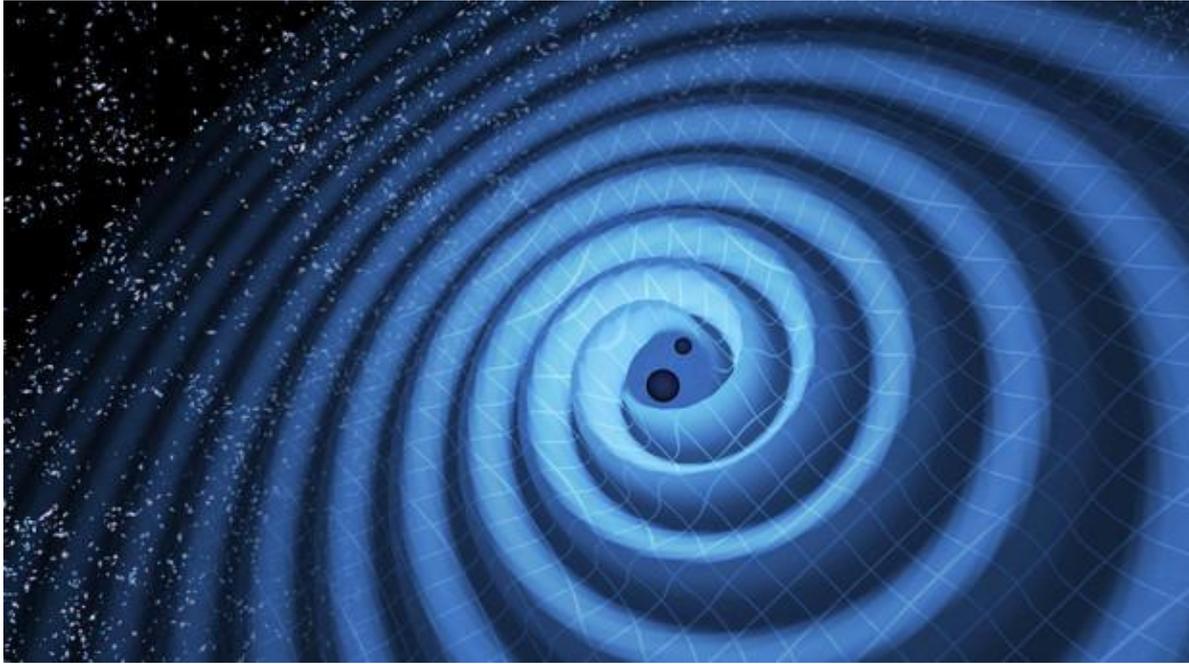# Tensor clustering fossils
(in modified gravity)

Based on **DY** PRD110(2024)10,103516
[arXiv2407.10450]

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2025/03/27 Mini-workshop on gravity and cosmology

Today's topic:
Gravitational Waves

How do we observe gravitational waves?

Constraints on GWs

□ Cosmic Microwave Background

- ✓ B-mode polarization $[f<10^{-16}\text{Hz}]$ (LiteBIRD,⋯)
- ✓ Curl-lensing $[10^{-16}\text{Hz}<f<10^{-13}\text{Hz}]$ (CMB S4,⋯)

□ Pulser Timing Array $[10^{-9}\text{Hz}<f<10^{-5}\text{Hz}]$ (NANOGrav,⋯)

□ Direct detection experiments $[10^{-3}\text{Hz}<f<10^{2}\text{Hz}]$ (LIGO, KAGRA, LISA,⋯)

□ Astrometry $[10^{-18}\text{Hz}<f<10^{-9}\text{Hz}]$ (Gaia, SKA, ngVLA,⋯)

□ Matter clustering $[f<10^{-18}\text{Hz}]$: Today's topic

Constraints on $\Omega_{\rm GW}$

$$\Omega_{\rm GW} \equiv \frac{1}{\rho_{\rm c}} \frac{{\rm d}\rho_{\rm GW}}{{\rm d}\ln k} \sim \frac{f}{H_0^2} \left\langle h^2 \right\rangle$$
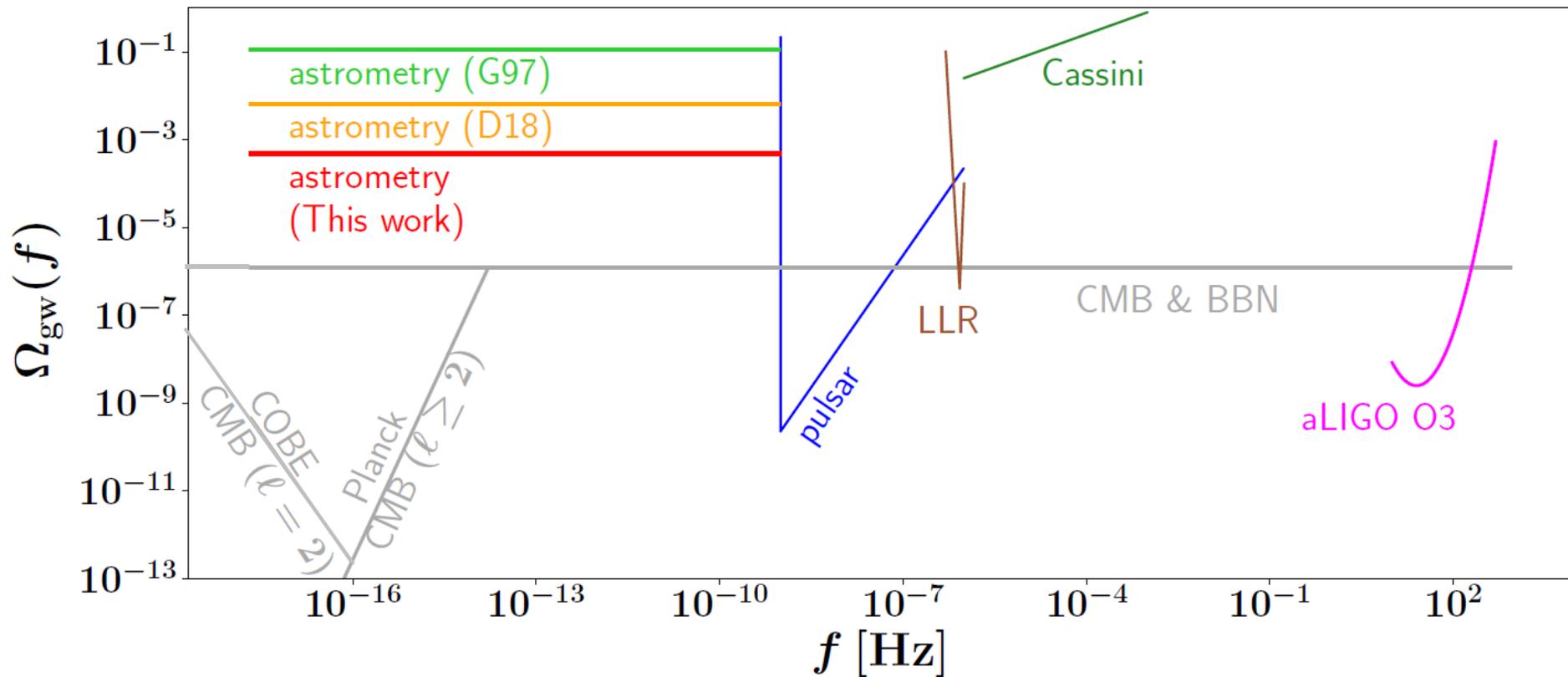
[Figure from Aoyama+**DY**+Shiraishi+Ouchi 2105.04039]

Constraints on Ω_{GW}

$$\Omega_{\mathrm{GW}} \equiv \frac{1}{\rho_{\mathrm{c}}} \frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\ln k} \sim \frac{f}{H_0^2} \langle h^2 \rangle$$
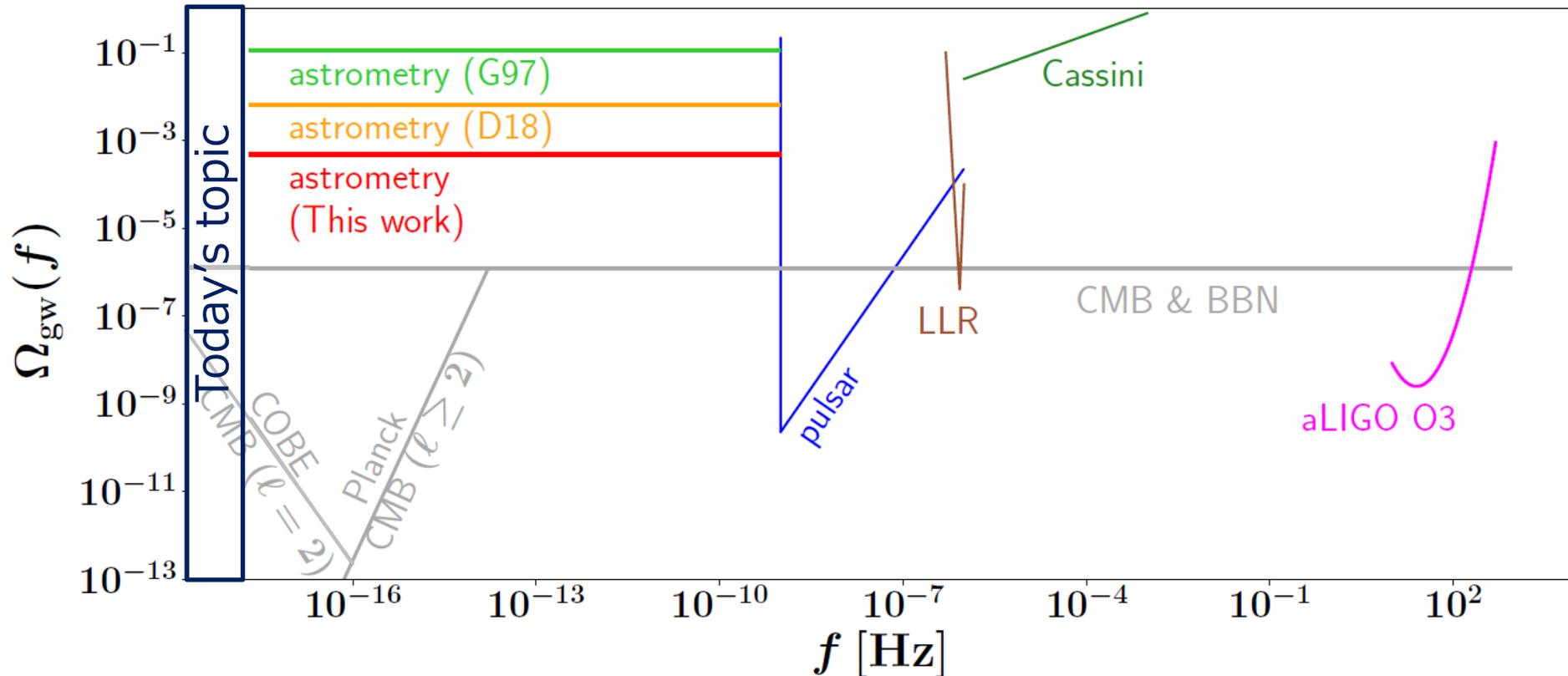
[Figure from Aoyama+**DY**+Shiraishi+Ouchi 2105.04039]

What is "tensor clustering *fossils*"?
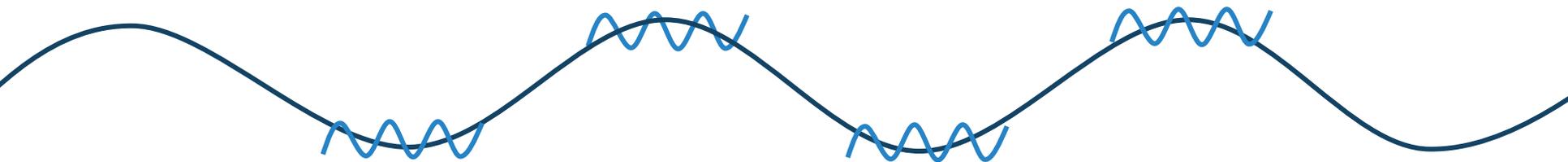
Wavenumber of
density contrast: k

Wavenumber of
GWs: k_{GW} ($\ll k$)
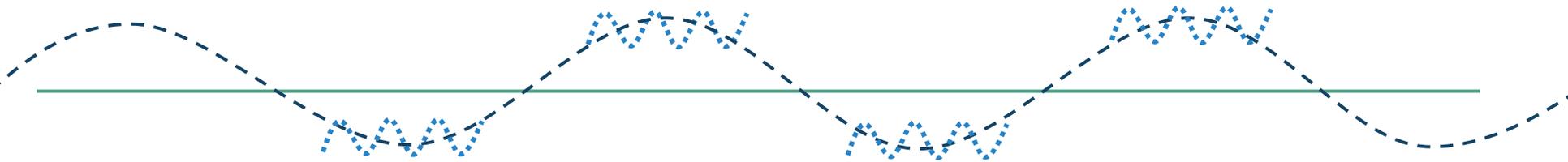
x^i=const. hypersurface

☐ Metric: FLRW + large-scale GWs (cosmological frame)

$$\mathrm{d}s^2 = a^2(\eta)\left[-\mathrm{d}\eta^2 + (\delta_{ij} + h_{ij})\,\mathrm{d}x^i\mathrm{d}x^j\right]$$

[Masui+Pen PRL105(2010)161302]

$\widetilde{x}^{i}$=const. hypersurface (LFF)
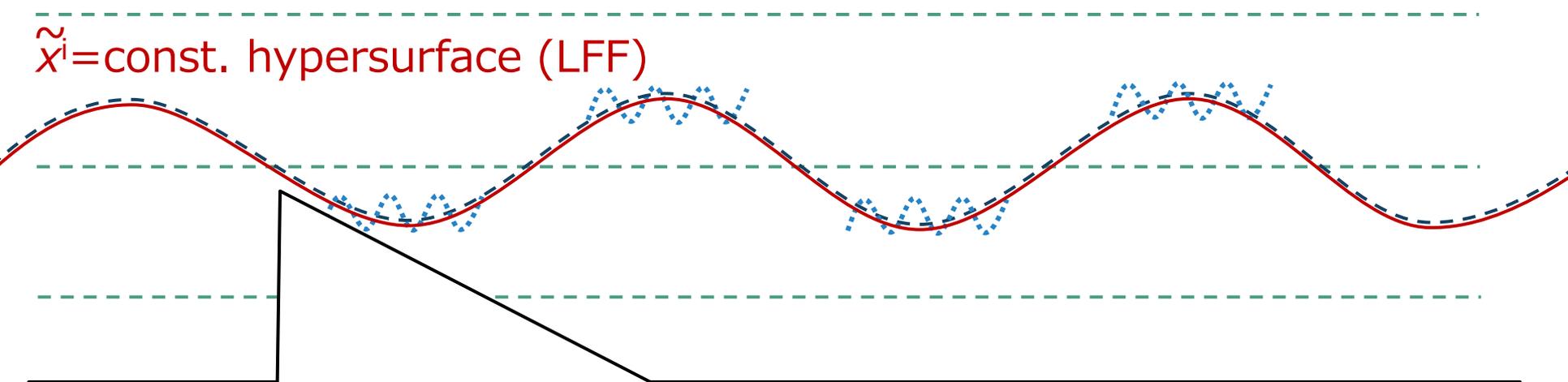
We can perform a coordinate transformation that the spacetime appears locally Minkowski at a point:

$$\widetilde{x}^{i} = x^{i} + \frac{1}{2} h^{i}{}_{j} x^{j}$$

(Equivalent principle)

$\widetilde{x}^i$=const. hypersurface (LFF)
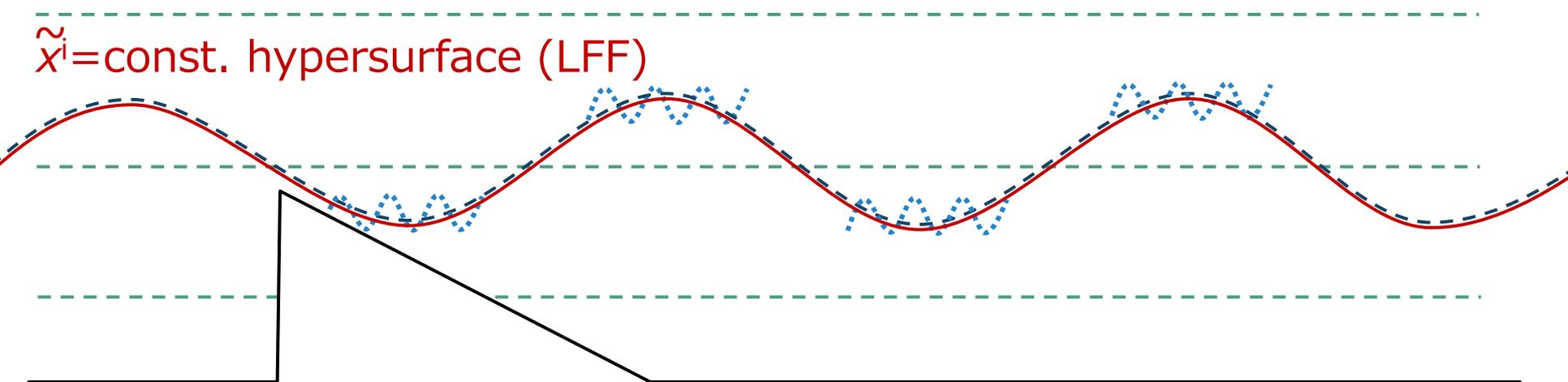
□ Locally Friedmann Frame (LFF):

$$\widetilde{x}^i = x^i + \frac{1}{2} h^i{}_j x^j$$

$$\mathrm{d}s^2 = a^2(\eta)\left[-\mathrm{d}\eta^2 + \delta_{ij}\mathrm{d}\widetilde{x}^i\mathrm{d}\widetilde{x}^j - \widetilde{x}^k \partial_i h_{jk}\mathrm{d}\widetilde{x}^j\mathrm{d}\widetilde{x}^k \right]$$

[Masui+Pen PRL105(2010)161302]

In locally Friedmann frame

statistical homogeneity
and isotropy!
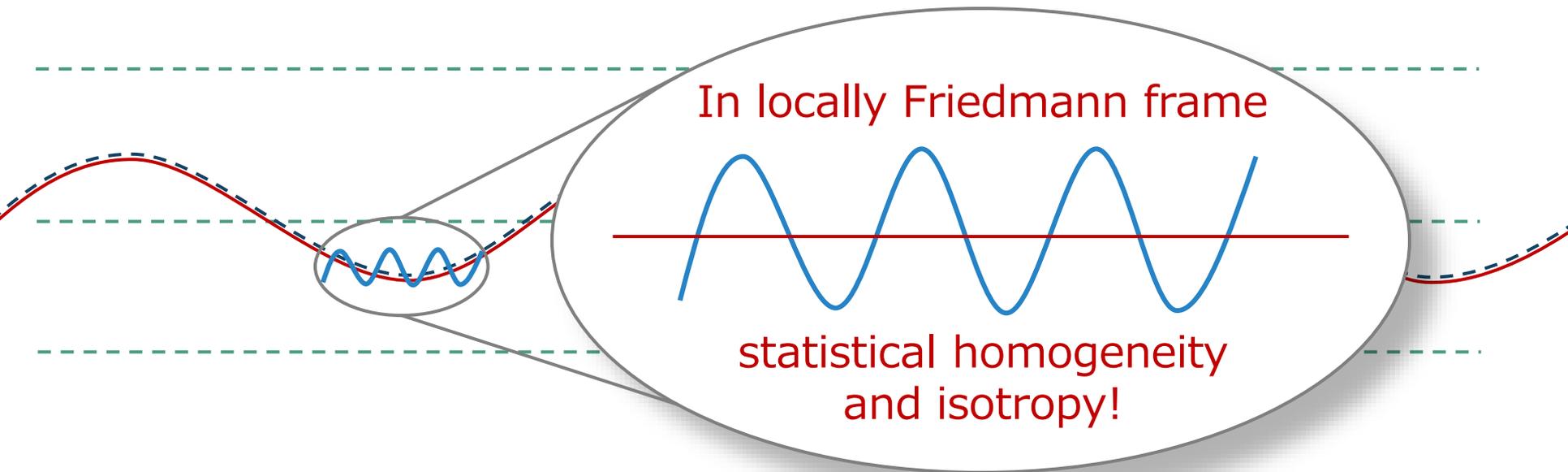
☐ Small-scale scalar modes must be **uncorrelated** with the long-wavelength GWs.
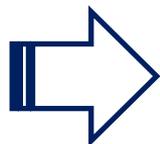
➡ Statistical homogeneity/isotropy should be **defined in locally Friedmann frame**.

Mechanism: Rough sketch 6/6

☐ When the long-wavelength GWs are considered, the isotropy **in the cosmological frame** is broken!

$$\widetilde{k}_i = k_i - \frac{1}{2}k_j h_i{}^j$$

anisotropy

$$\Rightarrow \quad \widetilde{P}_\delta(\widetilde{\boldsymbol{k}})\Big|_{h_{ij}} = P_\delta^{\mathrm{LFF}}(k) - \boxed{\frac{1}{2}\frac{\mathrm{d}P_\delta}{\mathrm{d}\ln k}h_{ij}\widehat{k}^i\widehat{k}^j} + \mathcal{O}(h^2)$$

GWs *tidally* imprint **local anisotropy** in the small-scale distribution of matter!

Detail analysis of tensor clustering fossils

Detail analysis: Setup

☐ Metric: FLRW background in the Newton gauge

$$\mathrm{d}s^2 = -\Big[1 + 2\Phi(t, \boldsymbol{x})\Big]\mathrm{d}t^2 + a^2(t)\bigg\{\Big[1 - 2\Psi(t, \boldsymbol{x})\Big]\delta_{ij} + h_{ij}(t, \boldsymbol{x})\bigg\}\mathrm{d}x^i\mathrm{d}x^j$$

☐ Matter: pressureless non-relativisitic matter

$$T^0{}_0 = -\rho_{\mathrm{m}}(t)\Big[1 + \delta(t, \boldsymbol{x})\Big],$$

$$T^0{}_i = a(t)\rho_{\mathrm{m}}(t)v_i(t, \boldsymbol{x}),$$

$$T^i{}_0 = -\frac{1}{a(t)}\rho_{\mathrm{m}}(t)\Big[\delta^{ij} - h^{ij}(t, \boldsymbol{x})\Big]v_j(t, \boldsymbol{x}),$$

$$T^i{}_j = 0.$$

Matter equations

[Dai+Jeong+Kamionkowski PRD88(2013)043507
Schmidt+Pajer+Zaldarriaga PRD89(2014)083507
DY PRD110(2024)10,103516]

☐ Continuity and Euler equations for pressureless non-relativistic matter:

$$\dot{\delta} + \frac{1}{a}\left(\delta^{ij} - \boxed{h^{ij}}\right)\partial_i v_j = 0,$$

$$\dot{v}_i + H v_i + \frac{1}{a}\partial_i \Phi = 0,$$

NOTE: We only keep **the *cross* interaction terms** between the scalar and tensor modes.

1st-order EoM of δ

❑ At **1st order**, the growth equation of the matter is the same as the standard one:

$$\ddot{\delta}^{(1)} + 2H\dot{\delta}^{(1)} - \frac{3}{2}H^2\delta^{(1)} = 0$$

For simplicity, we assume the matter-domination and the subhorizon limit ($k \gg 1/H$).

2nd-order EoM of δ

☐ At **2nd order**, the matter is affected by
the tidal interaction due to GWs:

$$\ddot{\delta}^{(2)} + 2H\dot{\delta}^{(2)} - \frac{3}{2}H^2\delta^{(2)} = \boxed{-\dot{h}^{ij}\frac{\partial_i\partial_j}{\partial^2}\dot{\delta}^{(1)}}$$

tidal interaction

For simplicity, we assume the matter-domination
and the subhorizon limit ($k \gg 1/H$).

δ$^{(2)}$ induced by tidal interaction

☐ The solution of the **2nd-order density contrast**
can be obtained by the Green function method:

$$\delta^{(2)}(\eta) = \int_0^\eta \mathrm{d}\bar\eta \left[-h^{ij\,\prime}(\bar\eta) \frac{\partial_i \partial_j}{\partial^2} \delta^{(1)\,\prime}(\bar\eta) \right] G_{\mathrm{ret}}(\eta, \bar\eta)$$

Source term from *tidal* interaction

For simplicity, we assume the matter-domination
and the subhorizon limit ($k \gg 1/H$).

δ⁽²⁾ induced by tidal interaction

[Dai+Jeong+Kamionkowski PRD88(2013)043507
Schmidt+Pajer+Zaldarriaga PRD89(2014)083507
DY PRD110(2024)10,103516]

☐ The solution of the **2nd-order density contrast** can be obtained by the Green function method:

(Using EoM of GWs and integrating by part)

$$\delta^{(2)}(\eta) = -\, h^{ij}(\eta)\frac{\partial_i\partial_j}{\partial^2}\delta^{(1)}(\eta)$$

$$+ \int_0^\eta \mathrm{d}\bar{\eta}\left[\partial^2 h^{ij}(\bar{\eta})\frac{\partial_i\partial_j}{\partial^2}\delta^{(1)}(\bar{\eta})\right]G_{\mathrm{ret}}(\eta,\bar{\eta})$$

For simplicity, we assume the matter-domination and the subhorizon limit ($k \gg 1/H$).

Final compact expression

[Dai+Jeong+Kamionkowski PRD88(2013)043507
Schmidt+Pajer+Zaldarriaga PRD89(2014)083507
DY PRD110(2024)10,103516]

$$\delta(\eta) = \left[1 - h^{ij}(\eta) \frac{\partial_i \partial_j}{\partial^2} \right] \delta^{(1)}(\eta)$$

$$+ \int_0^{\eta} \mathrm{d}\bar{\eta} \left[\partial^2 h^{ij}(\bar{\eta}) \frac{\partial_i \partial_j}{\partial^2} \delta^{(1)}(\bar{\eta}) \right] G_{\mathrm{ret}}(\eta, \bar{\eta})$$

Final compact expression

[Dai+Jeong+Kamionkowski PRD88(2013)043507
Schmidt+Pajer+Zaldarriaga PRD89(2014)083507
DY PRD110(2024)10,103516]

First correction term leads to the **anisotropic** matter power spectrum in the infinite wavelength limit.

$$\delta(\eta) = \left[1 - h^{ij}(\eta)\frac{\partial_i\partial_j}{\partial^2}\right]\delta^{(1)}(\eta)$$
$$+ \int_0^{\eta} \mathrm{d}\bar{\eta}\left[\partial^2 h^{ij}(\bar{\eta})\frac{\partial_i\partial_j}{\partial^2}\delta^{(1)}(\bar{\eta})\right]G_{\mathrm{ret}}(\eta,\bar{\eta})$$

Second correction term gives the nonzero contributions even **AFTER** the tensor mode itself decays away!
= **"Tensor Clustering Fossil"**

Infinite wavelength limit 1/2

☐ For $k_{GW} \to 0$, the 2nd-order density contrast is given by

Linear transfer function $\delta^{(1)}(\eta, \boldsymbol{k}) = T_\delta(\eta, k) \Phi_{\text{prim}}(\boldsymbol{k})$

$$\delta(\eta, \boldsymbol{k}) = \boxed{\mathcal{T}_\delta(k)} \Phi_{\text{prim}}(\boldsymbol{k}) \left[1 - \frac{1}{2} \boxed{\frac{\mathrm{d}\ln \mathcal{T}_\delta(k)}{\mathrm{d}\ln k}} h_{ij}^{\text{prim}} \widehat{k}^i \widehat{k}^j \right]$$

In the subhorizon limit, $k^2 \Phi \propto \delta \Rightarrow \mathrm{d}(\ln \mathcal{T}_\delta)/\mathrm{d}(\ln k) = 2$.

☐ The primordial correlation between large-scale tensor and small-scale scalar modes leads to

$$\widetilde{P}_\Phi(\widetilde{\boldsymbol{k}}) \Big|_{h_{ij}} = P_\Phi(k) \left[1 - \frac{1}{2} \frac{\mathrm{d}\ln P_\Phi}{\mathrm{d}\ln k} h_{ij}^{\text{prim}} \widehat{k}^i \widehat{k}^j \right]$$

Infinite wavelength limit 2/2

☐ Matter power spectrum in **cosmological frame**

$$\widetilde{P}_\delta(\widetilde{\boldsymbol{k}})\bigg|_{h_{ij}} = \mathcal{T}_\delta^2(k)\,\widetilde{P}_\Phi(\widetilde{\boldsymbol{k}})\bigg|_{h_{ij}}\left[1 - \frac{1}{2}\frac{\mathrm{d}\ln\mathcal{T}_\delta^2}{\mathrm{d}\ln k}h_{ij}^{\mathrm{prim}}\widehat{k}^i\widehat{k}^j\right]$$

$$P_\Phi(k)\left[1 - \frac{1}{2}\frac{\mathrm{d}\ln P_\Phi}{\mathrm{d}\ln k}h_{ij}^{\mathrm{prim}}\widehat{k}^i\widehat{k}^j\right]$$

$$= P_\delta(k)\left[1 - \frac{1}{2}\frac{\mathrm{d}\ln P_\delta}{\mathrm{d}\ln k}h_{ij}^{\mathrm{prim}}\widehat{k}^i\widehat{k}^j\right]$$

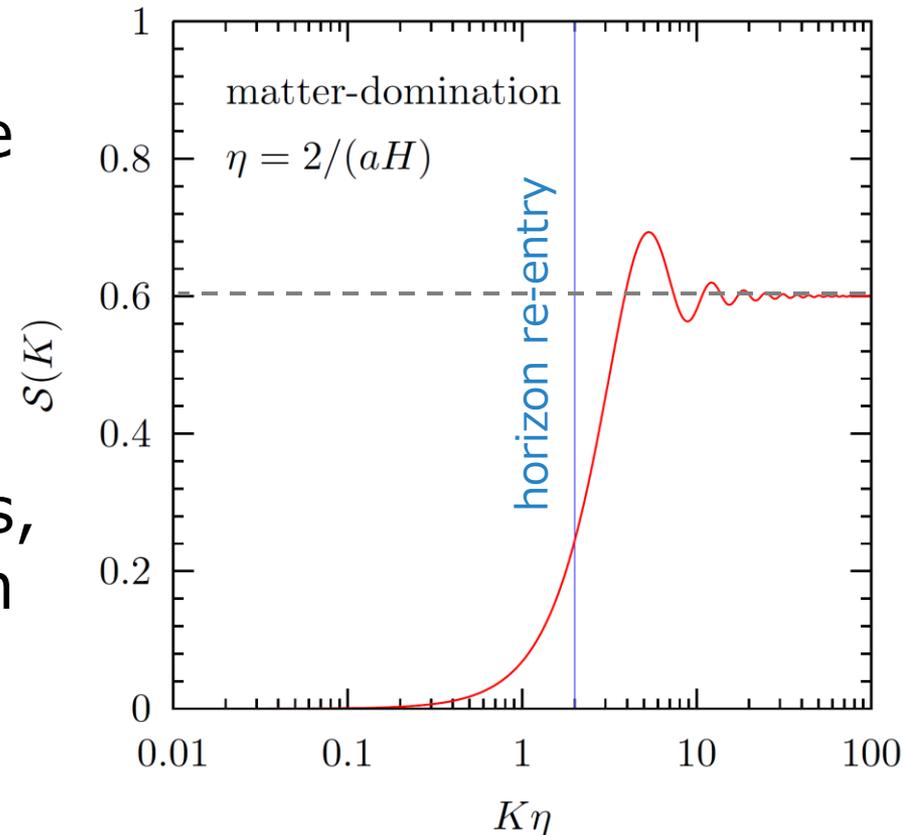This is the same as the expression in the rough sketch.

Fossil effect

$$\delta_{\text{fossil}}(\eta, \boldsymbol{k}) = -h_{ij}^{\text{prim}}(\boldsymbol{K}) \widehat{k}^i \widehat{k}^j \delta^{(1)}(\eta, \boldsymbol{k})\, \mathcal{S}(K\eta)$$

☐ When tensor mode re-enters horizon (Kη~1), tensor mode **interacts** with small-scale scalar modes.

☐ The interaction dies out as the tensor mode itself decays, but **this effect persists** even after the horizon re-entry.

matter-domination

$\eta = 2/(aH)$

horizon re-entry

$\mathcal{S}(K)$

$K\eta$

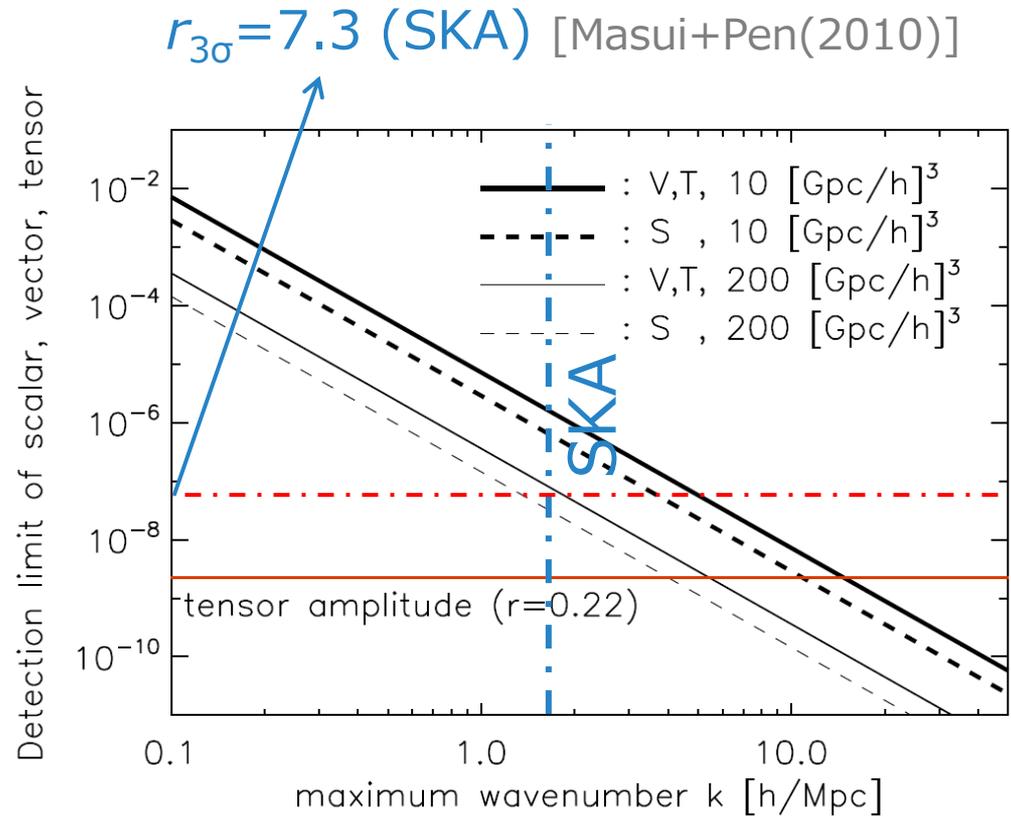[Figure from Dai+Jeong+Kamionkowski(2013)]

Detectability of
tensor clustering fossils

Detectability of tensor fossils ①

□ Detection of primordial GWs is probably **beyond** the reach of current galaxy surveys [$k_{max}/k_{min} < O(10^3)$].

⬇

Measurement with a **huge dynamic range** [$k_{max}/k_{min} > 10^4$] is needed!

$r_{3\sigma}$=7.3 (SKA) [Masui+Pen(2010)]

[Figure from Jeong+Kamionkowski PRL108(2012)251301]

# What observations could measure tensor fossils?

□ **21-cm line**

✓ Spectral line that is created by a hyperfine (spin-flip) transition in neutral hydrogen atoms.

✓ Wavelength = 21.1 cm, which is frequently observed in radio astronomy

λ = 21.1 cm
(ν = 1.4GHz: radio freq.)
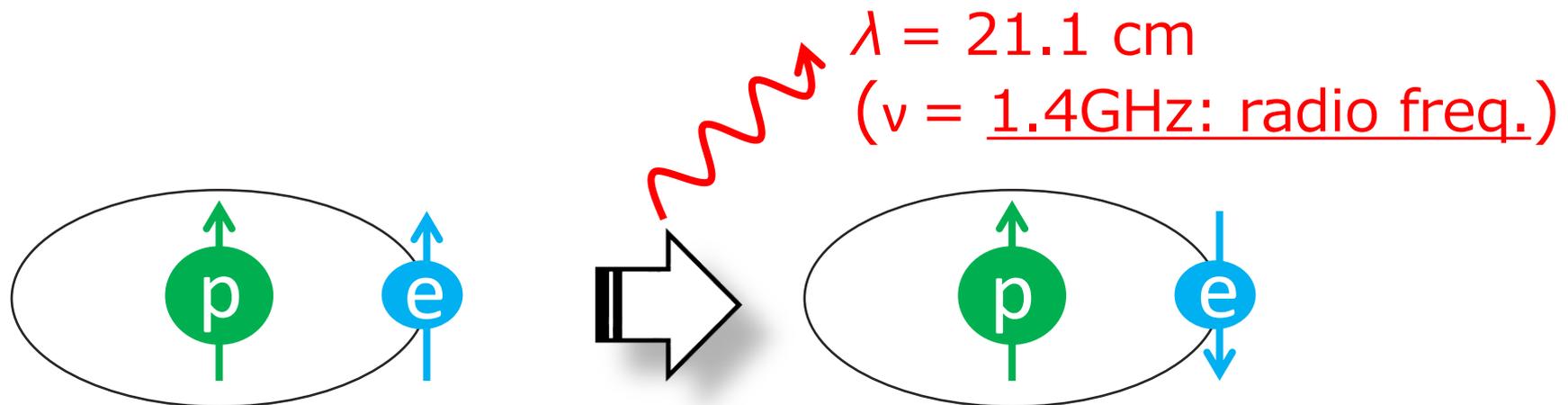
Why 21-cm line?

☐ Neutral hydrogen is the most **ubiquitous** baryonic matter in the high-redshift Universe.

⇨ We can observe large survey volume!
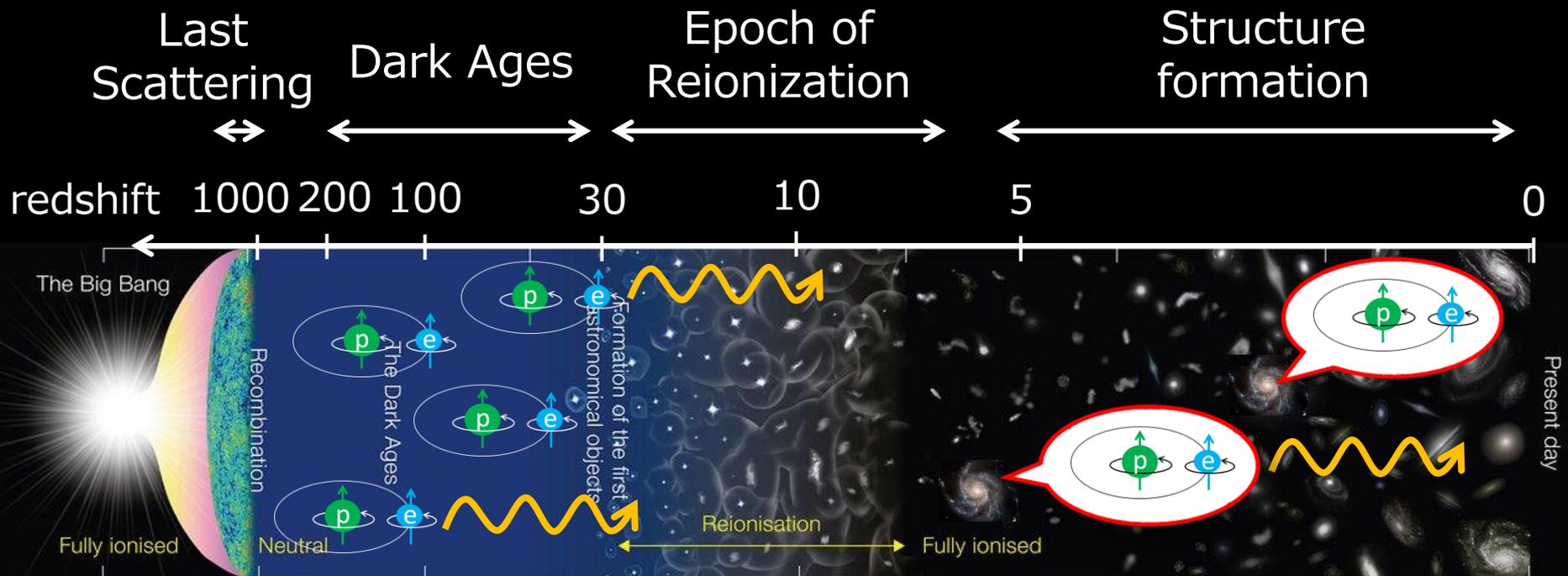
Moon-based observations

□ Lunar orbit

- DARE/DAPPER (NASA)
- DSL (China)
- NCLE (Netherland+China)
- CoDex (ESA)

□ Farside of the Moon

- FARSIDE (NASA)
- LCRT (NASA)
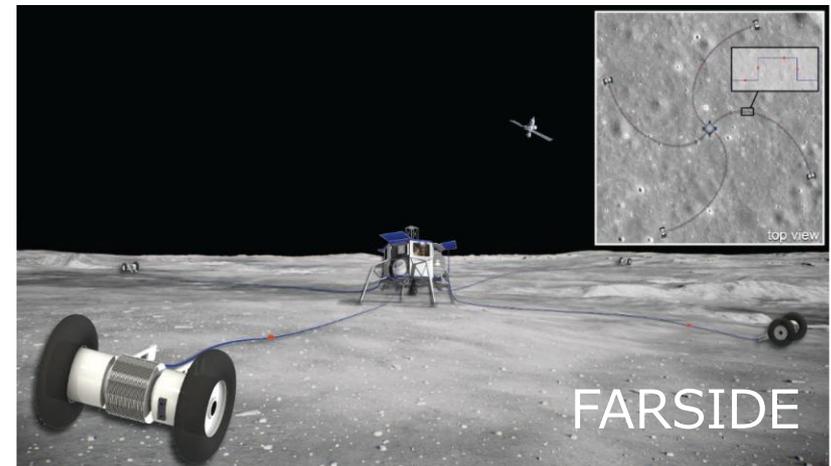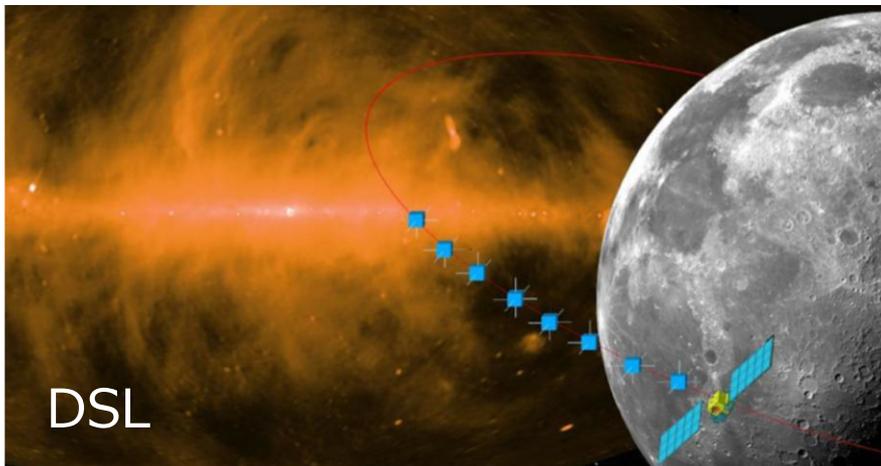- **TSUKUYOMI (JAXA)**

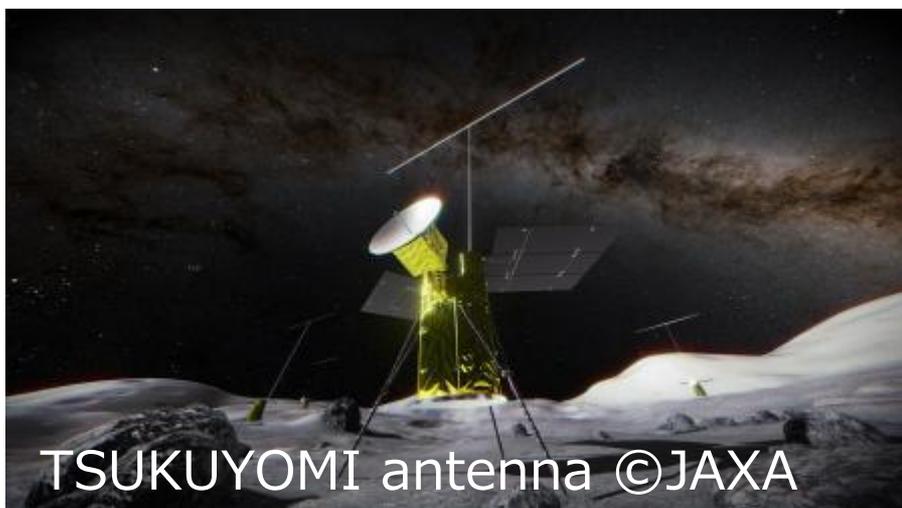DSL

FARSIDE

Moon-based observations

□ Lunar orbit

- DARE/DAPPER (NASA)
- DSL (China)
- NCLE (Netherland+China)
- CoDex (ESA)

□ Farside of the Moon

- FARSIDE (NASA)
- LCRT (NASA)
- **TSUKUYOMI (JAXA)**

TSUKUYOMI antenna ©JAXA

[MAINICHI newspaper online, 2022/11/26]

将来の月面天文台のイメージ
地球
②箱型装置の中の通信機で地球に情報を送信
宇宙誕生初期の水素ガスの電波
1メートル立方
5メートル
月面
①棒状のアンテナで電波をとらえる
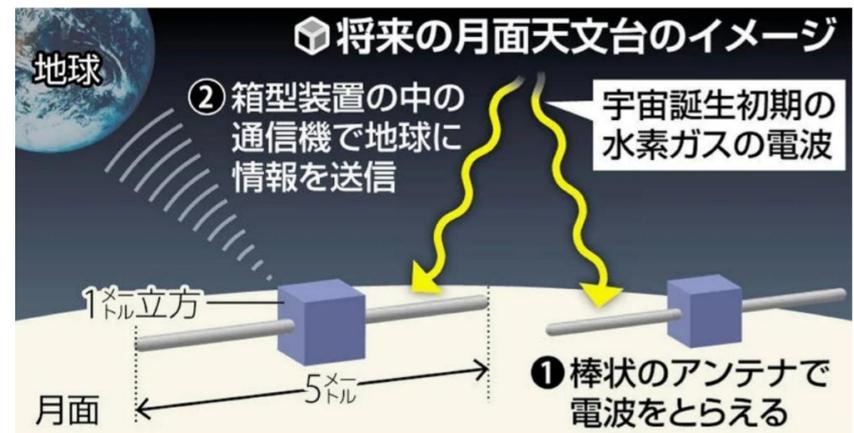
Detectability of tensor fossils ②

The **Moon**-based 21-cm line observations of neutral hydrogen during the dark ages can detect the primordial GWs with small *r*.

baseline b [km] ($k_{max} \propto b$)

[Figure from Cosmic Vision 21cm Collaboration 181009572]

What should we do next?
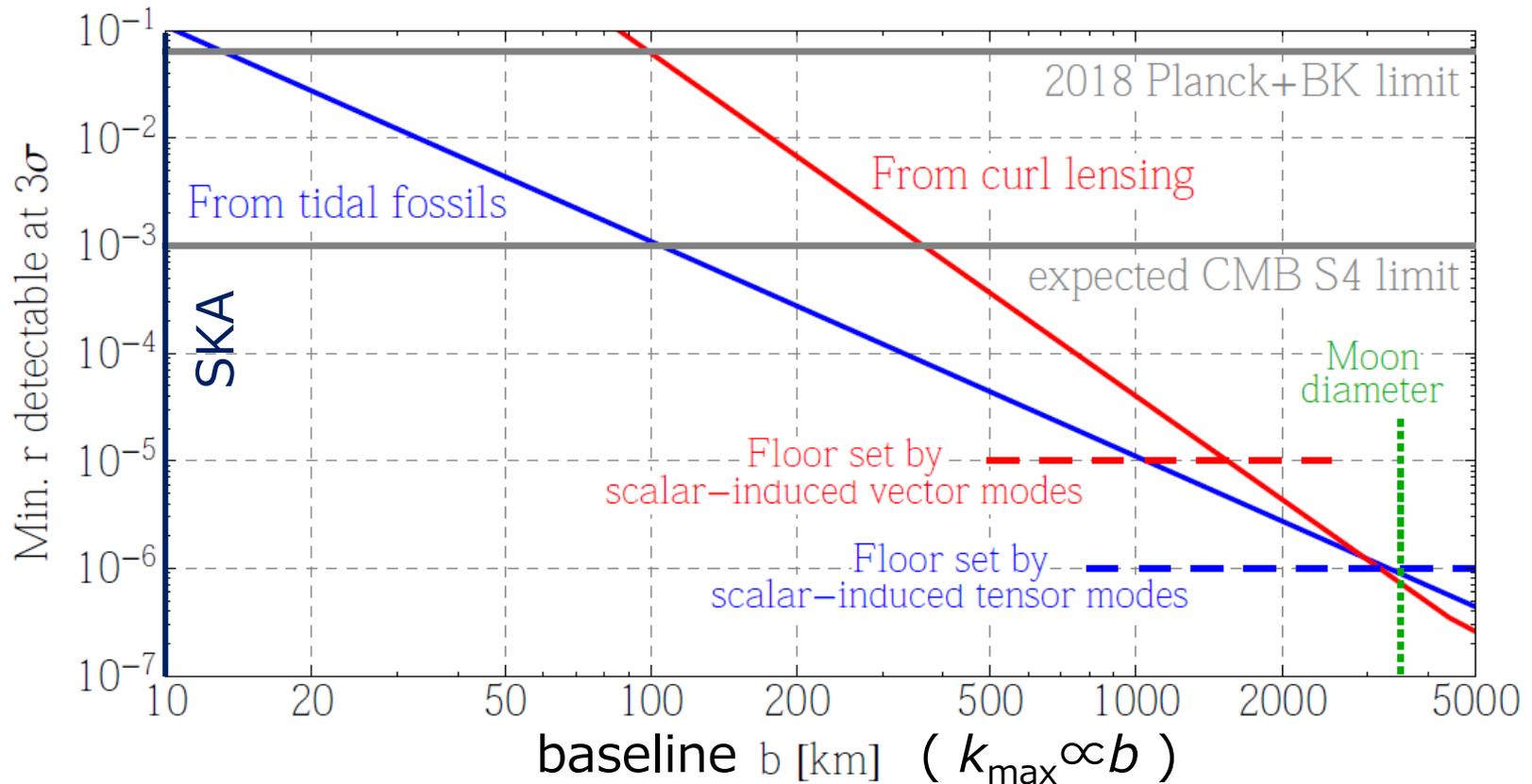
☐ Generic coupling between inflaton and spectator field [Jeong+Kamionkowski, PRL108(2012)251301]

$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle|_{h_p(\mathbf{K})} = f_p(\mathbf{k}_1, \mathbf{k}_2)h_p^*(\mathbf{K})\epsilon_{ij}^p k_1^i k_2^j \delta_{\mathbf{k}_{123}}^D$$

☐ Parity-odd gravitational waves
[Masui+Pen+Turok, PRL118(2017)22,221301]

$$h_{ab}(\mathbf{K}) = h_R(\mathbf{K})e_{ab}^R(\hat{\mathbf{K}}) + h_L(\mathbf{K})e_{ab}^L(\hat{\mathbf{K}})$$

☐ Test of GW propagation → Today's topic

Tensor clustering fossils in modified gravity

Which part is to be modified?

□ Strategy

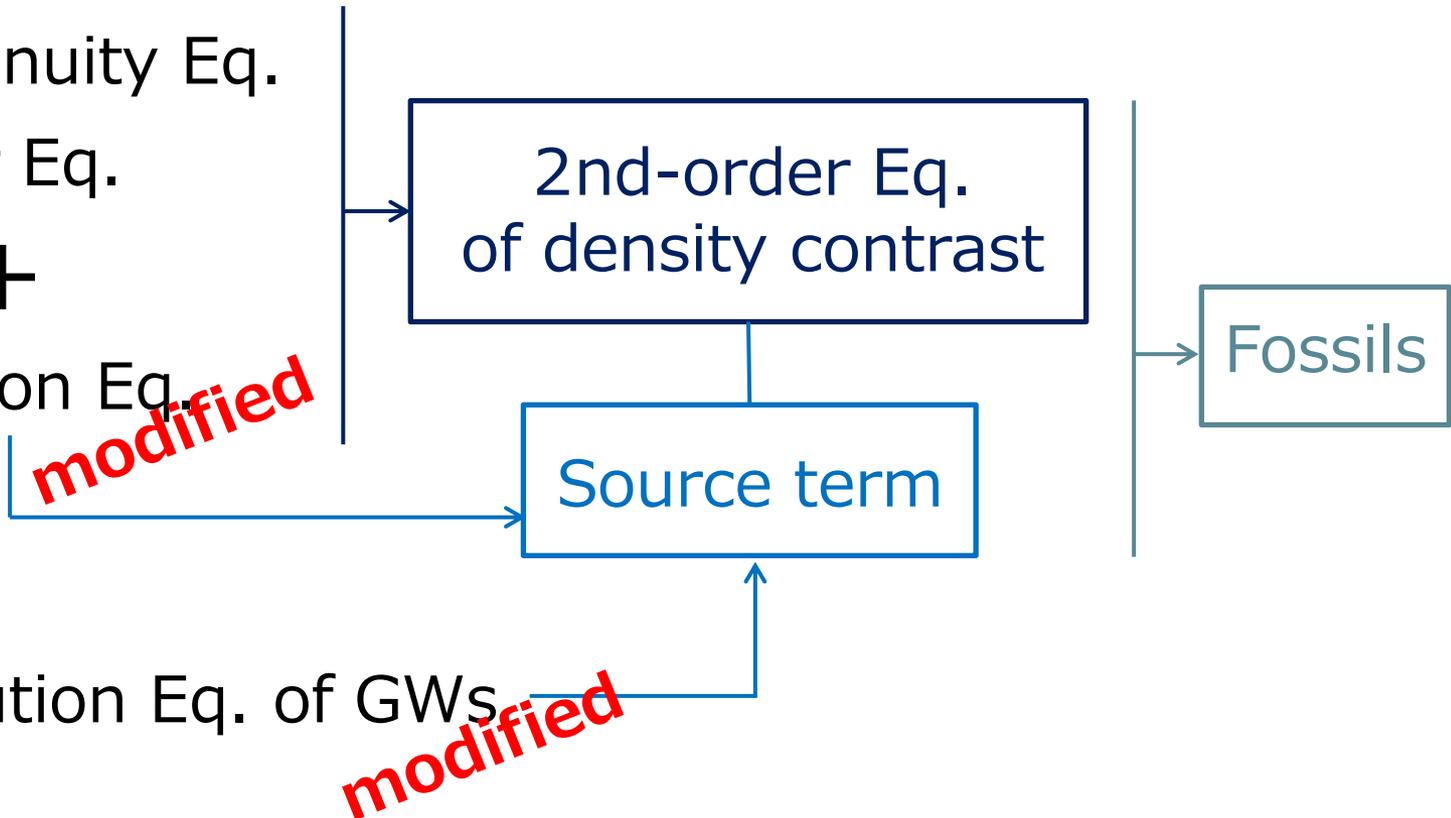✓ Continuity Eq.

✓ Euler Eq.

+

✓ Poisson Eq.

modified

✓ Evolution Eq. of GWs

modified

2nd-order Eq.
of density contrast

Source term

Fossils

Phenomenological model of modified Poisson equation

$\mu(t)$: Effective gravitational coupling

$$\frac{1}{a^2 H^2}\partial^2 \Phi(t,\boldsymbol{x}) = \frac{3}{2}\Omega_{\mathrm{m}}(t)\boxed{\mu(t)}\delta(t,\boldsymbol{x})$$

$$+ \left(-\boxed{\Gamma_1(t)}\frac{1}{a^2 H^2}\partial^2 h_{ij}(t,\boldsymbol{x}) + \boxed{\Gamma_2(t)}\frac{1}{H}\dot{h}_{ij}(t,\boldsymbol{x}) \right) \frac{\partial_i \partial_j}{\partial^2}\delta(t,\boldsymbol{x}) + \cdots$$

$\Gamma_1(t)$ & $\Gamma_2(t)$: Novel parameters characterizing tidal interactions

✓ GR limit: $\mu(t)=1$ and $\Gamma_1(t)=\Gamma_2(t)=0$

NOTE: When GR is considered, term such as $h^{ij}(d_i d_j/d^2)\delta$ appears. However, such term can be safely neglected in the subhorizon limit.

Phenomenological model of modified EoM of GWs

$\nu(t)$: Effective friction term

$$\ddot{h}_{ij} + (3 + \boxed{\nu}) H\dot{h}_{ij} - \boxed{c_{\mathrm{T}}^2}\frac{\partial^2}{a^2}h_{ij} = 0$$

$c_{\mathrm{T}}^2(t) = 1 + \alpha_{\mathrm{T}}(t)$: GW sound speed

✓ GR limit: $c_{\mathrm{T}}^2(t) = 1$ and $\nu(t) = 0$

Tensor clustering fossils in phenomenological models

$$\delta_{\text{fossil}}(\eta) = \int_0^{\eta} \mathrm{d}\bar{\eta}\, G_{\text{ret}}(\eta, \bar{\eta}) \left\{ \left[\boxed{c_{\text{T}}^2(\bar{\eta})} - \boxed{\Gamma_1(\bar{\eta})} \right] \partial^2 h^{ij}(\bar{\eta}) \right.$$
$$\left. + \left[\boxed{f(\bar{\eta}) - 1} - \boxed{\nu(\bar{\eta})} - \boxed{\Gamma_2(\bar{\eta})} \right] \mathcal{H}(\bar{\eta}) h^{ij\prime}(\bar{\eta}) \right\} \frac{\partial_i \partial_j}{\partial^2} \delta^{(1)}(\bar{\eta})$$

❑Tensor clustering fossils can be induced by three parts:

 ✓ c_{T}^2 & ν : propagation of large-scale GWs

 ✓ f-1　　　: deviation of structure growth rate
 　　　　　　　from the value in matter-domination

 ✓ Γ_1 & Γ_2 : tidal interactions from effective Poisson Eq.

Example: Horndeski theory

❑ Once the underlying gravity model is specified, we can calculate the clustering fossils by using the formula.

⇨ As a demonstration, let us consider the Horndeski class of modified gravity

= the most general scalar-tensor theory with the 2nd-order EoM

$$\mathcal{L} = P(\phi, X) - Q(\phi, X)\Box\phi + G_4(\phi, X)R - \frac{\partial G_4}{\partial X}\left(\nabla_\mu\nabla_\nu\phi\right)^2 + \cdots$$

Effective-Field-Theory (EFT) parameters

❑ Even complex full theories can be described by perturbed models with **a few** EFT parameters:

✓ Linear order [Bellini+ JCAP07,050(2014), Langlois+ JCAP05,033(2017),···]

$\alpha_K(t)$ Kineticity (kinetic term of additional field)

$\alpha_M(t)$ Planck-Mass run rate

$\alpha_T(t)$ Tensor speed excess

$\alpha_B(t)$ Braiding (Mixing between field and metric pert.)

✓ Nonlinear order [**DY**+Yokoyama+Tashiro PRD96,123516(2017)]

$\alpha_V(t)$ Veinstein screening (scalar self-interactions)

Phenomenological parameters in terms of EFT parameters

☐ Phenomenological parameters μ, Γ_1, and Γ_2, can be expressed in terms of the EFT parameters (although their dependence is quite complicated).

$$\mu = 1 + \alpha_{\text{T}} - \frac{[\alpha_{\text{B}}(1 + \alpha_{\text{T}}) + \alpha_{\text{T}} - \alpha_{\text{M}}]^2}{(1 + \alpha_{\text{B}})\left[\frac{\dot{H}}{H^2} + \alpha_{\text{B}}(1 + \alpha_{\text{T}}) + \alpha_{\text{T}} - \alpha_{\text{M}}\right] + \frac{\dot{\alpha}_{\text{B}}}{H} + \frac{3}{2}\Omega_{\text{m}}}$$

$$\Gamma_1 = \quad \cdots$$

$$\Gamma_2 = \quad \cdots$$

We derive the formula of the tensor clustering fossils in the context of the modified gravity.

Fossil effect [GW sound speed]

- When the deviation occurs **at η=η$_*$**, tensor mode interacts with small-scale scalar mode.

- Even in the case of modified gravity, **this effect persists** after tensor mode decays.

- The effect of α$_T$ largely **enhances** the amplitude of the tensor fossils.

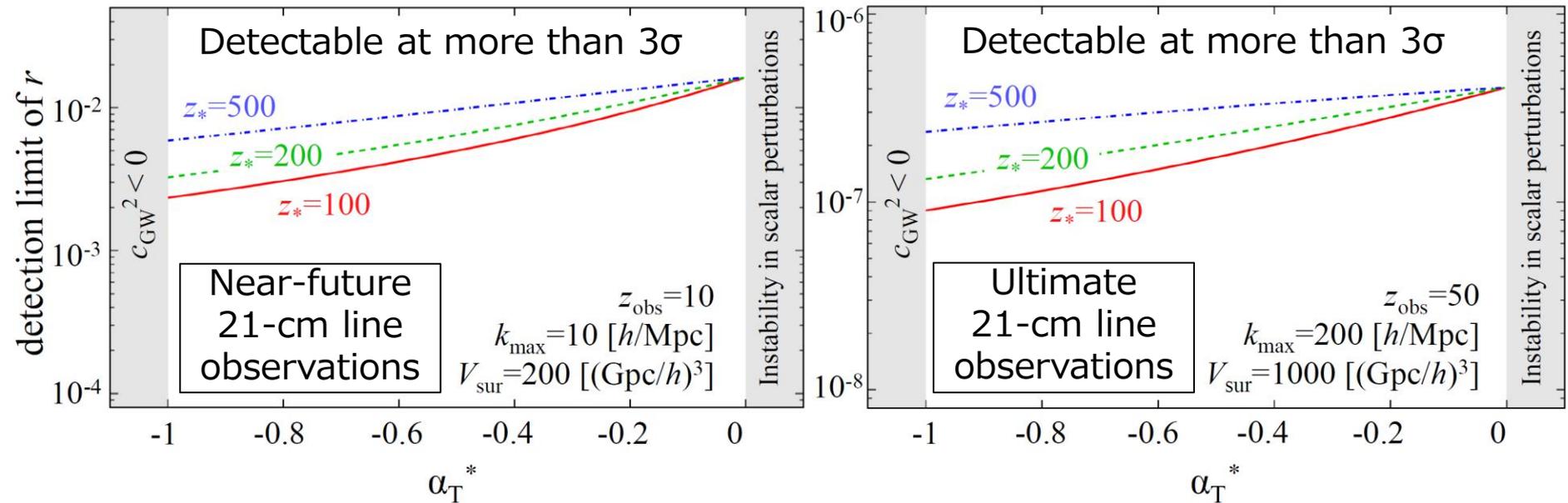$$\alpha_{\mathrm{T}}(\eta) = \alpha_{\mathrm{T}}^* \, \delta_{\mathrm{D}}(\ln \eta - \ln \eta_*)$$

[Other EFT parameters are taken to be zero.]
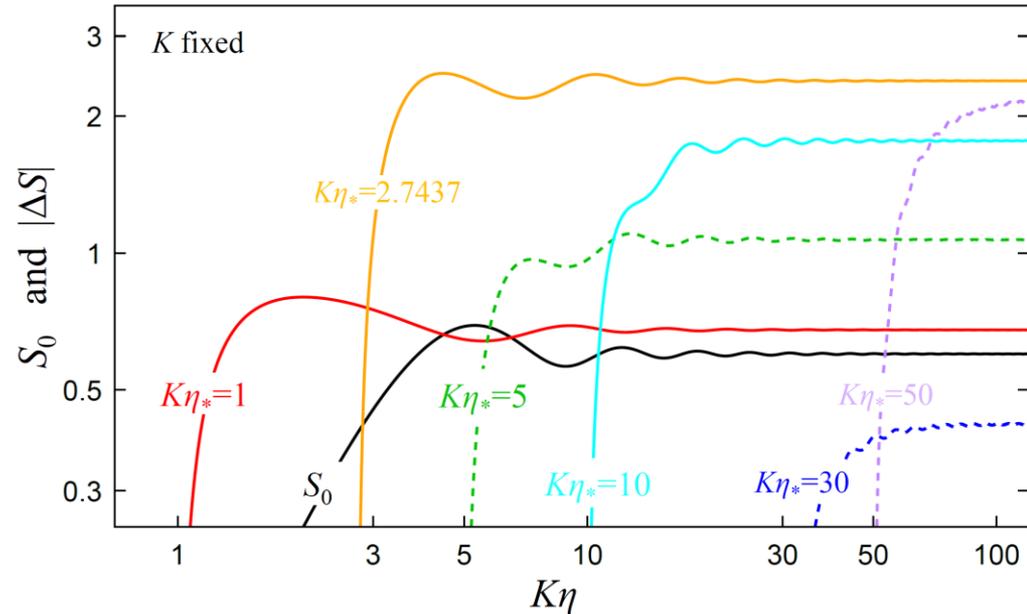
[**DY** PRD110(2024)10,103516]

Detectability of GWs with α_T

☐ The measurement with the extremely large survey volume, which can be achieved by 21-cm line obs., can detect the GWs with modified sound speed!

Summary

✓ Large-scale GWs tidally imprint **local anisotropy** in the small-scale distribution of matter.

✓ This imprint constitutes a fossilized map of GWs since it **persists even after the GWs decay**: **"tensor clustering fossils"**.

✓ Tensor clustering fossil is a novel tool to constrain large-scale GWs.

✓ We developed the formula applicable to the fossils in context of **the modified gravity**.

Summary: Future

✓ **Realistic effects of 21-cm line observations** such as redshift-space distortion, non-linear growth, and survey systematics should be considered.

✓ It can be used to extract the information of **three-point correlations** between primordial spectator fields and inflaton [Jeong+Kamionkowski, PRL108(2012)251301] .

✓ It would be interesting to consider **other GW contributions** such as tensor x tensor
[Bari+Ricciadone+Bartolo+Bertacca+Matarrese PRL129(2022)091301, JCAP07(2023)034]

Thank you!