Collective Coordinate and Collective Mass in nuclear reactions

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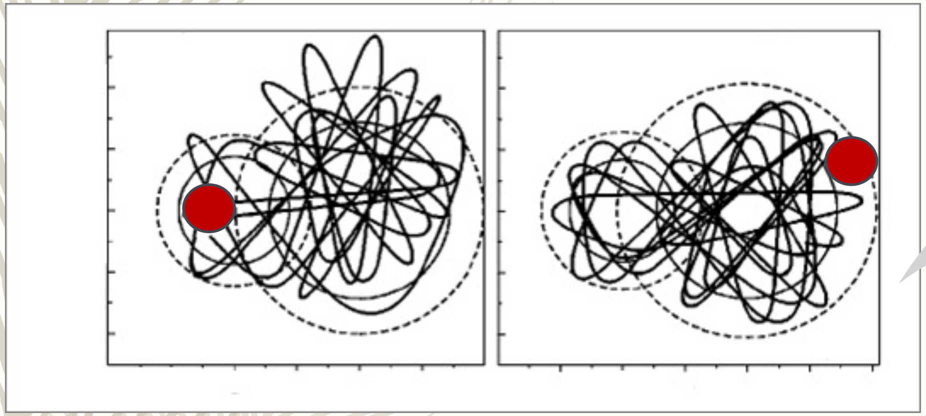
YITP Workshop "Fundamentals in density functional theory (DFT2022)"

Collective Dynamics

TDHF



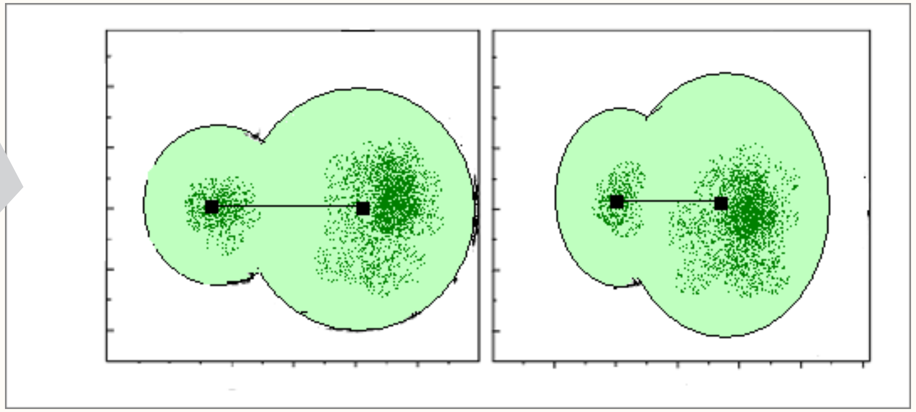
$$\Psi(t) = \frac{1}{\sqrt{A!}}\det\{\psi_1(t)\psi_2(t)\cdots\psi_A(t)\},$$

$$i\hbar\frac{\partial}{\partial t}\psi_j(r) = h(\rho)\psi_j(t).$$

P. Bonche et al., Phys. Rev. C 13, 1226 (1976).

J. Maruhn, P.-G. Reinhard, P. Stevenson, and A. Umar, Comput. Phys. Commun. 185, 2195 (2014).

ASCC



- Collective coordinates?
- Collective mass?
- Collective Hamiltonian?

Content:

- **Our Method ASCC**
  (Adiabatic Self-consistent Collective Coordinate)
- Results.
- Summary

Formulation of ASCC

Assume we found the optimal q and p, we use them to label wave function ψ(q, p):

$$\hat{P}\psi(q,p) = i\frac{\partial}{\partial q}\psi(q,p) \qquad \hat{Q}\psi(q,p) = -i\frac{\partial}{\partial p}\psi(q,p)$$

$$\psi(q,p) = (1 + i\hat{Q}p - \frac{1}{2}\hat{Q}^2p^2)\psi(q)$$

(require that the collective motion is slow)

Total energy is formally:

$$\langle\psi(q,p)|\hat{H}|\psi(p,q)\rangle = \langle q|\hat{H}|q\rangle + p\langle q|[i\hat{H},\hat{Q}]|q\rangle + \frac{1}{2}p^2\langle q|[[i\hat{H},\hat{Q}],i\hat{Q}]|q\rangle$$

$$I_{12} = \int_{t_1}^{t_2} dt\langle\psi(t)|i\frac{\partial}{\partial t} - \hat{H}|\psi(t)\rangle$$

variational principle:

$$\langle\delta\psi(q,p)|i\frac{\partial}{\partial t} - \hat{H}|\psi(q,p)\rangle \quad = \quad 0.$$

The ASCC equations

T. Nakatsukasa, K. Matsuyanagi, M. Matsuo, and K. Yabana, Rev. Mod. Phys. **88**, 045004 (2016).

$$\delta\langle\psi|H - \frac{\partial V}{\partial q}\hat{Q}(q)|\psi\rangle = 0,$$

$$\delta\langle\psi(q)|[H - \frac{\partial V}{\partial q}\hat{Q}(q), \frac{1}{i}\hat{P}(q)] - \frac{\partial^2 V}{\partial q^2}\hat{Q}(q)|\psi(q)\rangle = 0,$$

$$\delta\langle\psi(q)|[H - \frac{\partial V}{\partial q}\hat{Q}(q), i\hat{Q}(q)] - \frac{1}{M(q)}\hat{P}(q)|\psi(q)\rangle = 0,$$

The TDHF equation

$$\delta\langle\psi(t)|i\hbar\frac{\partial}{\partial t} - H|\psi(t)\rangle = 0$$

What is "P̂" and "Q̂" ?

Operator form of the optimal coordinate q and p:

$$\dot{q} = \frac{\partial H}{\partial p}$$

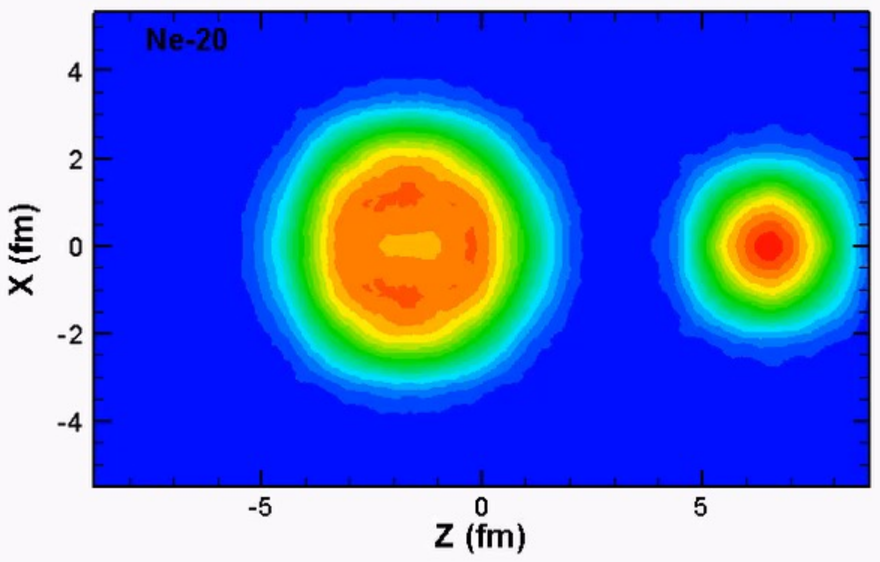
$$\dot{p} = -\frac{\partial H}{\partial q}$$

Weak canonicity condition for operator forms of q and p:

$$\langle \psi_q | \left[i\hat{P}, \hat{Q} \right] | \psi_q \rangle = 1$$

Collective fusion paths lead by "$\hat{Q}$"

$^{16}O + \alpha \rightarrow {}^{20}Ne$



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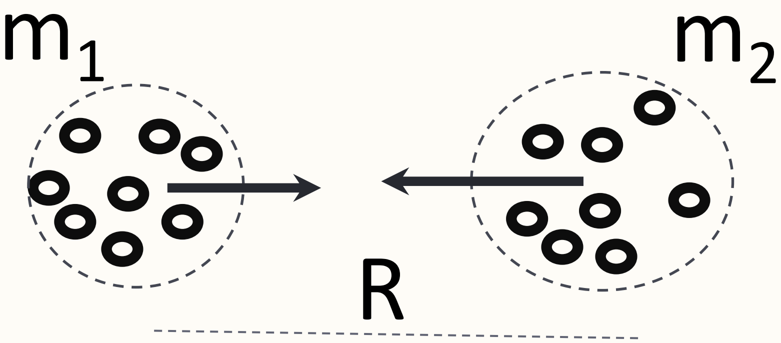
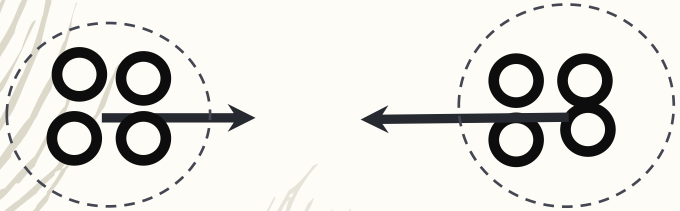
What is the M?



$$M_R \neq \frac{m_1 m_2}{m_1 + m_2}$$

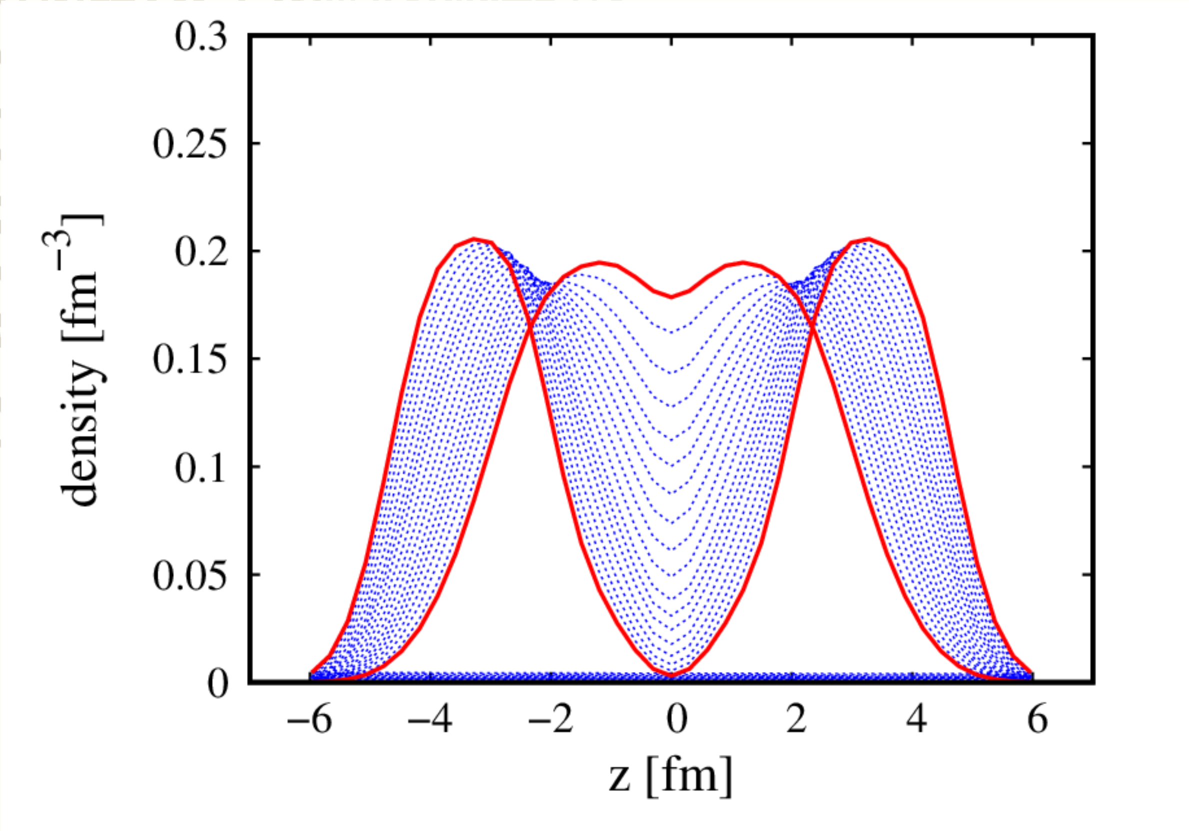
$$M_R = \ ?$$

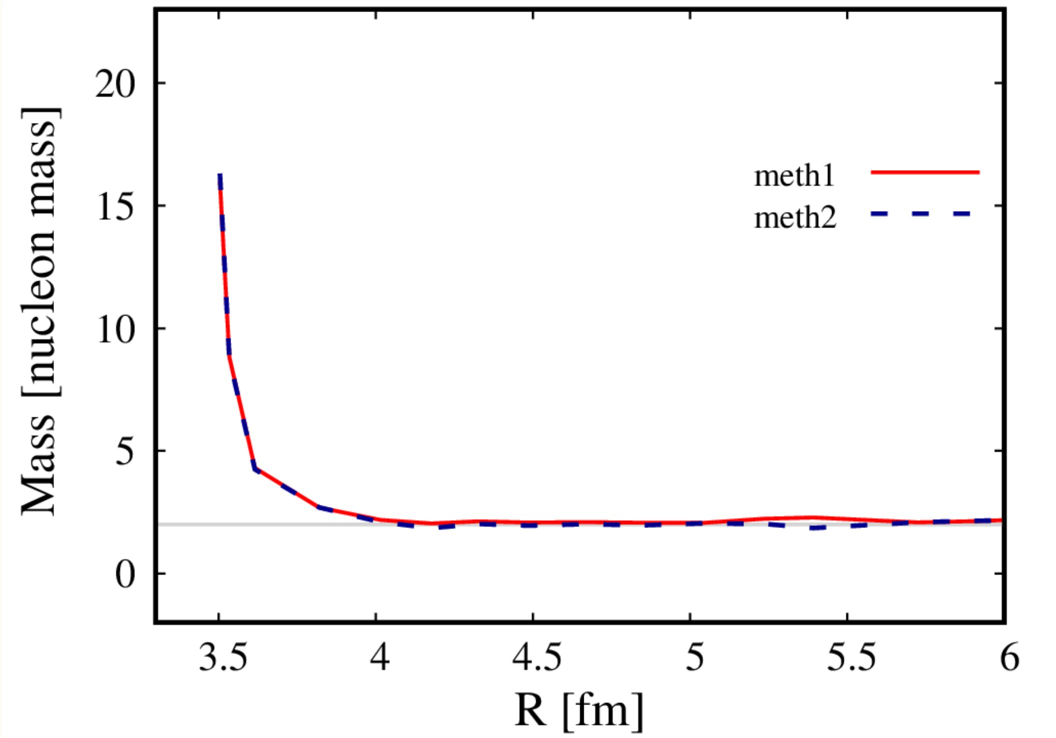
ASCC mass: fission path α + α → ^{8}Be



$m_1 = 4$ $m_2 = 4$

$$\frac{m_1 m_2}{m_1 + m_2} = 2$$





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The TDHF equation

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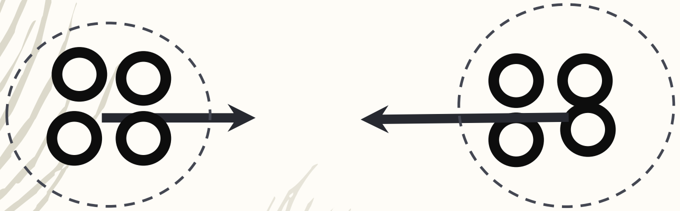
ASCC with Super fluidity (Preliminary)
fission path α + α → ^{8}Be



$m_1 = 4$ $m_2 = 4$

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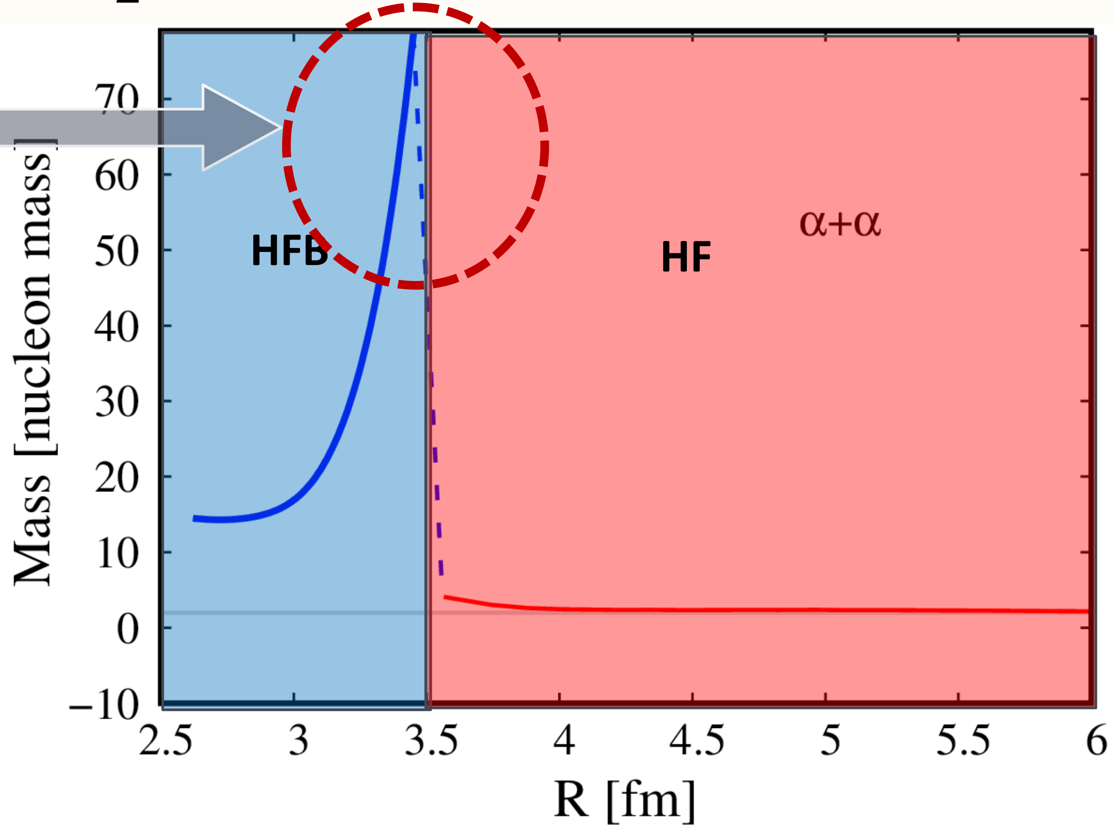
Pairing Rotation

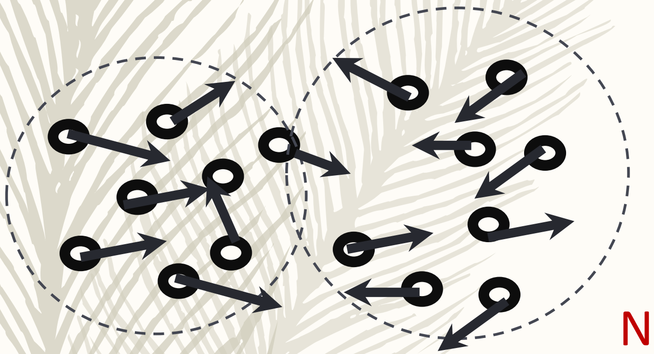


Summary

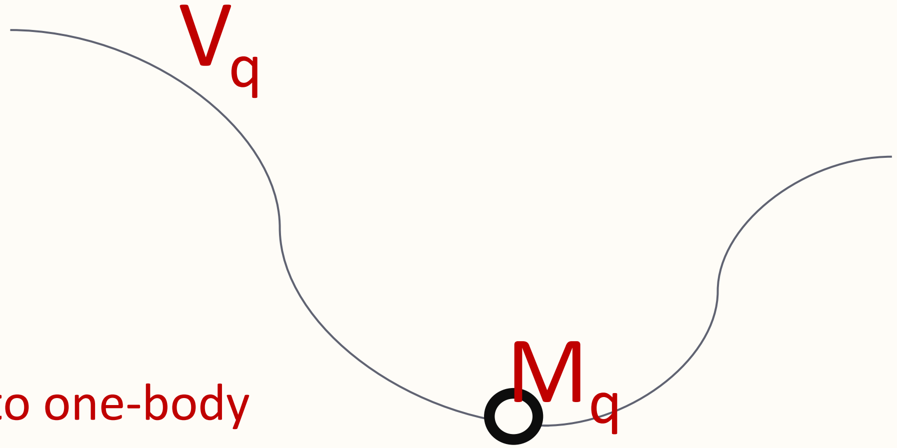
- *Using the ASCC method, we define an "optimal" collective coordinate.*
- With Skyrme *interaction applied, collectiv mass with respect to this coordinated is obtained.*

Perspective:

To construct collective Hamiltonian to quantize collective dynamics.



N-body dynamics simplified to one-body dynamics with ASCC V_q and M_q



$$\hat{H}_{\mathrm{coll}} = -\frac{1}{2}\frac{1}{\sqrt{M(R)}}\frac{d}{dR}\frac{1}{\sqrt{M(R)}}\frac{d}{dR} + V(q)$$

$$\hat{H}_{\mathrm{coll}}\psi(q) = E\psi(q)$$