Fundamentals in density functional theory (YITP Workshop DFT 2022) Yukawa Institute for Theoretical Physics, Kyoto University, December 07-20, 2022

Covariant Density Functional Theory with Localized Exchange Terms

Qiang ZHAO (赵强)

Center for Exotic Nuclear Studies, Institute for Basic Science



Collaborators: Jie Meng, Pengwei Zhao, Zhengxue Ren

Phys. Rev. C 106, 034315 (2022) (Editor's Suggestion)

Theoretical Framework

- Numerical Details
- Results and Discussions

D Summary

Covariant Density Functional Theory

Covariant Density Functional Theory (CDFT)

Meng, Relativistic Density Functional for Nuclear Structure, World Scientific, 2016

Meson-exchange: interact via exchange of mesons





Dirac equation

$$\begin{pmatrix} V_0 + S + M & \boldsymbol{\sigma} \cdot \boldsymbol{p} - \boldsymbol{\sigma} \cdot \boldsymbol{V} \\ \boldsymbol{\sigma} \cdot \boldsymbol{p} - \boldsymbol{\sigma} \cdot \boldsymbol{V} & V_0 - S - M \end{pmatrix} \begin{pmatrix} F_k \\ G_k \end{pmatrix} = \varepsilon_k \begin{pmatrix} F_k \\ G_k \end{pmatrix}$$

S: Scalar potential $V^{\mu} = (V^0, \boldsymbol{V})$: Vector potential

□ Advantages of CDFT: Lorentz symmetry

P. Ring PST 150, 014035 (2012)

Yukawa (1935)

- Large spin-orbit potential Time-odd fields
- Pseudospin symmetry

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Covariant Density Functional Theory

- Wide application of the covariant density functional theory (CDFT)
 - Ground state
 - ✓ Nuclear Mass Zhang et al., ADNDT 144, 101488 (2022)
 - ✓ Exotic Nuclei Meng and Ring, PRL 77, 3963 (1996)
 - Nuclear decay

Niu et al., PLB 723, 172 (2013) Zhao et al., PRC 90, 054326 (2014) Lim et al., PRC 93, 014314 (2016)

Nuclear fission

Lu, Zhao, and Zhou, PRC 85, 011301(R) (2012) Zhou, PS 91, 063008 (2016) Agbemava et al., PRC 95, 054324 (2017) Zhao et al., PRC 99, 054613 (2019) Ren et al., PRL 128, 172501 (2022)

Nuclear excitation

König and Ring PRL 71, 3079 (1993) Afanasjev and Abusara PRC 82, 034329 (2010) Peng et al., PRC 78, 024313 (2008) Zhao et al., PRL 107, 122501 (2011) Nikšić et al., PRC 66, 064302 (2002) Paar et al., PRL 103, 032502 (2009) Liang et al., PRL 101, 122502(2008) Niu et al., PLB 681, 315 (2009) Nikšić et al., PRC 79, 034303 (2009) Li et al., PRC 79, 054301 (2009) Nikšić et al., PRC 73, 034308 (2006) Yao et al., PRC 81, 044311 (2010)

Nuclear reaction

Ren, Zhao, and Meng, PLB 801, 135194 (2020) Ren, Zhao, and Meng, PRC 102, 044603 (2020)

But the exchange terms are not taken into account explicitly in most of the applications! □ Studies of the Relativistic Hartree-Fock (RHF) theory



J. Liu et al., PLB 806, 135524 (2020)

□ The RHF equation contains nonlocal exchange potentials

It becomes complicated and is not DFT

Exchange Terms in Relativistic Point-Coupling Model

Relativistic point-coupling model

Manakos(1989), Nikolaus(1992), Bürvenich (2002), Nikšić (2008), Zhao (2010)

Advantages

- > Avoid calculating the meson field
- Easily extend to beyond mean-field calculations
- Express the exchange terms as
 superpositions of direct terms Sulaksono(2003)
- Validity of localized exchange terms in RHF
 It can reasonably describe the splittings of Dirac
 mass and spin-isospin excitations.



Goal:

Develop a new covariant density functional with localized exchange terms

Theoretical Framework

- Numerical Details
- Results and Discussions

D Summary

Lagrangian Density

Lagrangian Density

$$\mathcal{L} = \mathcal{L}^{\mathrm{free}} + \mathcal{L}^{\mathrm{4f}} + \mathcal{L}^{\mathrm{der}} + \mathcal{L}^{\mathrm{em}}$$

$$-\frac{1}{2}\alpha_{PS}(\bar{\psi}\gamma_5\psi)(\bar{\psi}\gamma_5\psi)-\frac{1}{2}\alpha_{tPS}(\bar{\psi}\gamma_5\vec{\tau}\psi)(\bar{\psi}\gamma_5\vec{\tau}\psi)$$

$$-\frac{1}{2}\alpha_{PV}(\bar{\psi}\gamma_5\gamma_\mu\psi)(\bar{\psi}\gamma_5\gamma^\mu\psi) - \frac{1}{2}\alpha_{tPV}(\bar{\psi}\gamma_5\gamma_\mu\vec{\tau}\psi)(\bar{\psi}\gamma_5\gamma^\mu\vec{\tau}\psi)$$
$$\mathcal{L}^{der} = -\frac{1}{2}\delta_S\partial_\mu(\bar{\psi}\psi)\partial^\mu(\bar{\psi}\psi)$$

Pseudo-Vector

$$\mathcal{L}^{\rm em} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e \frac{1 - \tau_3}{2} \bar{\psi} \gamma_{\mu} \psi A^{\mu}$$

Hamiltonian

$$H = \int dx \ \mathcal{H} = \int x \left[\frac{\partial \mathcal{L}}{\partial \dot{\psi}} \dot{\psi} + \frac{\partial \mathcal{L}}{\partial \dot{A}_{\mu}} \dot{A}_{\mu} - \mathcal{L}
ight]$$

□ Slater determinant

$$|\Phi_0
angle = \prod_{lpha=1}^A c^\dagger_lpha |-
angle$$

Treatment of exchange terms

D Energy Functional $E = \langle \Phi_0 | H | \Phi_0 \rangle = E_k + E_{4f} + E_{der} + E_{em}$:

$$\begin{split} E_{4\mathrm{f}} &= E_{\mathrm{H}} + E_{\mathrm{F}} \\ &= \frac{1}{2} \int d\boldsymbol{r} \sum_{i;\alpha\beta} \alpha_{i}^{\mathrm{HF}} \Big[\bar{\psi}_{\alpha}(\boldsymbol{r}) (\mathcal{O}\Gamma)_{i} \psi_{\alpha}(\boldsymbol{r}) \Big] \Big[\bar{\psi}_{\beta}(\boldsymbol{r}) (\mathcal{O}\Gamma)^{i} \psi_{\beta}(\boldsymbol{r}) \Big] & \quad \boxed{\text{Direct term}} \\ &- \frac{1}{2} \int d\boldsymbol{r} \sum_{i;\alpha\beta} \alpha_{i}^{\mathrm{HF}} \Big[\bar{\psi}_{\alpha}(\boldsymbol{r}) (\mathcal{O}\Gamma)_{i} \psi_{\beta}(\boldsymbol{r}) \Big] \Big[\bar{\psi}_{\beta}(\boldsymbol{r}) (\mathcal{O}\Gamma)^{i} \psi_{\alpha}(\boldsymbol{r}) \Big] & \quad \boxed{\text{Exchange term}} \end{split}$$

□ Fierz transformation

Sulaksono et al., Ann. Phys. 306, 36 (2003) Greiner and Müller, Gauge Theory of Weak Interaction (2009)

$$[\bar{\psi}_{\alpha}(\mathcal{O}\Gamma)_{i}\psi_{\beta}][\bar{\psi}_{\beta}(\mathcal{O}\Gamma)^{i}\psi_{\alpha}] = \sum_{j}\Lambda_{ij}[\bar{\psi}_{\alpha}(\mathcal{O}\Gamma)_{j}\psi_{\alpha}][\bar{\psi}_{\beta}(\mathcal{O}\Gamma)^{j}\psi_{\beta}]$$

 \square E_{4f} can be re-expressed as

$$E_{4f} = \frac{1}{2} \int d\boldsymbol{r} \sum_{i;\alpha\beta} \alpha_i^{\rm H} \Big[\bar{\psi}_{\alpha}(\boldsymbol{r}) (\mathcal{O}\Gamma)_i \psi_{\alpha}(\boldsymbol{r}) \Big] \Big[\bar{\psi}_{\beta}(\boldsymbol{r}) (\mathcal{O}\Gamma)^i \psi_{\beta}(\boldsymbol{r}) \Big]$$

where

$$\alpha_i^{\rm H} = \sum_j C_{ij} \alpha_j^{\rm HF} = \sum_j (1 - \Lambda_{ji}) \alpha_j^{\rm HF}, \qquad i, j = S, tS, V, tV, T, tT, PS, tPS, PV, tPV$$

Localized Relativistic Hartree-Fock-Bogoliubov Eq.

Localized relativistic Hartree-Fock-Bogoliubov Equation

$$\begin{pmatrix} \hat{h}_D - \lambda & \hat{\Delta} \\ -\hat{\Delta}^* & -\hat{h}_D^* + \lambda \end{pmatrix} \begin{pmatrix} U_{\alpha}(\boldsymbol{r}) \\ V_{\alpha}(\boldsymbol{r}) \end{pmatrix} = E_{\alpha} \begin{pmatrix} U_{\alpha}(\boldsymbol{r}) \\ V_{\alpha}(\boldsymbol{r}) \end{pmatrix}$$

D Single-particle hamiltonian \hat{h}_D :

$$\hat{h}_D = \boldsymbol{\alpha} \cdot \boldsymbol{p} + \beta [M + S + \gamma_\mu V^\mu + \sigma_{\mu\nu} T^{\mu\nu}]$$

$$S = \alpha_{S}\rho_{S} + \alpha_{tS}\tau_{3}\rho_{tS} + \delta_{S}\Delta\rho_{S} \qquad \rho_{S} = \sum_{\alpha>0} \bar{V}_{\alpha}V_{\alpha}, \quad \rho_{tS} = \sum_{\alpha>0} \bar{V}_{\alpha}\vec{\tau}V_{\alpha}$$
$$V^{\mu} = \alpha_{V}j_{V}^{\mu} + \alpha_{tV}\tau_{3}j_{tV}^{\mu} + e\frac{1-\tau_{3}}{2}A^{\mu} \qquad j_{V}^{\mu} = \sum_{\alpha>0} \bar{V}_{\alpha}\gamma^{\mu}V_{\alpha}, \quad j_{tV}^{\mu} = \sum_{\alpha>0} \bar{V}_{\alpha}\gamma^{\mu}\vec{\tau}V_{\alpha}$$
$$T^{\mu\nu} = \alpha_{T}j_{T}^{\mu\nu} + \alpha_{tV}\tau_{3}j_{tT}^{\mu\nu} \qquad j_{T}^{\mu\nu} = \sum_{\alpha>0} \bar{V}_{\alpha}\sigma^{\mu\nu}V_{\alpha}, \quad j_{tT}^{\mu\nu} = \sum_{\alpha>0} \bar{V}_{\alpha}\sigma^{\mu\nu}\vec{\tau}V_{\alpha}$$

\square Pairing Field Δ (Separable pairing force)

Tian, Ma, and Ring, PLB 676, 44 (2009)

$$\Delta_{ab} = \frac{1}{2} \sum_{cd} \langle ab | V^{pp} | cd \rangle \kappa_{cd}$$

$$V^{pp} \left(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_1', \boldsymbol{r}_2' \right) = -G\delta \left(\boldsymbol{R} - \boldsymbol{R}' \right) P(\boldsymbol{r}) P\left(\boldsymbol{r}' \right) \frac{1}{2} \left(1 - P^{\sigma} \right), \qquad P(\boldsymbol{r}) = \frac{1}{(4\pi a^2)^{3/2}} e^{-r^2/4a^2}$$

Theoretical Framework

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□ Summary

Effective Lagrangian Density

 $\alpha_i^{\overline{\mathrm{HF}}}$

$$\mathcal{L} = \bar{\psi}(i\gamma_{\mu}\partial^{\mu} - M)\psi$$

$$- \frac{1}{2}\alpha_{S}(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_{V}(\bar{\psi}\gamma_{\mu}\psi)(\bar{\psi}\gamma^{\mu}\psi)$$

$$- \frac{1}{2}\alpha_{tS}(\bar{\psi}\vec{\psi})(\bar{\psi}\vec{\psi}\psi)$$

$$- \frac{1}{2}\alpha_{tS}(\bar{\psi}\vec{\tau}\psi)(\bar{\psi}\vec{\tau}\psi)$$

$$- \frac{1}{2}\alpha_{tV}(\bar{\psi}\gamma_{\mu}\vec{\tau}\psi)(\bar{\psi}\gamma^{\mu}\vec{\tau}\psi)$$

$$- \frac{1}{2}\alpha_{tV}(\bar{\psi}\gamma_{\mu}\vec{\tau}\psi)(\bar{\psi}\sigma^{\mu\nu}\psi) - \frac{1}{2}\delta_{S}\partial_{\mu}(\bar{\psi}\psi)\partial^{\mu}(\bar{\psi}\psi)$$

$$- \frac{1}{2}\alpha_{tT}(\bar{\psi}\sigma_{\mu\nu}\vec{\tau}\psi)(\bar{\psi}\sigma^{\mu\nu}\vec{\tau}\psi)$$

$$- \frac{1}{2}\alpha_{tS}(\bar{\psi}\gamma_{5}\psi)(\bar{\psi}\gamma_{5}\psi) - \frac{1}{2}\alpha_{tPS}(\bar{\psi}\gamma_{5}\vec{\tau}\psi)(\bar{\psi}\gamma_{5}\vec{\tau}\psi)$$

$$- \frac{1}{2}\alpha_{PV}(\bar{\psi}\gamma_{5}\gamma_{\mu}\psi)(\bar{\psi}\gamma_{5}\gamma^{\mu}\psi) - \frac{1}{2}\alpha_{tPV}(\bar{\psi}\gamma_{5}\gamma_{\mu}\vec{\tau}\psi)(\bar{\psi}\gamma_{5}\gamma^{\mu}\vec{\tau}\psi)$$

$$- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e\frac{1-\tau_{3}}{2}\bar{\psi}\gamma_{\mu}\psi A^{\mu}$$

$$11 \text{ coupling constants:} Determined by fitting to bulk properties of nuclear matter and finite nuclei$$



of nuclear matter and

Effective Lagrangian Density

$$\mathcal{L} = \bar{\psi}(i\gamma_{\mu}\partial^{\mu} - M)\psi$$

$$-\frac{1}{2}\alpha_{S}(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_{V}(\bar{\psi}\gamma_{\mu}\psi)(\bar{\psi}\gamma^{\mu}\psi)$$

$$-\frac{1}{2}\alpha_{iS}(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_{V}(\bar{\psi}\gamma_{\mu}\psi)(\bar{\psi}\gamma^{\mu}\psi)$$

$$-\frac{1}{2}\alpha_{iV}(\bar{\psi}\gamma_{\mu}\vec{\tau}\psi)(\bar{\psi}\gamma^{\mu}\vec{\tau}\psi)$$

$$-\frac{1}{2}\alpha_{iV}(\bar{\psi}\gamma_{\mu}\vec{\tau}\psi)(\bar{\psi}\sigma^{\mu\nu}\psi) - \frac{1}{2}\delta_{S}\partial_{\mu}(\bar{\psi}\psi)\partial^{\mu}(\bar{\psi}\psi)$$

$$-\frac{1}{2}\alpha_{iT}(\bar{\psi}\sigma_{\mu\nu}\psi)(\bar{\psi}\sigma^{\mu\nu}\psi) - \frac{1}{2}\delta_{S}\partial_{\mu}(\bar{\psi}\psi)\partial^{\mu}(\bar{\psi}\psi)$$

$$-\frac{1}{2}\alpha_{iT}(\bar{\psi}\sigma_{\mu\nu}\vec{\tau}\psi)(\bar{\psi}\sigma^{\mu\nu}\vec{\tau}\psi)$$

$$-\frac{1}{2}\alpha_{PS}(\bar{\psi}\gamma_{5}\psi)(\bar{\psi}\gamma_{5}\psi) - \frac{1}{2}\alpha_{iPS}(\bar{\psi}\gamma_{5}\vec{\tau}\psi)(\bar{\psi}\gamma_{5}\vec{\tau}\psi)$$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e\frac{1-\tau_{3}}{2}\bar{\psi}\gamma_{\mu}\psiA^{\mu}$$
Functional E_{CDF}

5 Independent parameters

5 Constraints

constants:

by fitting to bulk of nuclear matter and



$$\begin{array}{|c|c|c|c|c|} \hline & \mbox{Effective Lagrangian Density} \\ \mathcal{L} = \bar{\psi}(i\gamma_{\mu}\partial^{\mu} - M)\psi \\ \hline & \mbox{Inite Nuclei: Experimental data of bulk properties} \\ \hline & \mbox{Inite Nuclei: Experimental data of bulk properties} \\ \hline & \mbox{Inite Nuclei: Experimental data of bulk properties} \\ \hline & \mbox{Inite Nuclei: Experimental data of bulk properties} \\ \hline & \mbox{Inite Nuclei: Experimental data of bulk properties} \\ \hline & \mbox{Inite Nuclei: Experimental data of bulk properties} \\ \hline & \mbox{Inite Nuclei: Experimental data of bulk properties} \\ \hline & \mbox{Inite Nuclei: Experimental data of bulk properties} \\ \hline & \mbox{Inite Nuclei: Experimental data of bulk properties} \\ \hline & \mbox{Inite Nuclei: Experimental data of bulk properties} \\ \hline & \mbox{Inite Nuclei: Experimental data of bulk properties} \\ \hline & \mbox{Inite Nuclei: Experimental data of bulk properties} \\ \hline & \mbox{Inite Nuclei: Experimental data of bulk properties} \\ \hline & \mbox{Inite Nuclei: Experimental data of bulk properties} \\ \hline & \mbox{Inite Nuclei: Experimental data of bulk properties} \\ \hline & \mbox{Inite Nuclei: Nuclear Matter} \\ \hline & \mbox{Inite Nuclei} \\ \hline & \mbox{Inite$$

□ Parameters of CDF PCF-PK1:

parameter	value	parameter	value		
$lpha_S(ho_{ m sat.})~[{ m fm}^2]$	-6.315494	$lpha_V(ho_{ m sat.})~[{ m fm}^2]$	3.533254		
a_S	1.425216	a_V	0.8075938		
d_S	0.7498208	d_V	3.004506		
$\alpha_{tS}(ho_{ m sat.}) \ [{ m fm}^2]$	-1.980515	$lpha_{tV}(ho_{ m sat.}) \ [{ m fm}^2]$	2.975891		
a_{tS}	2.328043	a_{tV}	2.546886		
d_{tS}	0.7644323	d_{tV}	0.4547352		
$\alpha_T [{ m fm}^2]$	3.373807	$\delta_S ~[{ m fm}^4]$	-0.6658080		
$G \; [{ m MeV} \cdot { m fm}^3]$	657.5419				

Compare to existed local covariant density functionals

-											
		S	tS	V	tV	T	tT	PS	tPS	PV	tPV
Point-coupling	PCF-PK1	\checkmark	\checkmark		\checkmark	\checkmark	\checkmark	\checkmark	\checkmark		\checkmark
	PC-PK1	\checkmark			\checkmark						
	DD-PC1	\checkmark		\checkmark	\checkmark						
Meson-exchange		σ	δ	ω	ρ	π					
	$DD-ME\delta$	\checkmark	\checkmark	\checkmark	\checkmark						

Binding Energy of Reference Nuclei

Deviations of binding energies between theoretical and experimental values



RMSD of binding energies for 60 reference nuclei calculated by PCF-PK1 is 1.42 MeV, which is comparable with one of the most successful functional PC-PK1.

- Theoretical Framework
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Deformed Nuclei

 \blacksquare Binding Energy and deformation β_2 of Yb and U calculated by PCF-PK1



Exp. taken from Wang et al., CPC 41, 030003 (2017), Pritychenko et al., ADNDT 107, 1(2016)

PCF-PK1 well describe the binding energy and deformation of Yb and U isotopes.

Spurious Shell

Two-proton shell gap: Evaluate the size of shell gaps

$$\delta_{2p}(Z,N) = S_{2p}(Z,N) - S_{2p}(Z+2,N)$$

 S_{2p} : Two-proton separation energy



PCF-PK1 improves the description of shell structures around Z=58 and Z=92.

□ Strength distribution of Gamow-Teller resonances for ⁴⁸Ca, ⁹⁰Zr, and ²⁰⁸Pb



PCF-PK1 can self-consistently describe the GTR excitation energy without adjusting additional parameters.

- Theoretical Framework
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□ Summary

Summary

- Based on relativistic point-coupling model, a new covariant density functional PCF-PK1 are developed with localized exchange terms. It can
 - Well describe the binding energy of finite nuclei, and has the same precision as existed functional PC-PK1
 - > Eliminate the spurious shell closure at Z = 58 and Z = 92
 - Self-consistently describe the Gamow-Teller resonance

For meson-exchange interaction, the exchange terms can be taken into account by the orbital-dependent relativistic density functional theory. (In Preparation)

Thank you!