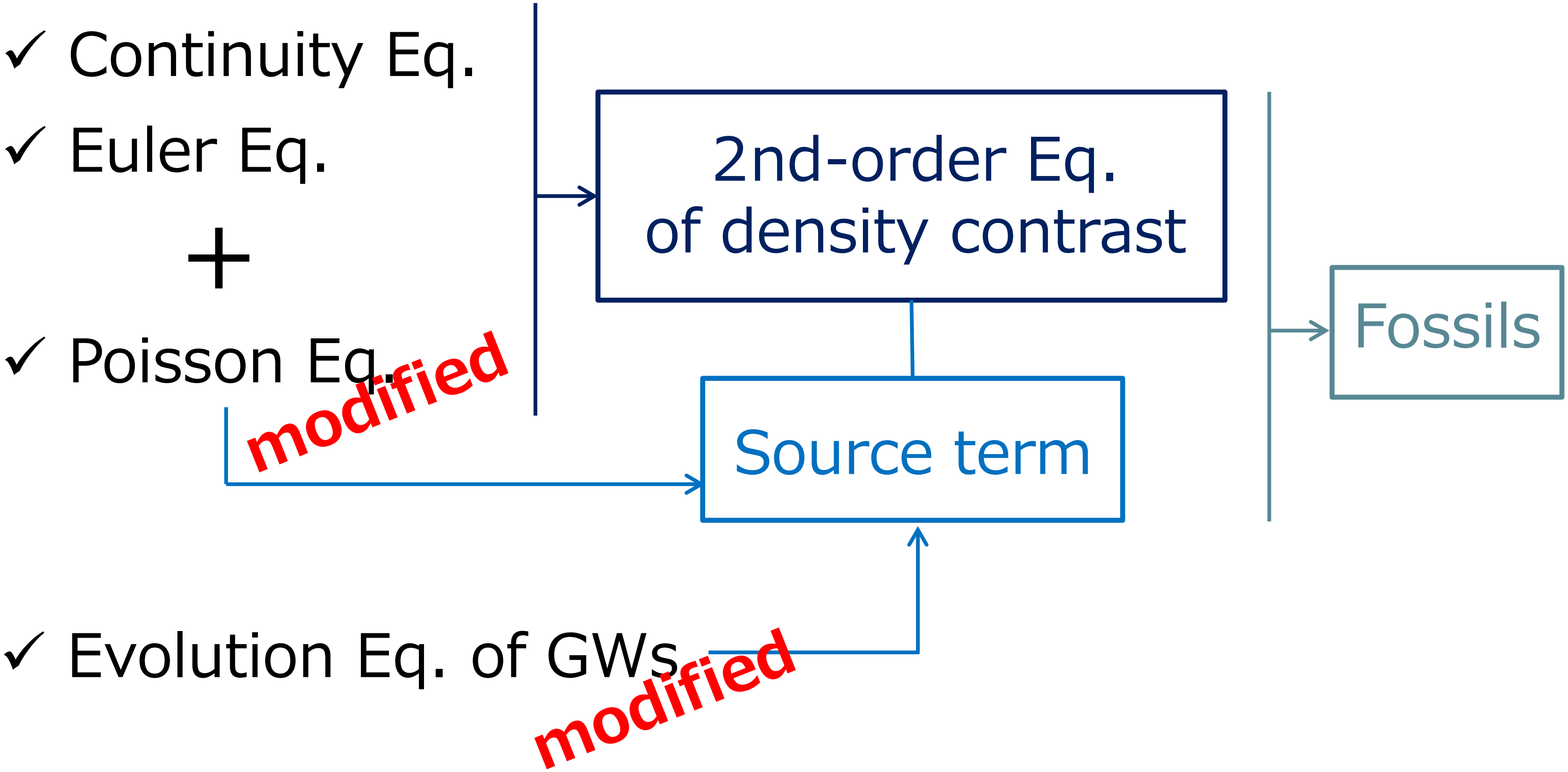
$$h_{ab}(\mathbf{K}) = h_R(\mathbf{K}) e_{ab}^R(\hat{\mathbf{K}}) + h_L(\mathbf{K}) e_{ab}^L(\hat{\mathbf{K}})$$

- Test of GW propagation → Today's topic

Tensor clustering fossils in modified gravity

Which part is to be modified?

- Strategy



Phenomenological model of modified Poisson equation

[**DY** 2407.10450]

$\mu(t)$: Effective gravitational coupling

$$\frac{1}{a^2H^2}\partial^2\Phi(t,\boldsymbol{x}) = \frac{3}{2}\Omega_{\rm m}(t)\boxed{\mu(t)}\delta(t,\boldsymbol{x})$$

$$+\left(-\boxed{\Gamma_1(t)}\frac{1}{a^2H^2}\partial^2 h_{ij}(t,\boldsymbol{x}) + \boxed{\Gamma_2(t)}\frac{1}{H}\dot{h}_{ij}(t,\boldsymbol{x})\right)\frac{\partial_i\partial_j}{\partial^2}\delta(t,\boldsymbol{x}) + \cdots$$

$\Gamma_1(t)$ & $\Gamma_2(t)$: Novel parameters characterizing tidal interactions

✓ GR limit: $\mu(t)=1$ and $\Gamma_1(t)=\Gamma_2(t)=0$

NOTE: When GR is considered, term such as $h^{ij}(d_i d_j/d^2)\delta$ appears. However, such term can be safely neglected in the subhorizon limit.

Phenomenological model of modified EoM of GWs

[**DY** 2407.10450]

$\nu(t)$: Effective friction term

$$\ddot{h}_{ij} + (3 + \nu)\, H\dot{h}_{ij} - c_{\mathrm{T}}^2 \frac{\partial^2}{a^2} h_{ij} = 0$$

$c_{\mathrm{T}}^2(t) = 1 + \alpha_{\mathrm{T}}(t)$: GW sound speed

✓ GR limit: $c_{\mathrm{T}}^2(t)=1$ and $\nu(t)=0$

Tensor clustering fossils in phenomenological models

[**DY** 2407.10450]

$$\delta_{\rm fossil}(\eta) = \int_0^{\eta} d\bar{\eta}\, G_{\rm ret}(\eta,\bar{\eta}) \Big\{ \Big[c_{\rm T}^2(\bar{\eta}) - \Gamma_1(\bar{\eta}) \Big] \partial^2 h^{ij}(\bar{\eta}) + \Big[f(\bar{\eta}) - 1 - \nu(\bar{\eta}) - \Gamma_2(\bar{\eta}) \Big] \mathcal{H}(\bar{\eta}) h^{ij\prime}(\bar{\eta}) \Big\} \frac{\partial_i \partial_j}{\partial^2} \delta^{(1)}(\bar{\eta})$$

- Tensor clustering fossils can be induced by three parts:
  - ✓ c_T^2 & ν : propagation of large-scale GWs
  - ✓ f-1 : deviation of structure growth rate from the value in matter-domination
  - ✓ Γ_1 & Γ_2 : tidal interactions from effective Poisson Eq.

Example: Horndeski theory

[Horndeski Int.J.Theor.Phys.10,363(1974),
Deffayet+ PRD84,063039(2011),
Kobayashi+Yamaguchi+Yokoyama PTP126,511(2011)]

- Once the underlying gravity model is specified, we can calculate the clustering fossils by using the formula.

⇨ As a demonstration, let us consider the Horndeski class of modified gravity

= the most general scalar-tensor theory with the 2nd-order EoM

$$\mathcal{L} = P(\phi, X) - Q(\phi, X)\Box\phi + G_4(\phi, X)R - \frac{\partial G_4}{\partial X}\left(\nabla_\mu \nabla_\nu \phi\right)^2 + \cdots$$

Effective-Field-Theory (EFT) parameters

- Even complex full theories can be described by perturbed models with **a few** EFT parameters:

✓ Linear order [Bellini+ JCAP07,050(2014), Langlois+ JCAP05,033(2017),⋯]

$\alpha_K(t)$ Kineticity (kinetic term of additional field)
$\alpha_M(t)$ Planck-Mass run rate
$\alpha_T(t)$ Tensor speed excess
$\alpha_B(t)$ Braiding (Mixing between field and metric pert.)

✓ Nonlinear order [**DY**+Yokoyama+Tashiro PRD96,123516(2017)]

$\alpha_V(t)$ Veinstein screening (scalar self-interactions)

Phenomenological parameters in terms of EFT parameters

[**DY** 2407.10450]

- Phenomenological parameters μ, Γ_1, and Γ_2, can be expressed in terms of the EFT parameters (although their dependence is quite complicated).

$$\mu = 1 + \alpha_{\rm T} - \frac{[\alpha_{\rm B}(1+\alpha_{\rm T}) + \alpha_{\rm T} - \alpha_{\rm M}]^2}{(1+\alpha_{\rm B})\left[\frac{\dot{H}}{H^2} + \alpha_{\rm B}(1+\alpha_{\rm T}) + \alpha_{\rm T} - \alpha_{\rm M}\right] + \frac{\dot{\alpha}_{\rm B}}{H} + \frac{3}{2}\Omega_{\rm m}}$$

$$\Gamma_1 = \cdots$$

$$\Gamma_2 = \cdots$$

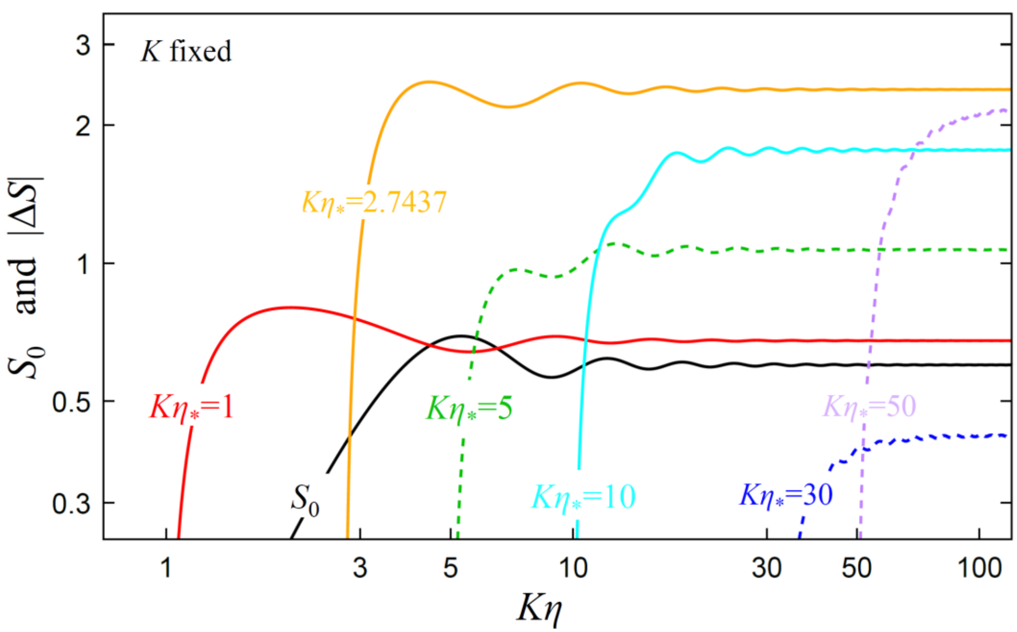
We derive the formula of the tensor clustering fossils in the context of the modified gravity.

Fossil effect [GW sound speed]

- When the deviation occurs **at $\eta=\eta_*$**, tensor mode interacts with small-scale scalar mode.
- Even in the case of modified gravity, **this effect persists** after tensor mode decays.
- The effect of α_T largely **enhances** the amplitude of the tensor fossils.

$$\alpha_{\rm T}(\eta) = \alpha_{\rm T}^*\,\delta_{\rm D}(\ln\eta - \ln\eta_*)$$

[Other EFT parameters are taken to be zero.]



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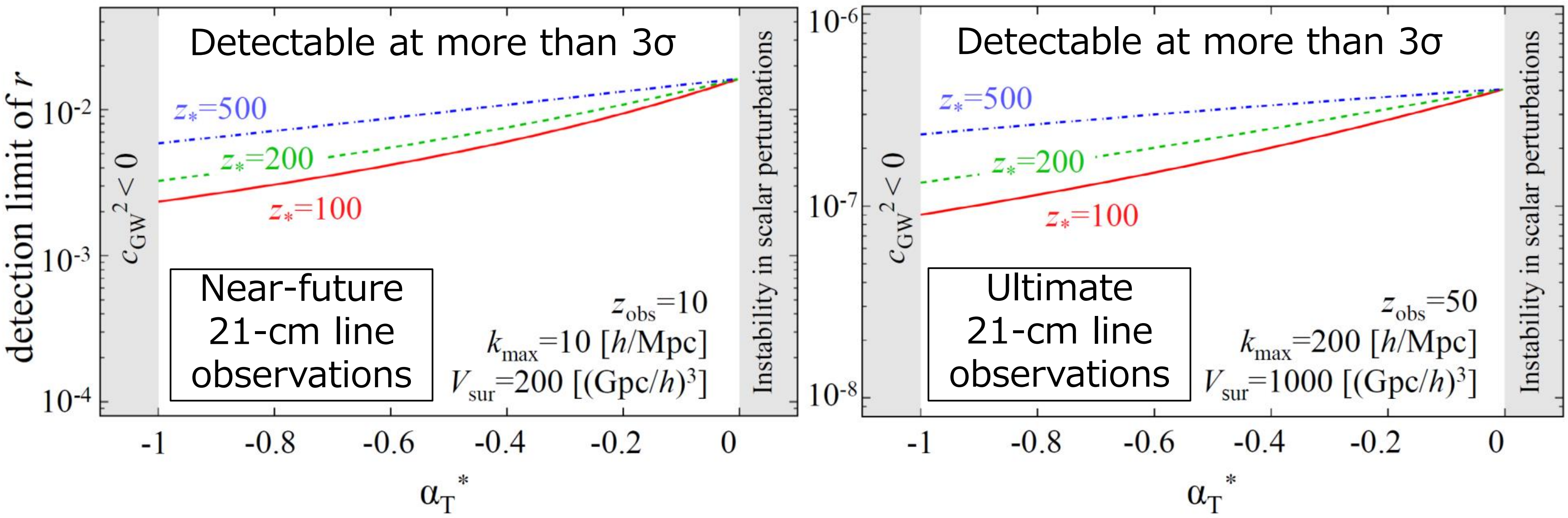
Detectability of GWs with α_T

[**DY** 2407.10450]



- The measurement with the extremely large survey volume, which can be achieved by 21-cm line obs., can detect the GWs with modified sound speed!

Summary

- ✓ Large-scale GWs tidally imprint **local anisotropy** in the small-scale distribution of matter.
- ✓ This imprint constitutes a fossilized map of GWs since it **persists even after the GWs decay**: **"tensor clustering fossils"**.
- ✓ Tensor clustering fossil is a novel tool to constrain large-scale GWs.
- ✓ We developed the formula applicable to the fossils in context of **the modified gravity**.

Summary: Future

- ✓ **Realistic effects of 21-cm line observations** such as redshift-space distortion, non-linear growth, and survey systematics should be considered.

- ✓ It can be used to extract the information of **three-point correlations** between primordial spectator fields and inflaton [Jeong+Kamionkowski, PRL108(2012)251301] .

- ✓ It would be interesting to consider **other GW contributions** such as tensor x tensor [Bari+Ricciadone+Bartolo+Bertacca+Matarrese PRL129(2022)091301, JCAP07(2023)034]

Thank you!

Simple model: α_T model

- All the EFT parameters except for α_T are taken to be zero.

  ⇨ The situation is drastically simplified.

$$\mathcal{S}(\eta, K) = \frac{1}{5}\int_0^{\eta} \mathrm{d}\bar{\eta} \left(1 + \frac{19}{4}\alpha_{\mathrm{T}}(\bar{\eta})\right) K^2 \bar{\eta}\, \mathcal{T}_h(\bar{\eta}, K; \alpha_{\mathrm{T}}) \left[1 - \left(\frac{\bar{\eta}}{\eta}\right)^5\right]$$

- We further assume that the deviation occurs only for a very short period during high-z MD:

$$\alpha_{\mathrm{T}}(\eta) = \alpha_{\mathrm{T}}^{*}\, \delta_{\mathrm{D}}(\ln \eta - \ln \eta_{*})$$

Small-scale effective Lagrangian

- We keep the scalar perturbations with the highest spatial derivatives and the tensor perturbations with all the time and spatial derivatives.
- We only consider the scalar-scalar-tensor three-point interaction terms with the highest derivatives.

$$\mathcal{L}_{\rm eff} = \frac{M^2 a}{2}\Big[4\Psi\partial^2\Phi - 2\left(1+\alpha_{\rm T}\right)\Psi\partial^2\Psi - a^2(\dot{h}_{ij})^2 + \left(1+\alpha_{\rm T}\right)\left(\partial_k h_{ij}\right)^2$$
$$+ 4H\left(\alpha_{\rm M}-\alpha_{\rm T}\right)\Psi\partial^2\pi - 4H\alpha_{\rm B}\Phi\partial^2\pi + H^2 c_{\pi\pi}\pi\partial^2\pi$$
$$+ 2\alpha_{\rm T}\dot{h}^{ij}\Psi\partial_i\partial_j\pi - 4\alpha_{\rm V}\dot{h}^{ij}\Phi\partial_i\partial_j\pi$$
$$- \frac{5}{a^2}\alpha_{\rm T}\partial^2 h^{ij}\pi\partial_i\partial_j\pi + H c_{\pi\pi\dot{h}}\dot{h}^{ij}\pi\partial_i\partial_j\pi - 2\alpha_{\rm V}\ddot{h}^{ij}\pi\partial_i\partial_j\pi\Big],$$